Fracture mechanics of homogeneous and heterogeneous brittle materials: An overview
The fracture of materials...

One generally wants to avoid it...
The fracture of materials...

One generally wants to avoid it...

But sometimes one wants it badly
How to predict the strength of brittle solids

M. Marder and J. Fineberg 1996
How to predict the strength of brittle solids

M. Marder and J. Fineberg 1996

$\sigma_{\text{ext}}$  

$E$ Young's modulus  

$a$ distance between atoms  

$\sigma_f$ stress at failure

Microscopic view
How to predict the strength of brittle solids

\[ \sigma_{\text{ext}} \]

\[ \sigma_F \sim E \]

M. Marder and J. Fineberg 1996

- Young’s modulus
- Distance between atoms
- Stress at failure

- Stiffness of elastic bonds \(~ E\)
- Deformation at failure \(~ a\)
- Force at failure \(~ E \cdot a\)

Failure: \[ \sigma_F \sim E \]
How to predict the strength of brittle solids

*M. Marder and J. Fineberg 1996*

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ (GPa)</th>
<th>$\sigma_f$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>200</td>
<td>0.1-2</td>
</tr>
<tr>
<td>Glass</td>
<td>70</td>
<td>300</td>
</tr>
<tr>
<td>$Al_2O_3$</td>
<td>400</td>
<td>&lt;100</td>
</tr>
</tbody>
</table>

-$E$ Young’s modulus
-$a$ distance between atoms
-$\sigma_f$ stress at failure

⇒ Stiffness of elastic bonds $\sim E$
⇒ Deformation at failure $\sim a$
⇒ Force at failure $\sim E \ a$

$\Rightarrow$ Failure: $\sigma_F \sim E$

$\Rightarrow$ Not like this!
Stress concentration at defects

C. E. Inglis 1913
Stress concentration at defects

\[ \sigma_A \sim \frac{\sigma_{ext}}{\sqrt{\rho}} \]

- Applied remote stress \( \sim \sigma_{ext} \)
- Local stress in A \( \sim \sigma_A \)
- Radius of curvature of the defect in A \( \sim \rho \)

C. E. Inglis 1913
Stress concentration at defects

C. E. Inglis 1913

\[ \sigma_A \sim \frac{\sigma_{ext}}{\sqrt{\rho}} \]

- Applied remote stress \( \sim \sigma_{ext} \)
- Local stress in A \( \sim \sigma_A \)
- Radius of curvature of the defect in A \( \sim \rho \)

Materials submitted to stress levels actually larger than the applied external stress
The slit crack model

\[ \sigma_{\text{ext}} \]

G. Irwin 1957
The slit crack model

G. Irwin 1957

Stress field diverges at the crack tip:

\[ \sigma \sim \frac{K}{\sqrt{r}} \]

- Stress intensity factor \( \sim K \)
- Distance to crack tip \( \sim r \)
The slit crack model

G. Irwin 1957

Stress field diverges at the crack tip:

\[ \sigma \sim \frac{K}{\sqrt{r}} \]

- Stress intensity factor \( \sim K \)
- Distance to crack tip \( \sim r \)

Stress based criterion for failure → Materials would have no resistance to failure

➤ Not like this neither!
A powerful predicting tool: Linear Elastic Fracture Mechanics

Predicting the stability of cracks in homogeneous media

A.A. Griffith 1920
J.R. Rice 1968
A powerful predicting tool: Linear Elastic Fracture Mechanics

Predicting the stability of cracks in homogeneous media

Energy balance:

\[ \delta W_{\text{ext}} = \delta E_{\text{el}} + \delta E_{\text{s}} \]

- Work of the external force
- Variation of elastic energy
- Variation of surface energy
A powerful predicting tool:
Linear Elastic Fracture Mechanics

Predicting the stability of cracks in homogeneous media

Energy balance:

$$\delta W_{\text{ext}} = \delta E_{\text{el}} + \delta E_s$$

Work of the external force
Variation of elastic energy
Variation of surface energy

Griffith's criterion:

Elastic energy release rate vs Fracture energy

$$G = \frac{\delta W_{\text{ext}} - \delta E_{\text{el}}}{\delta a.b}$$

$$G_c = \frac{\delta E_s}{\delta a.b}$$
A powerful predicting tool: Linear Elastic Fracture Mechanics

Predicting the stability of cracks in homogeneous media

Energy balance:

\[ \delta W_{\text{ext}} = \delta E_{\text{el}} + \delta E_s \]

- Work of the external force
- Variation of elastic energy
- Variation of surface energy

Griffith's criterion:

\[ G = \frac{\delta(W_{\text{ext}} - \delta E_{\text{el}})}{(\delta a.b)} \]
\[ G_c = \frac{\delta E_s}{(\delta a.b)} \]

- \( G < G_c \) \rightarrow Stable crack
- \( G = G_c \) \rightarrow Propagating crack

A.A. Griffith 1920
J.R. Rice 1968

A powerful predicting tool: Linear Elastic Fracture Mechanics

Predicting the stability of cracks in homogeneous media
A powerful predicting tool: Linear Elastic Fracture Mechanics

Predicting the stability of cracks in homogeneous media

A. A. Griffith 1920
J. R. Rice 1968

Energy balance:

\[ \delta W_{\text{ext}} = \delta E_{\text{el}} + \delta E_{\text{s}} \]

- Work of the external force
- Variation of elastic energy
- Variation of surface energy

Griffith's criterion:

- Elastic energy release rate vs Fracture energy

\[ G = \frac{\delta (W_{\text{ext}} - \delta E_{\text{el}})}{(\delta a. b)} \quad G_c = \frac{\delta E_{\text{s}}}{(\delta a. b)} \]

- \( G < G_c \) → Stable crack
- \( G = G_c \) → Propagating crack

Relation between stress intensity factor and energy release rate

\[ G = \frac{K^2}{E} \sim \sigma_{\text{ext}}^2 \]
A powerful predicting tool: Linear Elastic Fracture Mechanics

Predicting the stability of cracks in homogeneous media

Energy balance:
\[ \delta W_{\text{ext}} = \delta E_{\text{el}} + \delta E_{\text{s}} \]

Work of the external force
Variation of elastic energy
Variation of surface energy

Griffith's criterion:

Elastic energy release rate vs Fracture energy
\[ G = \frac{\delta (W_{\text{ext}} - \delta E_{\text{el}})}{(\delta a.b)} \quad G_c = \frac{\delta E_{\text{s}}}{(\delta a.b)} \]

Relation between stress intensity factor and energy release rate
\[ G = \frac{K^2}{E} \sim \sigma_{\text{ext}}^2 \]

Powerful predictive approach, but limited to homogeneous media
The crack tip as a magnifying glass of the material heterogeneities

Macroscopic failure properties strongly dependent on microscopic material features

The stress field diverges at the crack tip.
The crack tip as a magnifying glass of the material heterogeneities

Macroscopic failure properties strongly dependent on microscopic material features

Fracture as a complex and challenging multi-scale problem

Opportunities for a rational design of materials with improved failure properties
What are the effects of heterogeneities on the propagation of a crack?
What are the effects of heterogeneities on the propagation of a crack?

Pinning and deformation of the crack front:
Fracture Mechanics of heterogeneous materials

Heterogeneous field of fracture energy $G_c(M)$

$$G_c(M) = \langle G_c \rangle + \delta G_c(M)$$
Fracture Mechanics of heterogeneous materials

Heterogeneous field of fracture energy $G_c(M)$

$$G_c(M) = \langle G_c \rangle + \delta G_c(M)$$

Material elasticity: The crack front as an elastic interface

$$G(z) = G^{ext} + \frac{G^{ext}}{\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z' - z)^2} \, dz'$$

Hypothesis:
- Slow crack growth velocity
- Weekly heterogeneous material
- Very large sample

J. R. Rice 1985
Fracture Mechanics of heterogeneous materials

Heterogeneous field of fracture energy $G_c(M)$

$$G_c(M) = \langle G_c \rangle + \delta G_c(M)$$

Material elasticity: The crack front as an elastic interface

$$G(z) = G_{ext} + \frac{G_{ext}}{\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z' - z)^2} dz'$$

J. R. Rice 1985

Equation of motion for a crack

$$\mu \frac{\partial f(z,t)}{\partial t} \bigg|_M = G(M) - G_c(M)$$

Hypothesis:
- Slow crack growth velocity
- Weekly heterogeneous material
- Very large sample

L. B. Freund 1990
Fracture Mechanics of heterogeneous materials

Heterogeneous field of fracture energy $G_c(M)$

$$G_c(M) = \langle G_c \rangle + \delta G_c(M)$$

Material elasticity: The crack front as an elastic interface

$$G(z) = G_{ext} + \frac{G_{ext}}{\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} \, dz'$$

J. R. Rice 1985

Equation of motion for a crack

$$\mu \frac{\partial f(z,t)}{\partial t} = \left( G_{ext} - \langle G_c \rangle \right) + \frac{G_{ext}}{\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} \, dz' - \delta G_c(z,f(z,t))$$

Hypothesis:
- Slow crack growth velocity
- Weekly heterogeneous material
- Very large sample

Fracture Mechanics of heterogeneous materials

Heterogeneous field of fracture energy $G_c(M)$

$$G_c(M) = \langle G_c \rangle + \delta G_c(M)$$

Material elasticity: The crack front as an elastic interface

$$G(z) = G^{ext} + \frac{G^{ext}}{\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} dz'$$

\[ J. R. Rice 1985 \]

Hypothesis:
- Slow crack growth velocity
- Weekly heterogeneous material
- Very large sample

Equation of motion for a crack

$$\mu \frac{\partial f(z,t)}{\partial t} = k[v_m t - f(z,t)] + \frac{G^{ext}}{\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} dz' - \delta G_c(z, f(z,t))$$

For displacement control experiments: $G^{ext} = \langle G_c \rangle + k[v_m t - f(z,t)]$

Critical driving given by $G^{eff} = \langle G_c \rangle + k[v_m t - \langle f(z,t) \rangle_z] \quad \leftrightarrow \quad G^{eff} = \langle \delta G_c(z, f(z)) \rangle_z$
Thin film adhesive as a model system for exploring the failure of heterogeneous materials
Thin film adhesive as a model system for exploring the failure of heterogeneous materials

Equation of motion of the peeling front

\[
\mu \frac{\partial f(z, t)}{\partial t} \bigg|_M = G(z) - G_c(z, x = f(z, t))
\]

Local driving force:

\[
G(z) = G^{\text{ext}} + 4 \frac{G^{\text{ext}}}{\pi} \int \frac{f(z') - f(z)}{(z' - z)^2} dz'
\]

Applied driving force:

\[
G^{\text{ext}} = \frac{F_p}{b} (1 - \cos \theta_p)
\]
Thin film adhesive as a model system for exploring the failure of heterogeneous materials

Equation of motion of the peeling front

$$\frac{\partial f(z,t)}{\partial t} = G(z) - G_c(z,x = f(z,t))$$

Local driving force:

$$G(z) = G^\text{ext} + 4 \frac{G^\text{ext}}{\pi} \int \frac{f(z') - f(z)}{(z' - z)^2} dz'$$

Applied driving force:

$$G^\text{ext} = \frac{F_p}{b} (1 - \cos \theta_p)$$

Application: Toughness field invariant in the propagation direction

$$G[f(z)] \approx G_c(z)$$
Application to crack pinning by...

...a single heterogeneity

...a periodic array of heterogeneities

Application to material design

S. Xia, L. Ponson, G. Ravichandran and K. Bhattacharya 2012
and international patent 2011

Asymmetric adhesives

![Graph showing peel force per unit length vs. peel displacement with experimental and model results compared.](Image)
Application to material design

S. Xia, L. Ponson, G. Ravichandran and K. Bhattacharya 2012
and international patent 2011

Asymmetric adhesives

Determination of the optimal shape and distribution of pinning sites to achieve targeted properties
Crack propagation within disordered materials: Bridging the gap between experiments and theory

Experimental setup

Movie: Courtesy of S. Santucci

Crack front position measured through fast camera:
Characterizing the local crack dynamics

Position of the front at a given time

Matrix of waiting time $T_w$

Definition of clusters such as $T_w < C < T_w$>

Avalanche of size $S=5$
Distribution of local avalanche sizes

K. Maloy et al. 2006, K. Tallakstat et al. 2011

\[ P(S) \sim S^{-\gamma} \]
with \( \gamma \approx 1.55 \)
Comparison with the interface depinning model

White pixels correspond to

\[ T_w < C \langle T_w \rangle \]

and

\[ T_w > C \langle T_w \rangle \]

Depinning clusters

\[ P(S) \sim S^{-\tau} \]

with \( \tau_d \approx 1.55 \)

Pinning clusters

\[ P(S) \sim S^{-\tau} \]

with \( \tau_p \approx 1.65 \)

Energy transfer during brittle fracture

$t = 0^-$

$t = 0^+$

$t = t_{\text{end}}$
Energy transfer during brittle fracture

Total energy: \( E_{\text{tot}} = E_0 = E_{\text{pot}} + E_{\text{el}} + E_s \)

At time \( t=0^- \):
\[
\begin{align*}
E_m &= E_0 \\
E_s &= 0
\end{align*}
\]

At time \( t=0^+ \):
\[
\begin{align*}
E_m &= E_0 \\
E_s &= E_0
\end{align*}
\]

At time \( t=t_{\text{end}} \):
\[
\begin{align*}
E_m &= 0 \\
E_s &= E_0
\end{align*}
\]
Energy transfer during brittle fracture

Total energy: \( E_{\text{tot}} = E_0 = E_{\text{pot}} + E_{\text{el}} + E_s \)

At time \( t = 0 \):
\[
\begin{align*}
E_m &= E_0 \\
E_s &= 0
\end{align*}
\]

At time \( t = t_{\text{end}} \):
\[
\begin{align*}
E_m &= 0 \\
E_s &= E_0
\end{align*}
\]

Rate of energy transfer:
\[
P(t) = \frac{\delta E_s}{\delta t} = -\frac{\delta E_m}{\delta t} = b \int G(z,t) f(z,t) \, dz \approx b \langle G_c \rangle v_m
\]

\( P(t) \sim v_m(t) \)

Rate of energy transfer given by the average crack growth velocity
Energy transfer during brittle fracture

Total energy:
\[ E_{\text{tot}} = E_0 = E_{\text{pot}} + E_{\text{el}} + E_s \]

Rate of energy transfer:
\[ P(t) = \frac{\delta E_s}{\delta t} = -\frac{\delta E_m}{\delta t} = b \int G(z,t) \dot{f}(z,t) dz \approx b \langle G_c \rangle v_m \]

At time \( t = 0^- \):
\[ \begin{cases} E_m = E_0 \\ E_s = 0 \end{cases} \]

At time \( t = 0^+ \):
\[ \begin{cases} E_m = E_0 \\ E_s = 0 \end{cases} \]

At time \( t = t_{\text{end}} \):
\[ \begin{cases} E_m = E_0 \\ E_s = E_0 \end{cases} \]
Statistics of global avalanches

J. Barès, D. Bonamy et al. 2014

Avalanche size:

\[ S \sim \int_{t_1}^{t_2} v_m(t) \, dt \]

Avalanche duration: \( T = t_2 - t_1 \)

\[ P(S) \sim S^{-\tau} \]

with \( \tau \approx 1.40 \)
Statistics of *global* avalanches

**J. Barès, D. Bonamy et al. 2014**

Avalanche size:

\[ S \sim \int_{t_1}^{t_2} v_m(t) dt \]

Avalanche duration: \( T = t_2 - t_1 \)

Prediction of the interface depinning model:

\[ P(S) \sim S^{-\tau} \]

with \( \tau \approx 1.28 \)

Avalanche shape defined...

...at the global scale

Size vs duration

\[ S \sim T^\gamma \text{ with } \gamma^{exp} \approx 1.75 \]

Asymmetric shape and scaling exponent
\( \gamma^{th} = 1.80 \) consistent with the depinning model

See S. Santucci's talk at the conference
Interface depinning model: A relevant framework to describe the failure of brittle disordered materials?

Yes!

Depinning concepts capture:
- the avalanches dynamics at the *local* and *global* scale
- the self-affine roughness of crack fronts
  
  $\theta_{S.}\text{ Santucci et al. 2010}$

- the average crack dynamics
  
  Depinning transition: $v_m \sim (G - G_c)^\theta$
  
  $\text{L. Ponson 2009}$
Interface depinning model: A relevant framework to describe the failure of brittle disordered materials?

Yes!

Depinning concepts capture
- the avalanches dynamics at the local and global scale
- the self-affine roughness of crack fronts
  \[ v_m \sim (G - G_c)^\theta \]
  S. Santucci et al. 2010
- the average crack dynamics
  Depinning transition: \[ v_m \sim (G - G_c)^\theta \]
  L. Ponson 2009

But under some specific conditions
- Very large sample size
  Specimen width >> spatial extent of avalanches
- Scale separation between the material heterogeneity and the process zone
  Process zone size << material heterogeneity size
1. Extend the Fracture Mechanics framework to disordered systems

**Prediction on the mean crack velocity**

\[ v_{crack} = \left( \frac{\partial f(z,t)}{\partial t} \right)_{z,t} \]

Stable \quad Propagating

\[ G^{ext} - \langle G_c \rangle \]
1. Extend the Fracture Mechanics framework to disordered systems

**Prediction on the mean crack velocity**

\[
v_{\text{crack}} = \left( \frac{\partial f(z,t)}{\partial t} \right)_{z,t}
\]

If \( G^{\text{ext}} < \langle G_c \rangle + \Delta G_c \)

The crack front is pinned by the defects

If \( G^{\text{ext}} > \langle G_c \rangle + \Delta G_c \)

The crack propagates
1. Extend the Fracture Mechanics framework to disordered systems

**Prediction on the mean crack velocity**

If $G_{\text{ext}} < \langle G_c \rangle + \Delta G_c$ → The crack front is pinned by the defects

If $G_{\text{ext}} > \langle G_c \rangle + \Delta G_c$ → The crack propagates

$G_{\text{eff}}^c = \langle G_c \rangle + \Delta G_c$

Toughening effect
1. Extend the Fracture Mechanics framework to disordered systems

**Prediction on the mean crack velocity**

\[ V_{\text{crack}} \sim (G_{\text{ext}} - G_{c}^{\text{eff}})^{\theta} \]

with \( \theta = 0.75 \)

- If \( G_{\text{ext}} < <G_{c}> + \Delta G_{c} \) \( \rightarrow \) The crack front is pinned by the defects
- If \( G_{\text{ext}} > <G_{c}> + \Delta G_{c} \) \( \rightarrow \) The crack propagates

\[ G_{c}^{\text{eff}} = <G_{c}> + \Delta G_{c} \]

D. Ertas and M Kardar 1994 (RG, 1st loop)
P. Chauve et al. 2001 (RG, 2nd loop)
O. Duemmer and W. Krauth 2007 (Sim.)
2. Average crack dynamics: variations of the mean crack velocity with the driving force

Can we measure the depinning transition?

Experimental setup to measure the curves $v(G^{ext})$

- Slightly unstable geometry that enables to investigate a wide range of velocities
- Sandstone specimen: Brittle + heterogeneous materials

TDCB Sample

Clip gauge

Crack opening displacement

Crack
2. Average crack dynamics: variations of the mean crack velocity with the driving force

Can we measure the depinning transition?
Experimental setup to measure the curves $v(G^\text{ext})$

- Slightly unstable geometry that enables to investigate a wide range of velocities
- Sandstone specimen
  Brittle + heterogeneous materials
- Finite element analysis

Experimental values of:
- force
- crack opening displacement

$\Rightarrow v(G^\text{ext})$
2. Average crack dynamics: variations of the mean crack velocity with the driving force

Can we measure the depinning transition?

Fracture tests of brittle rocks
2. Average crack dynamics: variations of the mean crack velocity with the driving force

Can we measure the depinning transition?

Fracture tests of brittle rocks

L. Ponson 2009
2. Average crack dynamics: variations of the mean crack velocity with the driving force

Can we measure the depinning transition?
2. Average crack dynamics: variations of the mean crack velocity with the driving force

Can we measure the depinning transition?

Fracture tests of brittle rocks

\[ v_{\text{crack}} \sim (G^{\text{ext}} - G_{c}^{\text{eff}})^{\theta} \]

with \( \theta = 0.80 \pm 0.15 \)

L. Ponson 2009
Can we measure the depinning transition?

Fracture tests of brittle rocks

\[ v = v_0 e^{-\frac{c}{(G_{\text{ext}} - \langle G_c \rangle)^\mu}} \]
with \( \mu \approx 0.60 \)

\[ v_{\text{crack}} \sim (G_{\text{ext}}^\text{eff} - G_c^\text{eff})^\theta \]
with \( \theta = 0.80 \pm 0.15 \)
2. Average crack dynamics: variations of the mean crack velocity with the driving force

Coming back to the model..
2. Average crack dynamics: variations of the mean crack velocity with the driving force

Coming back to the model..

\[ v = v_0 e^{-\frac{C}{(G_{\text{ext}} - <G_c>)^\mu}} \]

with \( \mu \approx 0.60 \) (long range elasticity)

\[ T > 0 \]

Crack propagation is possible below the critical threshold through thermal activation processes

3. Intermittency and avalanches during crack propagation

Intermittent crack propagation at the transition and below
3. Intermittency and avalanches during crack propagation

Intermittent crack propagation at the transition and below

At the transition and below, the crack propagates through sudden jumps (avalanches) with no characteristic size (power law distributed)