Avalanches in elastic interfaces and sandpiles: mean field theory and beyond

P. Le Doussal

LPTENS

collab: K. Wiese, A. Rosso

A. Dobrinevski

M. Delorme, T. Thiery

- analytical calculations from first principles of avalanche observables

  precise predictions, test numerics (and experiments!)

- interface depinning, sandpiles, reaction-diffusion: same universality

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I - elastic interface, Middleton theorem, avalanche

- renormalized disorder, Functional RG at depinning, charact. scales

- avalanche size distribution mean field and beyond

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II. avalanche dynamics: mean field theory and Brownian Force Model (BFM)

- Results from BFM and ABBM: shape, size-duration, local sizes, q-asymmetry
- scaling relations, avalanche exponents, distribution at one loop and beyond
- shape at one loop: fixed duration, fixed size

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III. towards a unified theory of avalanches in sandpiles and interfaces ?

early work on FRG depinning,
PLD+KW+Chauve, Fedorenko,Rosso, Middleton, Rolley

avalanches from FRG: size distributions

Size distributions of shocks and static avalanches from the FRG
PLD, K. Wiese, arXiv:0812.1893, PRE, 79, 5 051106, (2009), arXiv:1111.3172, PRE 85 (2012) 061102.

size distribution: numerical + experimental tests

PLD, A.Middleton and K.Wiese, Phys. Rev. E 79, 050101, (2009).

Avalanche-size distribution at depinning: A numerical test of the theory
A. Rosso, PLD, K. Wiese, arXiv:0904.1123, PRB 80,144204 (2009).

avalanches dynamics: velocity distribution, duration, shape

Distribution of velocities in an avalanche
PLD, K. Wiese, arXiv:1104.2629, EPL 97 (2012) 46004

Avalanche dynamics of elastic interfaces
PLD, K. Wiese, arXiv:1302.4316, PRE 88 (2013) 022106.

Nonstationary dynamics of the ABBM model
A. Dobrinevski, PLD, K. Wiese, PRE 85, 031105 (2012).

Avalanches with Relaxation, and Barkhausen Noise: A Solvable Model, A. Dobrinevski, PLD, K. Wiese.PRE 85, 031105 (2012)

PhD thesis, A. Dobrinevski, arXiv1312.7156

A. Dobrinevski, PLD, KW arXiv 1407.7353+ in preparation

Equilibrium avalanches in spin glasses, PLD,KW+ Markus Mueller
PRB 85 (2012) PRE 88, 032106 (2013).

elastic interface: $x \in R^d$
$u(x,t)$

$$\overline{F(x,u)F(x',u')} = \delta(x-x')\Delta_0(u-u')$$

$$\eta_0 \partial_t u(x,t) = \nabla_x^2 u(x,t) + m^2(w(t) - u(x,t)) + F(u(x,t),x)$$

$f = m^2 w$ driving force

$L_m \sim 1/m$

$m \to 0$ critical

Long-range elasticity

$$(-\nabla_x^2 + m^2)u(x) \to \int d^d x' c(x,x') u(x')$$

$$c(q) = (q^2+\mu^2)^{\gamma/2} \qquad c(0) = m^2$$

SR $\gamma = 2$ $d_{uc} = 4$

LR $\gamma = 1$ $d_{uc} = 2$

$d_{uc} = 2\gamma$

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Long-range elasticity

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m not renormalized

$$(-\nabla_x^2 + m^2)u(x) \to \int d^dx' c(x,x') u(x')$$

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SR $\gamma = 2$ $d_{uc} = 4$

LR $\gamma = 1$ $d_{uc} = 2$

drive fixed distance from criticality

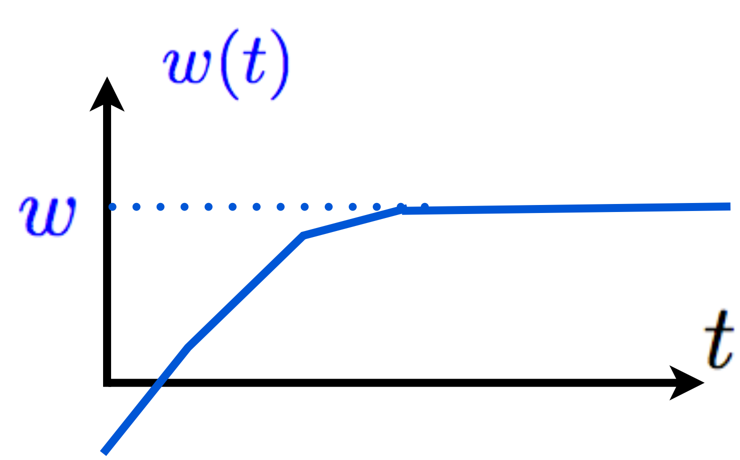
Middleton theorem - monotonicity $\dot{u}(x,t_0) \geq 0 \ \forall x$, $\dot{w}(t) \geq 0 \ \forall t \geq t_0$ $\Rightarrow$ $\dot{u}(x,t) \geq 0 \ \forall x$ $\forall t \geq t_0$

$c(x,x') \leq 0$

$\forall (x, x' \neq x)$

- partial order, memory loss, Middleton states

if driven forward from infinite left , stopped driving at w



$\Rightarrow$ interface converges to Middleton state $u(x;w)$

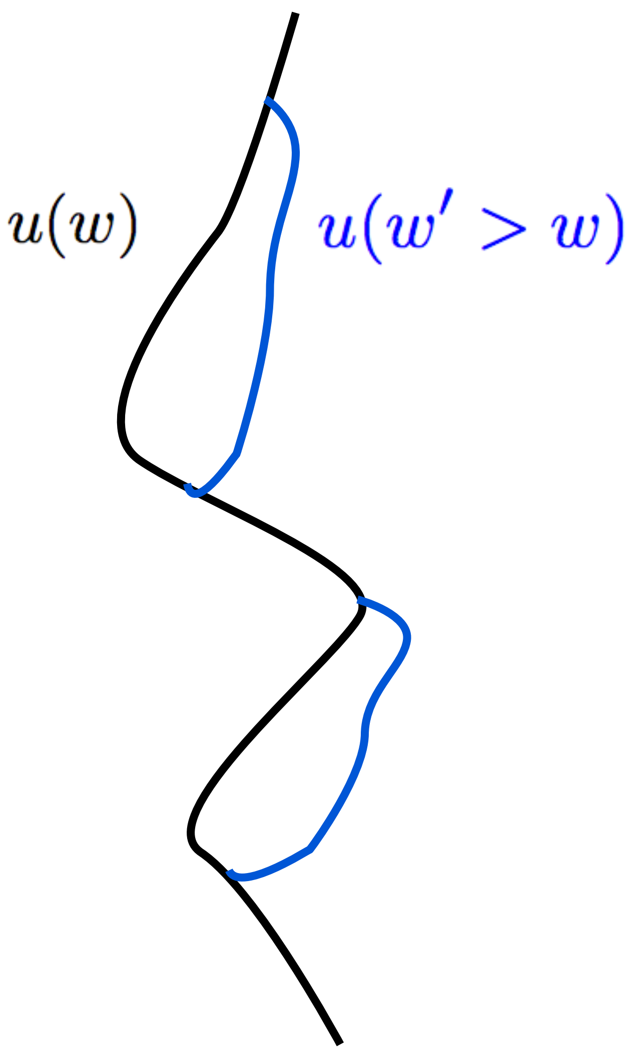
unique leftmost metastable state (for a fixed w)

$$u(x,t) \to_{t\to\infty} u(x;w)$$

$$u(w) = L^{-d} \int_x u(x,w)$$

should be tested in experiments (when possible!)

definition of avalanche = motion from a Middleton state to “the next one”



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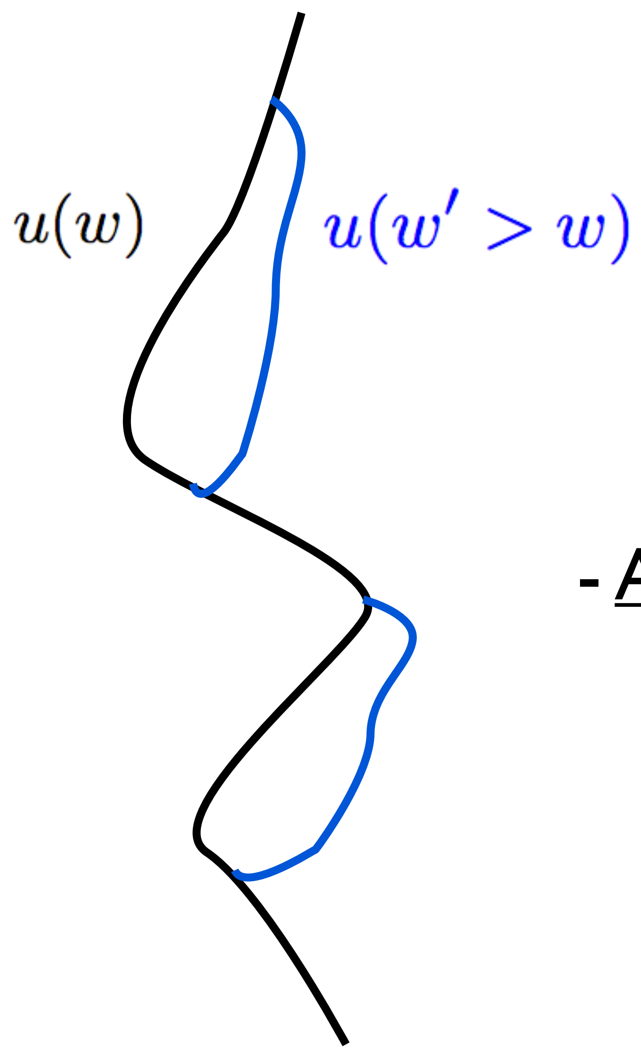


- smooth disorder discrete set w_i $\qquad w_{i+1} - w_i \sim L^{-d}$

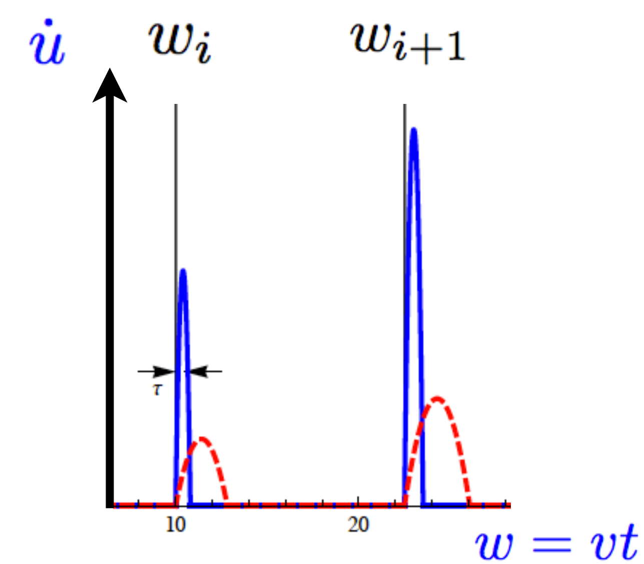
size $S_i = \int d^d x (u(x; w_i^+) - u(x, w_i^-)) = L^d (u(w_i^+) - u(w_i^-))$

$$\rho(S) = \sum_i \overline{\delta(S - S_i)\delta(w - w_i)} \qquad S > S_0$$

- rough disorder: avalanche any small scales

- <u>Avalanches in steady state:</u>

$$w(t) = vt \qquad v \to 0^+$$



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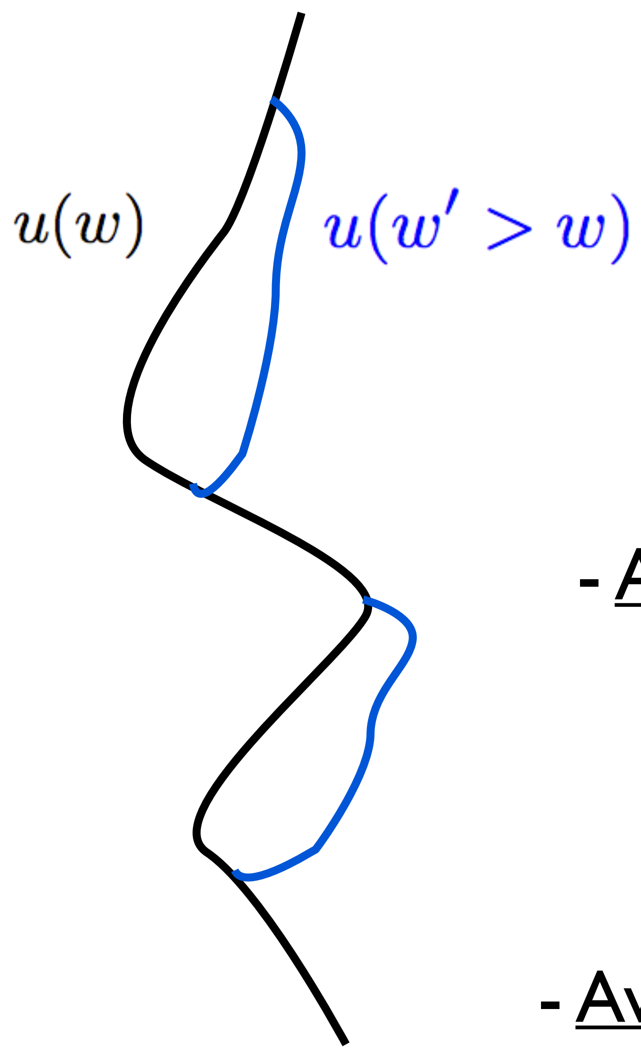

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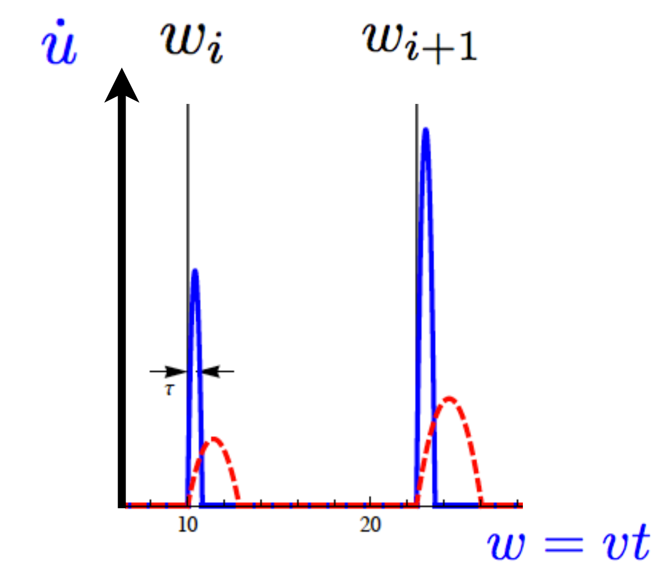
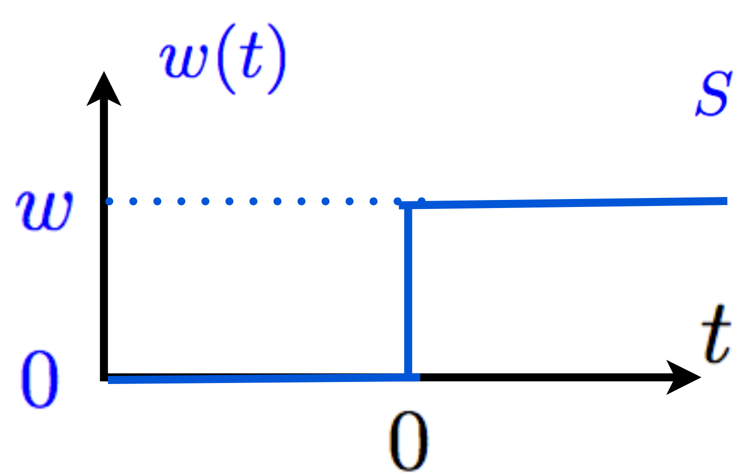
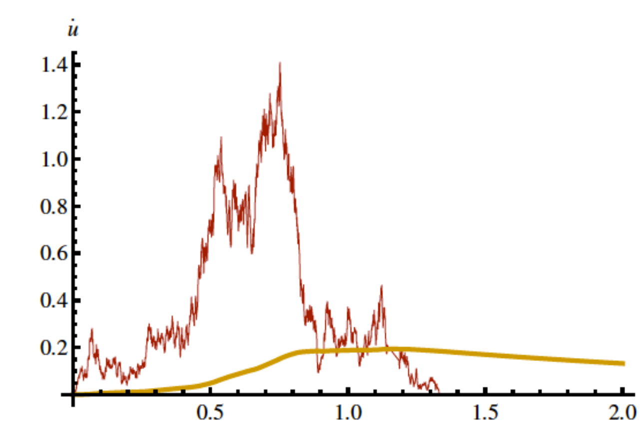


- Avalanches following a kick: $\dot w(t) = w\delta(t)$

at rest in Middleton state u(w=0) at t<0



$$S = L^d \int_0^{+\infty} \dot u(t) dt = L^d (u(w) - u(0))$$



in limit w=0^+ same as steady state avalanches

$$\rho(S) = \partial_w P_w(S)|_{w=0^+}$$

Functional RG and field theory

PLD, EPL (2006)   PLD, KW, EPL (2007)

$$\overline{(u(w) - w)(u(w) - w')}^c = m^{-4} L^d \Delta(w - w')$$

$$\Delta(w) \equiv \Delta_m(w)$$

renormalized disorder correlator

$$\Delta_m(w) \simeq_{m \to 0} m^{\epsilon - 2\zeta} \tilde{\Delta}^*(w m^\zeta)$$

obeys a differential FRG equation as m is varied

FRG fixed point:

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FRG fixed point:

$$\tilde{\Delta}^*(u) = \epsilon d_1(u) + \epsilon^2 d_2(u) + ..\qquad \epsilon = d_{uc} - d$$

All universal observables can be obtained in perturbation in $\tilde{\Delta}^*(u)$ i.e. in ϵ

Allows to calculate depinning critical exponents: two independent exponents

$$u \sim x^{\zeta}$$

SR $\zeta = \frac{\epsilon}{3}(1 + 0.1433\epsilon)$ $\epsilon = 4 - d$

LR 0.39735ϵ $\epsilon = 2 - d$

$$x \sim t^z$$

$$z = 2 - \frac{2}{9}\epsilon - 0.0432\epsilon^2$$

$-0.1133\epsilon^2$

Ferrero (2013)

SR d=1 $\zeta = 1.250 \pm 0.005$

$z = 1.433 \pm 0.007$

LR d=1 predicts 0.4 confirmed numerics

Rosso, Krauth $\zeta = 0.39..$

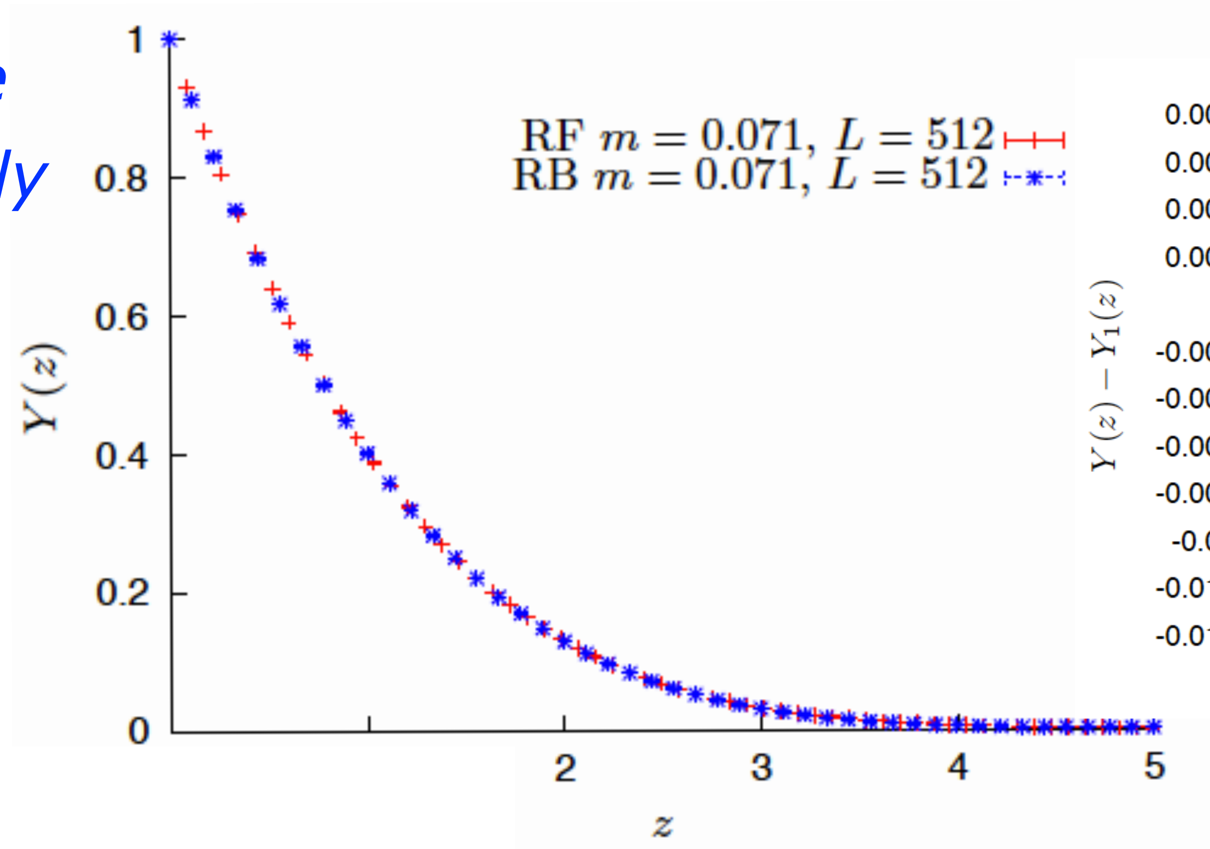
fracture: Ponson, Santucci,..

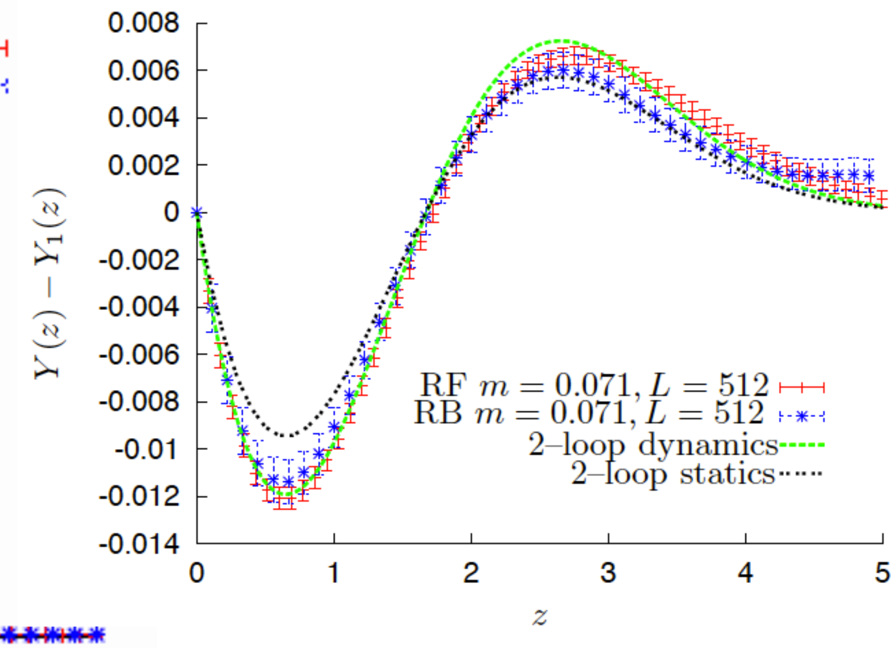
FRG fixed point at depinning: numerics and experiments

numerics: interface driven quasi-statically by quadratic well

A. Rosso, PLD, KW

condmat/0610821





contact line experiment:

E. Rolley, S. Moulinet, PLD, KW

EPL, 87 (2009) 56001

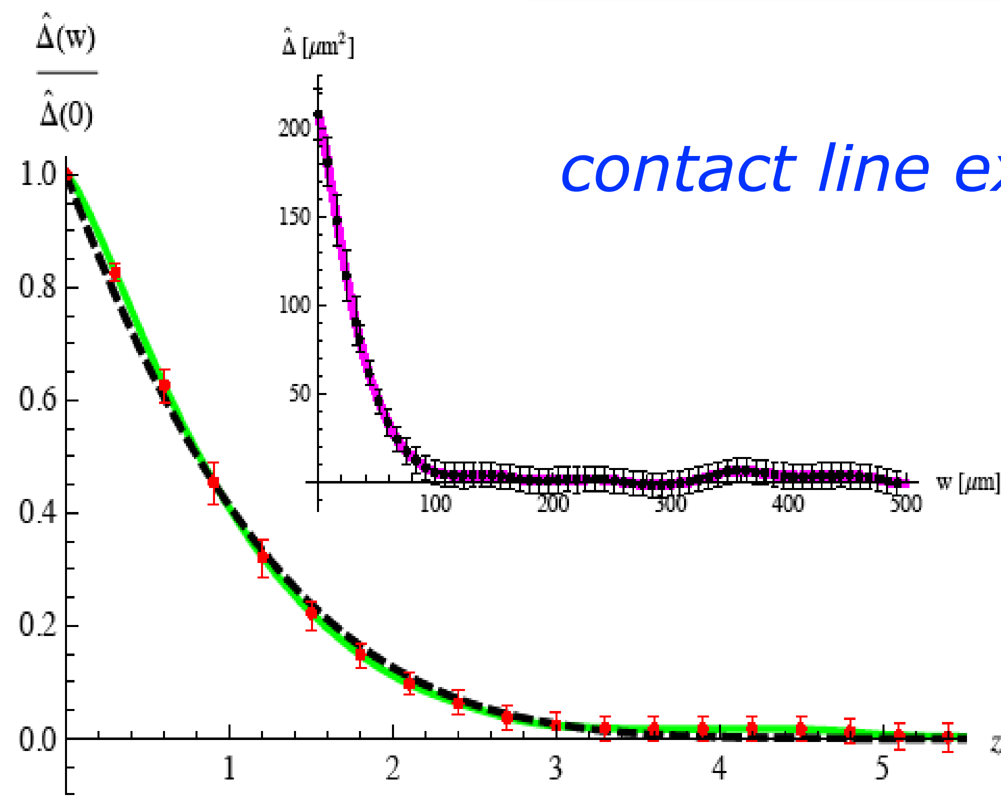


Fig. 6: Inset: The disorder correlator $\hat\Delta(w)$ for iso/Si at $v = 1\mu\text{m/s}$ up to $w = 35\mu\text{m}$, and then at $v = 10\mu\text{m/s}$ for $w > 35\mu\text{m}$, with error-bars as estimated from the experiment. Main plot: The rescaled disorder correlator $\hat\Delta(w)/\hat\Delta(0)$ (green/solid) with error bars (red). The dashed line is the 1-loop result from equation (6).

$$\hat\Delta(w - w') := \langle \overline{\mathsf{h}}_l(w)\overline{\mathsf{h}}_l(w')\rangle$$

$$\bar h_l(t) := \frac{1}{l}\int_0^l h(x,t)\,\mathrm{d}x$$

$$\mathsf{h}(x,w) := h(x, w/v) - h_0$$

FRG fixed point at depinning: numerics and experiments

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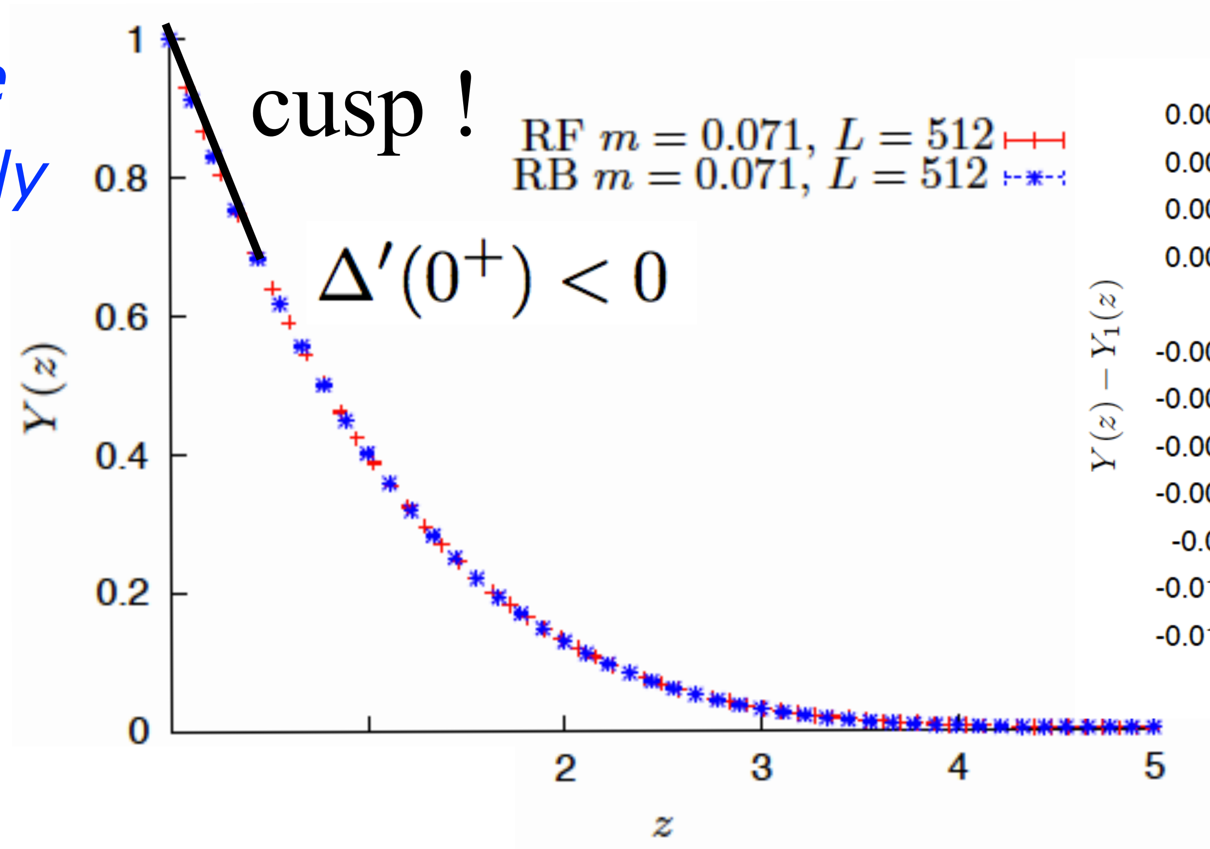

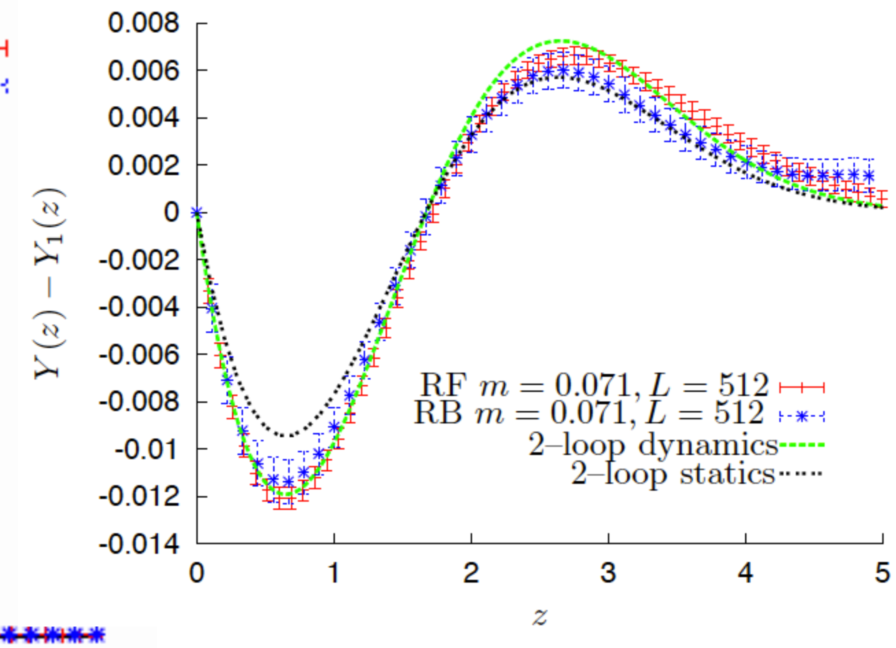

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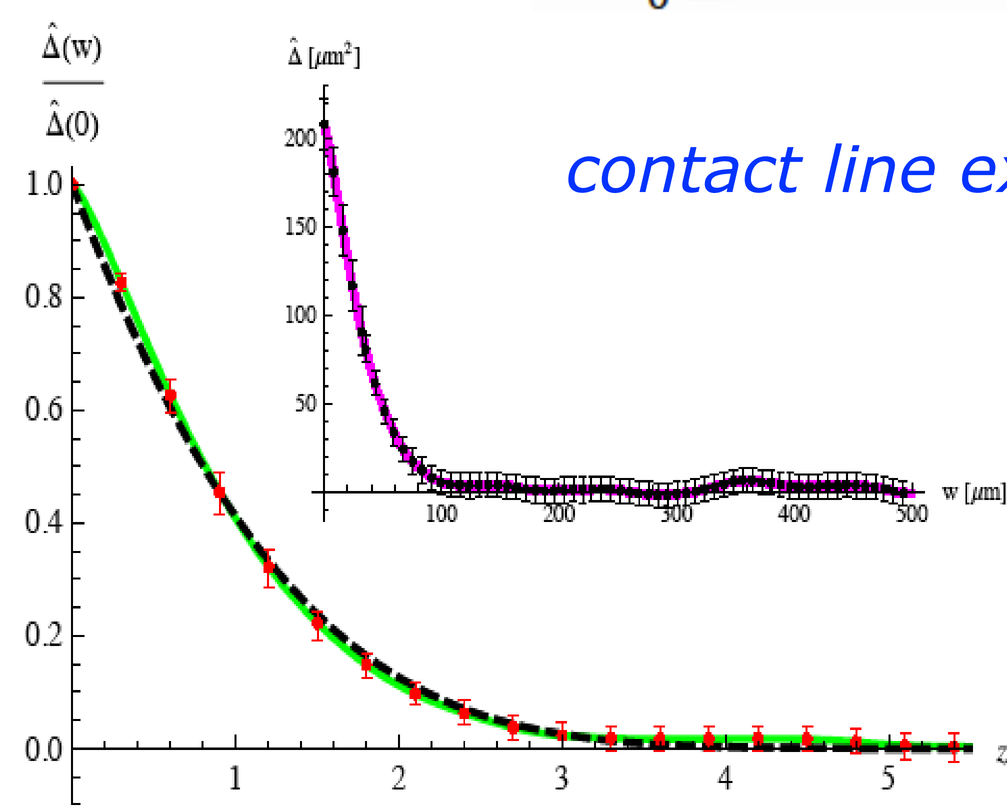

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Avalanches :

Two characteristic scales:

large size cutoff:

$$S_m := \frac{\langle S^2 \rangle}{2\langle S \rangle} = \frac{|\Delta'(0^+)|}{m^4} \simeq c_S m^{-(d+\zeta)}$$

relaxation time of large avalanches:

$$\tau_m = \frac{\eta_m}{m^2} \simeq c_\tau m^{-z}$$

from linear response function at q=0 and small frequency

$$1/R_{q,\omega} \simeq q^2 + m^2 + i\omega\eta_m + ..$$

$$\frac{1}{\eta_m} = \frac{\delta \dot{u}}{\delta f}$$

Mean field :

$$d \geq d_{uc}$$

$$S_m = \frac{\sigma}{m^4}$$

$$\tau_m = \frac{\eta}{m^2}$$

log(m) corrections in d=d_uc

avalanche size density $\rho(S) = \rho_0 P(S)$ $\rho_0 \langle S \rangle = L^d$ smooth disorder

$\rho(S) = \partial_w P_w(S)|_{w=0^+}$ $\rho(S) = \frac{L^d}{S_m^2} p(S/S_m)$ $P(S) = \frac{\langle S \rangle}{S_m^2} p(S/S_m)$

FRG yields $d = d_{uc}$ $p_{MF}(s) = \frac{1}{2\sqrt{\pi} s^{3/2}} e^{-s/4}$

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FRG yields $d = d_{uc}$ $p_{MF}(s) = \frac{1}{2\sqrt{\pi} s^{3/2}} e^{-s/4}$

$\gamma_E = 0.577216$

$d = 4 - \epsilon$

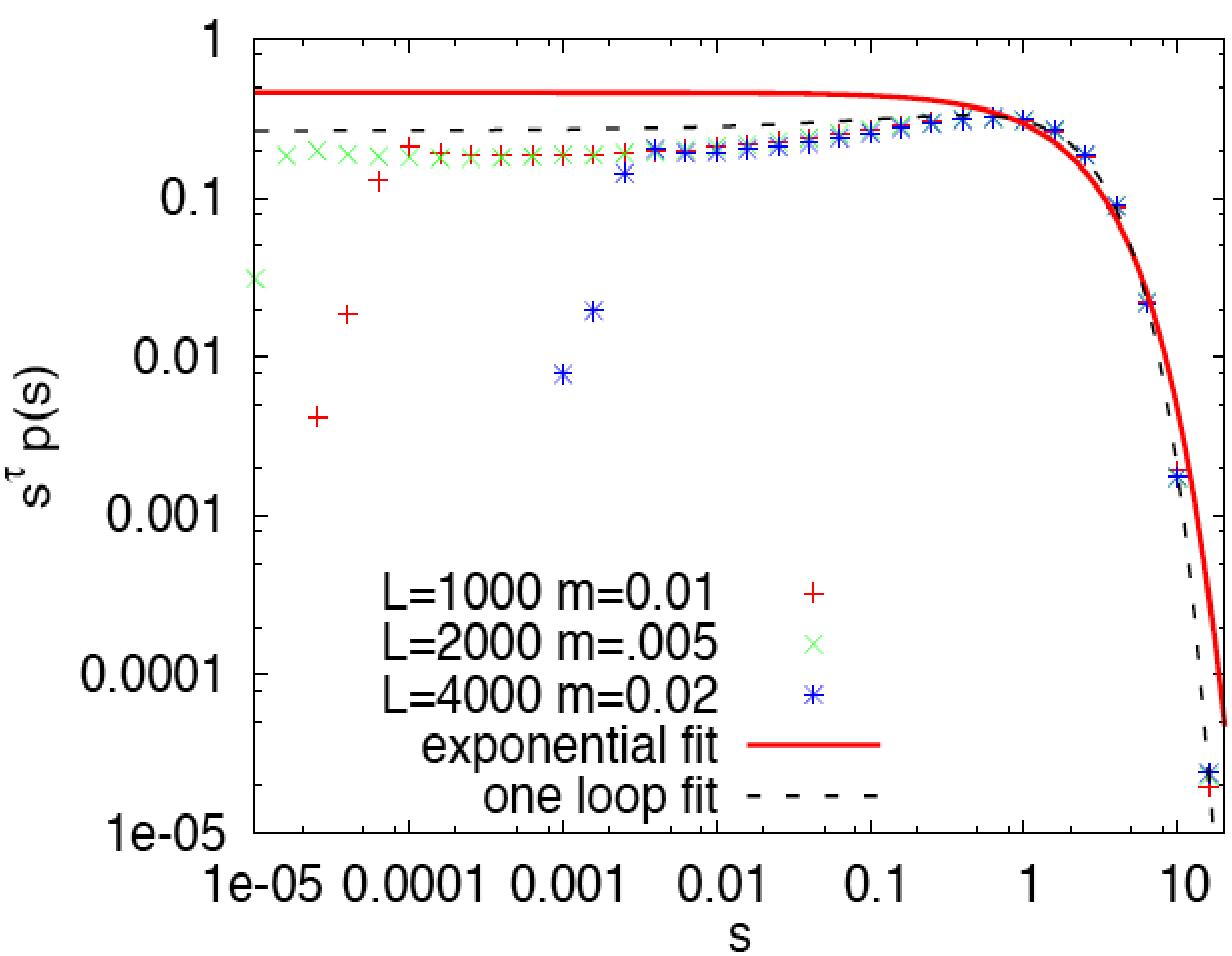
$$p(s) = \frac{A}{2\sqrt{\pi}} \frac{1}{s^\tau} e^{C s^{1/2} - \frac{B}{4} s^\delta}$$

$A = 1 - \frac{2 - 3\gamma_E}{36}\epsilon$

$B = 1 + \frac{2}{9}(1 + \frac{\gamma_E}{4})\epsilon$

$C = \frac{\sqrt{\pi}}{9}\epsilon$

$d = 1$



avalanche exponent:

$\delta = 1 + \frac{\epsilon}{6}$

$$\tau = \frac{3}{2} - \frac{\epsilon}{12}$$

agrees to $O(\epsilon)$ with Narayan-Fisher conjecture

$$\tau_{\rm conj} = 2 - \frac{2}{d + \zeta}$$

$$\tau_{\rm num}^{d=1} = 1.08 \pm 0.02$$

NF = 1.11

Mean-field theory of avalanche dynamics for driven interfaces

mean field theory of elastic interfaces: the Brownian Force Model (BFM)

"theorem": - for $d \geq d_{uc}$ the joint velocity distribution of all $\dot{u}(x,t)$ in an avalanche are described by the BFM

single avalanche, monotonicity $v \to 0^+$ $\dot{w} = 0^+$

large scale, only velocity, not position NOT a MFT for depinning

- for $d < d_{uc}$ the joint velocity distribution can be obtained in perturbation around the BFM in $d = d_{uc} - \epsilon$

BFM

$$\eta \partial_t \dot{u}(x,t) = \nabla_x^2 \dot{u} + m^2 (\dot{w}(t) - \dot{u}(x,t)) + \sqrt{\dot{u}(x,t)} \xi(x,t)$$

$\dot{w}(t) \geq 0$

$\dot{u}(x,t) \geq 0$

$$\overline{\xi(x,t)\xi(x',t')} = 2\sigma \delta^d(x-x')\delta(t-t')$$

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BFM $$\eta \partial_t \dot{u}(x,t) = \nabla_x^2 \dot{u} + m^2(\dot{w}(t) - \dot{u}(x,t)) + \sqrt{\dot{u}(x,t)}\xi(x,t)$$

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$$\sigma = -\Delta'(0^+) \equiv \sigma_m$$

renormalized disorder

$$\eta_0 \to \eta_m$$

log(m) diverg. at d_{uc}

- interface does not "revisit" same disorder

- renormalized disorder is rough (cusp) and locally Brownian

$$\Delta(0) - \Delta(u) = \sigma|u| + O(\Delta''(0))$$

neglect, higher order in ϵ

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BFM

$$\eta \partial_t \dot u(x,t) = \nabla_x^2 \dot u + m^2 (\dot w(t) - \dot u(x,t)) + \sqrt{\dot u(x,t)}\, \xi(x,t)$$

$\dot w(t) \geq 0$

$\dot u(x,t) \geq 0$

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renormalized disorder

$\eta_0 \to \eta_m$

log(m) diverg. at d_{uc}

time derivative of exact equation of motion $\sqrt{\dot u(x,t)}\,\xi(x,t) \equiv \partial_t F(u(x,t),x)$

$$\overline{\partial_t F(u(x,t),x)\partial_{t'} F(u(x',t'),x')} = \dot u(x,t)\partial_{t'} \overline{\partial_u F(u(x,t),x) F(u(x',t'),x')}$$

$$= \dot u(x,t)\Delta'(u(x,t)-u(x,t'))\delta^d(x-x') = \dot u(x,t)\partial_{t'}\Delta'(0^+)\mathrm{sgn}(t-t')\delta^d(x-x') + O(\Delta'') + ..$$

neglect, higher order in ϵ

dynamical action:

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_{\rm dis}$$

$$\mathcal{S}_0 = \int_{xt} \tilde u_{xt}(\eta\partial_t - \nabla_x^2 + m^2)\dot u_{xt}$$

$$\mathcal{S}_{\rm dis} = -\frac{1}{2}\int_{xtt'} \tilde u_{xt}\tilde u_{xt'}\partial_t\partial_{t'}\Delta\big(v(t{-}t') + u_{xt}{-}u_{xt'}\big).$$

expansion in derivatives:

$$\begin{aligned}&\partial_t\partial_{t'}\Delta(v(t-t') + u_{xt} - u_{xt'})\\ &\quad = (v+\dot u_{xt})\partial_{t'}\Delta'(v(t-t') + u_{xt} - u_{xt'})\\ &\quad = (v+\dot u_{xt})\Delta'(0^+)\partial_{t'}{\rm sgn}(t-t') + \dots\,.\end{aligned}$$

this is BFM

$$\mathcal{S}_{\rm dis} = -\sigma\int_{xtt'} \tilde u_{xt}\tilde u_{xt}(v+\dot u_{xt}) \qquad (302)$$

$$+\frac{1}{2}\int_{xtt'} \tilde u_{xt}\tilde u_{xt'}(v{+}\dot u_{xt})(v{+}\dot u_{xt'})\Delta''_{\rm reg}\big(v(t{-}t'){+}u_{xtt'}\big)\,.$$

$$\Delta(u) = -\sigma|u| + \Delta_{\rm reg}(u)$$

this is small, arises to O(d_c-d)

exact solution of BFM:

$$\eta \partial_t \dot{u}(x,t) = \nabla_x^2 \dot{u} + m^2(\dot{w}(t) - \dot{u}(x,t)) + \sqrt{\dot{u}(x,t)}\xi(x,t)$$

1 - center of mass obeys ABBM model : $L^d \dot{u}(t) = \int d^d x \ \dot{u}(x,t)$

$$\eta \partial_t L^d \dot{u}(t) = m^2(L^d \dot{w}(t) - L^d \dot{u}(t)) + \sqrt{L^d \dot{u}(t)}\xi(t)$$

Fokker-Planck equation methods

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Fokker-Planck equation methods

II - exact formula for generating function (Laplace transform of multi-point correlations)

Obtained from dynamical field theory, MSR action

$$\overline{e^{\int d^d x dt \lambda(x,t) \dot{u}(x,t)}} = e^{\int d^d x dt \dot{f}(x,t) \tilde{u}(x,t)} \qquad \dot{f}(t) = m^2 \dot{w}(t)$$

$\tilde{u}(x,t)$ solution of the "instanton equation" :

$$(\eta \partial_t + \nabla_x^2 - m^2)\tilde{u} + \sigma \tilde{u}^2 = -\lambda(x,t) \qquad \tilde{u}(x,+\infty) = 0$$

BFM: center of mass observables

following a finite kick $\dot{w}(t) = w\delta(t)$

- size distribution $S = \int_0^\infty dt \dot{u}(t)$ $\lambda(x,t) = \lambda\theta(t)$

$$\overline{e^{L^d \lambda S}} = e^{m^2 L^d w \tilde{u}}$$

$$-m^2 \tilde{u} + \sigma \tilde{u}^2 = -\lambda$$

dimensionless units

$$S_m = \frac{\sigma}{m^4}$$

$$P_w(S) = \frac{w L^d}{2\sqrt{\pi} S^{3/2}} e^{-\frac{(S - w L^d)^2}{4S}}$$

- "single" avalanche regime $w \sim L^{-d}$
- fixed w $L \to \infty$

converges to Gaussian

independent avalanches along x

$$\rho(S) = \partial_w P_w(S)|_{w=0^+} = \frac{L^d}{2\sqrt{\pi} S^{3/2}} e^{-\frac{S}{4}} \sim S^{-\tau} \qquad \tau = 3/2$$

BFM: center of mass observables

following a finite kick $\dot{w}(t) = w\delta(t)$

- size distribution

$$S = \int_0^\infty dt \dot{u}(t)$$

$$\lambda(x,t) = \lambda\theta(t)$$

$$\overline{e^{L^d \lambda S}} = e^{m^2 L^d w \tilde{u}}$$

$$-m^2 \tilde{u} + \sigma \tilde{u}^2 = -\lambda$$

dimensionless units

$$S_m = \frac{\sigma}{m^4}$$

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- duration distribution

units $\tau_m = \frac{\eta}{m^2}$

$$P_w(T) = \frac{w L^d e^{-w L^d/(e^T - 1)}}{4 \sinh^2(T/2)}$$

- fixed w $L \to \infty$ converges to Gumbel

maximum of w L^d independent events

$$\rho(T) := \partial_w P_w(T)|_{w=0^+} = \frac{L^d}{4 \sinh^2(T/2)} \sim T^{-\alpha} \qquad \alpha = 2$$

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$$\rho(T) := \partial_w P_w(T)|_{w=0^+} = \frac{L^d}{4\sinh^2(T/2)} \sim T^{-\alpha} \qquad \alpha = 2$$

- shape at fixed duration

$$L^d\langle\dot{u}_t\rangle_T = \frac{4\sinh(\frac{t}{2})\sinh(\frac{T-t}{2})}{\sinh(\frac{T}{2})} + w\left(\frac{\sinh(\frac{T-t}{2})}{\sinh(\frac{T}{2})}\right)^2$$

symmetric for w->0 skewed to beginning at finite w

- joint P(S,T)

$$\overline{S} = 2T\coth(T/2) - 4 \sim T^2 \qquad \gamma = 2 \qquad \sim T \text{ large T}$$

BFM: local or q-dependent observables

example

local size distribution

$$S^\phi = \int d^d x \phi(x) S(x)$$

$$S^\phi = \int d^{d-1} x S_{x_1=0,x}$$

$$\rho(S^\phi) = \frac{L^d}{(S_m^\phi)^2} p(S/S_m^\phi)$$

$$p(s) = \frac{2}{\pi s} K_{1/3}(2s/\sqrt{3}) \sim s^{-\tau_\phi}$$

$$\tau_\phi = \frac{4}{3}$$

finite q- asymmetry

$$A(t_1) := \frac{\overline{\dot{u}_{-T/2}\dot{u}_{q,t_1}\dot{u}_{-q,t_1}\dot{u}_{T/2}}}{\overline{\dot{u}_{-T/2}\dot{u}_{q,0}\dot{u}_{-q,0}\dot{u}_{T/2}}} .$$

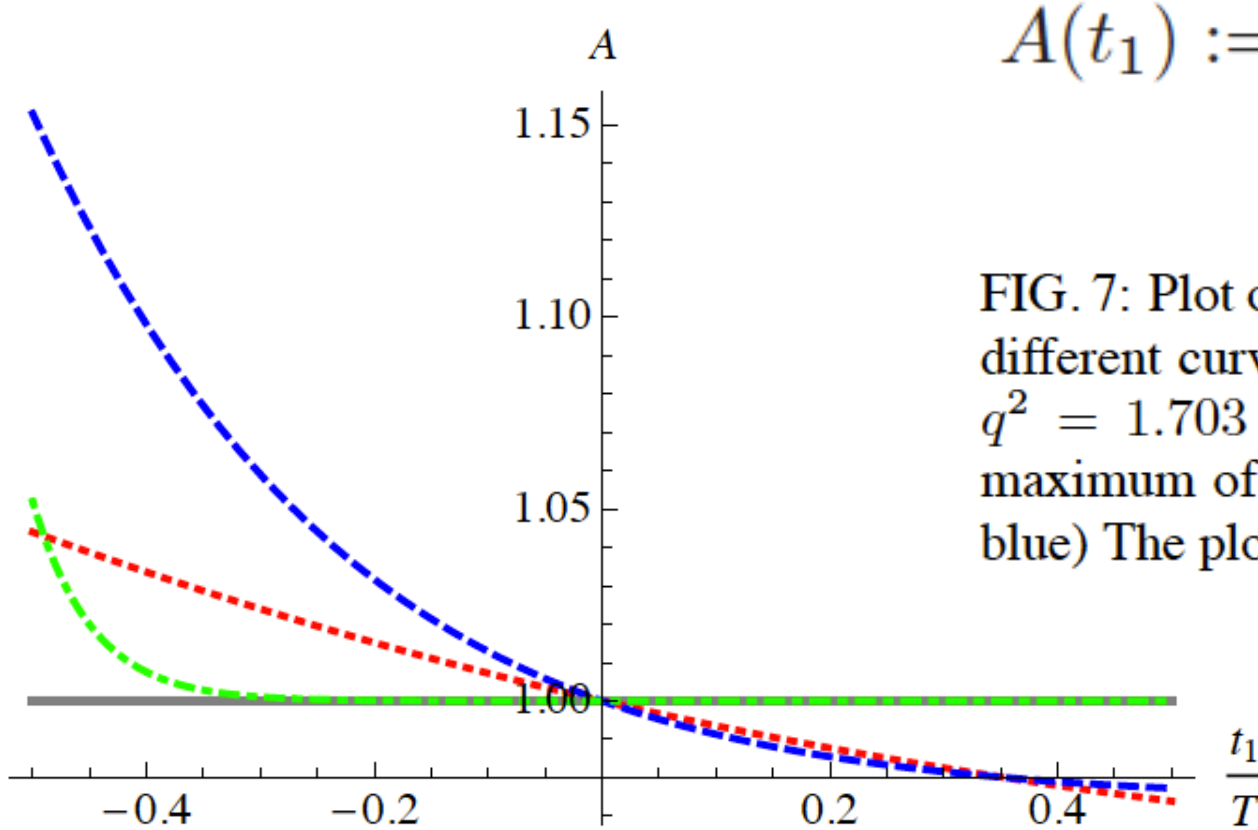

FIG. 7: Plot of the asymmetry ratio A defined in equation (291). The different curves are for $q^2 = 0$ (solid gray), $q^2 = 0.2$ (dotted red), $q^2 = 1.703$ (dashed blue), and $q^2 = 10$ (dot-dahed, green). The maximum of A at $t_1 = -T/2$ is attained for $q^2 = 1.703$ (dashed blue) The plot is for $T = 1$.

Beyond Mean-field theory

First step beyond MFT: Generalized Narayan-Fisher relations

idea : densities upon varying f(x,t) have a m=0 limit

$$\tau = 2 - \frac{2}{d + \zeta}$$

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look instead at density of avalanches per unit force $$f = m^2 w$$

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Now, assume that $\rho_f(S)$ has a finite limit as $m \to 0$ $\rho_f(S) = AS^{-\tau}$

equivalent to say that $\rho_0 \sim m^2$

$$\rho(S) = \frac{L^d}{S_m^2} p(S/S_m) \quad \Rightarrow \quad S_m^{\tau-2} \sim m^2$$

$$\boxed{\tau = 2 - \frac{2}{d+\zeta}}$$

$S_m \simeq c_S m^{-(d+\zeta)}$ $p(s) \sim s^{-\tau}$

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$S_m \simeq c_S m^{-(d+\zeta)}$ $\quad p(s) \sim s^{-\tau}$

more general:

e.g. local sizes exponent

$$\rho_f = \partial_{\dot{f}_{xt}} \overline{e^{\int_{xt} \lambda_{xt} \dot{u}_{xt}}}^{\dot{f}_{xt} = m^2 \dot{w}_{xt}} |_{f=0} = \langle \tilde{u}_{xt} \rangle_{\mathcal{S}[\tilde{u},\dot{u},\lambda]}$$

$$\tau_\phi = 2 - \frac{2}{d_\phi + \zeta}$$

has m=0 limit

$$\tau_{MF,d-1} = 4/3$$

distribution of velocity in an avalanche

Consider stationary driving

$w(t) = vt$

$$\dot{u}^{tot} = L^d \dot{u} = \int d^d x \ \dot{u}(x,t)$$

areal velocity has units

$$v_m^{tot} = S_m/\tau_m$$

- Mean-field (BFM) gives (dimless units)

$$P_v(\dot{u}^{tot}) = \frac{(\dot{u}^{tot})^{-1+L^d v}}{\Gamma(L^d v)} e^{-\dot{u}^{tot}}$$

$$\rho(\dot{u}^{tot}) = \partial_v P_v(\dot{u}^{tot})|_{v=0^+} = \frac{L^d}{\dot{u}^{tot}} e^{-\dot{u}^{tot}}$$

- regime "ABBM": $v \sim 1/L^d$ transition $v_c = 1/L^d$

$v < v_c$ interface still stops at discrete times

- fixed v large L: $\dot{u}^{tot} = L^d v + L^{d/2} \xi$

$$P(\xi) \sim e^{-\frac{\xi^2}{2}}$$

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- Beyond mean-field: $\rho(\dot{u}^{tot}) \sim (\dot{u}^{tot})^{-a}$ velocity exponent $a_{MF} = 1$

restoring units: $\rho(\dot{u}^{tot}) = \frac{L^d}{(v_m^{tot})^2} p(\dot{u}^{tot}/v_m^{tot})$

we have calculated p(u) to one-loop $a = 1 - \frac{2}{9}\epsilon$

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$$P_v(\dot{u}^{tot}) = \frac{(\dot{u}^{tot})^{-1+L^d v}}{\Gamma(L^d v)} e^{-\dot{u}^{tot}}$$

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we have calculated p(u) to one-loop $a = 1 - \frac{2}{9}\epsilon$

GNF argument:

$\rho_f(\dot{u}^{tot}) = \partial_f P_v(\dot{u}^{tot})|_{f=0^+}$

has m=0 limit

$\Rightarrow (v_m^{tot})^{a-2} \sim m^2 \Rightarrow$

$$a = 2 - \frac{2}{d+\zeta-z}$$

1-loop agrees with GNF

More: local velocity exponent

$$a_\phi = 2 - \frac{2}{d_\phi+\zeta-z}$$

Long Range elasticity: $c(q) = (q^2 + \mu^2)^{\gamma/2}$ $\quad \mu^\gamma = m^2$

$$S_\mu = \sigma_\mu / \mu^4 \sim \mu^{-(d+\zeta)} \qquad \tau_\mu = \eta_\mu / \mu^2 \sim \mu^{-z}$$

generalized NF: $S_\mu^{\tau-2} \sim m^2$ $\qquad \tau = 2 - \frac{\gamma}{d+\zeta}$

	$\mathcal{P}(S)$	$\mathcal{P}(S_\phi)$	$\mathcal{P}(T)$	$\mathcal{P}(\dot{u})$	$\mathcal{P}(\dot{u}_\phi)$
	$S^{-\tau}$	$S_\phi^{-\tau_\phi}$	$T^{-\gamma}$	$\dot{u}^{-\mathsf{a}}$	$\dot{u}_\phi^{-\mathsf{a}_\phi}$
short-ranged elasticity (SR)	$\tau = 2 - \frac{2}{d+\zeta}$	$\tau_\phi = 2 - \frac{2}{d_\phi+\zeta}$	$\gamma = 1 + \frac{d-2+\zeta}{z}$	$\mathsf{a} = 2 - \frac{2}{d+\zeta-z}$	$\mathsf{a}_\phi = 2 - \frac{2}{d_\phi+\zeta-z}$
long-ranged elasticity (LR)	$\tau = 2 - \frac{1}{d+\zeta}$	$\tau_\phi = 2 - \frac{1}{d_\phi+\zeta}$	$\gamma = 1 + \frac{d-1+\zeta}{z}$	$\mathsf{a} = 2 - \frac{1}{d+\zeta-z}$	$\mathsf{a}_\phi = 2 - \frac{1}{d_\phi+\zeta-z}$

	d	ζ	z	τ	τ_ϕ	α	a	a_ϕ	γ
SR	1	1.25	1.433	1.11	0.4	1.17	−0.45	12.9	1.57
	2	0.75	1.56	1.27	−0.67	1.48	0.32	4.47	1.76
	3	0.34	1.74	1.40	−3.88	1.77	0.75	3.43	1.92
LR	1	0.39	0.74	1.28	−0.56	1.53	0.46	4.86	1.88

2 1.6

TABLE II: Critical exponents obtained via the scaling relations. For the localized avalanche exponents we consider a point, $d_\phi = 0$.

Distribution of global velocity

with A. Kolton

$$\rho(\dot{u}^{tot}) \sim (\dot{u}^{tot})^{-\mathsf{a}}$$

$$\mathsf{a} = 2 - \frac{2}{d + \zeta - z}$$

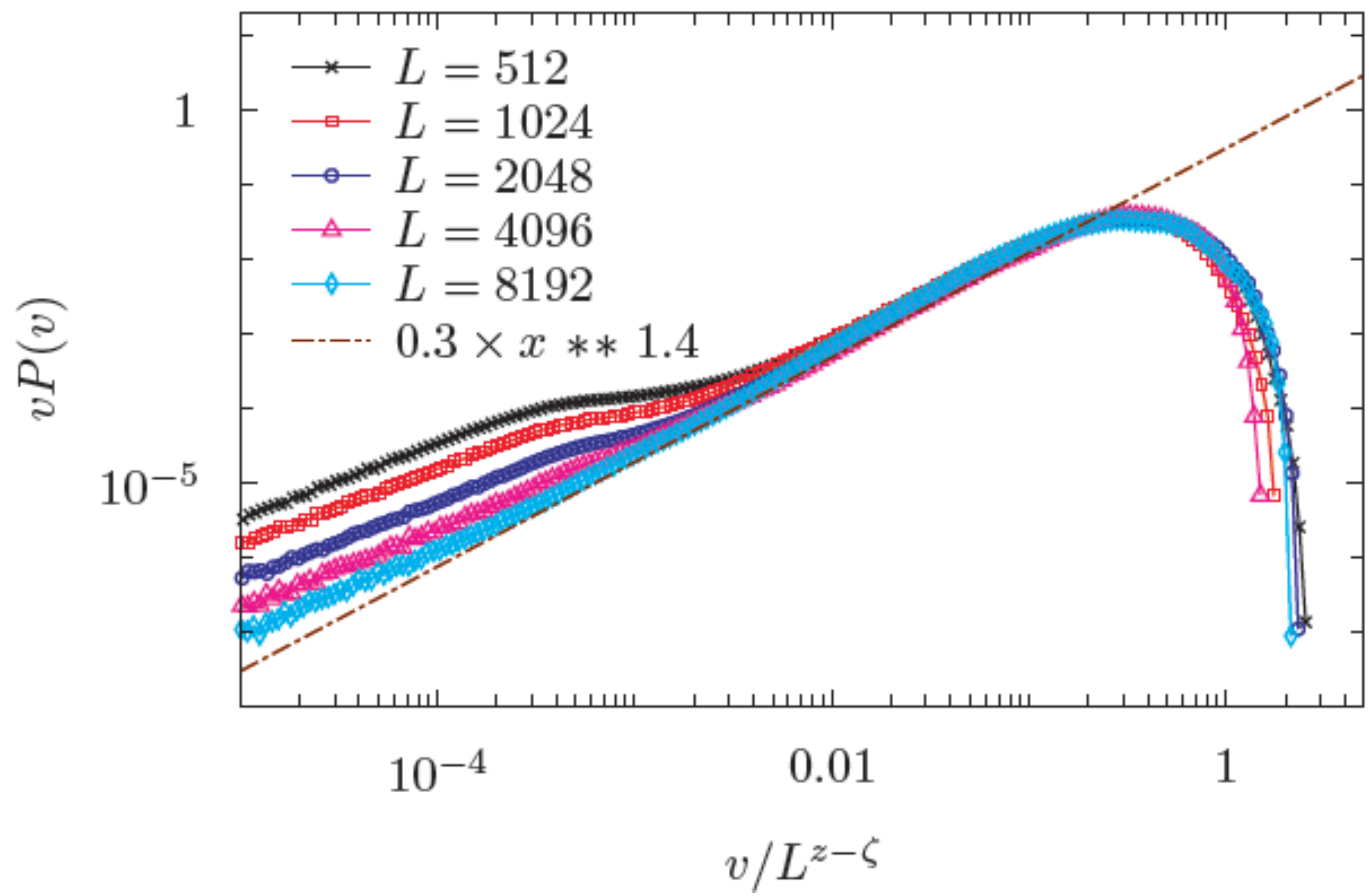
Ferrero (2013)

$\zeta = 1.250 \pm 0.005$

$z = 1.433 \pm 0.007$

$\mathsf{a}_{MF} = 1$

a= - 0.448 +/- 0.03



Shape at fixed duration

$$m \to 0$$

$$S_m \simeq c_S m^{-(d+\zeta)}$$

$$\tau_m \simeq c_\tau m^{-z}$$

$$L^d \langle \dot{u} \rangle_T = \frac{S_m}{\tau_m} s(\frac{t}{\tau_m}, \frac{T}{\tau_m})$$

- short time near beginning $t \ll \tau_m$
  independent of m,T

$$L^d \langle \dot{u} \rangle_T \sim t^{\gamma - 1} \qquad \gamma = \frac{d+\zeta}{z}$$

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- short time near beginning $t \ll \tau_m$ independent of m,T

$$L^d \langle \dot{u} \rangle_T \sim t^{\gamma - 1} \qquad \gamma = \frac{d+\zeta}{z}$$

- universal form both short times $T \ll \tau_m$

Mean-field:

$$s_{MF}(t,T) = 2Tx(1-x) \qquad x = t/T$$

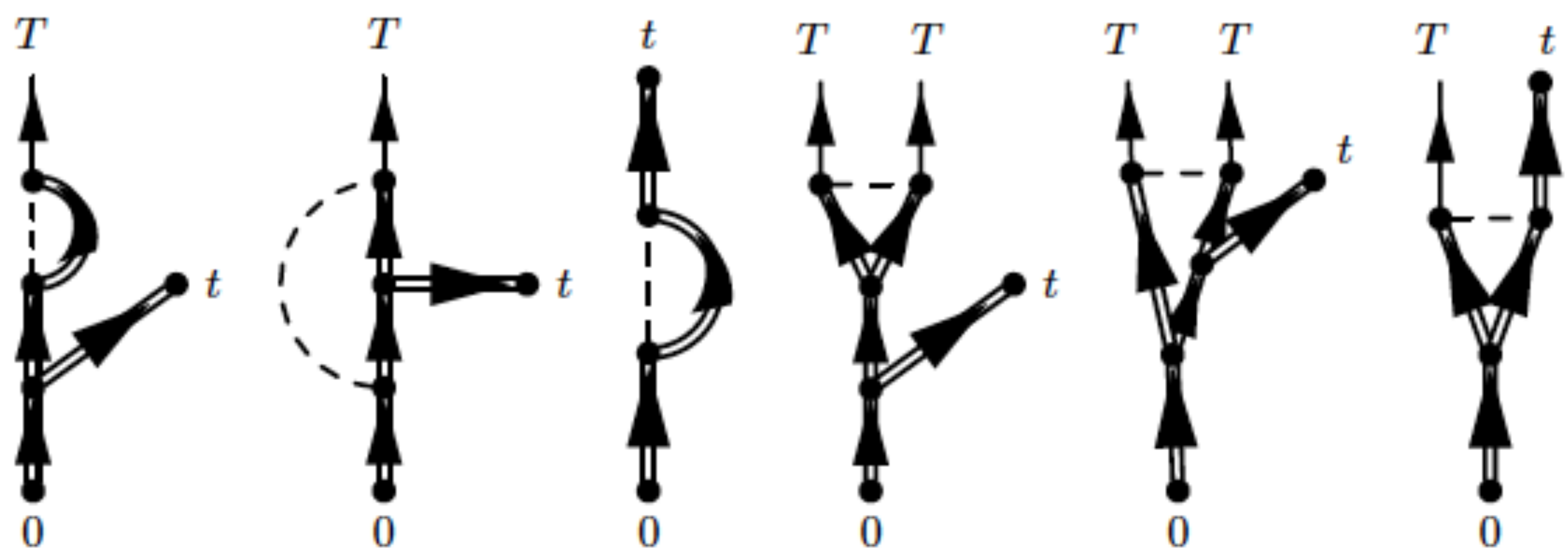

Fig. 1: Diagrammatic representation of the 1-loop corrections to the shape at fixed duration (28) (similarly for (34)). Solid lines are response functions, doubled for dressed ones, defined in [27]; they account for the non-vanishing expectation of $\tilde{u}_{xt}$ in Eq. (13). Dashed lines are g-vertices, the other vertices are σ. Internal times and the loop momentum are integrated over.

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Beyond mean-field: $$s(t,T) = 2(Tx(1-x))^{\gamma-1} f(x)$$

$$\langle S \rangle_T \simeq c T^\gamma$$

$c = 2\int_0^1 dx (x(1-x))^{\gamma-1} f(x)$

Shape at fixed duration

$$m \to 0 \qquad S_m \simeq c_S m^{-(d+\zeta)} \qquad \tau_m \simeq c_\tau m^{-z}$$

$$L^d \langle \dot{u} \rangle_T = \frac{S_m}{\tau_m} s(\frac{t}{\tau_m}, \frac{T}{\tau_m})$$

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$$c = 2\int_0^1 dx (x(1-x))^{\gamma-1} f(x)$$

$$f(x) = \mathcal{N} \exp(-\frac{16}{d_{uc}} \frac{\epsilon}{9} h(x))$$

SR: $\epsilon = 4 - d$ $\qquad d_{uc} = 4$

LR: $\epsilon = 2 - d$ $\qquad d_{uc} = 2$

$$h(x) = \mathrm{Li}_2(1-x) - \mathrm{Li}_2(\frac{1-x}{2}) + \frac{x\ln(2x)}{x-1} + \frac{(x+1)\ln(x+1)}{2(1-x)}$$

$$\mathcal{N}_{SR} = e^{-\frac{\epsilon}{9}(-1+\gamma_E - \frac{\pi^2}{3} - 2(\ln 2)^2)}$$

$$s(t,T) \approx T^{\gamma-1}(x(1-x))^{\gamma-1}\exp(\mathcal{A}(x-\frac{1}{2}))$$

$$\gamma = \frac{d+\zeta}{z}$$

$$\mathcal{A} = -0.084031\frac{4}{d_{uc}}\epsilon \qquad h'(1/2)$$

SR: $d = 1$

$\gamma = 1.57 \pm 0.01$

$\gamma_{1loop} = 2 - \frac{\epsilon}{9} \approx 1.66$

Evolution of the average avalanche shape
with the universality class

Lasse Laurson[1], Xavier Illa[2], Ste´phane Santucci[3], Ken Tore Tallakstad[4], Knut Jørgen Måløy[4] & Mikko J. Alava[1]

L. Laurson et al. Nat. Commun. 4 (2013) 2927

QEW numerics SR elasticity d=1 $\mathcal{A} = 0.08$

$$s(t,T) \approx T^{\gamma-1}(x(1-x))^{\gamma-1}\exp(\mathcal{A}(x-\frac{1}{2}))$$

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L. Laurson et al. Nat. Commun. 4 (2013) 2927

QEW numerics SR elasticity d=1 $\mathcal{A} = 0.08$

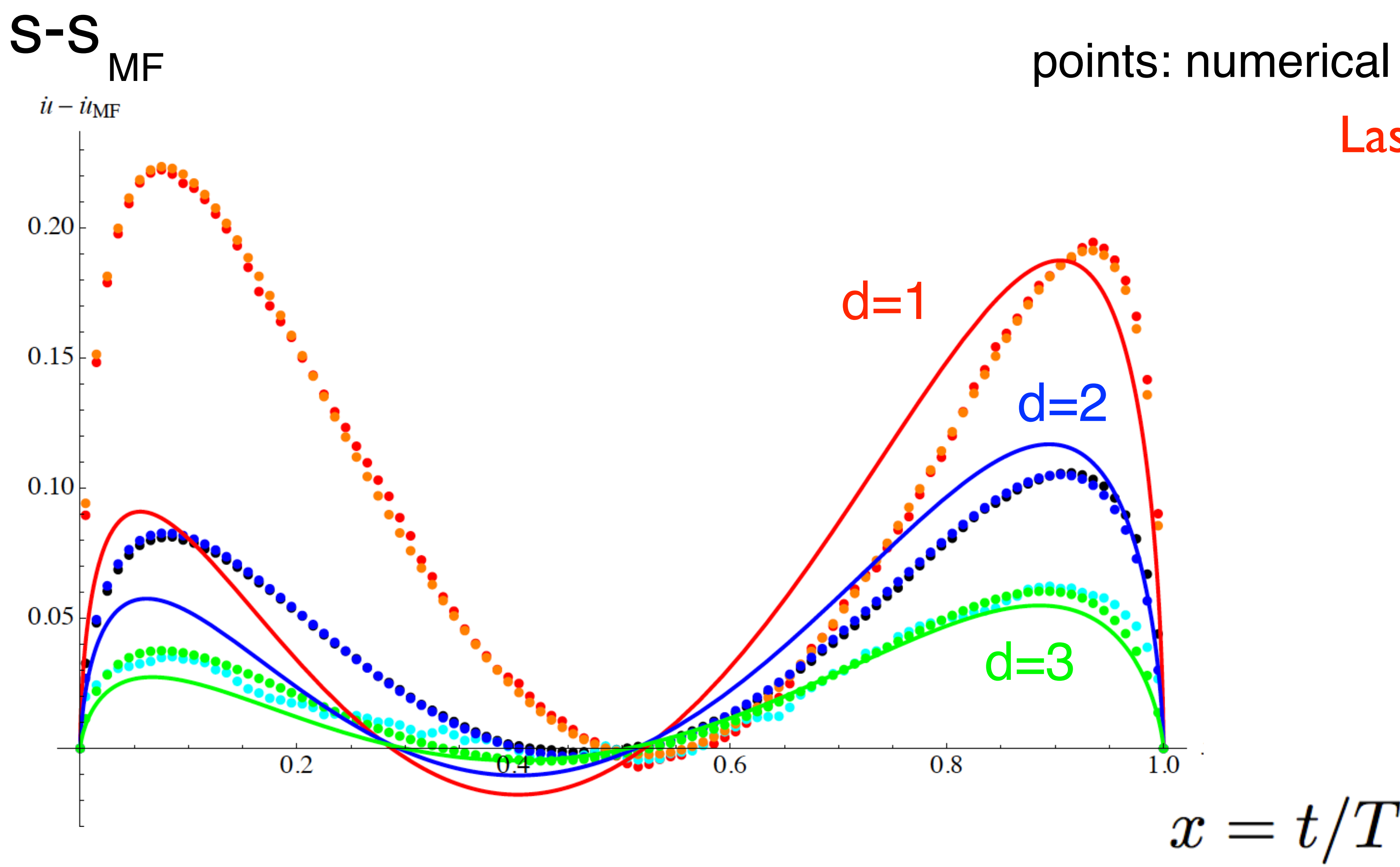
L. Laurson, private comm. d=2 $\mathcal{A} = -0.065$

Normalized shape minus normalized MF shape

solid lines: theory

points: numerical simulation

Lasse Laurson



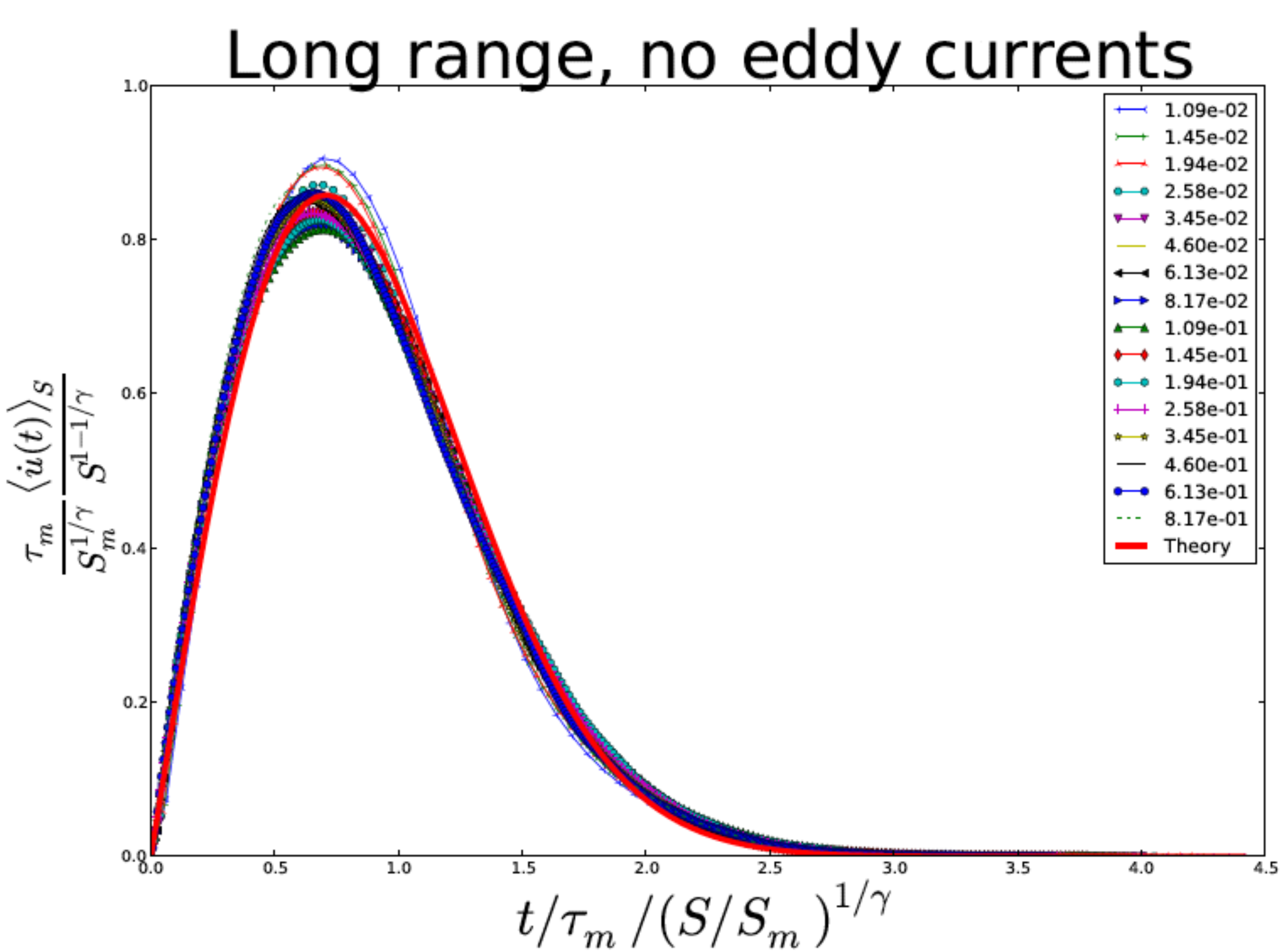
Shape at fixed size

$$L^d \langle \dot{u}(t) \rangle_S = \frac{S_m}{\tau_m} s(t/\tau_m, S/S_m)$$

$$\langle \dot{\mathrm{u}}(t) \rangle_S = \frac{S}{\tau_m} \left(\frac{S}{S_m} \right)^{-\frac{1}{\gamma}} f\left(\frac{t}{\tau_m} \left(\frac{S_m}{S} \right)^{\frac{1}{\gamma}} \right)$$

$$\int_0^\infty \mathrm{d}t\, f(t) \; = \; 1$$

Mean-field: $f_0(t) = 2te^{-t^2}$, $\gamma = 2$ independent of m



Gianfranco Durin

Barkhausen noise

$1\mu m$ polycrystals

$$S_m := \frac{\langle S^2 \rangle}{2 \langle S \rangle}$$

$\tau_m = 3.8 \times 10^{-5}$ s.

only parameter

m = k0 demag. field

Shape at fixed size

$$\langle \dot{u}(t) \rangle_S = \frac{S}{\tau_m} \left(\frac{S}{S_m} \right)^{-\frac{1}{\gamma}} f\left(\frac{t}{\tau_m} \left(\frac{S_m}{S} \right)^{\frac{1}{\gamma}} \right) \qquad \int_0^\infty \mathrm{d}t\, f(t) \;=\; 1$$

Mean-field: $$f_0(t) = 2te^{-t^2} \quad , \quad \gamma = 2$$

Beyond mean-field:

$$f(t) = f_0(t) - \frac{\varepsilon}{9}\delta f(t) \quad , \quad \gamma = 2 - \frac{\varepsilon}{9} \qquad \int_0^\infty \mathrm{d}t\, \delta f(t) = 0$$

$$\begin{aligned} \delta f(t) = \frac{f_0(t)}{4} \Big[& \pi \left(2t^2+1\right) \mathrm{erfi}(t) + 2\gamma_{\mathrm{E}} \left(1-t^2\right) - 4 \\ & - 2t^2 \left(2t^2+1\right) \, {}_2F_2\left(1,1;\frac{3}{2},2;t^2\right) \\ & - 2e^{t^2} \left(\sqrt{\pi} t \, \mathrm{erfc}(t) - \mathrm{Ei}\left(-t^2\right) \right) \Big] . \end{aligned}$$

$$f(t) \simeq_{t\to\infty} 2A' t^{\beta} e^{-Ct^{\delta}}$$

$$\delta = 2 + \frac{\varepsilon}{9}$$

$$\beta = 1 - \frac{\varepsilon}{18}$$

$$f(t) \simeq_{t\to 0} 2At^{\gamma-1}$$

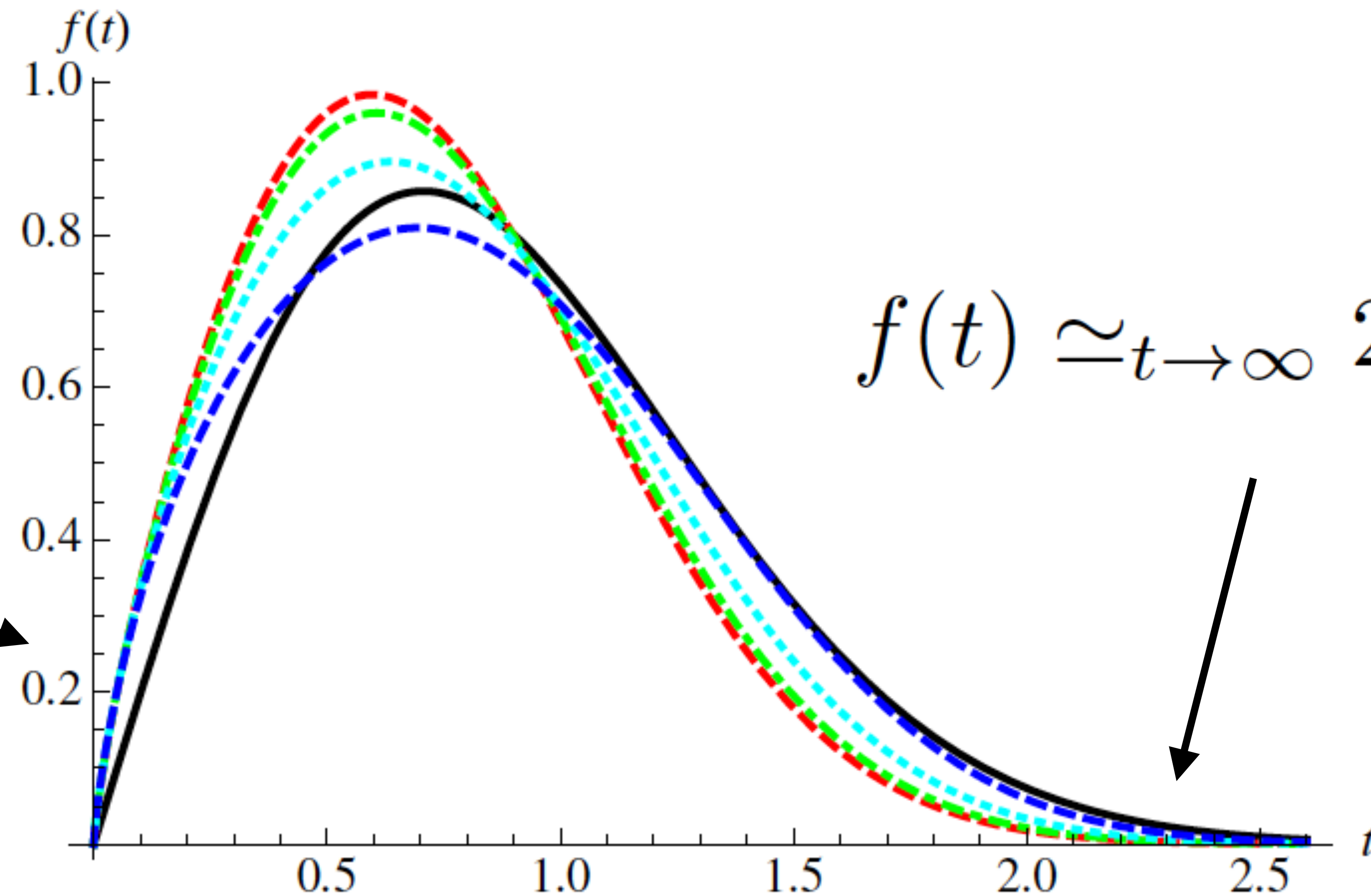


Fig. 3: The shape at fixed size, as given by Eq. (37). Mean field (black solid line). The remaining curves are for $\varepsilon = 2$: small $S/S_m = 0^+$ limit (red dashed) and $S/S_m = 1$, 10, 30 (green dot-dashed, cyan dotted, and blue dashed).

$$A = 1 + \tfrac{\varepsilon}{9}(1 - \gamma_{\mathrm{E}})$$

$$A' = 1 + \tfrac{\varepsilon}{36}(5 - 3\gamma_{\mathrm{E}} - \ln 4)$$

$$C = 1 + \tfrac{\varepsilon}{9}\ln 2$$

ABBM model + relaxation:

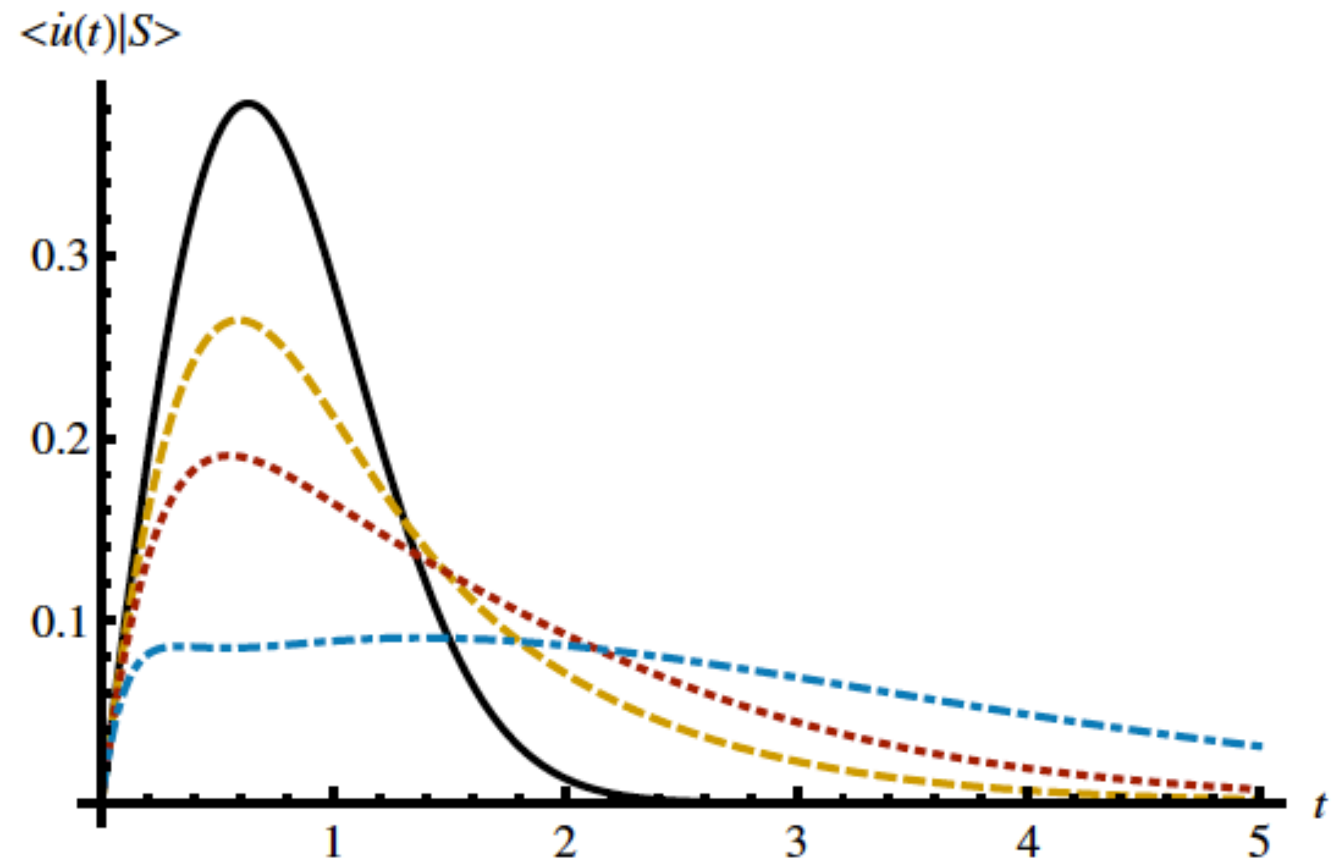
$$\eta \dot{u}(t) + a \int_{-\infty}^{t} \mathrm{d}s\, f(t-s)\dot{u}(s) = F\big(u(t)\big) + m^2\big[w(t) - u(t)\big].$$

Middleton OK => avalanches sizes unchanged,
splitted in subavalanches (aftershocks)

exactly solvable
by the "instanton" equation

analytical calculation of
the shape at fixed avalanche size
for exponential kernel

asymmetry in shapes (Zapperi
et al.) in Barhausen noise from
eddy currents



From sandpiles to interfaces

Sandpile models

integer height z(x,t)   x in square lattice

1) BTW model   x topples if $z(x,t) > z_c$ $z(x,t+1) = z(x,t) - 2d$

$z_c = 2d - 1$ $z(y,t+1) = z(y,t) + 1$

y n.neighbors

Sandpile models

integer height z(x,t)

1) BTW model x topples if $z(x,t) > z_c$

$z_c = 2d - 1$

$$z(x,t+1) = z(x,t) - 2d$$

$$z(y,t+1) = z(y,t) + 1$$

y n.neighbors

? periodic depinning, CDW

Narayan Middleton (1994)

Fedorenko,PLD,Wiese (2008)

Sandpile models

integer height z(x,t)

1) BTW model

x topples if $z(x,t) > z_c$

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2) Stochastic sandpiles

- rice pile model $z_c(x)$ random chosen at each toppling

  same rule

- Manna model $z_c = 1$ two grains given at randomly chosen neighbors y

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same rule chosen at each toppling

- Manna model $z_c = 1$ two grains given

at randomly chosen neighbors y

Manna class

? (random field) depinning, interface

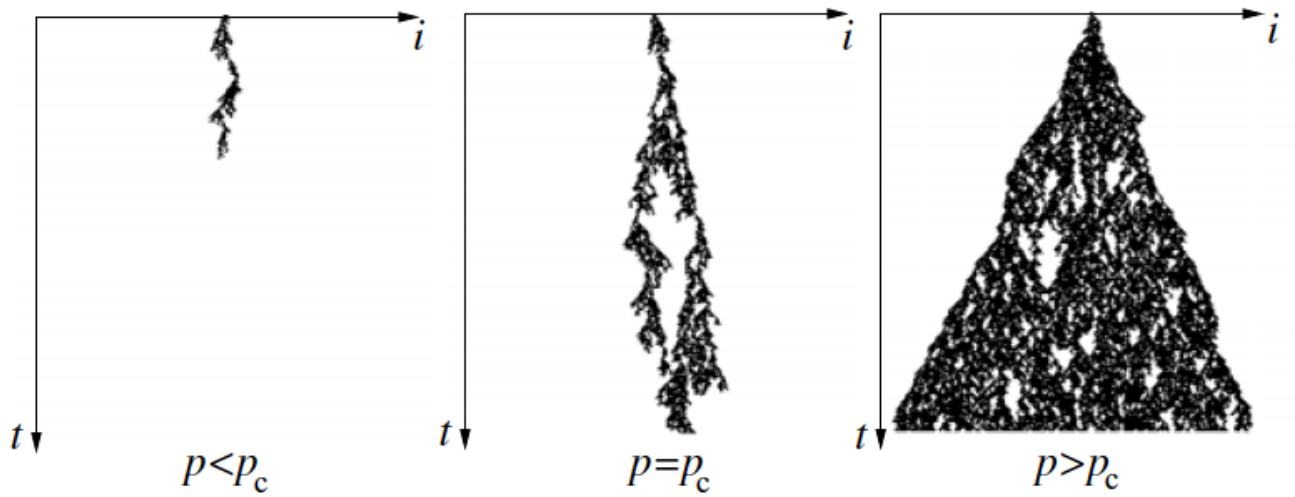
Paczuski, Boettcher, (1996)

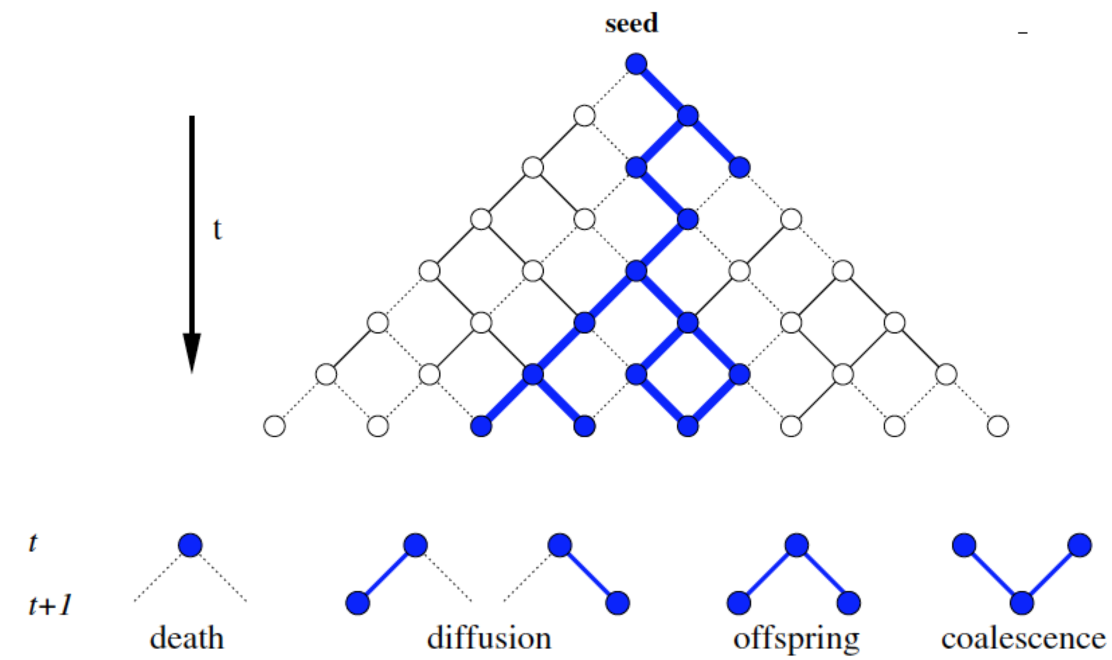
Alava Lauritsen (2001) …

toppling <=> activity

common framework = absorbing phase transitions

1) directed percolation (DP)





$\rho = 0$

inactive phase

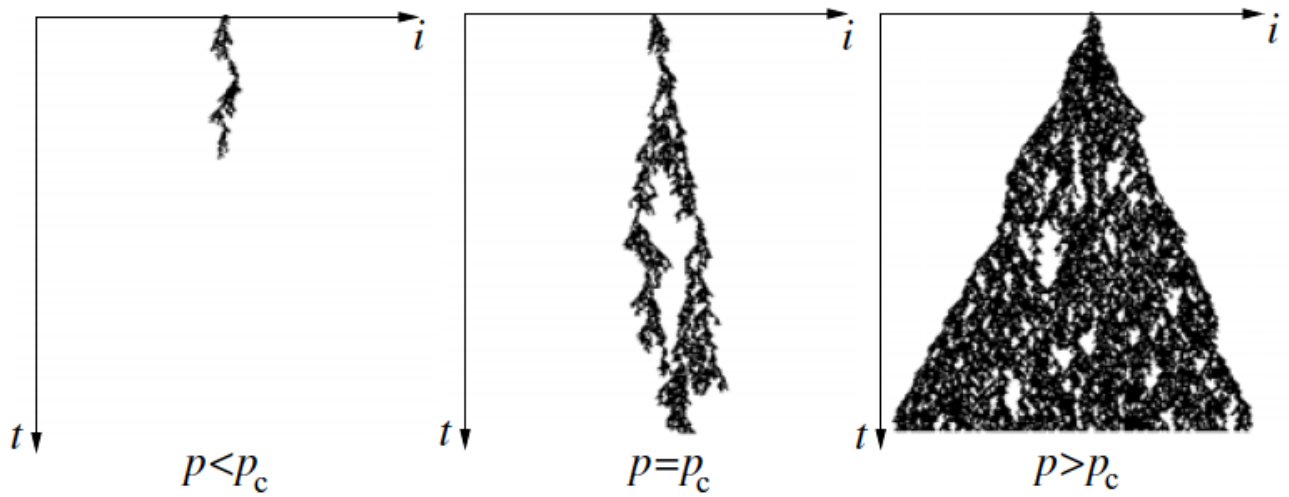
$\varrho \sim (p - p_c)^{\beta}$

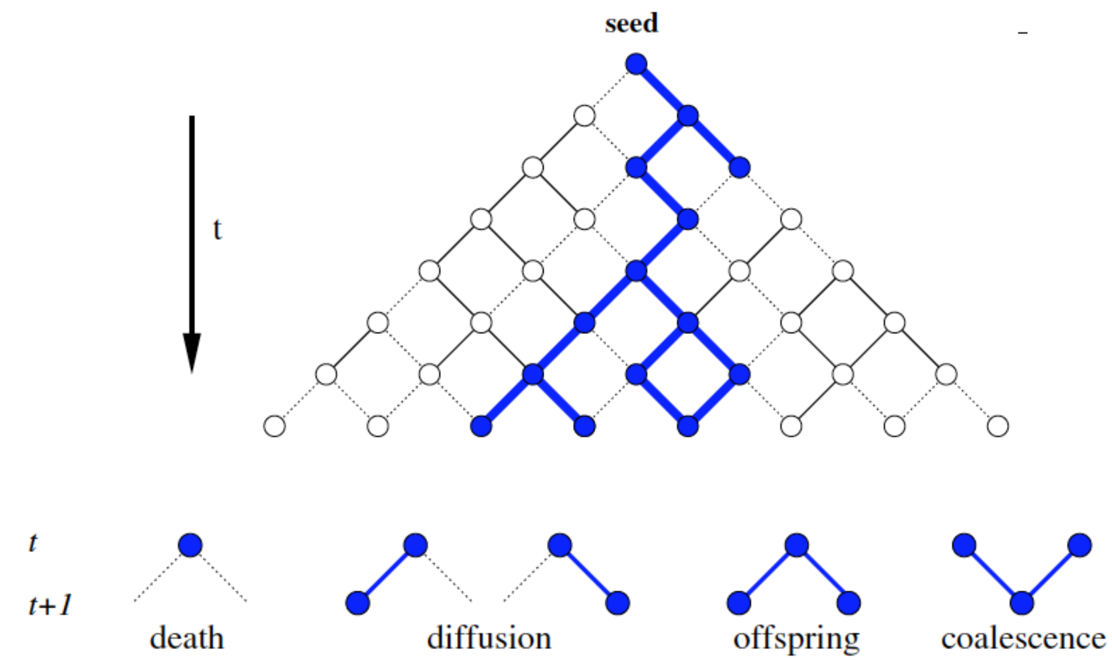
active phase

$\xi_{\perp} \sim (p - p_c)^{-\nu_{\perp}}$

$\xi_{||} \sim (p - p_c)^{-\nu_{||}}$

1) directed percolation (DP)





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upper-critical dim. dc=4

$$\partial_t \rho(x,t) = a\rho(x,t) - b\rho(x,t)^2 + \nabla^2 \rho(x,t) \quad + \eta(x,t)\sqrt{\rho(x,t)}$$

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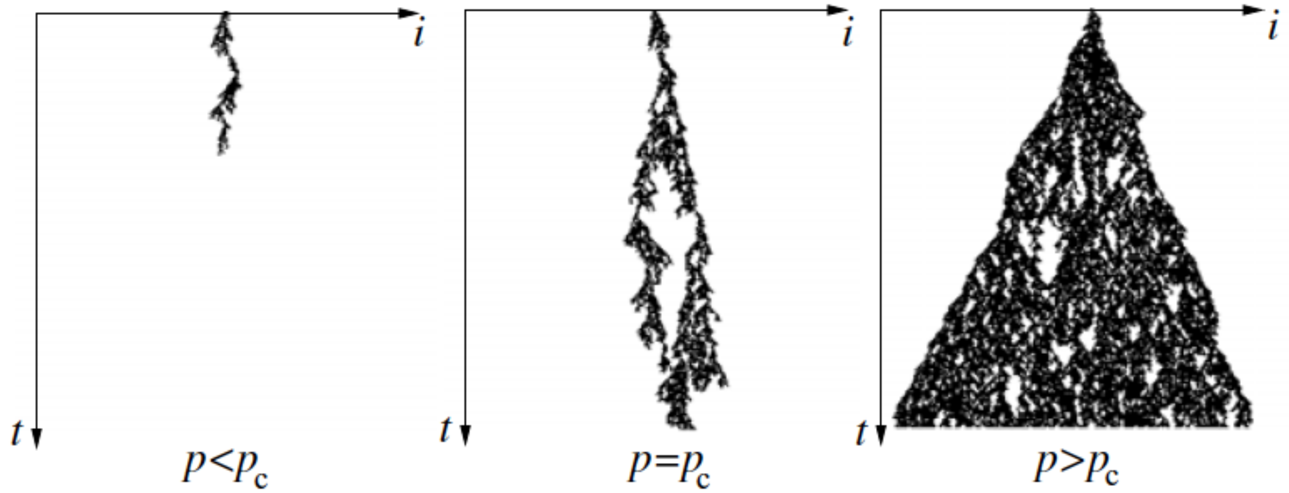

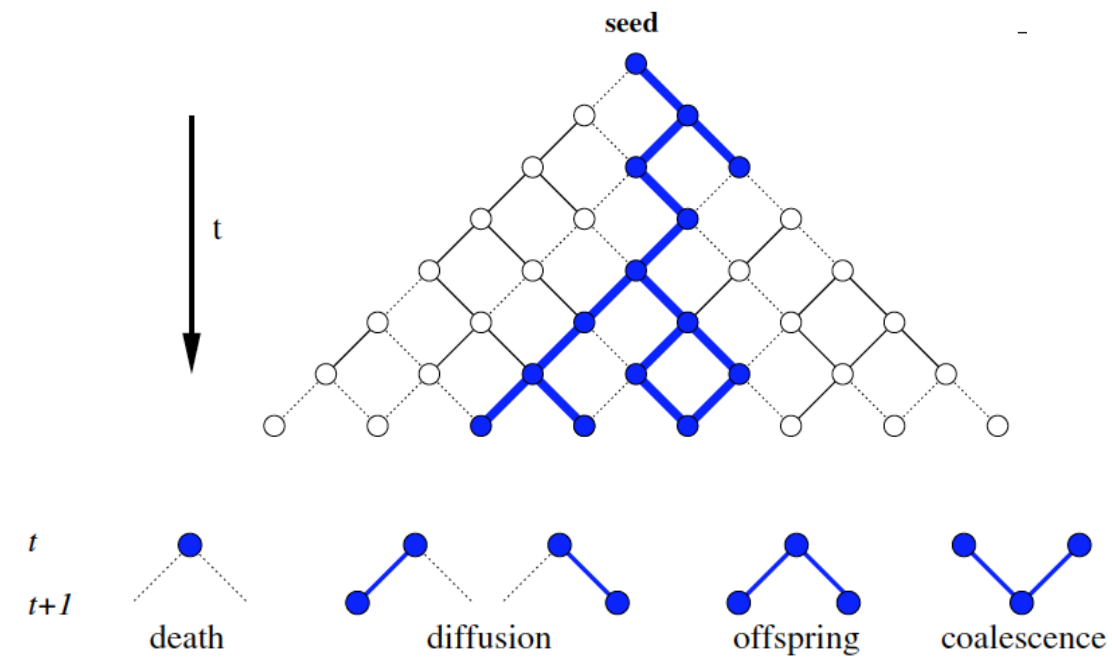

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$$\partial_t \phi(x,t) = (\nabla^2 - m^2)\rho(x,t)$$

infinite number of absorbing states

conservation of number of grains

“conserved” directed percolation (C-DP)

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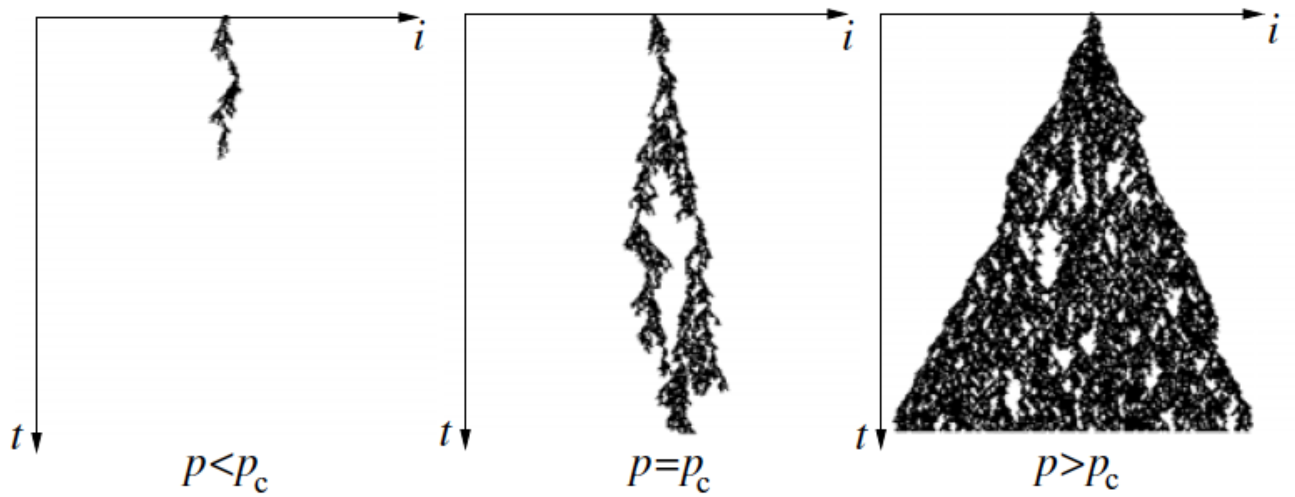

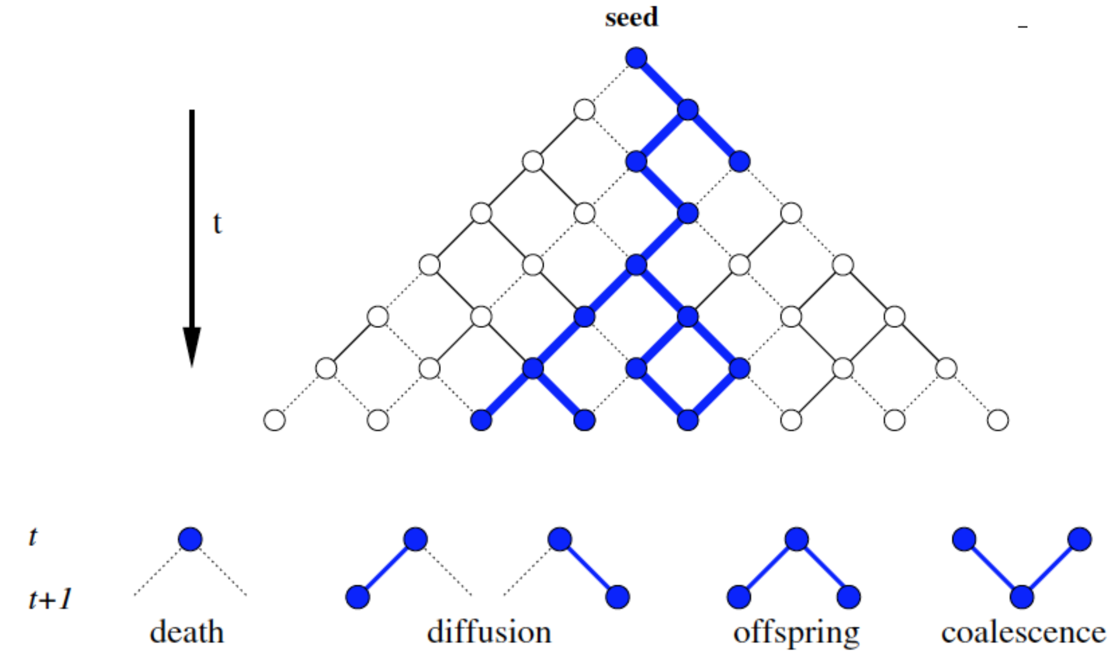

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infinite number of absorbing states

“conserved” directed percolation (C-DP)

conservation of number of grains

conjecture: Manna class <=> C-DP

Vespignani, Dickmann, Munoz, Zapperi (1998)

C-DP effective field theory for Manna sandpiles

Bonachela, Alava, Munoz (2009)

C-DP maps exactly to interface depinning !

PLD, K. Wiese arXiv 1410.1930

$$
\begin{aligned}
\partial_t \rho(x,t) &= a\rho(x,t) - b\rho(x,t)^2 + \nabla^2 \rho(x,t) \\
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$$\partial_t \phi(x,t) = (\nabla^2 - m^2)\rho(x,t) \ .$$

define force and velocity fields

$$\mathcal{F}(x,t) := \rho(x,t) - \phi(x,t) - \frac{a+m^2}{\gamma}$$

$$\rho(x,t) := \dot{u}(x,t) \ .$$

interface height
is total number of topplings

$$u(x,t) - u(x, t = 0) = \int_0^t \mathrm{d}t' \rho(x,t)$$

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quenched random force landscape is
Orstein-Uhlenbeck process

$$\partial_u F(u,x) = -\gamma F(u,x) + \tilde{\eta}(x,u)$$

$$\overline{F(u,x)F(u',x')} \to_{\gamma u, \gamma u' \gg 1} \delta^d(x-x')\Delta_0(u-u')$$

$$\Delta_0(u) = \frac{\mathrm{e}^{-\gamma|u|}}{2\gamma}$$

C-DP maps exactly to interface depinning !

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=> a single BIG universality class for:

- interface depinning (quenched edwards wilkinson)
- stochastic sandpile models
- conserved directed percolation (reaction-diffusions etc..)

=> same (avalanche, transition..) exponents !

Conclusion

- precise theory avalanches for elastic interfaces /with Middleton theorem

- correct MFT is BFM, reproduces ABBM for center of mass, L,w dependence

- exact solution BFM using instanton equation: many observables for arbitrary non-stationary driving w(t), finite q assymetry etc..

in progress: many more observables, spatial shape, P(l), other joint distributions, LR elasticity etc.. M. Delorme T. Thiery

- beyond MFT: scaling relations avalanche exponents

1loop calculation of scaling functions of many observables

avalanche shape at fixed T, at fixed S U-shape, joint S,T, second shape etc..

in progress: avalanche correlations T. Thiery

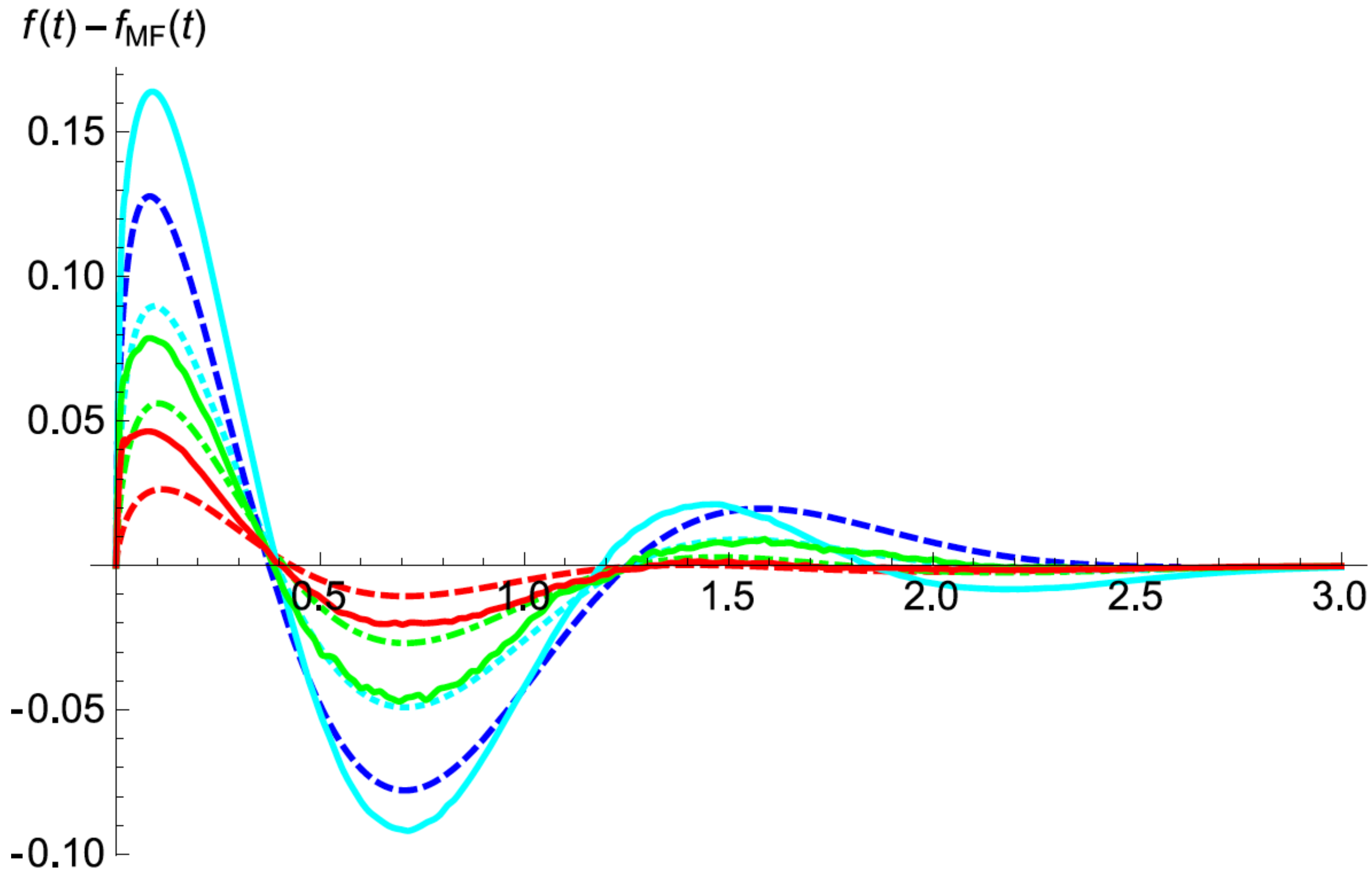
- open questions: finite v, non-monotonous, away from Middleton, etc..

....

- FRG for stochastic sandpiles, reaction-diffusion models, epidemics ..

- generalizations, stochastic PDE for quenched KPZ, glasses, yielding?

- BTW sandpiles <=> periodic depinning ??



f(t) − f_MF(t)
0.15
0.10
0.05
-0.05
-0.10
0.5
1.0
1.5
2.0
2.5
3.0