





Cracking the crack: What do the scaling properties of fracture surfaces tell us about material failure?

Laurent Ponson

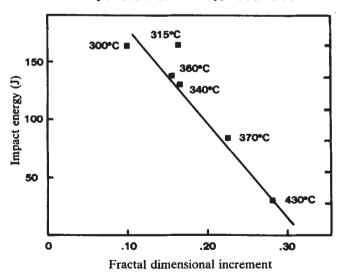
Institut Jean le Rond d'Alembert CNRS – Université Pierre et Marie Curie Paris, France



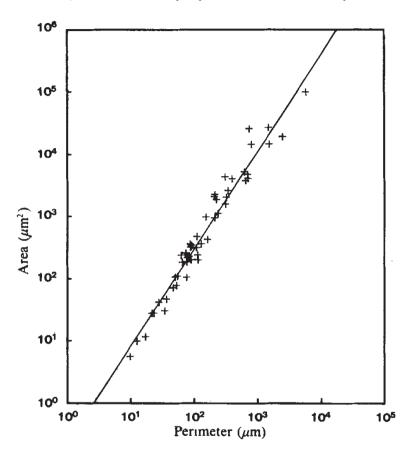
Fractal character of fracture surfaces of metals

Benoit B. Mandelbrot*, Dann E. Passoja† & Alvin J. Paullay‡

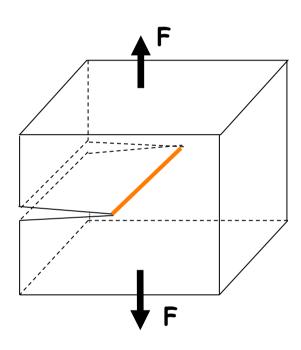
Fracture energy vs fractal dimension



Measurement of the fractal dimension of fracture surfaces



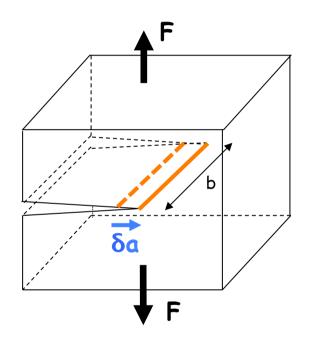
Predicting the stability of cracks in an idealized elastic homogeneous solid



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A.A. Griffith 1920 J.R. Rice 1968

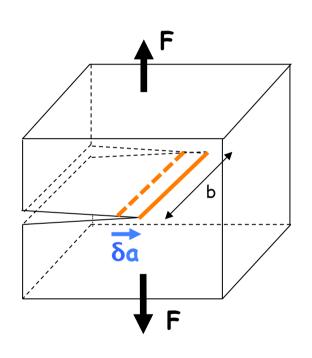
Energy balance:



 δW_F = δE_{el} + δE_{s} Work of the external force elastic energy surface energy

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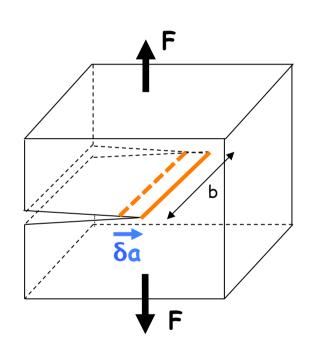
$$\delta W_F$$
 = δE_{el} + δE_s
Work of the external force elastic energy Variation of surface energy

Griffith's criterion:

Mechanical energy release rate	VS	Fracture energy
$\delta = \delta(W_{E} - \delta E_{el})/(\delta a_{el})$		$G_a = \delta E_a/(\delta a.b)$

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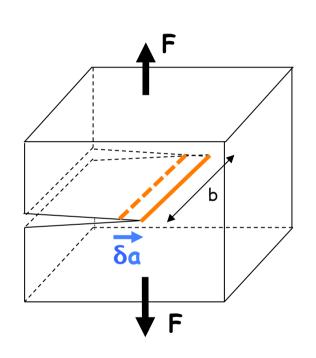
Griffith's criterion:

Mechanical energy release rate

$$G = \delta(W_F - \delta E_{el})/(\delta a.b)$$
 $G < G_c \longrightarrow \delta E_s/(\delta a.b)$
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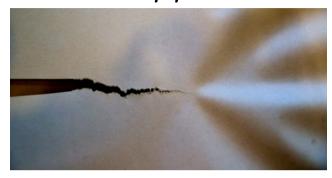
Mechanical energy release rate	VS	Fracture energy
$G = \delta(W_F - \delta E_{el})/(\delta a.b)$		$G_{c} = \delta E_{s}/(\delta a.b)$
G < G _c —	→ 5	Stable crack
G = G . —	→ P	Propagating crack

\longrightarrow But no hint on the actual value of fracture energy G_c

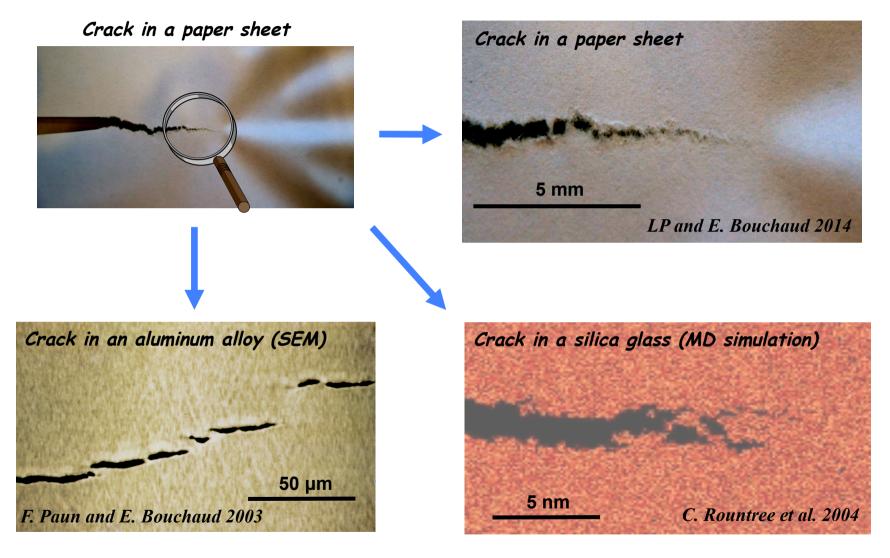
	Silica glass	Paper	Aluminum
Fracture energy	7 J/m ²	100 J/m ²	10 kJ/m ²

Let's have a closer look at the tip of cracks

Crack in a paper sheet

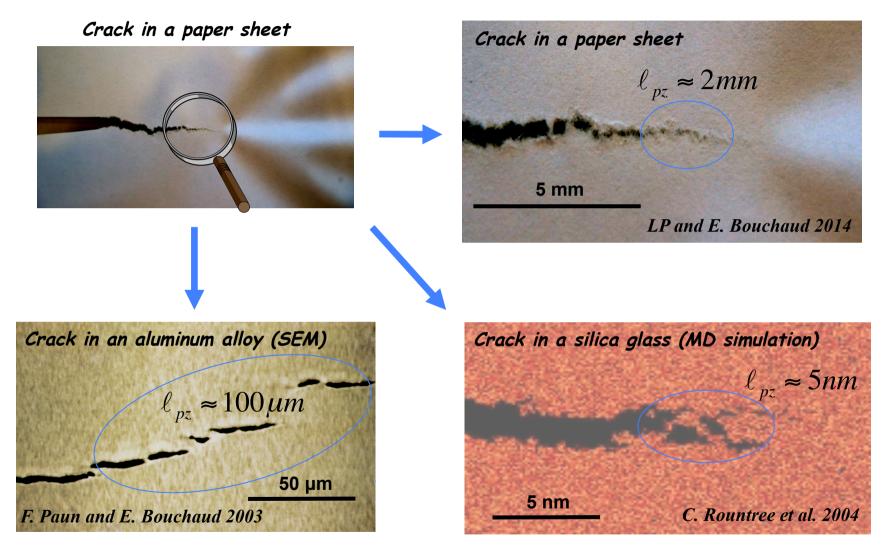


Let's have a closer look at the tip of cracks



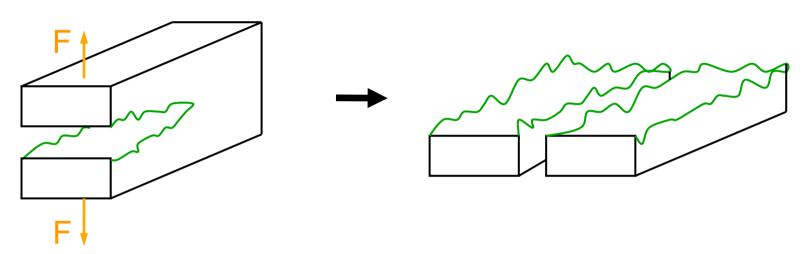
Crack propagation as a damage coalescence process taking place within some fracture process zone at the crack tip

Let's have a closer look at the tip of cracks

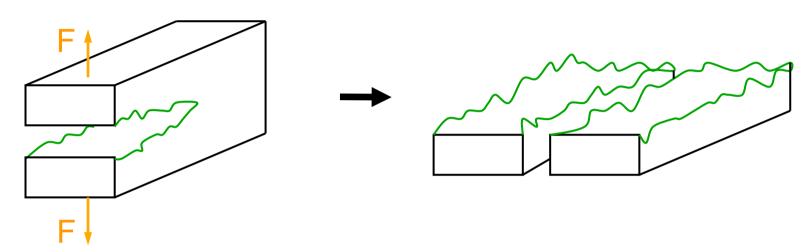


Crack propagation as a damage coalescence process taking place within some fracture process zone at the crack tip

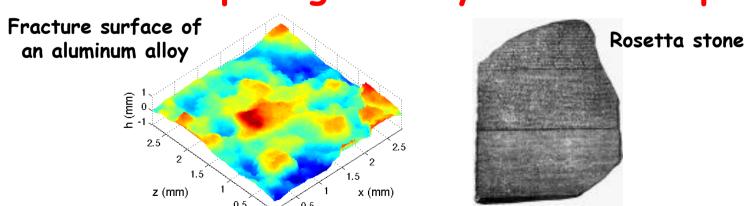
Statistical properties of crack roughness as a probe of the microscopic failure processes...



Statistical properties of crack roughness as a probe of the microscopic failure processes...



...if their complex geometry can be deciphered



Goal: Providing a statistical description of the roughness of cracks

Using it for (i) exploring the dissipative failure mechanisms

(ii) tracing back the history of the failure of a material

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Outline:

1- Roughness exponents: A signature of the failure mechanisms?

Persitent vs anti-persitent crack paths

Goal: Providing a statistical description of the roughness of cracks

Using it for (i) exploring the dissipative failure mechanisms (ii) tracing back the history of the failure of a material

Outline:

- 1- Roughness exponents: A signature of the failure mechanisms?
 - Persitent vs anti-persitent crack paths
- 2- Beyond the roughness exponent: Full statistics and fat tails in the height fluctuations of fracture surfaces
 - Gaussian vs non-Gaussian statistics of roughness

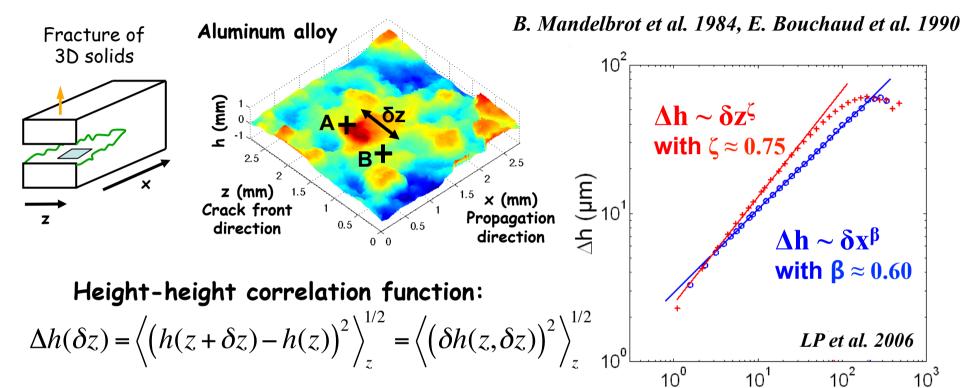
Goal: Providing a statistical description of the roughness of cracks

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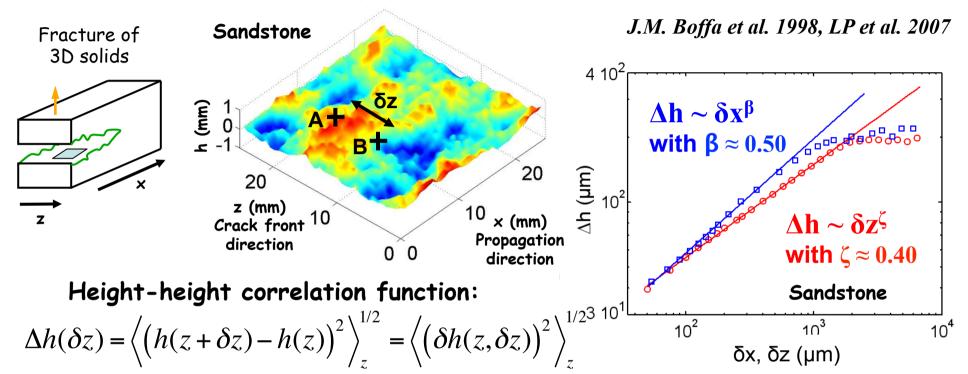
- 1- Roughness exponents: A signature of the failure mechanisms?
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- 3- Application: Measuring material toughness from the post-mortem analysis of fracture surfaces

The roughness exponent as a signature of the failure mechanisms

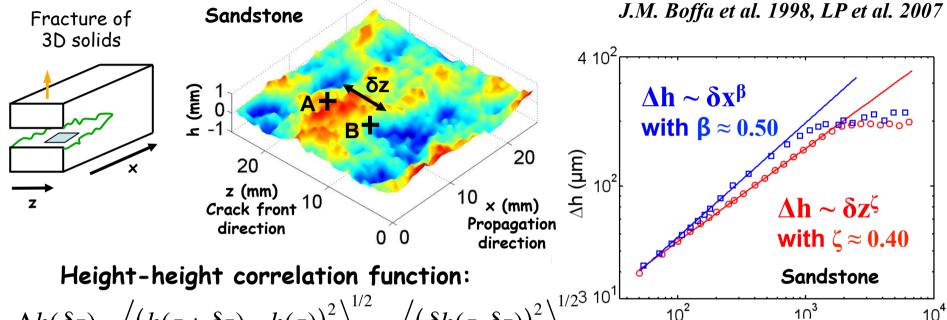


 δx , δz (µm)

The roughness exponent as a signature of the failure mechanisms



The roughness exponent as a signature of the failure mechanisms



$$\Delta h(\delta z) = \left\langle \left(h(z + \delta z) - h(z) \right)^2 \right\rangle_z^{1/2} = \left\langle \left(\delta h(z, \delta z) \right)^2 \right\rangle_z^{1/23 \cdot 10^{1/2}}$$

Two distinct classes of roughness

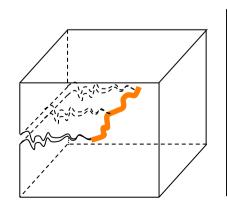
D. Bonamy et al. 2006, LP 2007

 δx , δz (µm)

Brittle failure

$$\zeta \approx 0.40$$
 $\beta \approx 0.50$

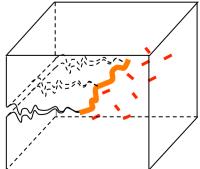
Ceramics, sandstone...



Failure by damage coalescence

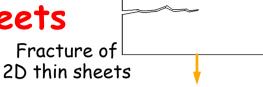
$$\zeta \approx 0.75$$
 $\beta \approx 0.60$

Metallic alloys, mortar, granite...



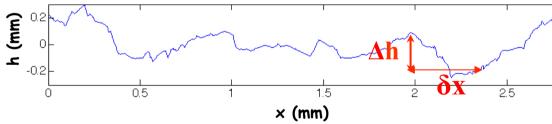
Interpretation of the value of the exponents: Crack paths in 2D thin sheets

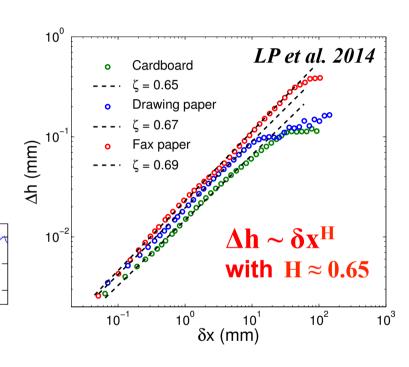
J. Kertész et al. 1993, T. Engøy et al. 1994, S. Santucci et al. 2007







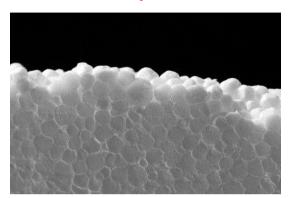




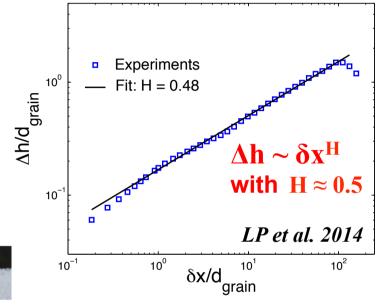
Interpretation of the value of the exponents: Crack paths in 2D thin sheets

Key assumption:

$$\ell_{\it pz} << d_{\it grain}$$



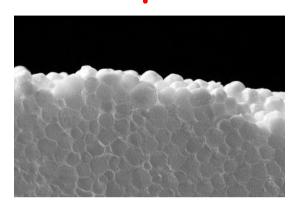
Fracture profile of a sheet of expanded polystyrene



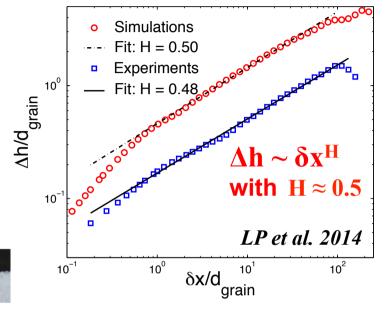
Interpretation of the value of the exponents: Crack paths in 2D thin sheets

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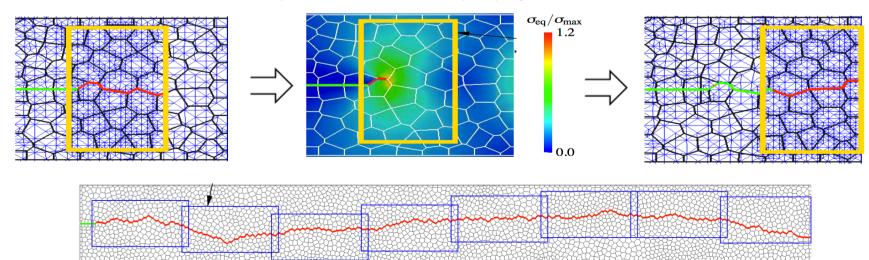
$$\ell_{pz} \ll d_{grain}$$



Fracture profile of a sheet of expanded polystyrene

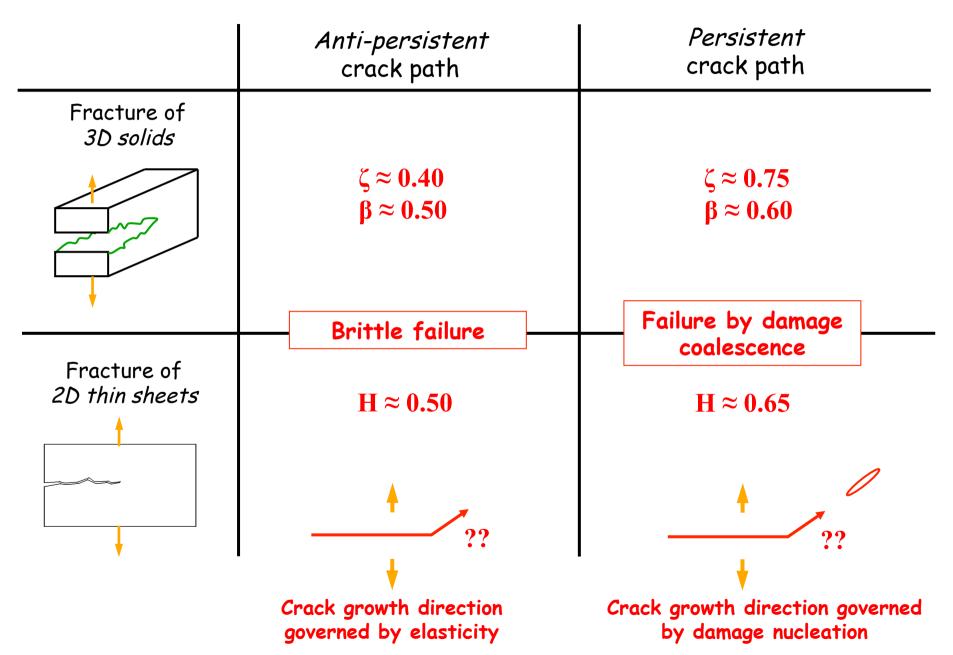


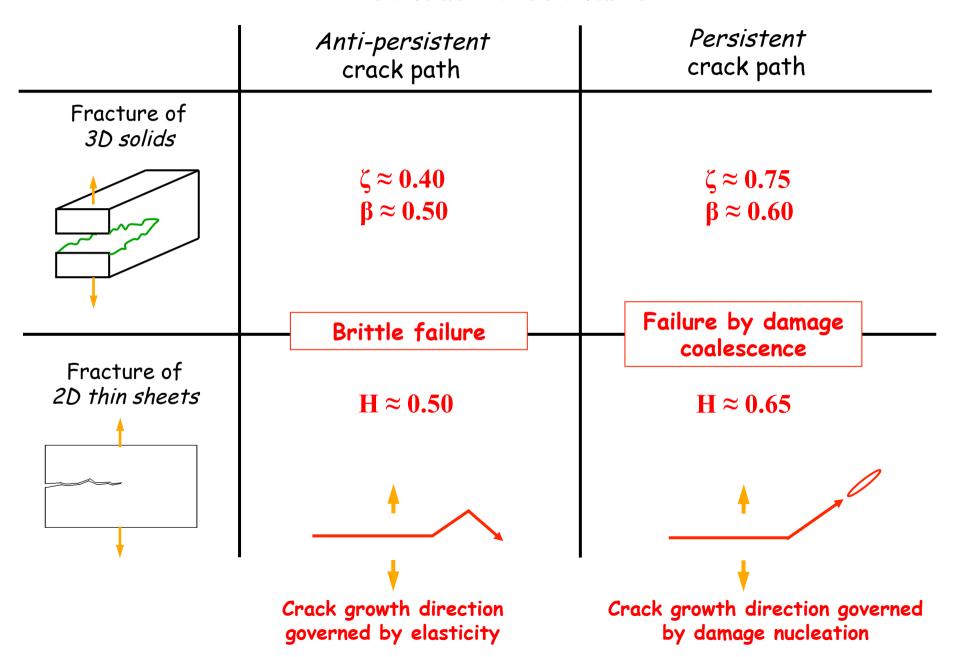
—— Simulation of the process of crack propagation via cohesive zone model

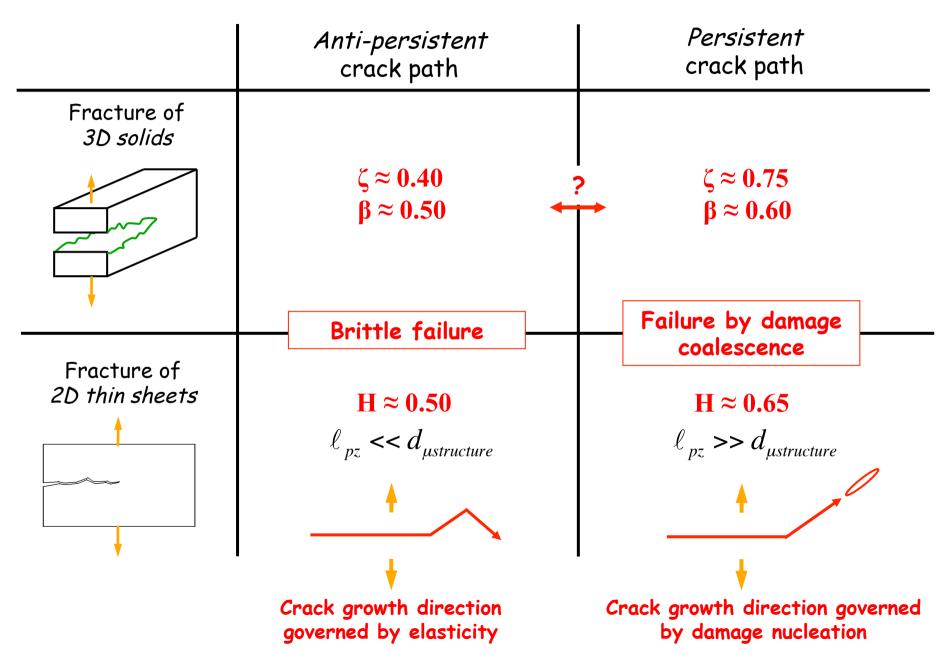


	<i>Anti-persistent</i> crack path	Persistent crack path
Fracture of 3D solids	$\zeta \approx 0.40$ $\beta \approx 0.50$	$\zeta \approx 0.75$ $\beta \approx 0.60$
Fracture of 2D thin sheets	H ≈ 0.50	H ≈ 0.65

	<i>Anti-persistent</i> crack path	<i>Persistent</i> crack path
Fracture of 3D solids		
	$\zeta \approx 0.40$ $\beta \approx 0.50$	$\zeta \approx 0.75$ $\beta \approx 0.60$
	Brittle failure	Failure by damage coalescence
	$\mathbf{H} \approx 0.50$	$H \approx 0.65$
	2D and 3D: Exponentscaptured by fracture mechanics based models	2D: Exponent capturedby damage coalescencebased models
	D. Bonamy et al. 2006, E. Katzav et al. 2007, L. Konate et al. 2014	M. Alava, S. Zapperi et al. 2006, E. Bouchbinder et al. 2007

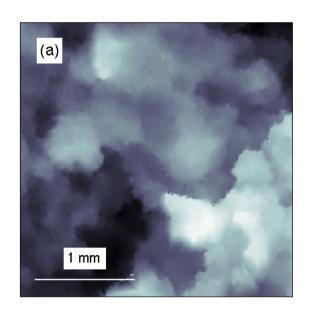


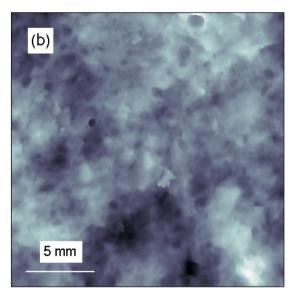


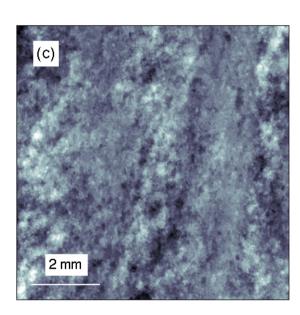


Beyond the value of the roughness exponent: Full statistics of fracture surfaces

The materials
One representative sample of each major class of failure mechanisms



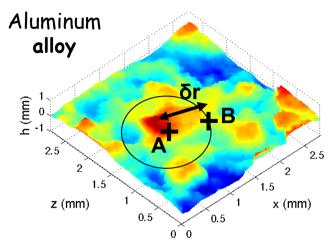




(a) Ductile: aluminum alloy (4% copper)

(b) Quasi-brittle: Mortar (c) Brittle: Glass ceramics

Statistics of height fluctuations

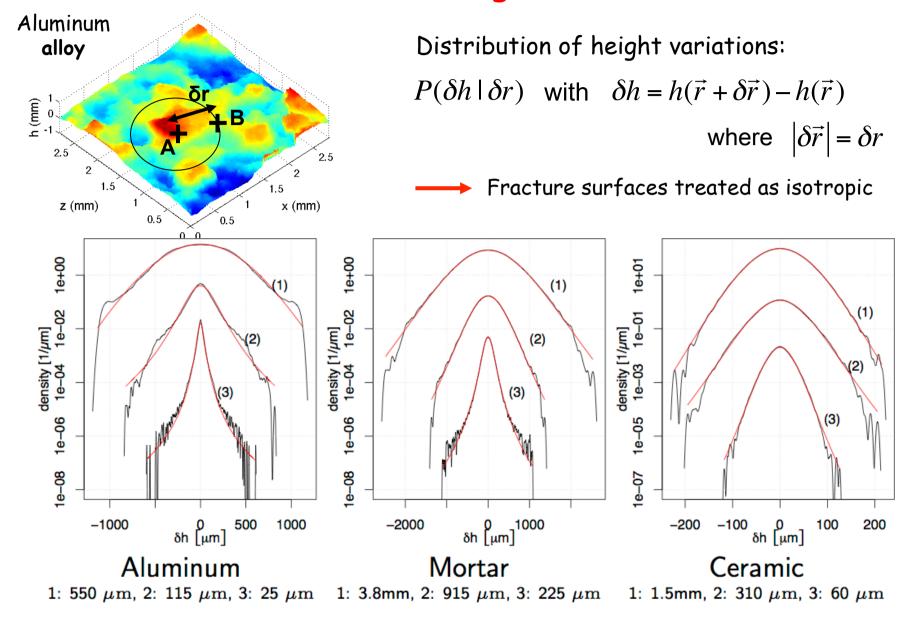


Distribution of height variations:

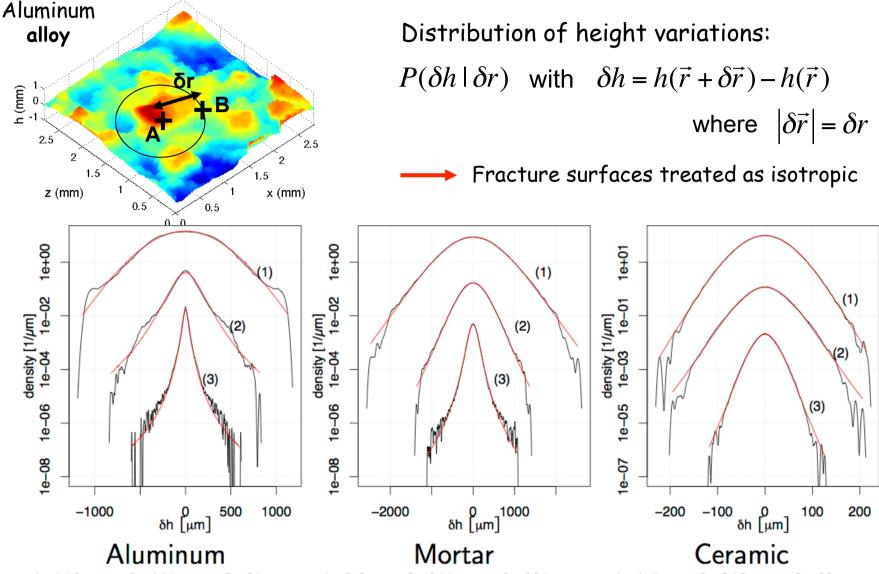
$$P(\delta h \mid \delta r) \quad \text{with} \quad \delta h = h(\vec{r} + \delta \vec{r}) - h(\vec{r})$$
 where $\left| \delta \vec{r} \right| = \delta r$

Fracture surfaces treated as isotropic

Statistics of height fluctuations



Statistics of height fluctuations



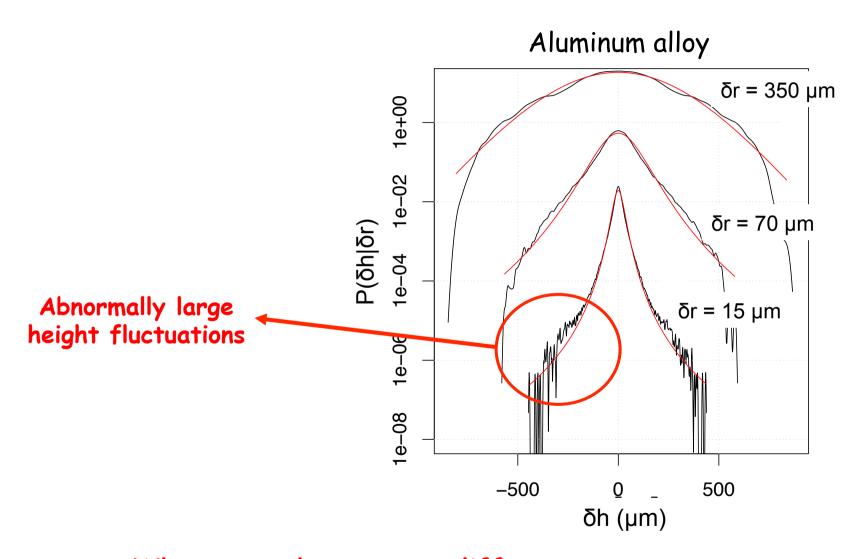
1: 550 μm , 2: 115 μm , 3: 25 μm 1: 3.8mm, 2: 915 μm , 3: 225 μm 1: 1.5mm, 2: 310 μm , 3: 60 μm

Aluminum and mortar: non-Gaussian at small scales



One exponent only insufficient to fully describe their statistics

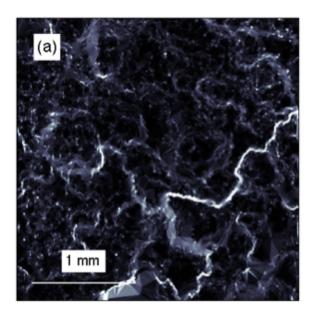
Origin of the fat tail statistics?



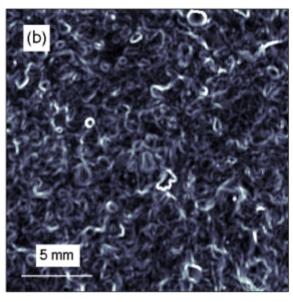
Where are these steep cliffs located on the fracture surface?

Spatial organization of the largest fluctuations

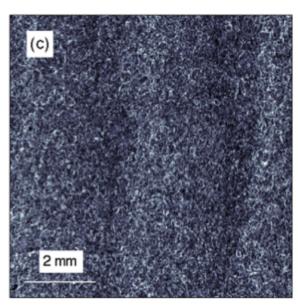
Operator
$$\omega_{\epsilon}(\mathbf{x}) = \frac{1}{2} \log \left(\langle \delta h(\mathbf{x}, \delta \mathbf{x})^2 \rangle_{|\delta \mathbf{x}| = \epsilon} \right)$$



Aluminum ($\varepsilon = 3\mu m$)



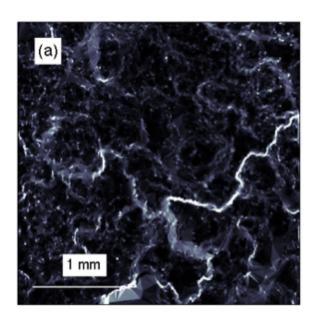
Mortar ($\varepsilon = 50 \mu m$)



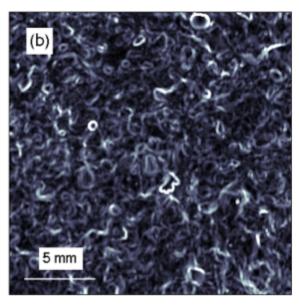
Ceramics ($\varepsilon = 50 \mu m$)

Spatial organization of the largest fluctuations

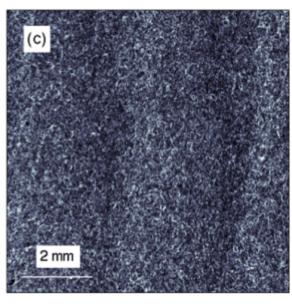
Operator
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Aluminum ($\varepsilon = 3\mu m$)



Mortar ($\varepsilon = 50 \mu m$)



Ceramics ($\varepsilon = 50 \mu m$)

Qualitatively: For the aluminum and mortar

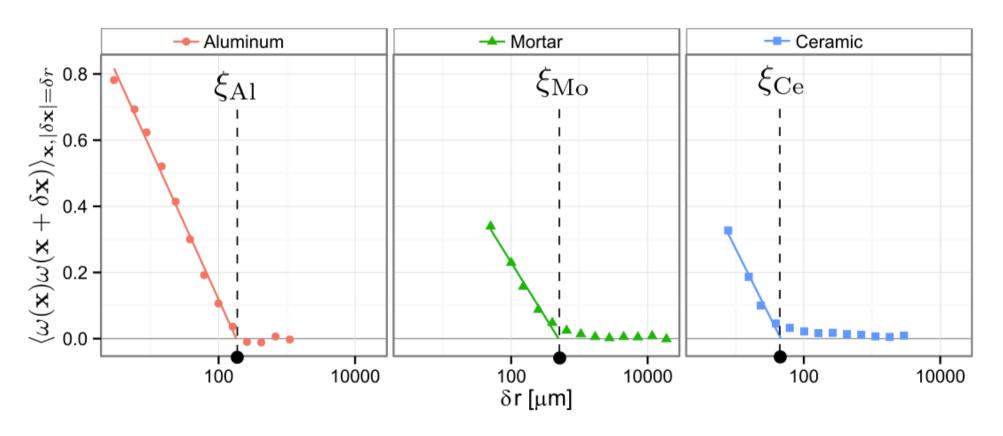
 \longrightarrow Large scale features, long-range correlation of ω

For the ceramics

→ Absence of large scale features

Spatial correlations of ω

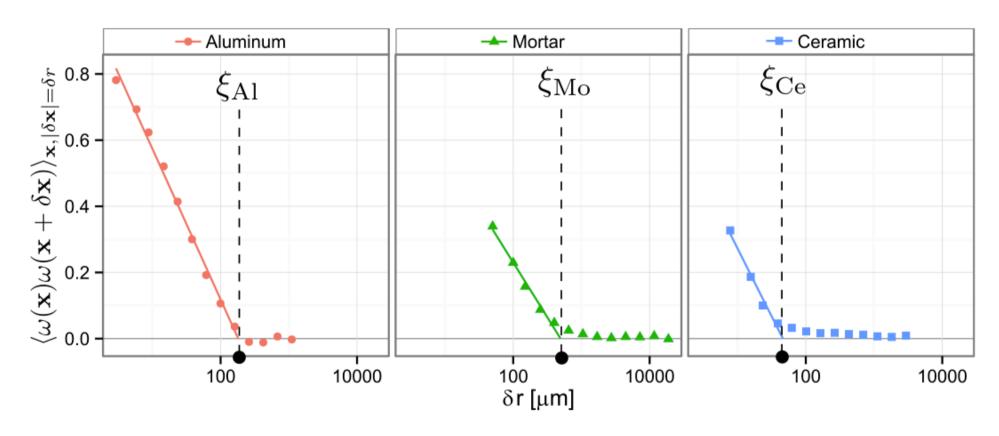
Characterized by its correlation function
$$C(\delta r) = \left\langle \omega(\vec{r})\omega(\vec{r} + \delta \vec{r}) \right\rangle_{\vec{r}, |\delta \vec{r}| = \delta r}$$



Revealing a cut-off length

Spatial correlations of ω

Characterized by its correlation function
$$C(\delta r) = \left\langle \omega(\vec{r})\omega(\vec{r} + \delta \vec{r}) \right\rangle_{\vec{r}, |\delta \vec{r}| = \delta r}$$



Revealing a cut-off length

 ξ of the order of the $\xi \approx \ell_{pz}$ process zone size

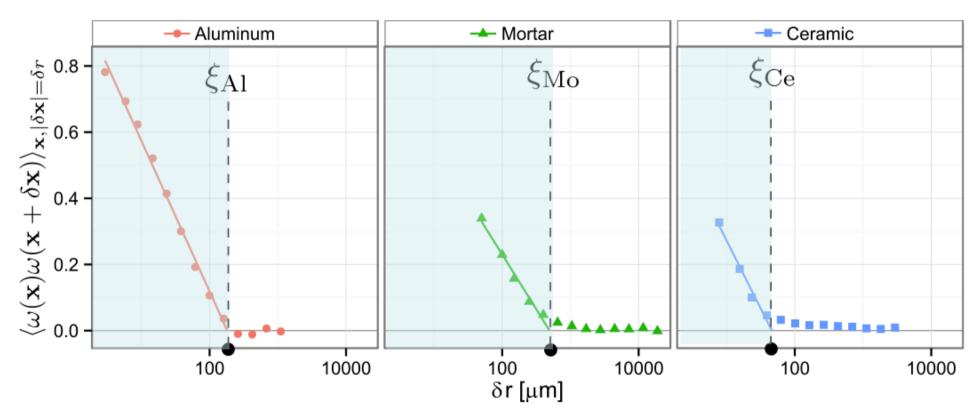
$$\longrightarrow$$
 Aluminum $\xi_{Al} \approx 180 \mu m$

→ Mortar
$$\xi_{Mortier} \approx 500 \mu m$$

→ Ceramics
$$\xi_{C\acute{e}ramique} \approx 50 \mu m$$

Spatial correlations of ω

Characterized by its correlation function
$$C(\delta r) = \left\langle \omega(\vec{r})\omega(\vec{r} + \delta \vec{r}) \right\rangle_{\vec{r}, |\delta \vec{r}| = \delta r}$$



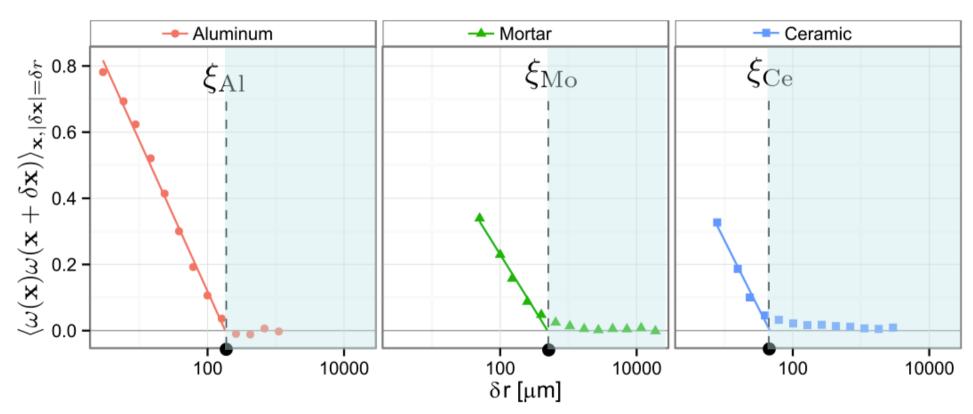
Defining two distinct ranges of length scales:

$$\longrightarrow \delta r < \xi \longrightarrow \left\langle \omega(\vec{r})\omega(\vec{r} + \delta \vec{r}) \right\rangle_{\vec{r}, |\delta \vec{r}| = \delta r} \approx -\lambda \log(\delta r / \xi)$$

Typical scales of damage processes

Spatial correlations of ω

Characterized by its correlation function
$$C(\delta r) = \left\langle \omega(\vec{r})\omega(\vec{r} + \delta \vec{r}) \right\rangle_{\vec{r}, |\delta \vec{r}| = \delta r}$$



Defining two distinct ranges of length scales:

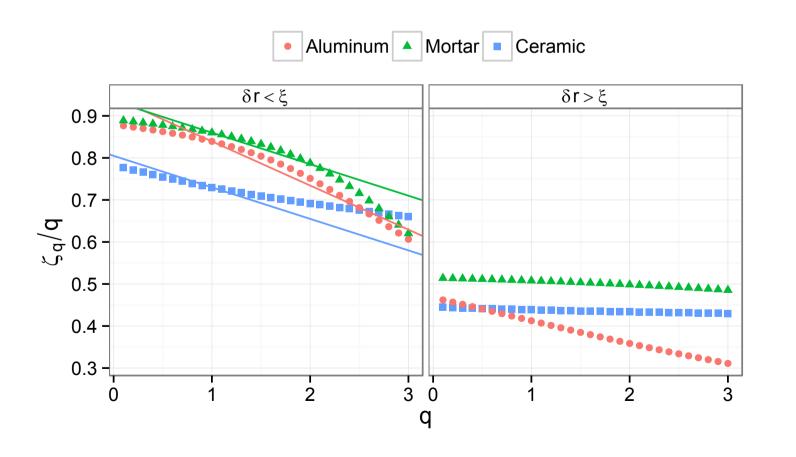
$$\longrightarrow \delta r < \xi \longrightarrow \left\langle \omega(\vec{r})\omega(\vec{r} + \delta \vec{r}) \right\rangle_{\vec{r}, |\delta \vec{r}| = \delta r} \approx -\lambda \log(\delta r / \xi)$$

$$\longrightarrow \delta r > \xi \longrightarrow \langle \omega(\vec{r})\omega(\vec{r} + \delta \vec{r}) \rangle_{\vec{r}, |\delta \vec{r}| = \delta r} \approx 0$$

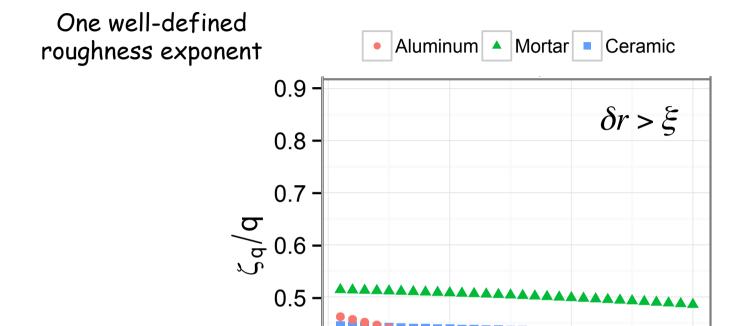
Roughness exponents

Natural extension of the roughness exponent: $\left\langle \left| \delta h(\vec{r}, \delta \vec{r}) \right|^q \right\rangle_{\vec{r}, |\delta \vec{r}| = \delta r} \sim \delta r^{\zeta_q}$

Computed for the two ranges of length scales $\rightarrow \delta r < \xi$



Roughness exponents at large scales



Mortar and ceramics: $\zeta_q / q \approx 0.45$

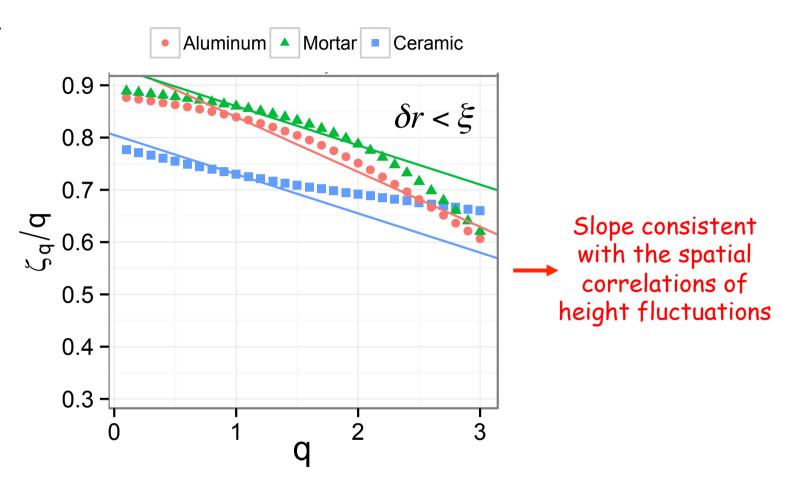
0.4 -

0.3

Low variations with q —— Consistent with (i) a mono-affine behavior (ii) a Gaussian distribution

Roughness exponents at small scales

Multi-affine Spectrum:



Simple multi-affine model: J.F. Muzy and E. Bacry 2002

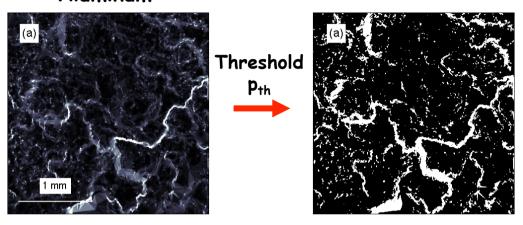
Multi-affine spectrum

Spatial correlation of slopes

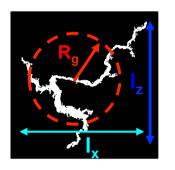
$$\xi_q / q \sim H - \frac{\lambda}{2} (q - 1)$$

$$\zeta_q / q \sim H - \frac{\lambda}{2} (q - 1)$$
 where λ given by $\langle \omega(\vec{r}) \omega(\vec{r} + \delta \vec{r}) \rangle_{\vec{r}, |\delta \vec{r}| = \delta r} \approx -\lambda \log(\delta r)$

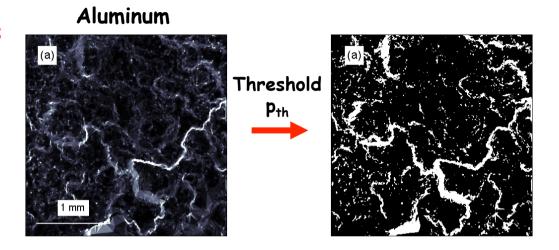
Aluminum



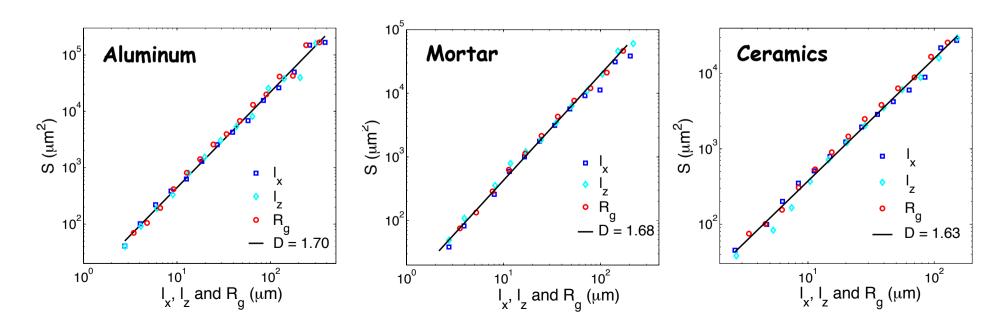
→ Fractal geometry of clusters



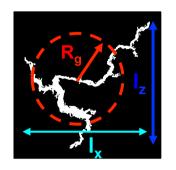
D ≈ 1.7 independent of the material



Surface vs caracteristic length of clusters



→ Fractal geometry of clusters

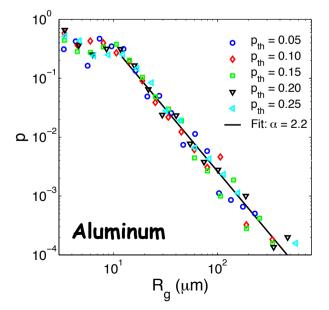


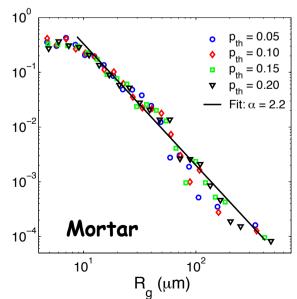
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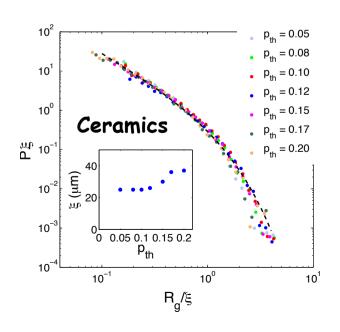
Aluminum (a) Threshold Pth

Power law distributed clusters

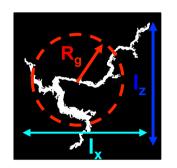
 $P(S) \sim S^{-\alpha}$ with $\alpha \approx 2.2$ independent of the material







→ Fractal geometry of clusters

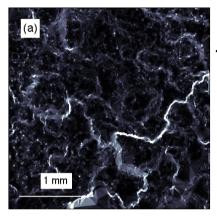


D≈ 1.7 independent of the material

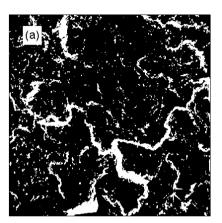
Power law distributed clusters

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Aluminum

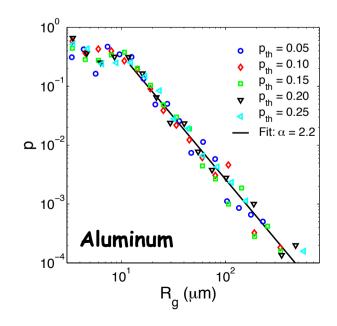


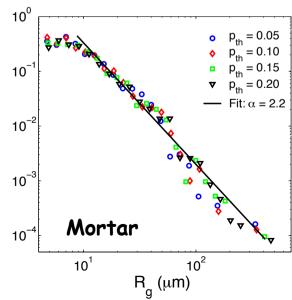


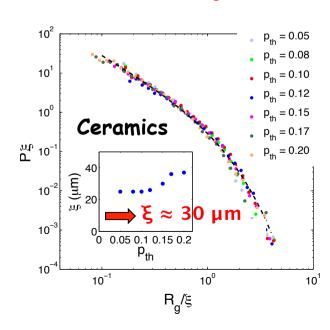


Cut-off length of the

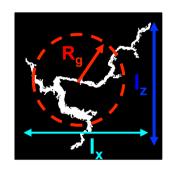
cluster size distribution consistent with ξ







--- Fractal geometry of clusters

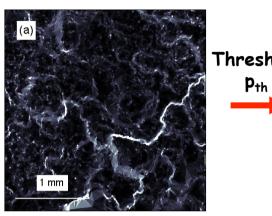


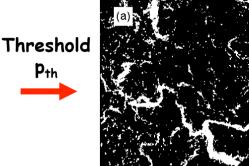
D ≈ 1.7 independent of the material

Power law distributed clusters

 $P(S) \sim S^{-\alpha}$ with $\alpha \approx 2.2$ independent of the material

Aluminum





Cut-off length of the

cluster size distribution

consistent with ξ

Interpretation:

Clusters reminiscent of the process of damage coalescence

Full statistics of 2D fracture surfaces: Summary

S. Vernède, LP and J.P. Bouchaud, 2014

Operator w

- Characterized the local intensity of height fluctuations δh
- Defined a cut-off length ξ

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- ω uncorrelated: $\langle \omega(x)\omega(y)\rangle = 0$
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δr < ξ

- Long range correlations of ω , with $\langle \omega(\vec{r})\omega(\vec{r}+\delta\vec{r})\rangle_{\vec{r},|\delta\vec{r}|=\delta r} \approx -\lambda \log(\delta r)$
- Multi-affine spectrum of the roughness
 Consistent with the spatial correlations of ω
- Universal geometrical properties of clusters of largest fluctuations

Towards a unified description of 2D fracture surfaces?

Three different failure behavior

Ductile - quasi-brittle - brittle

One description

- $\delta r < \xi \longrightarrow \text{Roughness signature of damage}$
- $\delta r > \xi$ \longrightarrow Roughness signature of the propagation of a brittle crack front

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Application S. Vernède and LP, French Patent 2014

• Length ξ — characteristic size of the dissipative failure mechanisms

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Application S. Vernède and LP, French Patent 2014

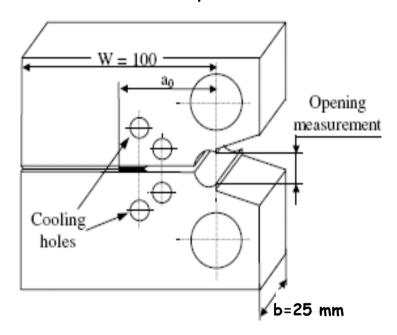
- Length ξ \longrightarrow characteristic size of the dissipative failure mechanisms
- Post-mortem measurement of the fracture energy

→ See A. Needleman's talk

Application: Post-mortem measurement of fracture toughness

S. Chapuliot et al. 2005

Same steel (A508) broken at different temperatures



Effect of temperature on fracture properties

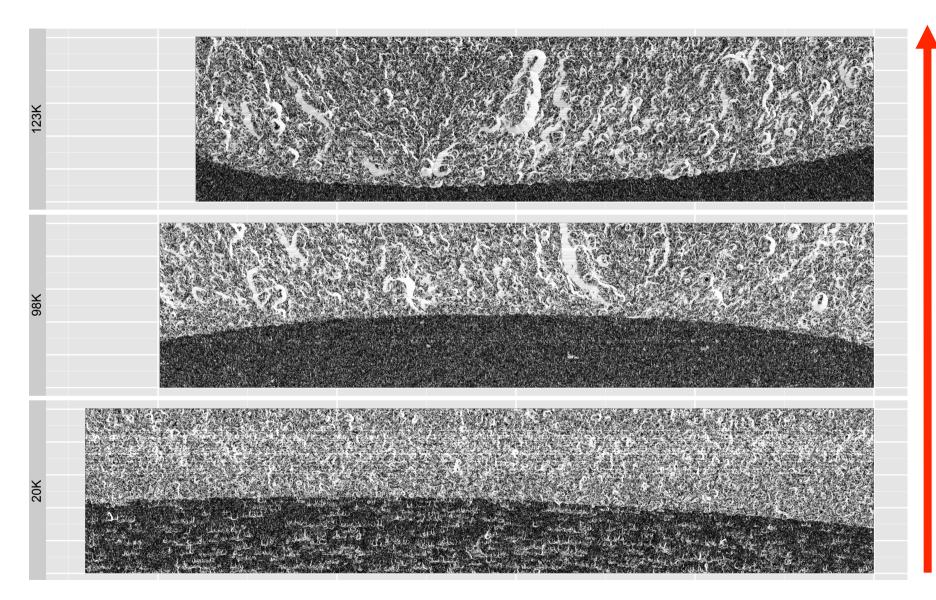
Temperature (°C)	σ _y (MPa) Yield Stress	J _{IC} (kJ/m²) Fracture energy	R _c (μm) Plastic process zone
-253°	1300	2.5	16
-175°	770	11.3	215
-150°	680	16	380

Plastic process zone:

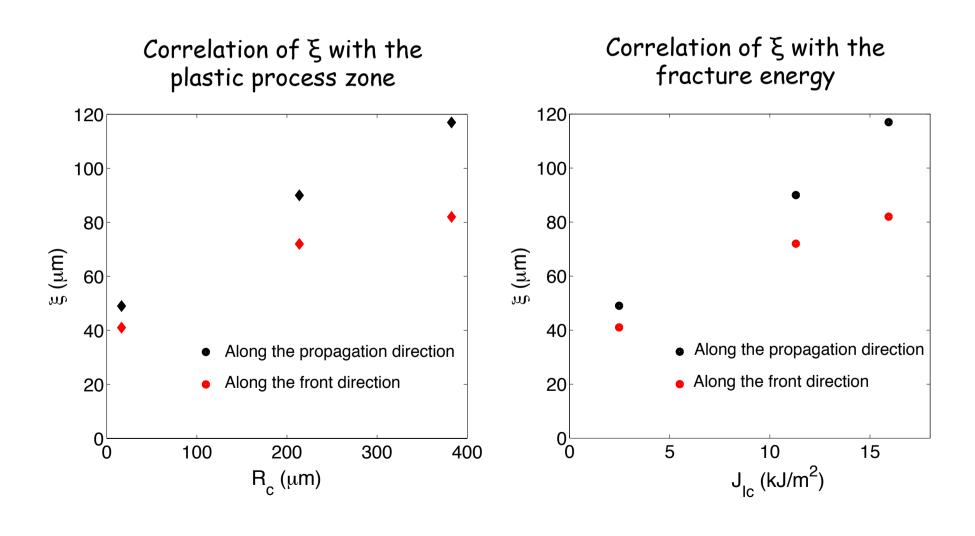
$$\longrightarrow R_c \sim \frac{J_{Ic}}{\sigma_Y^2}$$

Increasing temperature

Fields ω of local height fluctuations



Correlation of ξ with the material fracture properties



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To my collaborators

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Angelo Simone (Delft Univ., Netherlands)

Alan Needleman (North Texas Univ., USA)

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Emergence project (City of Paris)

Towards a unified description of fracture surfaces?

	<i>Anti-persistent</i> crack path	<i>Persistent</i> crack path
Fracture of 3D solids	δr < Lpz $ ζ ≈ 0.40 $ $ β ≈ 0.50$	$ δr > L_{pz} $ $ ζ ≈ 0.75 $ $ β ≈ 0.60 $
	Roughness signature of the propagation of a brittle crack front	Roughness signature of damage
Fracture of 2D thin sheets	$\ell_{\it pz} << d_{\it \mu structure}$	$\ell_{\it pz} >> d_{\it \mu structure}$
	H ≈ 0.50	H ≈ 0.65
↓	Crack growth direction governed by elasticity	Crack growth direction governed by damage nucleation