Critical scaling of avalanches, stress and spatial correlations in shear of disordered systems

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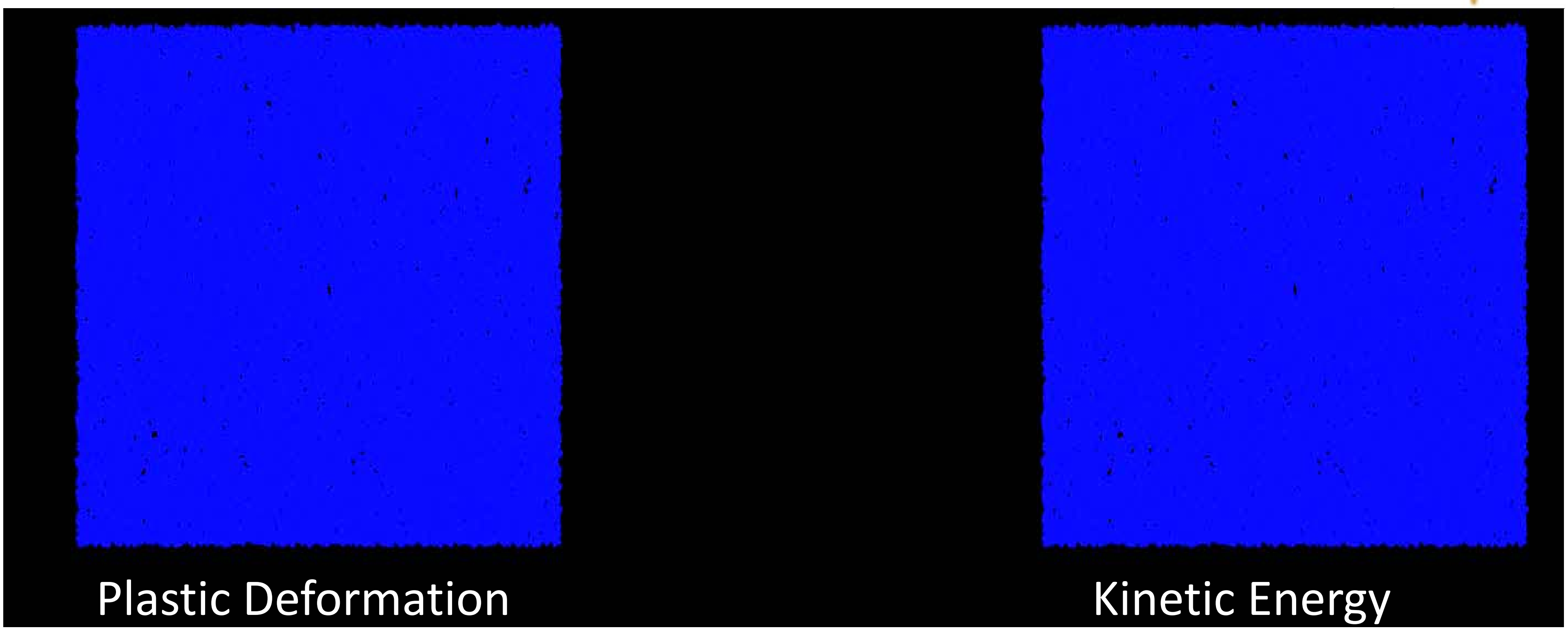


Motivation

Find power law distribution of events, avalanches, earthquakes in a wide variety of systems and on wide range of scales as long as they are driven slowly $\Rightarrow$ quasistatic

- Charge-density waves, fluid invasion, contact line motion, flux lattices, magnetic domains (Barkhausen noise), ...
- Deformation of solids, granular media, colloids, foams, ... (acoustic emission, dislocation bursts, cracks, earthquakes)

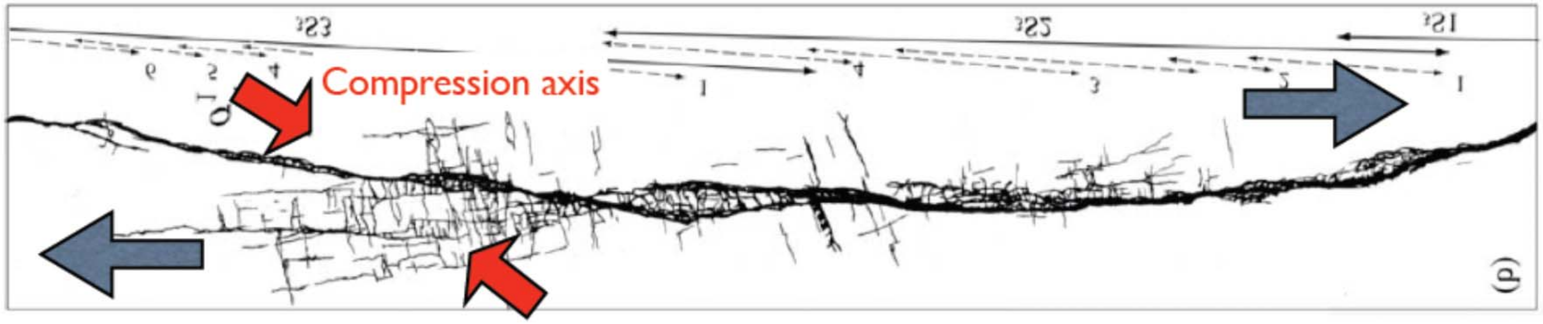
Above models usually studied with overdamped dynamics

Solids are generally underdamped at large scales

$\Rightarrow$ Does inertia drive system away from criticality?

$\Rightarrow$ Stress transmission and dynamic scaling exponents

$\Rightarrow$ Spatio-temporal correlations between avalanches



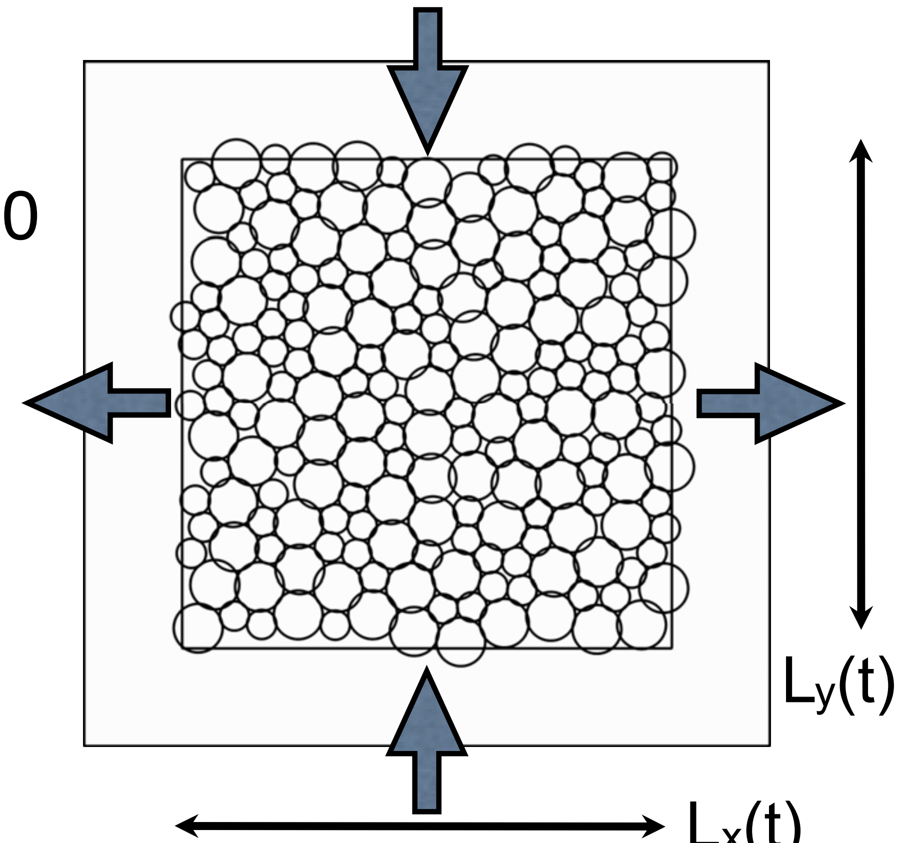
Athermal Simulations of Shear

Molecular Dynamics for 2D or 3D systems

- Binary Lennard-Jones to prevent crystallization
  mean diameter a, binding energy u, time $t_{LJ}=a(m/u)^{1/2}$

$$V(r)=4u\left[\left(\frac{a_{ij}}{r}\right)^{12}-\left(\frac{a_{ij}}{r}\right)^{6}\right]$$

- Quench at pressure p=0 to T=0
  $\Longrightarrow$ protocol not important
- Periodic boundaries
- Pure shear - fix area or vol.
- Fix strain rate or stop strain until event over
- Vary effect of inertia by adding damping $-\Gamma \mathbf{v_i}\, m/t_{LJ}$ or
  $-\Gamma\,(\mathbf{v_i}-\mathbf{v_j})\ f(\mathbf{r_i}-\mathbf{r_j})\ m/t_{LJ}$



Fix area $L_y(t)L_x(t)$

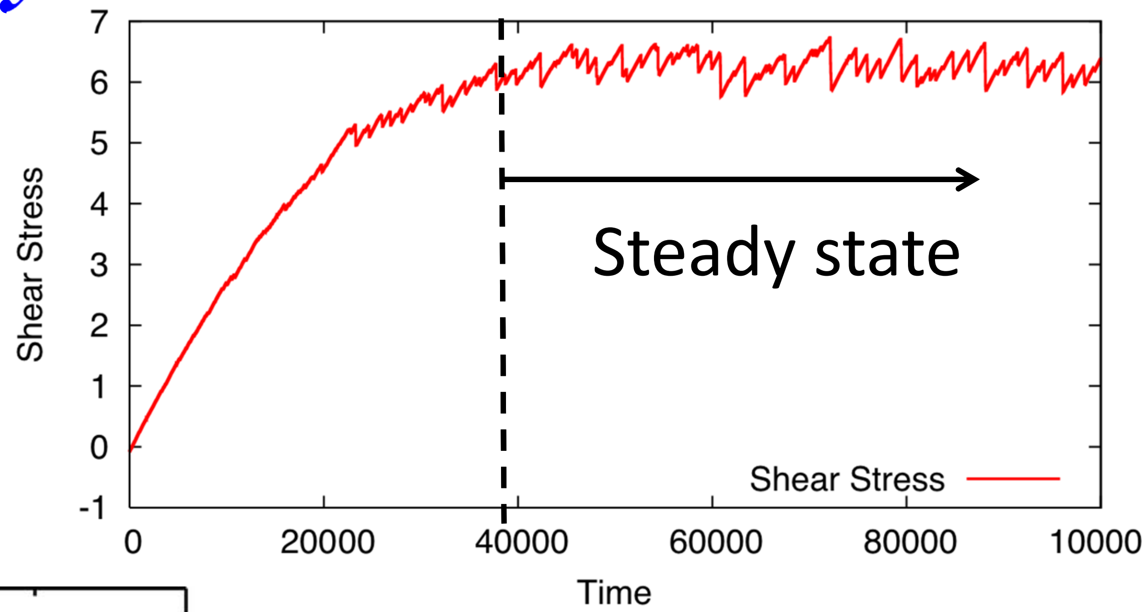
Steady-State Shear

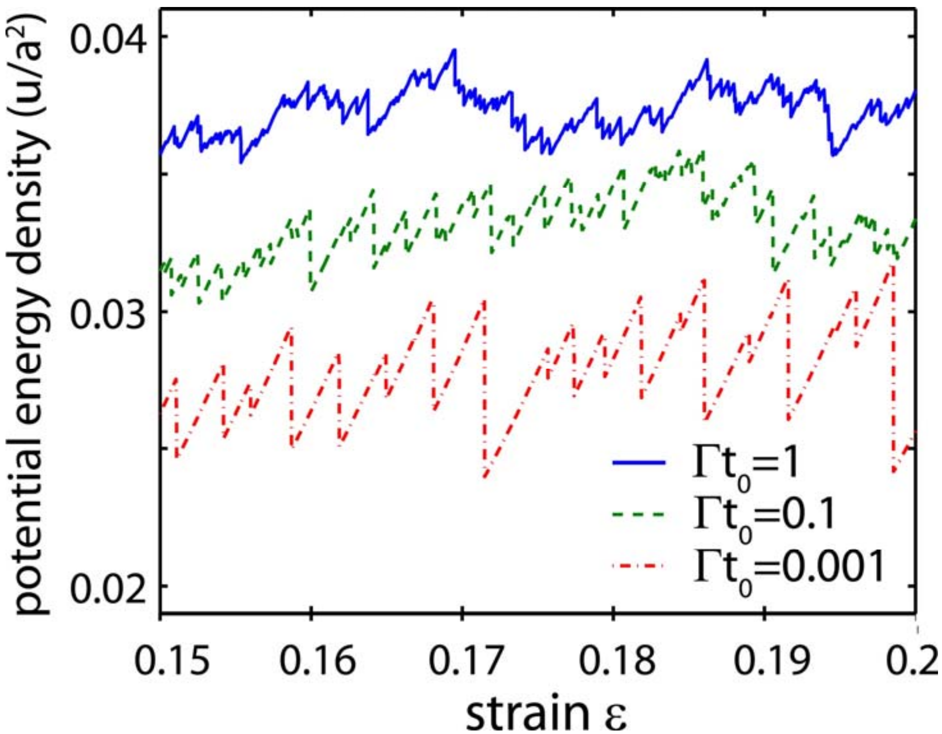
Elastic regions: stress & energy rise linearly

Plastic regions:

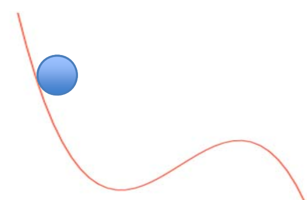
→ mechanical instability

→ energy & stress drop





Inertia changes energy ensemble ⟹ lower minima



Clear change in relative probability of large events

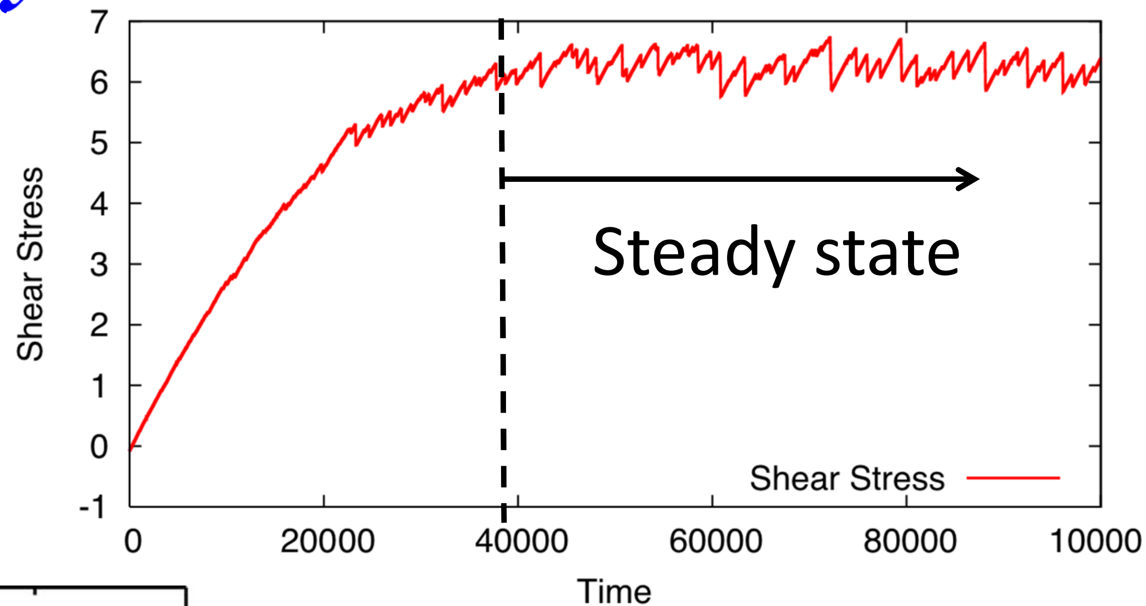
Steady-State Shear

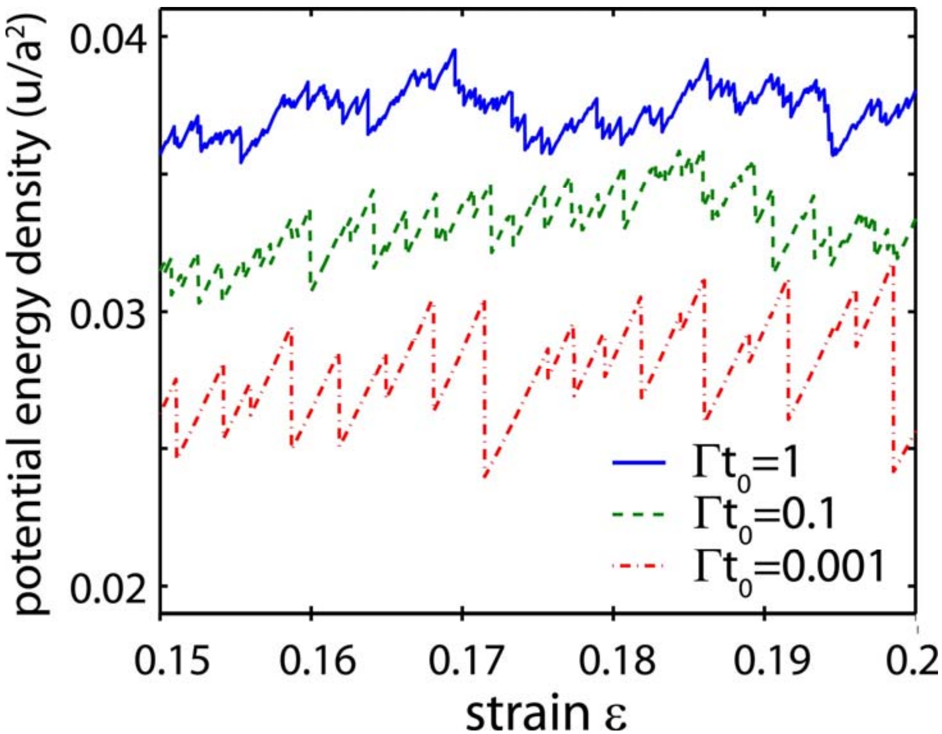
Elastic regions: stress & energy rise linearly

Plastic regions:

→ mechanical instability

→ energy & stress drop





Define extensive stress drop:

$\boldsymbol{S} = (\Delta\sigma_s)L^d\langle\sigma_s\rangle/4\mu$

d=dimension, μ=shear mod.

$S \propto E$ for large events

$\int dE\, E\, R(E,L) = \int dS\, S\, R(S,L)$

R = # events per unit E and strain in system of size L

Not usual normalized prob.

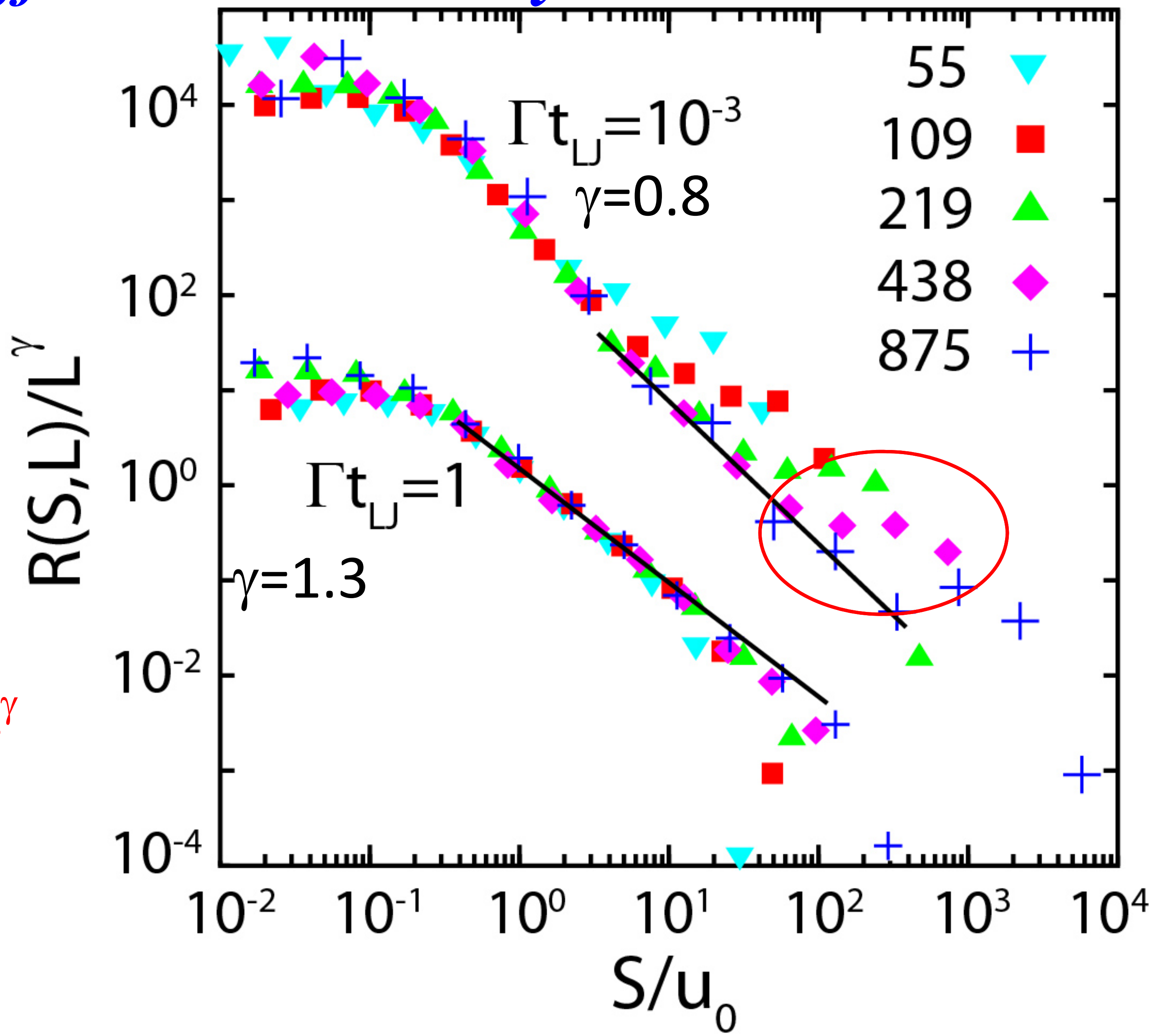
Overdamped and Underdamped Both Critical, Different Universality Classes

Range where $R \propto S^{-\tau}$

Largest event grows with increasing system size

Num. small events is not extensive: $\propto L^{\gamma}$ with $\gamma < d$

Inertia - bump at large events



Finite-Size Scaling Relations

Define R(X,L)=# of events of energy X=E,S in system of edge L per unit energy per unit strain

Expect: $R(E,L) = L^{\beta} g(E/L^{\alpha})$ – g universal scaling function

Mean yield stress $<\sigma>$ indep. of L $\rightarrow$ dissipation = $<\sigma>L^{d}$

$\int dE\, E\, R(E,L) = L^{\beta+2\alpha} \int dx\, x\, g(x) \sim L^{d} \quad \rightarrow \beta+2\alpha=d$

For small $x=E/L^{\alpha}$, $g(x) \sim x^{-\tau} \rightarrow R(E,L) \sim L^{\beta+\alpha\tau} E^{-\tau} \longrightarrow \gamma=\beta+\alpha\tau$

Usually $R(E,L) \sim L^{d}$, i.e. is extensive $\rightarrow \beta+\alpha\tau=d \rightarrow \tau=2$ $(\alpha\neq 0)$

Find small events are strongly suppressed by large events and $\gamma\equiv\beta+\alpha\tau \sim 1.3 << 2$ in 2d ($\gamma\sim 2<<3$ in 3d)

Determine τ, α, β, γ and test scaling relations

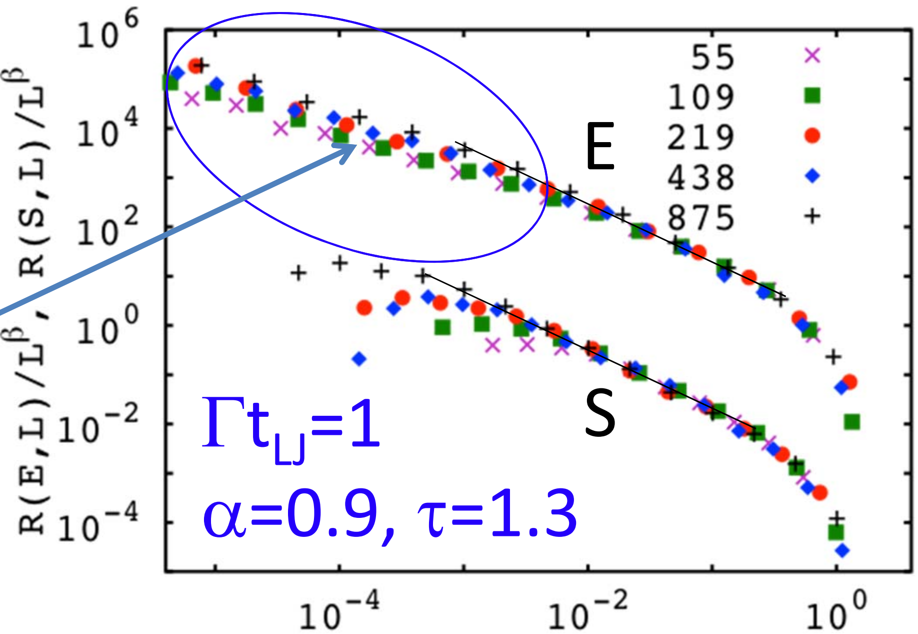
$\gamma=\beta+\alpha\tau$, $\beta+2\alpha=d$

Finite-Size Scaling

$\beta = d - 2\alpha$; $\gamma = \beta + \alpha\tau$

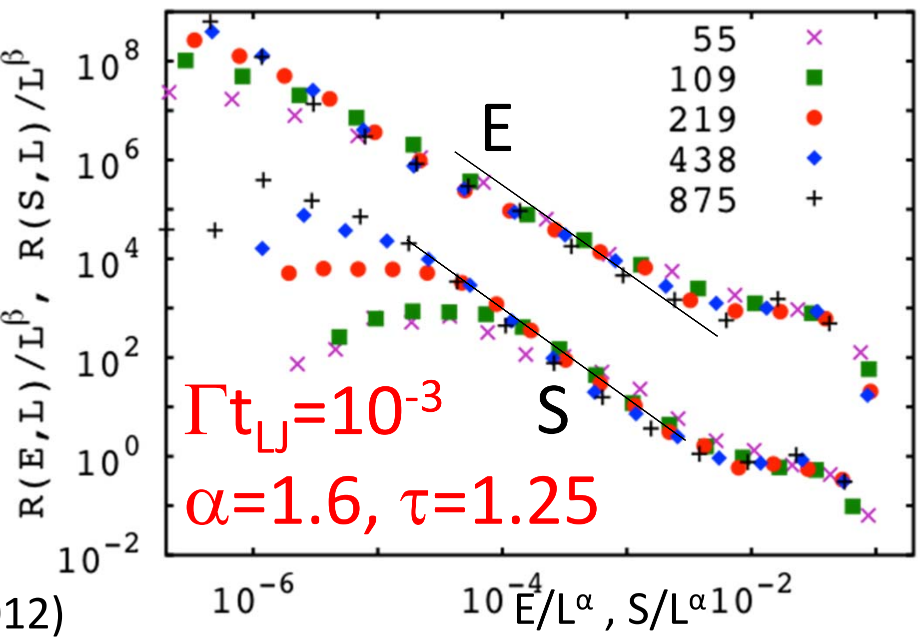
Overdamped

- E,S scale over similar range
- Large avalanches grow more slowly than system size~ **L$^{0.9}$**
- Noncritical power law in E dominated past studies



Underdamped

- Bump at large event size follows scaling law
- Large avalanches grow faster than system size **~ L$^{1.6}$**



Salerno, Maloney & Robbins, PRL **109**, 105703 (2012)

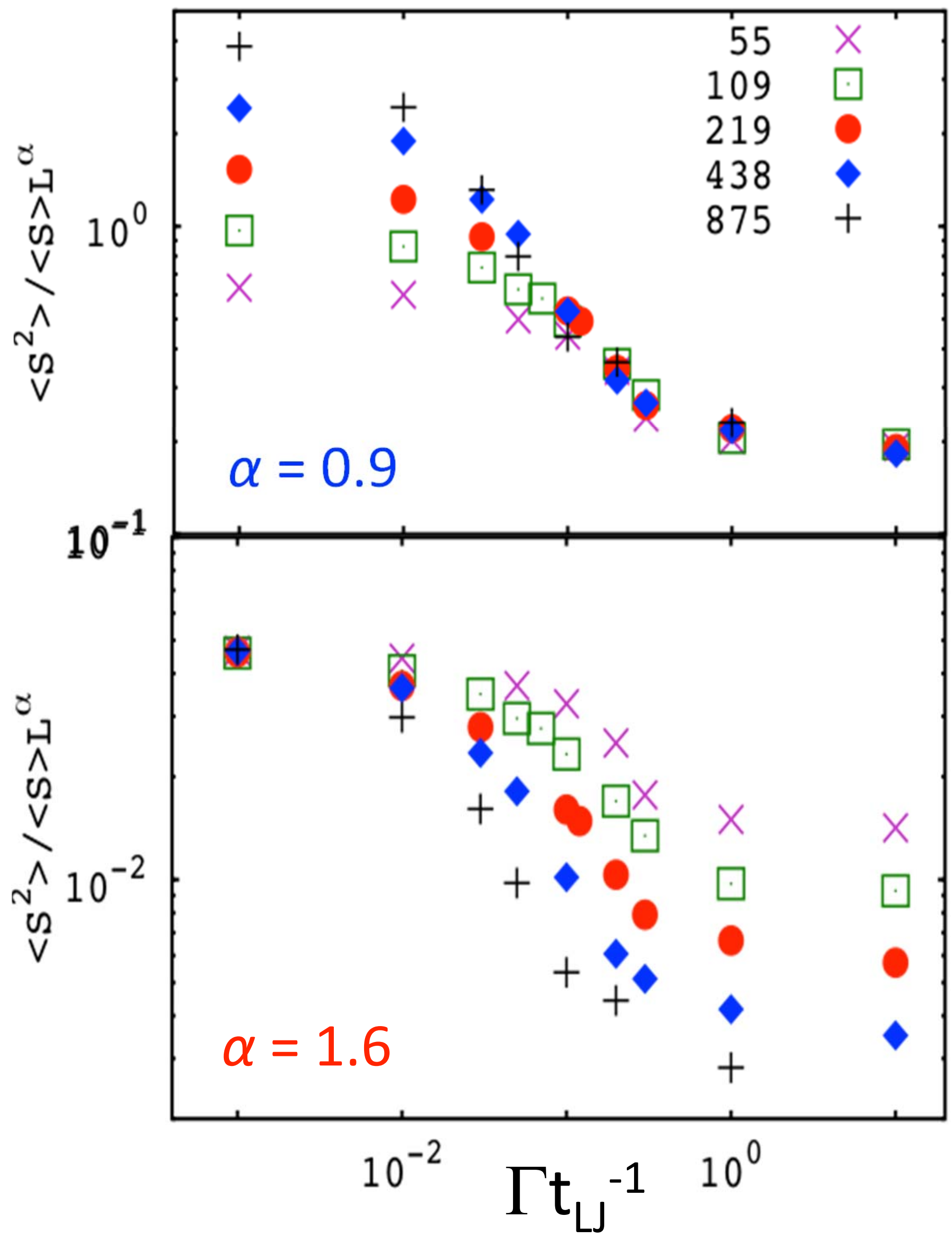
Transition Between Overdamped and Underdamped

If maximum energy $\sim L^{\alpha}$ then $\langle E^2\rangle / L^{\alpha}\langle E\rangle = \text{const}$

For $\Gamma > \Gamma_c = 0.1 t_{LJ}^{-1}$ all data scale with $\alpha = 0.9$ curves separate for lower damping

Small Γ, curves collapse for $\alpha = 1.6$

Multicritical point at $\Gamma_c = 0.1 t_{LJ}$

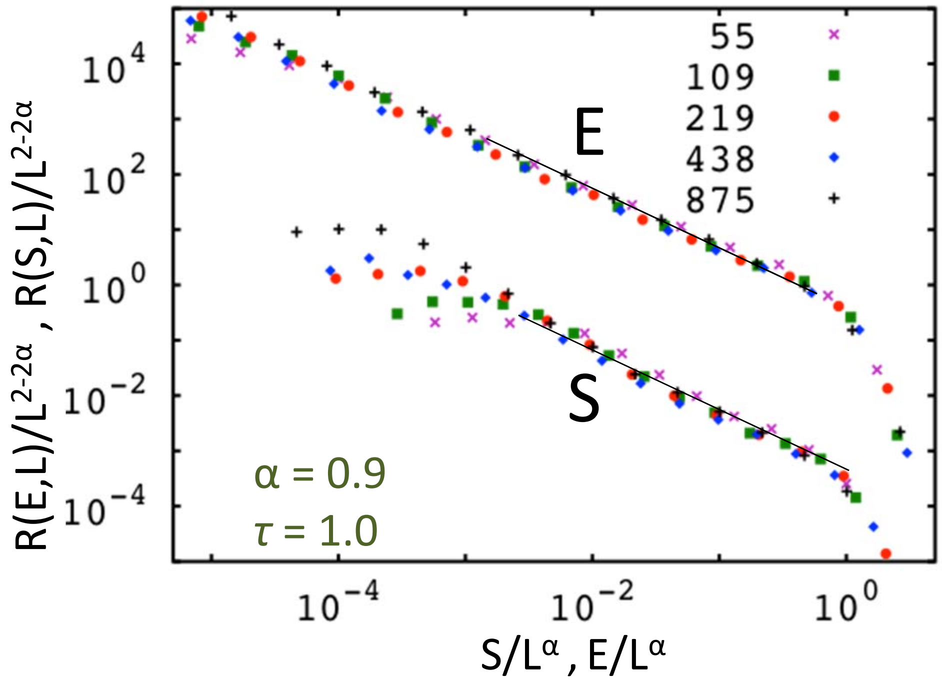


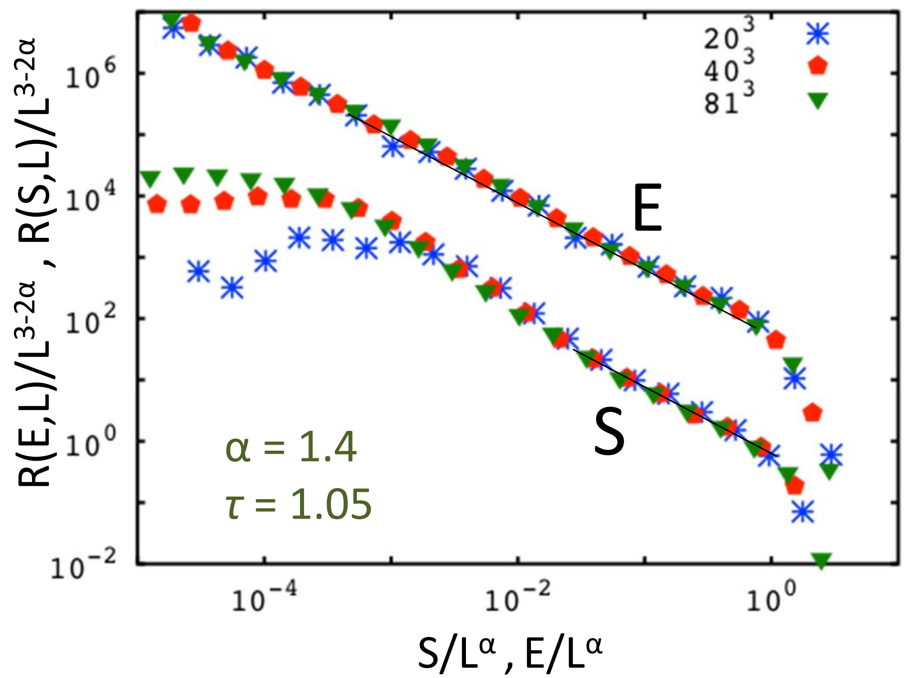
Scaling at Critical Damping

Near Γ=0.1 for 2D and 3D

- Consistent τ for 2D and 3D
  - 2D exponent $\tau \sim 1.0$
  - 3D exponent $\tau \sim 1.05$

- Unstable multi-critical point?
  - Systems near $\Gamma = 0.1$ seem to flow away from critical point as system size grows





Scaling of Stress Variations with L

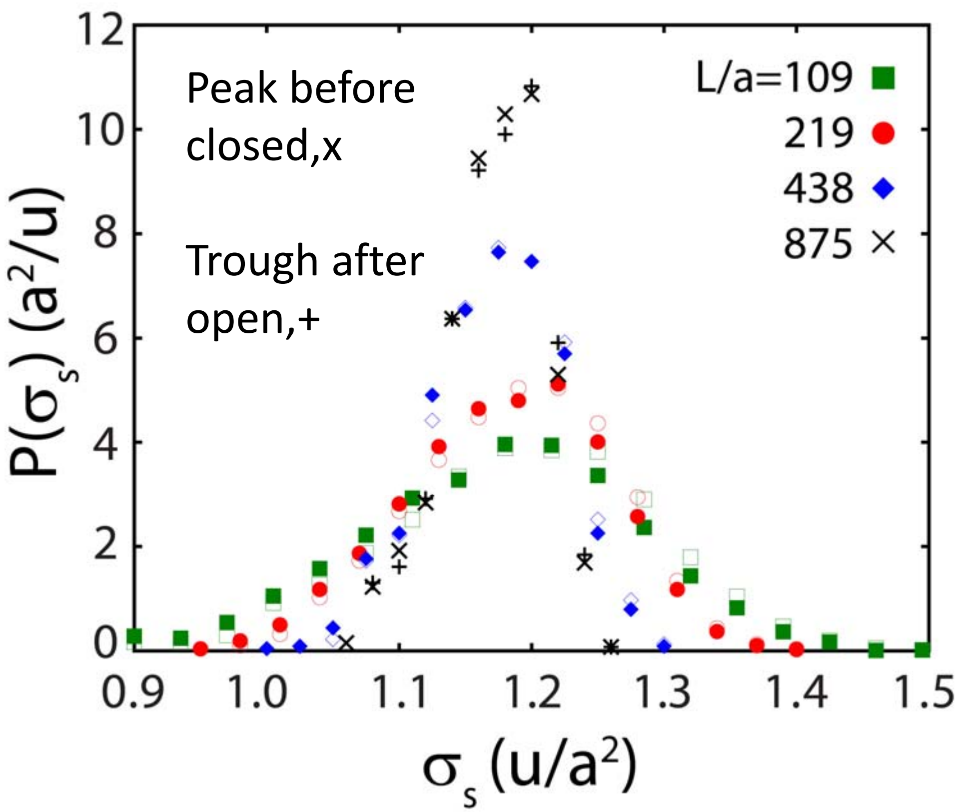
Does width narrow with rising L?

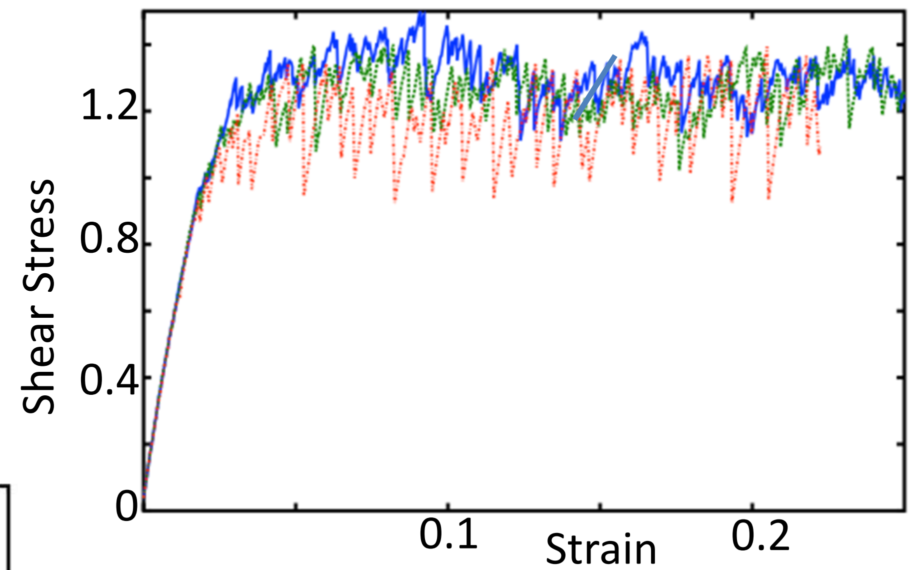
$<(\sigma_s - <\sigma_s>)^2> \sim L^{-2\phi}$

Or is there a gap like hysteresis in sand flow due to inertia?

Lower bound for stress variation

– largest events L^α/L^d , $\phi = d - \alpha$





Stress narrows in all cases

Even for underdamped, distribution of peaks before and troughs after avalanches overlap

No evidence of hysteresis

Salerno &Robbins, PRE **88**, 062206 (2013)

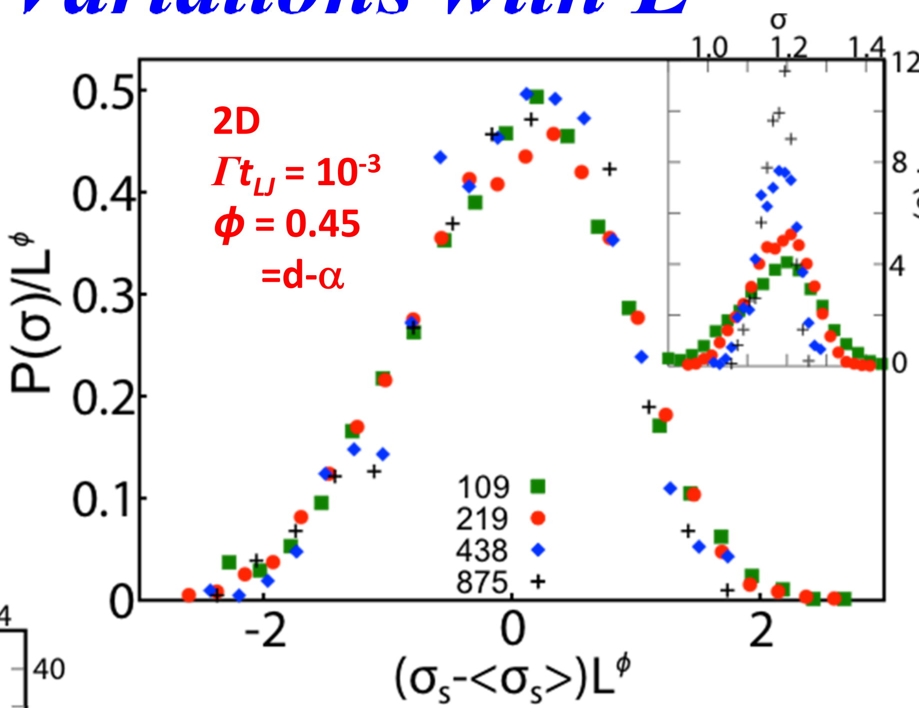
Scaling of Stress Variations with L

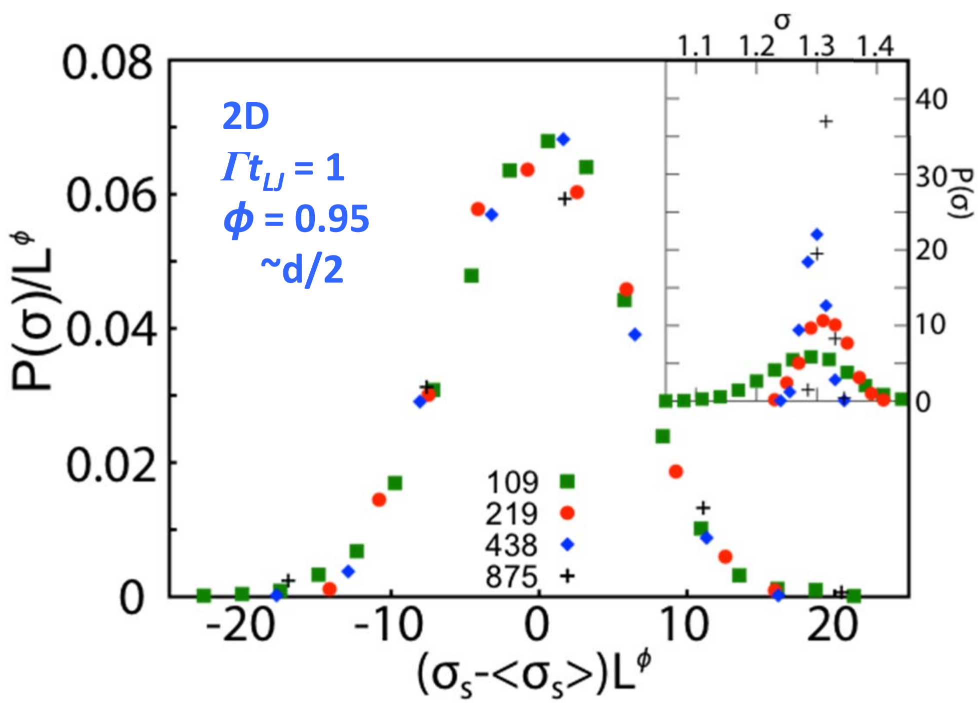
Does width narrow with rising L?

$\langle(\sigma_s - \langle\sigma_s\rangle)^2\rangle \sim L^{-2\phi}$

Lower bound for stress variation – largest events L^{α}/L^{d} , $\phi = d - \alpha$

But not quenched configuration, fluctuations in properties $\sim L^{-d/2}$





Find $\phi = \min(d-\alpha, d/2)$

For case where $\phi = d-\alpha$, reasonable that $\phi = 1/\nu$

correlation length $\xi \sim |\sigma - \sigma_c|^{-\nu}$

Salerno &Robbins, PRE **88**, 062206 (2013)

Quasistatic Exponents for 2D & 3D

Overdamped and underdamped in different universality classes
Separated by critical damping with own exponents

Salerno &Robbins, PRE **88**, 062206 (2013)

Γ	d	τ	α	γ	ϕ
1.0	2	1.3 ± 0.1	0.9 ± 0.05	1.3 ± 0.1	1.00 ± 0.1
0.1	2	1.0 ± 0.05	0.8 ± 0.1	1.2 ± 0.1	0.9 ± 0.1
0.001	2	1.25 ± 0.1	1.6 ± 0.1	0.8 ± 0.1	0.5 ± 0.1
1.0	3	1.3 ± 0.1	1.1 ± 0.1	2.1 ± 0.1	1.5 ± 0.2
0.1	3	1.05 ± 0.05	1.5 ± 0.1	1.6 ± 0.1	1.30 ± 0.1
0.001	3	1.2 ± 0.1	2.1 ± 0.2	1.3 ± 0.2	0.9 ± 0.1

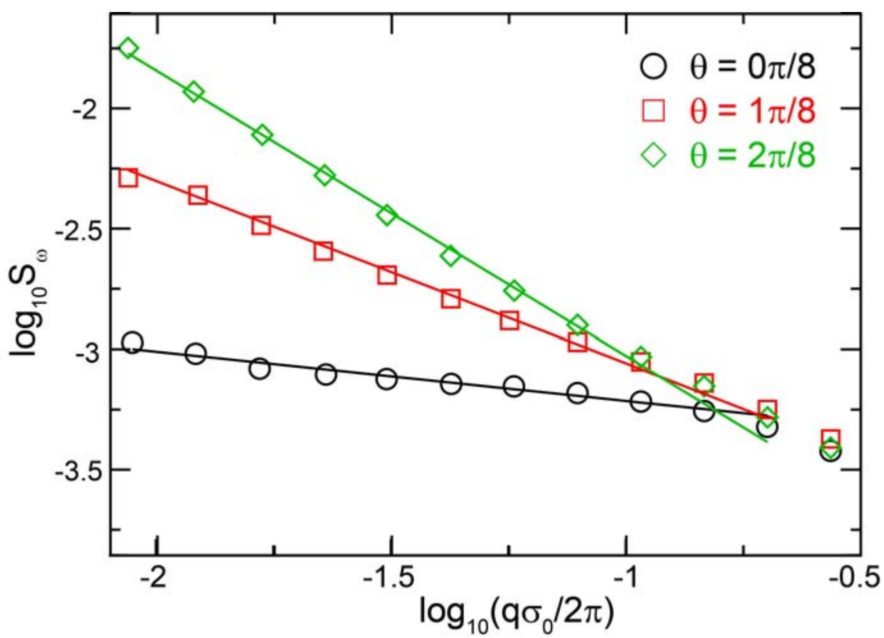
Past studies, overdamped, small L ($\leqslant$100), only reported α
2D: Maloney & Lemaître PRE 74, 016118, ’06 α~1
Lerner & Procaccia, PRE 79, 066109, 2009 α~0.75
3D: Bailey, et al. , PRL 98, 095501, ’07 α~1.4

Relation Between Energy and Spatial Size

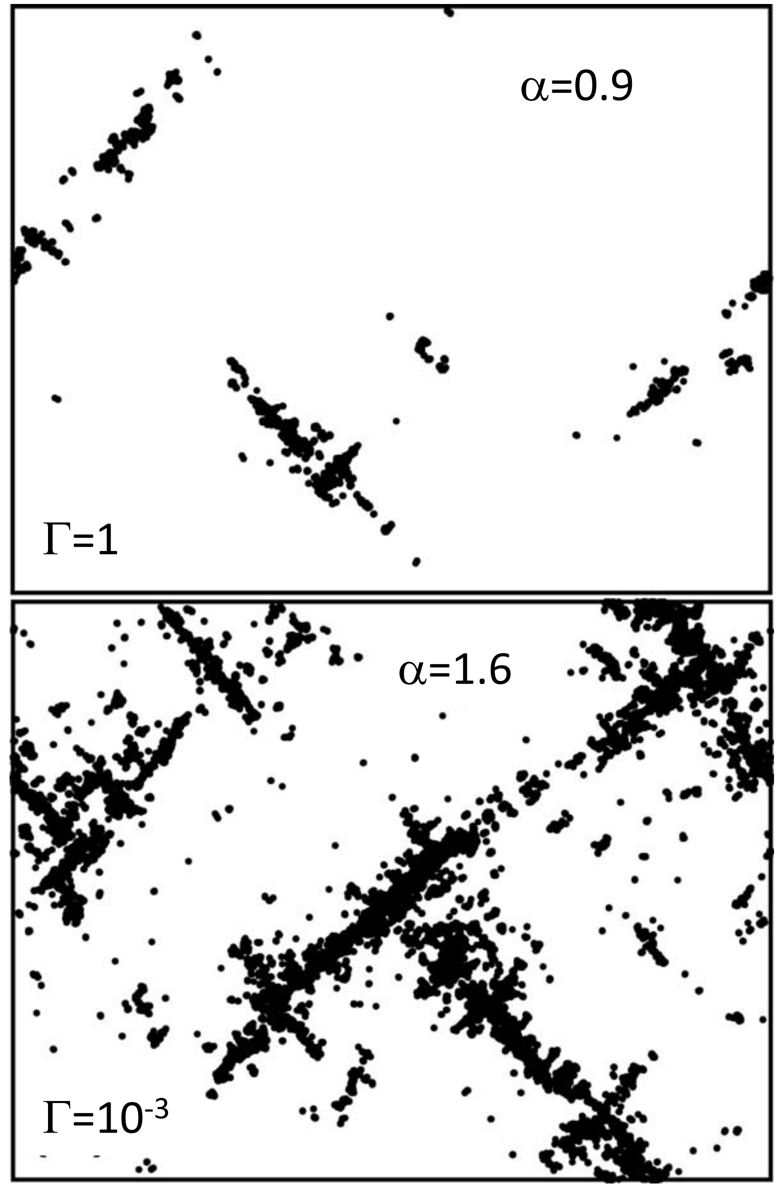
Find plastic area/volume $\propto$ E
$\Longrightarrow$Larger E deforms larger region
but not with larger strains same
in lattice models not earthquakes

Largest size event ~ L^{α}
$\rightarrow$is α a fractal dimension d_f?

Anisotropic scaling exponents!



Maloney & Robbins, PRL **102**, 225502 (2009)



Salerno &Robbins, PRE **88**, 062206 (2013)

Lattice Models → Mean Field Scaling to 2D

K. Dahmen, Y. Ben-Zion, J. Uhl, Phys. Rev. Lett. 102 175501 (2009), Nature Physics 7, 554 (2011)

Many lattice models and experiments show scaling consistent with mean-field exponents, $\tau = 1.5$

BUT models are all overdamped where we find $\tau = 1.3$

Above models have positive definite elastic coupling
- advance in one spot makes all spots more unstable

$\Longrightarrow$ Obey no-passing rule – unique pinned states that flow to from different initial conditions

Shear produces quadrupolar field
– suppress instability at sites normal to shear plane
– no-passing rule does not apply

Lattice Models with Quadrupolar Coupling

Talamali, Petaja, Vandembroucq, and Roux PRE, **84**, 016115 (2011) found τ=1.25, anisotropic clusters

Lin, Saade, Lerner, Rosso, Wyart Europhys Lett 105: 26003 (2014); Lin, Lerner, Rosso, Wyart, PNAS, (2014)

Random kicks from nearby avalanches cause states to diffuse toward instability

Probability that a kick of magnitude δ will cause instability goes to zero as $\delta^{\theta} \Longrightarrow$ leads to nonextensive scaling – γ<d

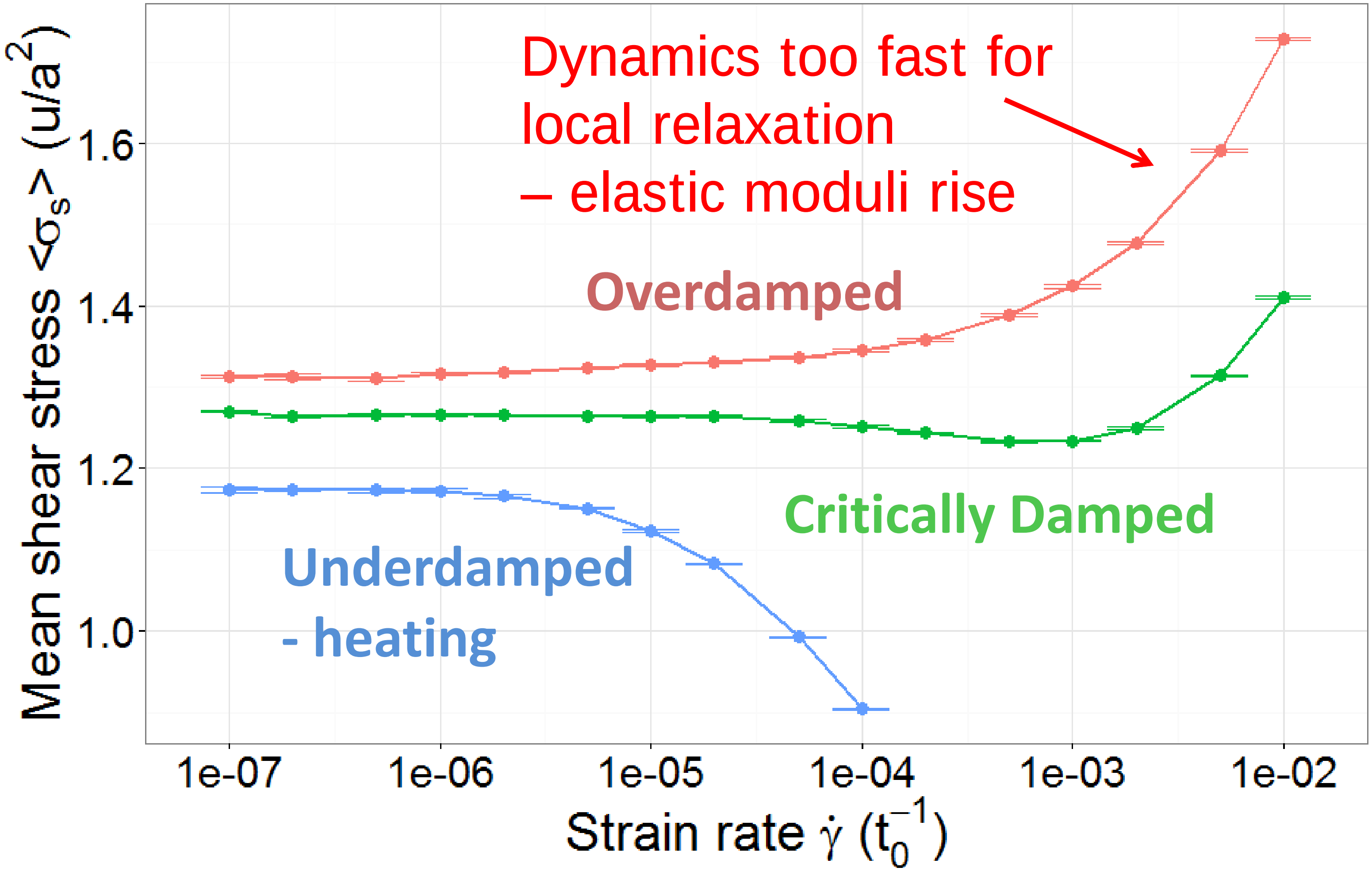
Find α=1.1 vs. 0.9 in 2D; 1.5 vs. 1.1 in 3D

τ=1.35 vs. 1.3 in 2D, 1.46 vs. 1.3 in 3d

Dynamic exponents strain rate $d\gamma/dt \sim (\sigma-\sigma_c)^{\beta}$

β=1.52-1.62 vs. 1.78 in 2D, 1.38 to 1.41 in 3d

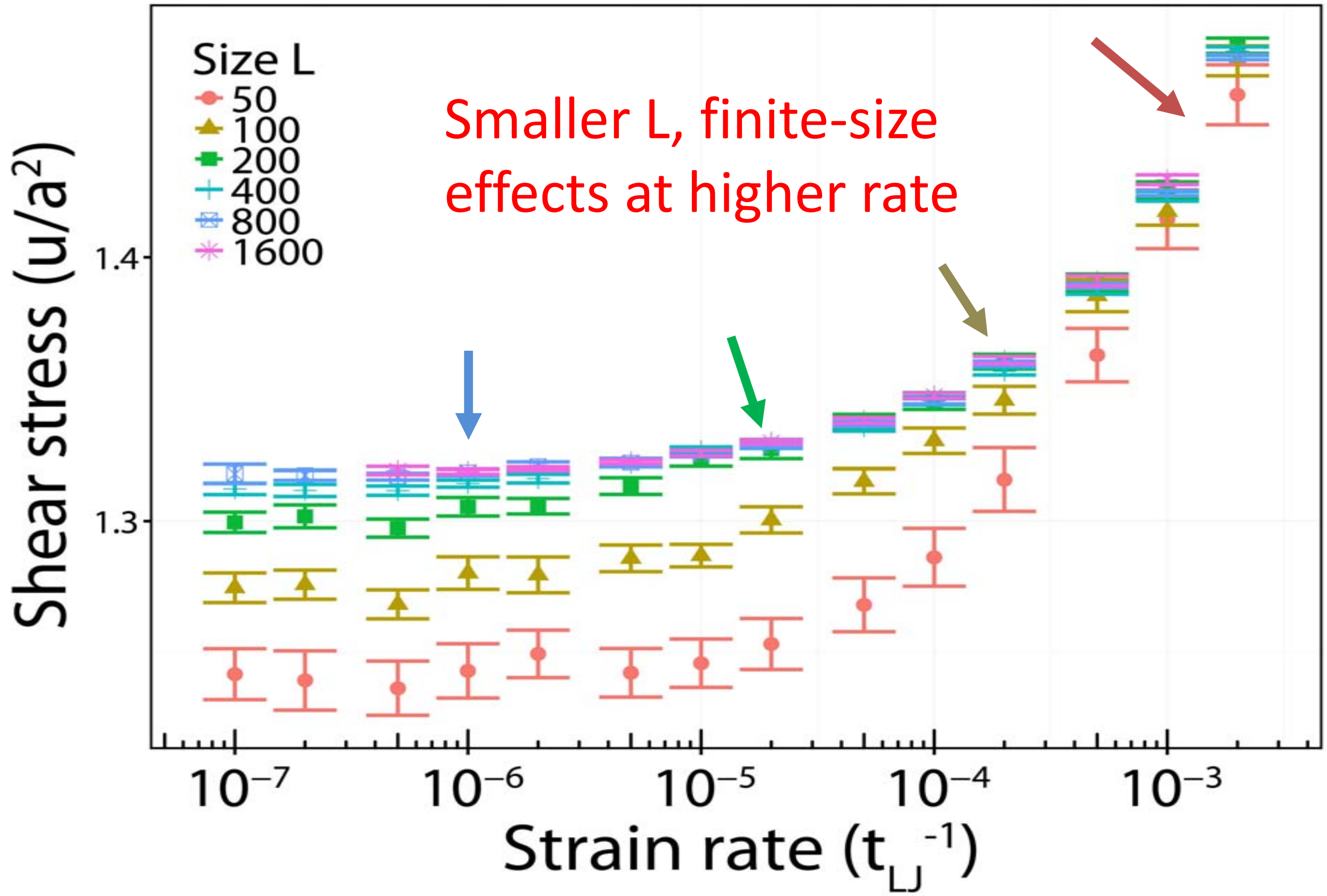
Stress vs. Constant Strain Rate



Critical damping takes energy out at same rate generated by plasticity

For Γ=1 see overdamped behavior but most modes underdamped

Stress vs. Strain Rate- Overdamped



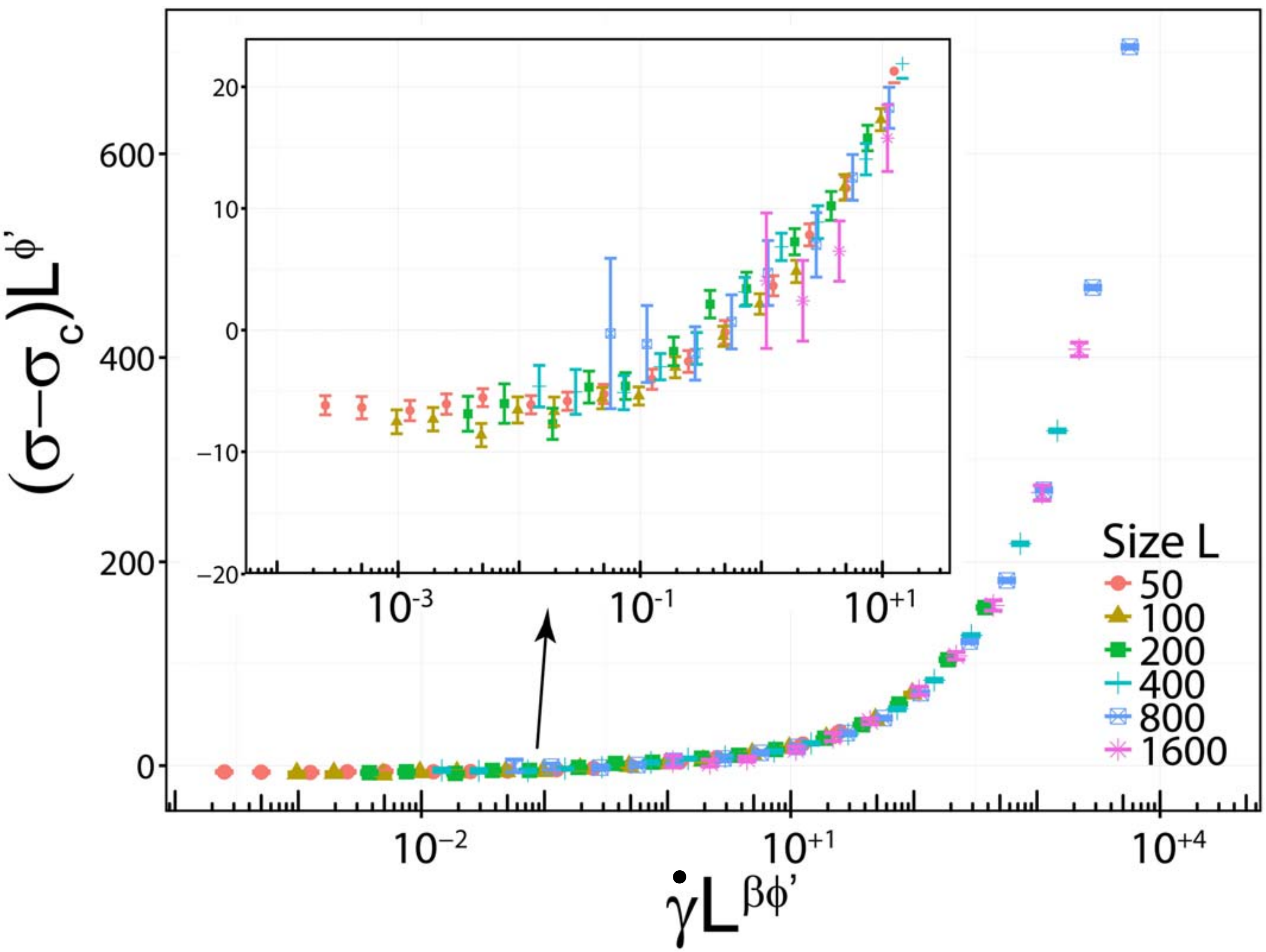
Finite-size scaling: $d\gamma/dt \sim L^y \, h[(\sigma-\sigma_c)L^{\phi'}]$

Large rate: $d\gamma/dt \sim L^y \, (\sigma-\sigma_c)^{\beta} L^{\beta\phi'}$ with $y=-\beta\phi'$

Small rate: $(\sigma-\sigma_c)L^{\phi'} \sim$ constant

- average random ensembles $\phi'=d-\alpha = 1.1 > d/2=\phi$

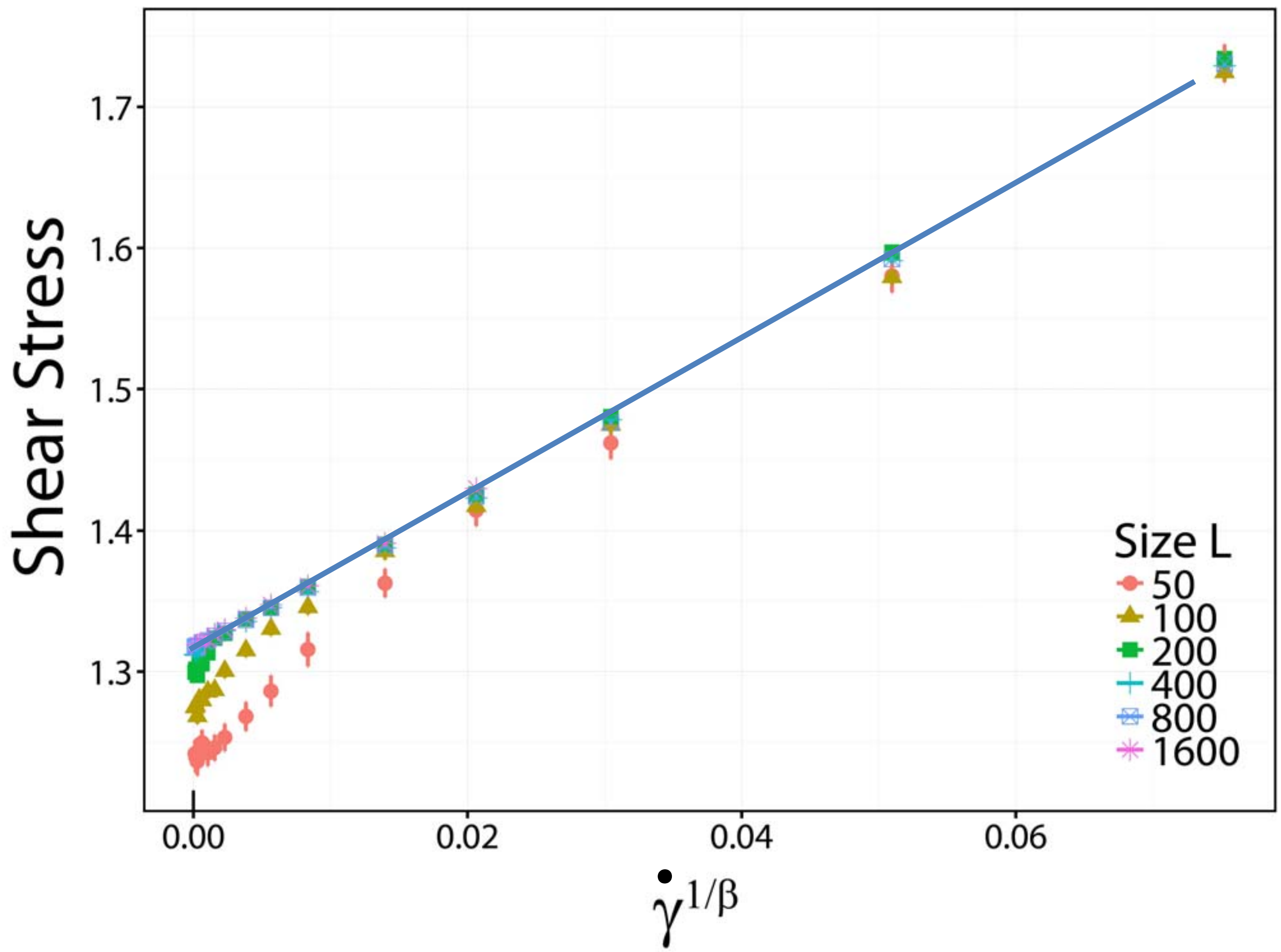
Stress vs. Strain Rate- Overdamped



Finite-size scaling: $d\gamma/dt \sim L^{-\beta\phi'}\, h[(\sigma-\sigma_c)L^{\phi'}]$

with σ_c=1.318; β= 1.78, ϕ'=d-α =1/ν = 1.09 > d/2=ϕ

Stress vs. Strain Rate- Overdamped

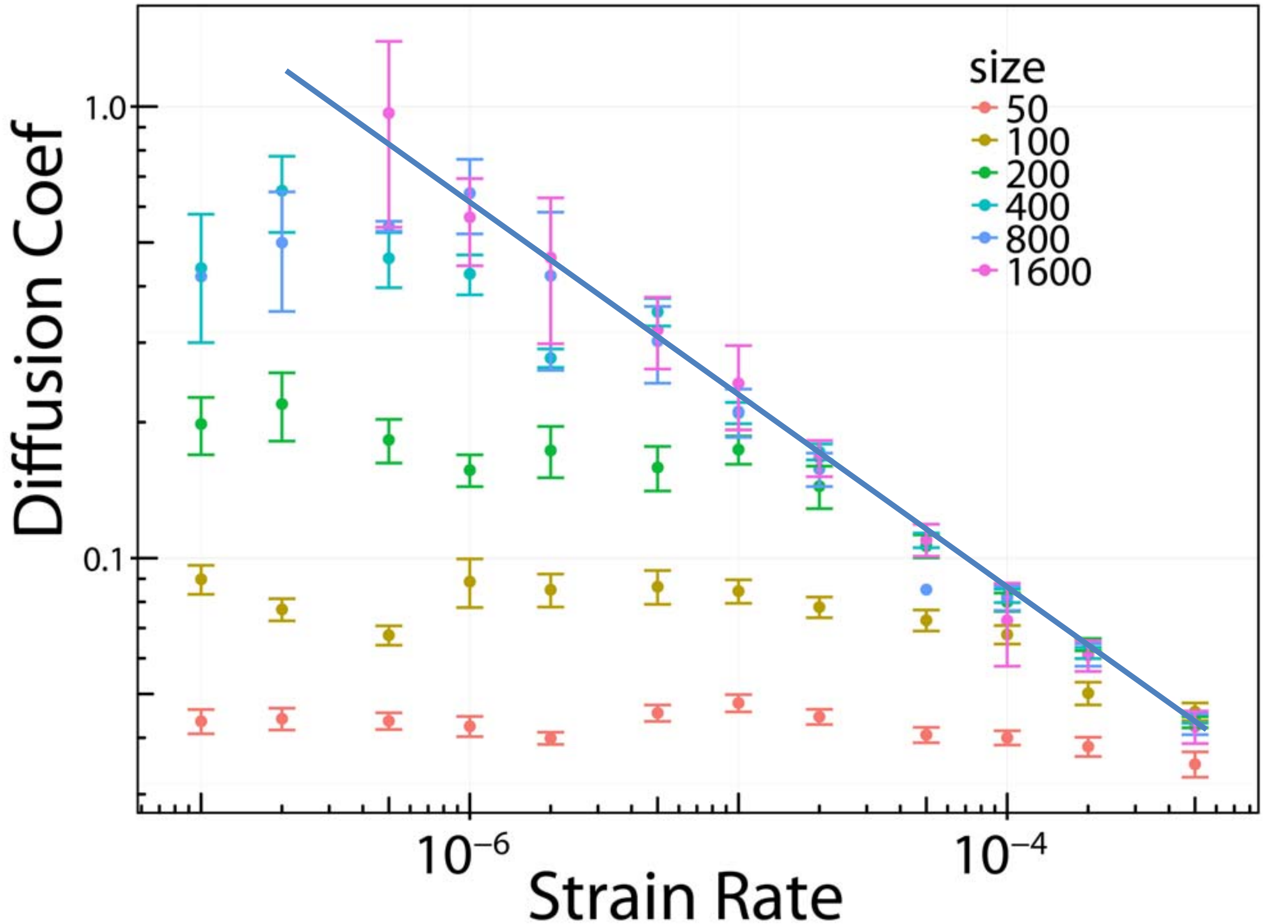


Finite-size scaling: $d\gamma/dt \sim L^{-\beta\phi'}\, h[(\sigma-\sigma_c)L^{\phi'}]$

with σ_c=1.318; β= 1.78, ϕ'=d-α = 1.09 > d/2=ϕ

Is ν=1/(d-α)=0.92?

Diffusion Constant vs. Rate

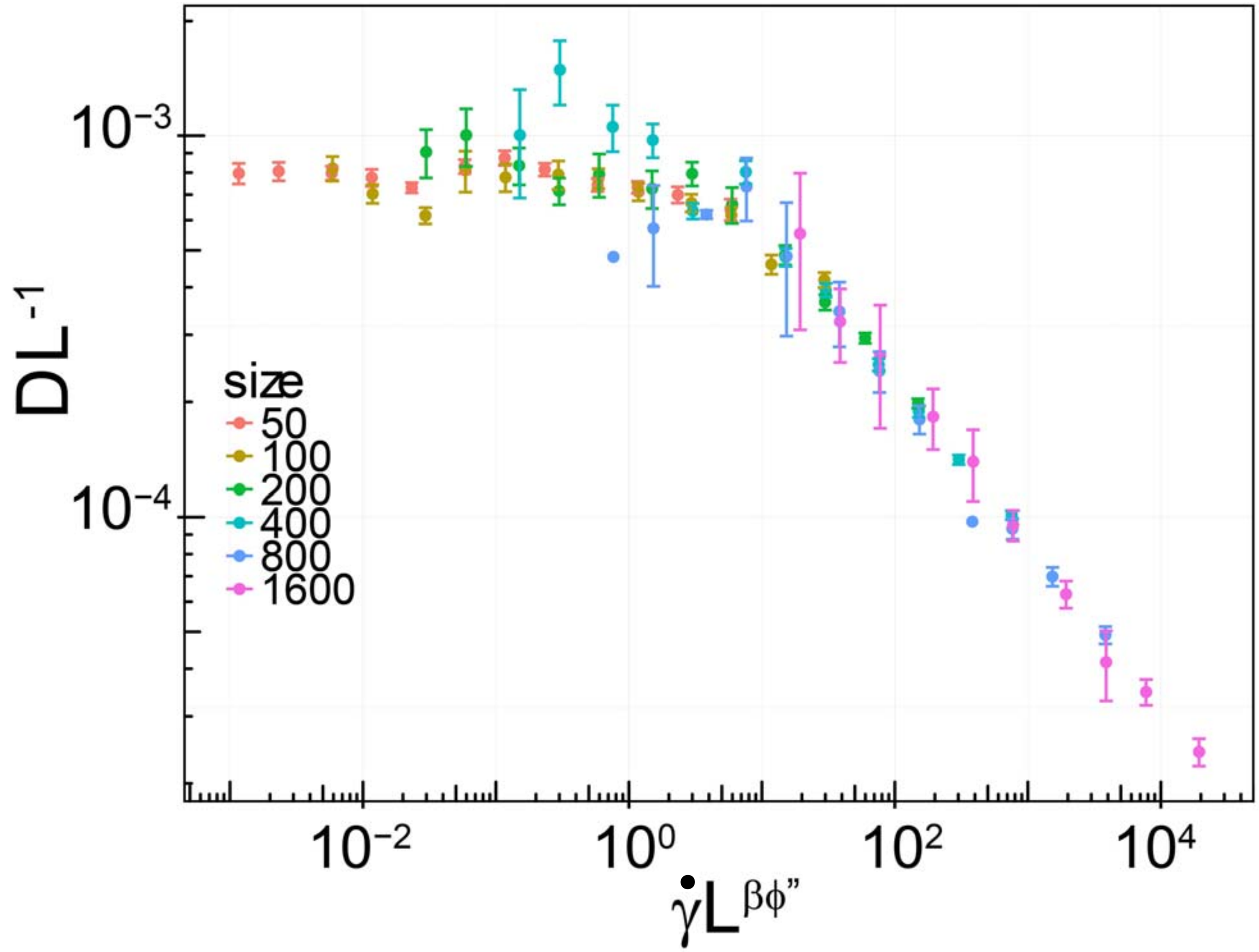


Non affine displacement dr scales as $dr^2=2D\, d\gamma$

where $d\gamma$ is change in strain and $\propto$ time

Find $D \propto L$ at low rates because of long range correlations

Diffusion Constant vs. Rate



See that diffusion scales as L for low rates.

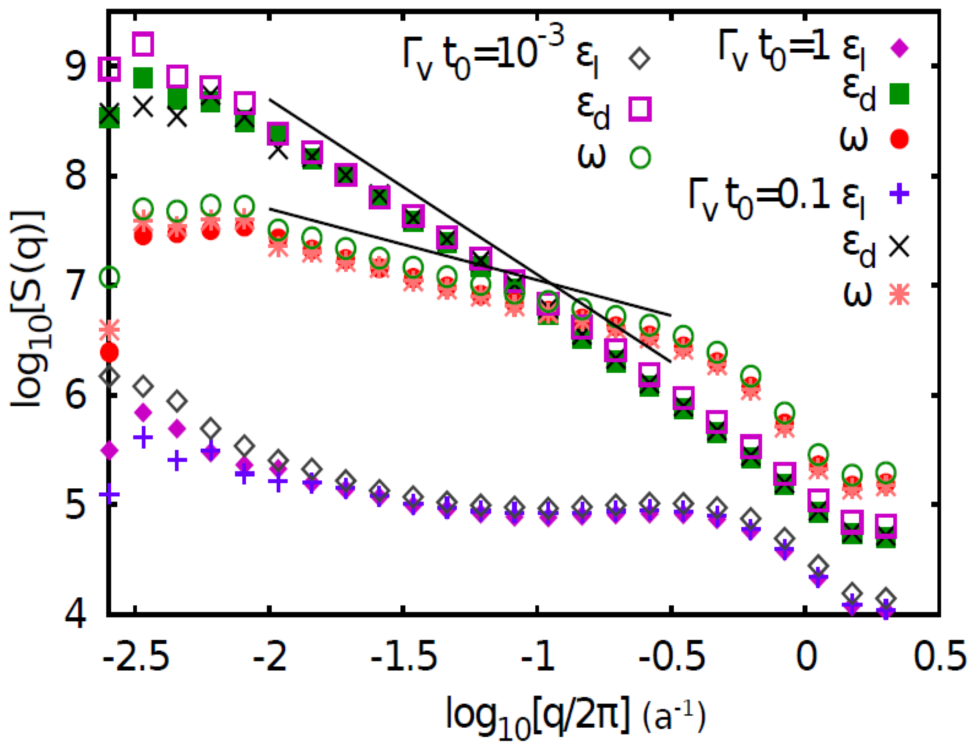
Find $\beta\phi''=2.34$ vs. $\beta\phi'=2$.

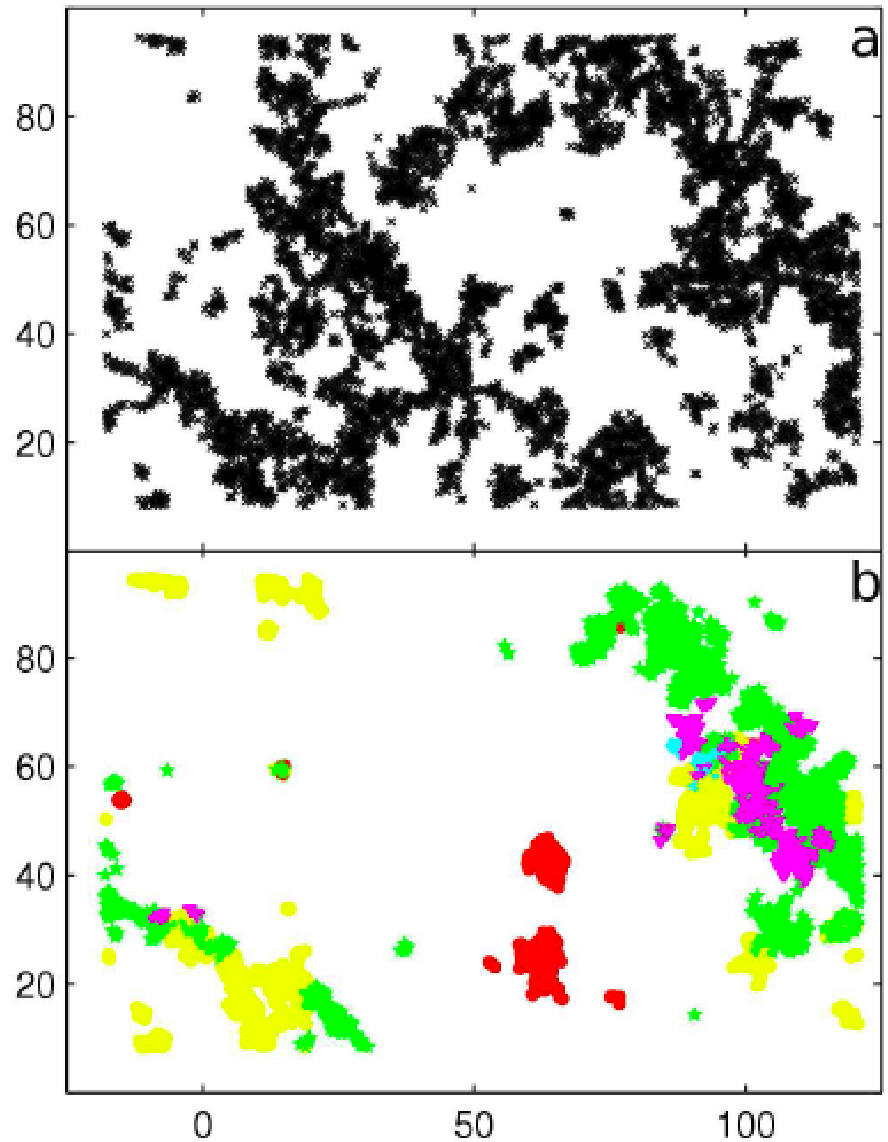
Since $\beta=1.78 \Longrightarrow \nu=1/\phi=0.76$ or 0.89

Plasticity From Same Initial State

Black-underdamped, colors different overdamped earthquakes in quasistatic limit

Same scaling of correlation function over given strain interval Independent of damping!



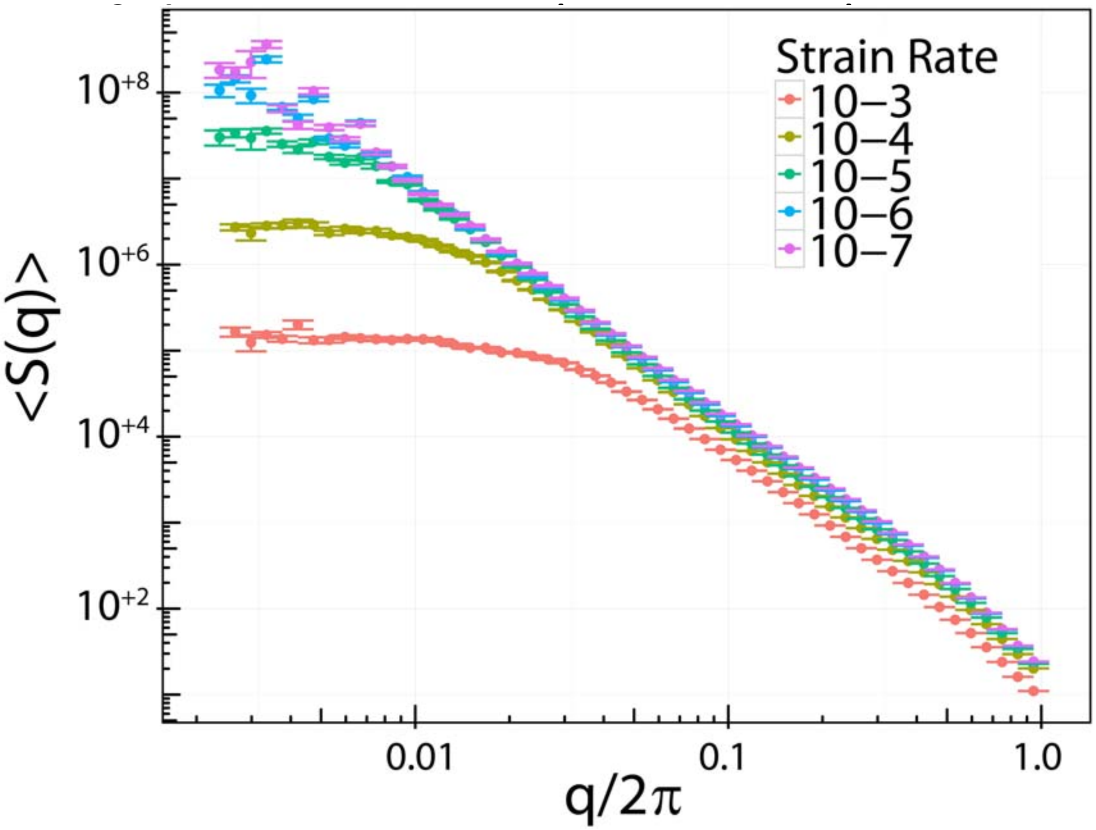


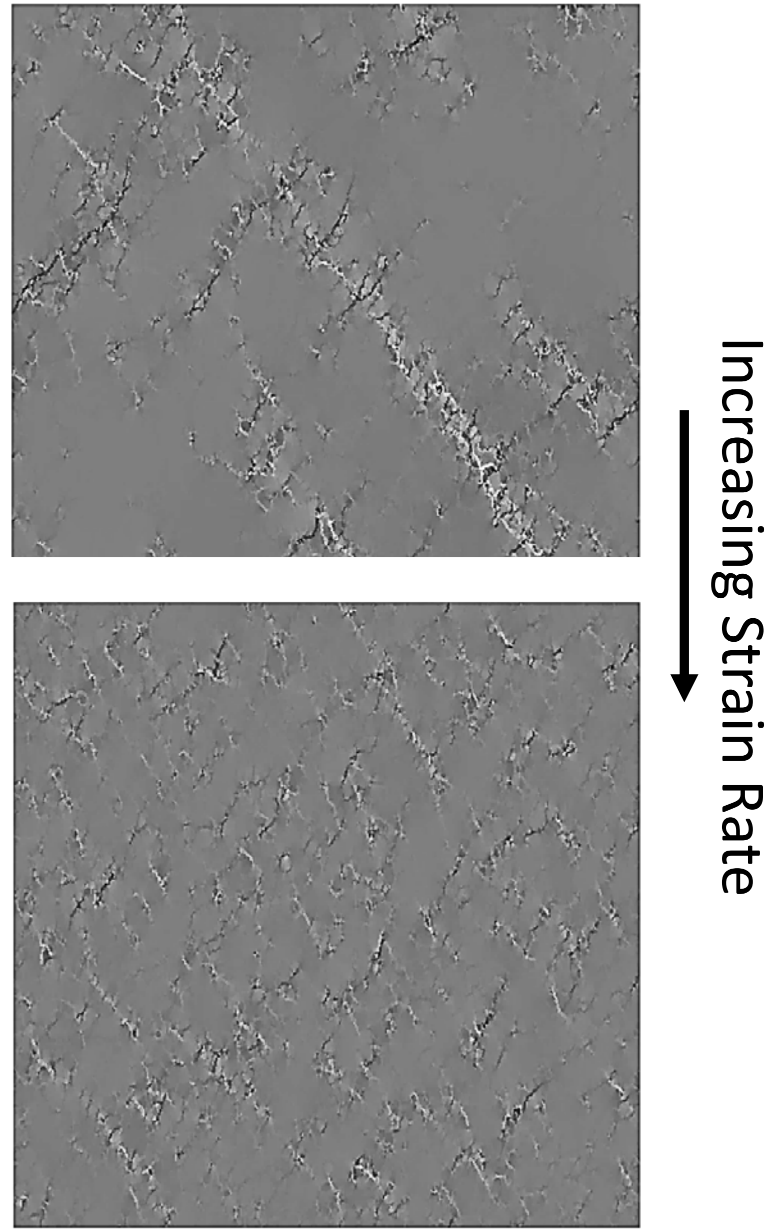
Finite Strain Rate → Limit Spatial Correlations

Pattern of plastic deformation has finite extent

Fourier transform of correlation in nonaffine displacement

$S(q) = \langle |u_x(q)|^2 + |u_y(q)|^2 \rangle$





Finite Strain Rate → Limit Spatial Correlations

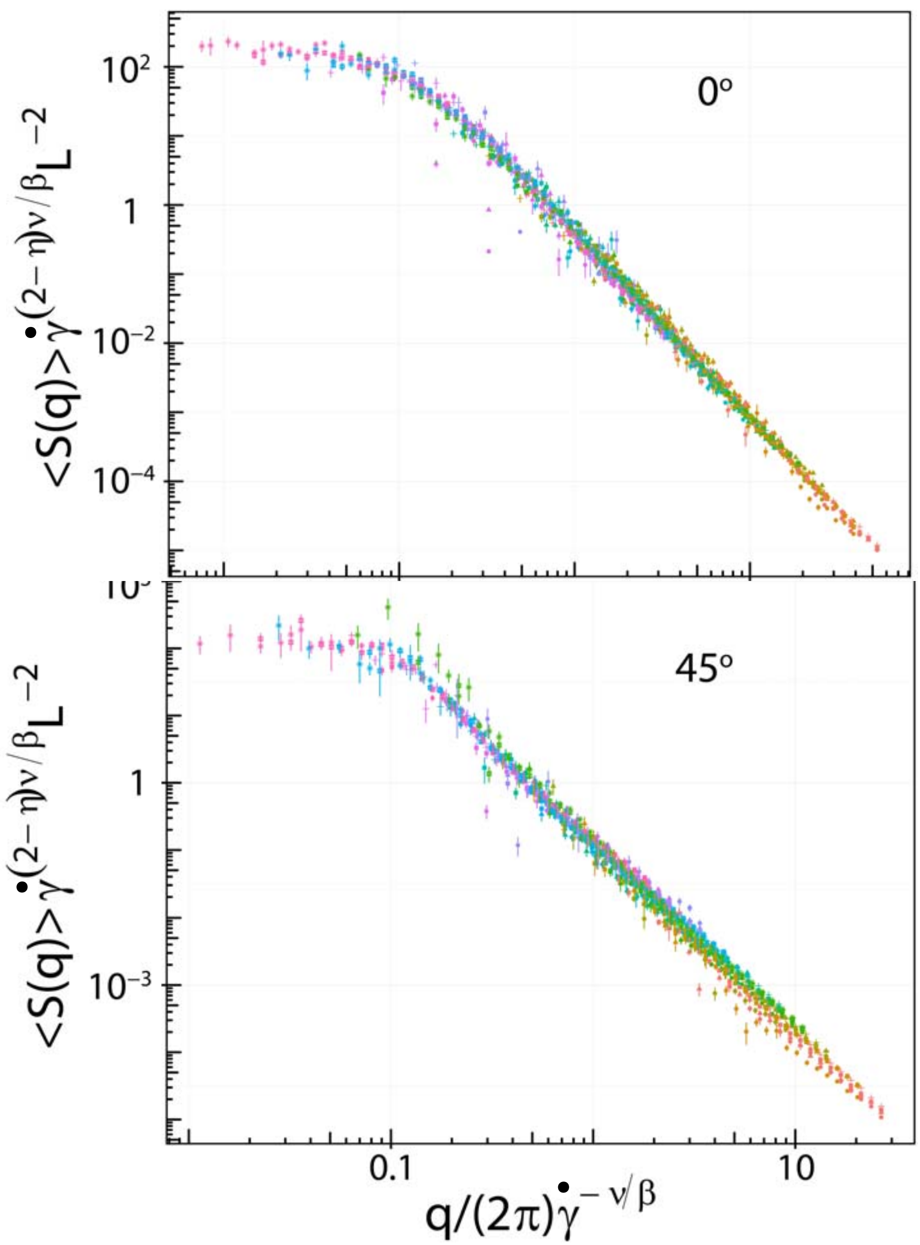
Finite size scaling collapse gives different exponents for different angles

θ=0,90 → ν/β=0.46

θ=±45 → ν/β=0.39

Multiple exponents needed to describe spatial correlations

⟹no single correlation length



Conclusions

- Define R(E,L)=# of events per unit energy per unit strain
Expect: $R(E,L) = L^{b}\, g(E/L^{\alpha})$ and $R(E,L) \sim L^{b+\alpha\tau}\, E^{-\tau}$ for small E
with scaling relations $\gamma = b+\alpha\tau$, $b+2\alpha=2$
- Always find subextensive # of small events $\gamma < d$
$\rightarrow$Large events suppress small events
- Plateau of large events for underdamped, no hysteresis
- Different universality classes in overdamped and underdamped limits - similar τ, very different α and γ
- Separated by critical damping – multicritical point
$\rightarrow$ just enough damping to remove energy as releasee
underdamped – unstable drop in stress with rising rate
- Overdamped – finite size scaling of dynamics gives β=1.78
Well-defined avalanche size, but multiple length scales
different exponents along 0,90 than +/-45
- All cases – location correlates over many avalanches