Criticality in martensites: plasticity or inertia?

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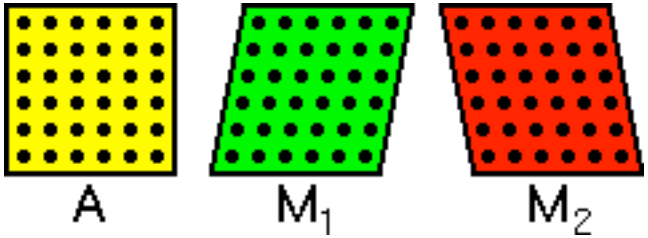
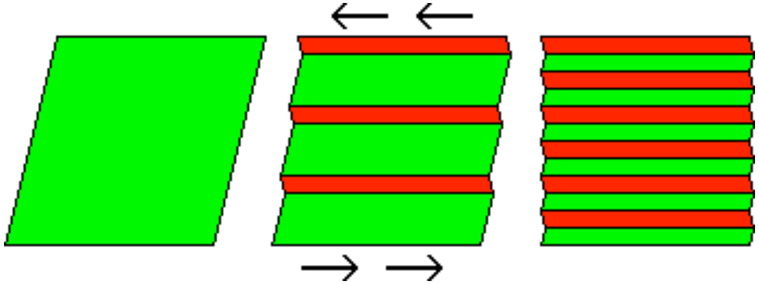
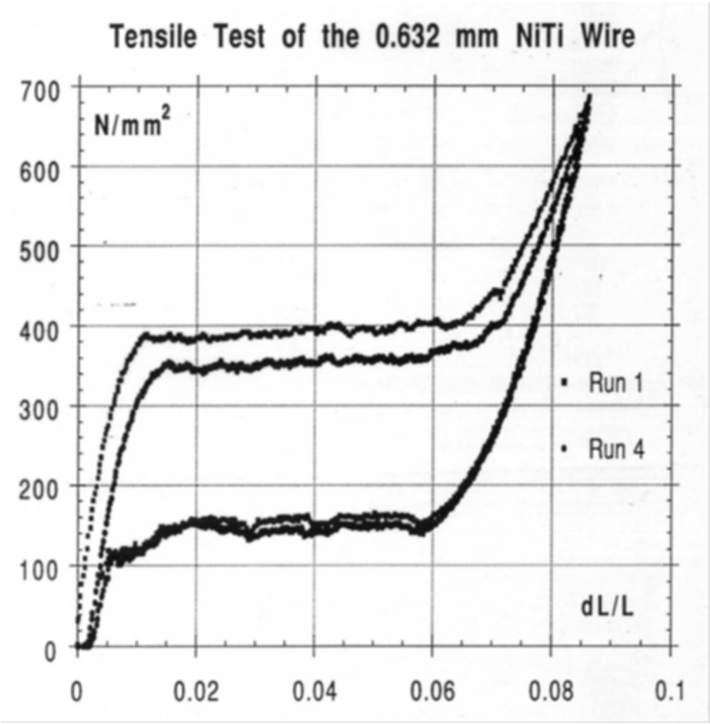
Joint work with F. Perez-Reche, G. Zanzotto, A. Finel and U. Salman

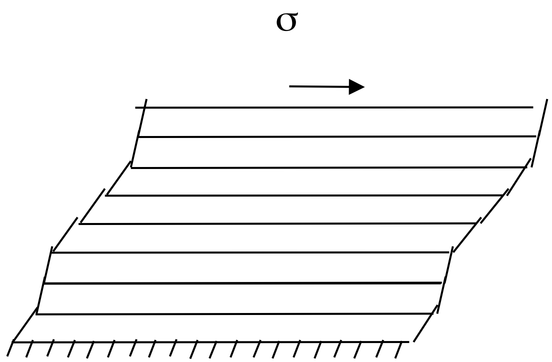
Martensitic phase transitions

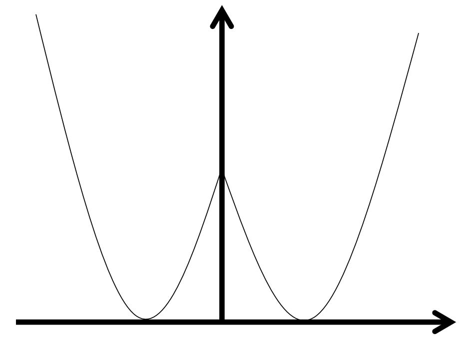


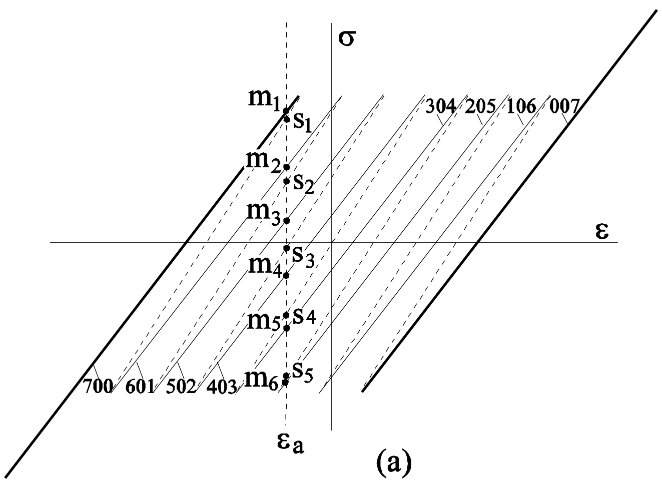




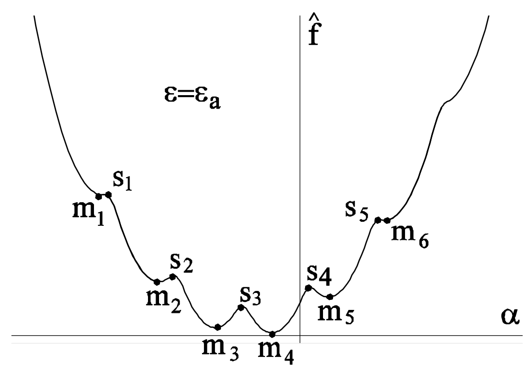
Double well energy







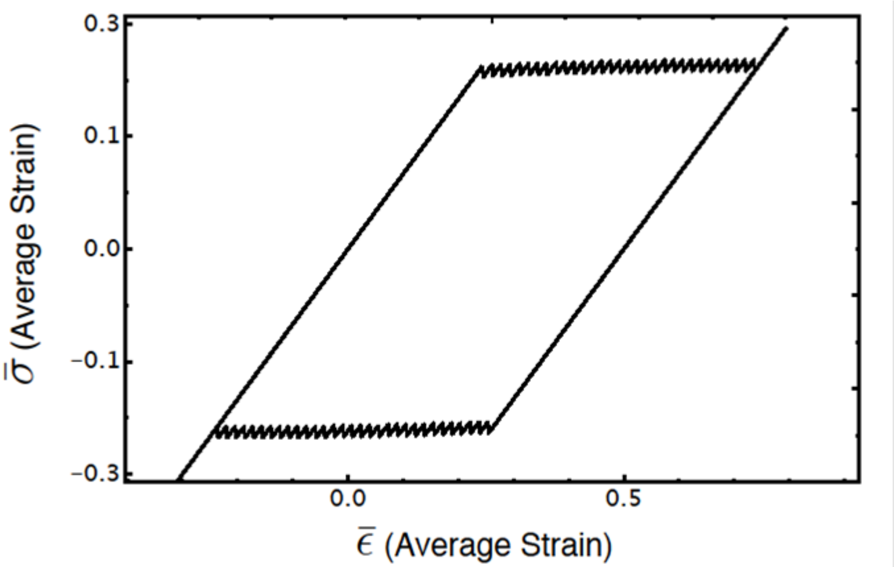
(a)

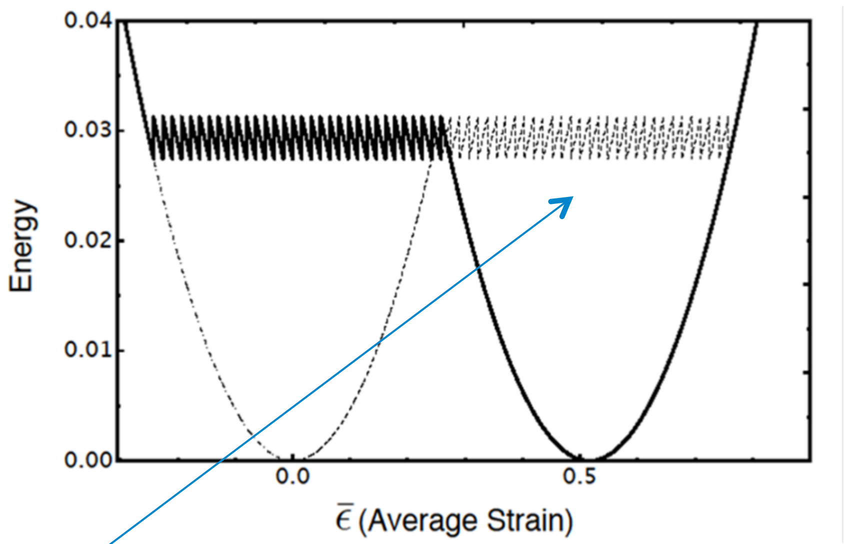


(b)

Over-damped dynamics

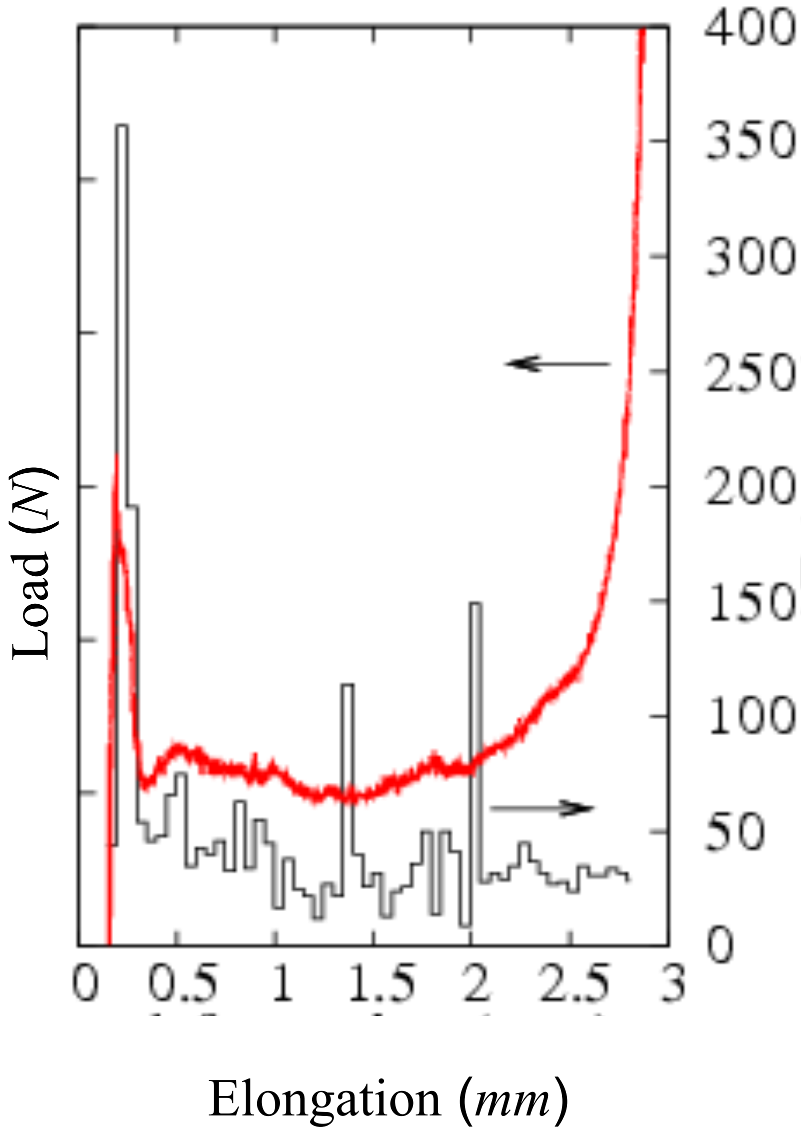
$$\nu\, \dot{\varepsilon}_i = -\frac{\partial \Phi}{\partial \varepsilon_i} + \sigma \qquad \Phi = N^{-1} \sum_{i=1}^{N} \left(f_0(\varepsilon_i) - \sigma\, \varepsilon_i \right)$$





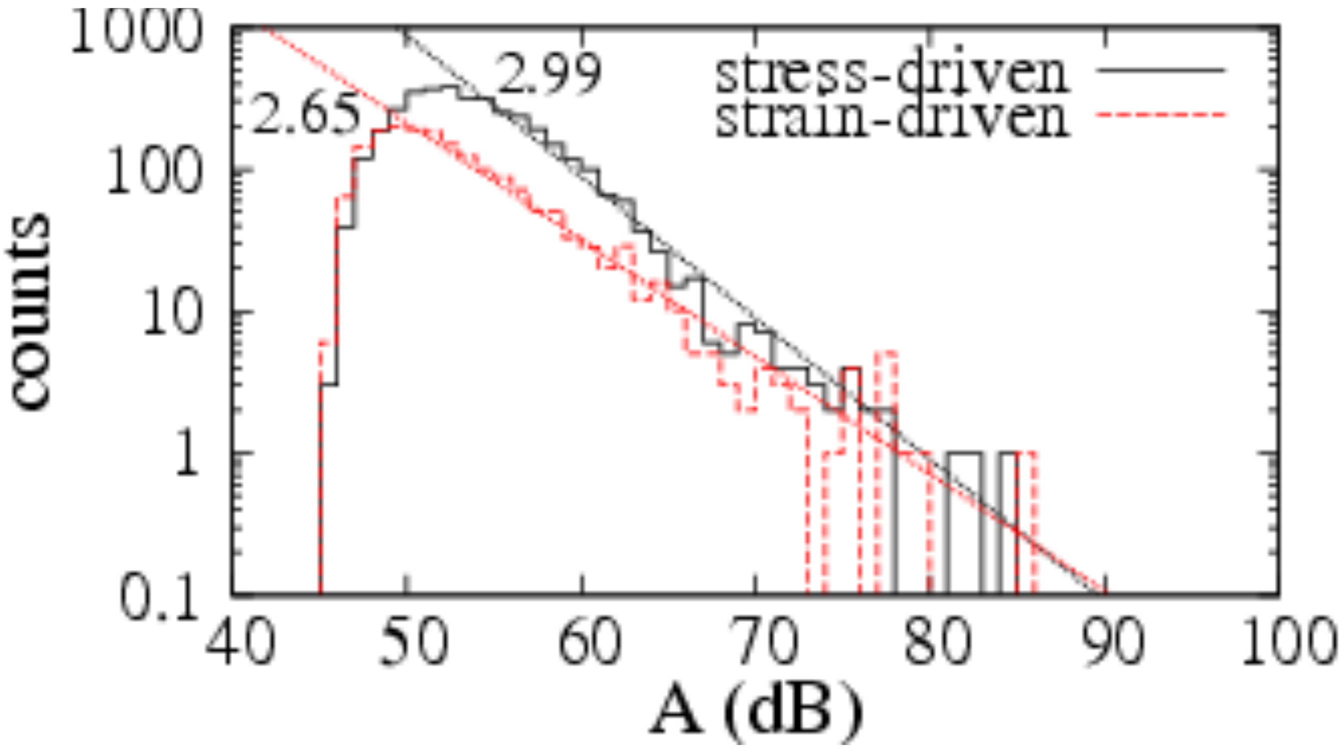
Marginally stable states

Martensitic phase transitions



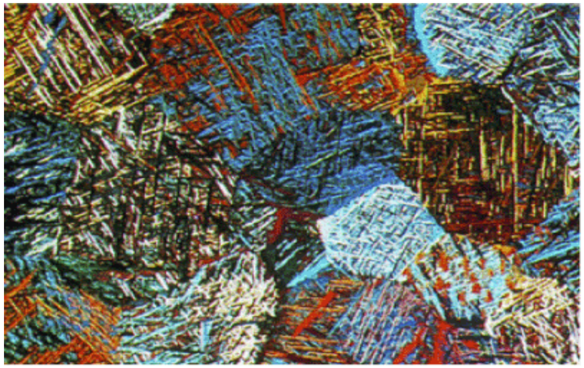
E.Vives et al,
PRB, 2009

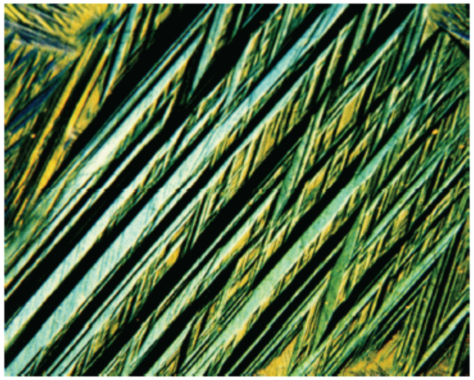
Statistics of avalanches

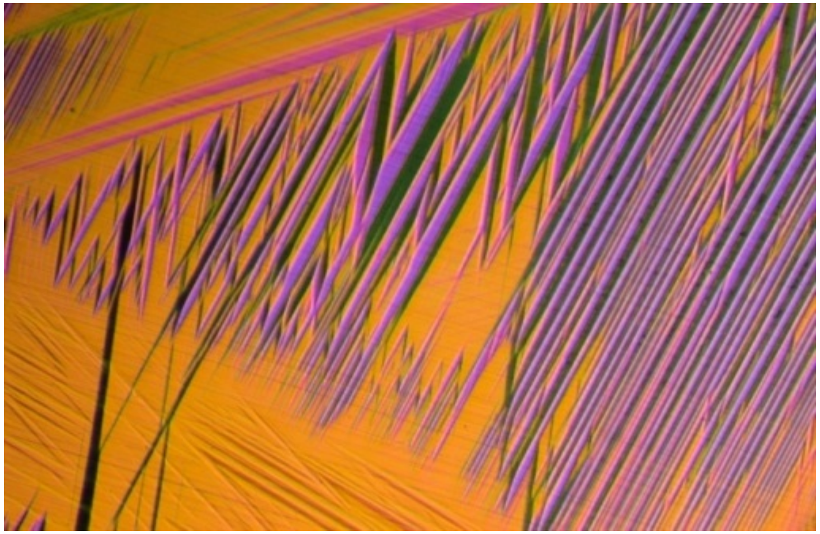


E.Vives et al, PRB, 2009

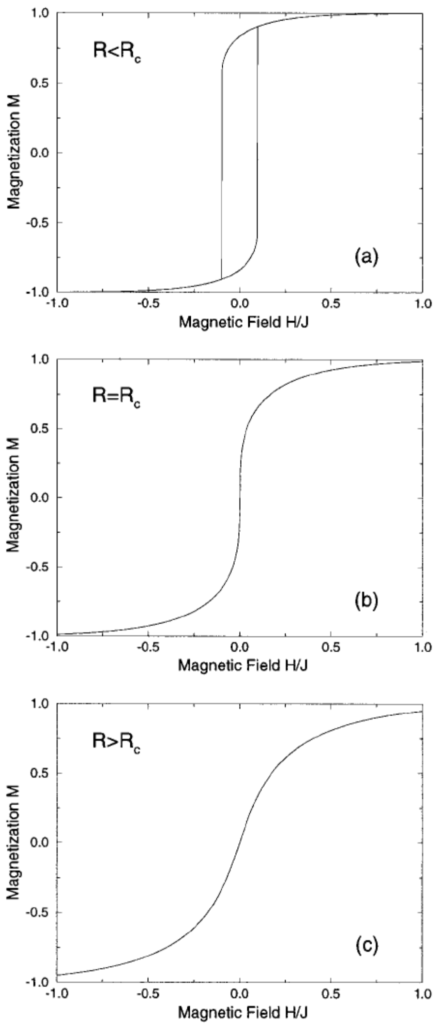
Spatial complexity

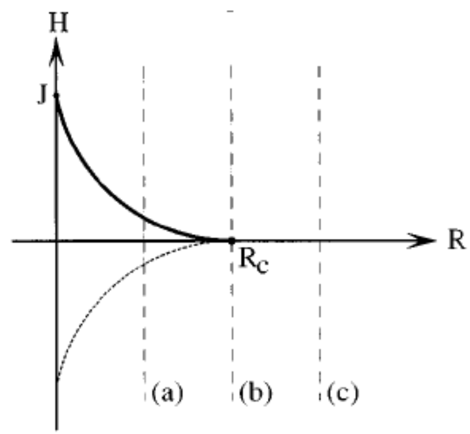


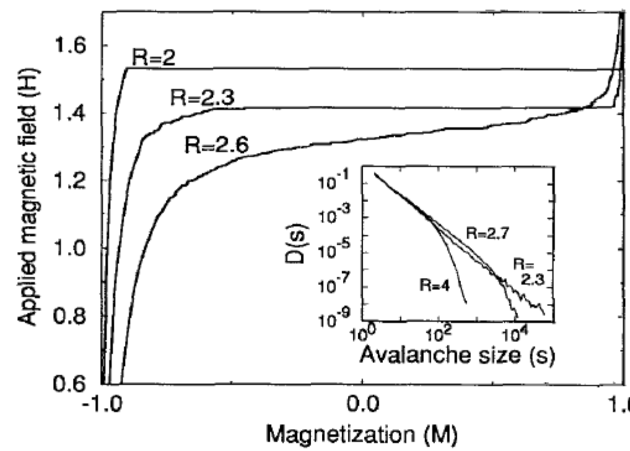




Random Field Ising model







$$\mathscr{H} = -\sum_{ij} J_{ij} s_i s_j - \sum_i (H + f_i) s_i$$

$$\rho(f_i) = \frac{1}{\sqrt{2\pi R}} \exp\left(-\frac{f_i^2}{2R^2} \right)$$

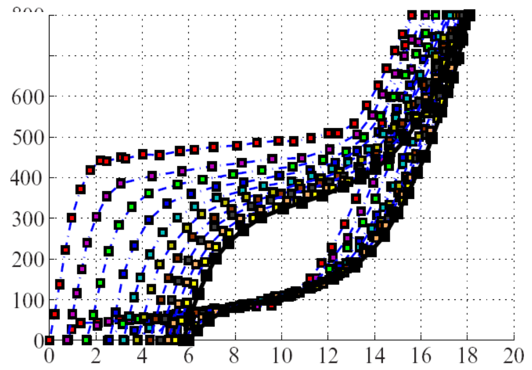
J. Sethna et al, PRL, 1993

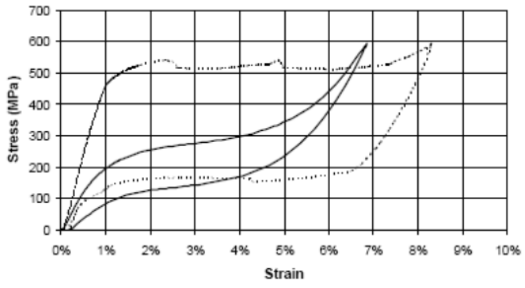
How the system finds the critical point?

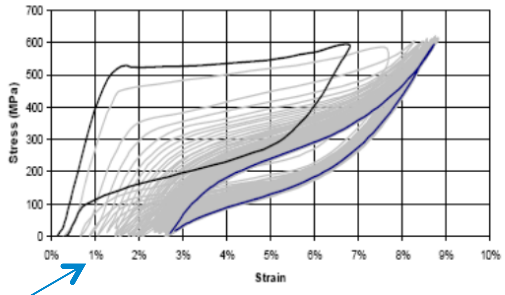
- Critical domain is anomalously large
- Limited plasticity
- Inertia

Plasticity?

Training







residual strain

Local energy

$$\boxed{\phi = \phi(\mathbf{C}; T)} \qquad C_{ab} = \mathbf{u}_a \cdot \mathbf{u}_b$$

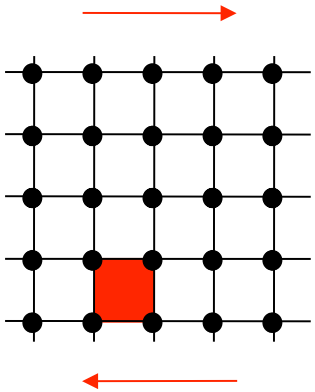
Invariant with respect to the change between lattice bases

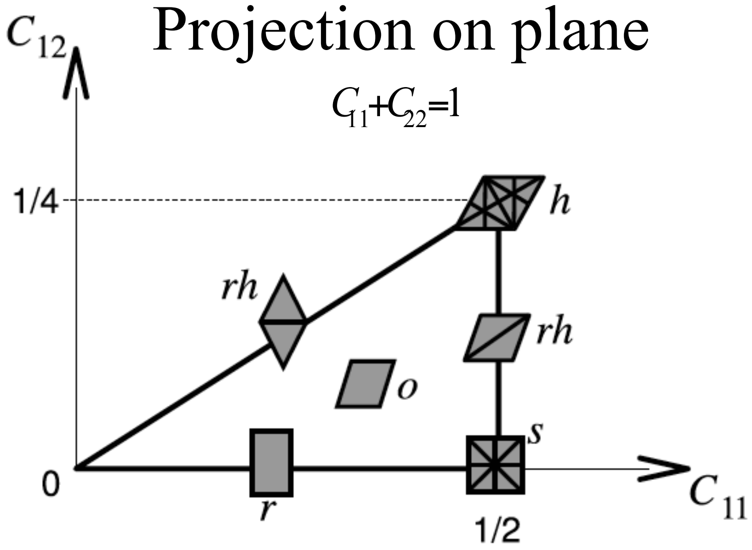
$$GL(2, \mathbb{Z}):$$

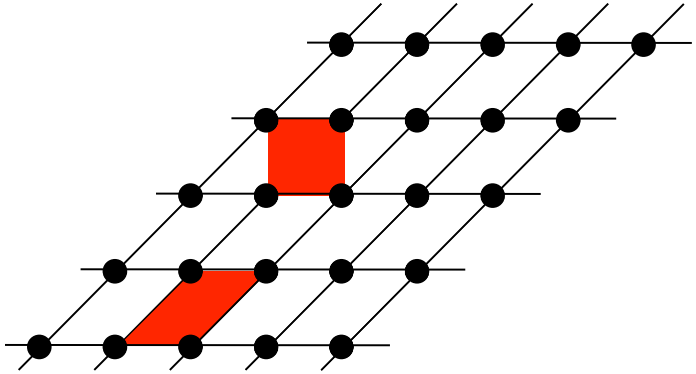
2×2 invertible matrices with integral entries

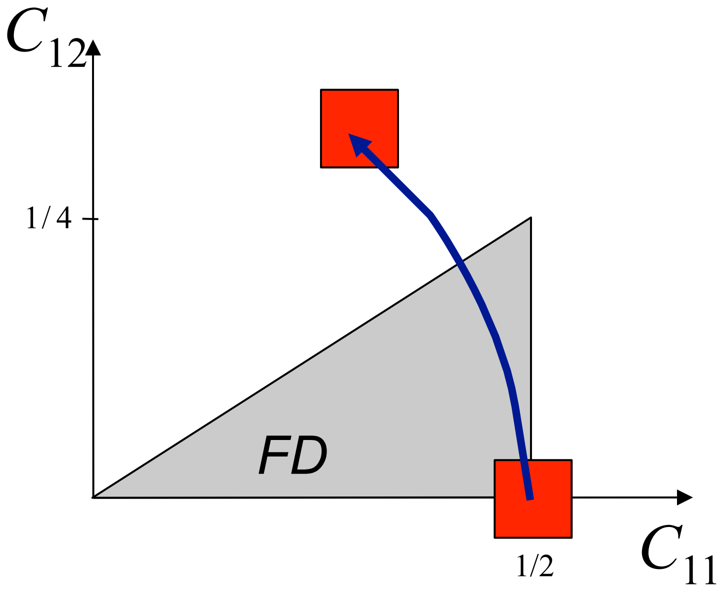
$$\phi(\mathbf{C}; T) = \phi(\mathbf{M}^t \mathbf{C} \mathbf{M}; T)$$

Plastic deformation

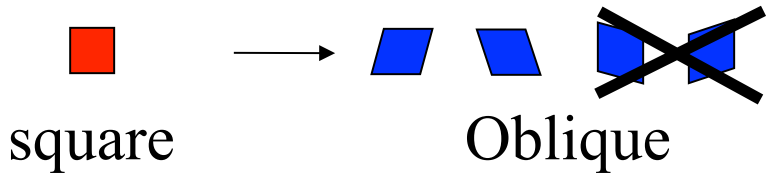


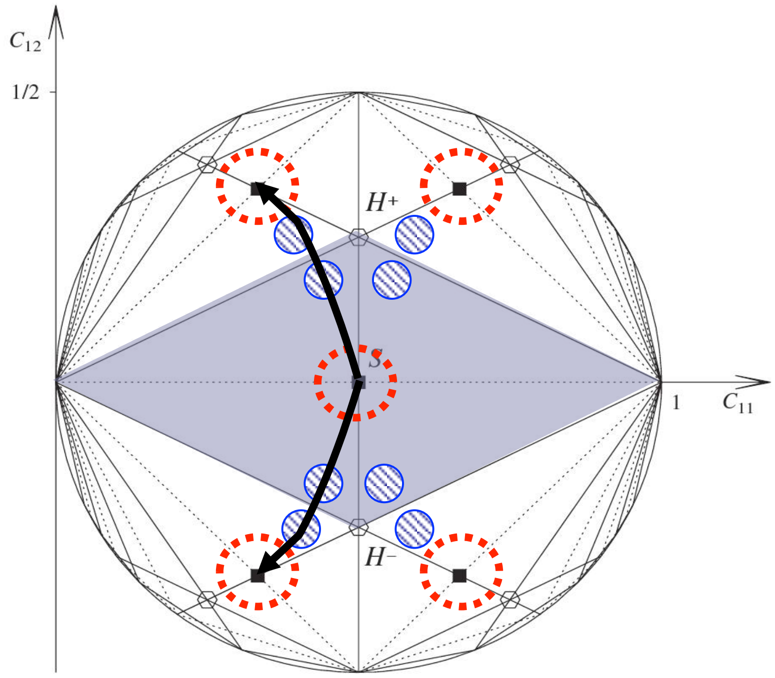






Energy structure





- Deformation along the path

$$\{e(\mathbf{C}), e'(\mathbf{C}), e''(\mathbf{C})\}$$

OP: Order parameter

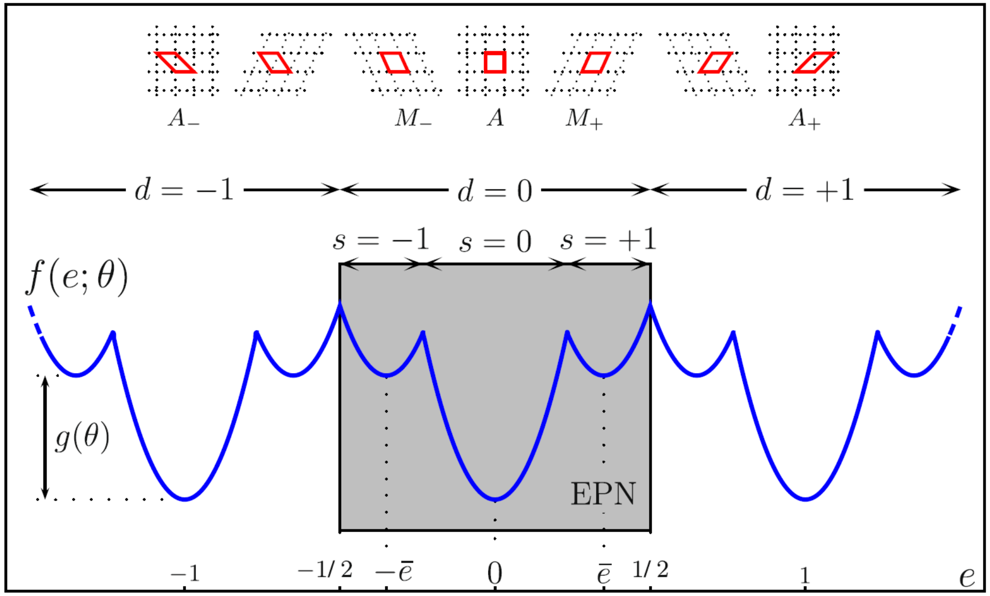
- Elimination of e' and e''
- Energy in terms of OP

$$\widetilde{\Phi}(e;\theta) = \frac{1}{2}\sum_{i,j=1}^{N} K_{ij} e_i e_j + \sum_{i=1}^{N} f(e_i;\theta)$$

elastic interaction

local energy elastic units

Energy accounting of plastic deformation



$$\widetilde{\Phi}(e;\theta) = \sum_{i=1}^{N} f(e_i;\theta) + \frac{1}{2}\sum_{i,j=1}^{N} K_{ij} e_i e_j$$

$$f(e;\theta) = \tfrac{1}{2}(e - w)^2 + g(\theta)s^2 \qquad\qquad w_i \quad (= d_i + \bar{e}s_i)$$

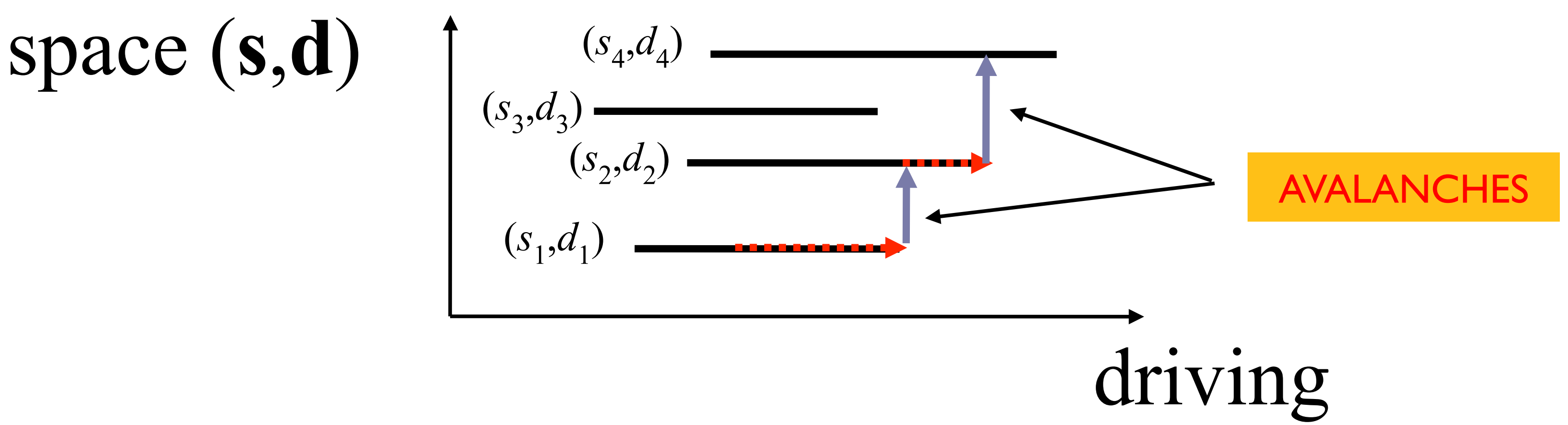
Dynamics of the order parameter

$$\frac{\partial \mathbf{e}}{\partial t} = -\gamma \frac{\partial \widetilde{\Phi}(\mathbf{e};\theta)}{\partial \mathbf{e}}$$

Slow driving: $\gamma \to \infty$

Dynamics projects as an automaton on the local minima of $\widetilde{\Phi}$

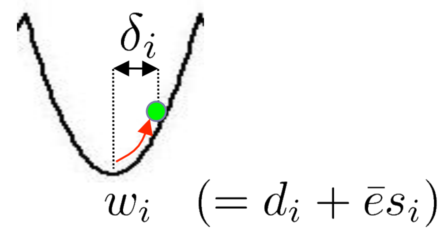
$$\frac{\partial \widetilde{\Phi}}{\partial \mathbf{e}} = 0 \;\Rightarrow\; \mathbf{e}(\mathbf{s},\mathbf{d}) = (\mathbf{1}+\mathbf{K})^{-1}(\mathbf{d}+\bar{e}\mathbf{s})$$

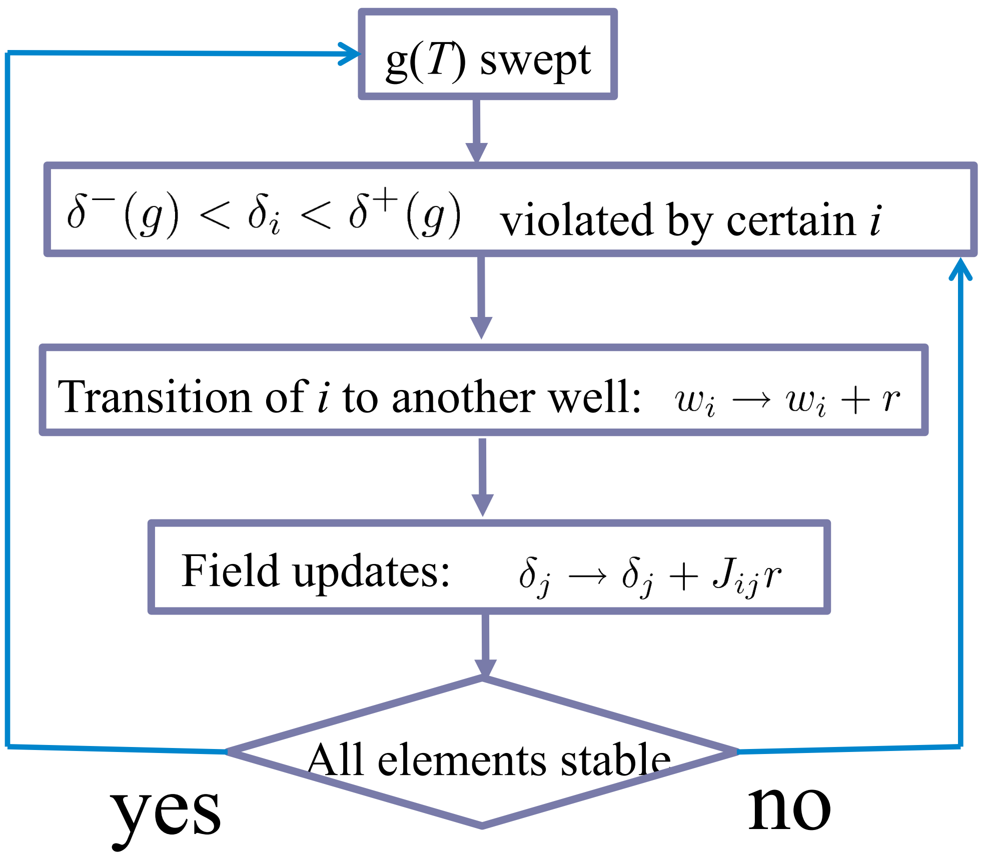


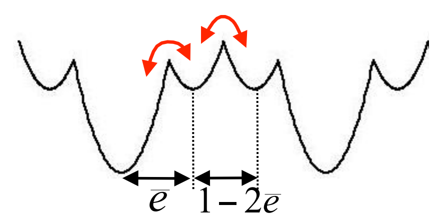
Automaton

$\delta_i = e_i - w_i$ elastic strain

$\delta^-(g) < \delta_i < \delta^+(g)$



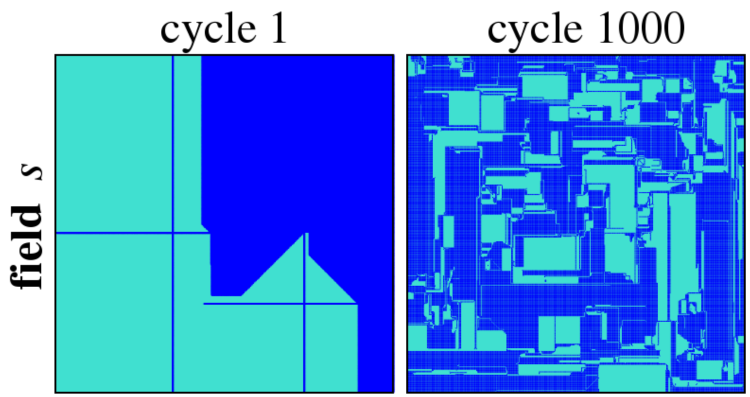




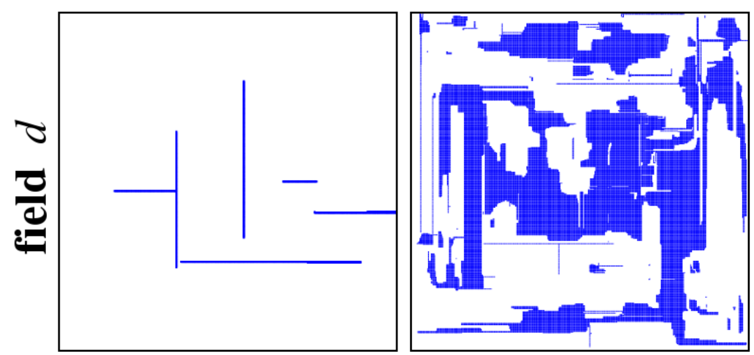
$$r = \begin{cases} \pm\bar{e}, & \text{phase} \\ \pm(1-2\bar{e}), & \text{slip} \end{cases}$$

Microstructure formation

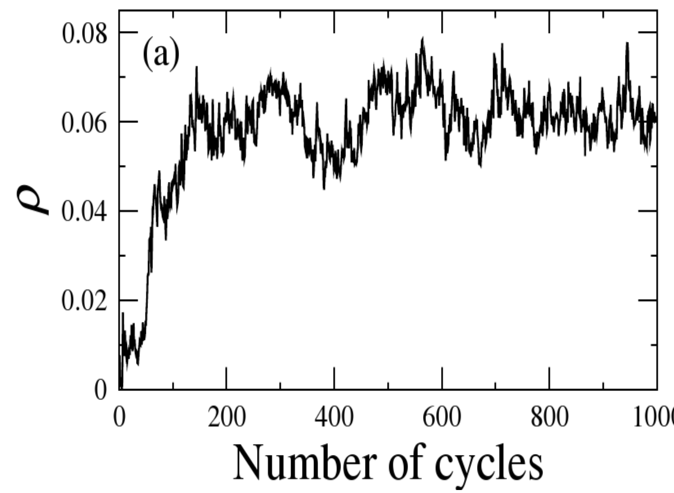
Transformation



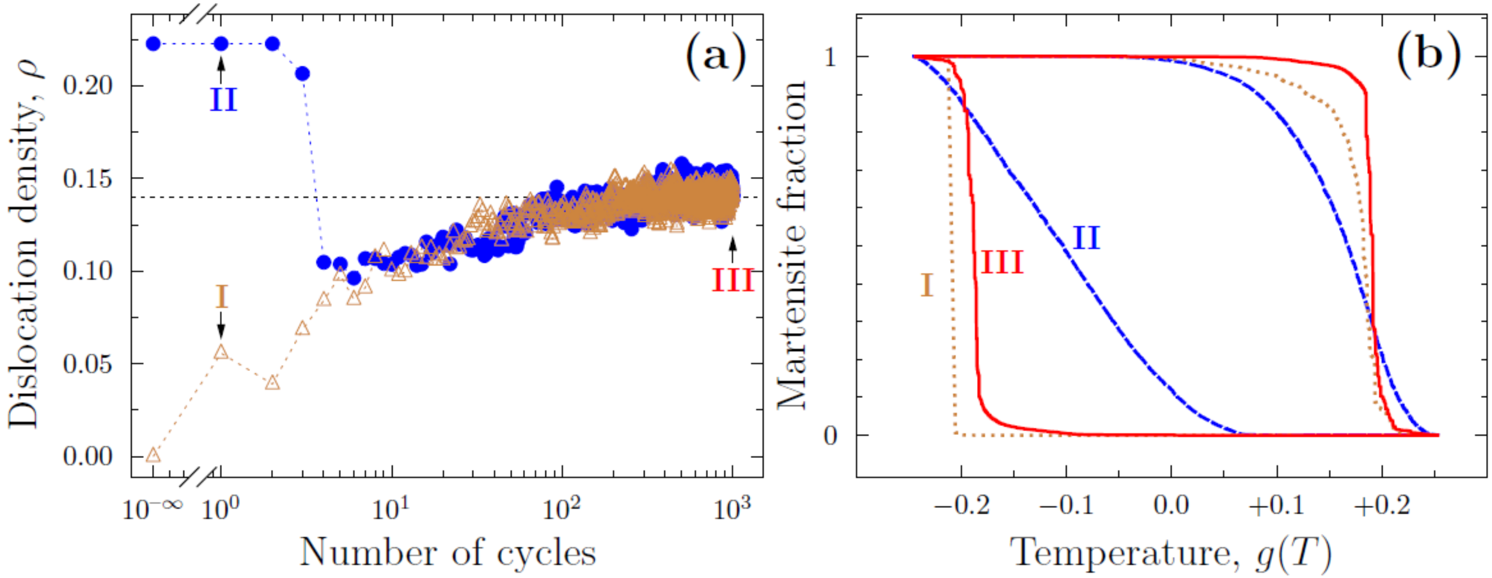
Plastic deformation



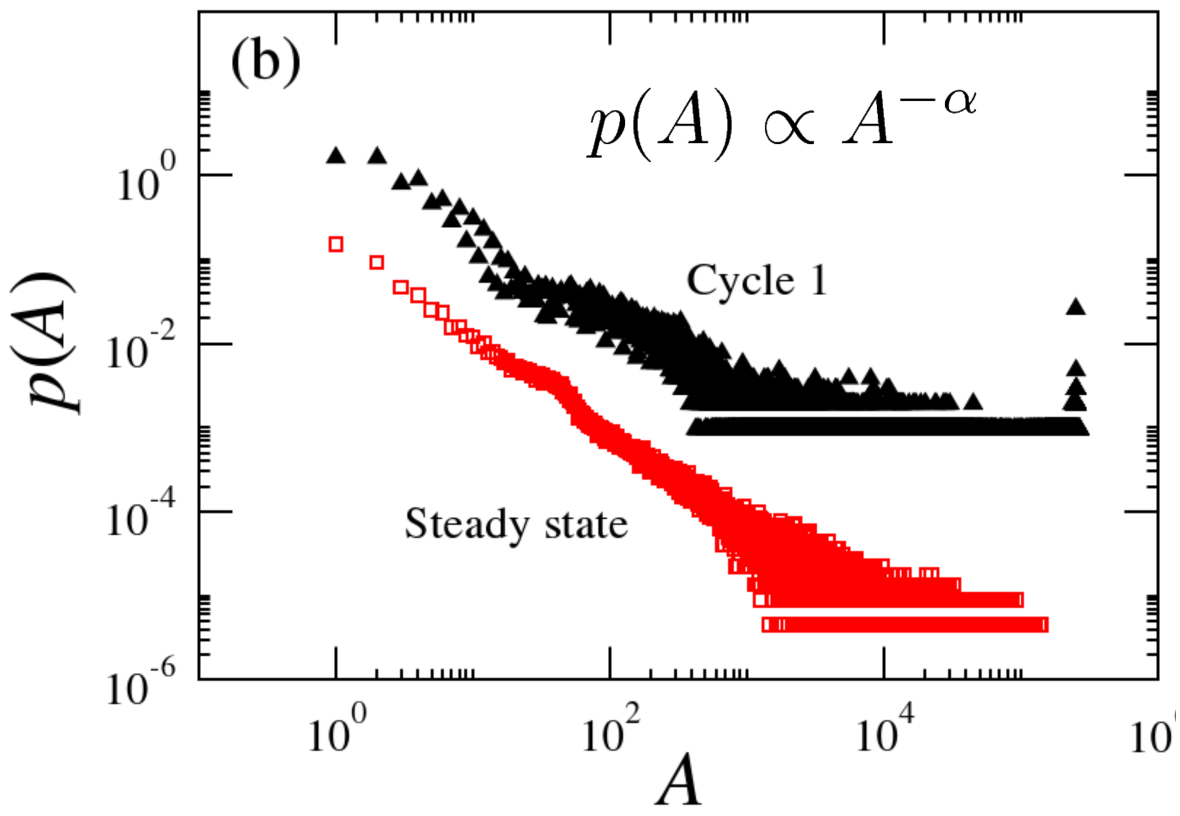
Density of dislocations



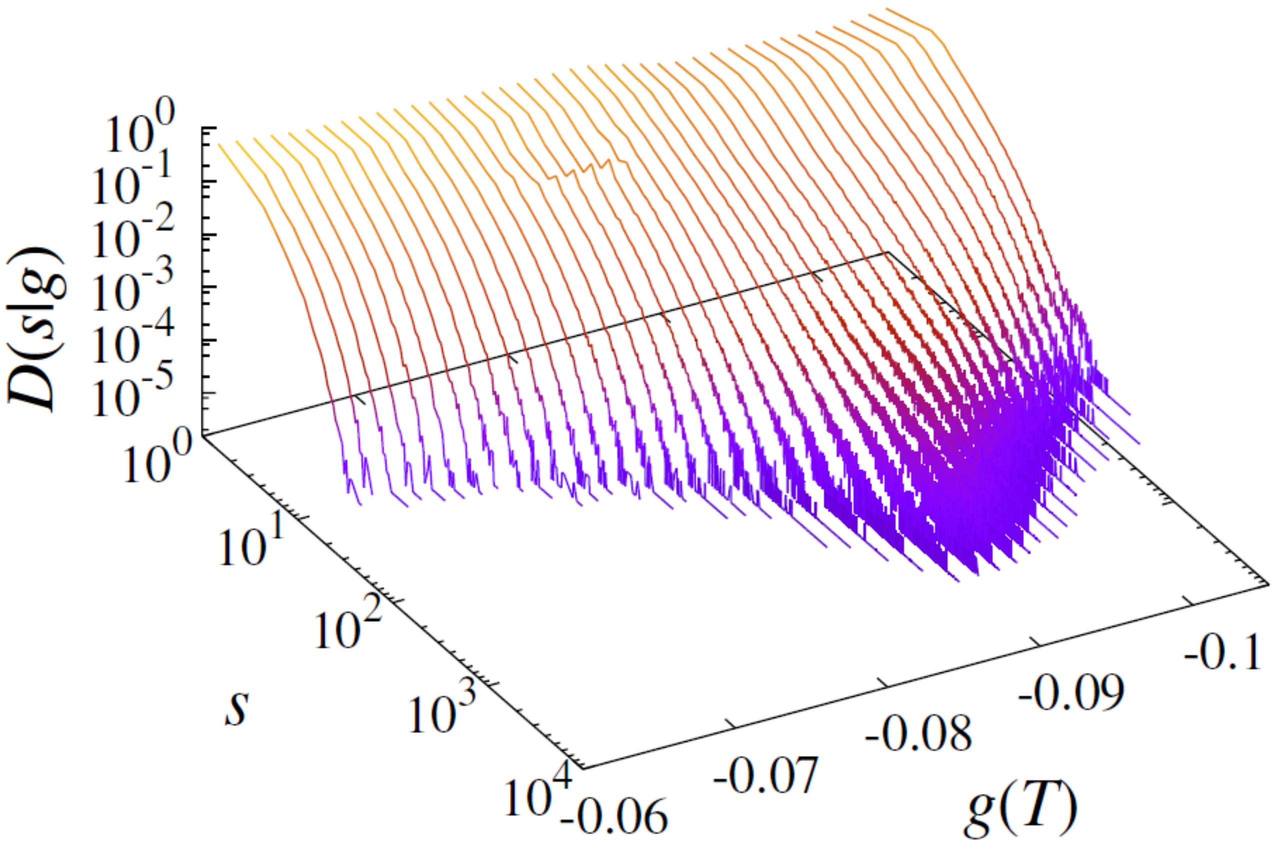
Shakedown state



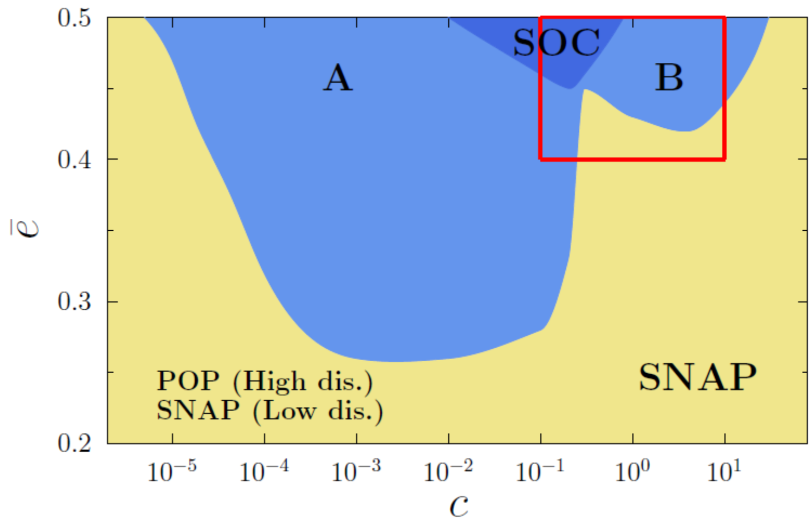
Power law behaviour

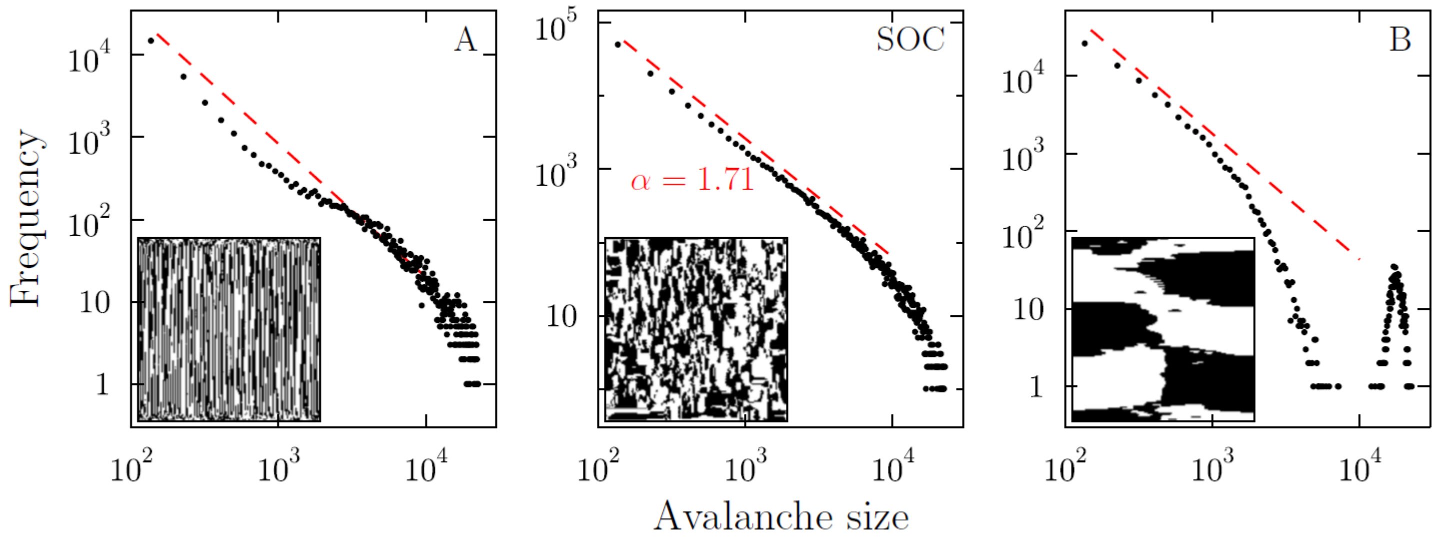


Fluctuation statistics varies along the cycle



Phase diagram

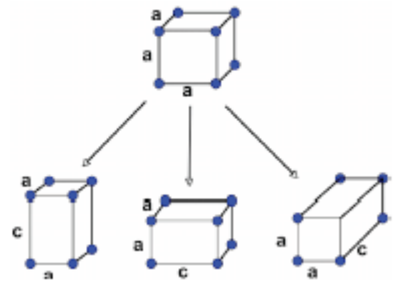




Inertia?

Direct continuum modeling

$$F_{\rm tetra} = A_2(T)\,(\epsilon_2^2+\epsilon_3^2) + A_3\,\epsilon_3(\epsilon_3^2-3\epsilon_2^2) + A_4\,(\epsilon_2^2+\epsilon_3^2)^2$$
$$F_{\rm grad} = \frac{\beta}{2}(|\nabla e_2|^2 + |\nabla e_3|^2)$$

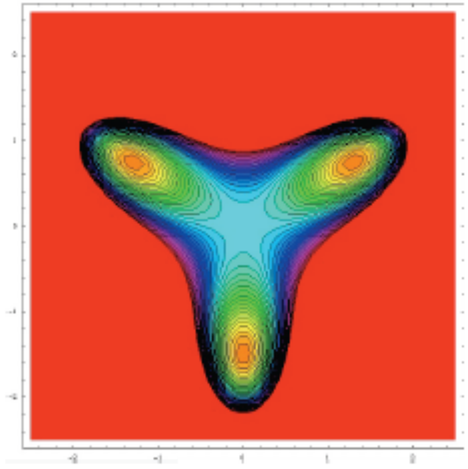


$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$e_2 = (e_{11} - e_{22})/\sqrt{2}$$
$$e_3 = (e_{11} + e_{22} - 2e_{33})/\sqrt{6}$$

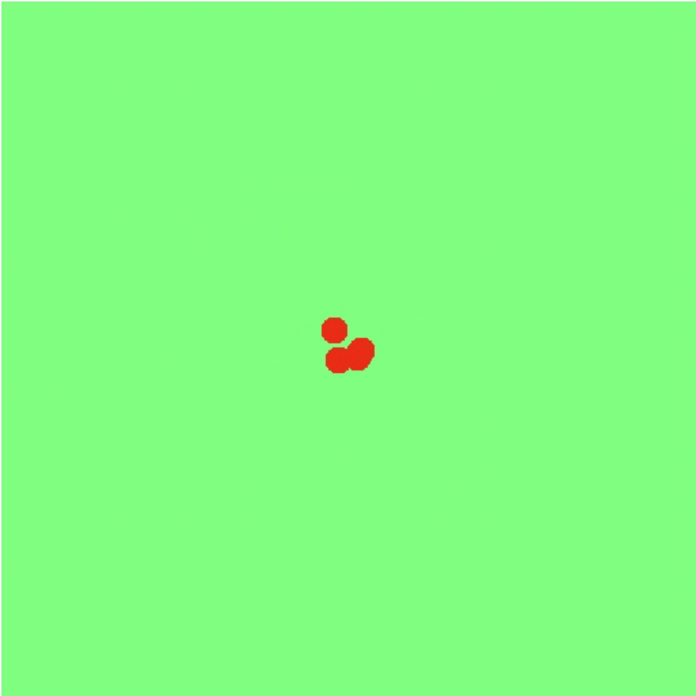
$$T = \frac{1}{2}\rho\,\dot{u_i}^2$$

$$R = \frac{1}{2}\gamma\,(\dot{e_2}^2 + \dot{e_3}^2)$$



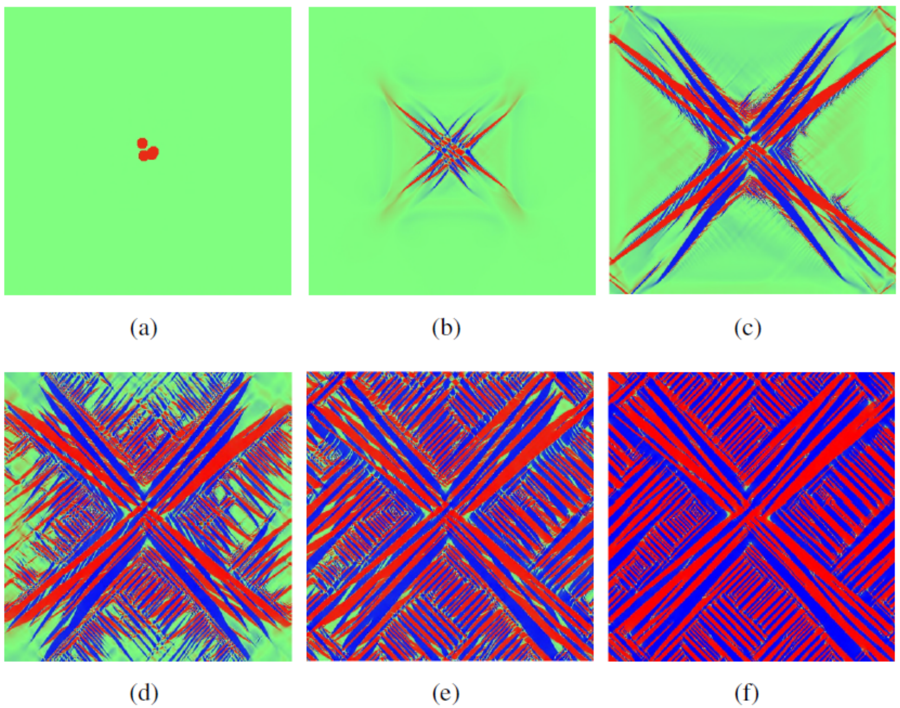
Numerical simulations

over-derdamped



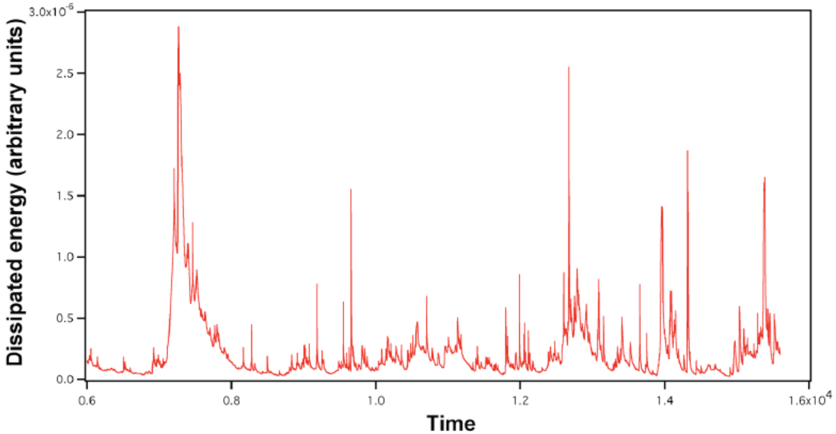
underdamped

Inertia-induced complexity



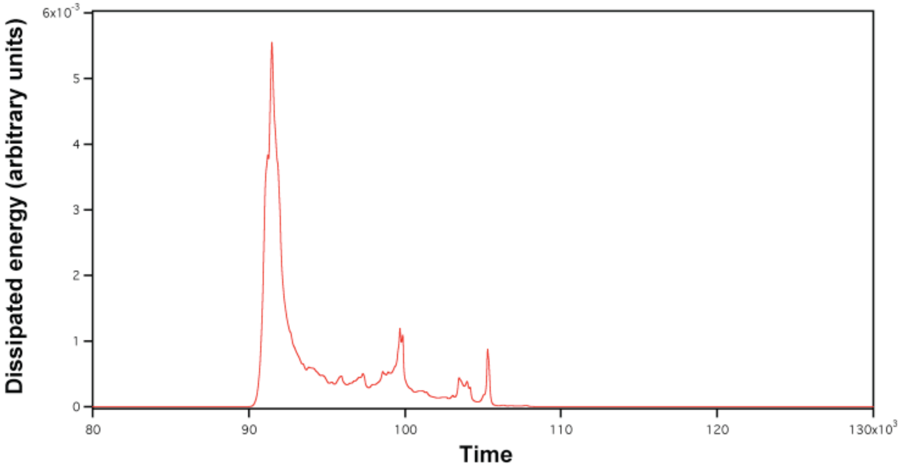
Cu-Zn-Al (3 mm x 2 mm)(Ortin - Lyon INSA)

Acoustic emission

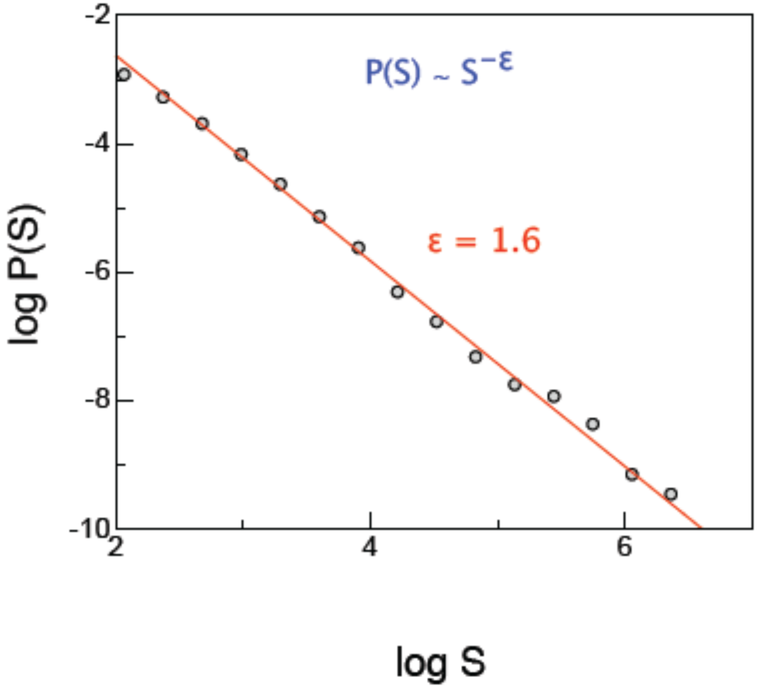


Inertial dynamics

Overdamped dynamics

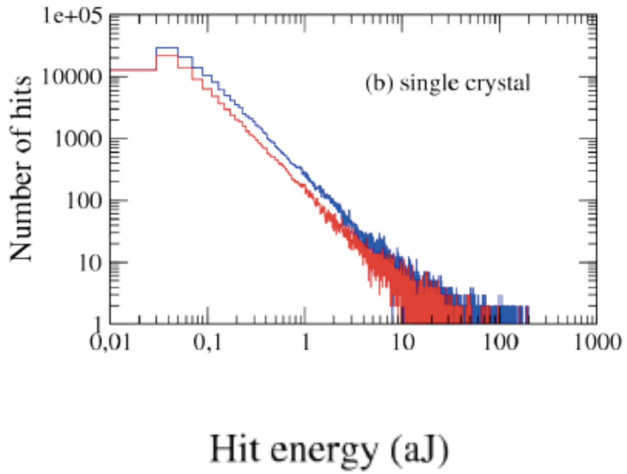


Critical regime



	ϵ
experiment	1.64 ± 0.1
simulation	1.60 ± 0.1

comparison with AE experiment on $Fe_{68.8}Pd_{31.2}$
(E. Bonnot et al., PRB, 2009)



Mass-spring chain

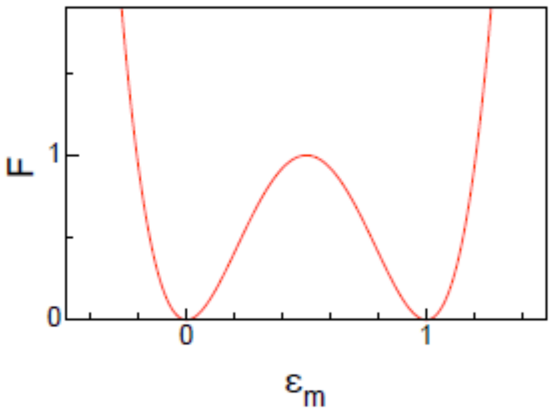
$$\rho \ddot{u}_i = \frac{1}{\epsilon}\left[\phi'(\frac{u_{i+1}-u_i}{\epsilon}) - \phi'(\frac{u_i - u_{i-1}}{\epsilon}) + \frac{\gamma}{\epsilon}(\dot{u}_{i+1} + \dot{u}_{i-1} - 2\dot{u}_i) + \frac{\mu}{\epsilon}(u_{i+2} + u_{i-2} - 2u_i)\right]$$

inertia NN interaction viscosity NNN interaction

Quasi-static driving

$$u_0 = u_1 = 0$$

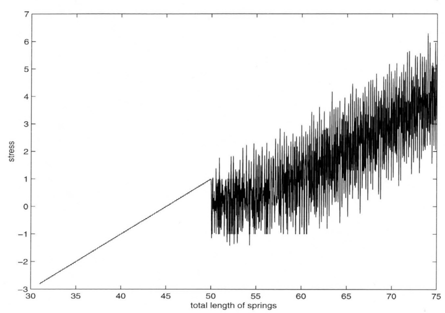
$$\dot{u}_{N-1} = \dot{u}_{N-2} = v$$

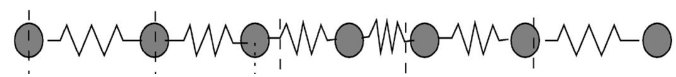


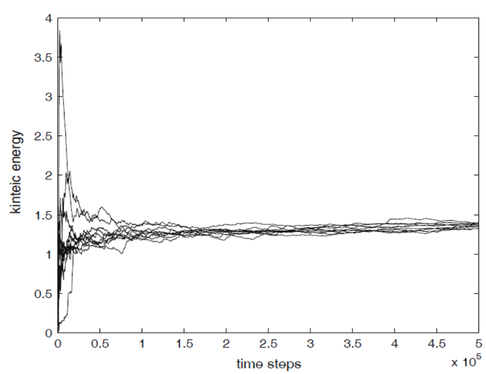
NN potential

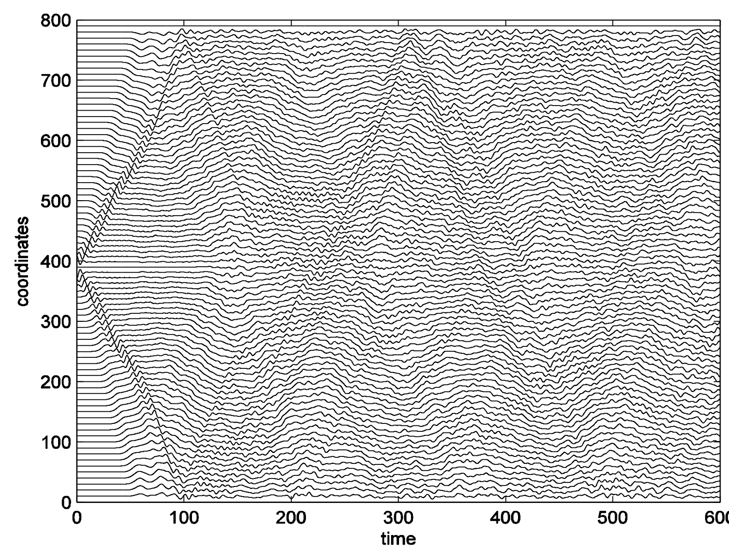
Hamiltonian system (FPU)

$$\rho \ddot{u}_i = \frac{1}{\epsilon}\left[\phi'\left(\frac{u_{i+1}-u_i}{\epsilon}\right)-\phi'\left(\frac{u_i-u_{i-1}}{\epsilon}\right)+\frac{\gamma}{\epsilon}(\dot{u}_{i+1}+\dot{u}_{i-1}-2\dot{u}_i)+\frac{\mu}{\epsilon}(u_{i+2}+u_{i-2}-2u_i)\right]$$

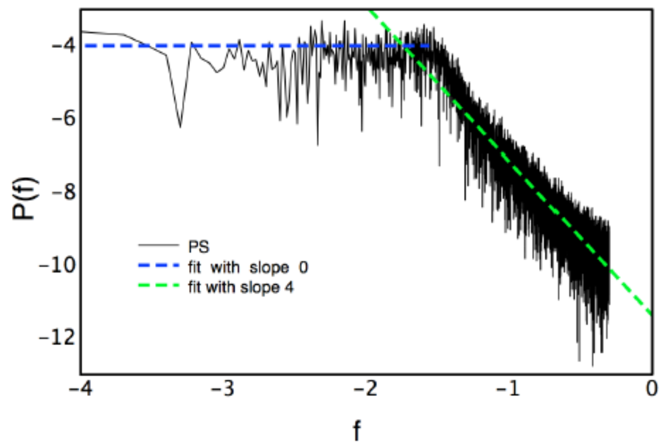


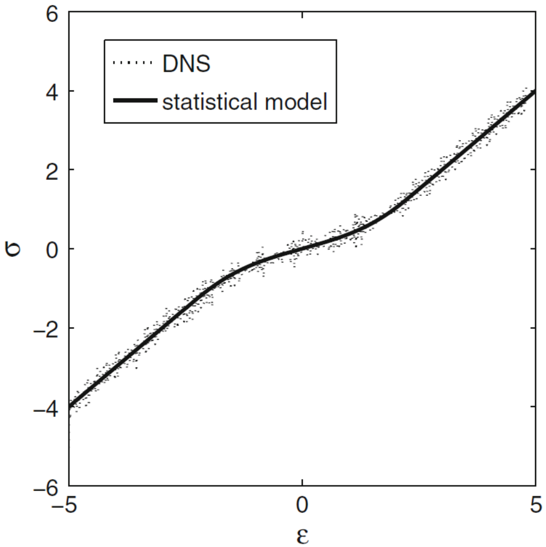






Thermodynamic behavior

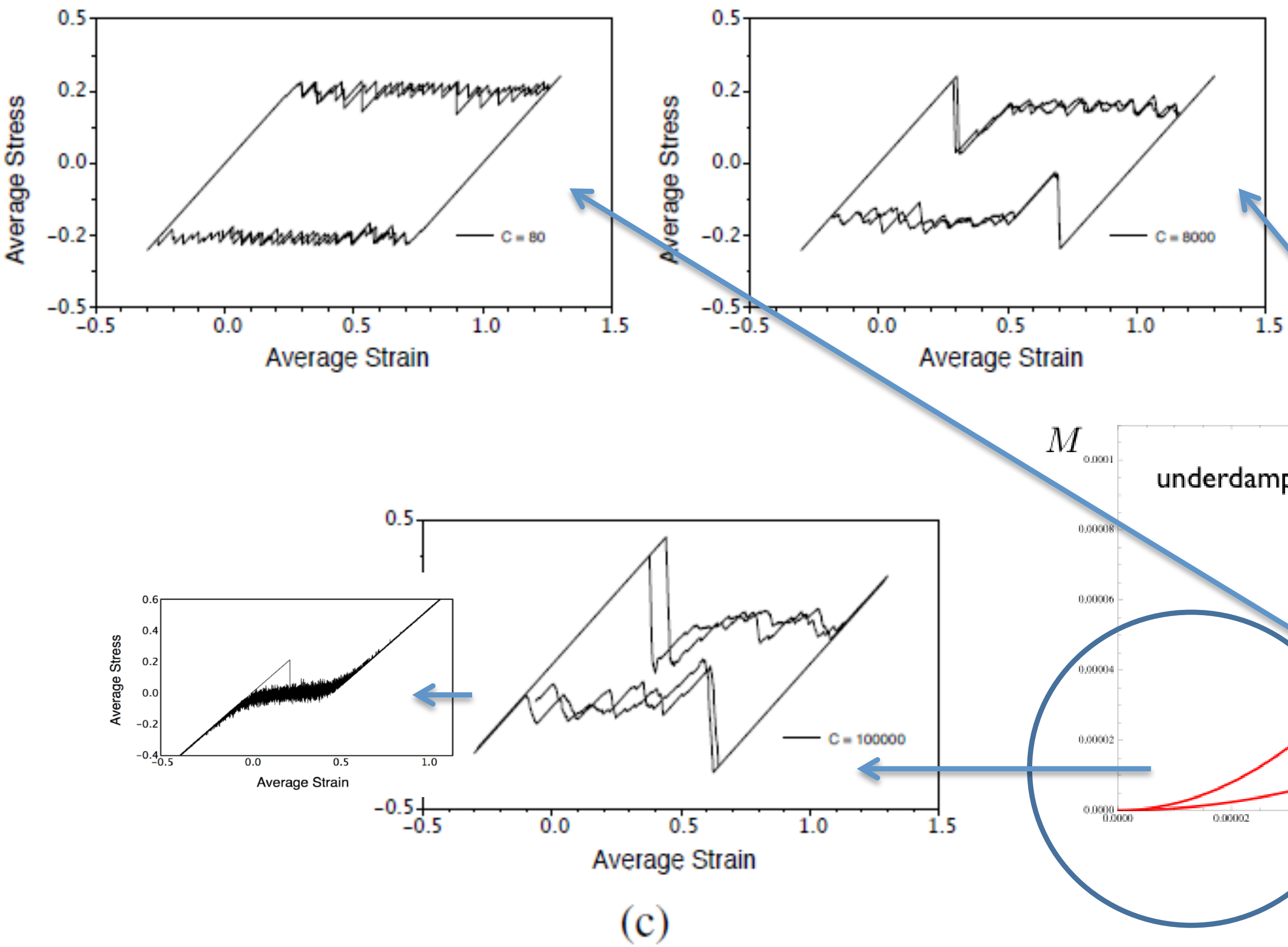




$$E_{\mathrm{p}} = \sum_{i=1}^{N} \frac{\mu_i}{2} (\epsilon_i + \alpha_i)^2$$

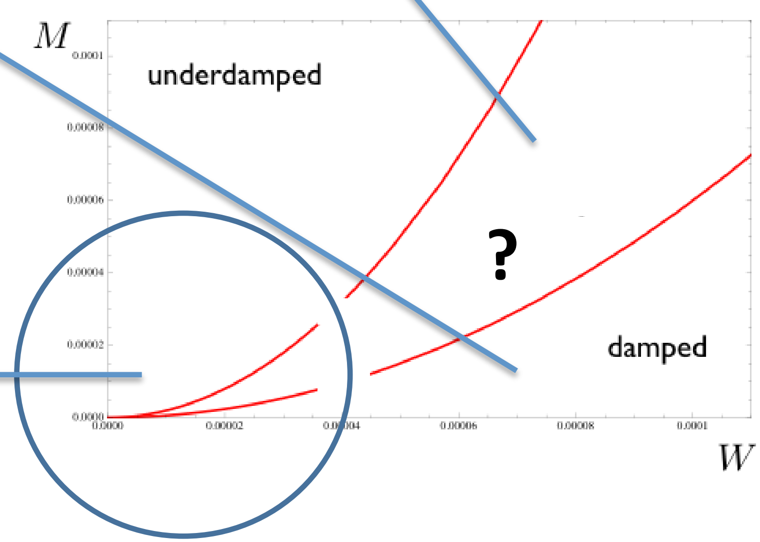
$$Z_{\mathrm{p}}(\bar{\epsilon}, \beta) = N^{-1/2} \frac{(2\pi)^{\frac{N-1}{2}}}{\beta^{\frac{N-1}{2}}} \sum_{p=0}^{N} \frac{N!}{p!(N-p)!} (\mu_1^{p/N} \mu_2^{1-p/N})^{-N/2} \left(\frac{p/N}{\mu_1} + \frac{1-p/N}{\mu_2} \right)^{-1/2}$$
$$\times \exp\left(-\frac{\beta}{2} N \left(\frac{p/N}{\mu_1} + \frac{1-p/N}{\mu_2} \right)^{-1} \left(\bar{\epsilon} + \frac{p}{N} a_1 + \left(1 - \frac{p}{N} \right) a_2 \right)^2 \right).$$

From under to over damping

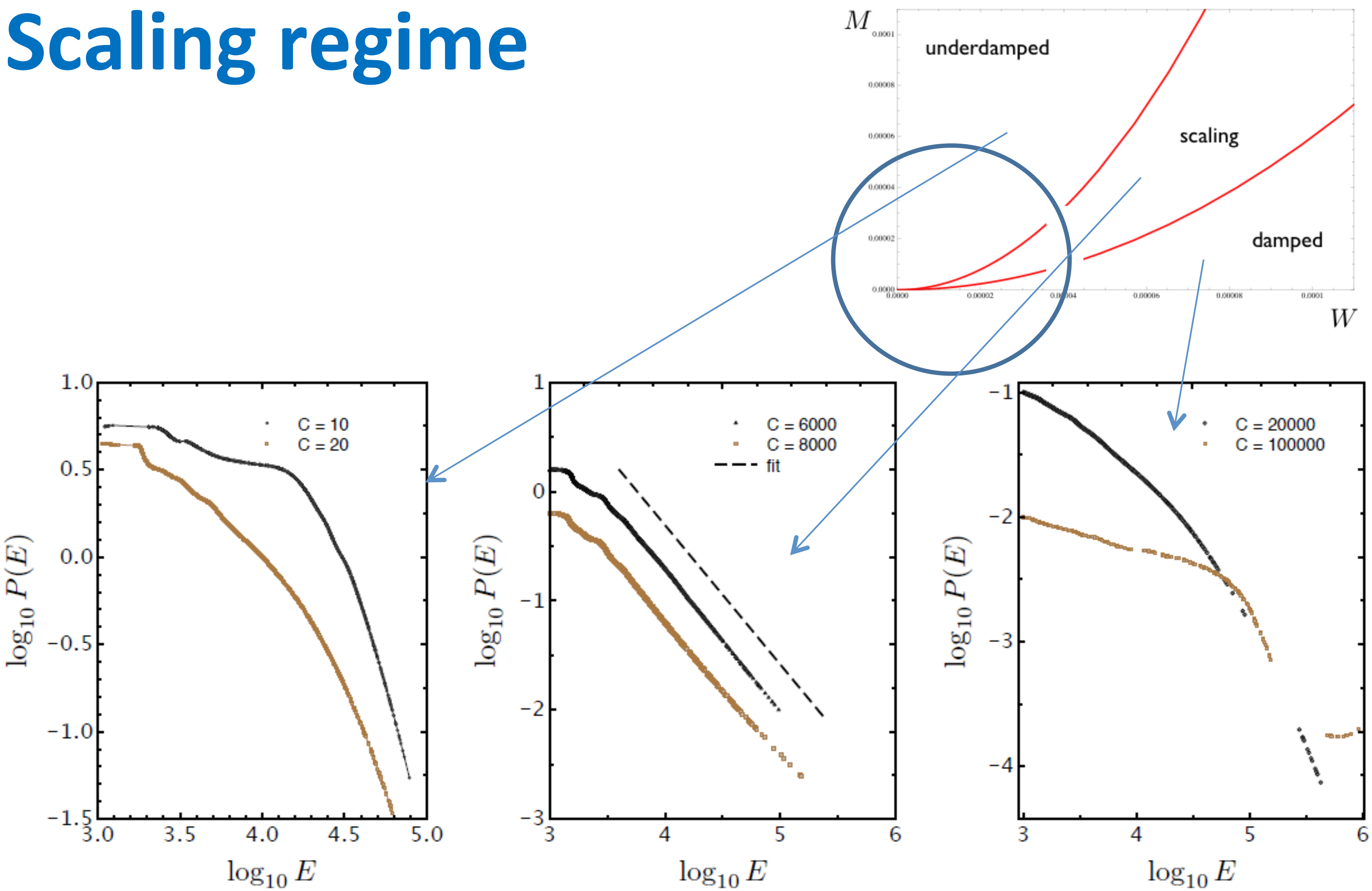


$$M = \frac{v^2 \rho}{\kappa}, W = \frac{\gamma v}{L \kappa}$$

$$C = M/W^2$$

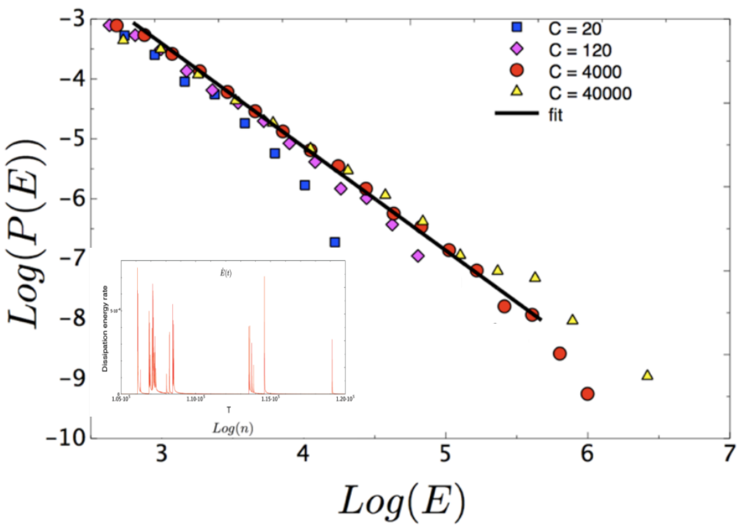


Scaling regime

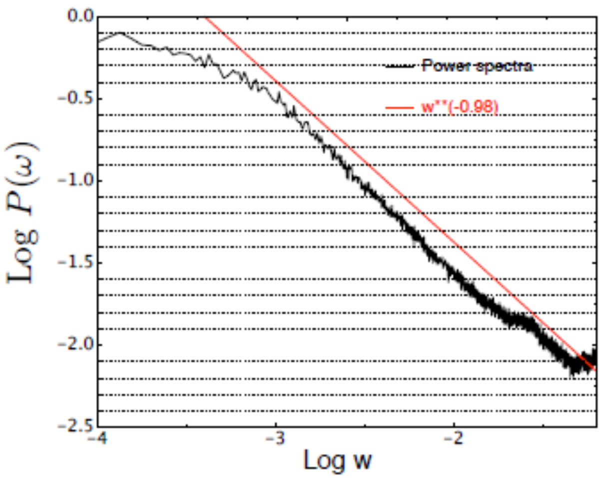


Scaling regime

$$P(E) \sim E^{-\epsilon} \quad \epsilon = 1.75 \ \pm 0.05$$



$$P(\omega) \sim \omega^{-\alpha} \quad \alpha = 0.98 \ \pm 0.1$$



Conclusions

- There are (at least) two paths to scaling in martensites. The domineering mechanism depends on micro-parameters: transformation strain, elastic moduli, barrier structure.

- Limited plasticity allows the system to generate (after training) a statistically stable configuration of defects which makes subsequent phase transformation scale free.

- Even small inertia can become crucial if the system is close to marginal stability because it can generate a dynamic disorder (ergodicity on a subset of the configurational space?)