Depinning-like models of amorphous plasticity

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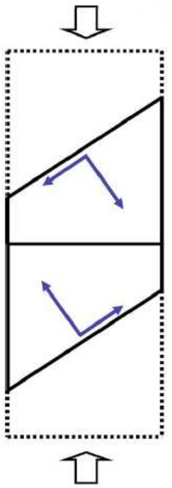
KITP, UC Santa Barbara
22 october 2014

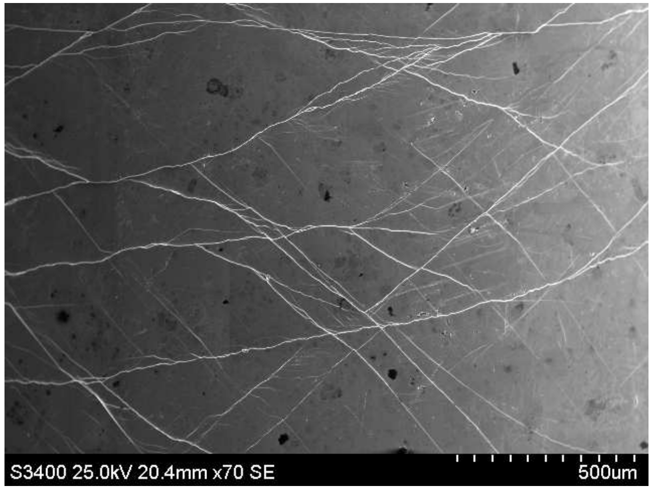
Plasticity of metallic glasses

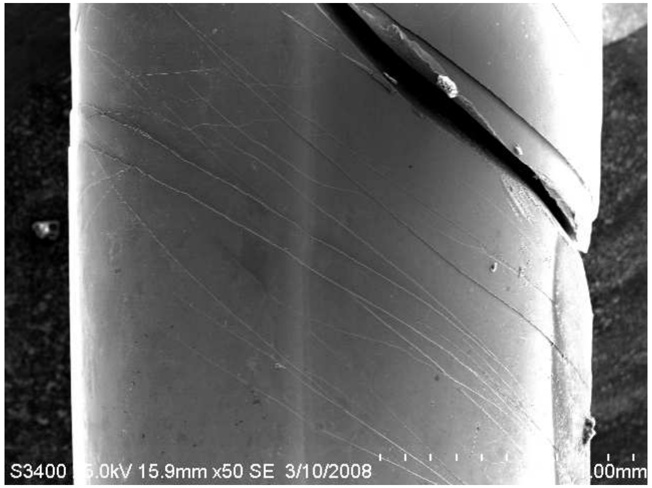
From metallic to glassy properties High yield strength but brittle

Shear banding Cracks initiate on shear bands – Localization of plastic strain is a key issue

Dependence on age and quenching rate







J. Lewandowski et al. Nature **5**, 15 (2006)

J. Zhang et al. Scripta Mat. **61**, 1145 (2009)

Plastic deformation results from series of local reorganizations

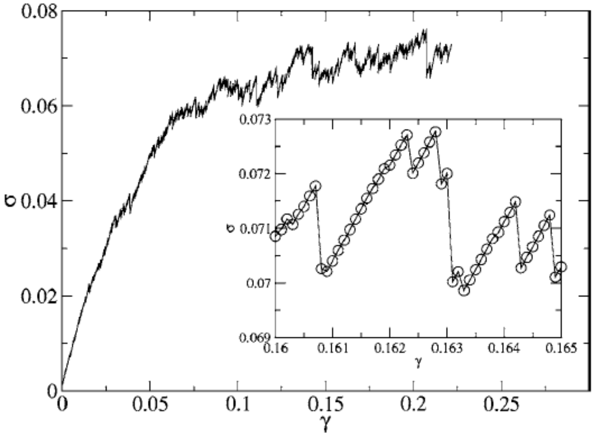


FIG. 2. Stress vs strain curve for a 200×200 system of harmonic discs. The event at γ=0.1631 will be discussed further below in Sec. IV. Note the smooth, roughly linear elastic segments interrupted by the discrete plastic events.

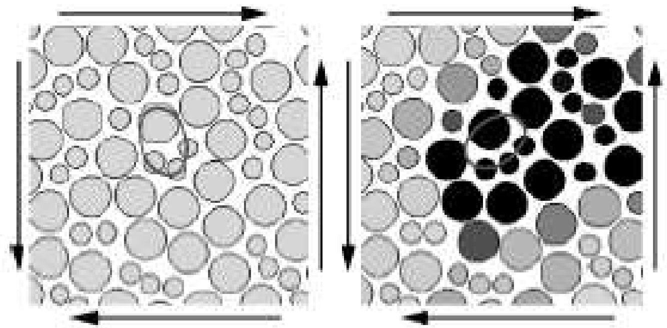
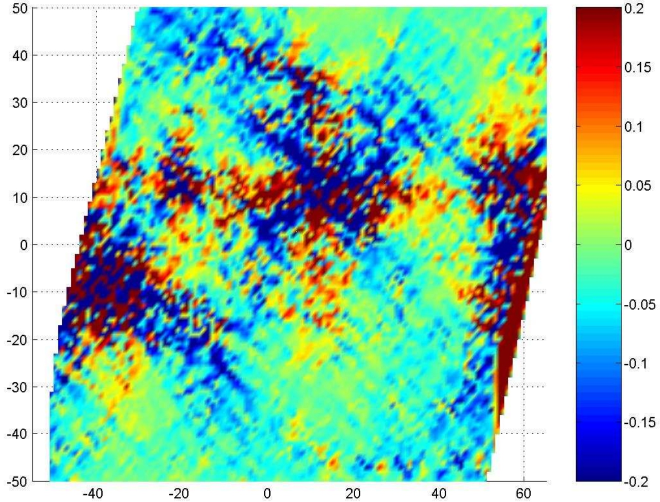


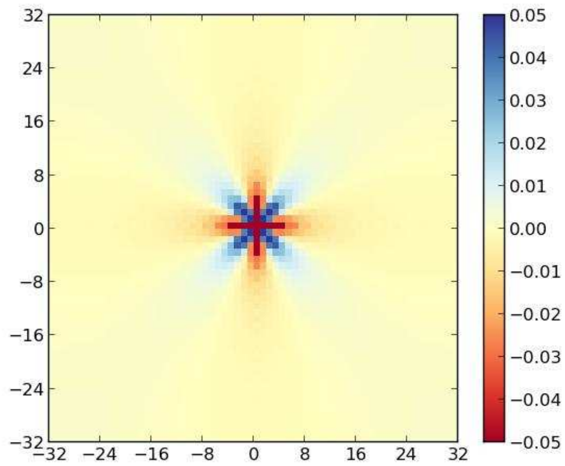
FIG. 8. Closeup picture of a shear transformation zone before and after undergoing transformation. Molecules after transformation are shaded according to their values of D^2_{min} using the same gray scale as in Fig. 7. The direction of the externally applied shear stress is shown by the arrows. The ovals are included solely as guides for the eye.

Maloney and Lemaitre PRE 06, Falk and Langer PRE 98

Local reorganizations induce anisotropic internal stress



Stress map induced by a plastic event, Tanguy et al. PRB 06

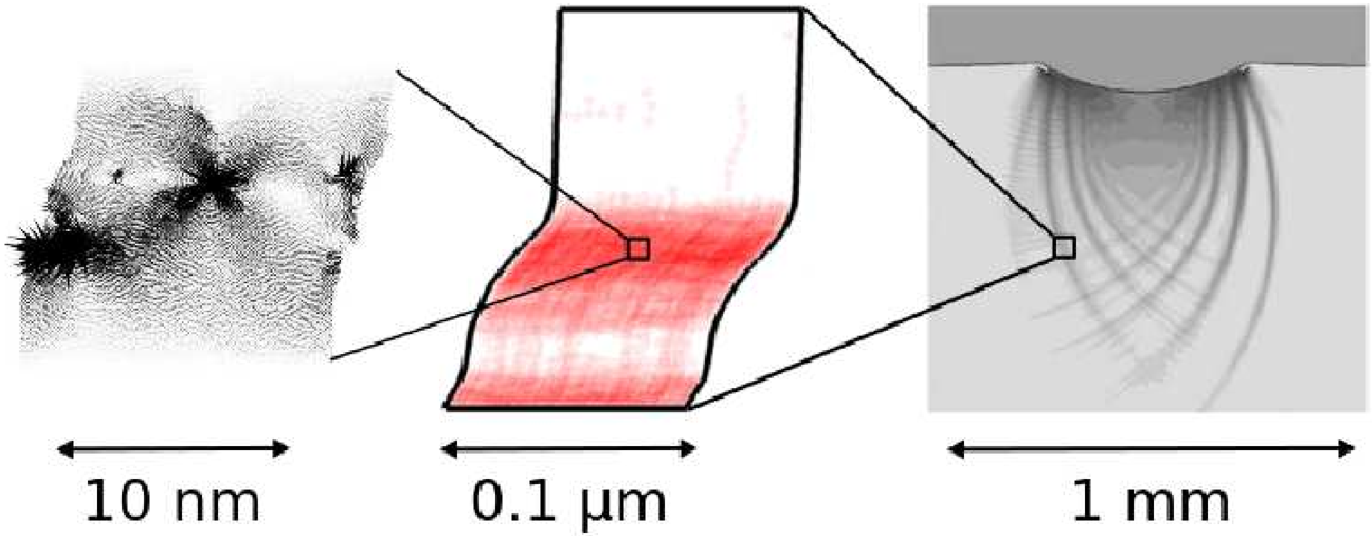


Quadrupolar stress field induced by a plastic event

The (far field) internal stress induced by a plastic reorganization obeys a quadrupolar symmetry (Eshelby inclusion):

$$\sigma_{xy}(r,\theta) = A\frac{\cos 4\theta}{r^2} \; ; \qquad A = \frac{2\mu^*}{\pi}\mathcal{S}\gamma_p$$

Modeling strategy – Up-scaling



Can we build at mesoscopic scale a *minimal* model that reproduces at large scale the important features of amorphous plasticity ?

Two main ingredients: structural disorder and elastic interactions

$$\mu \frac{\partial \varepsilon_p}{\partial t} = \Sigma^{ext} + \sigma^{el}\left[\mathbf{x}, \{\varepsilon_p(\mathbf{x})\}\right] - \sigma^{Y}\left[\mathbf{x}, \varepsilon_p(\mathbf{x})\right]$$

Rodney, Tanguy and Vandembroucq MSMSE 2011

A cellular automaton for amorphous plasticity

Discretization on a lattice at mesoscopic scale

A scalar plastic criterion $\sigma > \sigma_Y$

Structural disorder Local plastic criterion : $\sigma(i,j) > \overline{\sigma_Y} + \delta\sigma_Y(i,j)$

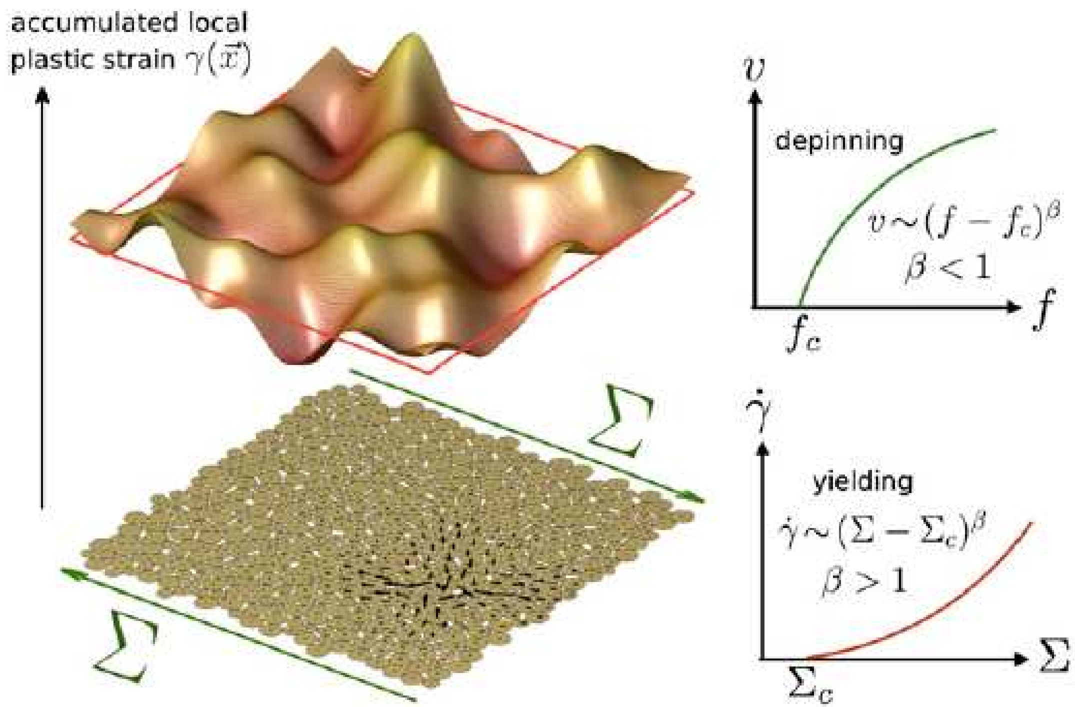
Local reorganization local slip increment $\delta\gamma$ and **update** of local plastic threshold: $\delta\sigma_Y$ is **renewed** from the distribution of random barriers

Anisotropic elastic response A local slip induces a stress redistribution σ^{el} all over the system, $\sigma^{el} \propto \mu\delta\gamma \cos 4\theta / r^2$

Extremal dynamics only the weakest site is advanced per simulation step; equivalent of athermal quasi-static atomistic simulations.

Parameters of the model: $\mu = 1$, $\delta\sigma_Y \in rand[0,1]$, $\delta\gamma \in rand[0,\mathbf{d}]$; $\mathbf{d}$ sets the amplitude of the elastic interactions.

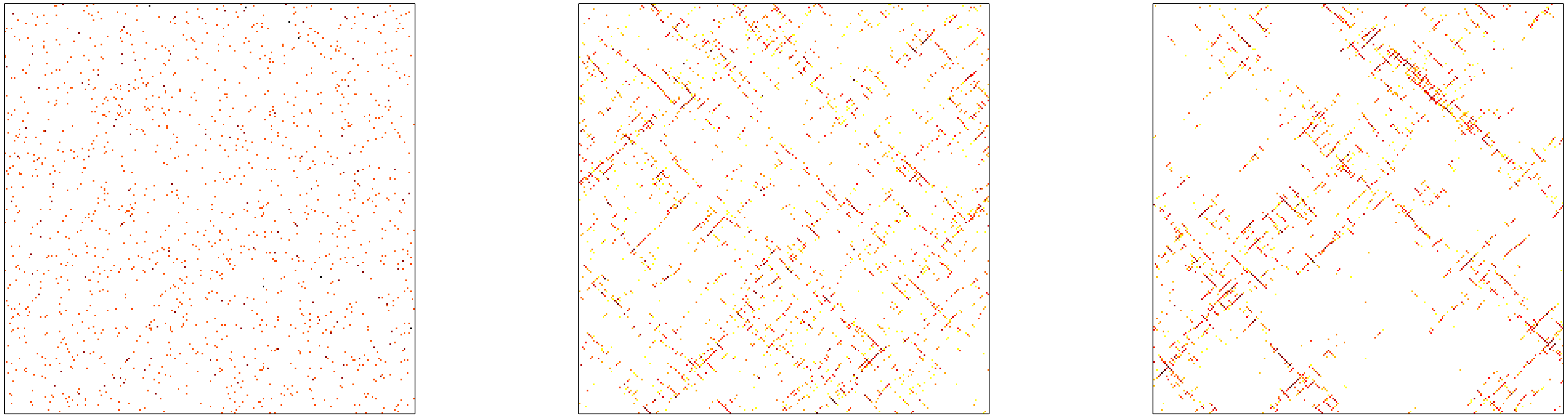
Depinning vs Plastic yielding: scaling properties



J. Lin, E. Lerner, A. Rosso and M. Wyart, PNAS **111**, 40 (2014)

Some specific features of plastic yielding models

Development of localization

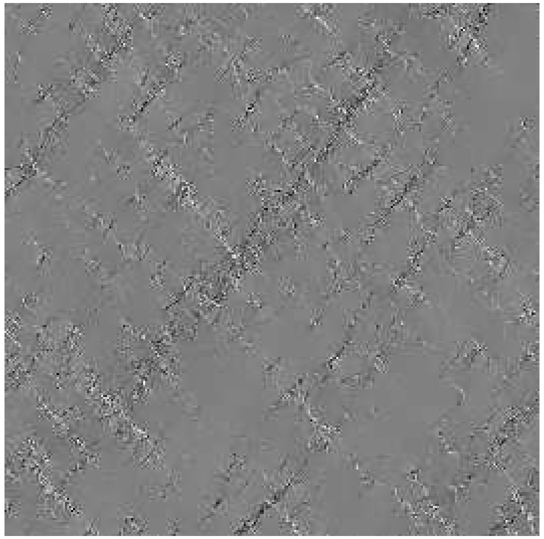


During the transient/hardening stage, plastic deformation gets progressively more and more correlated with the same quadrupolar symmetry as the elastic interaction.

Localization *vs* diffusion

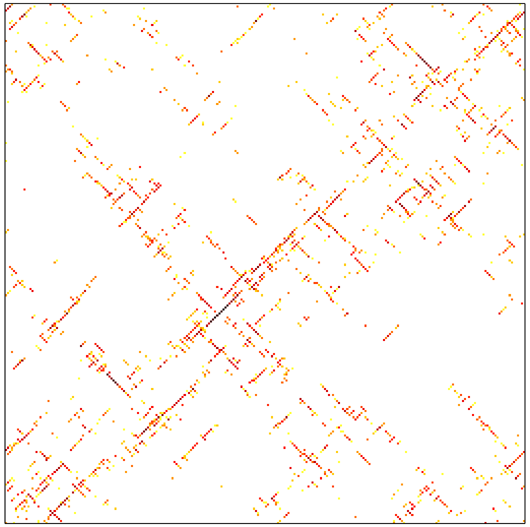
In the stationary regime, plastic deformation remains localized but, localization patterns are not persistent, rather they diffuse throughout the system.

Localization *vs* diffusion



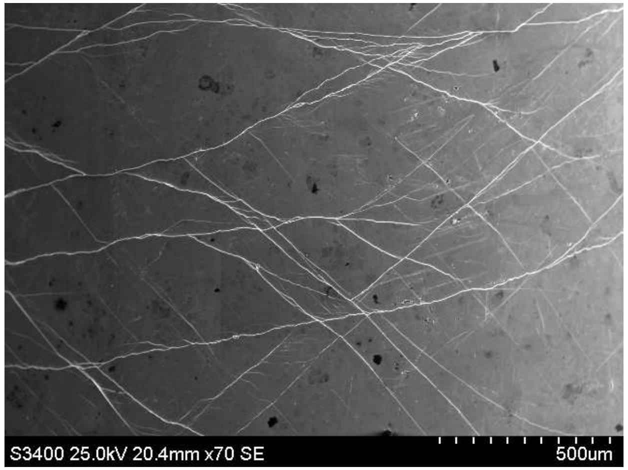
Atomistic scale

Maloney, Robbins, JPCM 08, PRL 09



Mesoscopic Scale

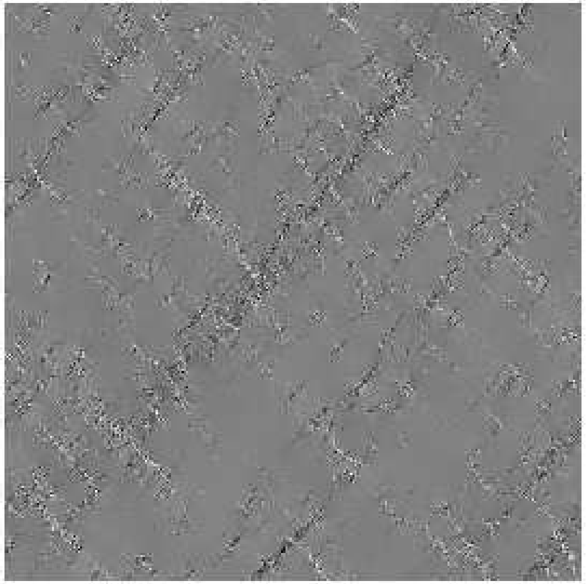
Talamali et al, PRE 11, C.R Meca. 12



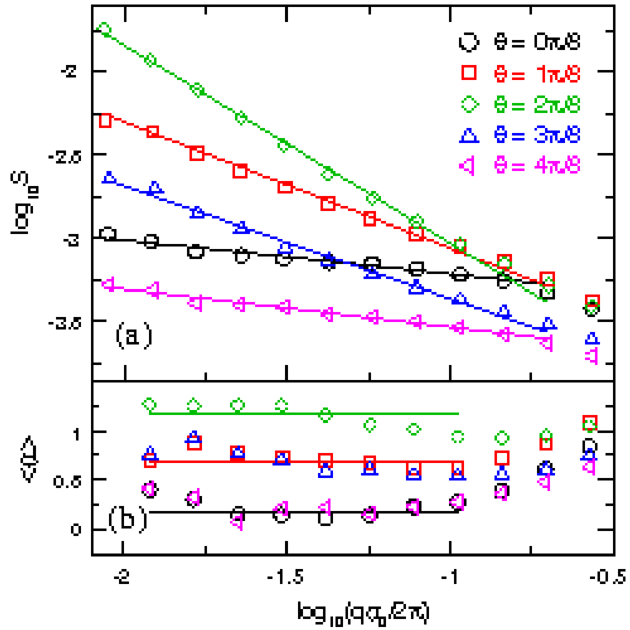
Experimental Results

Zhang et al, Scripta Mat. 09

Anisotropy of internal stress induces localization of plasticity



Localization of plastic strain field

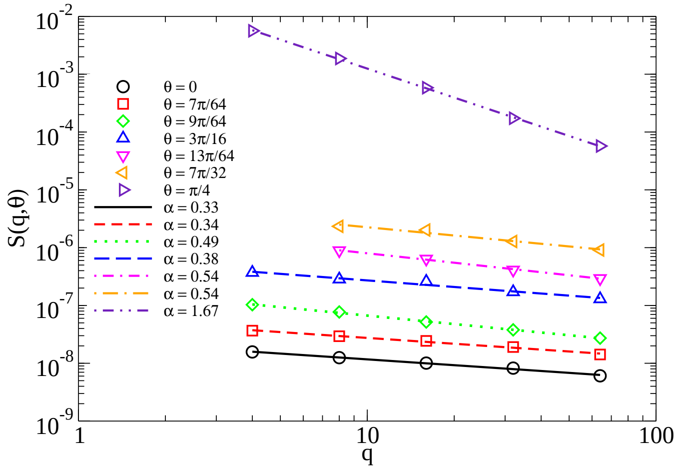


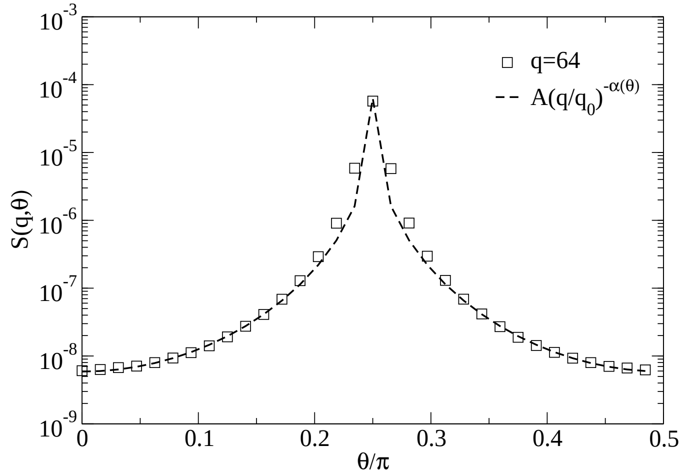
Anisotropic scaling of the plastic strain correlations

$$S(q,\theta) \propto q^{-\alpha(\theta)}\ , \quad \alpha(0) \approx -0.18\ , \quad \alpha(\pi/4) \approx -1.38$$

Maloney, Robbins, JPCM 08, PRL 09

Anisotropic strain correlation



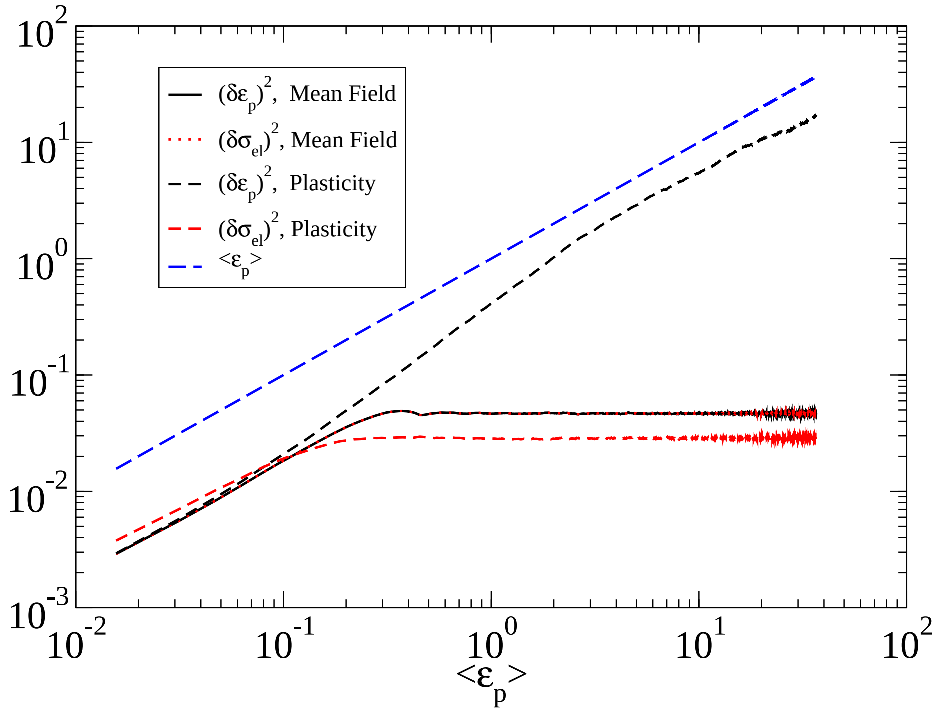


Power spectrum of plastic deformation exhibits **scaling** and **quadrupolar symmetry**

$$S(q,\theta) \propto q^{-\alpha(\theta)} \,, \quad \alpha(0) \approx -0.3 \,, \quad \alpha(\pi/4) \approx -1.7$$

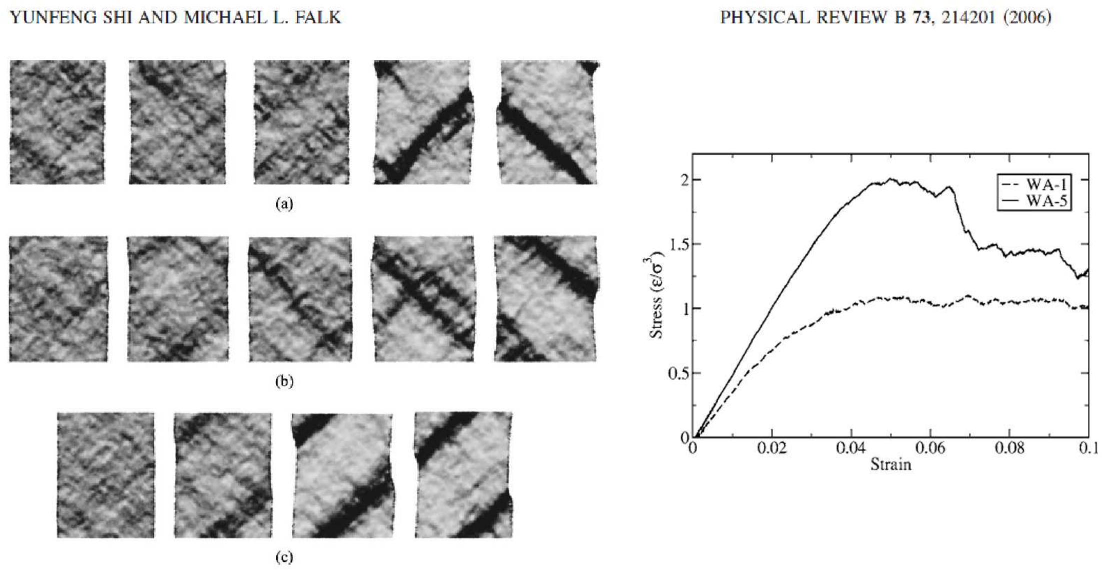
Talamali et al, C.R. Mecanique 2012

Family-Vicsek vs diffusive regime



Plastic strain fluctuations do not saturate : diffusive regime

Aging or rate dependent Shear Banding



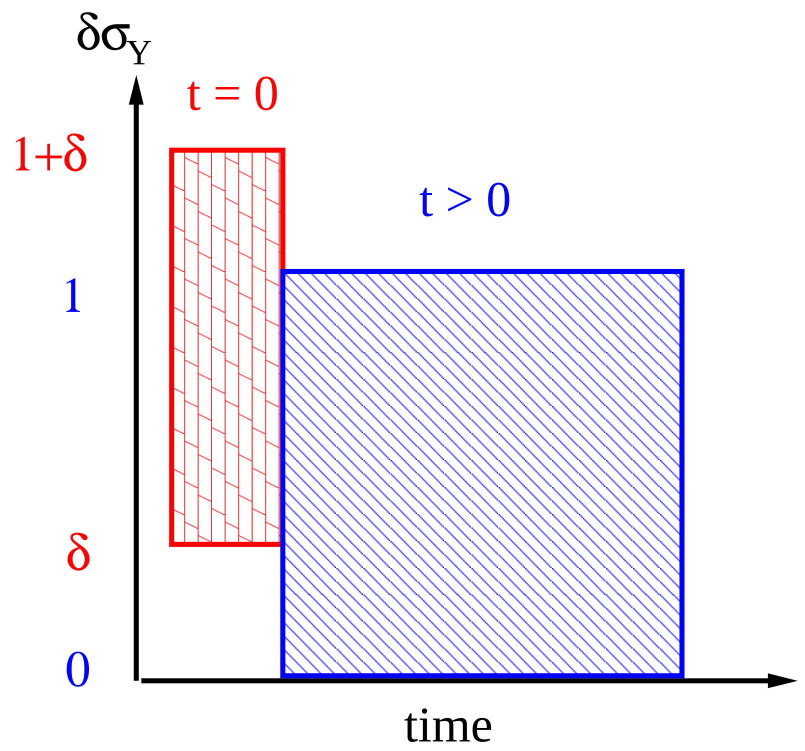
Results from atomistic simulations (Varnik 04, Rottler 05, Shi 06)

The more relaxed the amorphous structure, the more prone to shear banding, the higher the stress peak (the higher the energy barriers)

Logarithmic evolution with age of the stress peak (Rottler PRL 05)

Recipe: mimicking structural aging

- Introduction of an age-like parameter δ
- Initial state characterized by a biased distribution of barriers $\delta\sigma_Y^i \in rand[\delta, 1+\delta]$
- Under shear $\delta\sigma_Y^i$ still renewed in $rand[0,1]$;

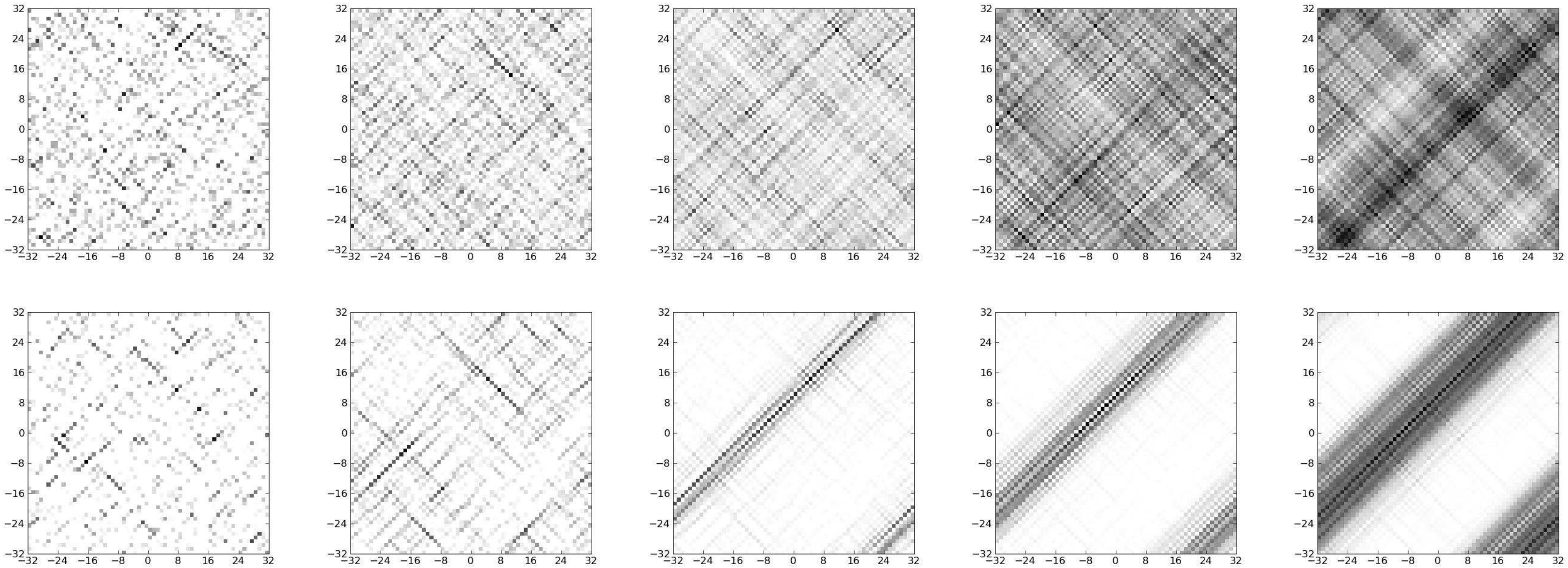


Random barriers

Aged structure = higher energy barriers ?

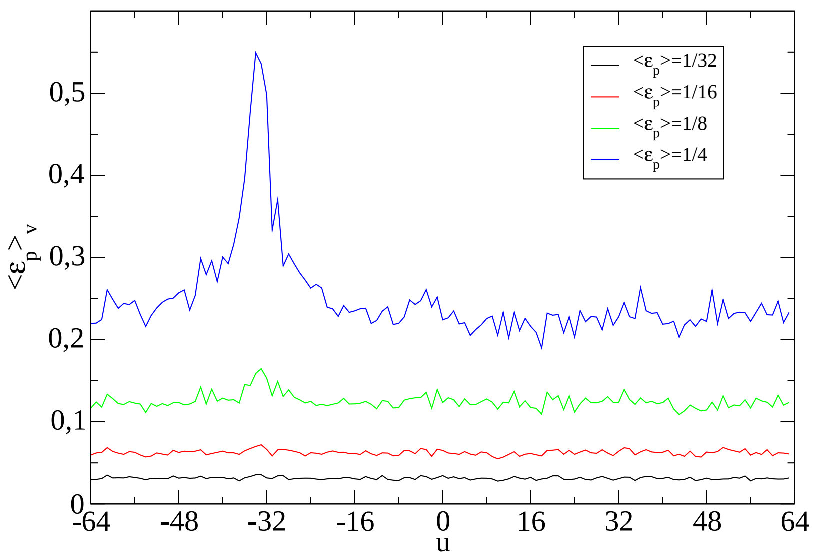
Don't change anything but the initial condition

Aging effect on Shear Banding

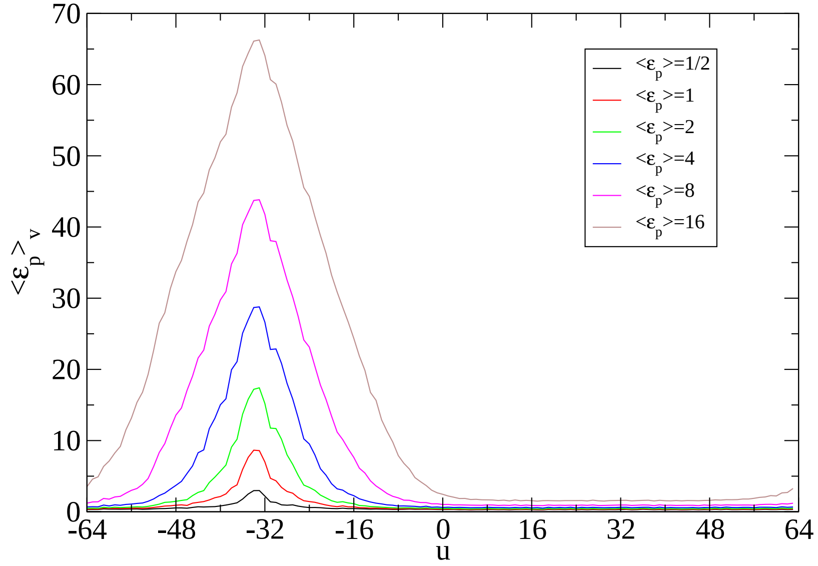


Maps of plastic strain obtained from left to right at $\langle \varepsilon_p \rangle = 1/16$, $1/4$, 1, 4 and 16 and with a bias value $\delta = 0$ **(top, no aging)** and $\delta = 0.5$ **(bottom, aging)** with a slip increment $d = 0.3$.

Nucleation and broadening of a Shear Band

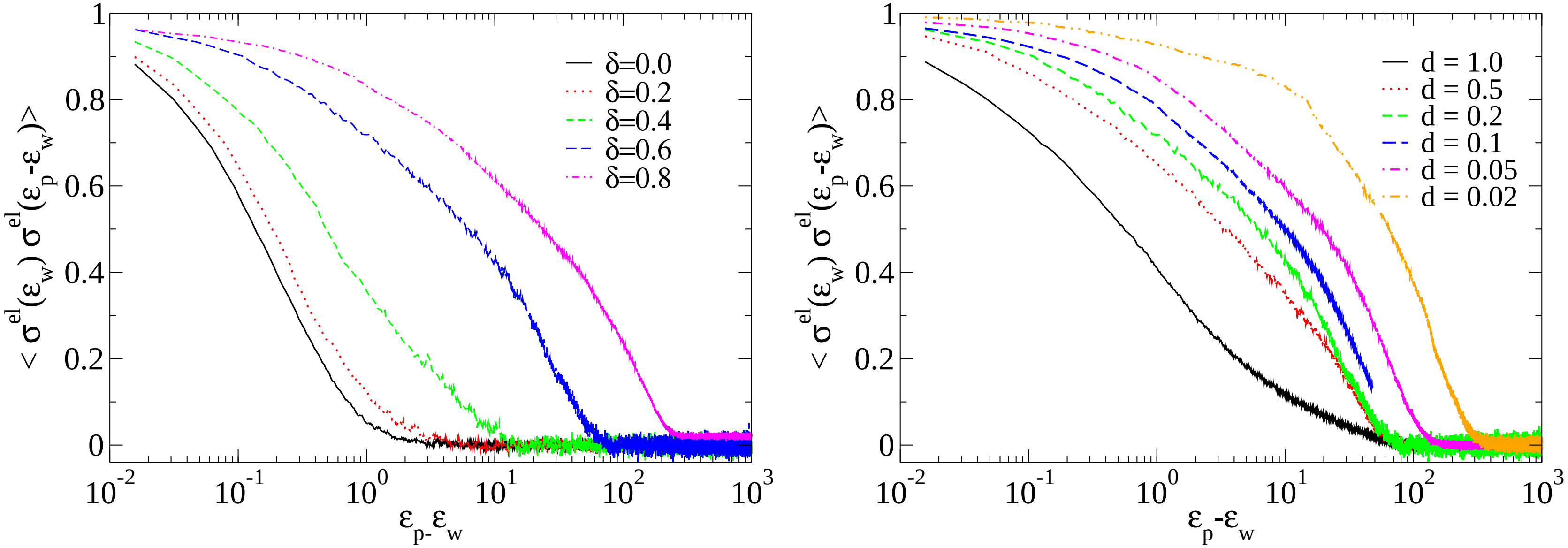


$\langle \varepsilon_p \rangle = 1/32,\ 1/16,\ 1/8,\ 1/4$



$\langle \varepsilon_p \rangle = 1/2,\ 1,\ 2,\ 4,\ 8,\ 16$

Slow relaxation of shear-banding

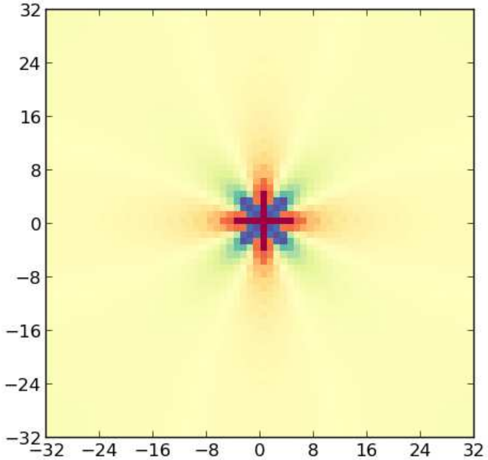


Effect of age δ and mechanical noise d on the relaxation of 2-points Stress-Stress relaxation:

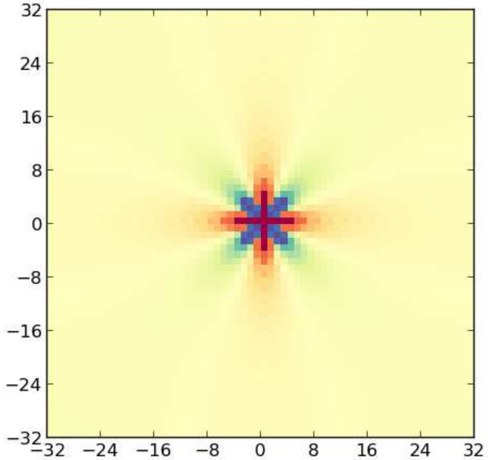
$$C_\sigma(\varepsilon_w, \varepsilon_p) = \frac{\langle \sigma_{res}(\varepsilon_w, x)\sigma_{res}(\varepsilon_p, x)\rangle_x}{(\langle \sigma_{res}(\varepsilon_w, x)\sigma_{res}(\varepsilon_w, x)\rangle_x \langle \sigma_{res}(\varepsilon_p, x)\sigma_{res}(\varepsilon_p, x)\rangle_x)^{1/2}} \quad (1)$$

Plastic yielding vs Mean Field depinning

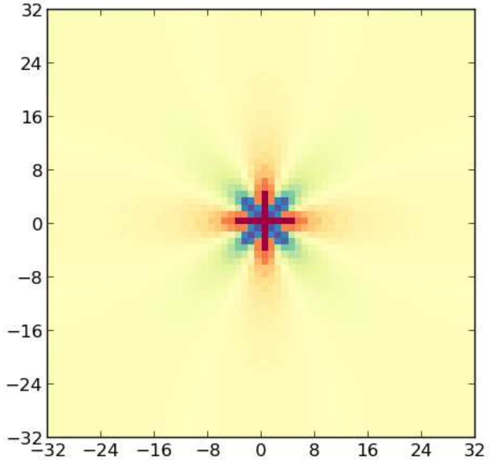
From quadrupolar interaction to Mean Field



$a = 0$ (Eshelby)



$a = 0.01$

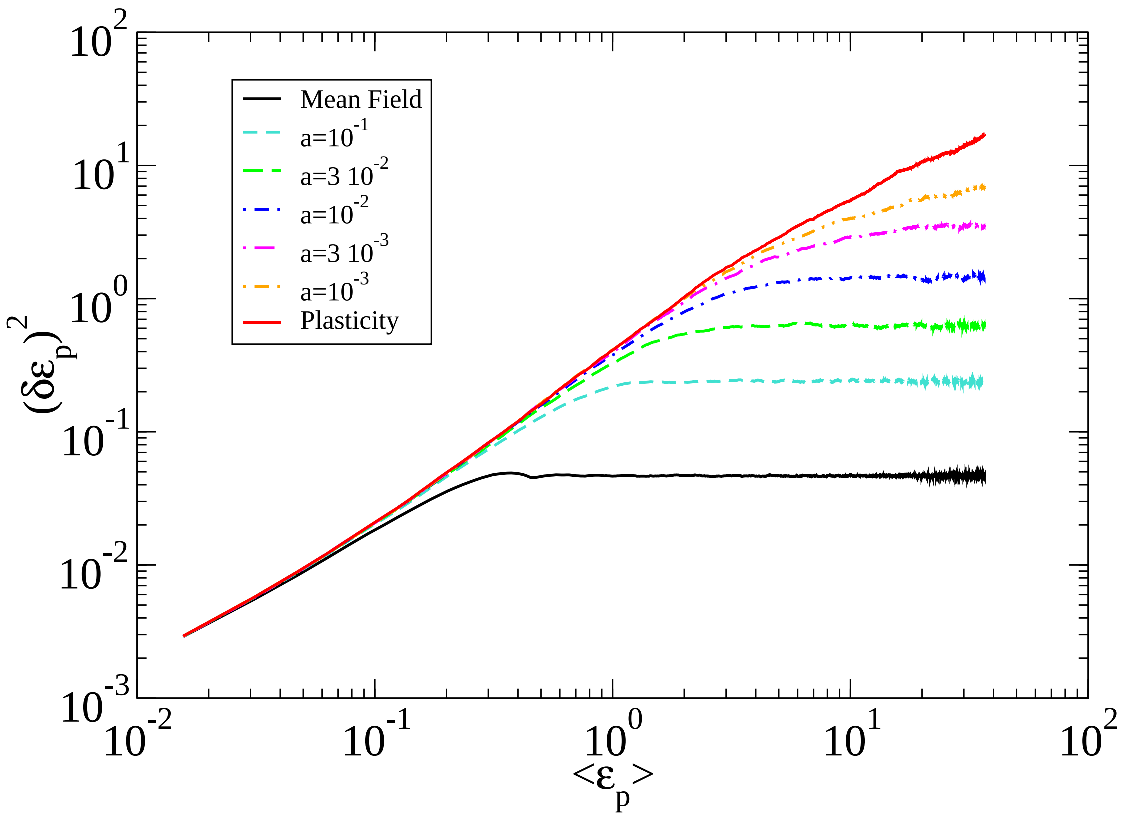


$a = 0.1$

$$G = (1 - a)G_Q + aG_{MF}$$

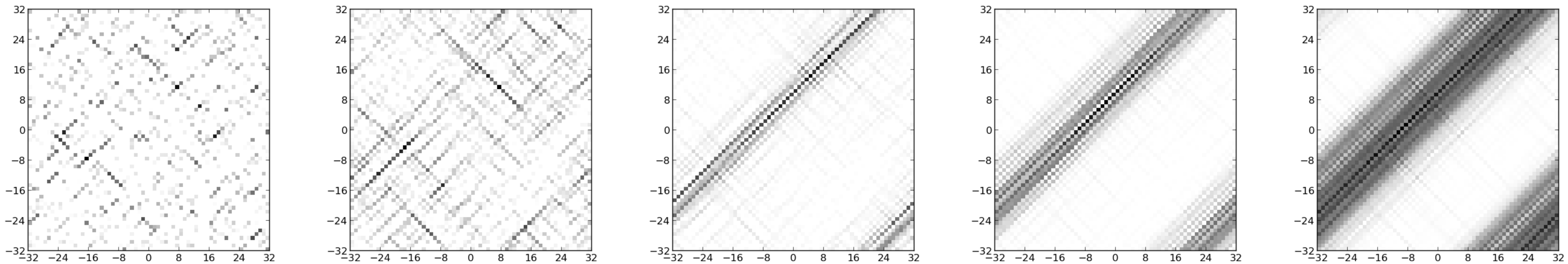
Eshelby long-range anisotropic interaction biased by a small MF contribution

Mean Field effect on diffusive regime



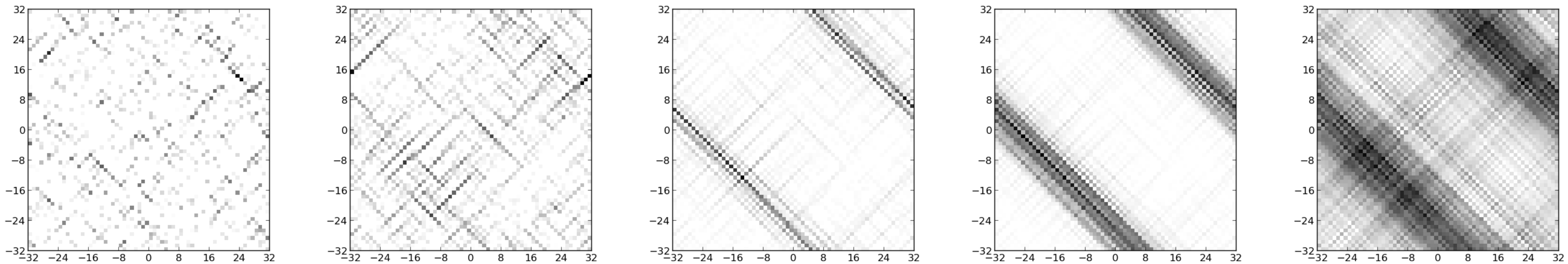
Saturation is eventually recovered

Mean Field effect on shear-banding



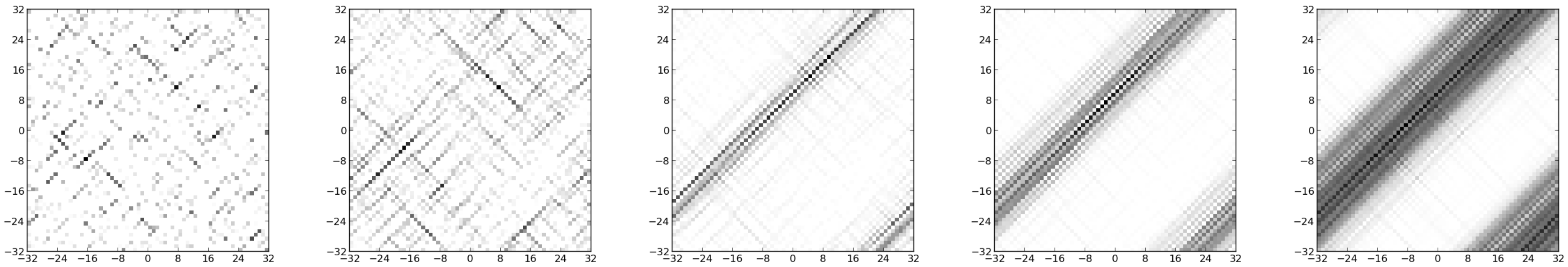
bare quadrupolar interaction

$\langle \varepsilon_p \rangle = 1/16, \quad \langle \varepsilon_p \rangle = 1/4, \quad \langle \varepsilon_p \rangle = 1, \quad \langle \varepsilon_p \rangle = 4, \quad \langle \varepsilon_p \rangle = 16$



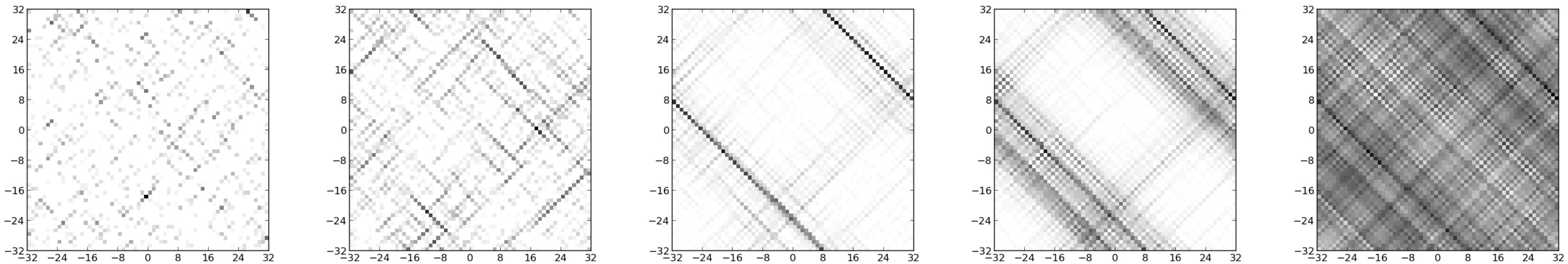
Mean-Field weighted quadrupolar interaction $a = 10^{-3}$

Mean Field effect on shear-banding



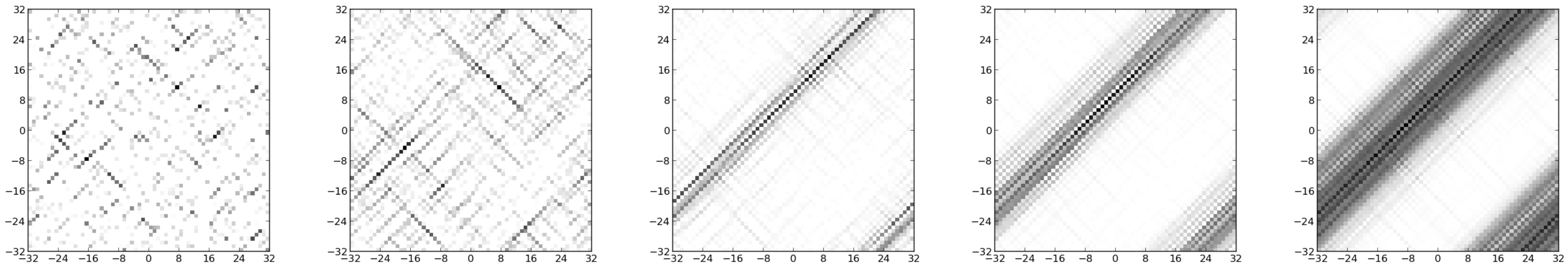
bare quadrupolar interaction

$\langle \varepsilon_p \rangle = 1/16, \quad \langle \varepsilon_p \rangle = 1/4, \quad \langle \varepsilon_p \rangle = 1, \quad \langle \varepsilon_p \rangle = 4, \quad \langle \varepsilon_p \rangle = 16$



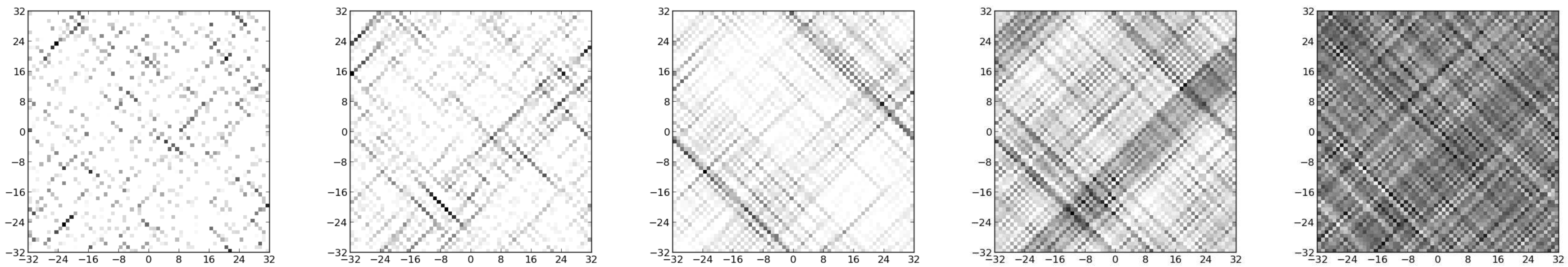
Mean-Field weighted quadrupolar interaction $a = 3\ 10^{-3}$

Mean Field effect on shear-banding



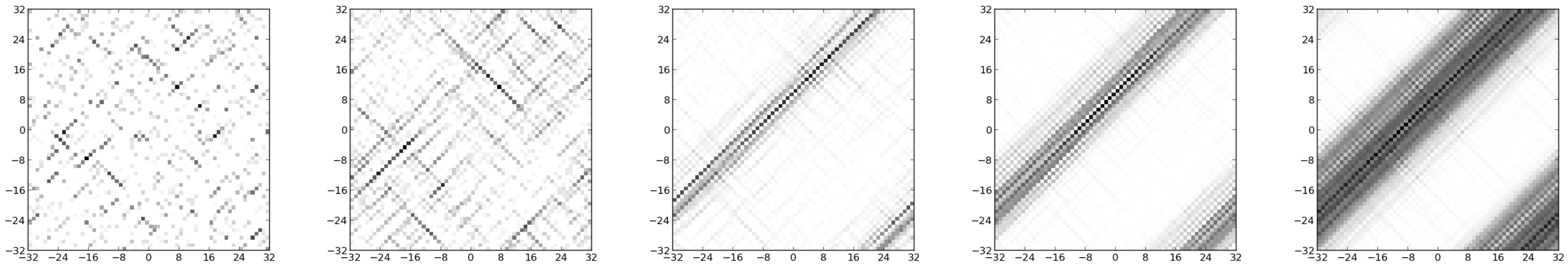
bare quadrupolar interaction

$\langle \varepsilon_p \rangle = 1/16$, $\langle \varepsilon_p \rangle = 1/4$, $\langle \varepsilon_p \rangle = 1$, $\langle \varepsilon_p \rangle = 4$, $\langle \varepsilon_p \rangle = 16$



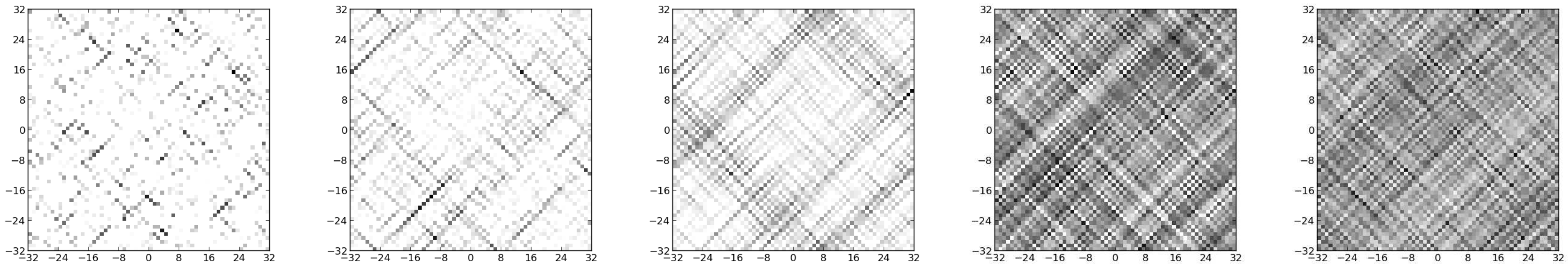
Mean-Field weighted quadrupolar interaction $a = 10^{-2}$

Mean Field effect on shear-banding



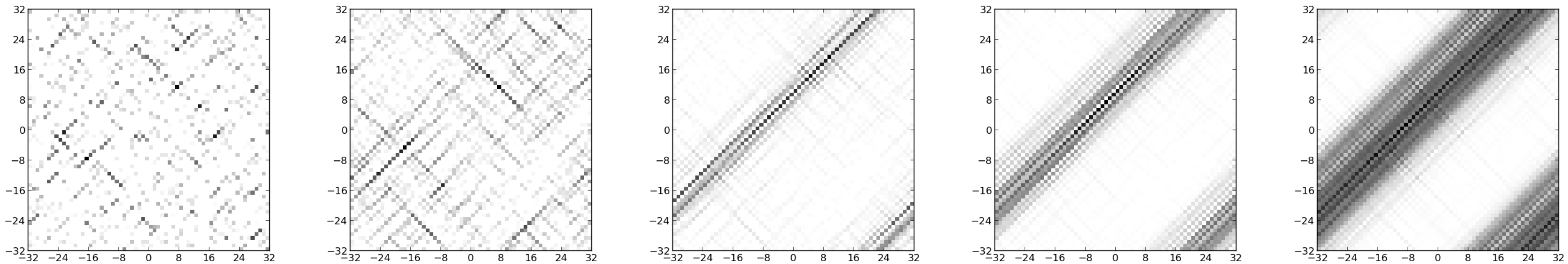
bare quadrupolar interaction

$\langle \varepsilon_p \rangle = 1/16$, $\langle \varepsilon_p \rangle = 1/4$, $\langle \varepsilon_p \rangle = 1$, $\langle \varepsilon_p \rangle = 4$, $\langle \varepsilon_p \rangle = 16$



Mean-Field weighted quadrupolar interaction $a = 3\ 10^{-2}$

Mean Field effect on shear-banding



bare quadrupolar interaction

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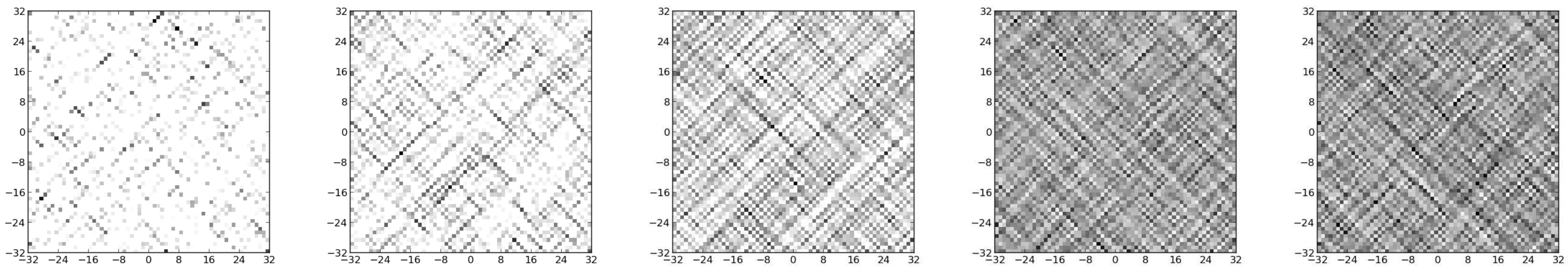
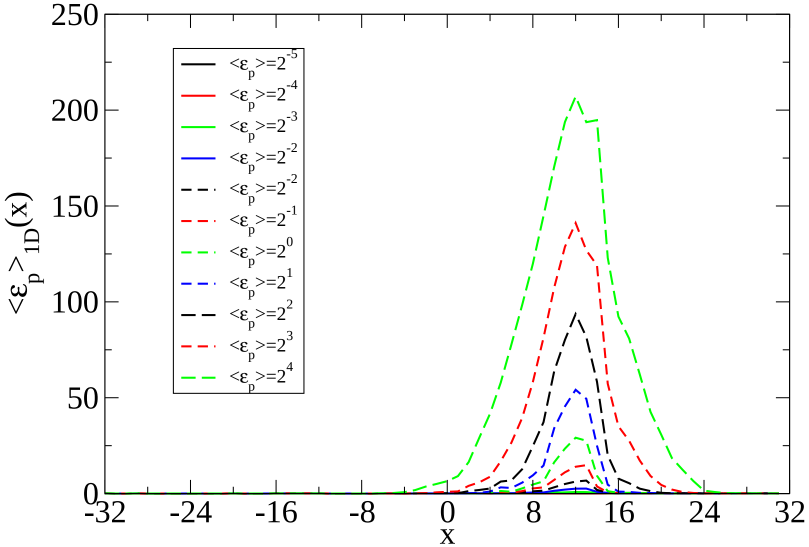
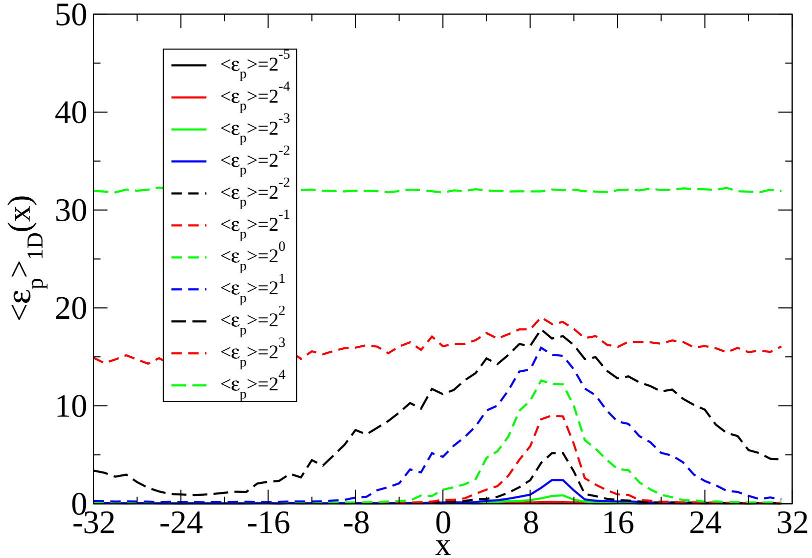


Mean-Field weighted quadrupolar interaction $a = 10^{-1}$

Mean Field effect on shear-banding



Quadrupolar interaction

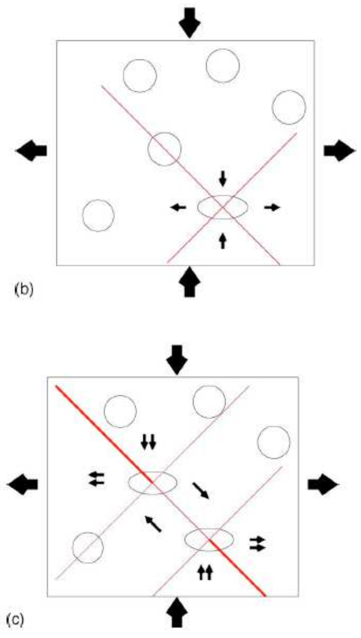


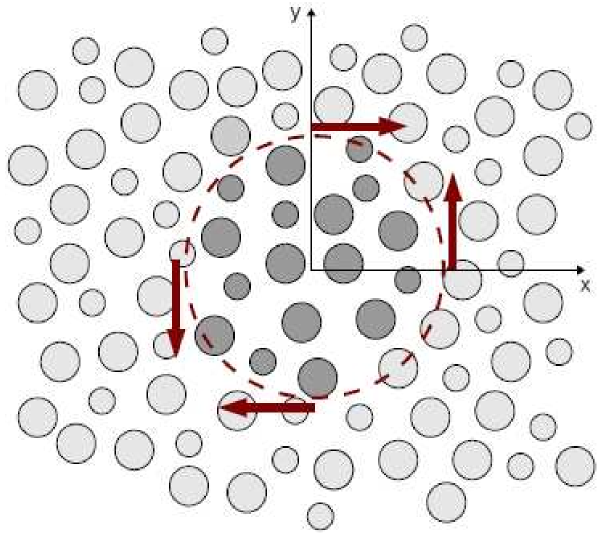
$a = 10^{-2}$

Shear-banding behavior is gradually smeared out by Mean Field

From depinning to plastic yielding: A soft modes perspective

Plastic events induce quadrupolar stress redistribution

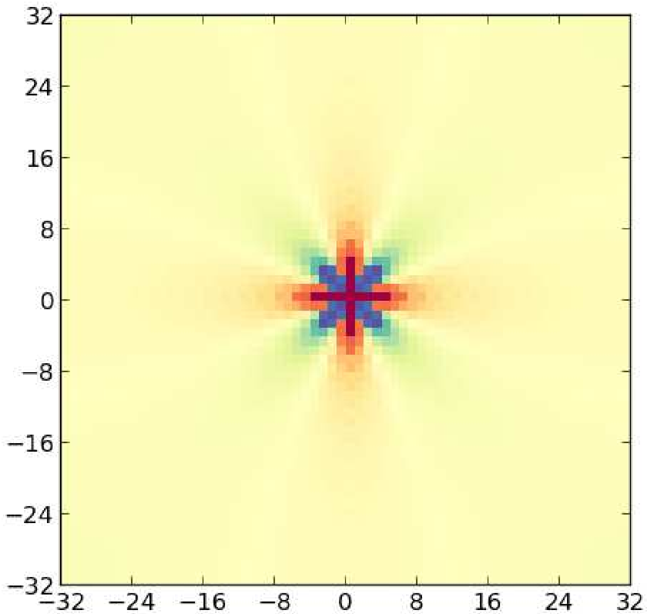


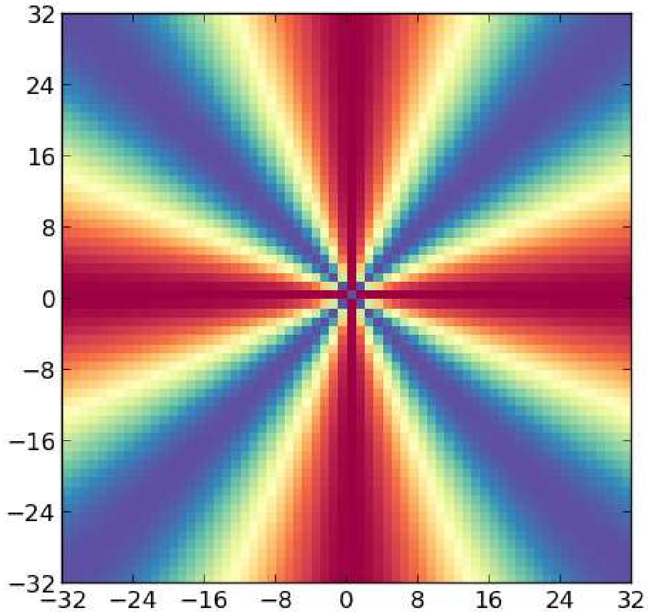


Maloney and Lemaître PRE 06 Puosi, Rottler and Barrat PRE 14

Back to continuum mechanics, Stress induced by an inclusion that experienced a plastic strain γ_p (Eshelby 57)

Biperiodic boundary conditions – Eshelby inclusion





Real space:

$$G(r,\theta) = -A\gamma_p \frac{a^2}{r^2}\cos(4\theta)$$

$$G(0,0) = -A\gamma_p/2$$

Fourier space:

$$\widetilde{G}(\rho,\phi) = -\frac{A\gamma_p}{4}\pi a^2\,[1+\cos(4\phi)]$$

$$\widetilde{G}(0,0) = 0.0$$

See also Procaccia et al PREs 2013, Bukridis and Zapperi PRE 2013

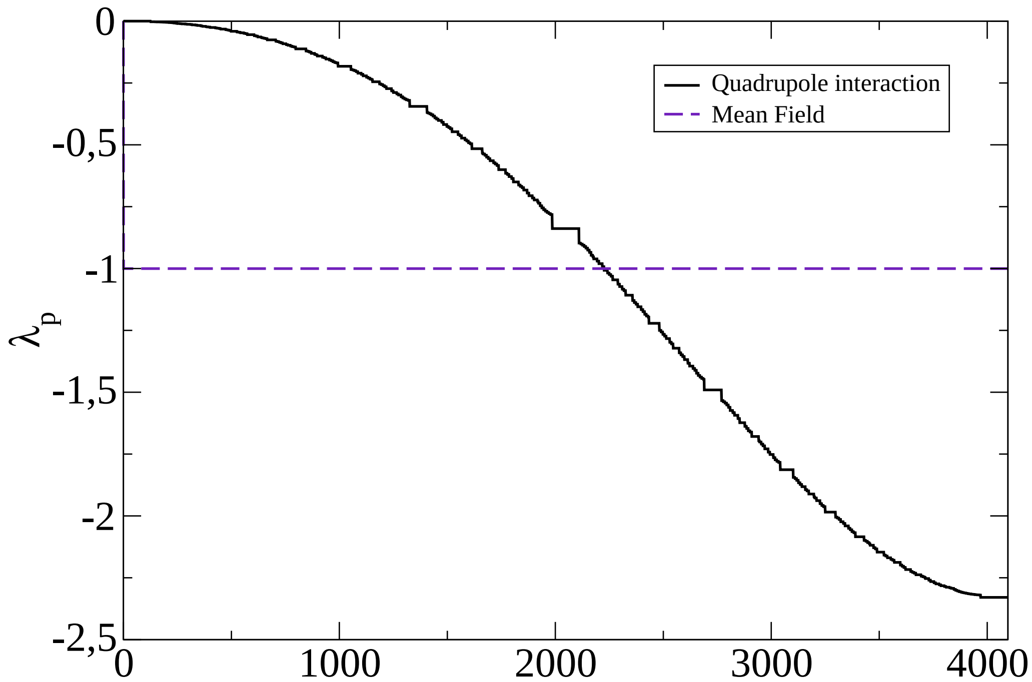
Soft modes and quadrupolar interaction

- Elastic stress : $\sigma^{el} = G * \varepsilon_p$
- Bi-periodic conditions $\to$ circulant matrix, eigenvalues given by $\widetilde{G}$, eigenmodes are Fourier modes
- The spectrum of eigenvalues is given by the Fourier transform of the quadrupolar interaction

$$\lambda_{pq} = \widetilde{G}_{pq} = -A\,(\cos(4\phi_{pq}) + 1) = -8A \left(\frac{p^2 - q^2}{p^2 + q^2} \right)^2$$

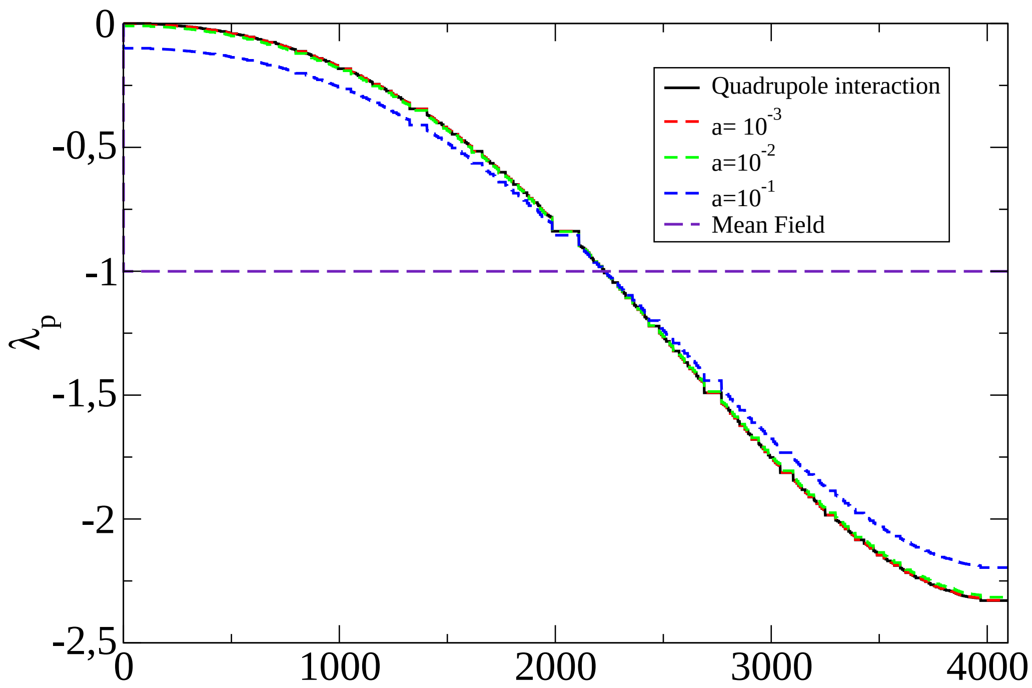
$(2N - 1)$ zero eigenvalues : translation + soft modes (shear-bands)

Spectrum of eigenvalues: quadrupolar vs Mean Field interaction

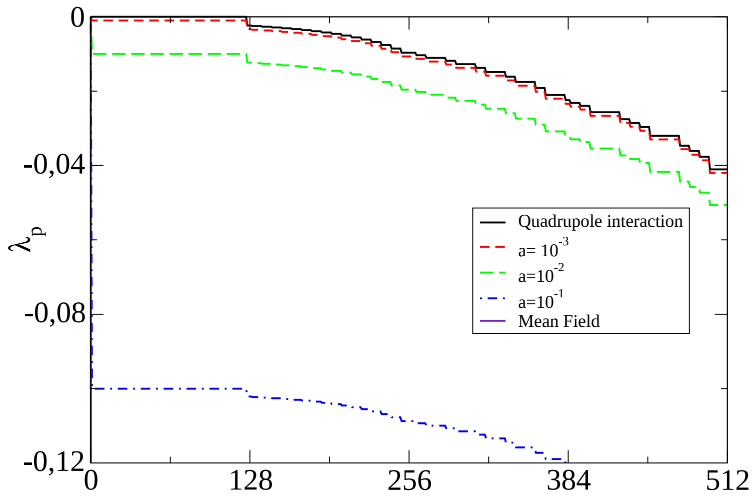


Quadrupolar interaction characterized by an abundance of soft modes

Spectrum of eigenvalues: effect of Mean Field



Spectrum of eigenvalues: effect of Mean Field



The introduction of a Mean Field contribution opens a gap in the eigenvalue spectrum of the stress redistribution operator

Mean Field finite stiffness brings back the model toward depinning

Summary

- Depinning-like lattice model of amorphous plasticity
- Scaling properties (see Wyart PNAS 2014)
- Specific features that reproduce plastic phenomenology: anisotropic srain correlation, diffusion, shear-banding
- Specificity of the model controlled by the presence of soft modes of the propagator
- A new class / a sub-class of depinning models ?

Baret, Vandembroucq, Roux, PRL 2002

Talamali, Petäjä, Vandembroucq, Roux, PRE 2011, C.R. Mécanique 2012

Vandembroucq and Roux PRB 2011

Tyukodi, Patinet, Roux, Vandembroucq (in preparation)