Fracture size effects

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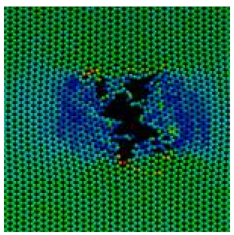
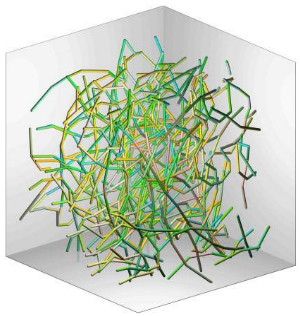
Failure across scales

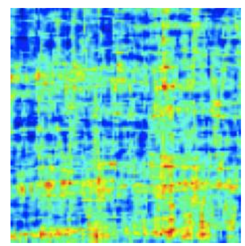
10^{-9} 10^{-6} 10^{-3} 1 [m]



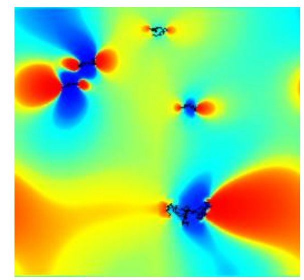
Atomic scale cracks



Dislocations

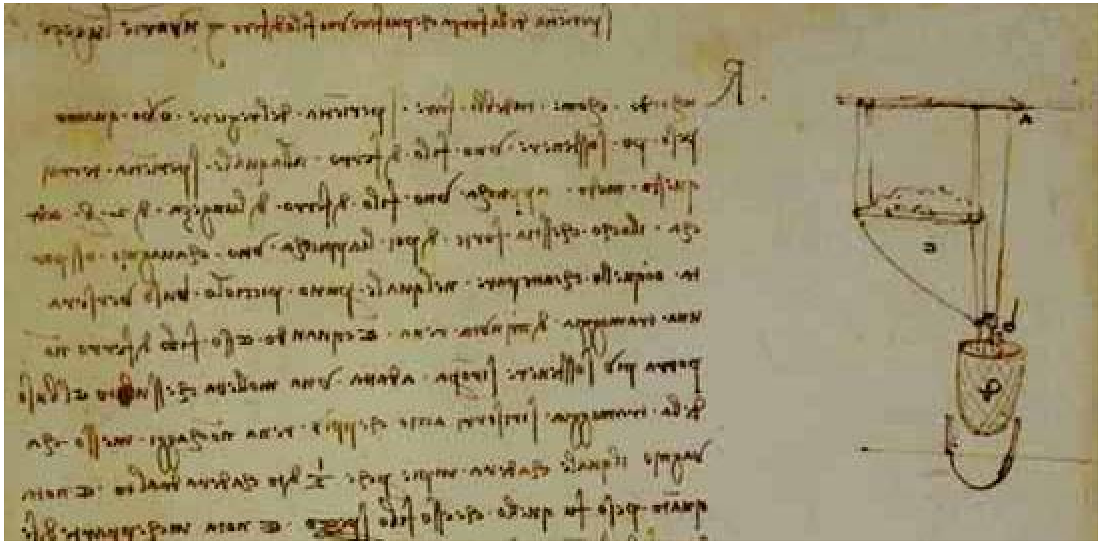


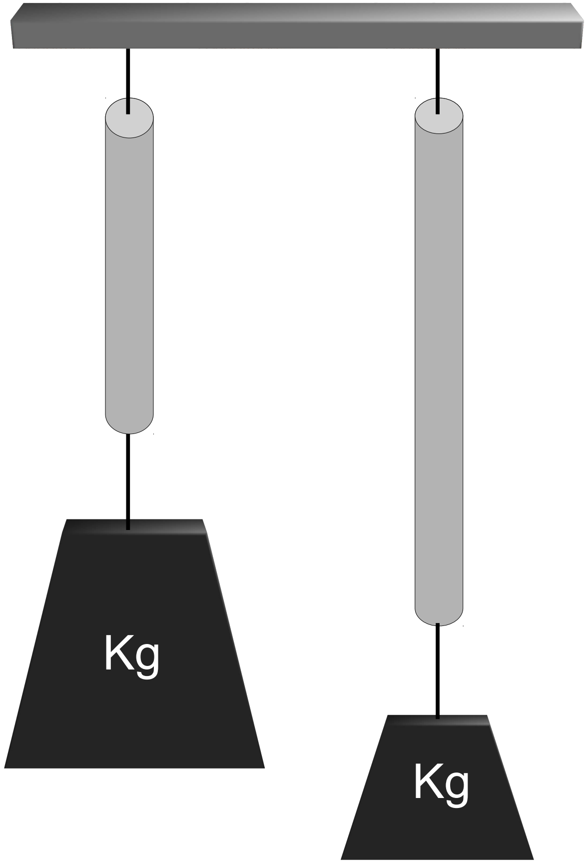
Shear bands in amorphous media



Interacting cracks

Size effects: an old problem



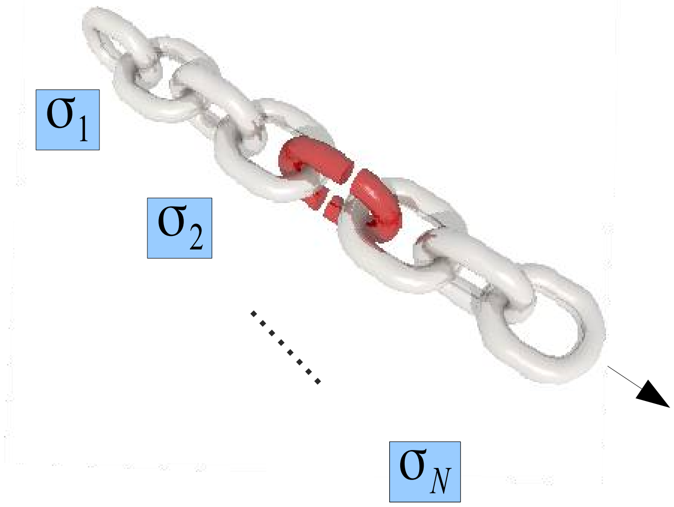


Leonardo da Vinci, test of metal wires:

Longer wires are easier to break.

The weakest link hypothesis

The strength of corresponds to the strength of the weakest link, the minimum of σ_i



$$S_1(\sigma) \equiv \int_{\sigma}^{\infty} dx\, p(x)$$

Survival Distribution

$$S_N(\sigma) = S_1(\sigma)^N$$

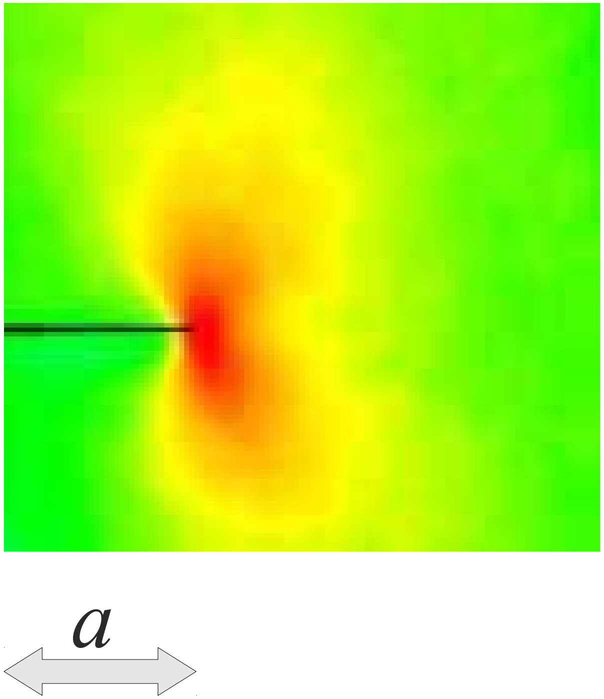
for N links

If the low stress tail scales as: $S_1(\sigma) \simeq 1 - (\sigma/\sigma_0)^{\mu}$

$$S_N(\sigma) \simeq e^{-N(\sigma/\sigma_0)^{\mu}}$$

Weibull distribution

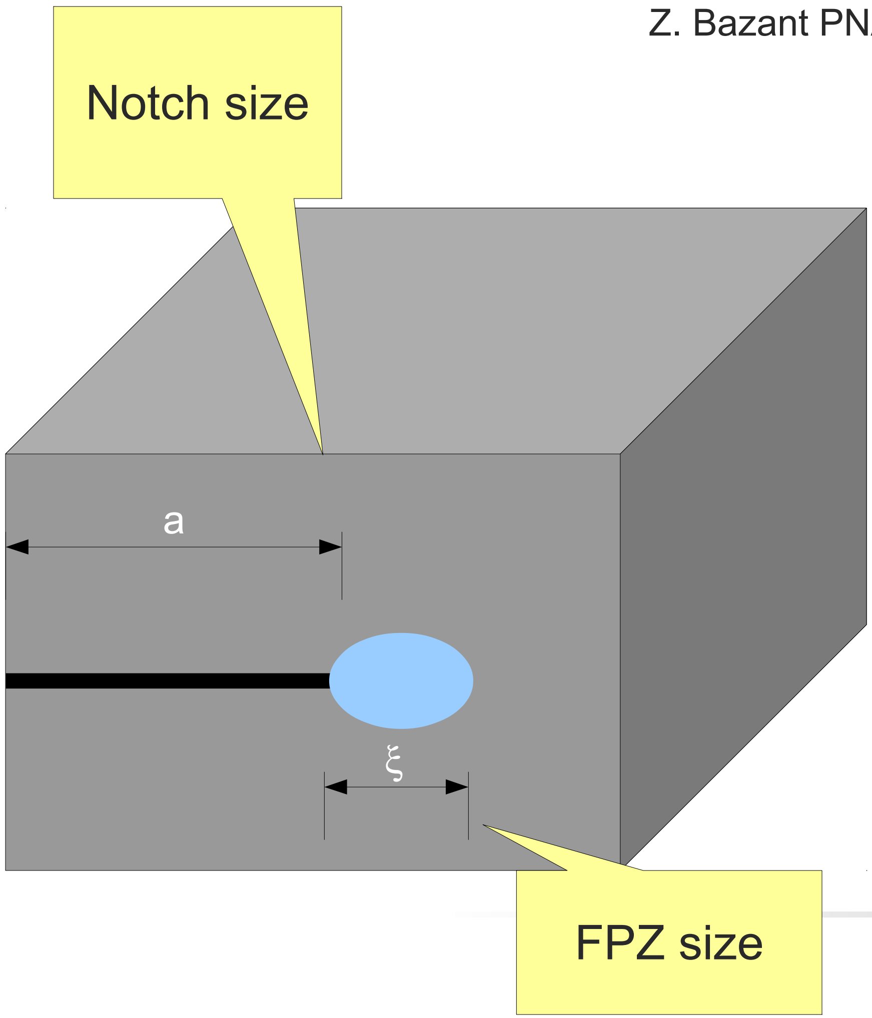
Stress concentration



According to Griffith theory a crack of length a is unstable at stress:

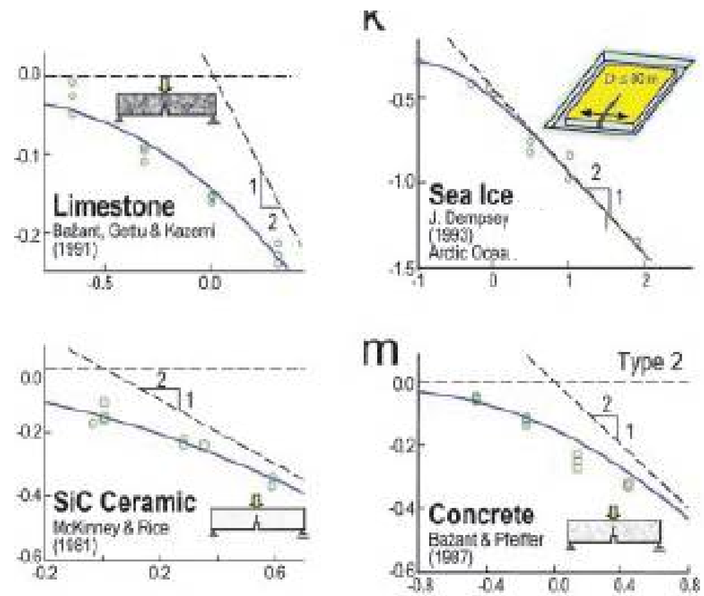
$$\sigma_c = \frac{K_c}{\sqrt{a}}$$

Energetic size effect



Z. Bazant PNAS 101, 13400, 2004

$a \sim L \quad \Rightarrow \quad \sigma_c = \frac{K_c}{\sqrt{L+\xi}}$



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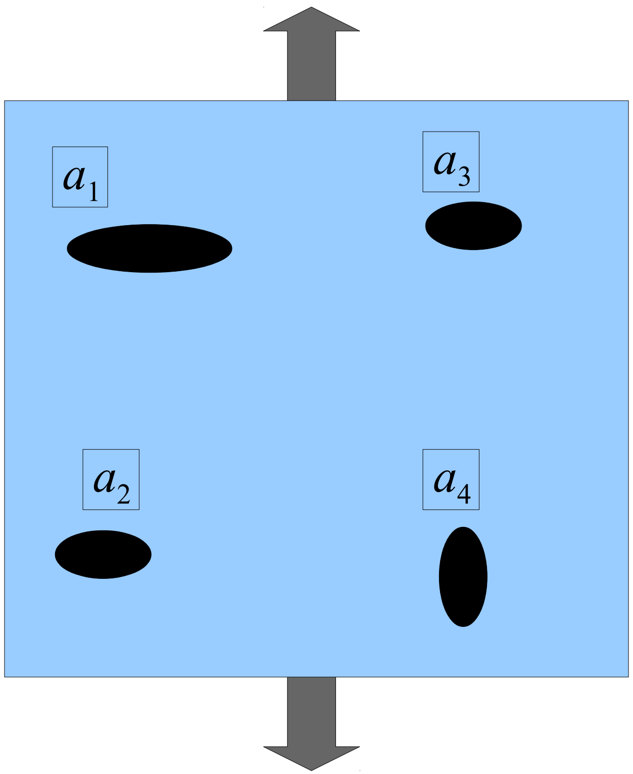
Crack nucleation (largest crack)



With a random micro-crack population, the material fails when the largest crack becomes unstable. The distribution of the maximum over N cracks:

$$P_N(a) \simeq 1 - e^{-NP(a)}$$

1) For an **exponential** tail we obtain asymptotically

$$S_N(\sigma) \simeq \exp[-N\, e^{-(\sigma_0/\sigma)^2}]$$

Duxbury-Leath-Beale (**DLB**) distribution.

2) For **power law** crack distribution we obtain

$$S_N(\sigma) \simeq \exp[-N\, c\, \sigma^{\alpha}]$$ **Weibull** distribution

Duxbury-Leath-Beale (DLB) 1987, Ray-Chakrabarti 1985, Freudenthal (1967)

Renormalization Group for fracture

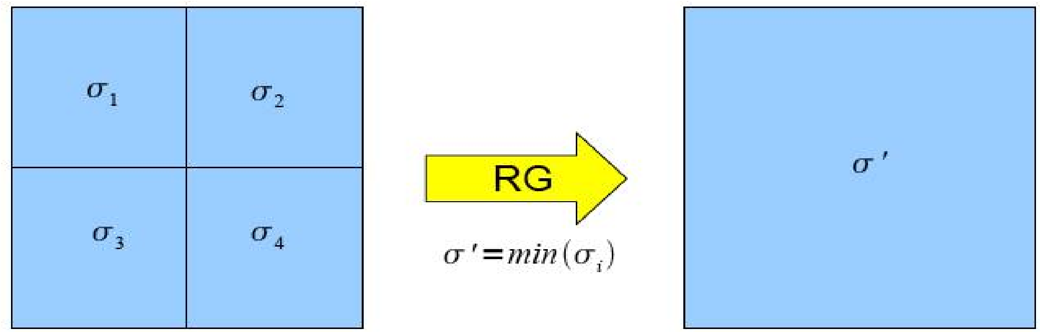
(i) Coarse graining:

$$S_L(\sigma) = \left[S_{L/2}(\sigma)\right]^4$$

(ii) Rescaling:

$$\sigma = A_L\sigma + B_L$$

(iii) Fixed point:



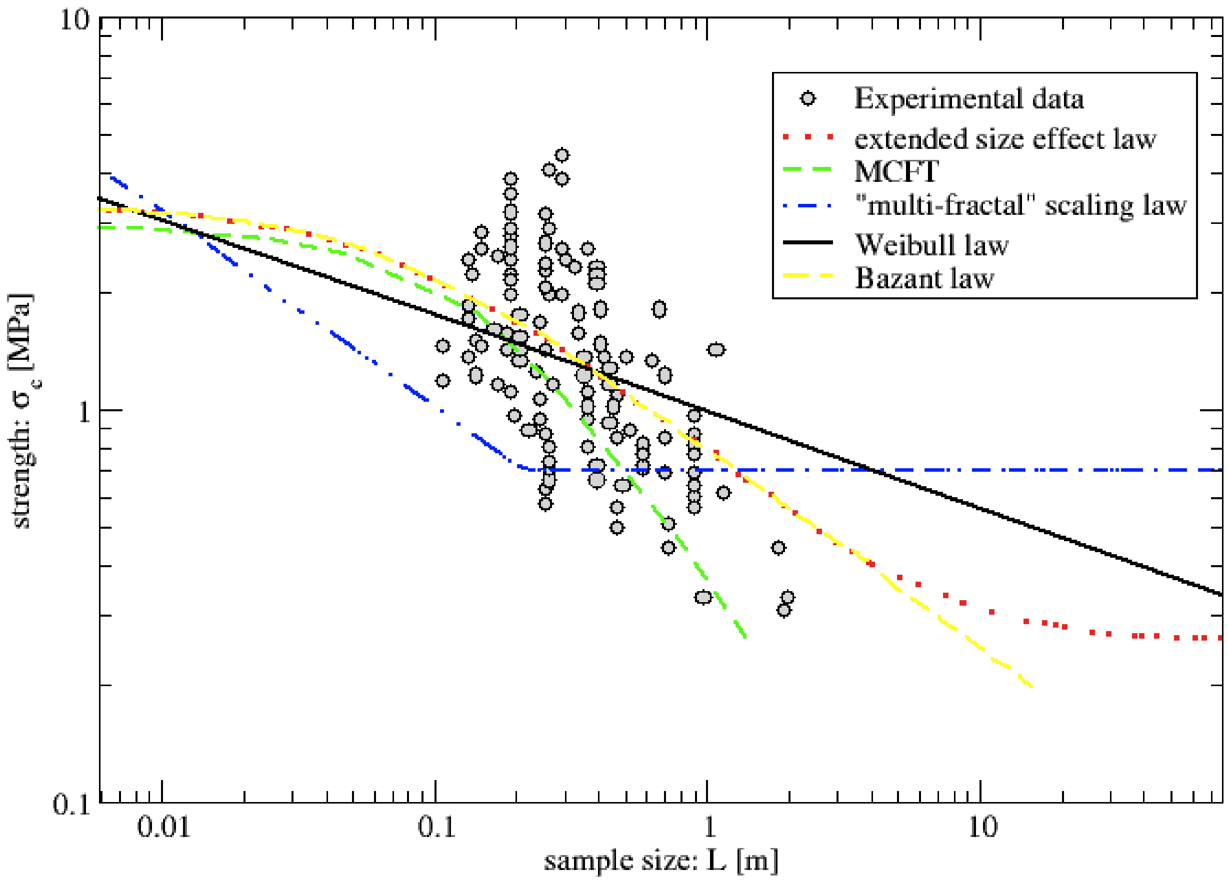
$$S^*(\sigma) = \mathcal{R}[S^*(\sigma)] = \left[S^*(a\sigma + b)\right]^4$$

Weibull

Gumbel

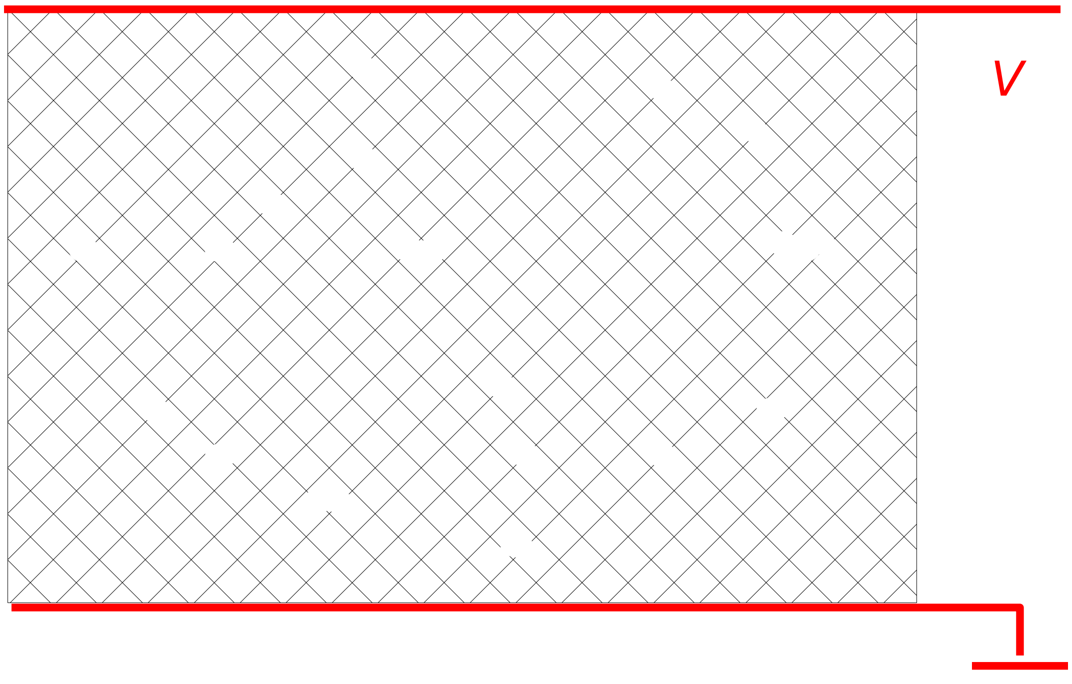
Extreme Value Theory is the “free” RG fixed point

Testing the theories



Z. P. Bazant and A. Yavari, Engineering Fracture Mechanics 72 1 (2005).

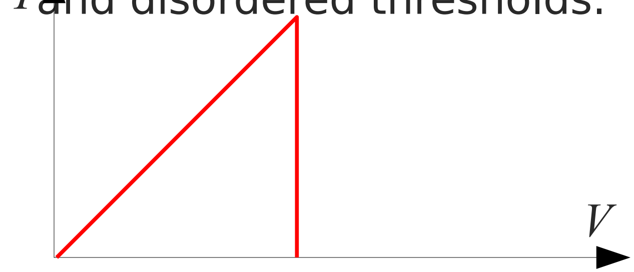
Modeling fracture with fuse models



- A external current is applied through the bus bars to a a resistor network
- Local current are obtained solving Kirchhoff equations

$$\sum_i \sigma_{ij}(V_i - V_j) = 0$$

- Fuses have unit conductivity and disordered thresholds.



As the current *I* is raised, fuses burn until a spanning crack is nucleated

de Arcangelis, Redner & Herrmann
J. Phys (Paris) Lett 46, 585 (1985)

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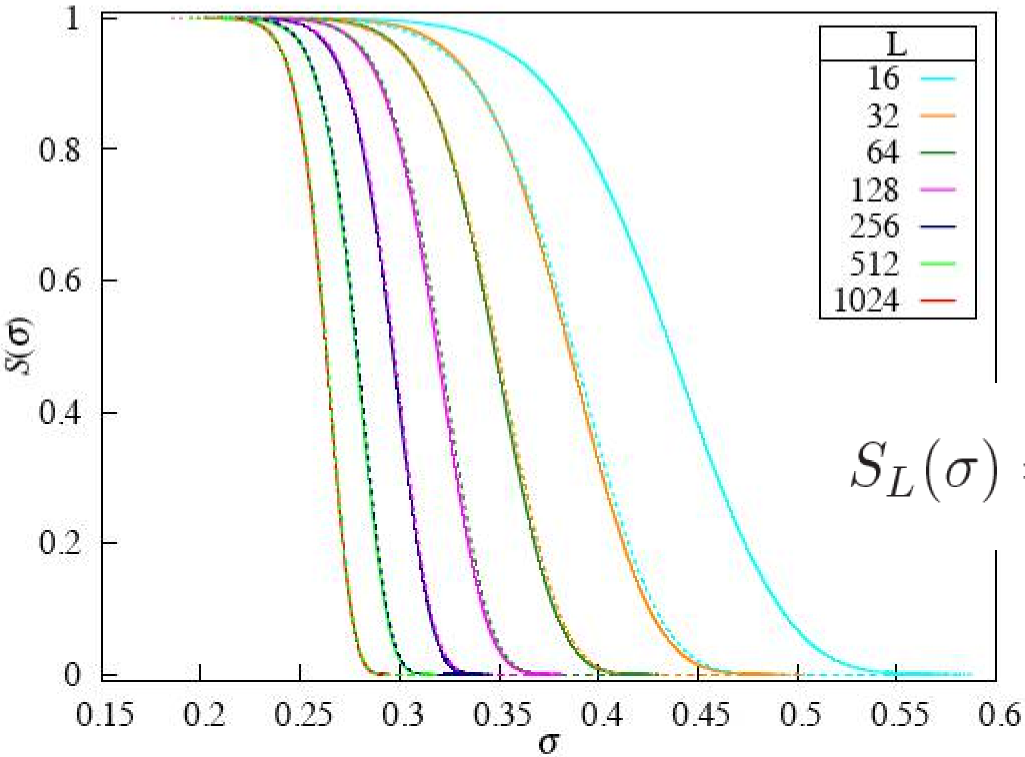
Testing renormalization group



$$S_L(\sigma) = \left[S_{L/2}(\sigma)\right]^4$$

No visible corrections from interactions: EVT works!

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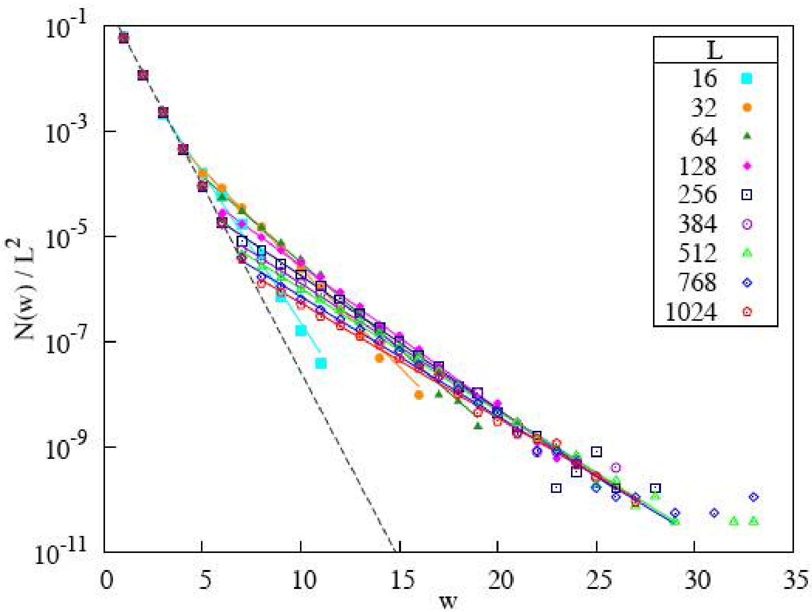
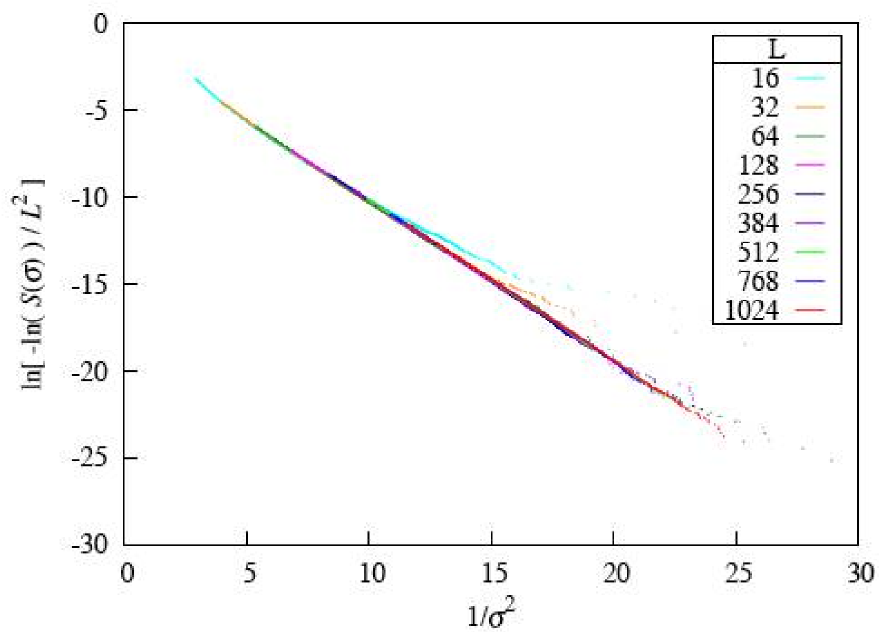
Testing the largest crack model

Random dilution: exponential crack distribution





Cracks distribution is exponential: **DLB failure distribution**

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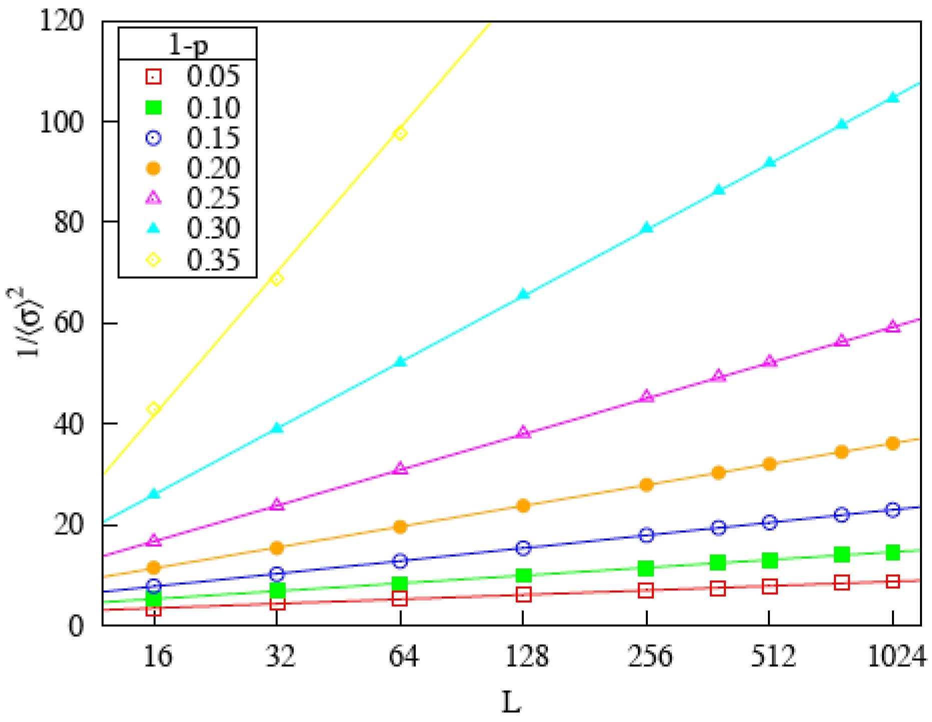
Size effects: effect of disorder



Change the initial concentration of broken bonds

$$\langle \sigma \rangle = \sigma_0 / \sqrt{\log(L^2)}$$

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Asymptotic properties

The DLB distribution converges to the **Gumbel** distribution:

$$D_L(\sigma) \equiv \exp[-L^2 e^{-(\sigma_0/\sigma)^2}]$$

$$\lim_{L\to\infty} D_L(A_L\sigma + B_L) = \Lambda(\sigma)$$

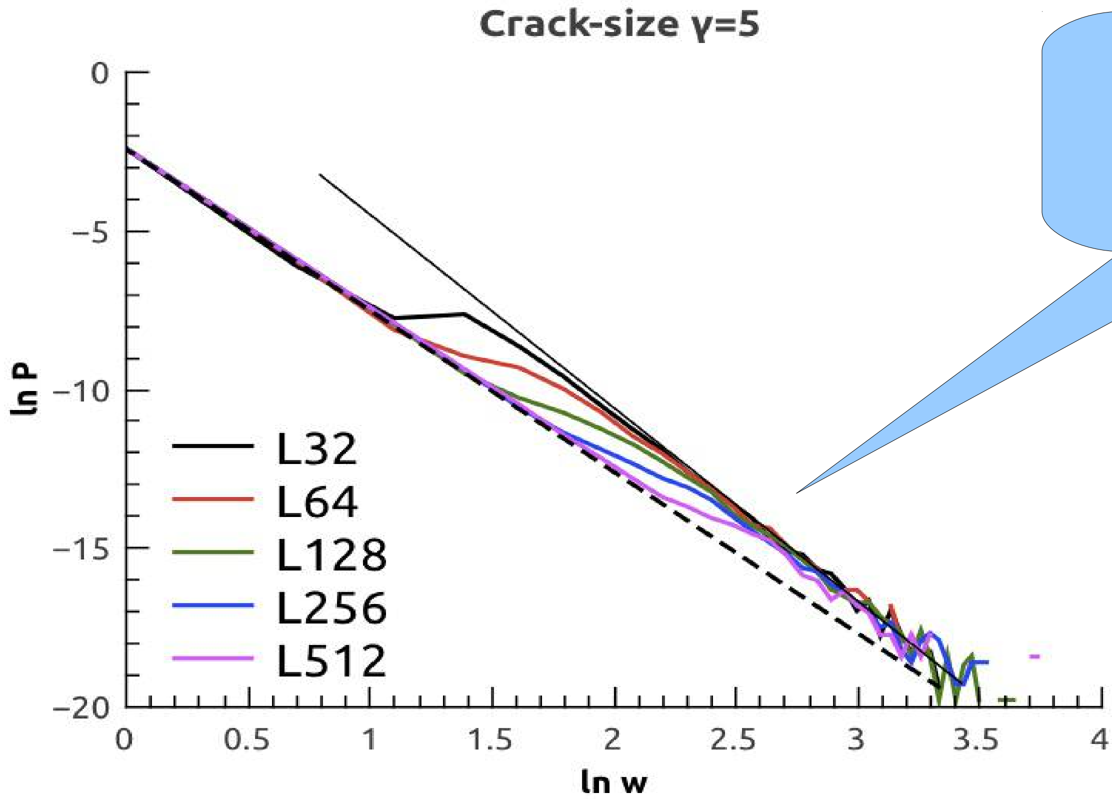
$$\Lambda(\sigma) \equiv \exp[-e^{\sigma}], \quad \sigma \in \Re$$

Direct proof: $A_L = \sigma_0/(2(\log(L^2))^{3/2})$ and $B_L = \sigma_0/\sqrt{\log(L^2)}$

Gumbel has a tail for negative values, but stress is positive!

Gumbel gives a very bad approximation for the tails in finite systems.
It is better to use DLB for extrapolations.

Power law cracks



Power law changes at peak load!

Z. Bertalan,A. Shekhawat JP Sethna SZ, Phys Rev Appl 2014

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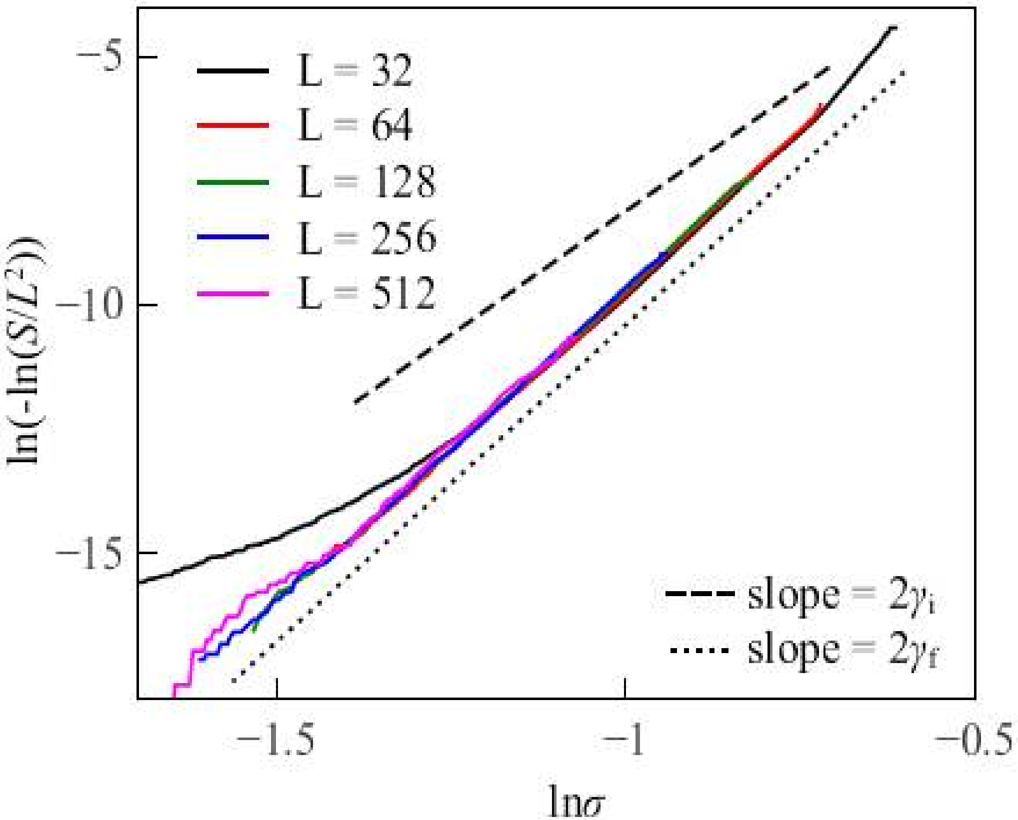
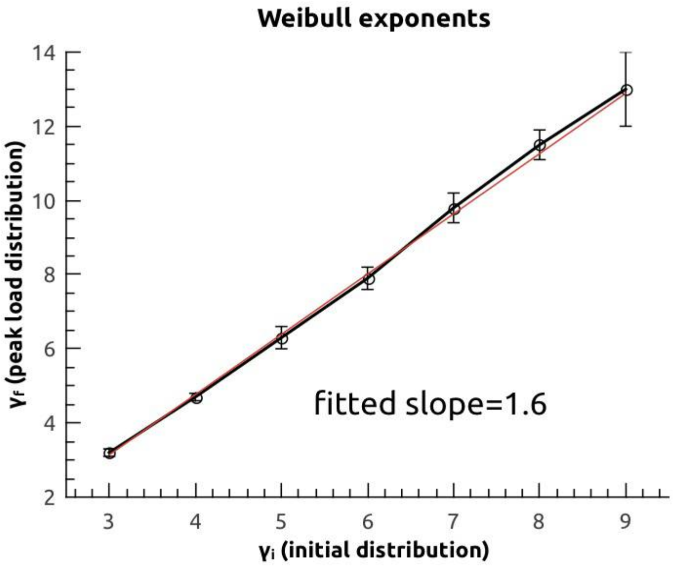
Power law cracks yield Weibull distribution





Z. Bertalan,A. Shekhawat JP Sethna SZ, Phys Rev Appl 2014

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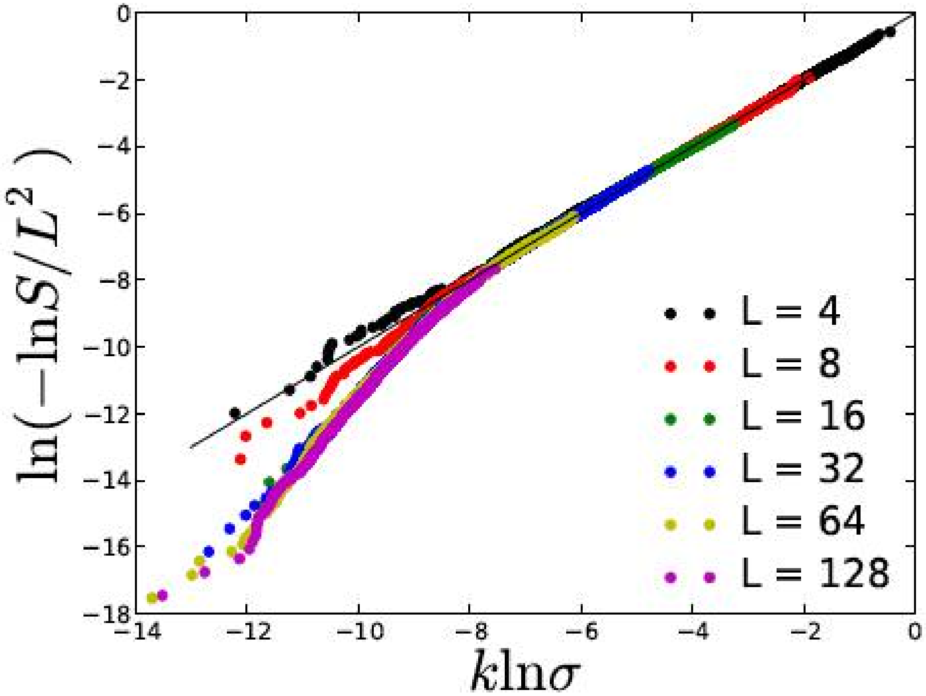
Stability of Weibull distribution in two dimensions?

Weibull distributed thresholds

$$S_{L=1} = \exp(-\sigma^k)$$

Increase L

Deviation from Weibull



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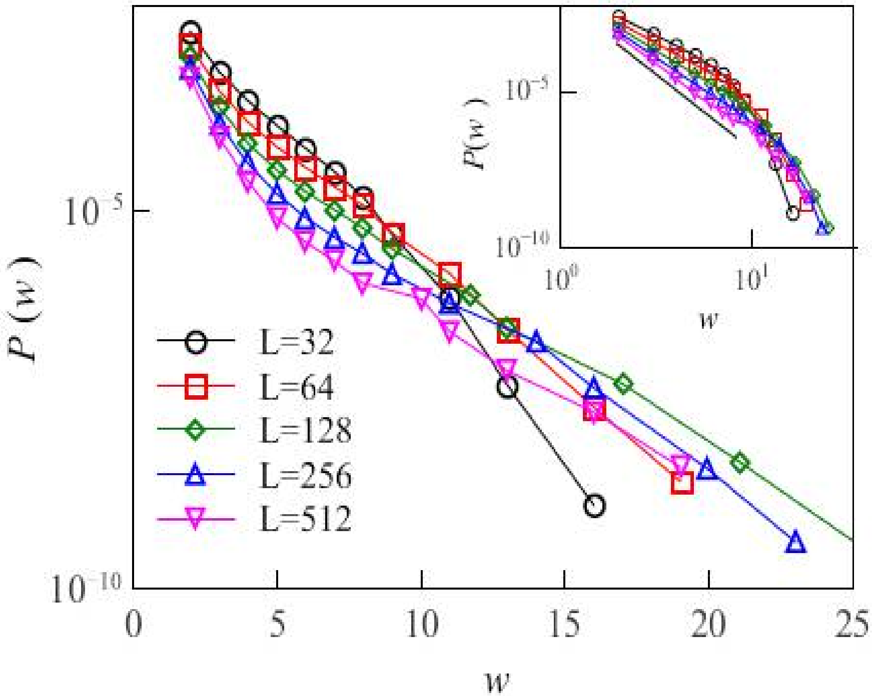
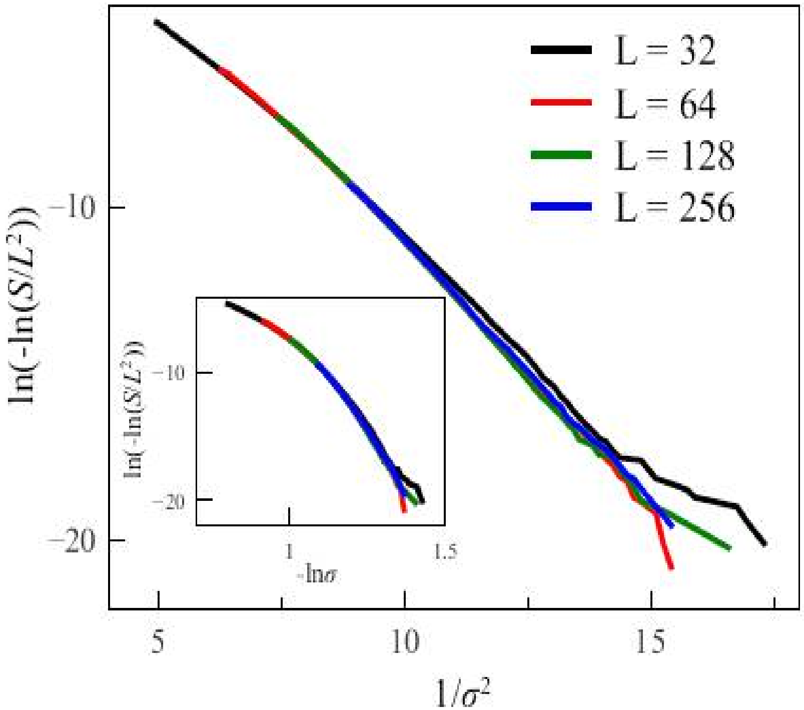
Stress enhancement around cracks rules failure statistics





Crack distribution is exponential: **DLB failure distribution**

Z. Bertalan,A. Shekhawat JP Sethna SZ, Phys Rev Appl 2014

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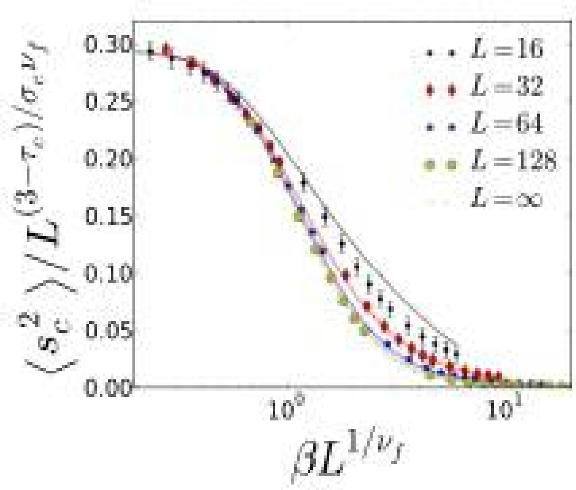
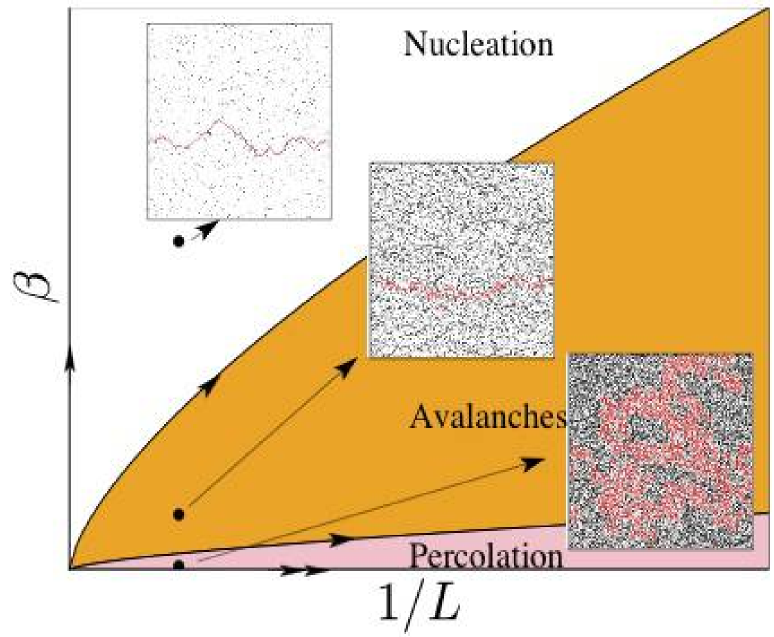
From Percolation to Nucleation

$$S_1(x) = 1 - x^\beta$$

$\beta \to 0$ Percolation

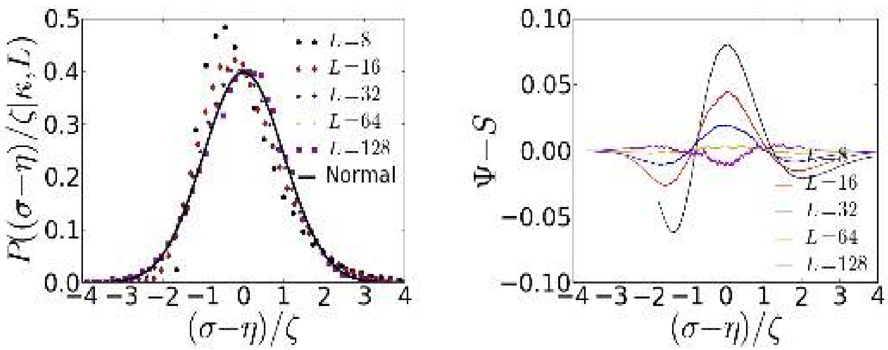




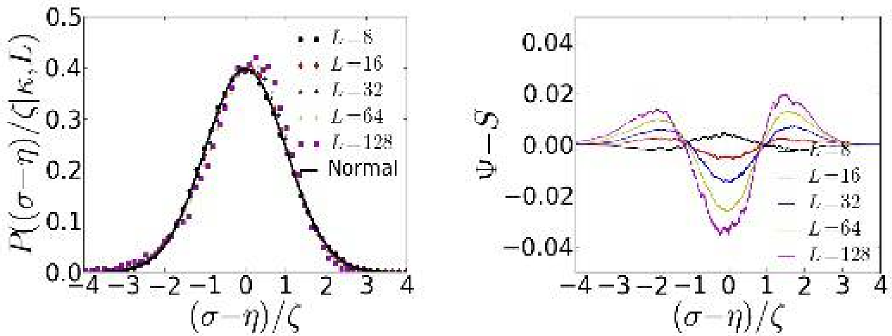
A. Shekhawat, SZ, J. P. Sethna PRL 2013

Strong disorder: crossover

k=0.5



k=3



Weibull

↓

Gaussian

↓

Nucleation (DLB?)

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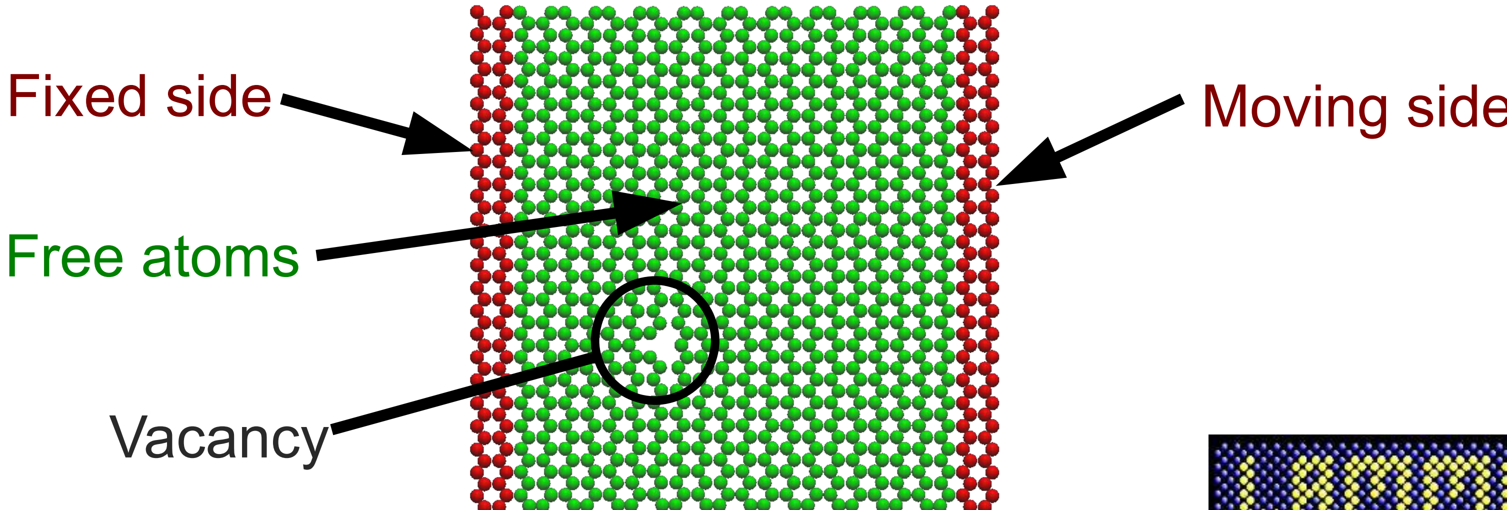
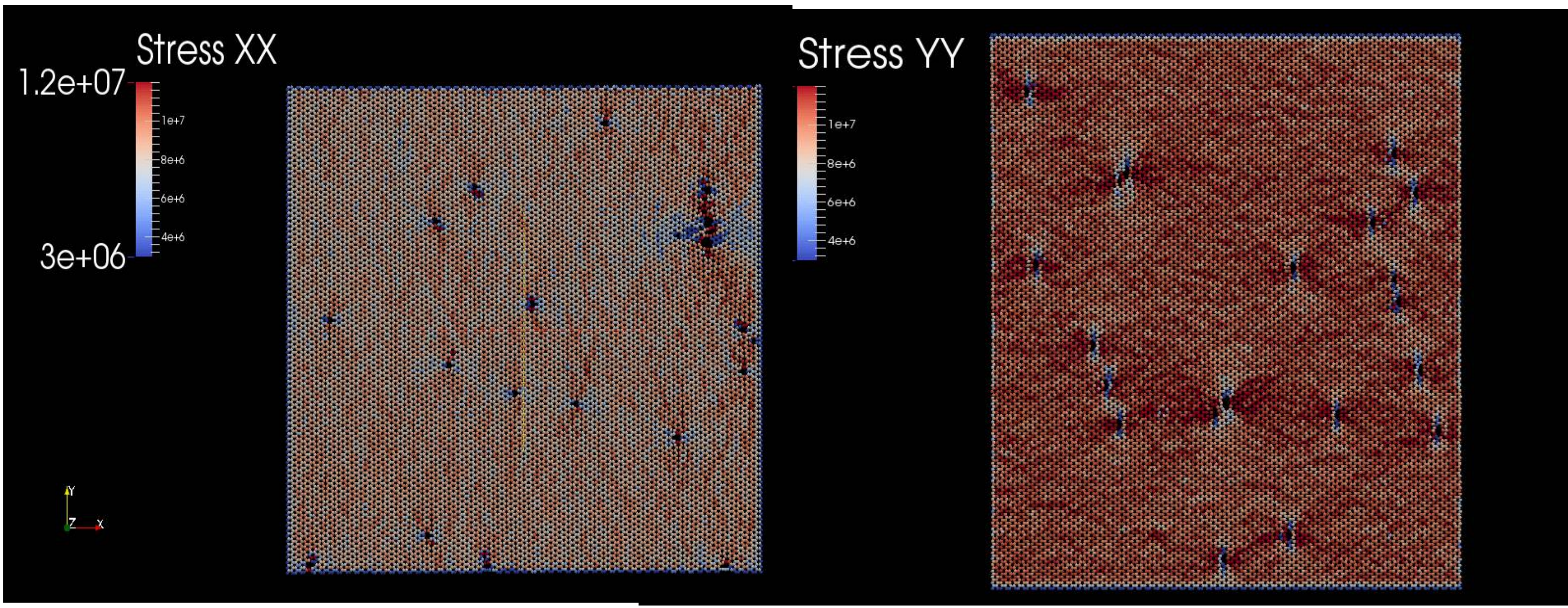
Size effects in graphene fracture

Using standard MD tools: C-C: AIREBO[1] (+ tuning)
Defects: random vacancies in lattice
Deformation: constant engineering strain rate

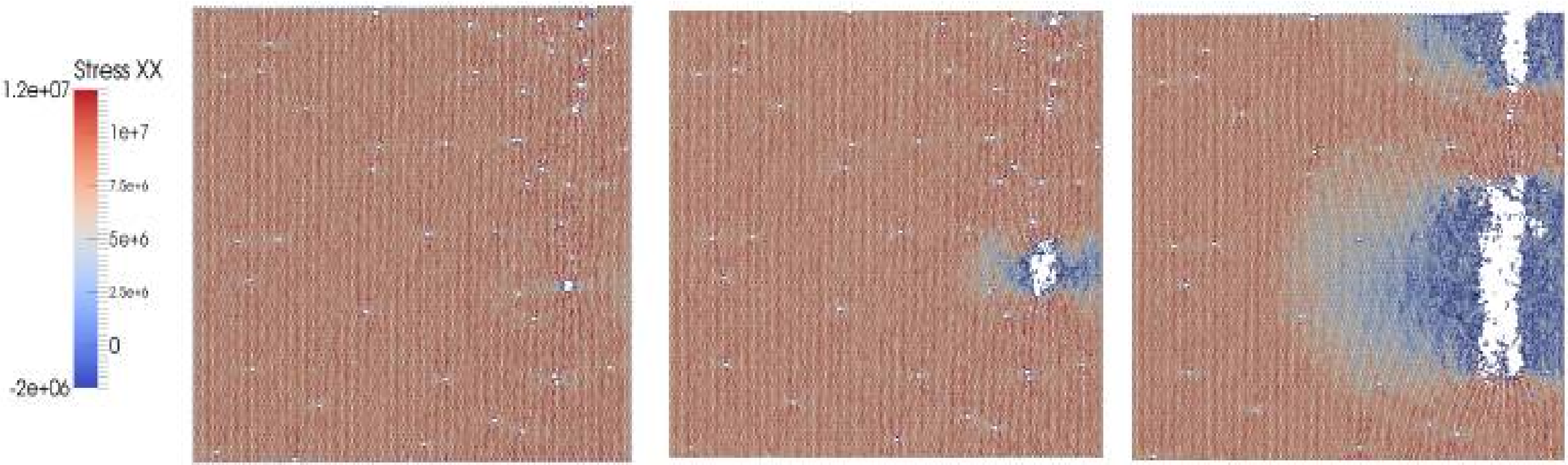


LAMMPS

Stress localization



Crack nucleation



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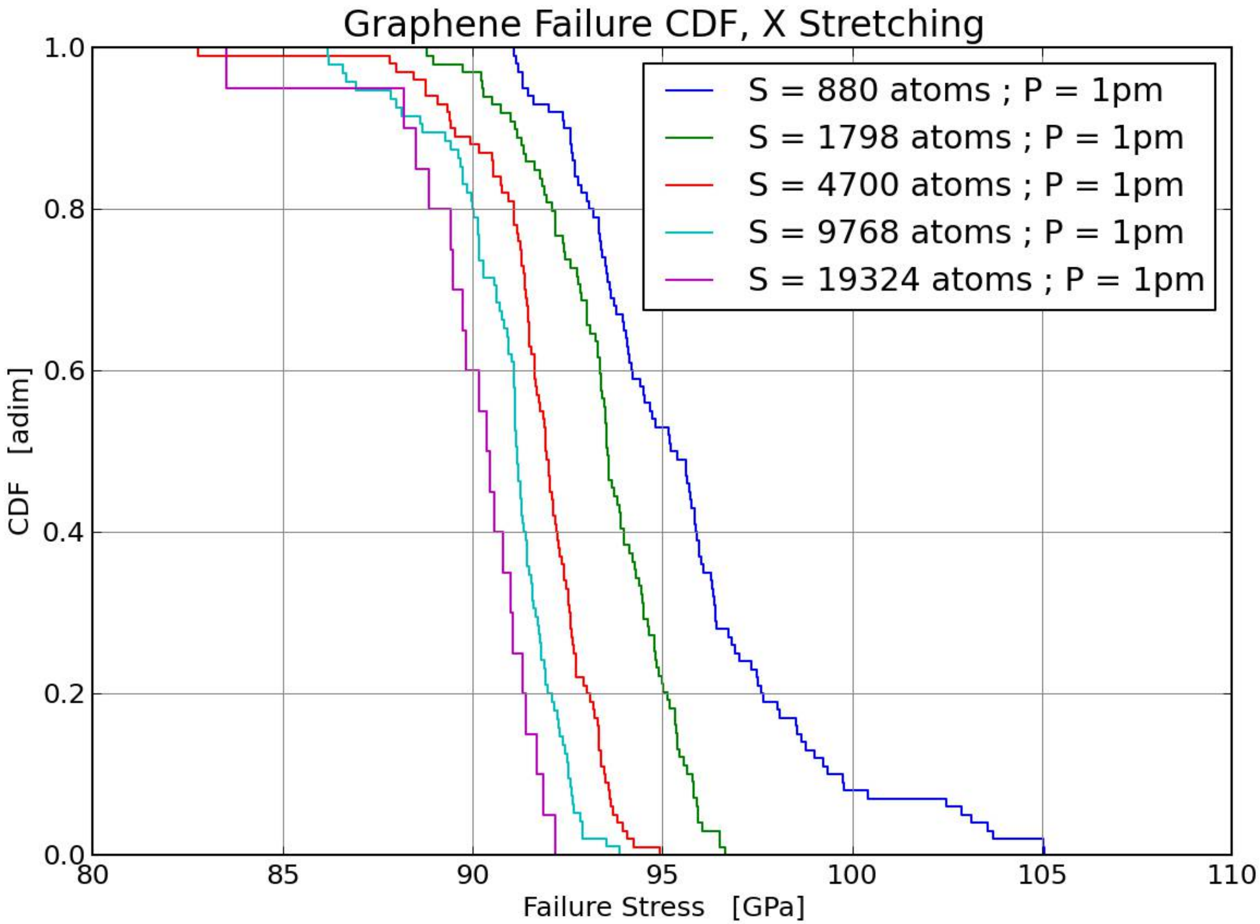
Size effects



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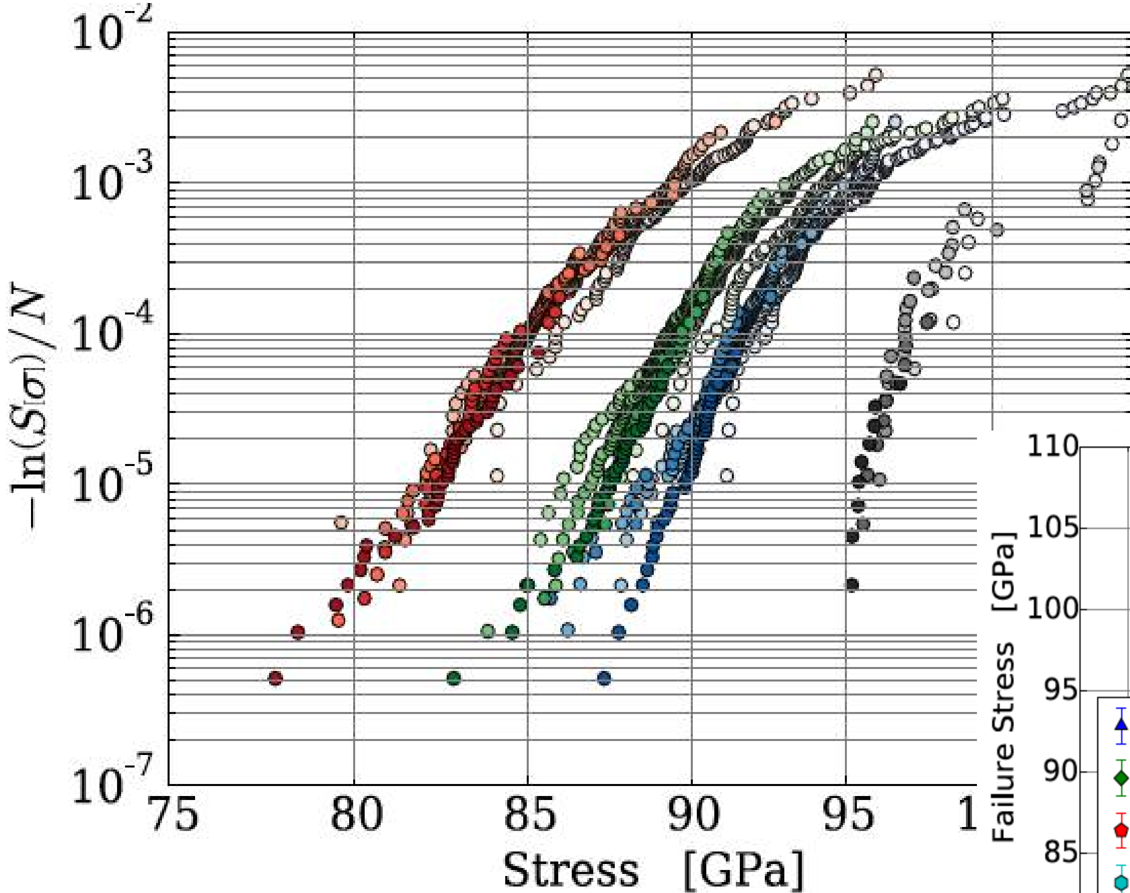
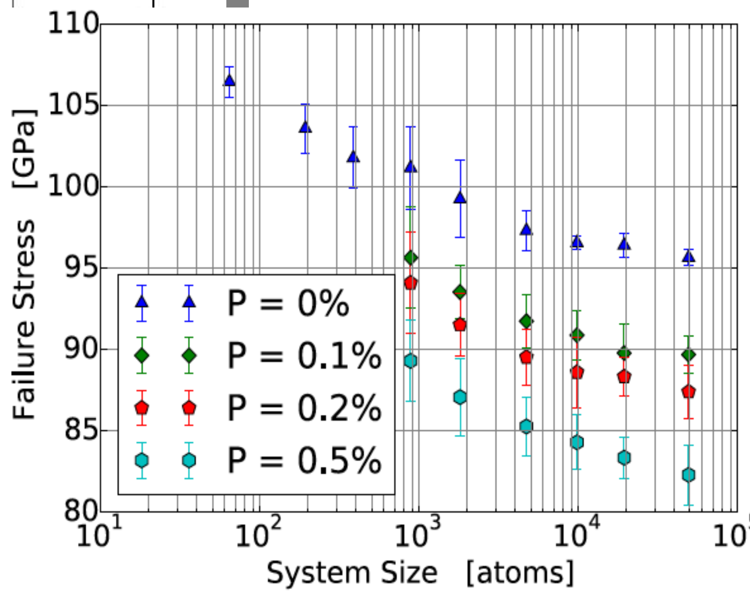
Extreme value scaling





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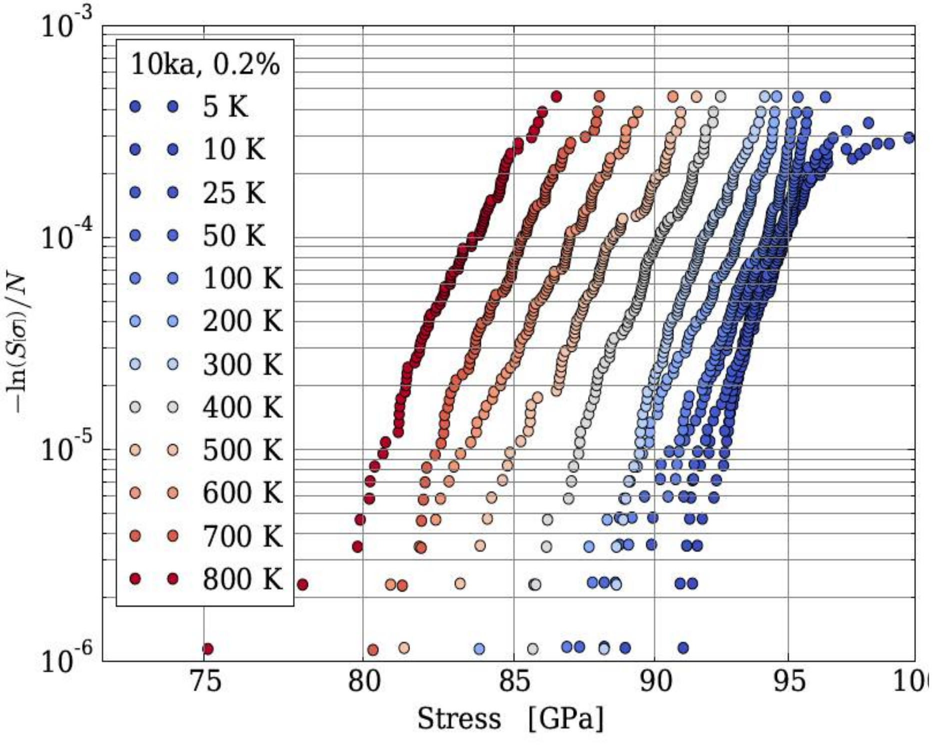
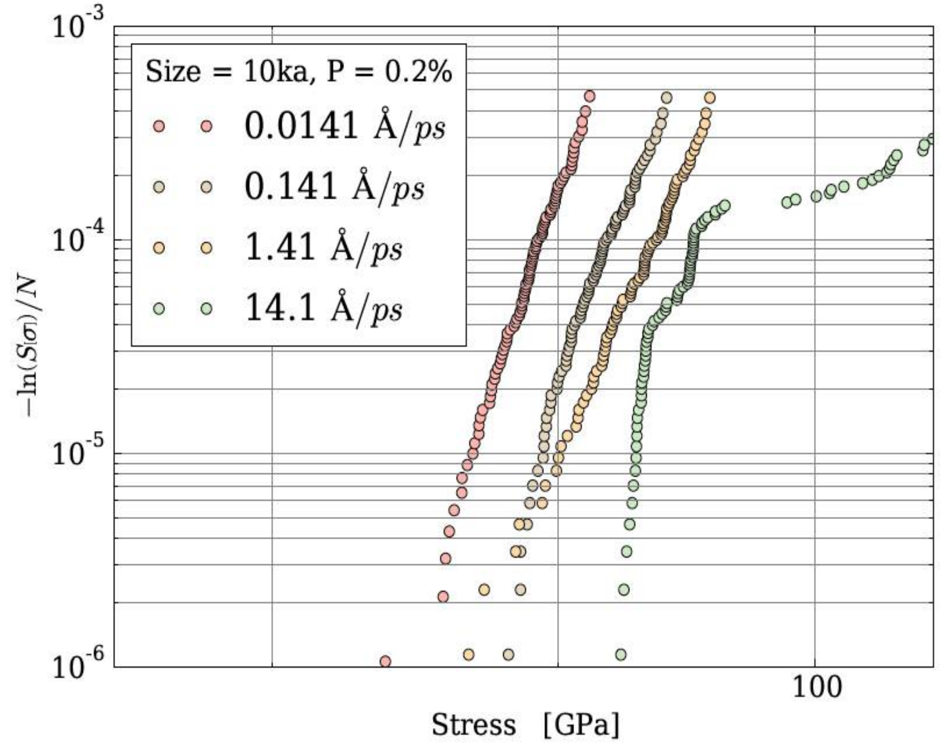
Temperature & rate dependence





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Conclusions

- Size effects can be understood using analogies with phase transition.
- RG for fracture yields non interacting EVT fixed point, nucleation theory works well.
- For strong disorder there is a crossover from percolation to nucleation
- Graphene fracture scales as EVT but with thermal and rate dependence.

Thanks

CNR
Milano

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Sellerio

Alessandro
Taloni

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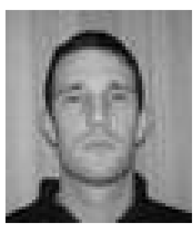
Torino

Zsolt
Bertalan

Aalto
Helsinki



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Sethna

Announcements

Three postdoctoral positions available!!!

Research topics include computational models of plasticity in crystalline and amorphous materials and other rheological problems including active fluids, fracture of disordered media, frictional sliding, domain wall dynamics and biomechanics of tissues and other biological related problems.

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