Genomics, Networks, and Computational Concepts for Polytopic SUSY Representation Theory

S. James Gates, Jr.

Brown Theoretical Physics Center Director, Ford Foundation Professor of Physics, Affiliate Mathematics Professor, and Watson Institute for International & Public Affairs Faculty Fellow

BTPC Director Office, Rm 110, Barus Hall, 340 Brook St., Providence, RI 02912
t: 401-863-6452  e: sylvester_gates@brown.edu  w: https://sites.brown.edu/sjgates
Graphs, Networks & Polytopes In Science
The structure of DNA

DNA is a double helix formed by base pairs attached to a sugar-phosphate backbone.
Tempo and mode of genome evolution

• Sequenced **264 complete genomes** (12 populations x 11 time points x 2 clones).
The network graph formed by Wikipedia editors (edges) contributing to different Wikipedia language versions (vertices) during one month in summer 2013.

By Computermacgyver - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=30349051

Graph theory in sociology: Moreno Sociogram (1953).

https://en.wikipedia.org/wiki/Graph_theory
Salt
Diamond C$_{18}$
<table>
<thead>
<tr>
<th><strong>Faujasite</strong></th>
<th><strong>General</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Category</td>
<td>Zeolite</td>
</tr>
<tr>
<td>Formula</td>
<td>(Na₂, Ca, Mg)₃₅[Al₇Si₁₇O₄₈]·32(H₂O)[¹]</td>
</tr>
<tr>
<td>(repeating</td>
<td></td>
</tr>
<tr>
<td>unit)</td>
<td></td>
</tr>
<tr>
<td>Strunz</td>
<td>9.GD.30</td>
</tr>
<tr>
<td>classification</td>
<td></td>
</tr>
<tr>
<td>Crystal</td>
<td>Cubic</td>
</tr>
<tr>
<td>system</td>
<td></td>
</tr>
<tr>
<td>Crystal class</td>
<td>Hexoctahedral (m̅3m)</td>
</tr>
<tr>
<td></td>
<td>H–M Symbol (4/m 3 2/m)</td>
</tr>
<tr>
<td>Space group</td>
<td>Fd₃m</td>
</tr>
<tr>
<td>Unit cell</td>
<td>a = 24.638–24.65 Å, Z = 32</td>
</tr>
</tbody>
</table>
Chemistry (Wikipedia)

The truncated octahedron exists in the structure of the faujasite crystals.
BLAST finds regions of similarity between biological sequences. The program compares nucleotide or protein sequences to sequence databases and calculates the statistical significance.

https://blast.ncbi.nlm.nih.gov/Blast.cgi?
PROGRAM=blastn&PAGE_TYPE=BlastSearch&LINK_LOC=blasthome
Standard Model & Supersymmetry Extension
### Fields of the Standard Model

<table>
<thead>
<tr>
<th>Force Carrier Fields</th>
<th>Fermions</th>
<th>Bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matter Fields</td>
<td><img src="image1" alt="Fermions" /></td>
<td><img src="image2" alt="Bosons" /></td>
</tr>
</tbody>
</table>

#### Fermions
- $\nu_e$, $\nu_{\mu}$, $\nu_{\tau}$
- $e^-$, $\mu^-$, $\tau^-$
- $u_R$, $d_R$, $s_R$, $b_R$

#### Bosons
- $g$, $\gamma$, $Z^0$, $W^\pm$
Precision in Measurement

The Best Known Number In All Of Science
Electron’s Magnetic Property: The “g-Value”

Measured $g$: 2.002 324 303 9
Theoretical $g$: 2.002 324 304 4
Delta $g$: 0.000 000 004 0

These values from about a decade old and are given in example of experimental v. theoretical g-Values.
Is string theory phenomenologically viable?

S. James Gates Jr

String theory is entering an era in which its theoretical constructs will be confronted by experimental data. Some cherished ideas just might fail to pass the test.
“Thus, if Nature is kind enough to provide light superpartners, one might still expect about a century to pass before a superparticle is directly observed.”

– SJG in Physics Today,

“SUSY, I strongly believe, will in the end be figuratively like Mark Twain who is often misquoted as having said, "The reports of my death have been greatly exaggerated.”

– SJG 2008 in Waves and Packets:
https://multibriefs.com/briefs/nsbp/extrapage.html
Feature: Supersymmetry

Sticking with SUSY

When CERN's Large Hadron Collider failed to uncover evidence of new "superpartner" particles during its first run, some claimed that the theory that predicts them — known as supersymmetry, or SUSY — should be abandoned. S James Gates, Jr, however, argues that giving up on SUSY now would be like concluding that giant sequoia trees do not exist after surveying only the east coast of North America, and that there is more at stake than meets the eye.
“In my view, the current situation is akin to that of an explorer who, having scoured the eastern seaboard of North America, concludes that no groves of *Sequoiadendron giganteum* exist on the entire continent. As with this hypothetical hunt for giant sequoia trees, finding evidence for SUSY depends on the observer looking in the right place.”

Fields of the Standard Model

<table>
<thead>
<tr>
<th>Force Carrier Fields</th>
<th>Fermions</th>
<th>Bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>v_e, v_μ, v_τ</td>
<td></td>
<td>g</td>
</tr>
<tr>
<td>e, μ, τ</td>
<td></td>
<td>γ</td>
</tr>
<tr>
<td>e, μ, τ</td>
<td></td>
<td>Z^0</td>
</tr>
<tr>
<td>v_e, v_μ, v_τ</td>
<td></td>
<td>H^0</td>
</tr>
<tr>
<td>e, μ, τ</td>
<td></td>
<td>W^+</td>
</tr>
</tbody>
</table>
Fields of the Minimal SUSY Standard Model

<table>
<thead>
<tr>
<th>Force Carrier Fields</th>
<th>Fermions</th>
<th>Bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force Carrier Fields</td>
<td><img src="image" alt="Fermions" /></td>
<td><img src="image" alt="Bosons" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Matter Fields</th>
<th>Fermions</th>
<th>Bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matter Fields</td>
<td><img src="image" alt="Fermions" /></td>
<td><img src="image" alt="Bosons" /></td>
</tr>
</tbody>
</table>
Precision in Measurement

The Best Known Number In All Of Science
Electron’s Magnetic Property: The “g-Value”

**NIST CODATA recommended $g$-factor values**\(^{[12]}\)

<table>
<thead>
<tr>
<th>Particle</th>
<th>Symbol</th>
<th>$g$-factor</th>
<th>Relative standard uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron</td>
<td>$g_e$</td>
<td>$-2.002,319,304,362,56(35)$</td>
<td>$1.7 \times 10^{-13}^{[8]}$</td>
</tr>
</tbody>
</table>

These values from about a decade old and are given in example of experimental v. theoretical g-Values.
Long-Awaited Muon Measurement Boosts Evidence for New Physics

Initial data from the Muon g-2 experiment have excited particle physicists searching for undiscovered subatomic particles and forces

By Daniel Garisto on April 7, 2021
Classical Mechanics vs. Quantum Mechanics
Classical Mechanics vs. Quantum Mechanics
Classical Mechanics vs. Quantum Mechanics
Classical Mechanics vs. Quantum Mechanics
Classical Mechanics vs. Quantum Mechanics
Classical Mechanics vs. Quantum Mechanics
Classical Mechanics vs. Quantum Mechanics
Classical Mechanics vs. Quantum Mechanics
Classical Mechanics vs. Quantum Mechanics
SU(3): A Golden Gateway Understanding to the Polytopes Undergirding the Symmetries of the Standard Model
The Gell-Mann su(3) “Lambda-Matrices”

\[
\begin{align*}
\lambda_1 &= \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_2 &= \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, & \lambda_3 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
\lambda_4 &= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & \lambda_5 &= \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, & \lambda_6 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\
\lambda_7 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, & \lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.
\end{align*}
\]

su(3) structure constants

su(3) “d-symbols”
Eigenvalues and eigenvectors

\[ Av = \lambda v, \quad |A - \lambda I| = 0 \]

\[ |A - \lambda I| = (\lambda_1 - \lambda)(\lambda_2 - \lambda) \cdots (\lambda_n - \lambda), \]

\[ A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}. \]

\[ |A - \lambda I| = \begin{vmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{vmatrix} = 3 - 4\lambda + \lambda^2. \]

\[ v_{\lambda=1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad v_{\lambda=3} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \]
Review of Representation Tools for the su(3) Lie Algebra

Commutators

$$[T_i, T_j] = i f_{ij}^k T_k$$,

$$T_i = \frac{1}{2} \lambda_i$$, su(3) generators

Totally anti-symmetric structure constants $f_{ij}^k$ specified by

$$f_{123} = 1, \quad f_{147} = f_{165} = f_{246} = f_{257} = f_{345} = f_{376} = \frac{1}{2},$$

$$f_{458} = f_{678} = \frac{\sqrt{3}}{2},$$

$T_3$ and $T_8$ commute, $\rightarrow$ simultaneous eigenvectors

$$T_3 |t_3, t_8\rangle = t_3 |t_3, t_8\rangle, \quad T_8 |t_3, t_8\rangle = t_8 |t_3, t_8\rangle.$$
The Nonvanishing Values of $f_{ijk}$ and $d_{ijk}$

<table>
<thead>
<tr>
<th>$[ijk]$</th>
<th>$f_{ijk}$</th>
<th>$(ijk)$</th>
<th>$d_{ijk}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>123</td>
<td>1</td>
<td>118</td>
<td>$1/\sqrt{3}$</td>
</tr>
<tr>
<td>147</td>
<td>$\frac{1}{2}$</td>
<td>146</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>156</td>
<td>$-\frac{1}{2}$</td>
<td>157</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>246</td>
<td>$\frac{1}{2}$</td>
<td>228</td>
<td>$1/\sqrt{3}$</td>
</tr>
<tr>
<td>257</td>
<td>$\frac{1}{3}$</td>
<td>247</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>345</td>
<td>$\frac{1}{2}$</td>
<td>256</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>367</td>
<td>$-\frac{1}{2}$</td>
<td>338</td>
<td>$1/\sqrt{3}$</td>
</tr>
<tr>
<td>458</td>
<td>$\sqrt{3}/2$</td>
<td>344</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>678</td>
<td>$\sqrt{3}/2$</td>
<td>355</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>366</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>377</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>448</td>
<td>$-1/2 \sqrt{3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>558</td>
<td>$-1/2 \sqrt{3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>668</td>
<td>$-1/2 \sqrt{3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>778</td>
<td>$-1/2 \sqrt{3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>888</td>
<td>$-1/\sqrt{3}$</td>
</tr>
</tbody>
</table>
“Raising operators” and “Lowering operators,”

\[ T_\pm = T_1 \pm iT_2 , \quad U_\pm = T_6 \pm iT_7 , \quad V_\pm = T_4 \pm iT_5 , \]

generate ‘motions’ between the eigenvectors. Also

\[ [T_+, T_-] = 2T_3 , \quad [U_+, U_-] = \sqrt{3}T_8 - T_3 , \quad [V_+, V_-] = \sqrt{3}T_8 + T_3 , \]

along with a number of commutators which have the property that the matrices \( T_3 \) and \( T_8 \) do not appear on the rhs of them.

\[ [T_\pm , T_\pm ] = [U_\pm , U_\pm ] = [V_\pm , V_\pm ] = 0 . \]
Counting Representations

\[ d_{SU(2)} = (2j + 1) \]

\[ d_{SU(3)} = \frac{1}{2}(p + 1)(q + 1)(p + q + 2) \]

Weyl discovered that for the simple Lie algebras the dimension of a representation \( \Gamma(w^h) \) labeled by the highest weight \( w^h \) is given the formula

\[ \dim \Gamma(w^h) = \prod_{\mu > 0} [\mu \cdot (w^h + \mu^+/2)]/[\mu \cdot (\mu^+/2)]. \]

Here the product is to be taken over all positive root vectors and \( \mu^+ \) is the sum of all positive roots.
\[ p = 1, \quad q = 0 \quad \text{and} \quad p = 0, \quad q = 1 \]
$p = 1, \quad q = 1$
\[ s = 1 \]

\[ s = 0 \]

\[ s = -1 \]

\[ q = 1 \]

\[ q = -1 \]

\[ q = 0 \]
$p = 3$, $q = 0$
A typical representation $[(p, q) = (7, 3)]$ with multiplicities.
Can Polytopes Undergird Supersymmetry Extension of the Standard Model?
\[ \mathcal{V}(x^a, \theta^\alpha) = v^{(0)}(x^a) + \theta^\alpha v^{(1)}_\alpha(x^a) \\
+ \theta^\alpha \theta^\beta \left[ C_{\alpha \beta} v^{(2)}_1(x^a) + i(\gamma^5)_{\alpha \beta} v^{(2)}_2(x^a) + i(\gamma^5 \gamma^b)_{\alpha \beta} v^{(2)}_b(x^a) \right] \\
+ \theta^\alpha \theta^\beta \theta^\gamma C_{\alpha \beta} C_{\gamma \delta} v^{(3)}(x^a) + \theta^\alpha \theta^\beta \theta^\gamma \theta^\delta C_{\alpha \beta} C_{\gamma \delta} v^{(4)}(x^a) \]

<table>
<thead>
<tr>
<th>Level</th>
<th>Adinkra nodes</th>
<th>Component fields</th>
<th>Irrep(s) in so(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>\mathcal{V}</td>
<td>f(x^a)</td>
<td>{1}</td>
</tr>
<tr>
<td>1</td>
<td>D_\alpha \mathcal{V}</td>
<td>\psi_\alpha(x^a)</td>
<td>{4}</td>
</tr>
<tr>
<td>2</td>
<td>D_{[\alpha D_\beta]} \mathcal{V}</td>
<td>g(x^a), h(x^a), \nu_b(x^a)</td>
<td>{1}, {1}, {4}</td>
</tr>
<tr>
<td>3</td>
<td>D_{[\alpha D_\beta D_\gamma]} \mathcal{V}</td>
<td>\chi^\delta(x^a)</td>
<td>{4}</td>
</tr>
<tr>
<td>4</td>
<td>D_{[\alpha D_\beta D_\gamma D_\delta]} \mathcal{V}</td>
<td>N(x^a)</td>
<td>{1}</td>
</tr>
</tbody>
</table>

Explicit Relations between Adinkra Nodes, Component Fields, and Irreps
Eleven Dimensional SUGRA Limit of M-Theory

<table>
<thead>
<tr>
<th>FERMION</th>
<th>BOSON</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Psi_\mu^\alpha$</td>
<td>$h_{\mu \nu}$</td>
</tr>
<tr>
<td></td>
<td>$A_{\mu \nu \rho}$</td>
</tr>
</tbody>
</table>
Superfield Component Decompositions and the Scan for Prepotential Supermultiplets in 10D Superspaces

S. James Gates Jr., Yangrui Hu, S.-N. Hazel Mak

The first complete and explicit SO(1,9) Lorentz descriptions of all component fields contained in $\mathcal{N} = 1$, $\mathcal{N} = 2A$, and $\mathcal{N} = 2B$ unconstrained scalar 10D superfields are presented. These are made possible by the discovery of the relation of the superfield component expansion as a consequence of the branching rules of irreducible representations in one ordinary Lie algebra into one of its Lie subalgebras. Adinkra graphs for ten dimensional superspaces are defined for the first time, whose nodes depict spin bundle representations of SO(1,9). An analog of Breitenlohner's approach is implemented to scan for superfields that contain graviton(s) and gravitino(s), which are the candidates for the prepotential superfields of 10D off-shell supergravity theories and separately abelian Yang–Mills theories are similarly treated. Version three contains additional content, both historical and conceptual, which broaden the reach of the scan in the 10D Yang–Mills case.
Adinkra Foundation of Component Decomposition and the Scan for Superconformal Multiplets in 11D, N = 1 Superspace

S. James Gates Jr., Yangrui Hu, S.-N. Hazel Mak

For the first time in the physics literature, the Lorentz representations of all 2,147,483,648 bosonic degrees of freedom and 2,147,483,648 fermionic degrees of freedom in an unconstrained eleven dimensional scalar superfield are presented. Comparisons of the conceptual bases for this advance in terms of component field, superfield, and adinkra arguments, respectively, are made. These highlight the computational efficiency of the adinkra-based approach over the others. It is noted at level sixteen in the 11D, N = 1 scalar superfield, the \{65\} representation of SO(1,10), the conformal graviton, is present. Thus, Adinkra-based arguments suggest the surprising possibility that the 11D, N = 1 scalar superfield alone might describe a Poincare supergravity prepotential in analogy to one of the off-shell versions of 4D, N = 1 superfield supergravity.
Advancing to Adynkrafields: Young Tableaux to Component Fields of the 10D, N = 1 Scalar Superfield

S. James Gates Jr., Yangrui Hu, S.-N. Hazel Mak

Starting from higher dimensional adinkras constructed with nodes referenced by Dynkin Labels, we define "adynkra." These suggest a computationally direct way to describe the component fields contained within supermultiplets in all superspaces. We explicitly discuss the cases of ten dimensional superspaces. We show this is possible by replacing conventional $\theta$-expansions by expansions over Young Tableaux and component fields by Dynkin Labels. Without the need to introduce $\sigma$-matrices, this permits rapid passages from Adynkra $\rightarrow$ Young Tableaux $\rightarrow$ Component Field Index Structures for both bosonic and fermionic fields while increasing computational efficiency compared to the starting point that uses superfields. In order to reach our goal, this work introduces a new graphical method, "tying rules," that provides an alternative to Littlewood's 1950 mathematical results which proved branching rules result from using a specific Schur function series. The ultimate point of this line of reasoning is the introduction of mathematical expansions based on Young Tableaux and that are algorithmically superior to superfields. The expansions are given the name of "adynkrafields" as they combine the concepts of adinkras and Dynkin Labels.
“Learning to use coding is like putting on the maths version of the Iron Man suit.”

“We required Mathematica to handle the over four billion functions! We have codes and apps for this.”
<table>
<thead>
<tr>
<th>Level #</th>
<th>Component</th>
<th>Field</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>19</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>6</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>99</td>
<td>99</td>
<td>99</td>
</tr>
<tr>
<td>8</td>
<td>106</td>
<td>106</td>
<td>106</td>
</tr>
<tr>
<td>9</td>
<td>173</td>
<td>173</td>
<td>173</td>
</tr>
<tr>
<td>10</td>
<td>171</td>
<td>171</td>
<td>171</td>
</tr>
<tr>
<td>11</td>
<td>247</td>
<td>247</td>
<td>247</td>
</tr>
<tr>
<td>12</td>
<td>225</td>
<td>225</td>
<td>225</td>
</tr>
<tr>
<td>13</td>
<td>296</td>
<td>296</td>
<td>296</td>
</tr>
</tbody>
</table>

\[ N_{\text{Bosonic Fields}} = 1,198 \]

\[ h_{\mu \nu}, A_{\mu \nu \rho}, \ldots \]

\[ N_{\text{Fermionic Fields}} = 1,186 \]

\[ \Psi_{\mu \alpha}, \ldots \]
The conceptual background for obtaining an adinkra from a superfield.

Each supersymmetric quantum field theory has a “shadow” in supersymmetric quantum mechanics obtained by dimensionally reducing all of the spatial dimensions in the field theory.

**Adinkras: A Graphical Technology for Supersymmetric Representation Theory**

*Michael Faux, S. J. Gates Jr*
The conceptual background for obtaining an adinkra from a superfield.
SUSY holography

SUSY holography: reduce 4D models to 1D, 1D models encode the structure of 4D models (e.g. gadgets are the same).

How \((4D, \mathcal{N} = 2 \rightarrow 1D, N = 8)\):

- \(\text{field}(t, x, y, z) \rightarrow \text{field}(\tau)\)
- \(D^i_a (a = 1, \ldots, 4, i = 1, 2) \rightarrow D^i_l (l = 1, \ldots, 8)\)

Figure 8
Adinkra Diagram for 4D, $\mathcal{N} = 1$
Bits Naturally Arise From The Geometry Of Hypercubes

To a small part, the appearance of the SDEC’s is not mysterious. Bits naturally appear in any situation where cubical geometry is relevant. The vertices of a cube can always be written in the form

\[(\pm 1, \pm 1, \pm 1, \ldots, \pm 1)\]

or re-written in the form

\[((-1)^{p_1}, (-1)^{p_2}, (-1)^{p_3}, \ldots, (-1)^{p_d})\]

where the exponents are bits since they take on values 1 or 0. Thus any vertex has an ‘address’ that is a string of bits

\[(p_1, p_2, p_3, \ldots, p_d)\]

the information theoretic definition of a ‘word.’
Symbols of power
Adinkras and the nature of reality

Cell control  Fighting cancer with physics
Fits and starts  Why randomness does not rule our lives
Who, what, when?  Deciding upon a discovery
‘Gemstone Cutting’
SUSY Crystals
An adinkra with some nodes collapsed with multiplicity shown

An adinkra with all nodes shown

From 1D, $\mathcal{N} = 4$ Adinkra to 4D, $\mathcal{N} = 1$ Adinkra
Dynkin Labels For The Nodes

Adinkra Diagram for 4D, $\mathcal{N} = 1$
Separating a Reducible Adinkra Into Its Two Irreducible Components

From 4D, $\mathcal{N} = 1$ Reducible Adinkra to Irreducible Adinkras (Vector and Chiral Supermultiplets)
The 4D, N = 1 Minimal Supermultiplet Zoo
There exists ten distinct minimal off-shell 4D, $\mathcal{N} = 1$ supermultiplets as indicated below:

(S01.) Chiral Supermultiplet: $(A, B, \psi_a, F, G)$ ,

(S02.) Hodge – Dual #1 Chiral Supermultiplet: $(\hat{A}, \hat{B}, \psi_a, f_{\mu \nu \rho}, \hat{G})$ ,

(S03.) Hodge – Dual #2 Chiral Supermultiplet: $(\tilde{A}, \tilde{B}, \psi_a, \tilde{F}, g_{\mu \nu \rho})$ ,

(S04.) Hodge – Dual #3 Chiral Supermultiplet: $(\tilde{A}, \tilde{B}, \psi_a, \tilde{f}_{\mu \nu \rho}, \tilde{g}_{\mu \nu \rho})$ ,

(S05.) Tensor Supermultiplet: $(\varphi, B_{\mu \nu}, \chi_a)$ ,

(S06.) Axial – Tensor Supermultiplet: $(\tilde{\varphi}, \tilde{B}_{\mu \nu}, \tilde{\chi}_a)$ ,

(S07.) Vector Supermultiplet: $(A_\mu, \lambda_b, d)$ ,

(S08.) Axial – Vector Supermultiplet: $(U_\mu, \lambda_b, \hat{d})$ ,

(S09.) Hodge – Dual Vector Supermultiplet: $(\hat{A}_\mu, \lambda_b, \hat{d}_{\mu \nu \rho})$ ,

(S10.) Hodge – Dual Axial – Vector Supermultiplet: $(\hat{U}_\mu, \lambda_b, \hat{d}_{\mu \nu \rho})$ .

Hodge duality relates some of the supermultiplets.
Parity duality relates some of the supermultiplets.
There exists ten distinct minimal off-shell 4D, $\mathcal{N} = 1$ supermultiplets as indicated below:

(S01.) Chiral Supermultiplet: $(A, B, \psi_a, F, G)$,

(S02.) Hodge – Dual #1 Chiral Supermultiplet: $(\hat{A}, \hat{B}, \psi_a, f_{\mu \nu \rho}, \hat{G})$,

(S03.) Hodge – Dual #2 Chiral Supermultiplet: $(\tilde{A}, \tilde{B}, \psi_a, \tilde{F}, g_{\mu \nu \rho})$,

(S04.) Hodge – Dual #3 Chiral Supermultiplet: $(\tilde{A}, \tilde{B}, \psi_a, f_{\mu \nu \rho}, \tilde{g}_{\mu \nu \rho})$,

(S05.) Tensor Supermultiplet: $(\varphi, B_{\mu \nu}, \chi_a)$,

(S06.) Axial – Tensor Supermultiplet: $(\hat{\varphi}, \hat{B}_{\mu \nu}, \hat{\chi}_a)$,

(S07.) Vector Supermultiplet: $(A_{\mu}, \lambda_b, d)$,

(S08.) Axial – Vector Supermultiplet: $(U_{\mu}, \hat{\lambda}_b, \hat{d})$,

(S09.) Hodge – Dual Vector Supermultiplet: $(\tilde{A}_{\mu}, \tilde{\lambda}_b, \tilde{d}_{\mu \nu \rho})$,

(S10.) Hodge – Dual Axial – Vector Supermultiplet: $(\tilde{U}_{\mu}, \tilde{\lambda}_b, \tilde{d}_{\mu \nu \rho})$.

Hodge duality relates some of the supermultiplets.
Parity duality relates some of the supermultiplets.
There exists ten distinct minimal off-shell 4D, $\mathcal{N} = 1$ supermultiplets as indicated below:

(S01.) Chiral Supermultiplet: $(A, B, \psi_a, F, G)$,

(S02.) Hodge – Dual #1 Chiral Supermultiplet: $(\tilde{A}, \tilde{B}, \psi_a, f_{\mu \nu \rho}, \tilde{G})$,

(S03.) Hodge – Dual #2 Chiral Supermultiplet: $(\tilde{A}, \tilde{B}, \psi_a, \tilde{F}, g_{\mu \nu \rho})$,

(S04.) Hodge – Dual #3 Chiral Supermultiplet: $(\check{A}, \check{B}, \psi_a, \check{f}_{\mu \nu})$,

(S05.) Tensor Supermultiplet: $(\varphi, B_{\mu \nu}, \chi_a)$,

(S06.) Axial – Tensor Supermultiplet: $(\check{\varphi}, \check{B}_{\mu \nu}, \check{\chi}_a)$,

(S07.) Vector Supermultiplet: $(A_\mu, \lambda_b, d)$,

(S08.) Axial – Vector Supermultiplet: $(U_\mu, \lambda_b, \hat{d})$,

(S09.) Hodge – Dual Vector Supermultiplet: $(\tilde{A}_\mu, \tilde{\lambda}_b, \tilde{d}_{\mu \nu \rho})$,

(S10.) Hodge – Dual Axial – Vector Supermultiplet: $(\check{U}_\mu, \check{\lambda}_b, \check{d}_{\mu \nu \rho})$.

Hodge duality relates some of the supermultiplets.
Parity duality relates some of the supermultiplets..
Using the ‘Hanging Garden Theorem’ yields.

A ‘valise’ adiinkra.
Chiral Supermultiplet: \((A, B, \psi_a, F, G)\)

\[ D_a A = \psi_a, \quad D_a B = i (\gamma^5)_a^b \psi_b, \]
\[ D_a \psi_b = i (\gamma^\mu)_{ab} \partial_\mu A - (\gamma^5 \gamma^\mu)_{ab} \partial_\mu B - i C_{ab} F + (\gamma^5)_{ab} G, \]
\[ D_a F = (\gamma^\mu)_{a}^b \partial_\mu \psi_b, \quad D_a G = i (\gamma^5 \gamma^\mu)_{a}^b \partial_\mu \psi_b, \]

Hodge – Dual #3 Chiral Supermultiplet: \((A, B, \psi_a, \mathbf{f}_{\mu \nu \rho}, \mathbf{g}_{\mu \nu \rho})\)

\[ D_a A = \psi_a, \quad D_a B = i (\gamma^5)_a^b \psi_b, \]
\[ D_a \psi_b = i (\gamma^\mu)_{ab} \partial_\mu A - (\gamma^5 \gamma^\mu)_{ab} \partial_\mu B \]
\[ - i \frac{1}{3!} C_{ab} (\epsilon^\nu_{\mu \rho} \partial_\sigma \mathbf{f}_{\mu \nu \rho}) \quad + \quad \frac{1}{3!} (\gamma^5)_{ab} (\epsilon^\nu_{\mu \rho} \partial_\sigma \mathbf{g}_{\mu \nu \rho}), \]
\[ D_a \mathbf{f}_{\mu \nu \rho} = - (\gamma^\sigma)_{a}^b \epsilon_{\sigma \mu \nu \rho} \psi_b, \quad D_a \mathbf{g}_{\mu \nu \rho} = - (\gamma^5 \gamma^\sigma)_{a}^b \epsilon_{\sigma \mu \nu \rho} \psi_b. \]
\[ D_1 \Phi_1 = i \Psi_1, \quad D_2 \Phi_1 = i \Psi_2, \quad D_3 \Phi_1 = \chi_0 i \Psi_3, \quad D_4 \Phi_1 = -i \Psi_4 \]
\[ D_1 \Phi_2 = i \Psi_2, \quad D_2 \Phi_2 = -i \Psi_1, \quad D_3 \Phi_2 = \chi_0 i \Psi_4, \quad D_4 \Phi_2 = i \Psi_3 \]
\[ D_1 \Phi_3 = i \Psi_3, \quad D_2 \Phi_3 = -i \Psi_4, \quad D_3 \Phi_3 = -\chi_0 i \Psi_1, \quad D_4 \Phi_3 = -i \Psi_2 \]
\[ D_1 \Phi_4 = i \Psi_4, \quad D_2 \Phi_4 = i \Psi_3, \quad D_3 \Phi_4 = -\chi_0 i \Psi_2, \quad D_4 \Phi_4 = i \Psi_1 \]

\[ D_1 \Psi_1 = \dot{\Phi}_1, \quad D_2 \Psi_1 = -\dot{\Phi}_2, \quad D_3 \Psi_1 = -\chi_0 \dot{\Phi}_3, \quad D_4 \Psi_1 = \dot{\Phi}_4 \]
\[ D_1 \Psi_2 = \dot{\Phi}_2, \quad D_2 \Psi_2 = \dot{\Phi}_1, \quad D_3 \Psi_2 = -\chi_0 \dot{\Phi}_4, \quad D_4 \Psi_2 = -\dot{\Phi}_3 \]
\[ D_1 \Psi_3 = \dot{\Phi}_3, \quad D_2 \Psi_3 = \dot{\Phi}_4, \quad D_3 \Psi_3 = \chi_0 \dot{\Phi}_1, \quad D_4 \Psi_3 = \dot{\Phi}_2 \]
\[ D_1 \Psi_4 = \dot{\Phi}_4, \quad D_2 \Psi_4 = -\dot{\Phi}_3, \quad D_3 \Psi_4 = \chi_0 \dot{\Phi}_2, \quad D_4 \Psi_4 = -\dot{\Phi}_1 \]
\[ D_I \Phi_i = i(L_I)_i^j \hat{j} \Psi_j, \quad D_I \Psi_j = (R_I)_j^i \hat{i} \Phi_i, \]
\[ (L_I)_i^j (R_J)_j^k + (L_J)_i^j (R_I)_j^k = 2 \delta_{IJ} \delta_i^k, \]
\[ (R_I)_j^i (L_J)_i^k + (R_J)_j^i (L_I)_i^k = 2 \delta_{IJ} \delta_j^k. \]
These equations show why there appears to be no polytopic basis for SUSY representation theory. As

\[ D_I \Phi_i = i (L_I)_{ij} \hat{\psi}_j, \quad D_I \hat{\psi}_j = (R_I)_{ij} \hat{\Phi}_i, \]

there is no obvious way to introduce eigenvalues!
A Closer Look at
L-matrices & R-matrices
The Coxeter Group BC(4)

Restricting our further considerations only to such signed permutation L-matrix solutions of the system (2.4), we note that all such matrices factorize

\[(L_i)_i^k = (S^{(1)})_i^\hat{k} (P_{(1)})_i^\hat{k}, \quad \text{for each fixed } I = 1, 2, \ldots, N.\]

Here, the sign-matrix \(S^{(1)}\) is a diagonal \(d \times d\) matrix with only \(\pm 1\) entries on the diagonal, and each \(P_{(1)}\) is a matrix representation of a permutation of \(d\) objects.

\[
(S^{(1)})_i^\hat{k} = \begin{bmatrix}
(-1)^{b_1} & 0 & 0 & \cdots \\
0 & (-1)^{b_2} & 0 & \cdots \\
0 & 0 & (-1)^{b_3} & \cdots \\
\vdots & \vdots & \vdots & \ddots 
\end{bmatrix} \leftrightarrow \left( R_1 = \sum_{i=1}^{d} b_i 2^{i-1} \right)_b = \left[ b_1 b_2 \cdots b_d \right]_2 \text{ (binary “word”) (reversed)}
\]
\[ (L_1)_i^k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = (10)_b \langle 1423 \rangle = \langle 1423 \rangle; \]

\[ (L_2)_i^k = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = (12)_b \langle 2314 \rangle = \langle 2314 \rangle; \]

\[ (L_3)_i^k = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = (6)_b \langle 3241 \rangle = \langle 3241 \rangle; \]

\[ (L_4)_i^k = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = (0)_b \langle 4132 \rangle = \langle 4132 \rangle. \]
\[\{CM\} \equiv \{ <1423>, <2314>, <3241>, <4132> \} \]
\[\{VM\} \equiv \{ <2413>, <1324>, <3142>, <4231> \} \]
\[\{TM\} \equiv \{ <1342>, <2431>, <3124>, <4213> \} \]
\[\{VM_1\} \equiv \{ <1432>, <2341>, <3214>, <4123> \} \]
\[\{VM_2\} \equiv \{ <1243>, <2134>, <3421>, <4312> \} \]
\[\{VM_3\} \equiv \{ <1234>, <2143>, <3412>, <4321> \} \]
Four Examples of Symbolic Permutation Notation

\[
\begin{align*}
\langle 1234 \rangle & \Rightarrow \langle 1234 \rangle \Rightarrow 1 \rightarrow 1, \ 2 \rightarrow 2, \ 3 \rightarrow 3, \ 4 \rightarrow 4, \\
\langle 1324 \rangle & \Rightarrow \langle 1234 \rangle \Rightarrow 1 \rightarrow 1, \ 2 \rightarrow 3, \ 3 \rightarrow 2, \ 4 \rightarrow 4, \\
\langle 1342 \rangle & \Rightarrow \langle 1234 \rangle \Rightarrow 1 \rightarrow 1, \ 2 \rightarrow 3, \ 3 \rightarrow 4, \ 4 \rightarrow 2, \\
\langle 2143 \rangle & \Rightarrow \langle 1234 \rangle \Rightarrow 1 \rightarrow 2, \ 2 \rightarrow 1, \ 3 \rightarrow 4, \ 4 \rightarrow 3,
\end{align*}
\]
\[ \{S_4\} = \{P_1\} \cup \{P_2\} \cup \{P_3\} \cup \{P_4\} \cup \{P_5\} \cup \{P_6\}, \]

\[ \{P_1\} \equiv \{< 1423 >, < 2314 >, < 3241 >, < 4132 > \} \]

\[ \{P_2\} \equiv \{< 1342 >, < 2431 >, < 3124 >, < 4213 > \} \]

\[ \{P_3\} \equiv \{< 2413 >, < 1324 >, < 3142 >, < 4231 > \} \]

\[ \{P_4\} \equiv \{< 1432 >, < 2341 >, < 3214 >, < 4123 > \} \]

\[ \{P_5\} \equiv \{< 1243 >, < 2134 >, < 3421 >, < 4312 > \} \]

\[ \{P_6\} \equiv \{< 1234 >, < 2143 >, < 3412 >, < 4321 > \} \]
Bruhat Weak Ordering in $\mathbb{B}C_4$

24 nodes
36 links
### Truncated Octahedron

| Type | Archimedean solid
| Uniform polyhedron |
|------|------------------|

#### Elements

\[
\begin{align*}
A &= (6 + 12\sqrt{3})a^2 \\
V &= 8\sqrt{2}a^3
\end{align*}
\]

\[
A \approx 26.784\,6097\,a^2 \\
V \approx 11.313\,7085\,a^3.
\]

<table>
<thead>
<tr>
<th>Faces by sides</th>
<th>6{4}+8{6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conway notation</td>
<td>tO</td>
</tr>
<tr>
<td>Schläfli symbols</td>
<td>t{3,4}</td>
</tr>
<tr>
<td>Wythoff symbol</td>
<td>2\ 4\ 1\ 3</td>
</tr>
<tr>
<td>Coxeter diagram</td>
<td></td>
</tr>
<tr>
<td>Symmetry group</td>
<td>Oh, B_{3}, [4,3], (*432), order 48</td>
</tr>
<tr>
<td>Rotation group</td>
<td>O, [4,3]^+, (432), order 24</td>
</tr>
<tr>
<td>Dihedral angle</td>
<td>4-6: (\arccos\left(-\frac{1}{\sqrt{3}}\right) = 125^\circ15'51'')</td>
</tr>
</tbody>
</table>
Permutahedron
Permutahedron & CM Elements
Permutahedron & TM Elements
Permutahedron & VM Elements
Permutahedron & VM₂ Elements
Permutahedron & VM₃ Elements
Teaching Apps To ‘See’
Using Eigenvalues & Traces
The 300 “Correlators” of the $S_4$ Permutahedron

$$A_{\ell x}[\mathcal{P}[a|A], \mathcal{P}[b|B]]$$

For this purpose, there is introduced a function denoted by $A_{\ell x}[\mathcal{P}[a|A], \mathcal{P}[b|B]]$ that we will call a “two-point correlator.” This function assigns a number of the minimal Bruhat distance between the elements $\mathcal{P}[a|A]$ and $\mathcal{P}[b|B]$ contained in $S_4$. This means this is a symmetric $24 \times 24$ matrix with 300 possible entries. The symbol $\mathcal{P}[a|A]$ is meant to denote the $a$-th element in the dissected subset $A$. If the two subsets are such that $A = B$, we will call these “intra-quartet correlators”, and if the two subsets are such that $A \neq B$, we then call these “inter-quartet correlators.”
Permutahedron & Correlator
Calculations:
Counting Links
The 300 “Correlators” of the $S_4$ Permutahedron

\[ A_{\lambda \chi} [\mathcal{P}[a|A], \mathcal{P}[b|B]] \]

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Traces</th>
<th>4</th>
<th>8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>(12, 0, 0, 0)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>(\mathcal{P}_1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(\mathcal{P}_4</td>
</tr>
<tr>
<td>(12, -4, 0, 0)</td>
<td>(\mathcal{P}_2</td>
<td>\mathcal{P}_3)</td>
<td>(\mathcal{P}_1</td>
<td>\mathcal{P}_2) (\mathcal{P}_1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(\mathcal{P}_2</td>
<td>\mathcal{P}_6) (\mathcal{P}_3</td>
<td>\mathcal{P}_4) \nonumber</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>(12, -4, -4, 0)</td>
<td>(\mathcal{P}_1</td>
<td>\mathcal{P}_5) (\mathcal{P}_2</td>
<td>\mathcal{P}_4) \nonumber</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\mathcal{P}_3</td>
<td>\mathcal{P}_6) \nonumber</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>(12, -8, 0, 0)</td>
<td>(\mathcal{P}_1</td>
<td>\mathcal{P}_4) (\mathcal{P}_5</td>
<td>\mathcal{P}_6) \nonumber</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-</td>
</tr>
</tbody>
</table>
The 300 “Correlators” of the $S_4$ Permutahedron

<table>
<thead>
<tr>
<th>$\mathcal{P}_{[1]}$</th>
<th>$\langle 1423 \rangle$</th>
<th>$\langle 2314 \rangle$</th>
<th>$\langle 3241 \rangle$</th>
<th>$\langle 4132 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 1423 \rangle$</td>
<td>0</td>
<td>4</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>$\langle 2314 \rangle$</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>$\langle 3241 \rangle$</td>
<td>6</td>
<td>2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>$\langle 4132 \rangle$</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 1:** $\{CM\} - \{CM\}$ Two - Point Correlator Values

<table>
<thead>
<tr>
<th>$\mathcal{P}_{[2]}$</th>
<th>$\langle 1342 \rangle$</th>
<th>$\langle 2431 \rangle$</th>
<th>$\langle 3124 \rangle$</th>
<th>$\langle 4213 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 1342 \rangle$</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$\langle 2431 \rangle$</td>
<td>6</td>
<td>0</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$\langle 3124 \rangle$</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>$\langle 4213 \rangle$</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 2:** $\{TM\} - \{TM\}$ Two - Point Correlator Values
### The 300 "Correlators" of the $S_4$ Permutahedron

<table>
<thead>
<tr>
<th>$\mathcal{P}_{[3]}$</th>
<th>$\langle 1324 \rangle$</th>
<th>$\langle 2413 \rangle$</th>
<th>$\langle 3142 \rangle$</th>
<th>$\langle 4231 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 1324 \rangle$</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>$\langle 2413 \rangle$</td>
<td>4</td>
<td>0</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>$\langle 3142 \rangle$</td>
<td>2</td>
<td>6</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>$\langle 4231 \rangle$</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 1:** $\{CM\} - \{CM\}$ Two-Point Correlator Values

<table>
<thead>
<tr>
<th>$\mathcal{P}_{[2]}$</th>
<th>$\langle 1342 \rangle$</th>
<th>$\langle 2431 \rangle$</th>
<th>$\langle 3124 \rangle$</th>
<th>$\langle 4213 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 1342 \rangle$</td>
<td>0</td>
<td>6</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$\langle 2431 \rangle$</td>
<td>6</td>
<td>0</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$\langle 3124 \rangle$</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>$\langle 4213 \rangle$</td>
<td>4</td>
<td>2</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 2:** $\{TM\} - \{TM\}$ Two-Point Correlator Values
### The 300 “Correlators” of the $S_4$ Permutahedron

<table>
<thead>
<tr>
<th>$\mathcal{P}^{[3]}$</th>
<th>$\langle 1324 \rangle$</th>
<th>$\langle 2413 \rangle$</th>
<th>$\langle 3142 \rangle$</th>
<th>$\langle 4231 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 1324 \rangle$</td>
<td>0</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>$\langle 2413 \rangle$</td>
<td>4</td>
<td>0</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>$\langle 3142 \rangle$</td>
<td>2</td>
<td>6</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>$\langle 4231 \rangle$</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 3:** $\{VM\} - \{VM\}$ Two-Point Correlator Values

<table>
<thead>
<tr>
<th>$\mathcal{P}^{[4]}$</th>
<th>$\langle 1432 \rangle$</th>
<th>$\langle 2341 \rangle$</th>
<th>$\langle 3214 \rangle$</th>
<th>$\langle 4123 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 1432 \rangle$</td>
<td>0</td>
<td>6</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$\langle 2341 \rangle$</td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$\langle 3214 \rangle$</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>$\langle 4123 \rangle$</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 4:** $\{VM\}_1 - \{VM\}_1$ Two-Point Correlator Values
The 300 “Correlators” of the $S_4$ Permutahedron

<table>
<thead>
<tr>
<th>$\mathcal{P}_{[5]}$</th>
<th>$\langle 1243 \rangle$</th>
<th>$\langle 2134 \rangle$</th>
<th>$\langle 3421 \rangle$</th>
<th>$\langle 4312 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 1243 \rangle$</td>
<td>0</td>
<td>2</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>$\langle 2134 \rangle$</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>$\langle 3421 \rangle$</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$\langle 4312 \rangle$</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 5:** $\{VM\}_2 - \{VM\}_2$ Two – Point Correlator Values

<table>
<thead>
<tr>
<th>$\mathcal{P}_{[6]}$</th>
<th>$\langle 1234 \rangle$</th>
<th>$\langle 2143 \rangle$</th>
<th>$\langle 3412 \rangle$</th>
<th>$\langle 4321 \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\langle 1234 \rangle$</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>$\langle 2143 \rangle$</td>
<td>2</td>
<td>0</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>$\langle 3412 \rangle$</td>
<td>4</td>
<td>6</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$\langle 4321 \rangle$</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 6:** $\{VM\}_3 - \{VM\}_3$ Two – Point Correlator Values
The 300 “Correlators” of the $S_4$ Permutahedron

$$A_{\ell x}[\mathcal{P}_{[a|1]}, \mathcal{P}_{[b|1]}] = \begin{bmatrix} 0 & 4 & 6 & 2 \\ 4 & 0 & 2 & 6 \\ 6 & 2 & 0 & 4 \\ 2 & 6 & 4 & 0 \end{bmatrix}$$

It is clear that the trace of this matrix vanishes and a direct calculation of its eigenvalues yield \{12, 0, -4, -8\} expressed in descending order.
There are interesting symmetries of the permutahedron that lead to the results of the SUSY quartets 2-point ‘correlators’ forms to resemble a **Sudoku**-like game!

This is surprisingly true for the inter-quartet ‘correlators’ as well as for the intra-quartet ‘correlators.’
Next Stop: 4D, N = 2 SUSY & The 40,320 Nodes & 141,120 Edges Of The “Hexipentiruncitruncated 7-simplex”
### Omnitruncated 7-simplex

<table>
<thead>
<tr>
<th>Type</th>
<th>uniform 7-polytope</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schlӓfli symbol</td>
<td>t₀,₁,₂,₃,₄,₅,₆{₃⁶}</td>
</tr>
<tr>
<td>Coxeter-Dynkin diagrams</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>6-faces</td>
<td></td>
</tr>
<tr>
<td>5-faces</td>
<td></td>
</tr>
<tr>
<td>4-faces</td>
<td></td>
</tr>
<tr>
<td>Cells</td>
<td></td>
</tr>
<tr>
<td>Faces</td>
<td></td>
</tr>
<tr>
<td>Edges</td>
<td>141120</td>
</tr>
<tr>
<td>Vertices</td>
<td>40320</td>
</tr>
<tr>
<td>Vertex figure</td>
<td>Irr. 6-simplex</td>
</tr>
<tr>
<td>Coxeter group</td>
<td>A₇, [[3⁶]], order 80640</td>
</tr>
<tr>
<td>Properties</td>
<td>convex</td>
</tr>
</tbody>
</table>

Hexipentiruncitruncitruncated 7-simplex
Omnitruncated 7-simplex, also known as the

Hexipentiruncitruncated 7-simplex, aka the 8-permutahedron

Hexipentiruncitruncated 7-simplex
What’s Teaching Got To Do With It?
Adinkras: A Graphical Technology for Supersymmetric Representation Theory

Michael Faux, S. J. Gates Jr

We present a symbolic method for organizing the representation theory of one-dimensional superalgebras. This relies on special objects, which we have called adinkra symbols, which supply tangible geometric forms to the still-emerging mathematical basis underlying supersymmetry.
The 300 "Correlators" Suggests 4D, $\mathcal{N} = 1$ SUSY Is a Solution to a Set of Sudoku Puzzles

Aleksander J. Cianciara, S. James Gates Jr, Yangrui Hu, Renee Kirk

A conjecture is made that the weight space for 4D, $\mathcal{N}$–extended supersymmetrical representations is embedded within the permutahedra associated with permutation groups $\mathfrak{S}_d$. Adinkras and Coxeter Groups associated with minimal representations of 4D, $\mathcal{N} = 1$ supersymmetry provide evidence supporting this conjecture. It is shown the appearance of the mathematics of 4D, $\mathcal{N} = 1$ minimal off–shell supersymmetry representations is equivalent to solving a four color problem on the truncated octahedron. This observation suggest an entirely new way to approach the off–shell SUSY auxiliary field problem based on IT algorithms probing the properties of $\mathfrak{S}_d$. 
A Note On Exemplary Off–Shell Constructions Of 4D, $\mathcal{N} = 2$ Supersymmetry Representations

Devin D. Bristow, John H. Caporaletti, Aleksander J. Cianciara, S. James Gates Jr., Delina Levine, Gabriel Yerger

We continue the search for rules that govern when off–shell 4D, $\mathcal{N} = 1$ supermultiplets can be combined to form off–shell 4D, $\mathcal{N} = 2$ supermultiplets. We study the $S_8$ permutations and Height Yielding Matrix Numbers (HYMN) embedded within the adinkras that correspond to these putative 4D, $\mathcal{N} = 2$ supermultiplets off–shell supermultiplets. Even though the HYMN definition was designed to distinguish between the raising and lowering of nodes in one dimensional valises supermultiplets, they are shown to accurately select out which combinations of off–shell 4D, $\mathcal{N} = 1$ supermultiplets correspond to off–shell 4D, $\mathcal{N} = 2$ supermultiplets. Only the combinations of the chiral + vector and chiral + tensor are found to have valises in the same class. This is consistent with the well known structure of 4D, $\mathcal{N} = 2$ supermultiplets.
ArXiv-al Bibliographical Links


(2.) https://arxiv.org/abs/2012.13308

(3.) https://arxiv.org/abs/2012.14015
Adinkra Links

2010 Non-technical Article On Adinkras

2021 Non-technical Lecture Using Adinkras Solving 40 Year Old/4.2+ Billion Unknowns In 11D Supergravity Problem
https://www.youtube.com/watch?v=TA7h_XrDqd4

2021 Technical Lecture Using Adinkras Solving 40 Year Old/4.2+ Billion Unknowns In 11D Supergravity Problem
https://www.youtube.com/watch?v=Qnx22BCGc9Q

2021 Gemant Award Links

https://www.eurekalert.org/news-releases/925909

https://mailchi.mp/16cd339bca72/
nithec-sab-member-honoured-with-2021-andrew-gemant-award
Acknowledgments

The Great Course (formerly The Teaching Company) played a role in my conceptualization of the CGI content in Superstring Theory: The DNA of Reality and Mr. Kenneth Griggs animated them in


and most of the CGI media in this presentation uses these.
It is my pleasure to inform you that you have been selected to receive the annual Edward Dolan Award by the Department of Mathematical Sciences. This award is given to a graduate student who exhibits exceptional mathematical talent.
Some SSTPRS Alums

Brian Keating
@DrBrianKeating | https://briankeating.com

Delilah Gates
Black Hole Initiative
Harvard University

Delilah Gates: Black Hole Basics!

https://www.youtube.com/watch?v=HyYrV9PK8p8
Some SSTPRS Alums

Ibrahima Bah

Assistant Professor

PhD, University of Michigan

Bloomberg 463
410-516-4122
Iboubah@jhu.edu
Some SSTPRS Alums

SSTPRS
Research Session - 2005

Standing: Xiaolong Liu, Prof. Leopoldo Pando Zayas, Prof. Vincent Rodgers, Chris Negron, Jeff Hansen, Stephen Colodner, Brislin Thomas, Osaro Harriott, Quentin A Collier, Nichole Kiefer, Stephen Gliske, Prof. Jim Gates
Seated: Ninad Jag, Ibrahima Bah, Leo Rodriguez, Nick Romano
Some SSTPRS Alums

Some SSTPRS Alums

Her Key to Modeling Brains: Ignore the Right Details

SSTPRS Links

https://cmns.umd.edu/news-events/features/3598


https://www.aapt.org/Membership/spotlight_july2020.cfm

https://btpc.brown.edu/2020/08/05/a-virtual-spin-on-a-summer-program-connects-students-from-afar/

https://sites.brown.edu/sjgates/sstprs/
Acknowledgments

Research supported by the endowment of the Ford Foundation Professorship endowment at Brown University.

I acknowledge my postdoctoral researcher, Dr. Konstantinos Koutrolikos, and Ph.D. students, Yangrui Hu, Aleksander Cianciara. In addition, gratitude must be expressed to the efforts of SSTPRS

https://sites.brown.edu/sjgates/sstprs/

undergraduate research interns.

Finally, I acknowledge S. J. Gates, III for technical assistance in preparation of the presentation deck.
2021 Gemant Award Links

https://www.eurekalert.org/news-releases/925909

Diamond: 3 Tetrahedra/Unit Cell
Constructing The Octahedron
Constructing The Octahedron & Its Truncation
### Properties of the Tetrahedron

#### Face area

\[ A_0 = \frac{\sqrt{3}}{4} a^2 \]

#### Surface area\(^3\)

\[ A = 4A_0 = \sqrt{3}a^2 \]

#### Height of pyramid\(^4\)

\[ h = \frac{\sqrt{6}}{3} a = \sqrt{\frac{2}{3}} a \]

#### Centroid to vertex distance

\[ \frac{3}{4} h = \frac{\sqrt{6}}{4} a = \sqrt{\frac{3}{8}} a \]

#### Edge to opposite edge distance

\[ l = \frac{1}{\sqrt{2}} a \]

#### Volume\(^3\)

\[ V = \frac{1}{3} A_0 h = \frac{\sqrt{2}}{12} a^3 = \frac{a^3}{6\sqrt{2}} \]

#### Face-vertex-edge angle

\[ \arccos \left( \frac{1}{\sqrt{3}} \right) = \arctan \left( \sqrt{2} \right) \]

(\text{approx. } 54.7356°)

#### Face-edge-face angle, i.e., "dihedral angle"\(^3\)

\[ \arccos \left( \frac{1}{3} \right) = \arctan \left( 2\sqrt{2} \right) \]

(\text{approx. } 70.5288°)

---

\(^3\) Source: [Wikipedia](https://en.wikipedia.org/wiki/Tetrahedron)

\(^4\) Source: [MathWorld](http://mathworld.wolfram.com/Tetrahedron.html)
Tetrahedron Vertex Unit Vectors

\[ v_1 = \left( \sqrt{\frac{8}{9}}, 0, -\frac{1}{3} \right) \]

\[ v_2 = \left( -\sqrt{\frac{2}{9}}, \sqrt{\frac{2}{3}}, -\frac{1}{3} \right) \]

\[ v_3 = \left( -\sqrt{\frac{2}{9}}, -\sqrt{\frac{2}{3}}, -\frac{1}{3} \right) \]

\[ v_4 = (0, 0, 1) \]