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Genomics, Networks, and Computational Concepts for Polytopic SUSY Representation Theory

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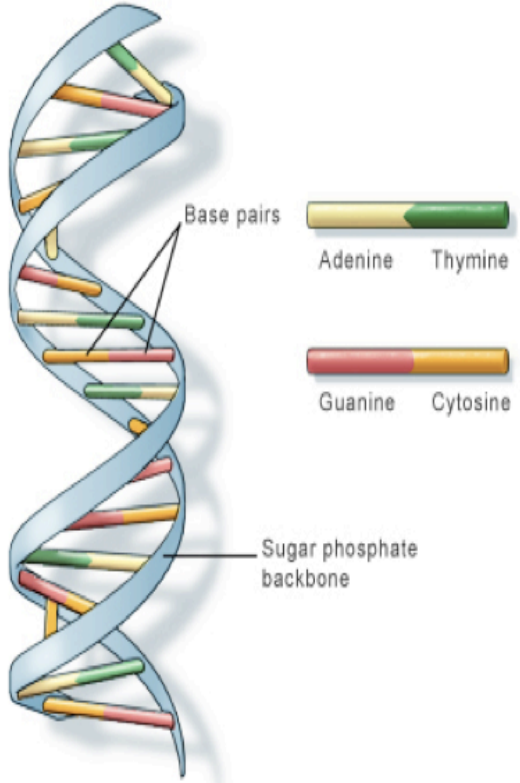
Graphs, Networks & Polytopes In Science

The structure of DNA

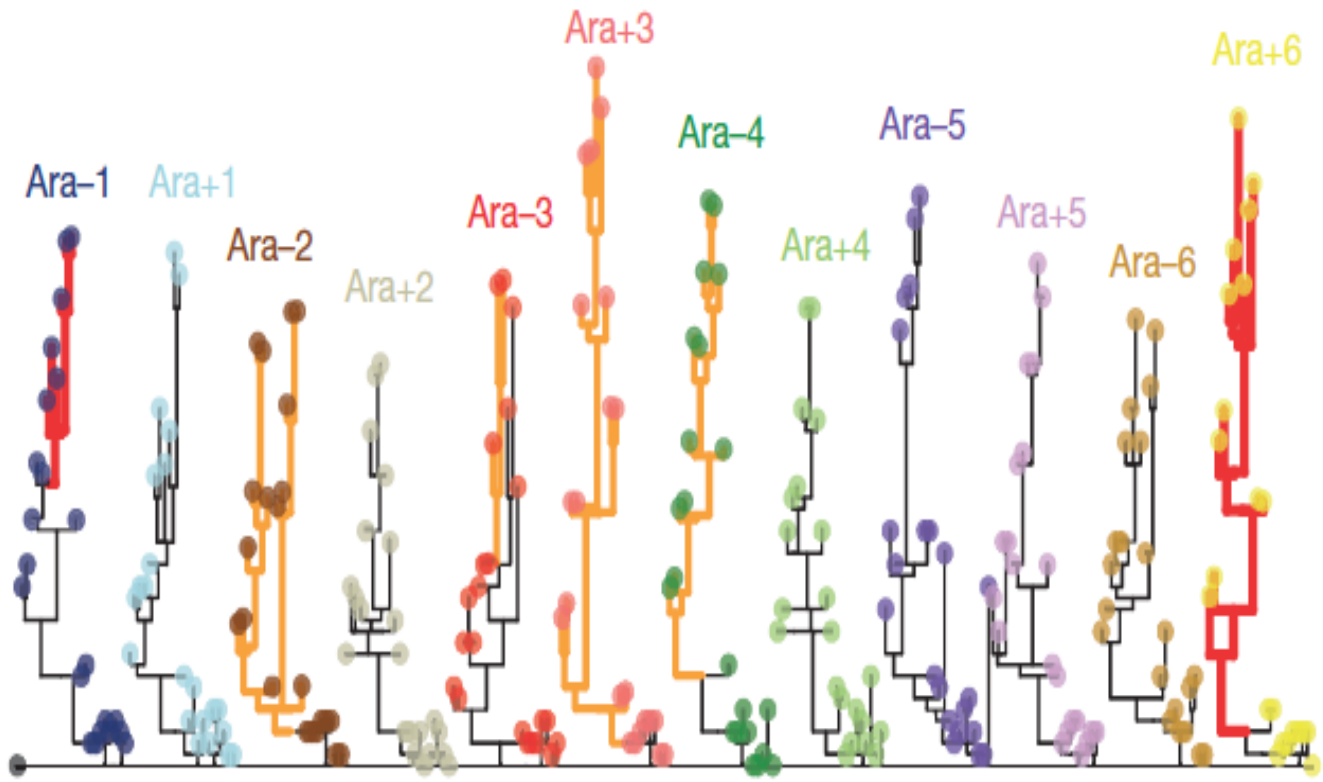
DNA is a double helix formed by base pairs attached to a sugar-phosphate backbone.



U.S. National Library of Medicine

Tempo and mode of genome evolution

- Sequenced **264 complete genomes** (12 populations x 11 time points x 2 clones).



Slide courtesy of R. Lenski

Tenaillon et al 2016 Nature

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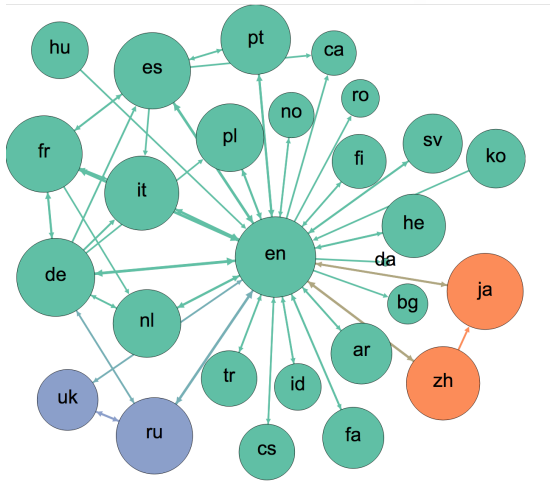
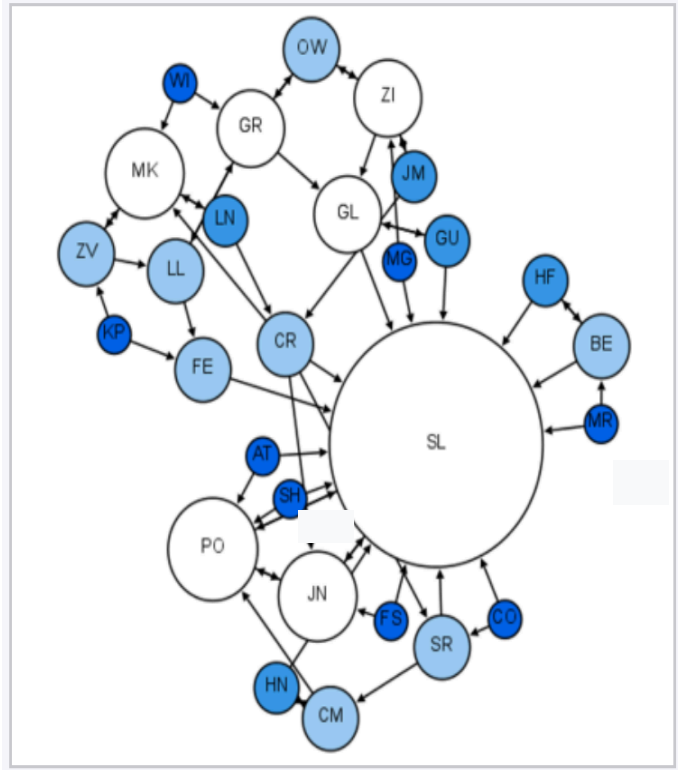


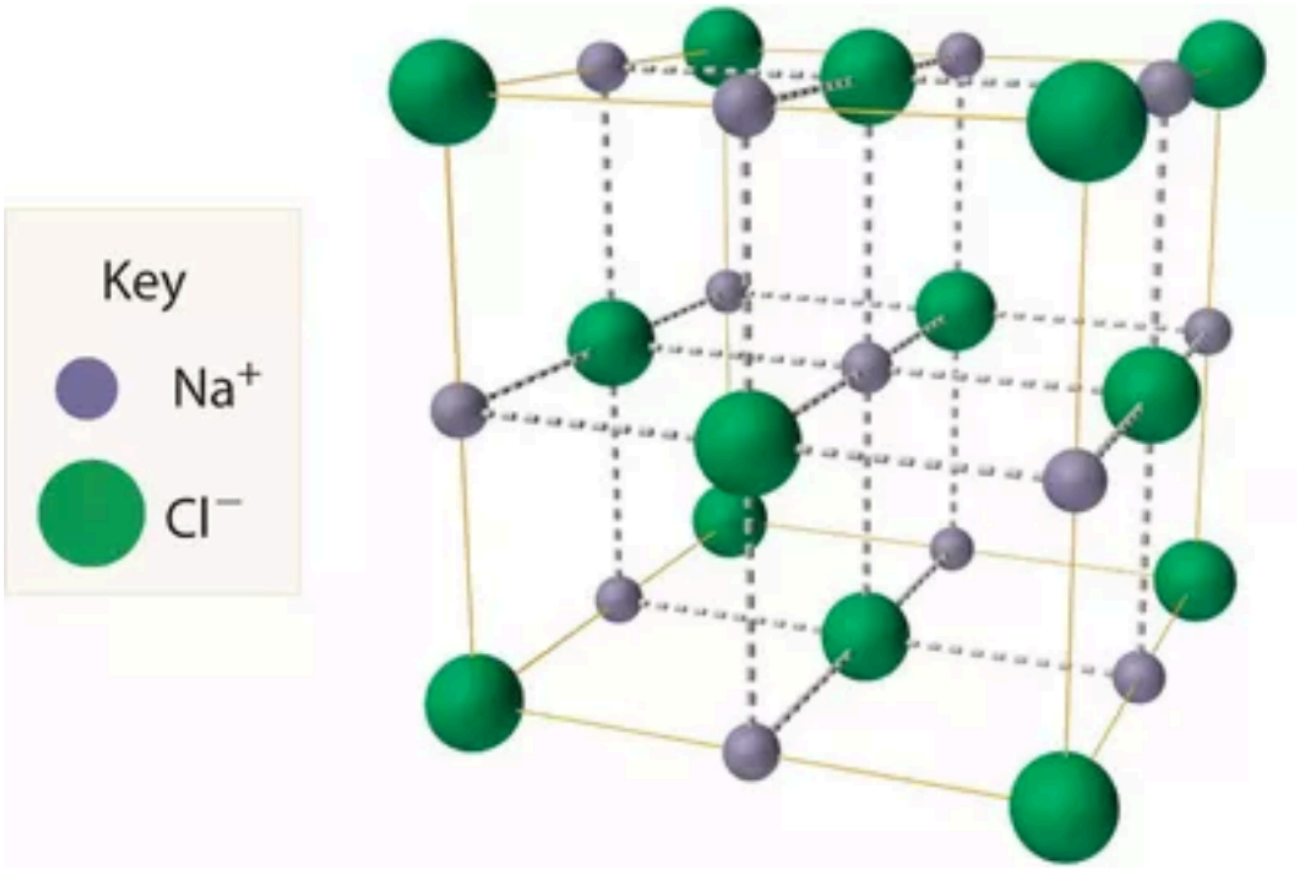
The network graph formed by Wikipedia editors (edges) contributing to different Wikipedia language versions (vertices) during one month in summer 2013.

By Computermacgyver - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=30349051

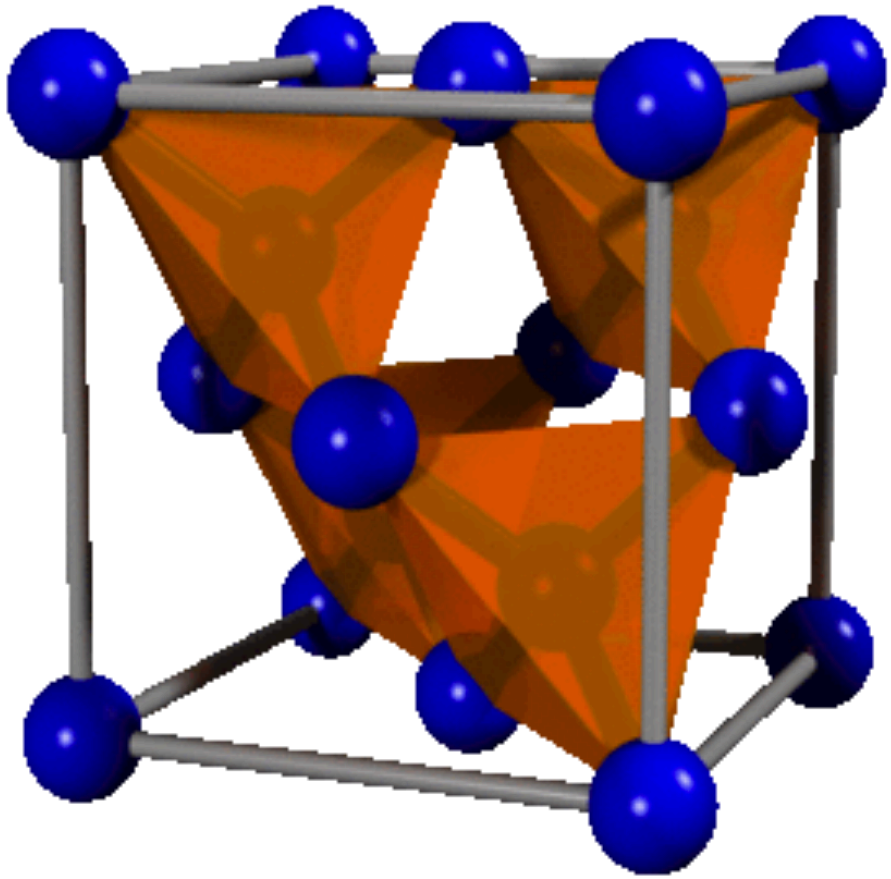


Graph theory in sociology: Moreno Sociogram (1953).

https://en.wikipedia.org/wiki/Graph_theory



Salt



Diamond C_{18}

Chemistry (Wikipedia)

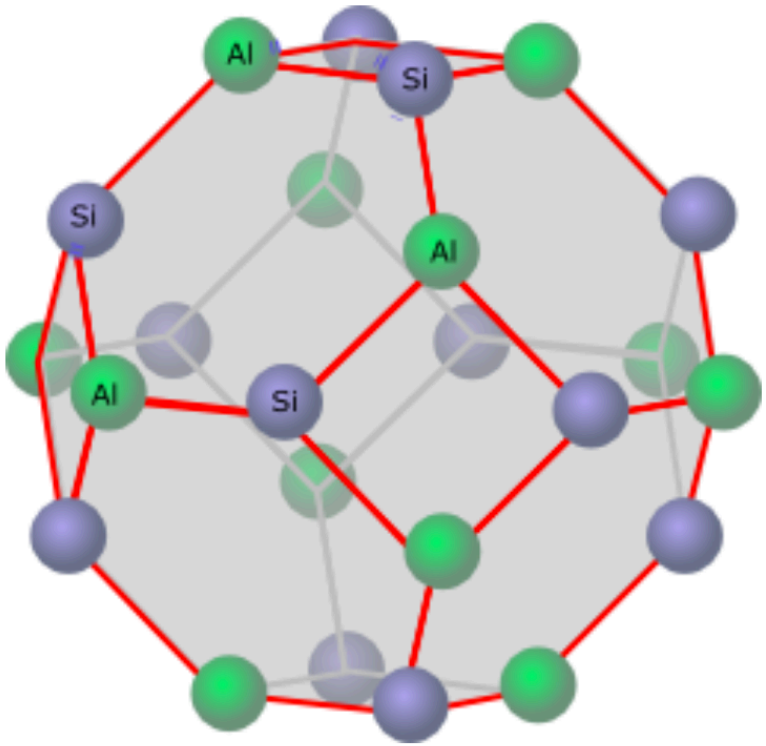
Faujasite

General

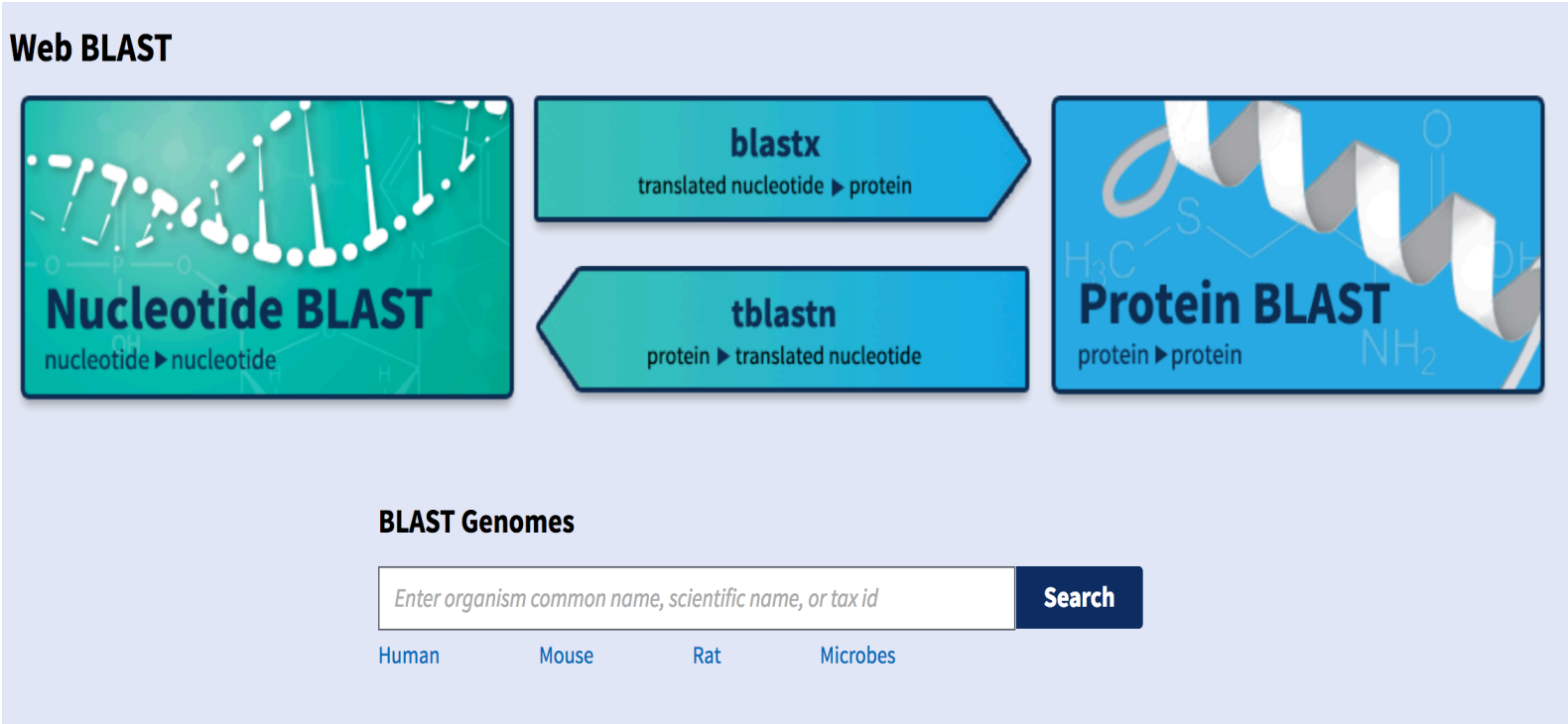
General	
Category	Zeolite
Formula (repeating unit)	$(Na_2,Ca,Mg)_{3.5}[Al_7Si_{17}O_{48}]\cdot 32(H_2O)$[1]
Strunz classification	9.GD.30
Crystal system	Cubic
Crystal class	Hexoctahedral ($m\bar{3}m$) H–M Symbol ($4/m\ \bar{3}\ 2/m$)
Space group	$Fd\bar{3}m$
Unit cell	a = 24.638–24.65 Å, Z = 32

Chemistry (Wikipedia)

The *truncated octahedron* exists in the structure of the faujasite crystals.



Basic Local Alignment Search Tool



BLAST finds regions of similarity between biological sequences. The program compares nucleotide or protein sequences to sequence databases and calculates the statistical significance.

https://blast.ncbi.nlm.nih.gov/Blast.cgi

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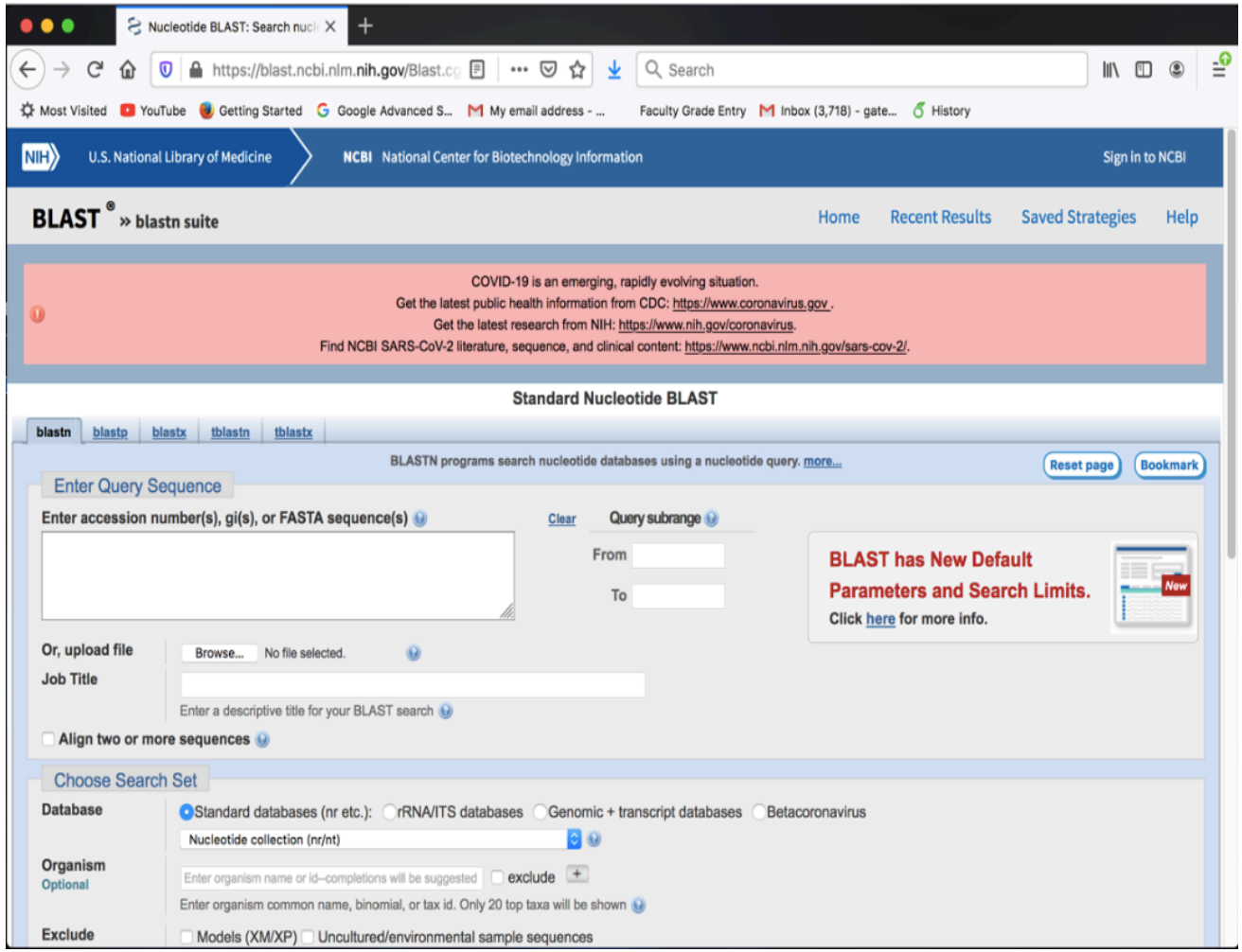

https://blast.ncbi.nlm.nih.gov/Blast.cgi?PROGRAM=blastn&PAGE_TYPE=BlastSearch&LINK_LOC=blasthome

Standard Model & Supersymmetry Extension

Fields of the Standard Model

	Fermions	Bosons
Force Carrier Fields		g, γ, Z^0, $W^\pm$, H^0
Matter Fields	$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$ e_R μ_R τ_R ν_{eR} $\nu_{\mu R}$ $\nu_{\tau R}$ $\begin{pmatrix}u\\ d\end{pmatrix}_L \begin{pmatrix}c\\ s\end{pmatrix}_L \begin{pmatrix}t\\ b\end{pmatrix}_L$ u_R c_R t_R d_R s_R b_R	

Precision in Measurement

The Best Known Number In All Of Science
Electron's Magnetic Property: The "g-Value"

Measured g:	2.002 324 303 9
Theoretical g:	2.002 324 304 4
Delta g	0.000 000 004 0

These values from about a decade old and are given in example of experimental v. theoretical g-Values.

Is string theory phenomenologically viable?

S. James Gates Jr

String theory is entering an era in which its theoretical constructs will be confronted by experimental data. Some cherished ideas just might fail to pass the test.

"Thus, if Nature is kind enough to provide light superpartners, one might still expect about a century to pass before a superparticle is directly observed."

- SJG in *Physics Today, 59N6 (2006) 54.*

"SUSY, I strongly believe, will in the end be figuratively like Mark Twain who is often misquoted as having said, "The reports of my death have been greatly exaggerated."

- SJG 2008 in Waves and Packets: https://multibriefs.com/briefs/nsbp/extrapage.html

Feature: Supersymmetry

physicsworld.com

Sticking with SUSY

When CERN's Large Hadron Collider failed to uncover evidence of new "superpartner" particles during its first run, some claimed that the theory that predicts them – known as supersymmetry, or SUSY – should be abandoned. **S James Gates, Jr**, however, argues that giving up on SUSY now would be like concluding that giant sequoia trees do not exist after surveying only the east coast of North America, and that there is more at stake than meets the eye

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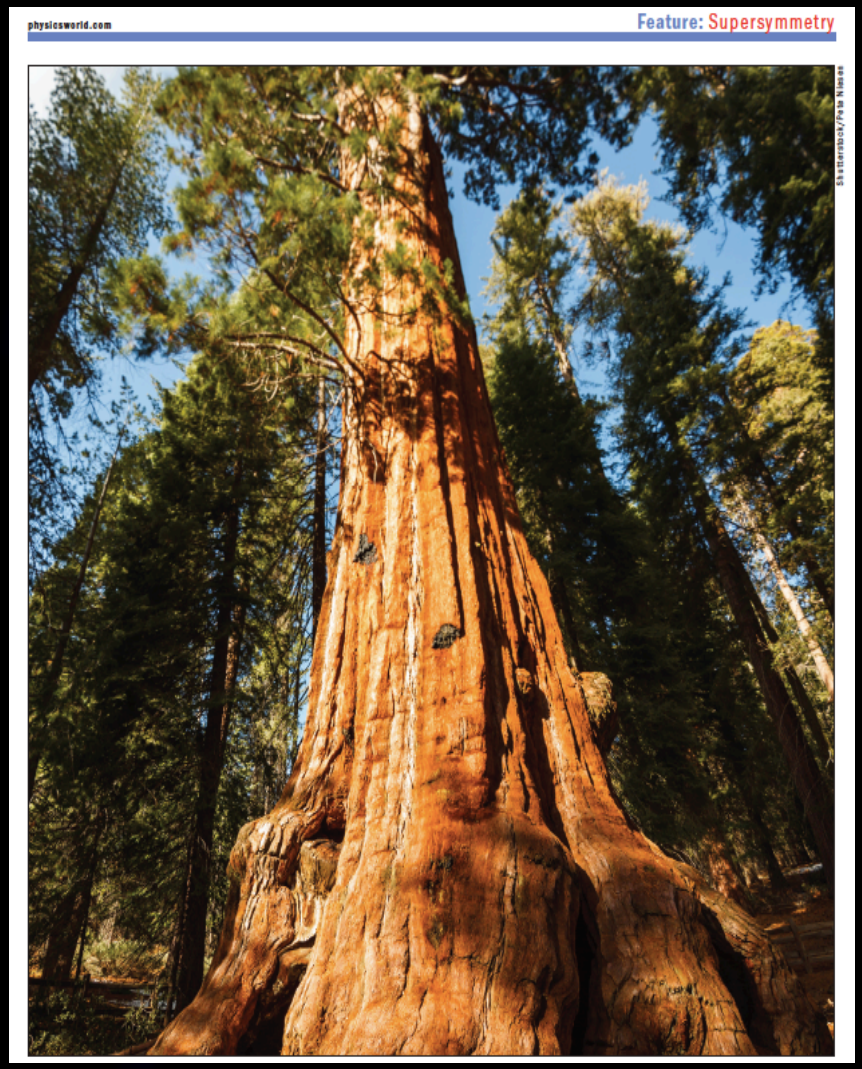

"In my view, the current situation is akin to that of an explorer who, having scoured the eastern seaboard of North America, concludes that no groves of *Sequoiadendron giganteum* exist on the entire continent. As with this hypothetical hunt for giant sequoia trees, finding evidence for SUSY depends on the observer looking in the right place."

- SJG in *Physics World, (2014).*

Fields of the Standard Model

	Fermions	Bosons
Force Carrier Fields		g, γ, Z^0, $W^\pm$, H^0
Matter Fields	$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$ e_R μ_R τ_R ν_{eR} $\nu_{\mu R}$ $\nu_{\tau R}$ $\begin{pmatrix}u\\ d\end{pmatrix}_L \begin{pmatrix}c\\ s\end{pmatrix}_L \begin{pmatrix}t\\ b\end{pmatrix}_L$ u_R c_R t_R d_R s_R b_R	

Fields of the Minimal SUSY Standard Model

	Fermions	Bosons
Force Carrier Fields	$\tilde{g}$ $\tilde{\gamma}$ $\tilde{Z}^0$ $\tilde{W}^\pm$ $\tilde{H}^0$ $\tilde{h}^0$ $\tilde{A}^0$ $\tilde{H}^\pm$	g γ Z^0 $W^\pm$ H^0 h^0 A^0 $H^\pm$
Matter Fields	$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix}_L \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix}_L$ e_R μ_R τ_R ν_{e_R} ν_{μ_R} ν_{τ_R} $\begin{pmatrix}u\\ d\end{pmatrix}_L \begin{pmatrix}c\\ s\end{pmatrix}_L \begin{pmatrix}t\\ b\end{pmatrix}_L$ u_R c_R t_R d_R s_R b_R	$\begin{pmatrix}\tilde{\nu}_{e_L}\\ \tilde{e}_L\end{pmatrix} \begin{pmatrix}\tilde{\nu}_{\mu_L}\\ \tilde{\mu}_L\end{pmatrix} \begin{pmatrix}\tilde{\nu}_{\tau_L}\\ \tilde{\tau}_L\end{pmatrix}$ $\tilde{e}_R$ $\tilde{\mu}_R$ $\tilde{\tau}_R$ $\tilde{\nu}_{e_R}$ $\tilde{\nu}_{\mu_R}$ $\tilde{\nu}_{\tau_R}$ $\begin{pmatrix}\tilde{u}\\ \tilde{d}\end{pmatrix}_L \begin{pmatrix}\tilde{c}\\ \tilde{s}\end{pmatrix}_L \begin{pmatrix}\tilde{t}\\ \tilde{b}\end{pmatrix}_L$ $\tilde{u}_R$ $\tilde{c}_R$ $\tilde{t}_R$ $\tilde{d}_R$ $\tilde{s}_R$ $\tilde{b}_R$

Precision in Measurement

The Best Known Number In All Of Science
Electron's Magnetic Property: The "g-Value"

NIST CODATA recommended *g*-factor values[12]

Particle	Symbol	*g*-factor	Relative standard uncertainty
electron	g_e	–2.002 319 304 362 56(35)	1.7×10^{-13}[8]

These values from about a decade old and are given in example of experimental v. theoretical g-Values.

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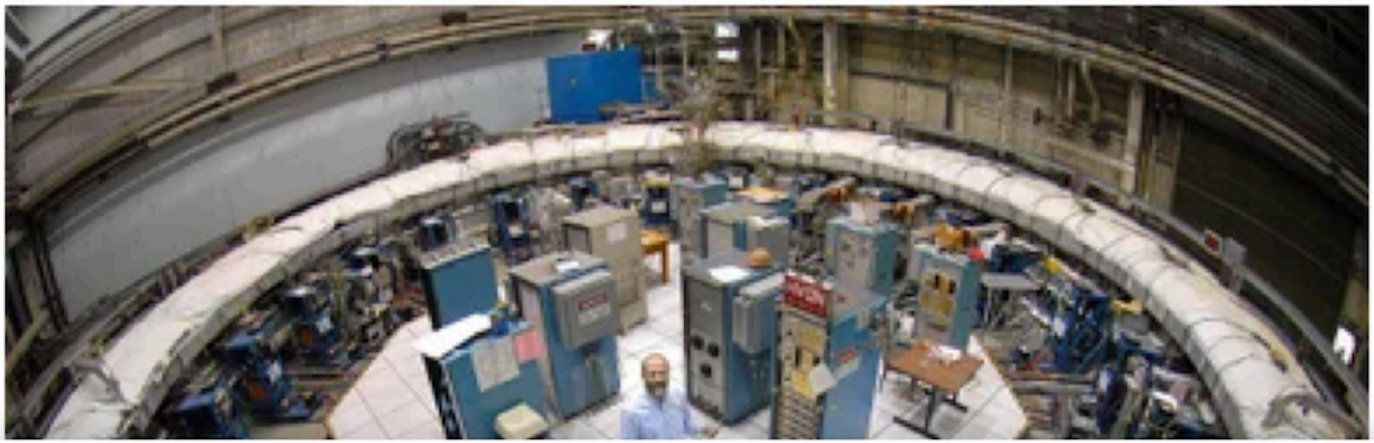
Long-Awaited Muon Measurement Boosts Evidence for New Physics

Initial data from the Muon g-2 experiment have excited particle physicists searching for undiscovered subatomic particles and forces

By Daniel Garisto on April 7, 2021



Classical Mechanics vs. Quantum Mechanics

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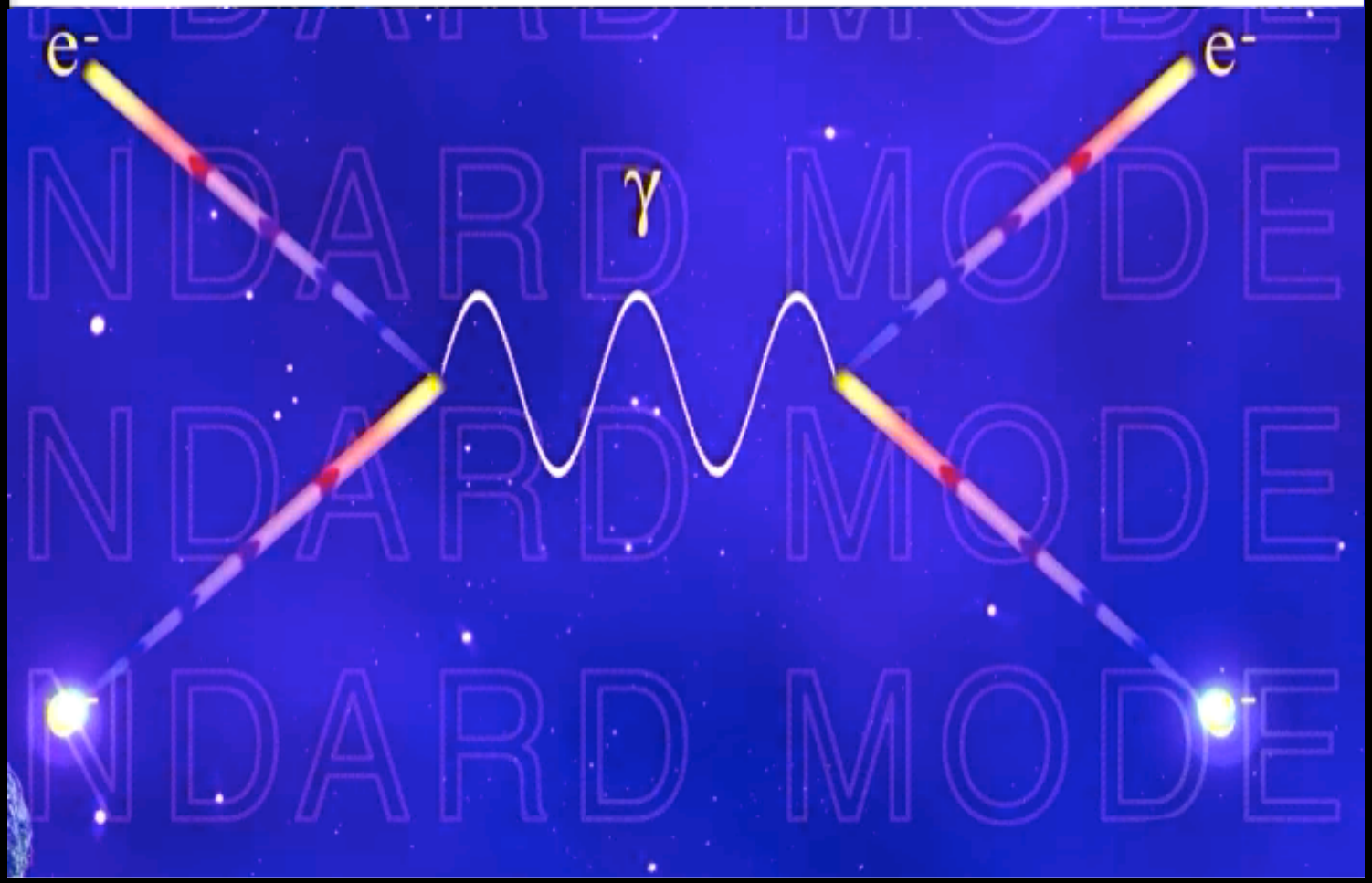
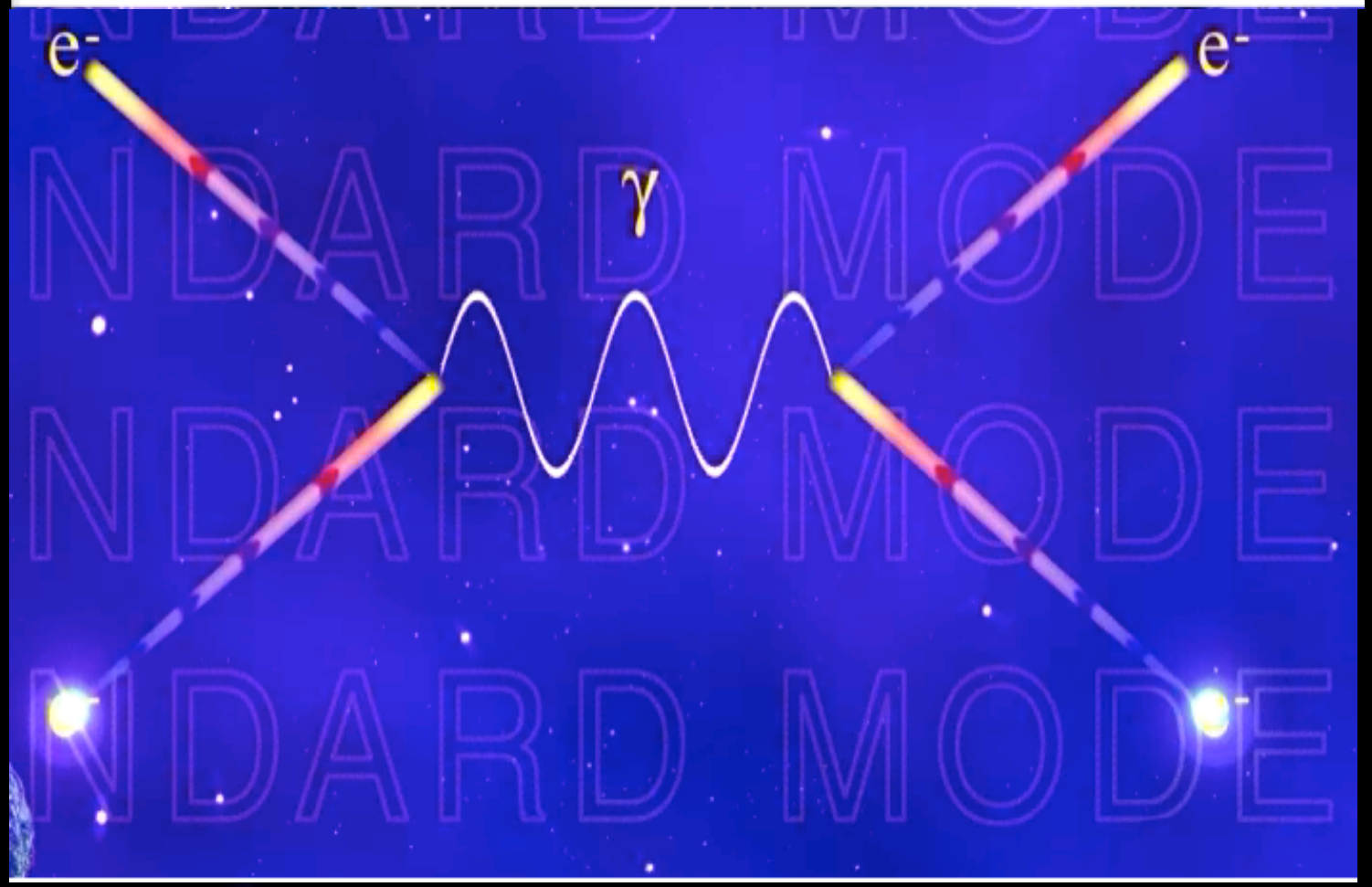


Classical Mechanics vs. Quantum Mechanics



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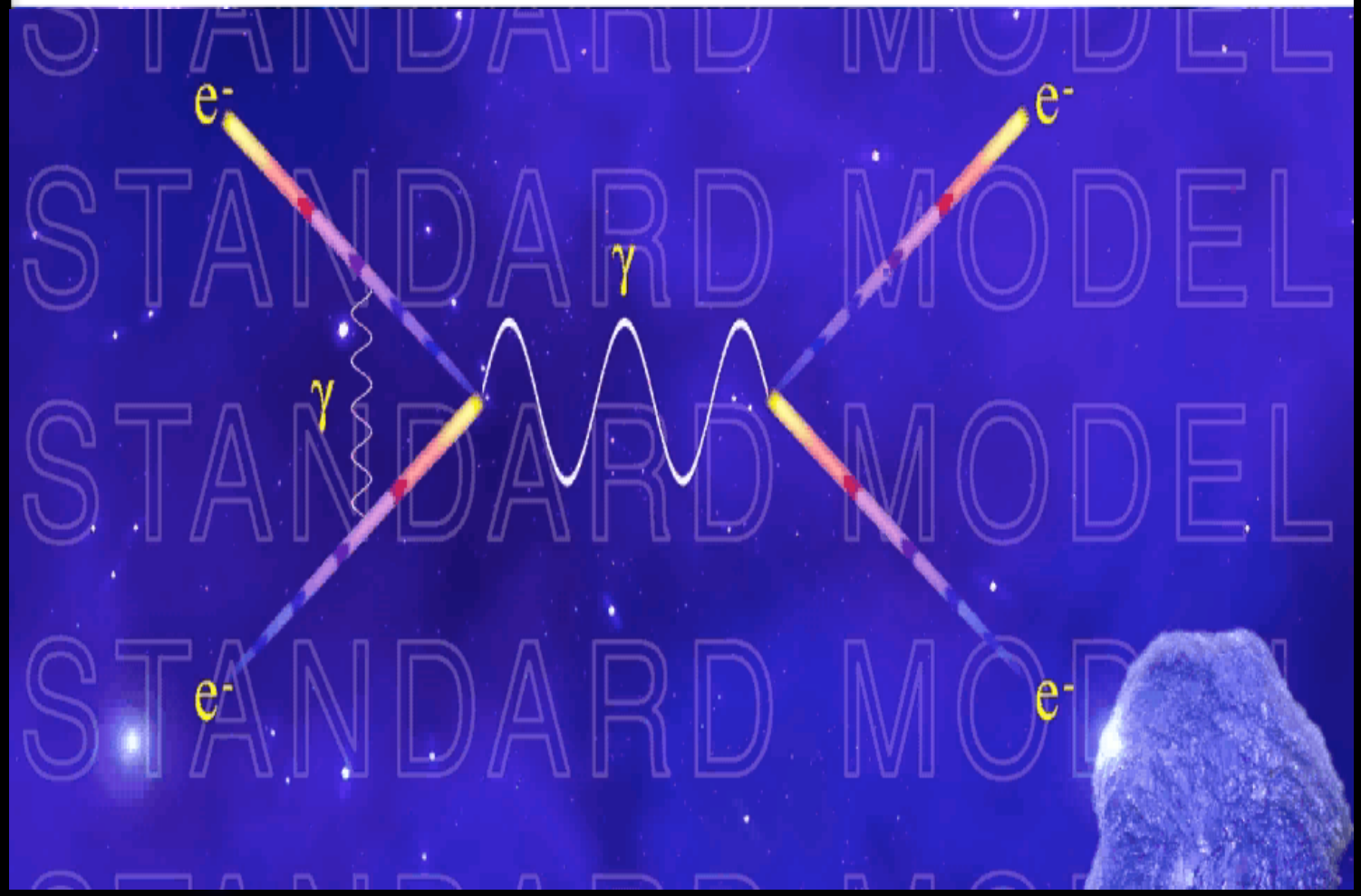

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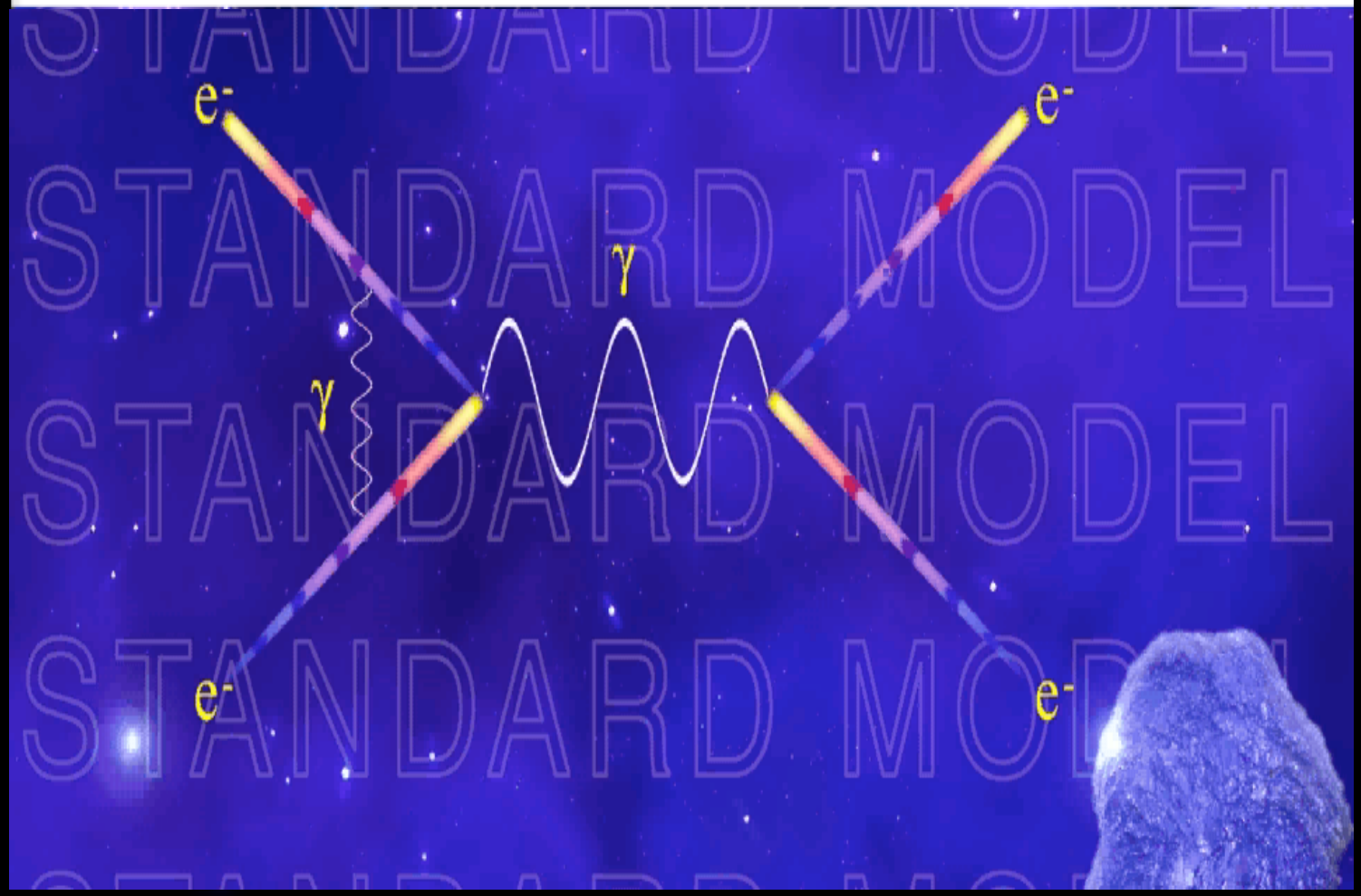

Classical Mechanics vs. Quantum Mechanics

Classical Mechanics vs. Quantum Mechanics

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SU(3): A Golden Gateway Understanding to the Polytopes Undergirding the Symmetries of the Standard Model

The Gell-Mann su(3) "Lambda-Matrices"

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

$$[\tfrac{1}{2}\lambda_i, \tfrac{1}{2}\lambda_j] = if_{ijk}(\tfrac{1}{2}\lambda_k)$$ su(3) structure constants

$$\{\lambda_i, \lambda_j\} = \tfrac{4}{3}\,\delta_{ij}\mathbf{1} + 2d_{ijk}\lambda_k$$ su(3) "d-symbols"

Eigenvalues and eigenvectors

$$A\mathbf{v} = \lambda\mathbf{v}, \qquad |A - \lambda I| = 0$$

$$|A - \lambda I| = (\lambda_1 - \lambda)(\lambda_2 - \lambda)\cdots(\lambda_n - \lambda),$$

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}.$$

$$|A - \lambda I| = \begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 3 - 4\lambda + \lambda^2.$$

$$\mathbf{v}_{\lambda=1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{v}_{\lambda=3} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Review of Representation Tools for the su(3) Lie Algebra

Commutators

$$[\boldsymbol{T}_i\,,\,\boldsymbol{T}_j] \;=\; i\,f_{ij}{}^k\,\boldsymbol{T}_k \quad ,$$

$$\boldsymbol{T}_i = \tfrac{1}{2}\boldsymbol{\lambda}_i \qquad \text{su(3) generators}$$

Totally anti-symmetric structure constants $f_{ij}{}^k$ specified by

$$f_{123} \;=\; 1, \;\; , \;\; f_{147} \;=\; f_{165} \;=\; f_{246} \;=\; f_{257} \;=\; f_{345} \;=\; f_{376} \;=\; \frac{1}{2} \;\; ,$$

$$f_{458} \;=\; f_{678} \;=\; \frac{\sqrt{3}}{2} \;\; ,$$

$\boldsymbol{T}_3$ and $\boldsymbol{T}_8$ commute, $\rightarrow$ simultaneous eigenvectors

$$\boldsymbol{T}_3|t_3,\,t_8\rangle \;=\; t_3\,|t_3,\,t_8\rangle \quad , \quad \boldsymbol{T}_8|t_3,\,t_8\rangle \;=\; t_8\,|t_3,\,t_8\rangle \quad .$$

$$\boldsymbol{T}_3\,|\tfrac{1}{2},\,\tfrac{1}{2\sqrt{3}}> \;=\; \tfrac{1}{2}\,|\tfrac{1}{2},\,\tfrac{1}{2\sqrt{3}}> \quad , \quad \boldsymbol{T}_8\,|\tfrac{1}{2},\,\tfrac{1}{2\sqrt{3}}> \;=\; \tfrac{1}{2\sqrt{3}}\,|\tfrac{1}{2},\,\tfrac{1}{2\sqrt{3}}> \quad ,$$

$$\boldsymbol{T}_3\,|-\tfrac{1}{2},\,\tfrac{1}{2\sqrt{3}}> \;=\; -\tfrac{1}{2}\,|\tfrac{1}{2},\,\tfrac{1}{2\sqrt{3}}> \quad , \quad \boldsymbol{T}_8\,|-\tfrac{1}{2},\,\tfrac{1}{2\sqrt{3}}> \;=\; \tfrac{1}{2\sqrt{3}}\,|-\tfrac{1}{2},\,\tfrac{1}{2\sqrt{3}}> \quad ,$$

$$\boldsymbol{T}_3\,|0,\,-\tfrac{1}{\sqrt{3}}> \;=\; 0 \quad , \quad \boldsymbol{T}_8\,|0,\,\tfrac{1}{2\sqrt{3}}> \;=\; -\tfrac{1}{\sqrt{3}}\,|0,\,-\tfrac{1}{\sqrt{3}}> \quad ,$$

The Nonvanishing Values of f_{ijk} and d_{ijk}

$[i\,j\,k]$	f_{ijk}	$(i\,j\,k)$	d_{ijk}
123	1	118	$1/\sqrt{3}$
147	$\frac{1}{2}$	146	$\frac{1}{2}$
156	$-\frac{1}{2}$	157	$\frac{1}{2}$
246	$\frac{1}{2}$	228	$1/\sqrt{3}$
257	$\frac{1}{2}$	247	$-\frac{1}{2}$
345	$\frac{1}{2}$	256	$\frac{1}{2}$
367	$-\frac{1}{2}$	338	$1/\sqrt{3}$
458	$\sqrt{3}/2$	344	$\frac{1}{2}$
678	$\sqrt{3}/2$	355	$\frac{1}{2}$
		366	$-\frac{1}{2}$
		377	$-\frac{1}{2}$
		448	$-1/2\sqrt{3}$
		558	$-1/2\sqrt{3}$
		668	$-1/2\sqrt{3}$
		778	$-1/2\sqrt{3}$
		888	$-1/\sqrt{3}$

"Raising operators" and "Lowering operators,"

$$T_\pm = T_1 \pm iT_2 \ , \ U_\pm = T_6 \pm iT_7 \ , \ V_\pm = T_4 \pm iT_5 \ ,$$

generate 'motions' between the eigenvectors. Also

$$[T_+ , T_-] = 2T_3 \ , \ [U_+ , U_-] = \sqrt{3}T_8 - T_3 \ , \ [V_+ , V_-] = \sqrt{3}T_8 + T_3 \ ,$$

along with a number of commutators which have the property that the matrices T_3 and T_8 do not appear on the rhs of them.

$$[T_\pm , T_\pm] = [U_\pm , U_\pm] = [V_\pm , V_\pm] = 0 \ .$$

Counting Representations

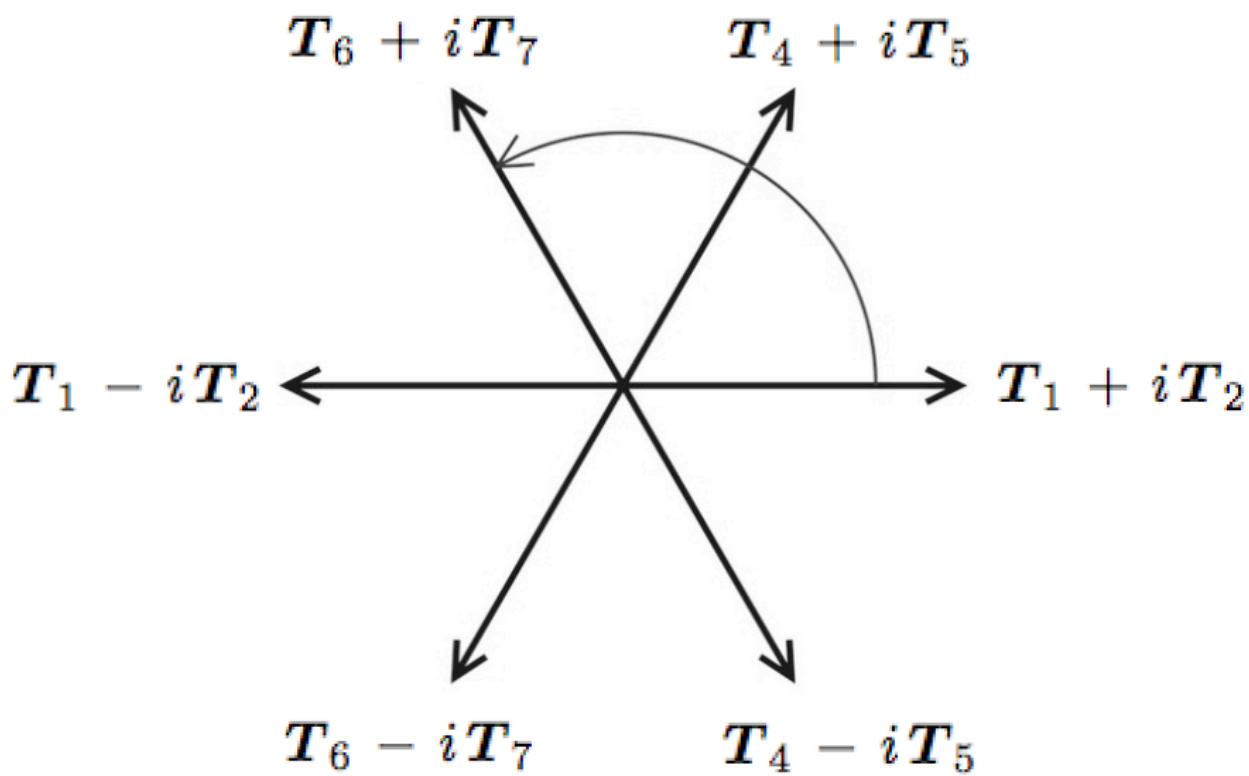
$$\boldsymbol{d}_{SU(2)} = (2j+1)$$

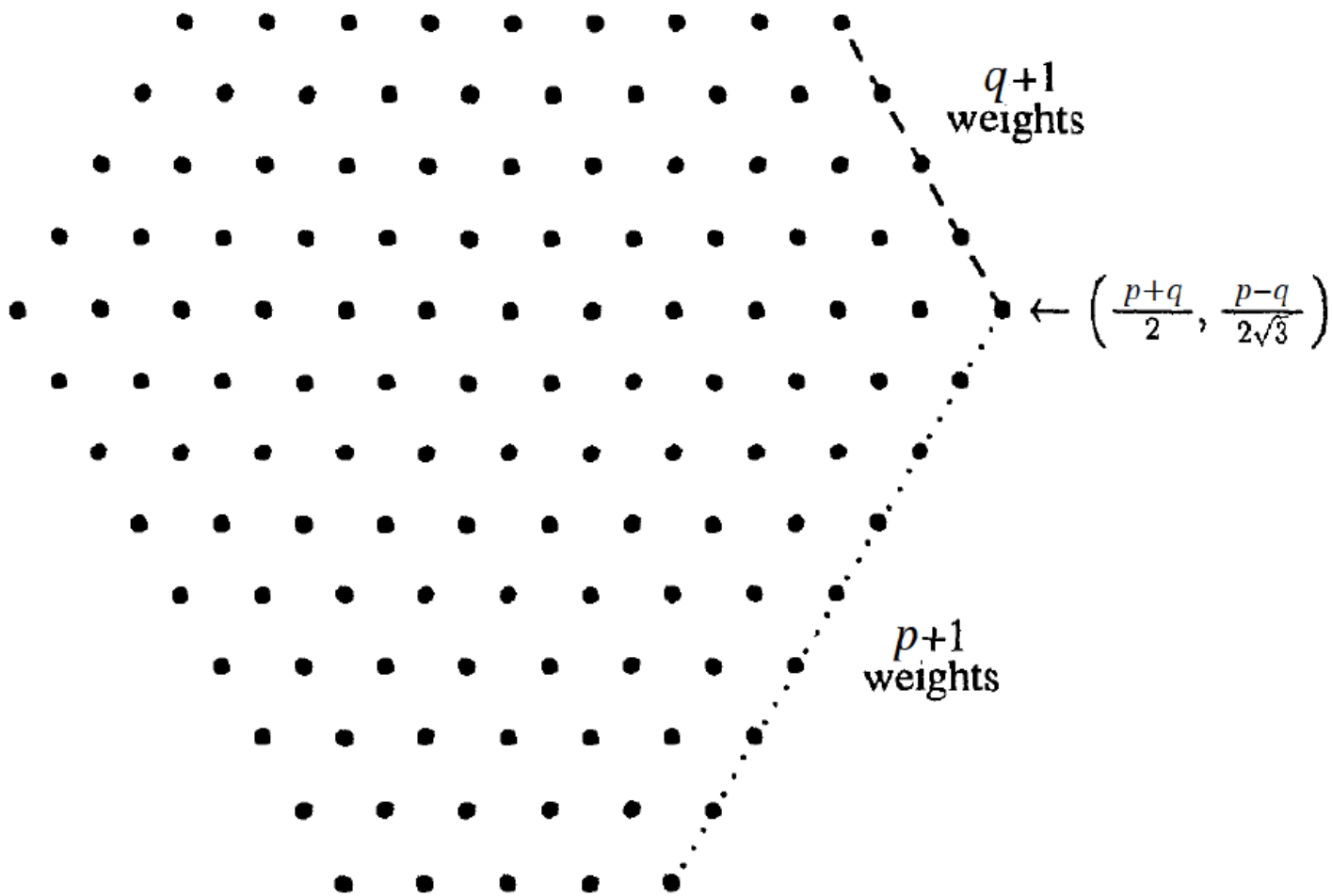
$$\boldsymbol{d}_{SU(3)} = \tfrac{1}{2}(p+1)(q+1)(p+q+2)$$

Weyl discovered that for the simple Lie algebras the dimension of a representation $\Gamma(w^h)$ labeled by the highest weight w^h is given the formula

$$\dim \Gamma(w^h) = \prod_{\mu>0} [\mu \cdot (w^h + \mu^+/2)]/[\mu \cdot (\mu^+/2)].$$

Here the product is to be taken over all *positive* root vectors and μ^+ is the sum of all positive roots


$T_6 + iT_7$
$T_4 + iT_5$
$T_1 - iT_2$
$T_1 + iT_2$
$T_6 - iT_7$
$T_4 - iT_5$


$q+1$
weights
$\left(\frac{p+q}{2}, \frac{p-q}{2\sqrt{3}}\right)$
$p+1$
weights

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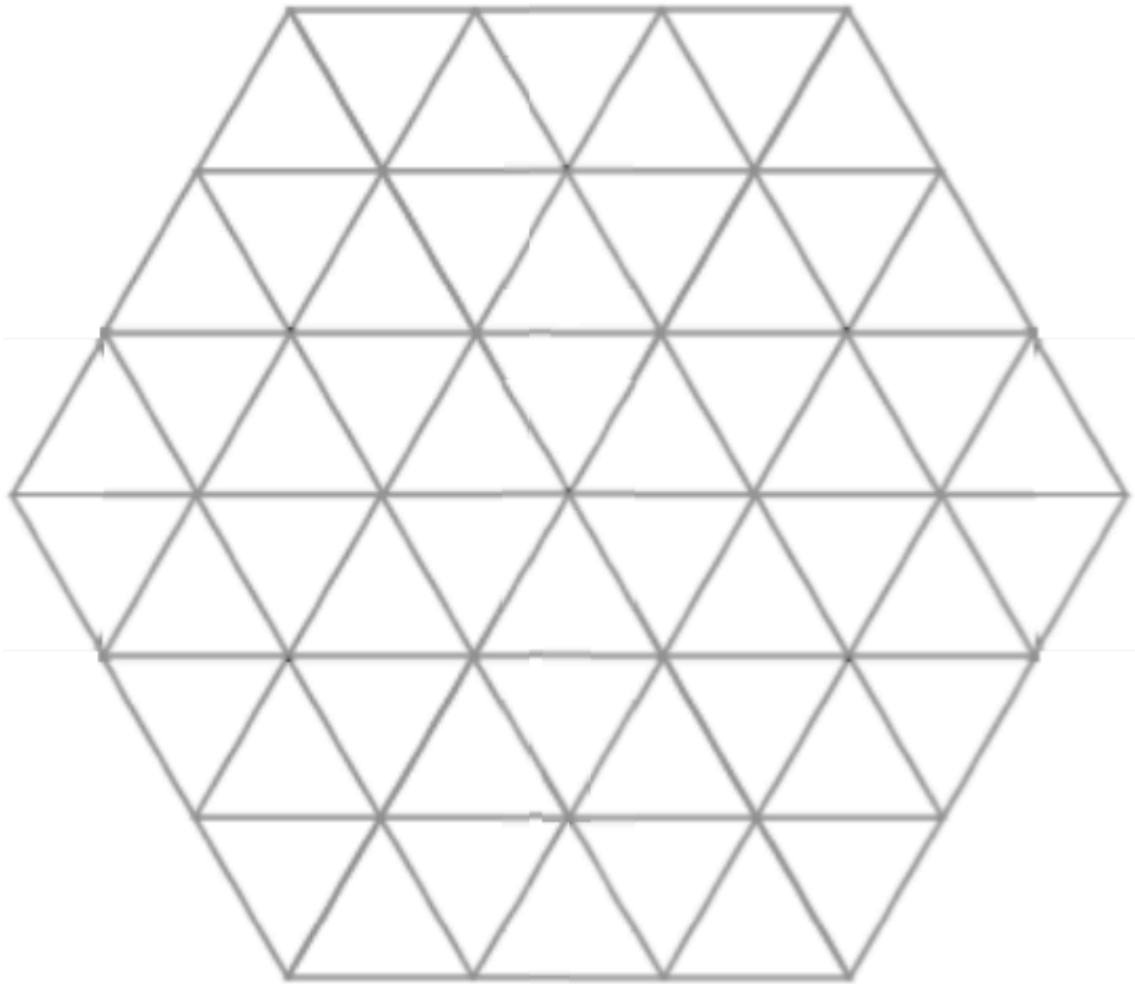

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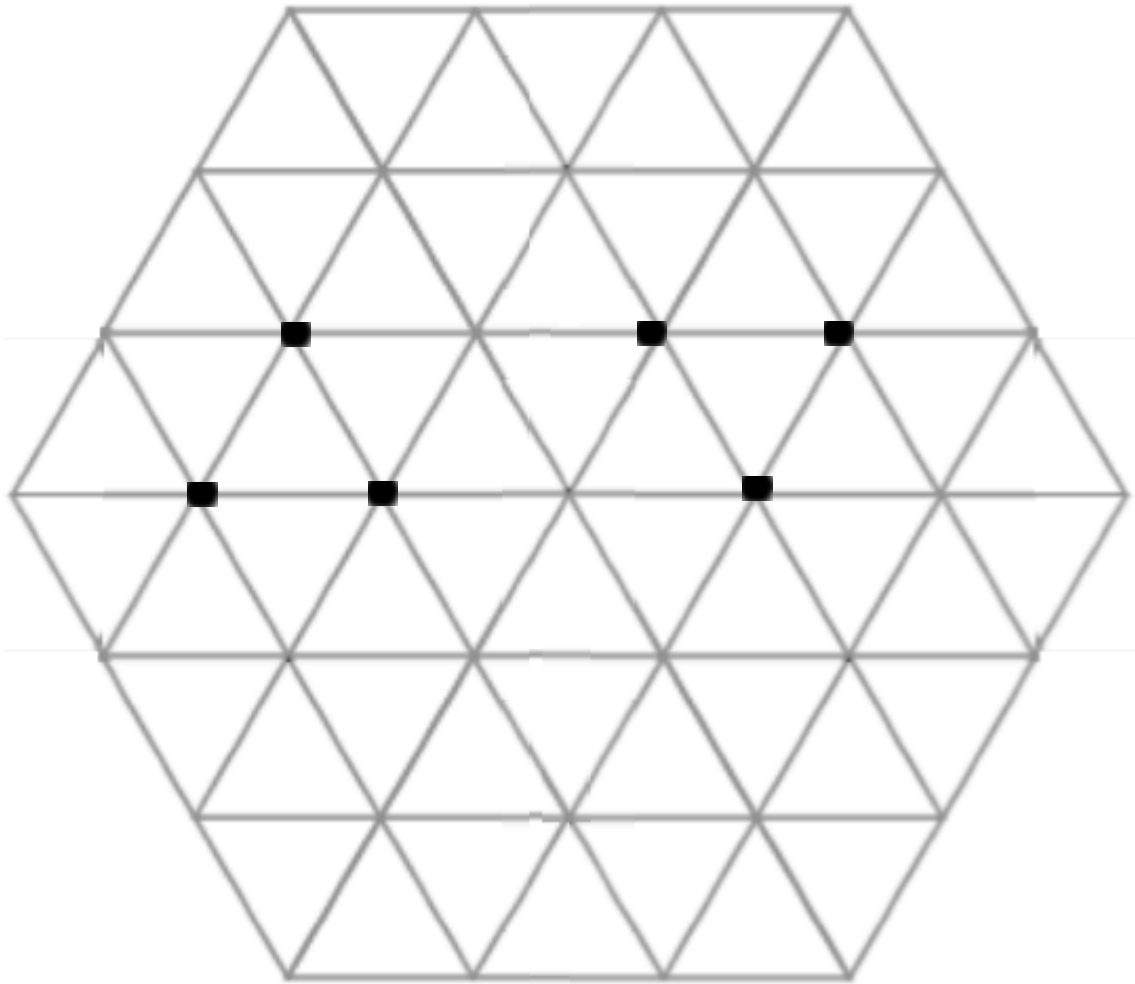

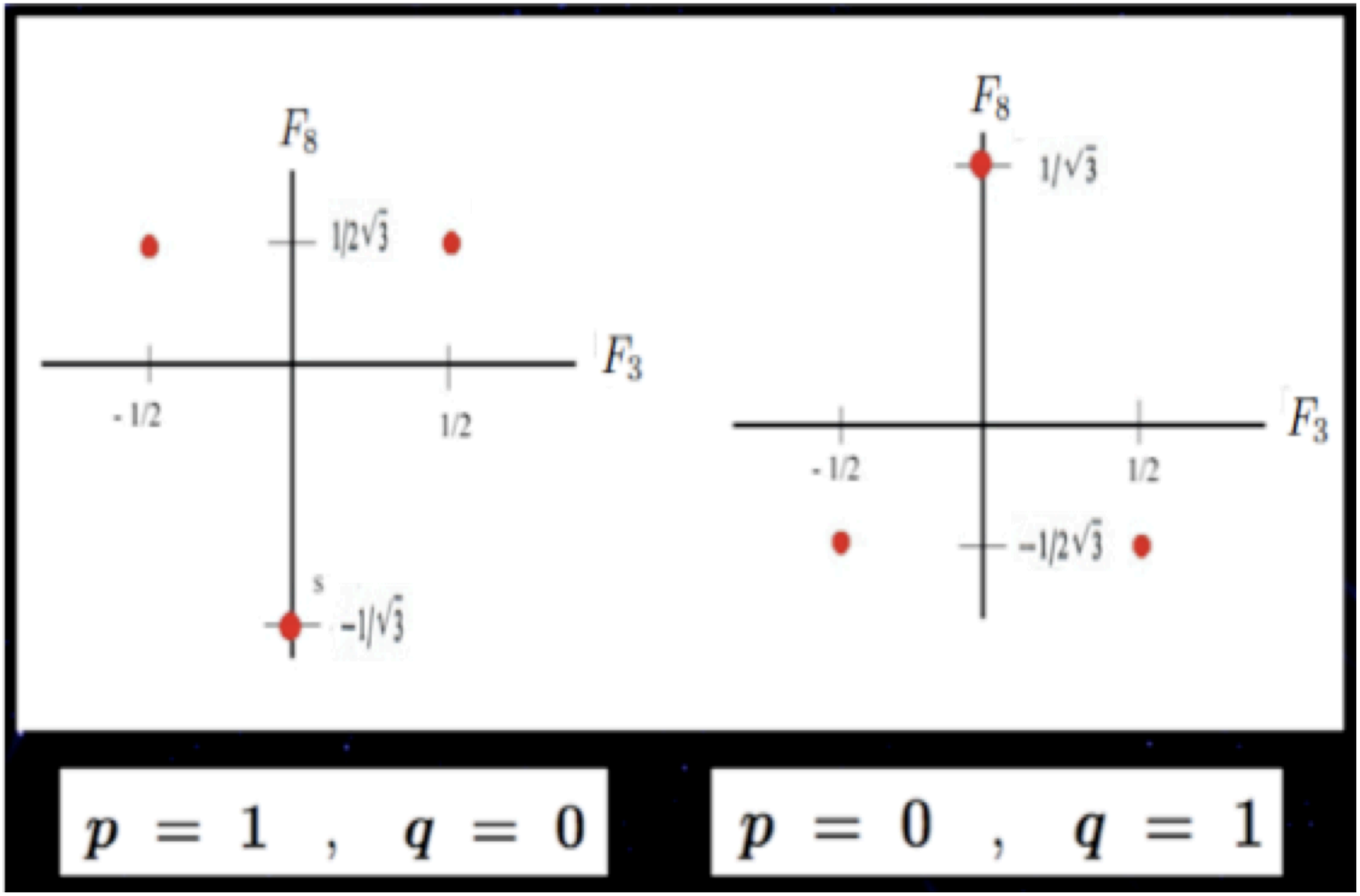
F_8
$1/2\sqrt{3}$
F_3
- 1/2
1/2
$-1/\sqrt{3}$
F_8
$1/\sqrt{3}$
F_3
- 1/2
1/2
$-1/2\sqrt{3}$
$p = 1 \quad , \quad q = 0$
$p = 0 \quad , \quad q = 1$

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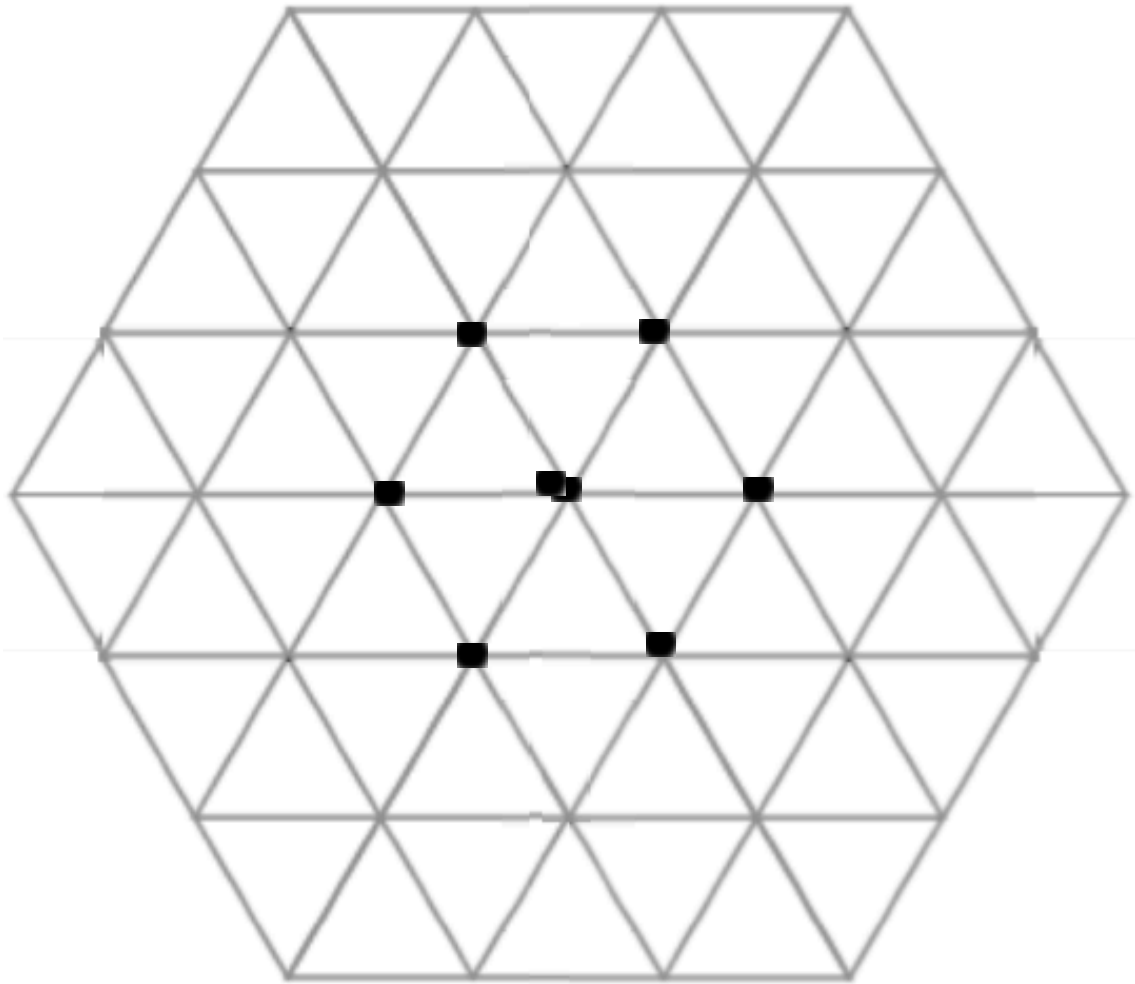
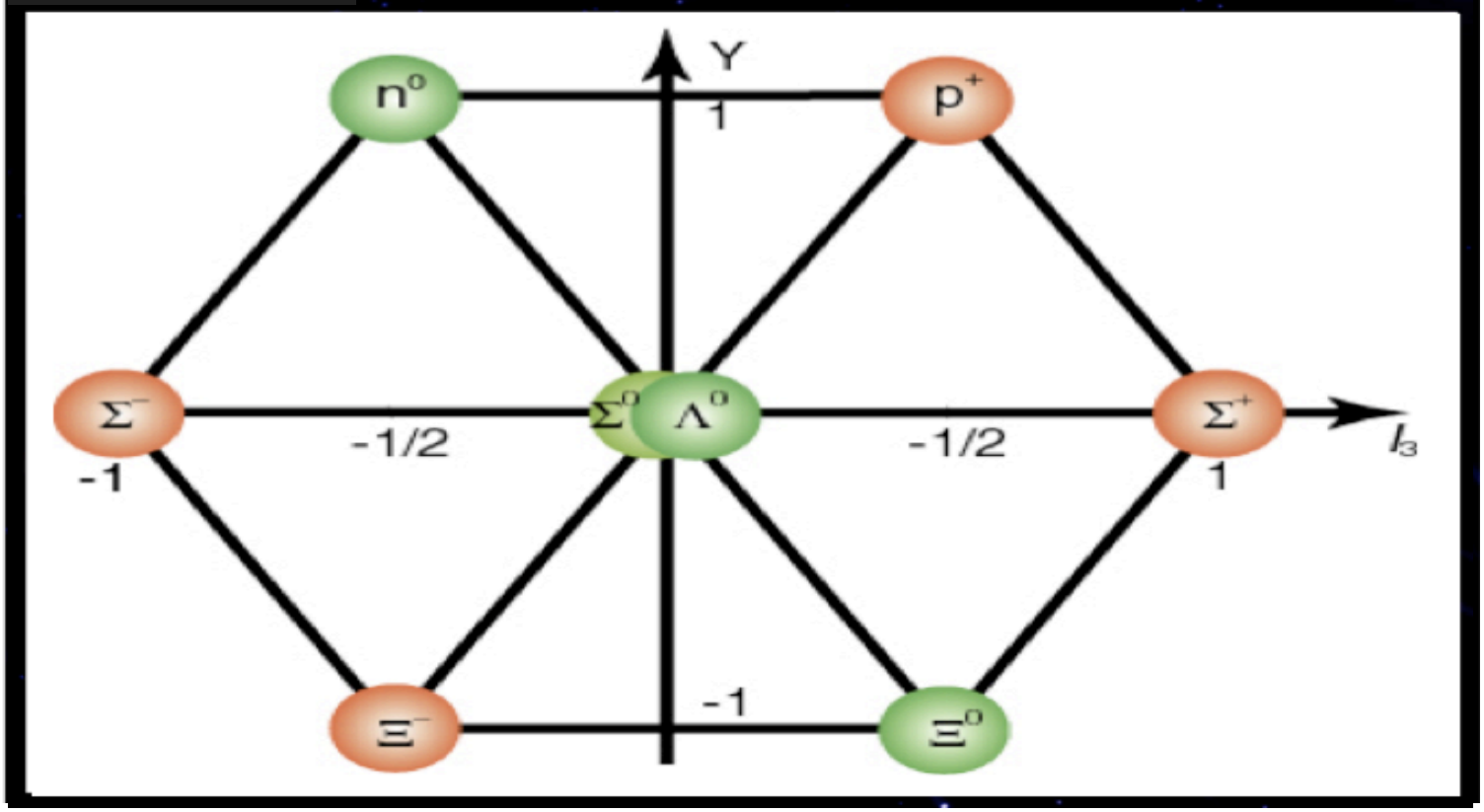
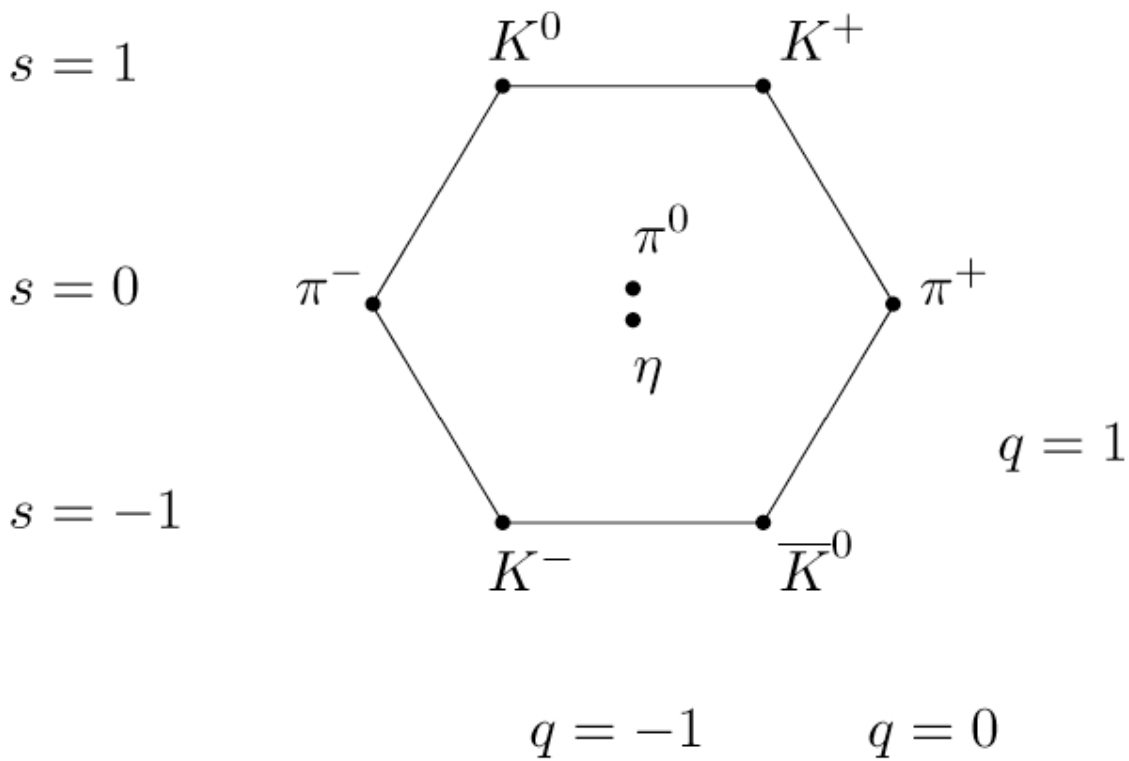


$$p = 1 \quad , \quad q = 1$$




$s = 1$
$s = 0$
$s = -1$
K^0
K^+
π^-
π^0
η
π^+
$q = 1$
K^-
$\overline{K}^0$
$q = -1$
$q = 0$

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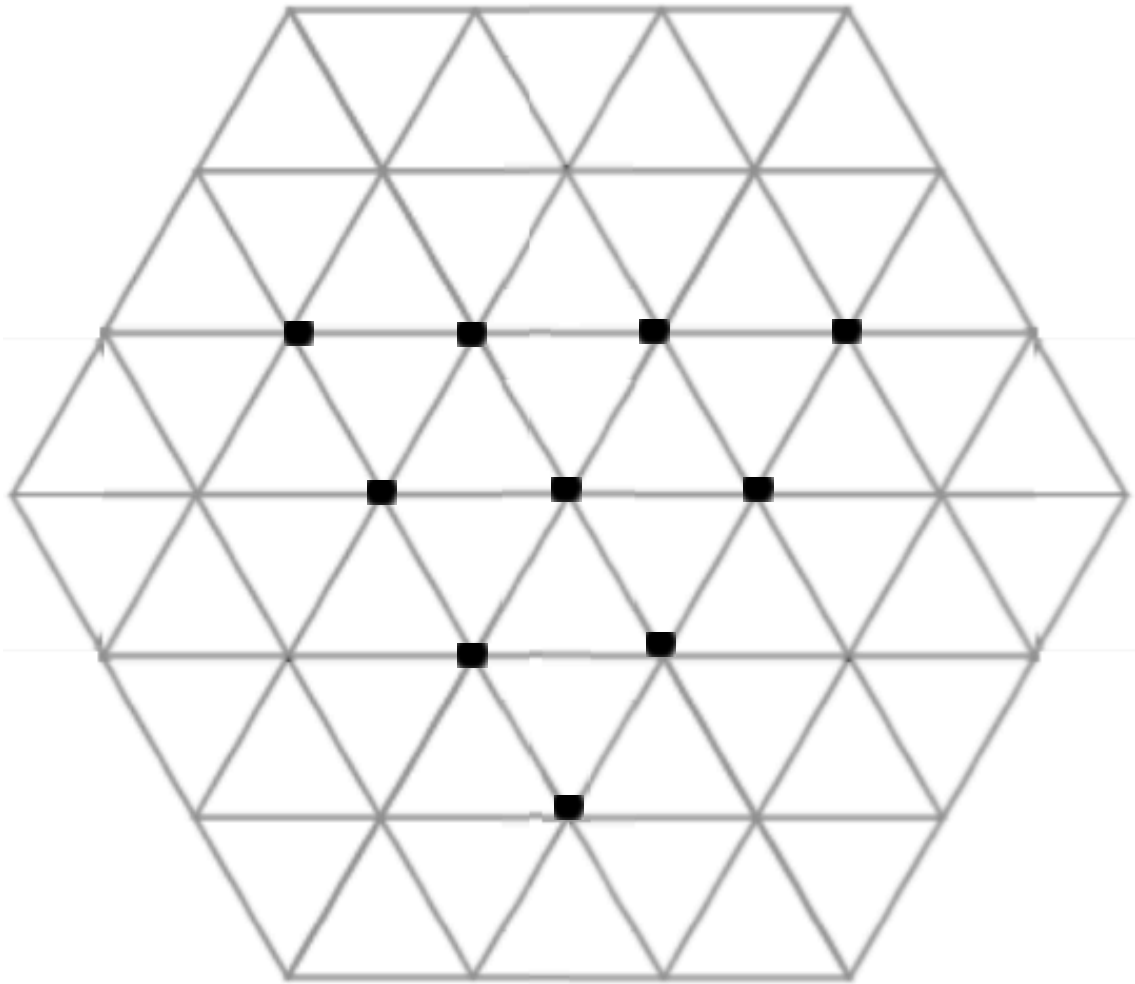

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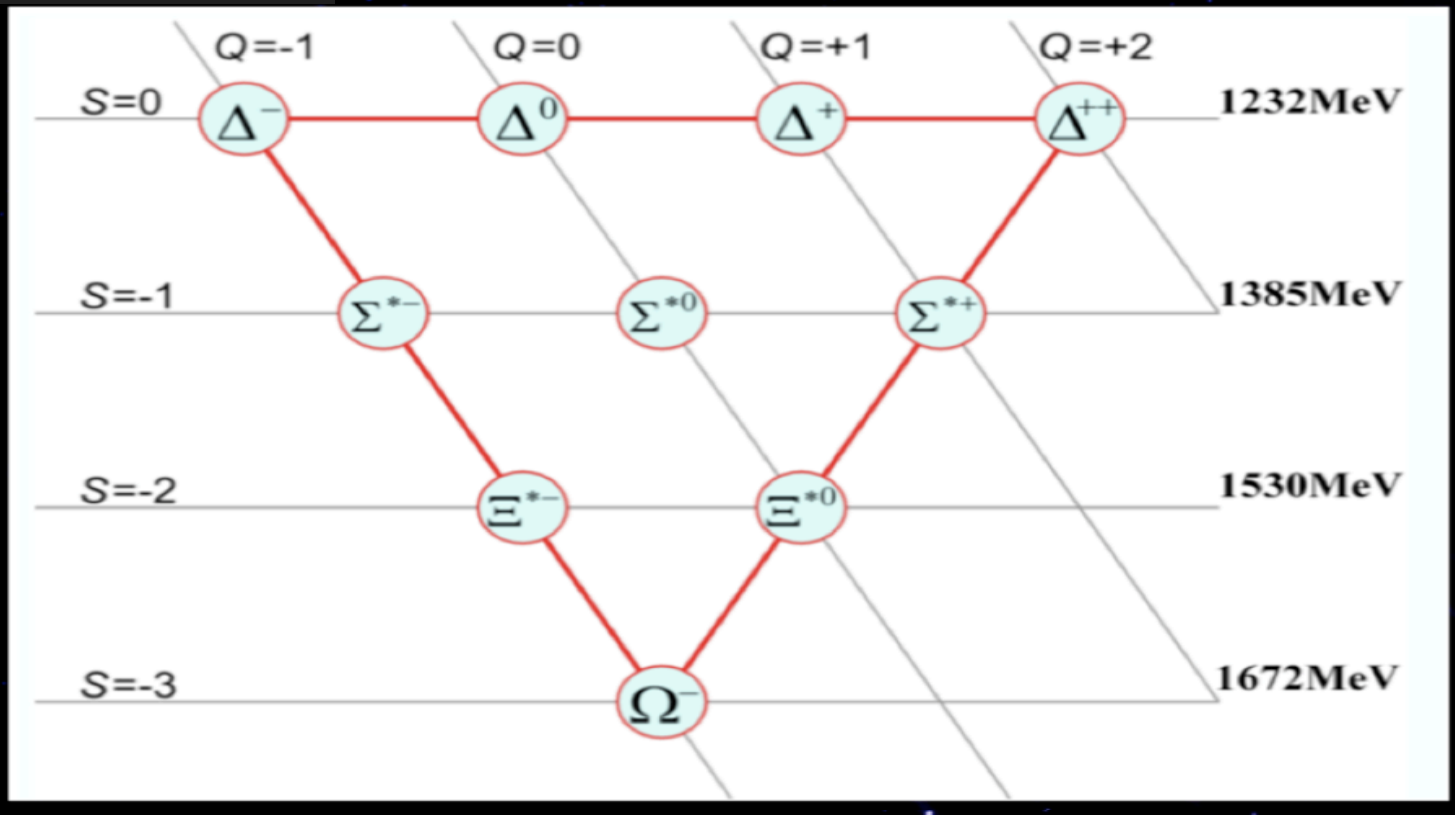
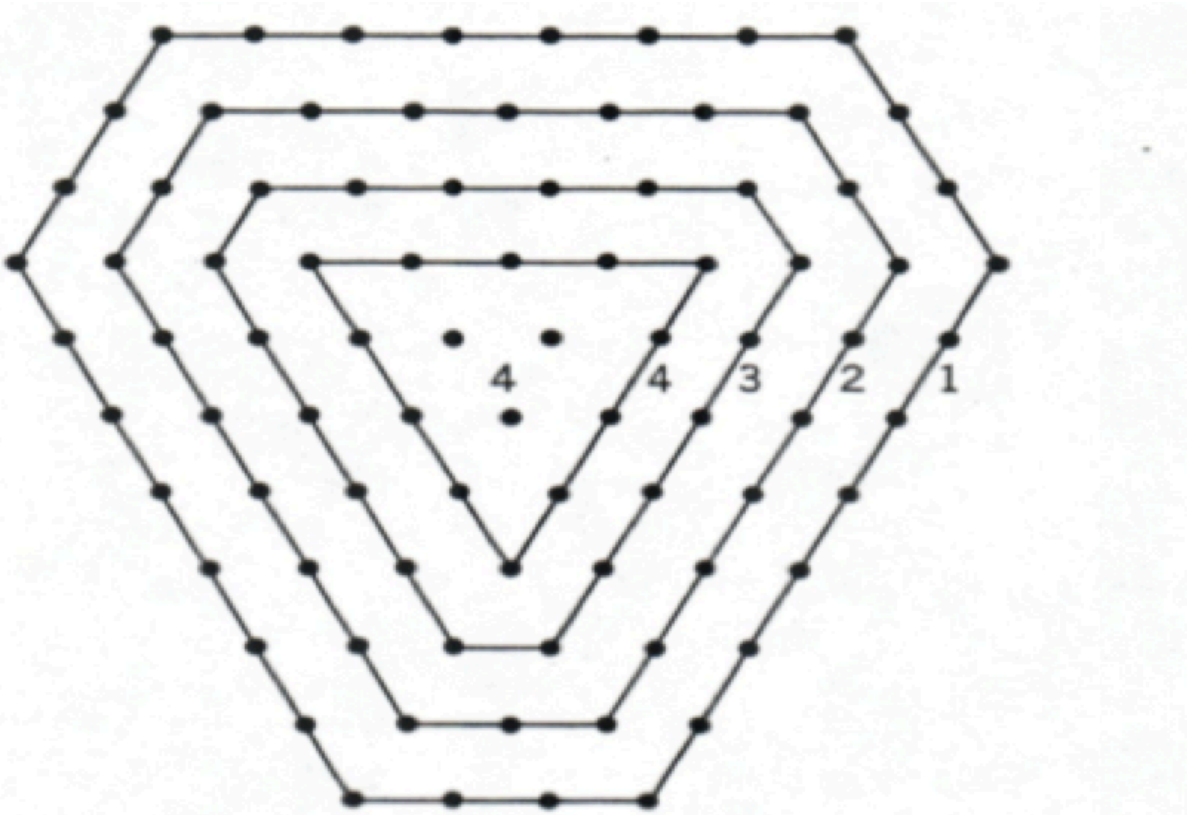
$$p = 3 \quad , \quad q = 0$$





A typical representation [(p, q) = (7, 3)] with multiplicities.

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Can Polytopes Undergird Supersymmetry Extension of the Standard Model?

$$\mathcal{V}(x^{\underline{a}},\theta^\alpha) \;=\; v^{(0)}(x^{\underline{a}}) \;+\; \theta^\alpha\, v^{(1)}_\alpha(x^{\underline{a}})$$
$$+\; \theta^\alpha\theta^\beta\Big[\, C_{\alpha\beta}\, v^{(2)}_1(x^{\underline{a}}) \;+\; i(\gamma^5)_{\alpha\beta}\, v^{(2)}_2(x^{\underline{a}}) \;+\; i(\gamma^5\gamma^{\underline{b}})_{\alpha\beta}\, v^{(2)}_{\underline{b}}(x^{\underline{a}})\,\Big]$$
$$+\; \theta^\alpha\theta^\beta\theta^\gamma C_{\alpha\beta}C_{\gamma\delta}\, v^{(3)\delta}(x^{\underline{a}}) \;+\; \theta^\alpha\theta^\beta\theta^\gamma\theta^\delta C_{\alpha\beta}C_{\gamma\delta}\, v^{(4)}(x^{\underline{a}})$$

Level	Adinkra nodes	Component fields	Irrep(s) in $\mathfrak{so}(4)$
0	$\mathcal{V}\vert$	$f(x^{\underline{a}})$	$\{1\}$
1	$\mathrm{D}_\alpha\mathcal{V}\vert$	$\psi_\alpha(x^{\underline{a}})$	$\{4\}$
2	$\mathrm{D}_{[\alpha}\mathrm{D}_{\beta]}\mathcal{V}\vert$	$g(x^{\underline{a}}),\ h(x^{\underline{a}}),\ v_{\underline{b}}(x^{\underline{a}})$	$\{1\},\ \{1\},\ \{4\}$
3	$\mathrm{D}_{[\alpha}\mathrm{D}_\beta\mathrm{D}_{\gamma]}\mathcal{V}\vert$	$\chi^\delta(x^{\underline{a}})$	$\{4\}$
4	$\mathrm{D}_{[\alpha}\mathrm{D}_\beta\mathrm{D}_\gamma\mathrm{D}_{\delta]}\mathcal{V}\vert$	$N(x^{\underline{a}})$	$\{1\}$

Explicit Relations between Adinkra Nodes, Component Fields, and Irreps

Eleven Dimensional SUGRA Limit of M-Theory

	FERMION	BOSON
	$\Psi_{\mu}{}^{\alpha}$	$h_{\mu\nu}$ $A_{\mu\nu\rho}$

arXiv.org > hep-th > arXiv:1911.00807

High Energy Physics - Theory

[Submitted on 3 Nov 2019 (v1), last revised 30 Jan 2020 (this version, v3)]

Superfield Component Decompositions and the Scan for Prepotential Supermultiplets in 10D Superspaces

S. James Gates Jr., Yangrui Hu, S.-N. Hazel Mak

The first complete and explicit SO(1,9) Lorentz descriptions of all component fields contained in $\mathcal{N} = 1$, $\mathcal{N} = 2$A, and $\mathcal{N} = 2$B unconstrained scalar 10D superfields are presented. These are made possible by the discovery of the relation of the superfield component expansion as a consequence of the branching rules of irreducible representations in one ordinary Lie algebra into one of its Lie subalgebras. Adinkra graphs for ten dimensional superspaces are defined for the first time, whose nodes depict spin bundle representations of SO(1,9). An analog of Breitenlohner's approach is implemented to scan for superfields that contain graviton(s) and gravitino(s), which are the candidates for the prepotential superfields of 10D off-shell supergravity theories and separately abelian Yang-Mills theories are similarly treated. Version three contains additional content, both historical and conceptual, which broaden the reach of the scan in the 10D Yang-Mills case.

arXiv.org > hep-th > arXiv:2002.08502

High Energy Physics – Theory

[Submitted on 20 Feb 2020 (v1), last revised 12 May 2020 (this version, v7)]

Adinkra Foundation of Component Decomposition and the Scan for Superconformal Multiplets in 11D, N = 1 Superspace

S. James Gates Jr., Yangrui Hu, S.-N. Hazel Mak

For the first time in the physics literature, the Lorentz representations of all 2,147,483,648 bosonic degrees of freedom and 2,147,483,648 fermionic degrees of freedom in an unconstrained eleven dimensional scalar superfield are presented. Comparisons of the conceptual bases for this advance in terms of component field, superfield, and adinkra arguments, respectively, are made. These highlight the computational efficiency of the adinkra-based approach over the others. It is noted at level sixteen in the 11D, N = 1 scalar superfield, the {65} representation of SO(1,10), the conformal graviton, is present. Thus, Adinkra-based arguments suggest the surprising possibility that the 11D, N = 1 scalar superfield alone might describe a Poincare supergravity prepotential in analogy to one of the off-shell versions of 4D, N = 1 superfield supergravity.

arXiv.org > hep-th > arXiv:2002.08502

arXiv.org > hep-th > arXiv:2006.03609

High Energy Physics – Theory

[Submitted on 5 Jun 2020 (v1), last revised 20 Jul 2020 (this version, v2)]

Advening to Adynkrafields: Young Tableaux to Component Fields of the 10D, N = 1 Scalar Superfield

S. James Gates Jr., Yangrui Hu, S.-N. Hazel Mak

Starting from higher dimensional adinkras constructed with nodes referenced by Dynkin Labels, we define "adynkras." These suggest a computationally direct way to describe the component fields contained within supermultiplets in all superspaces. We explicitly discuss the cases of ten dimensional superspaces. We show this is possible by replacing conventional θ–expansions by expansions over Young Tableaux and component fields by Dynkin Labels. Without the need to introduce σ–matrices, this permits rapid passages from Adynkras → Young Tableaux → Component Field Index Structures for both bosonic and fermionic fields while increasing computational efficiency compared to the starting point that uses superfields. In order to reach our goal, this work introduces a new graphical method, "tying rules," that provides an alternative to Littlewood's 1950 mathematical results which proved branching rules result from using a specific Schur function series. The ultimate point of this line of reasoning is the introduction of mathematical expansions based on Young Tableaux and that are algorithmically superior to superfields. The expansions are given the name of "adynkrafields" as they combine the concepts of adinkras and Dynkin Labels.

"Learning to use coding is like putting on the maths version of the Iron Man suit."

"We required Mathematica to handle the over four billion functions! We have codes and apps for this."

Level #	Component Field Count
0	1
1	1
2	3
3	3
4	8
5	9
6	19
7	23
8	49
9	55
10	99
11	106
12	173
13	171
14	247
15	225
16	296

$$N_{Bosonic\ Fields} = 1,198$$

$$h_{\mu\nu}, A_{\mu\nu\rho}, \ldots$$

$$N_{Fermionic\ Fields} = 1,186$$

$$\Psi_{\mu}{}^{\alpha}, \ldots$$

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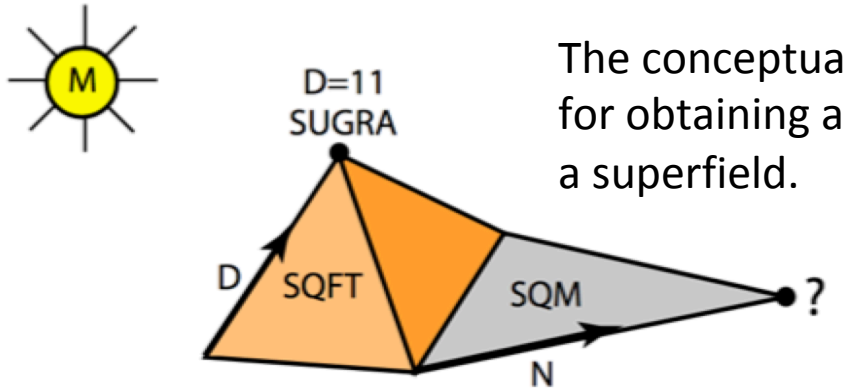
arXiv.org > hep-th > arXiv:hep-th/0408004

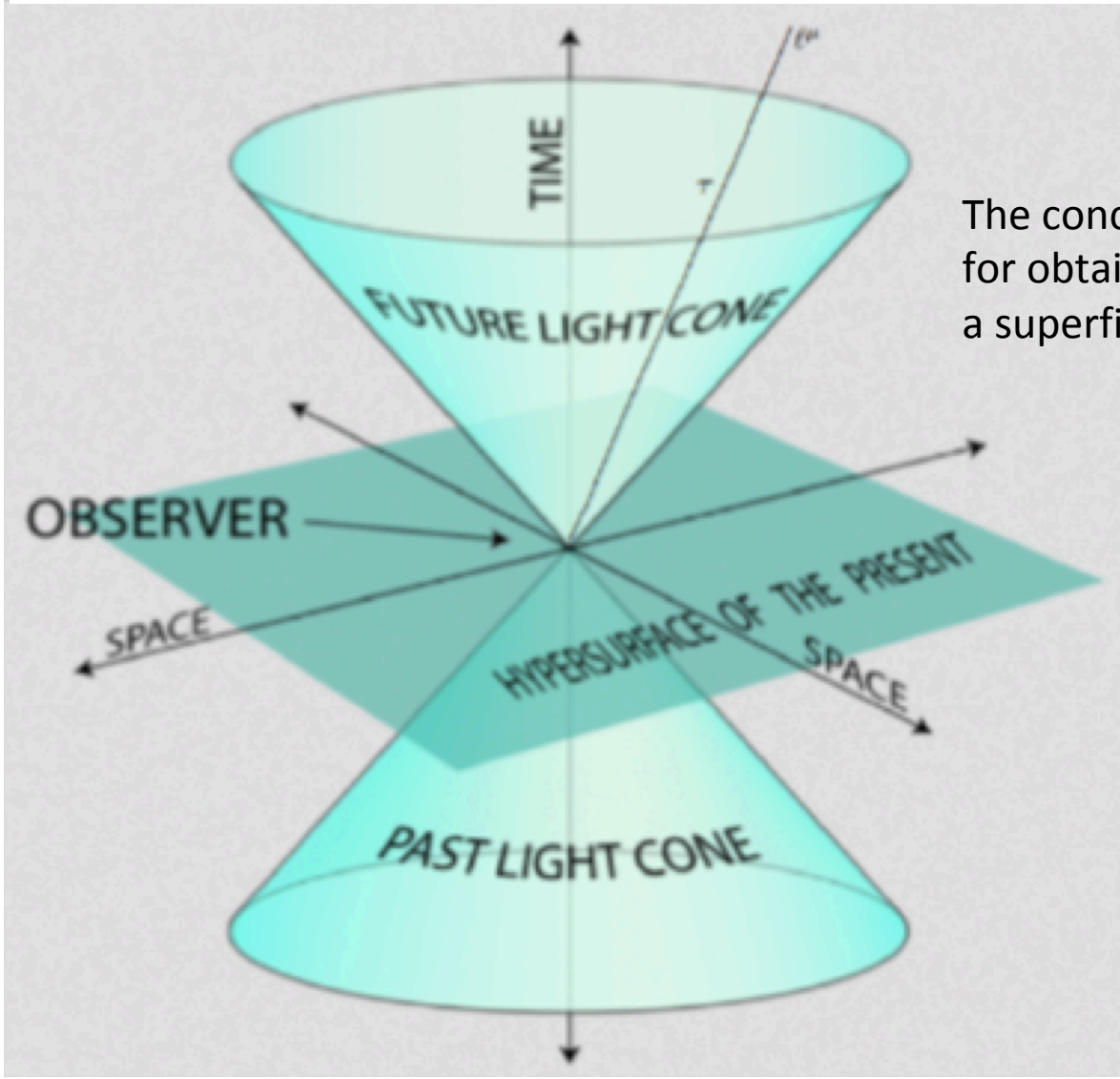


The conceptual background for obtaining an adinkra from a superfield.

Each supersymmetric quantum field theory has a "shadow" in supersymmetric quantum mechanics obtained by dimensionally reducing all of the spatial dimensions in the field theory.

Adinkras: A Graphical Technology for Supersymmetric Representation Theory

Michael Faux, S. J. Gates Jr



The conceptual background for obtaining an adinkra from a superfield.

SUSY holography

SUSY holography: reduce 4D models to 1D, 1D models encode the structure of 4D models (e.g. gadgets are the same).

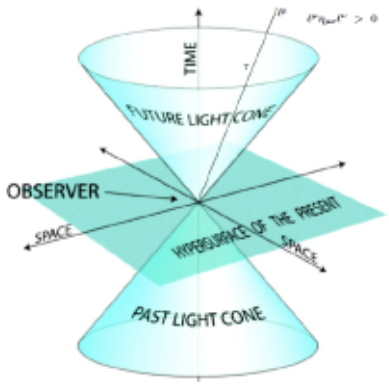
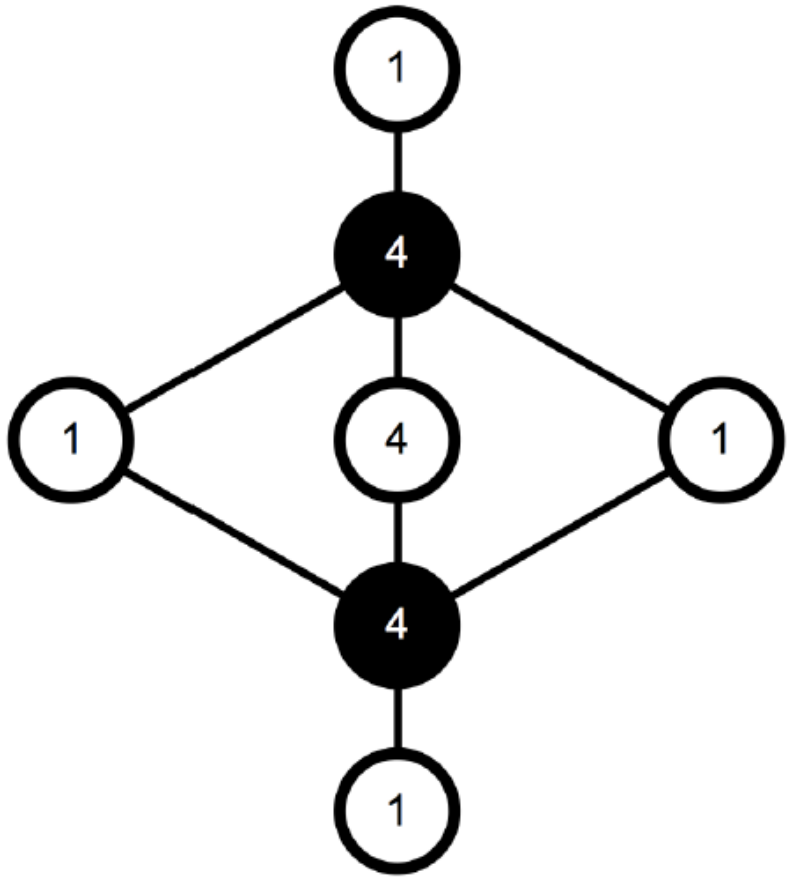


Figure 8

How (4D, $\mathcal{N} = 2 \to$ 1D, $N = 8$):

- field$(t, x, y, z) \to$ field(τ)
- D_a^i $(a = 1, \ldots, 4, i = 1, 2) \to D_I$ $(I = 1, \ldots, 8)$



Adinkra Diagram for 4D, $\mathcal{N} = 1$

Error

Correcting

Codes

Control

Folding

Bits Naturally Arise From The Geometry Of Hypercubes

To a small part, the appearance of the SDEC's is not mysterious. Bits naturally appear in any situation where cubical geometry is relevant. The vertices of a cube can always be written in the form

$$(\pm 1, \ \pm 1, \ \pm 1, \ \ldots, \ \pm 1)$$

or re-written in the form

$$((-1)^{p_1}, \ (-1)^{p_2}, \ (-1)^{p_3}, \ \ldots, \ (-1)^{p_d})$$

where the exponents are bits since they take on values 1 or 0. Thus any vertex has an 'address' that is a string of bits

$$(\boldsymbol{p}_1, \ \boldsymbol{p}_2, \ \boldsymbol{p}_3, \ \ldots, \ \boldsymbol{p}_d)$$

the information theoretic definition of a 'word.'

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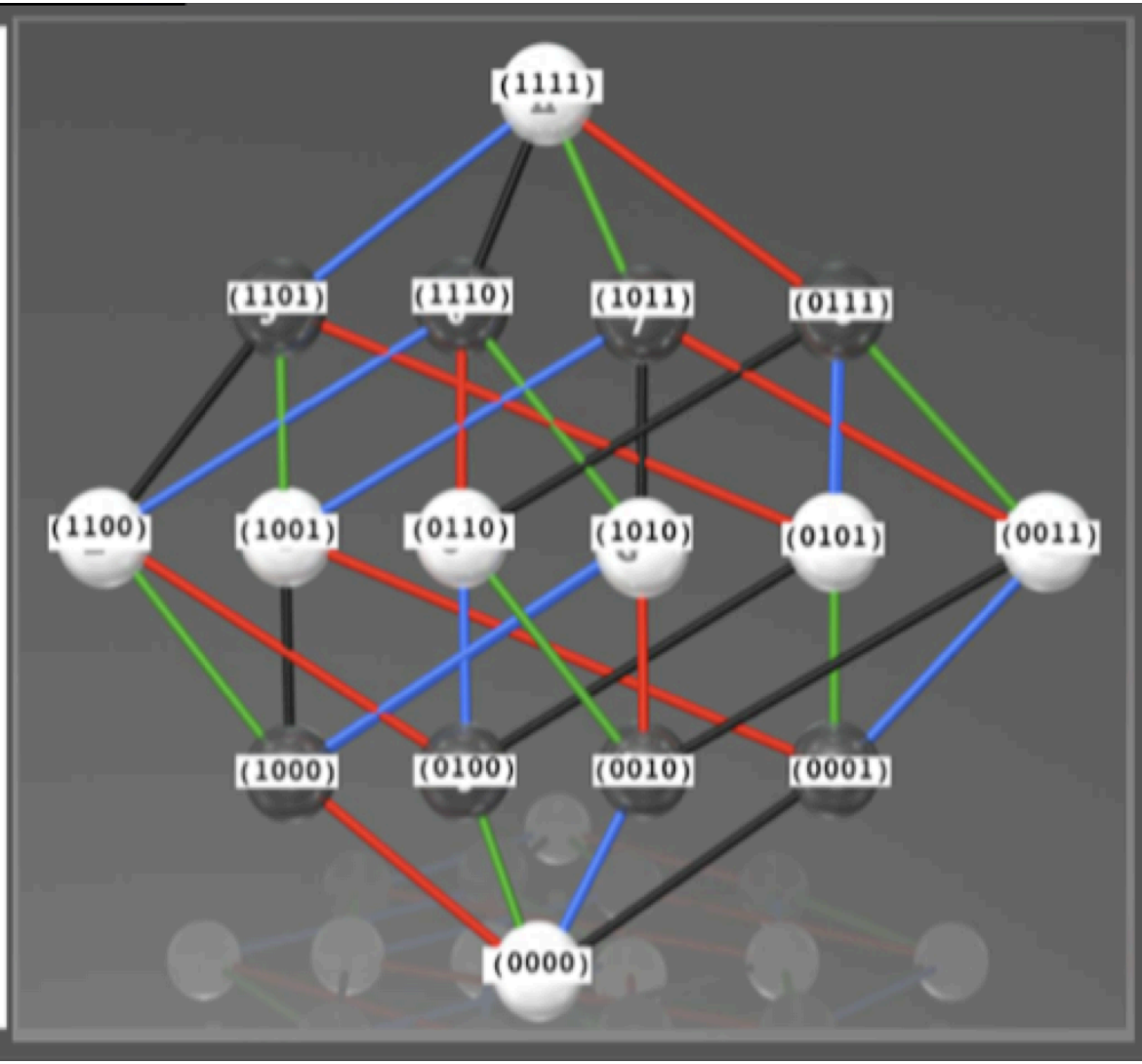
Error

Correcting

Codes

Control

Folding



Error

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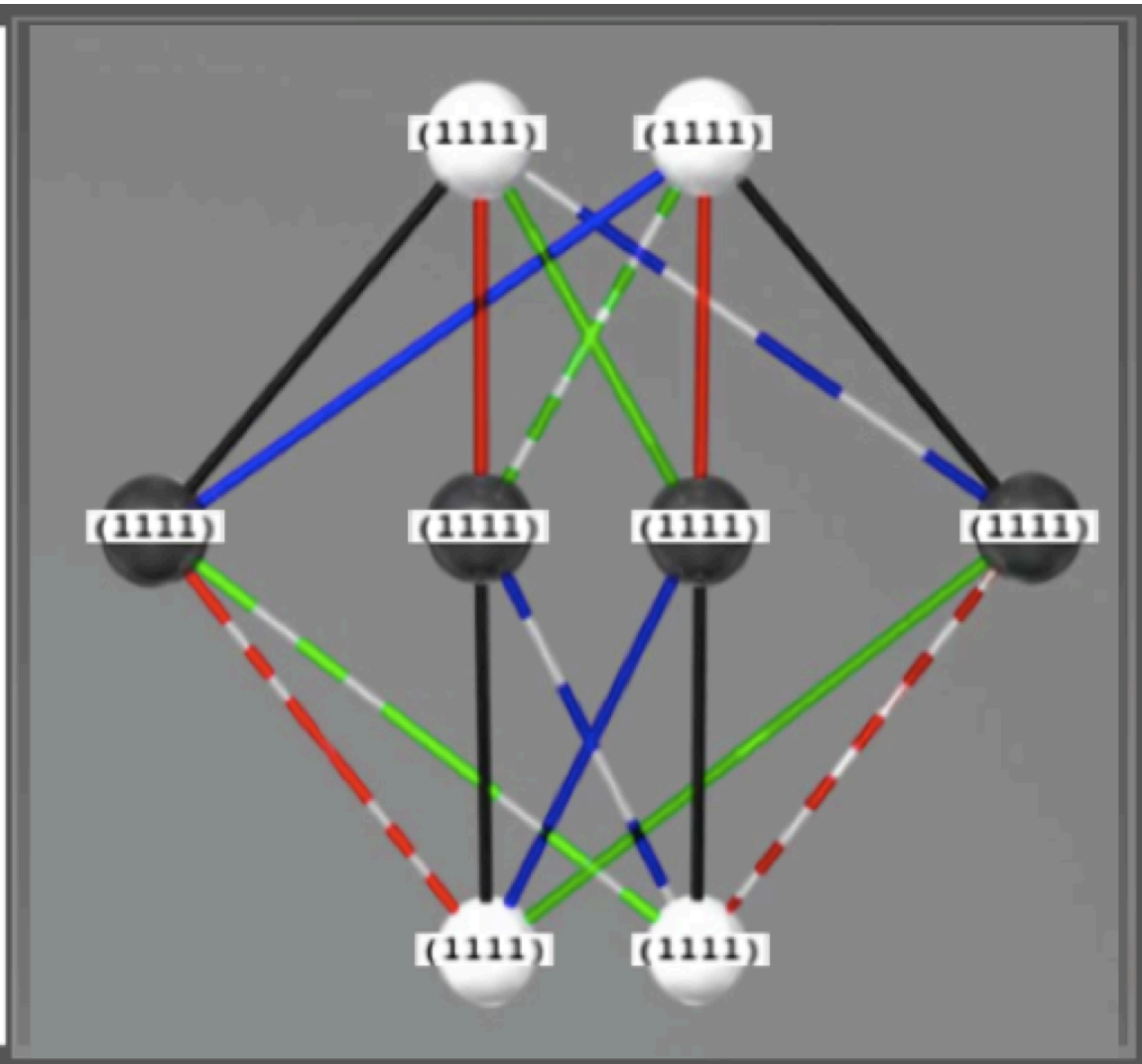
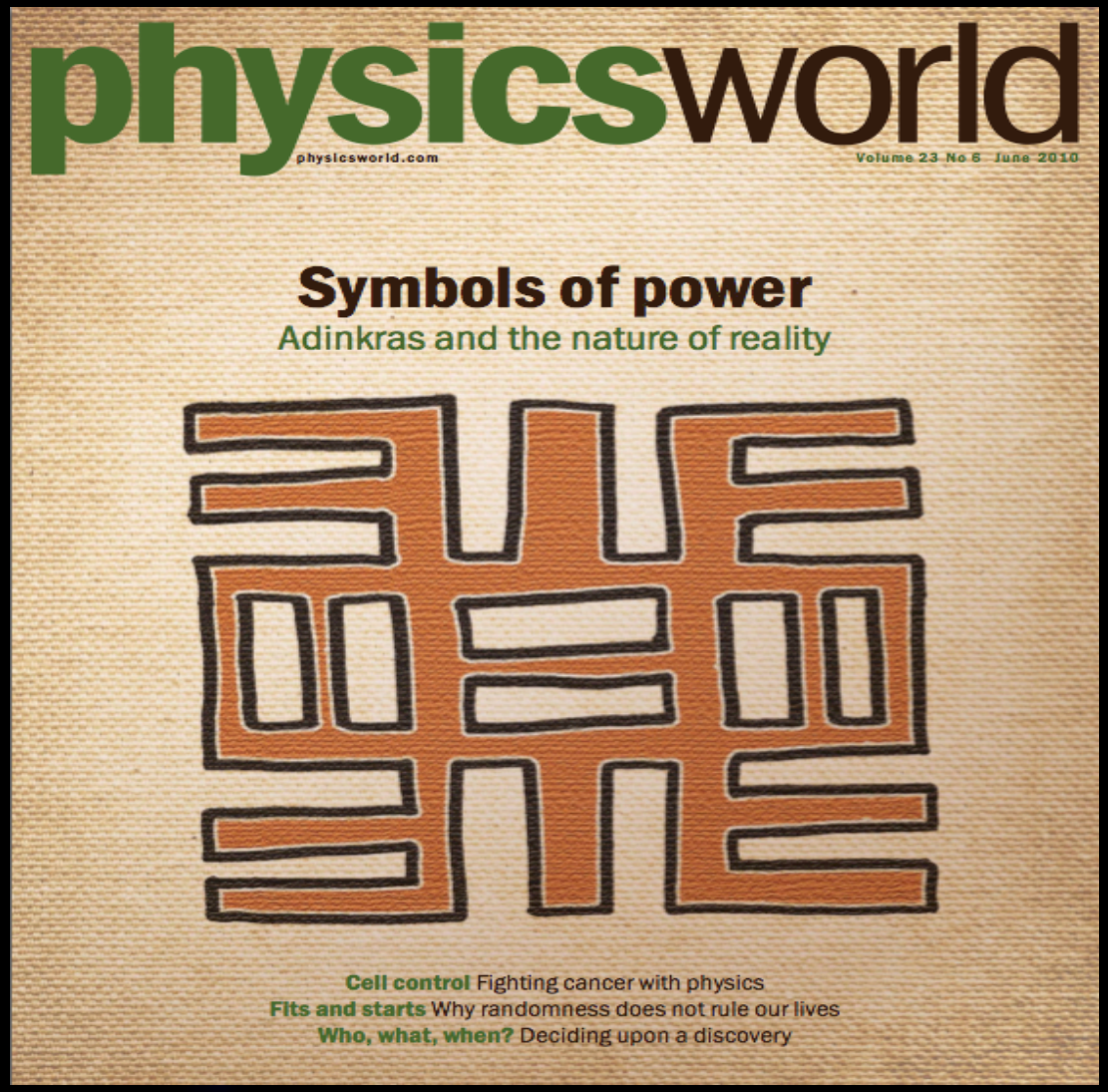
Folding

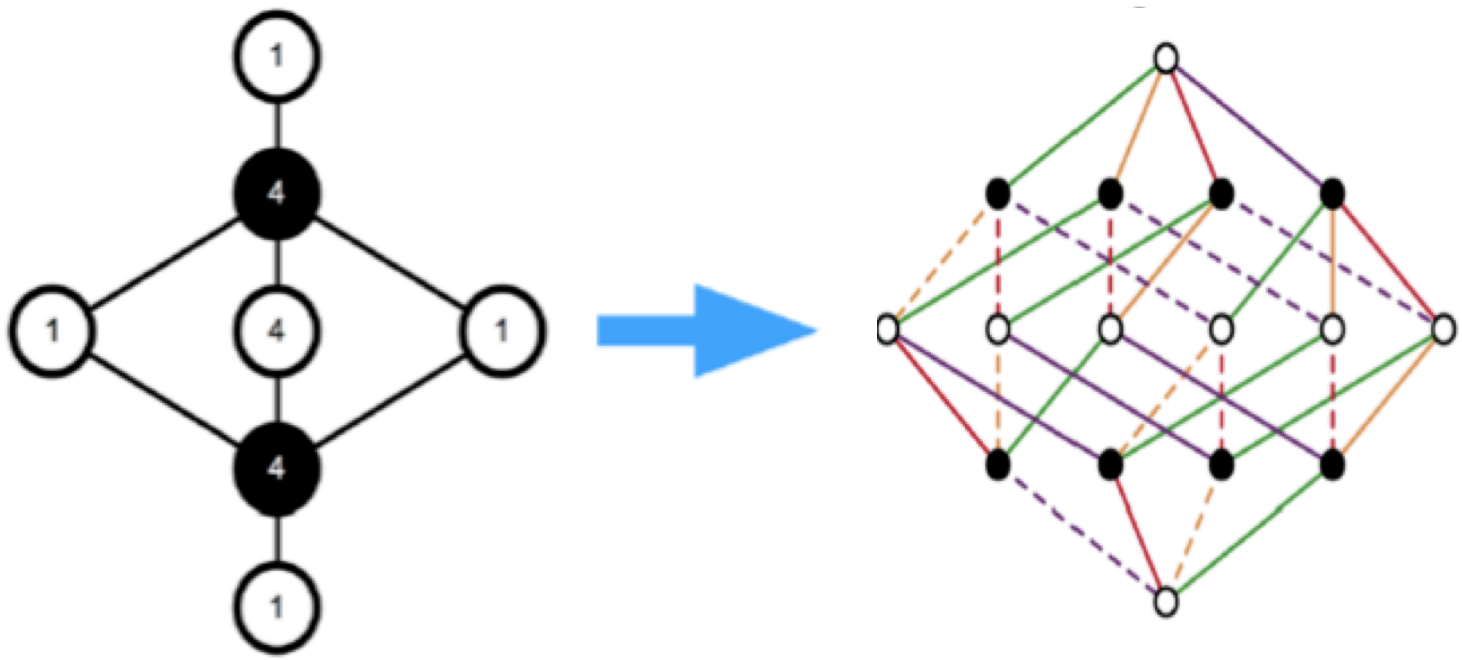



physicsworld
physicsworld.com
Volume 23 No 6 June 2010
Symbols of power
Adinkras and the nature of reality
Cell control Fighting cancer with physics
Fits and starts Why randomness does not rule our lives
Who, what, when? Deciding upon a discovery

‘Gemstone Cutting’
SUSY Crystals

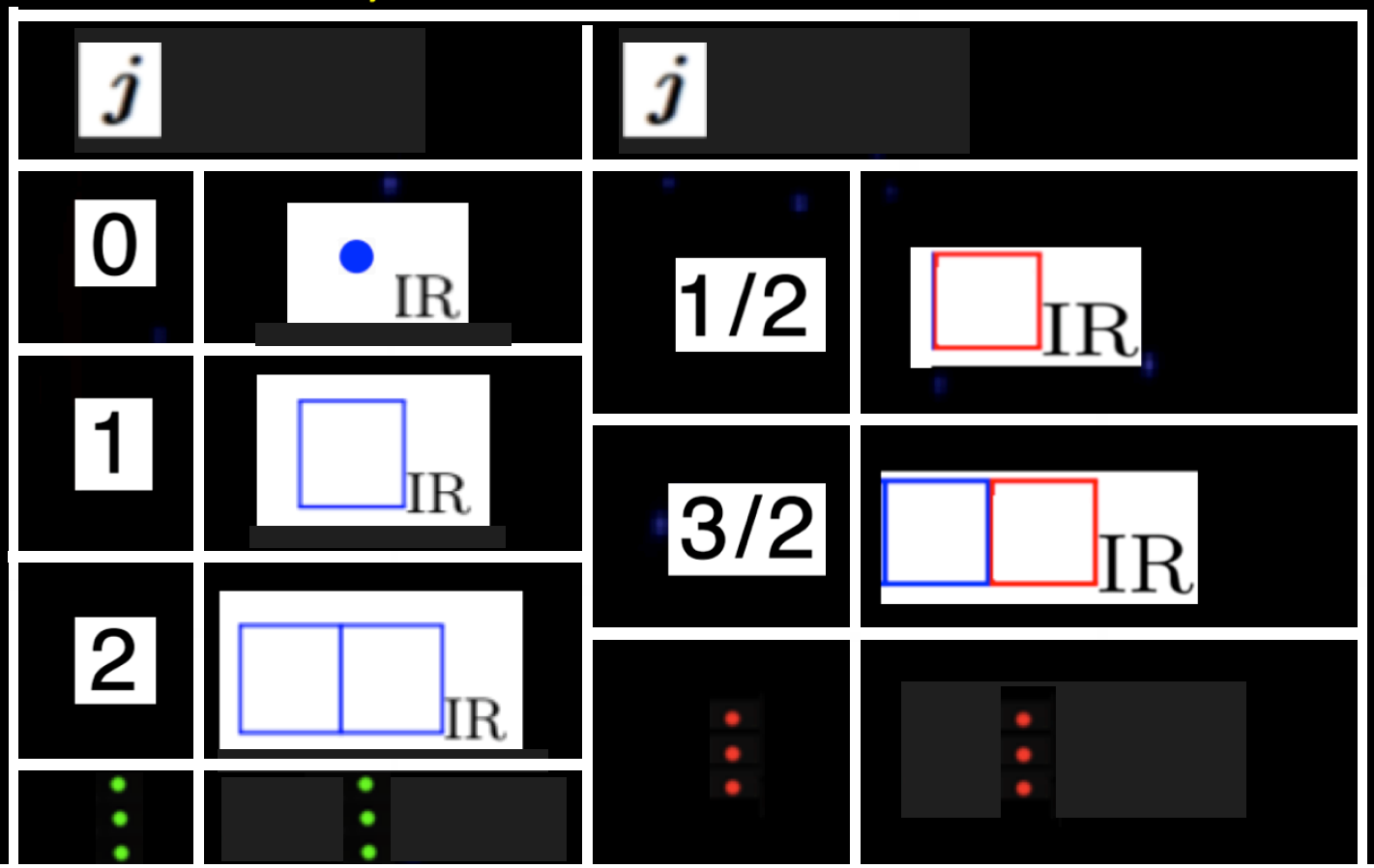
An adinkra with some nodes collapsed with multiplicity shown

An adinkra with all nodes shown



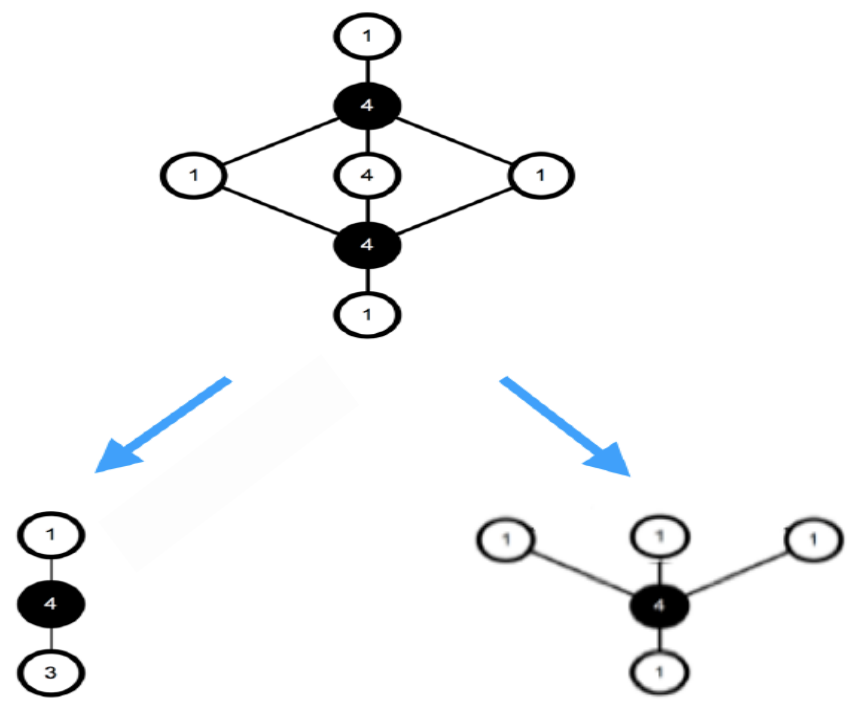
From 1D, $\mathcal{N} = 4$ Adinkra to 4D, $\mathcal{N} = 1$ Adinkra

Dynkin Labels For The Nodes



Adinkra Diagram for 4D, $\mathcal{N} = 1$

Separating a Reducible Adinkra Into Its Two Irreducible Components



From 4D, $\mathcal{N} = 1$ Reducible Adinkra to Irreducible Adinkras

(Vector and Chiral Supermultiplets)

The 4D, N = 1 Minimal Supermultiplet Zoo

There exists ten distinct minimal off-shell 4D, $\mathcal{N} = 1$ supermultiplets as indicated below:

(S01.) Chiral Supermultiplet : (A, B, ψ_a, F, G) ,

(S02.) Hodge – Dual #1 Chiral Supermultiplet : $(\widehat{A}, \widehat{B}, \psi_a, \mathrm{f}_{\mu\nu\rho}, \widehat{G})$,

(S03.) Hodge – Dual #2 Chiral Supermultiplet : $(\widetilde{A}, \widetilde{B}, \psi_a, \widehat{F}, \mathrm{g}_{\mu\nu\rho})$,

(S04.) Hodge – Dual #3 Chiral Supermultiplet : $(\check{A}, \check{B}, \psi_a, \check{\mathrm{f}}_{\mu\nu\rho}, \check{\mathrm{g}}_{\mu\nu\rho})$,

(S05.) Tensor Supermultiplet : $(\varphi, B_{\mu\nu}, \chi_a)$,

(S06.) Axial – Tensor Supermultiplet : $(\widehat{\varphi}, \widehat{B}_{\mu\nu}, \widehat{\chi}_a)$,

(S07.) Vector Supermultiplet : $(A_\mu, \lambda_b, \mathrm{d})$,

(S08.) Axial – Vector Supermultiplet : $(U_\mu, \widehat{\lambda}_b, \widehat{\mathrm{d}})$,

(S09.) Hodge – Dual Vector Supermultiplet : $(\widetilde{A}_\mu, \widetilde{\lambda}_b, \widetilde{\mathrm{d}}_{\mu\nu\rho})$,

(S10.) Hodge – Dual Axial – Vector Supermultiplet : $(\breve{U}_\mu, \breve{\lambda}_b, \breve{\mathrm{d}}_{\mu\nu\rho})$.

Hodge duality relates some of the supermultiplets.

Parity duality relates some of the supermultiplets..

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(S04.) Hodge – Dual #3 Chiral Supermultiplet : $(\breve{A}, \breve{B}, \psi_a, \breve{\mathrm{f}}_{\mu\nu\rho}, \breve{\mathrm{g}}_{\mu\nu\rho})$,

(S05.) Tensor Supermultiplet : $(\varphi, B_{\mu\nu}, \chi_a)$,

(S06.) Axial – Tensor Supermultiplet : $(\widehat{\varphi}, \widehat{B}_{\mu\nu}, \widehat{\chi}_a)$,

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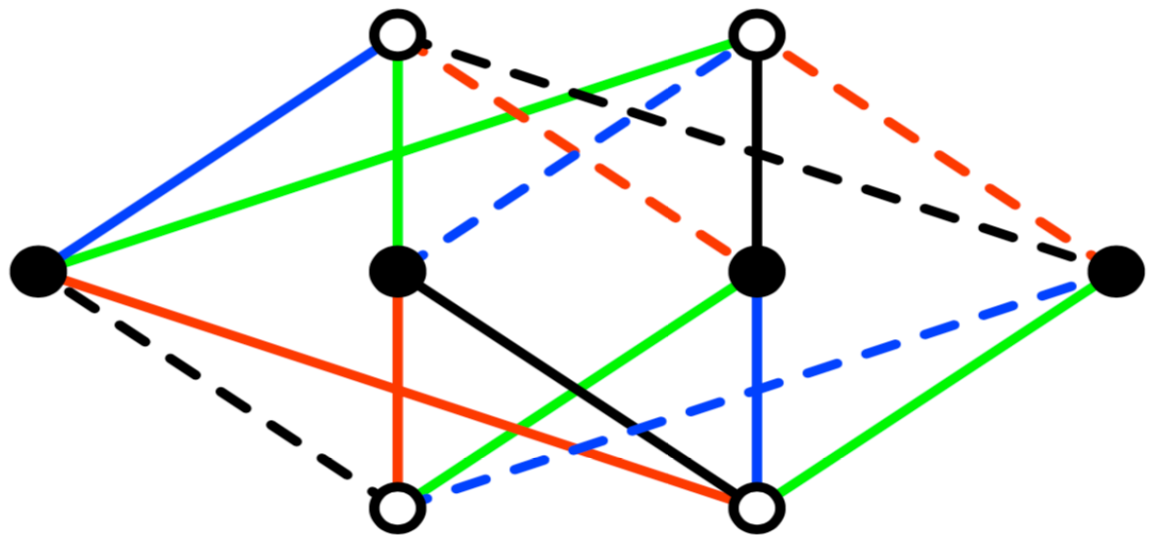
Hodge duality relates some of the supermultiplets.

Parity duality relates some of the supermultiplets..

S01



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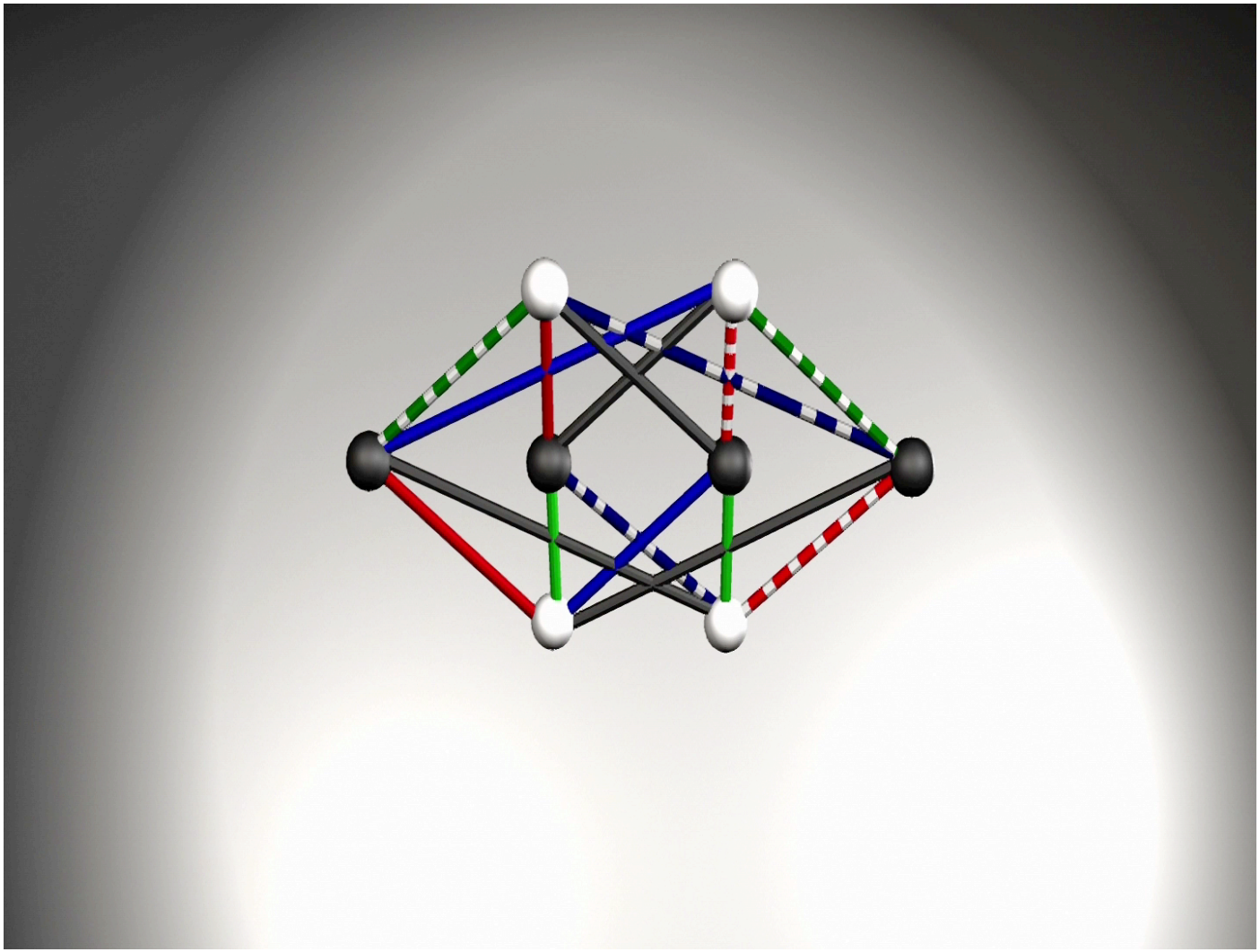

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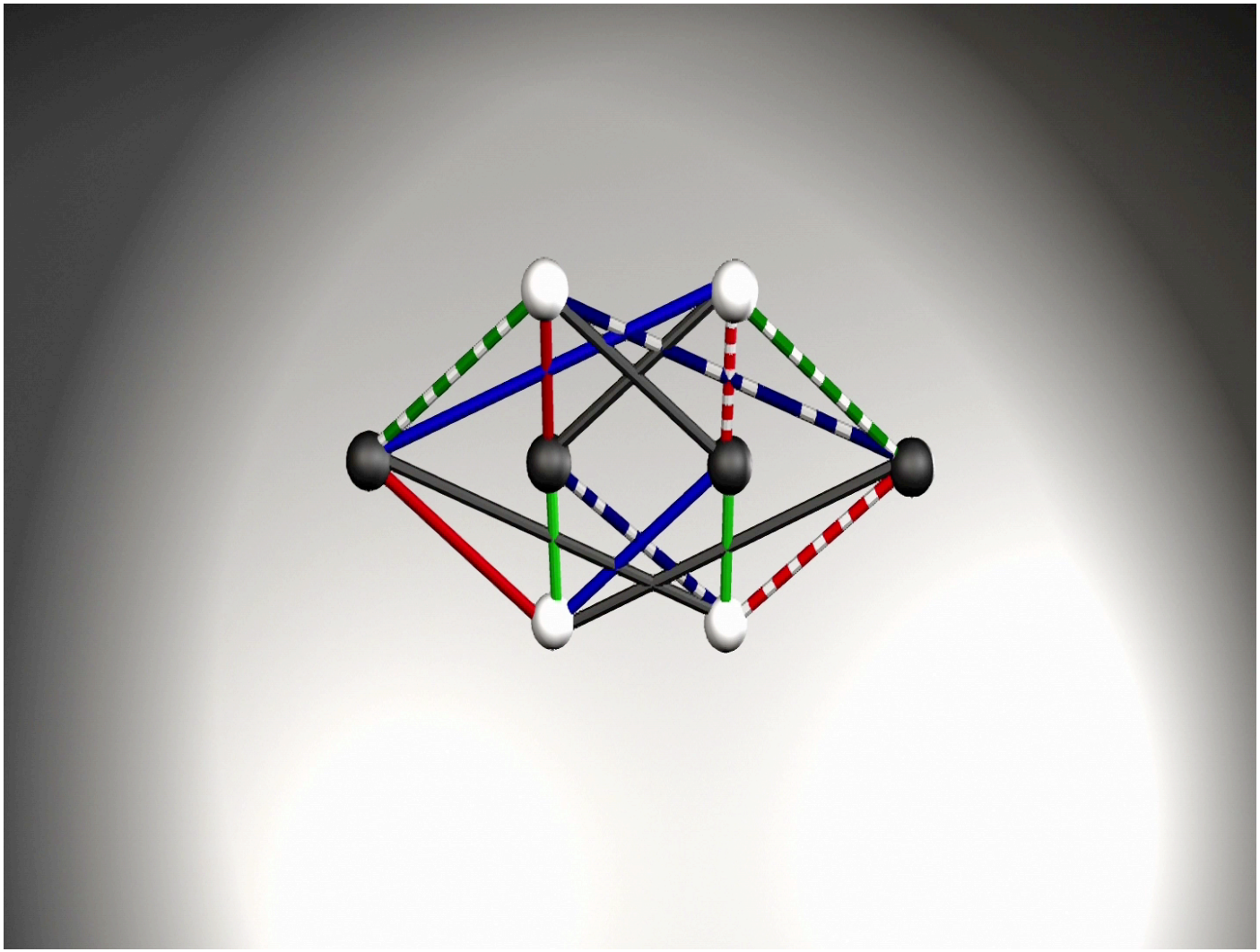
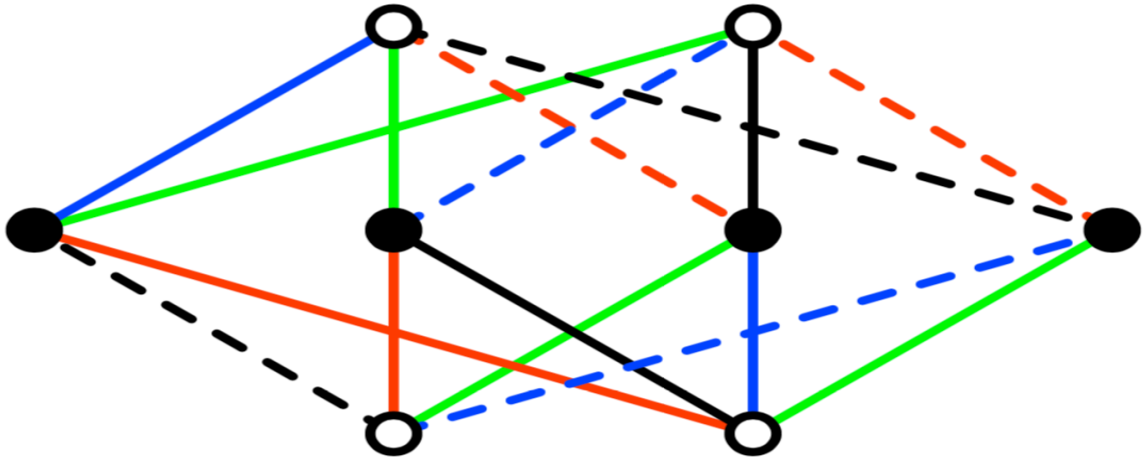
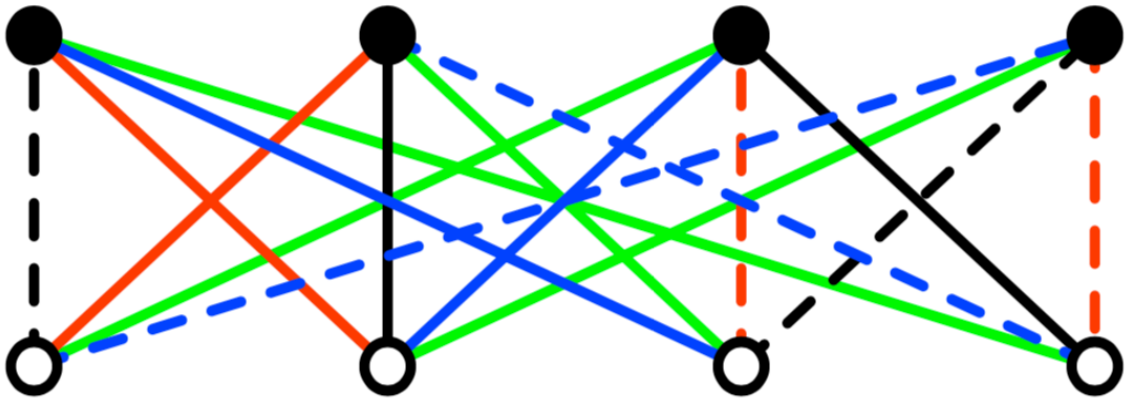


S01



Using the 'Hanging Garden Theorem' yields.

S04



A 'valise' adiinkra.

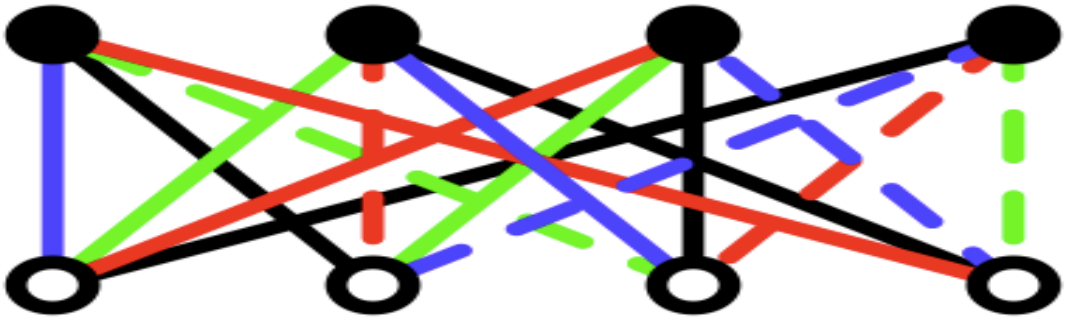
***Chiral Supermultiplet* : (A, B, ψ_a, F, G)**

$$
\begin{aligned}
\mathrm{D}_a A &= \psi_a \quad , \quad \mathrm{D}_a B = i\,(\gamma^5)_a{}^b\,\psi_b \quad , \\
\mathrm{D}_a \psi_b &= i\,(\gamma^\mu)_{a\,b}\,\partial_\mu A - (\gamma^5\gamma^\mu)_{a\,b}\,\partial_\mu B - i\,C_{a\,b}\,F + (\gamma^5)_{a\,b} G \ , \\
\mathrm{D}_a F &= (\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \quad , \quad \mathrm{D}_a G = i\,(\gamma^5\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \quad ,
\end{aligned}
$$

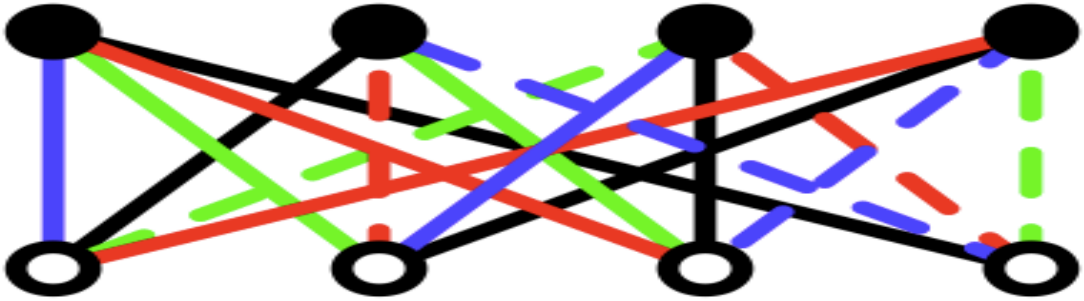
***Hodge – Dual #3 Chiral Supermultiplet* : $(A, B, \psi_a, \mathrm{f}_{\mu\nu\rho}, \mathrm{g}_{\mu\nu\rho})$**

$$
\begin{aligned}
\mathrm{D}_a A &= \psi_a \quad , \quad \mathrm{D}_a B = i\,(\gamma^5)_a{}^b\,\psi_b \quad , \\
\mathrm{D}_a \psi_b &= i\,(\gamma^\mu)_{a\,b}\,\partial_\mu A - (\gamma^5\gamma^\mu)_{a\,b}\,\partial_\mu B \\
&\quad - i\,\frac{1}{3!}\,C_{a\,b}\,(\epsilon^{\sigma\mu\nu\rho}\,\partial_\sigma \mathrm{f}_{\mu\nu\rho}) + \frac{1}{3!}\,(\gamma^5)_{a\,b}\,(\epsilon^{\sigma\mu\nu\rho}\,\partial_\sigma \mathrm{g}_{\mu\nu\rho}) \ , \\
\mathrm{D}_a \mathrm{f}_{\mu\nu\rho} &= -\,(\gamma^\sigma)_a{}^b\,\epsilon_{\sigma\mu\nu\rho}\,\psi_b \quad , \quad \mathrm{D}_a \mathrm{g}_{\mu\nu\rho} = -\,(\gamma^5\gamma^\sigma)_a{}^b\,\epsilon_{\sigma\mu\nu\rho}\,\psi_b \quad .
\end{aligned}
$$

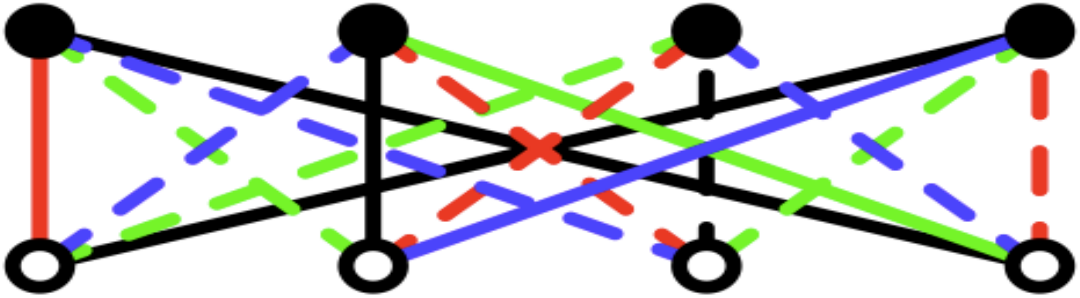
S04



S05



S10



$$D_1\Phi_1 = i\Psi_1, \quad D_2\Phi_1 = i\Psi_2, \quad D_3\Phi_1 = \chi_0 i\Psi_3, \quad D_4\Phi_1 = -i\Psi_4$$

$$D_1\Phi_2 = i\Psi_2, \quad D_2\Phi_2 = -i\Psi_1, \quad D_3\Phi_2 = \chi_0 i\Psi_4, \quad D_4\Phi_2 = i\Psi_3$$

$$D_1\Phi_3 = i\Psi_3, \quad D_2\Phi_3 = -i\Psi_4, \quad D_3\Phi_3 = -\chi_0 i\Psi_1, \quad D_4\Phi_3 = -i\Psi_2$$

$$D_1\Phi_4 = i\Psi_4, \quad D_2\Phi_4 = i\Psi_3, \quad D_3\Phi_4 = -\chi_0 i\Psi_2, \quad D_4\Phi_4 = i\Psi_1$$

$$D_1\Psi_1 = \dot{\Phi}_1, \quad D_2\Psi_1 = -\dot{\Phi}_2, \quad D_3\Psi_1 = -\chi_0\dot{\Phi}_3, \quad D_4\Psi_1 = \dot{\Phi}_4$$

$$D_1\Psi_2 = \dot{\Phi}_2, \quad D_2\Psi_2 = \dot{\Phi}_1, \quad D_3\Psi_2 = -\chi_0\dot{\Phi}_4, \quad D_4\Psi_2 = -\dot{\Phi}_3$$

$$D_1\Psi_3 = \dot{\Phi}_3, \quad D_2\Psi_3 = \dot{\Phi}_4, \quad D_3\Psi_3 = \chi_0\dot{\Phi}_1, \quad D_4\Psi_3 = \dot{\Phi}_2$$

$$D_1\Psi_4 = \dot{\Phi}_4, \quad D_2\Psi_4 = -\dot{\Phi}_3, \quad D_3\Psi_4 = \chi_0\dot{\Phi}_2, \quad D_4\Psi_4 = -\dot{\Phi}_1$$

$$\mathrm{D_I}\Phi_i = i(\mathrm{L_I})_i{}^{\hat{j}}\Psi_{\hat{j}}, \quad \mathrm{D_I}\Psi_{\hat{j}} = (\mathrm{R_I})_{\hat{j}}{}^{i}\dot{\Phi}_i,$$

$$(\mathrm{L_I})_i{}^{\hat{j}}(\mathrm{R_J})_{\hat{j}}{}^{k} + (\mathrm{L_J})_i{}^{\hat{j}}(\mathrm{R_I})_{\hat{j}}{}^{k} = 2\delta_{\mathrm{IJ}}\delta_i{}^{k},$$

$$(\mathrm{R_I})_{\hat{j}}{}^{i}(\mathrm{L_J})_i{}^{\hat{k}} + (\mathrm{R_J})_{\hat{j}}{}^{i}(\mathrm{L_I})_i{}^{\hat{k}} = 2\delta_{\mathrm{IJ}}\delta_{\hat{j}}{}^{\hat{k}}.$$

$$D_I \Phi_i = i(L_I)_i{}^{\hat{j}} \Psi_{\hat{j}}, \quad D_I \Psi_{\hat{j}} = (R_I)_{\hat{j}}{}^{i} \dot{\Phi}_i,$$

These equations show why there appears to be no polytopic basis for SUSY representation theory. As

$\Phi_i \quad \Psi_{\hat{j}}$ analogous to $\mathbf{v}$

D_I analogous to A

there is no obvious way to introduce eigenvalues!

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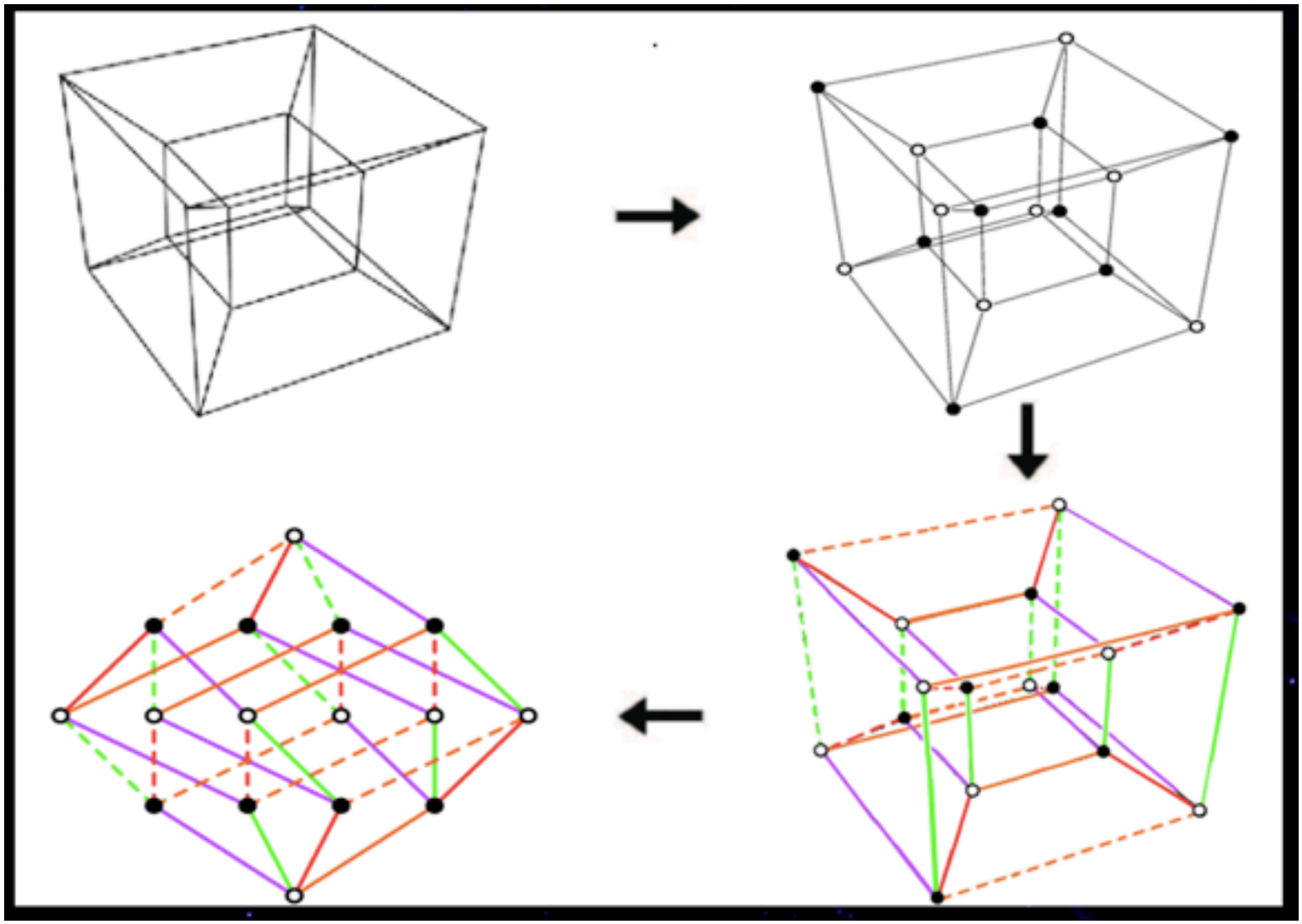
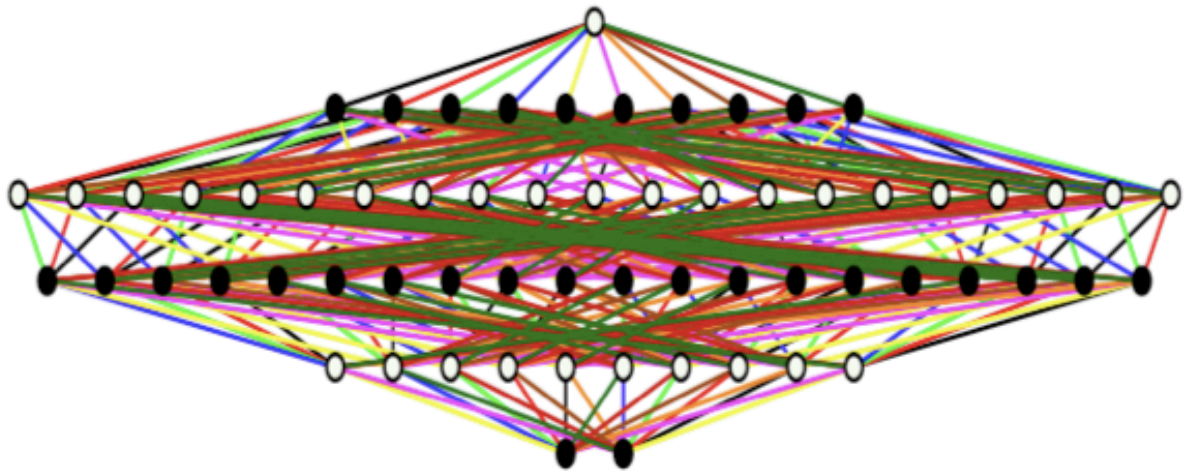
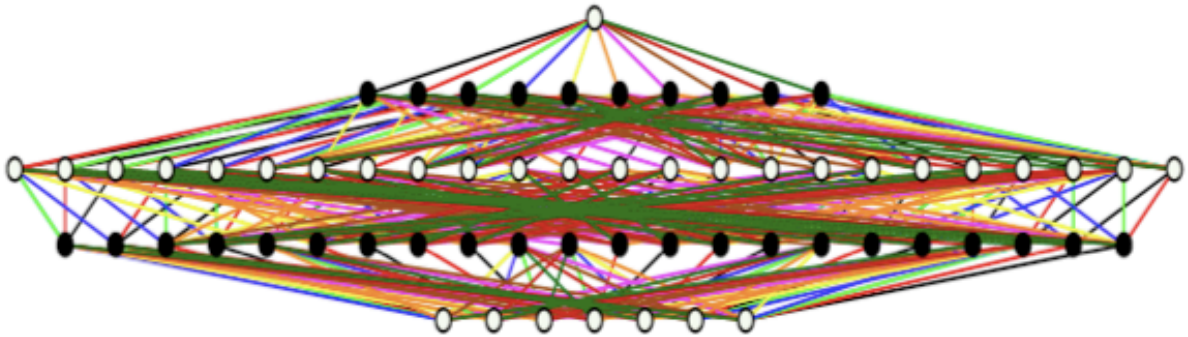


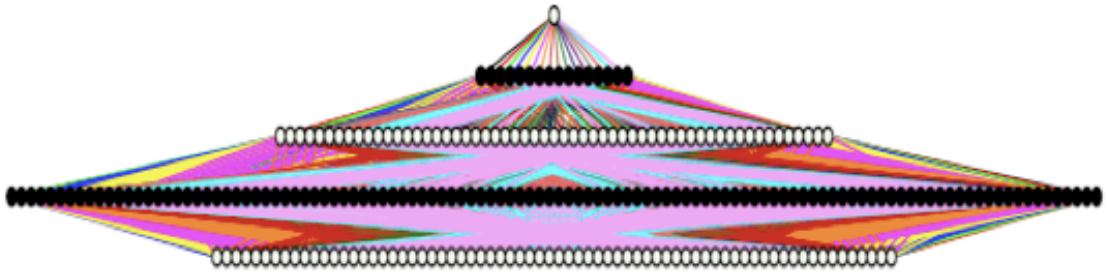
D_{10} :



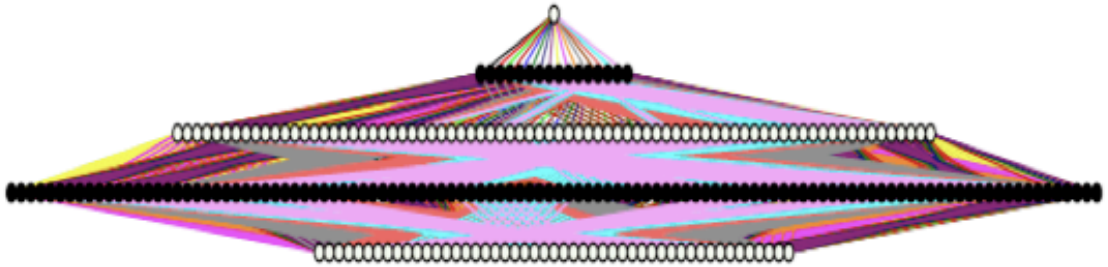
$E_8 \times I^2$:

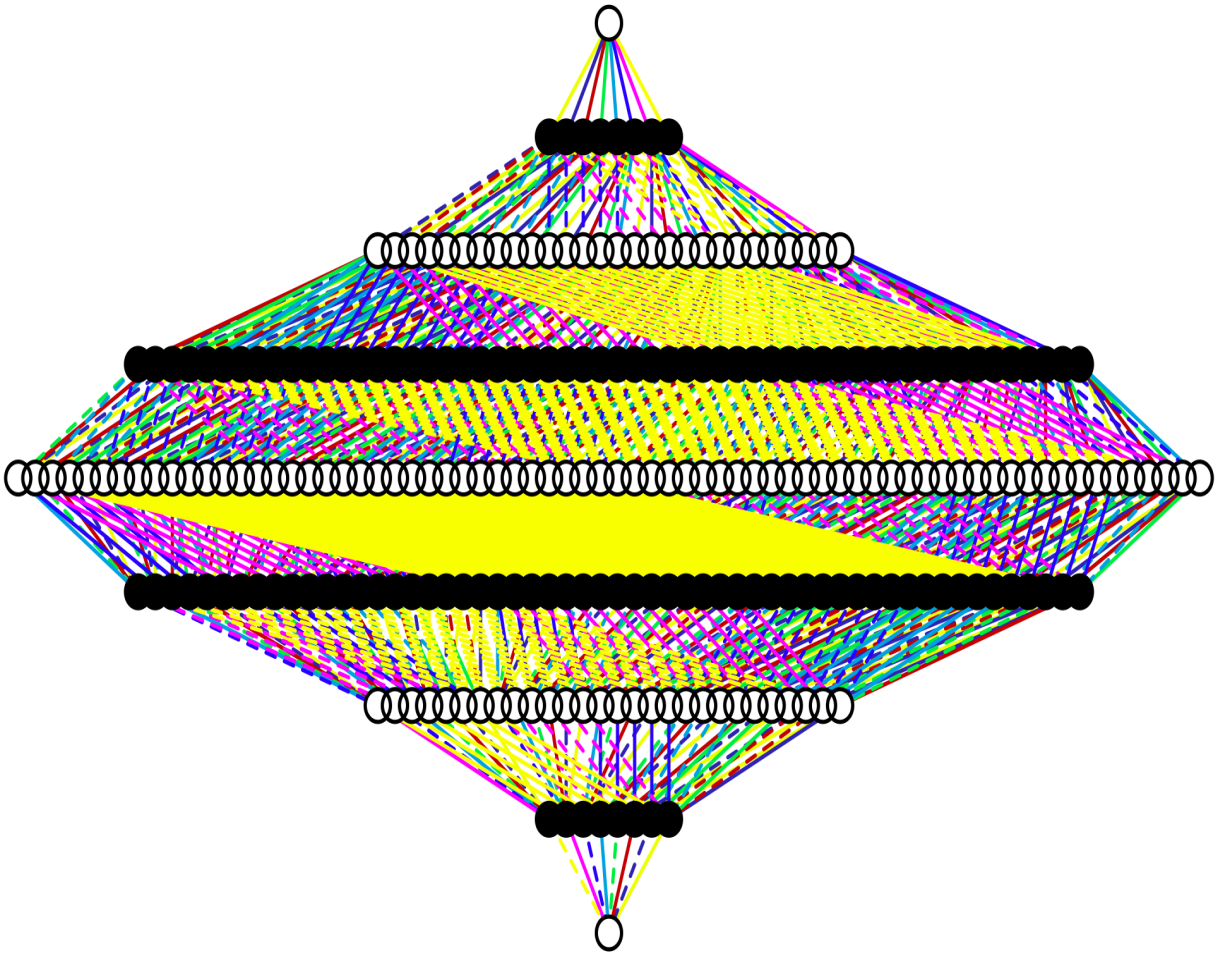


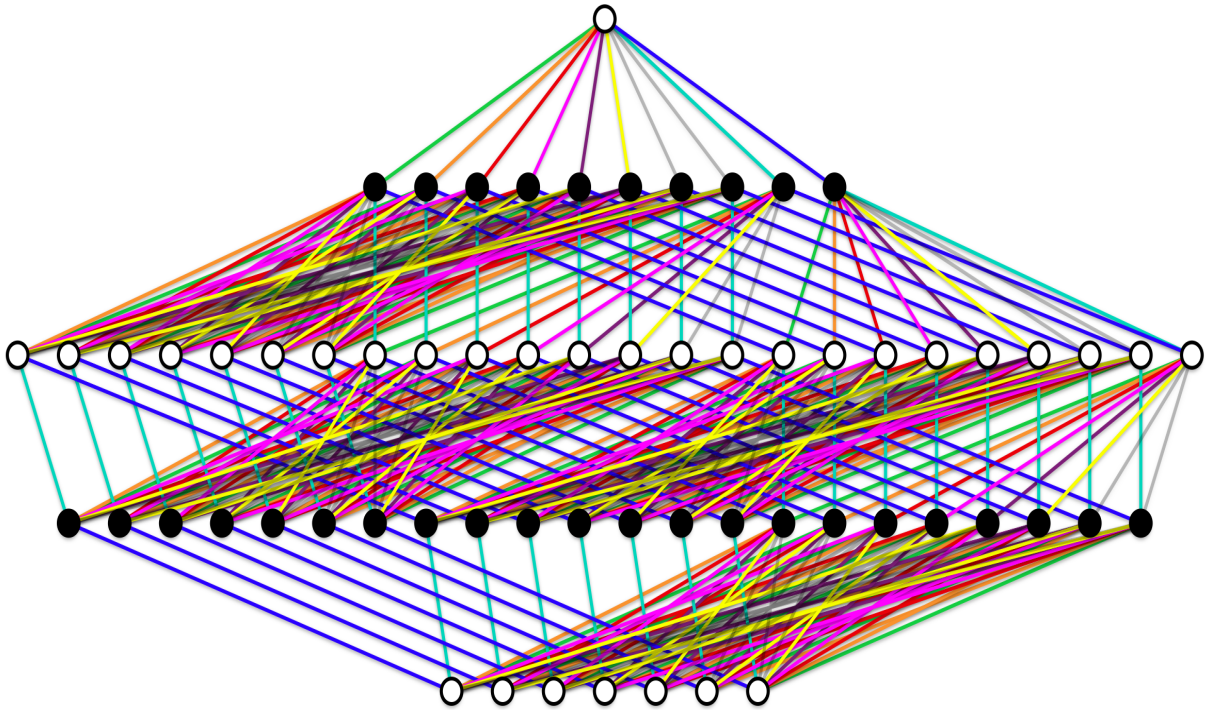
E_{16} :

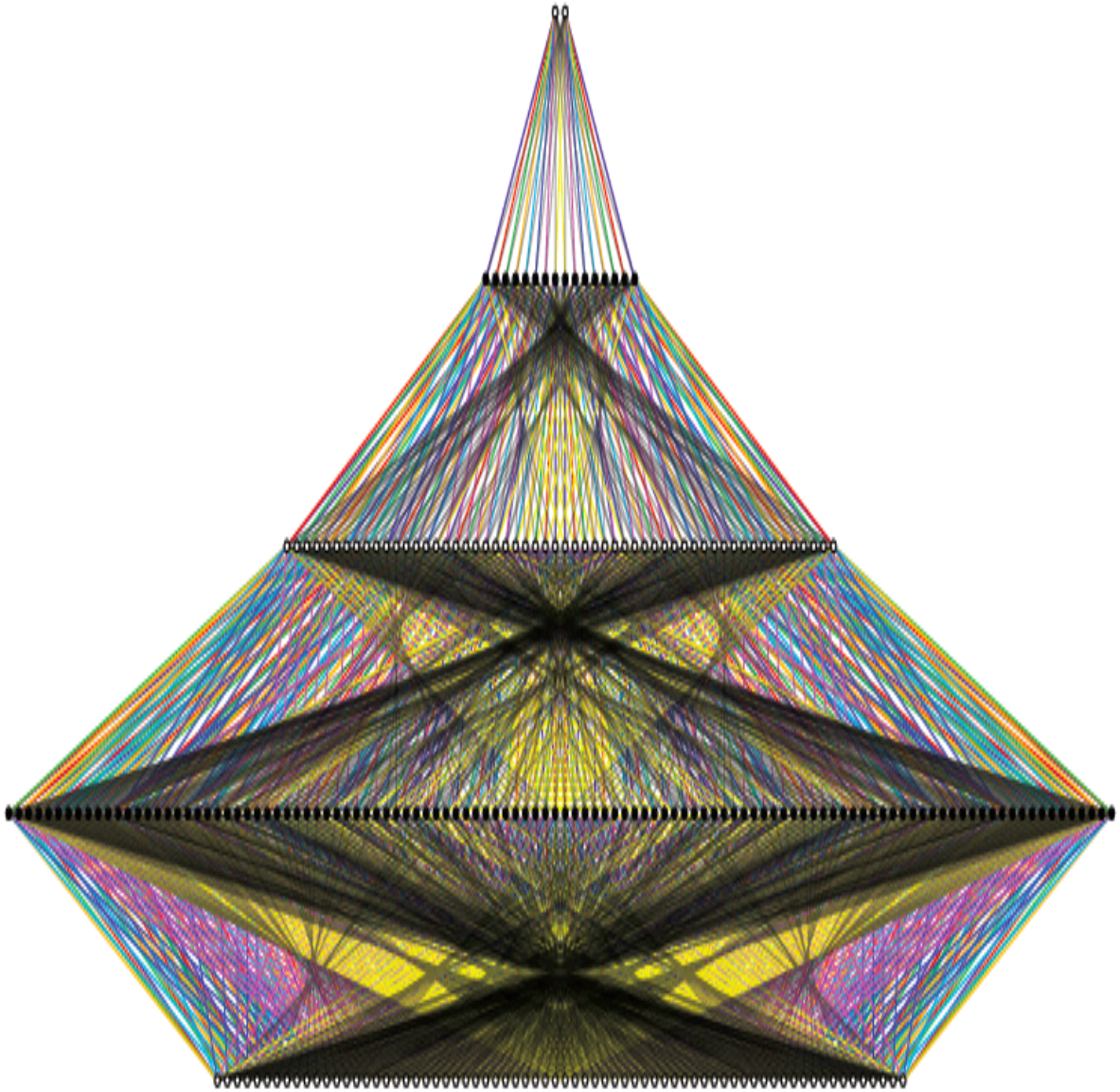


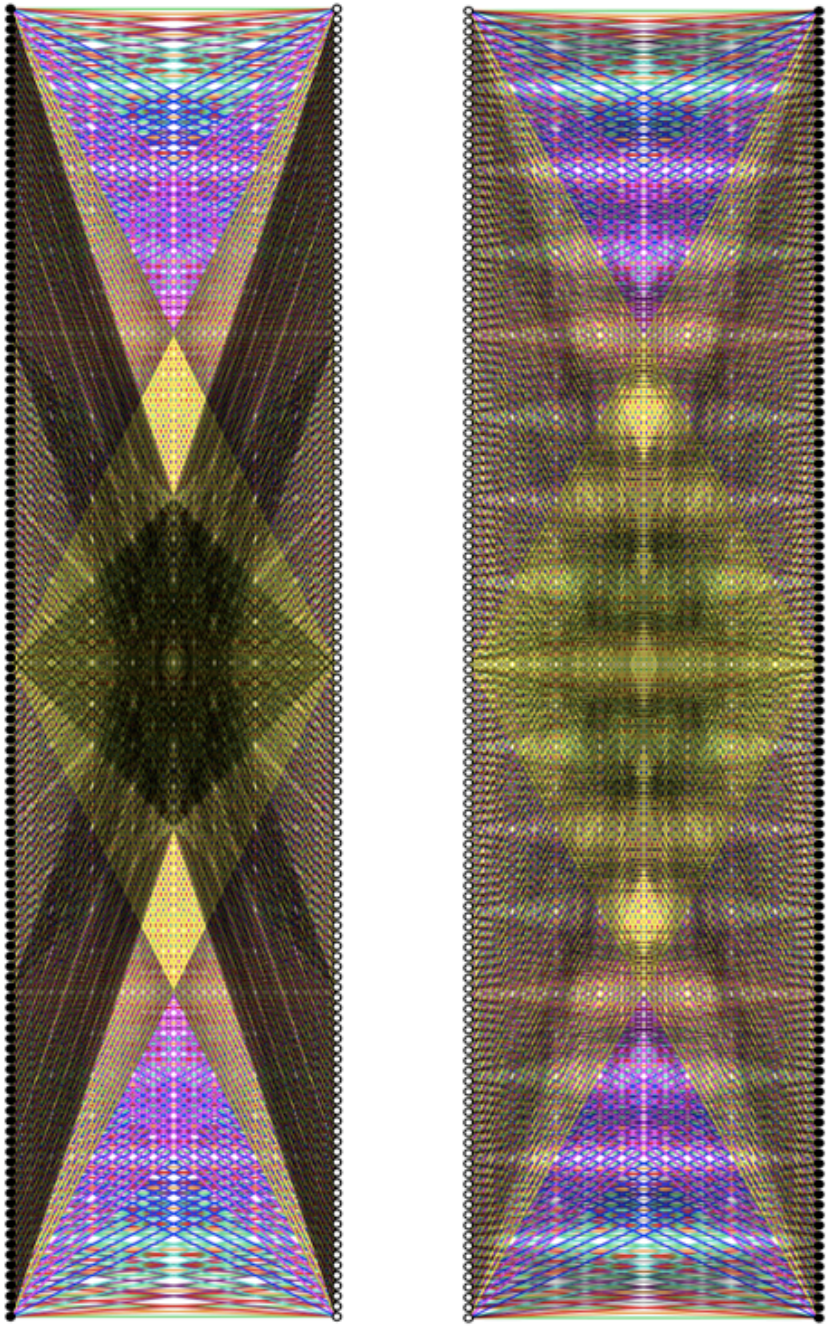
$E_8 \times E_8$:











A Closer Look at L-matrices & R-matrices

The Coxeter Group BC(4)

Restricting our further considerations only to such signed permutation L-matrix solutions of the system (2.4), we note that all such matrices factorize

$$(\mathrm{L}_{\mathrm{I}})_i^{\ \hat{k}} \;=\; (\mathcal{S}^{(\mathrm{I})})_i^{\ \hat{\ell}}\,(\mathcal{P}_{(\mathrm{I})})_{\hat{\ell}}^{\ \hat{k}}, \qquad \text{for each fixed } \mathrm{I} = 1, 2, \ldots, N.$$

Here, the sign-matrix $\mathcal{S}^{(\mathrm{I})}$ is a diagonal $\mathrm{d} \times \mathrm{d}$ matrix with only ± 1 entries on the diagonal, and each $\mathcal{P}_{(\mathrm{I})}$ is a matrix representation of a permutation of d objects.

$$(\mathcal{S}^{(\mathrm{I})})_i^{\ \hat{\ell}} = \begin{bmatrix} (-1)^{b_1} & 0 & 0 & \cdots \\ 0 & (-1)^{b_2} & 0 & \cdots \\ 0 & 0 & (-1)^{b_3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad \leftrightarrow \quad \left(\boldsymbol{\mathcal{R}}_{\mathrm{I}} = \sum_{i=1}^{\mathrm{d}} b_i\, 2^{i-1}\right)_b \;=\; \underset{\substack{\text{(binary "word")} \\ \text{(reversed)}}}{[b_1 b_2 \cdots b_{\mathrm{d}}]_2}$$

$$(\mathrm{L}_1)_i^{\ \hat{k}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = (10)_b\langle 1423\rangle = \langle 1\bar{4}2\bar{3}\rangle \ ;$$

$$(\mathrm{L}_2)_i^{\ \hat{k}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = (12)_b\langle 2314\rangle = \langle 23\bar{1}\bar{4}\rangle \ ;$$

$$(\mathrm{L}_3)_i^{\ \hat{k}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = (6)_b\langle 3241\rangle \ = \langle 3\bar{2}\bar{4}1\rangle \ ;$$

$$(\mathrm{L}_4)_i^{\ \hat{k}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = (0)_b\langle 4132\rangle \ = \langle 4132\rangle \ .$$

$$\{CM\} \equiv \{<1423>, <2314>, <3241>, <4132>\}$$

$$\{VM\} \equiv \{<2413>, <1324>, <3142>, <4231>\}$$

$$\{TM\} \equiv \{<1342>, <2431>, <3124>, <4213>\}$$

$$\{VM_1\} \equiv \{<1432>, <2341>, <3214>, <4123>\}$$

$$\{VM_2\} \equiv \{<1243>, <2134>, <3421>, <4312>\}$$

$$\{VM_3\} \equiv \{<1234>, <2143>, <3412>, <4321>\}$$

Four Examples of Symbolic Permutation Notation

$$\begin{matrix}\langle 1234\rangle \\ ()\end{matrix} \Rightarrow \begin{matrix}\langle 1234\rangle \\ \langle 1234\rangle\end{matrix} \Rightarrow 1\to 1,\ 2\to 2,\ 3\to 3,\ 4\to 4,$$

$$\begin{matrix}\langle 1324\rangle \\ (23)\end{matrix} \Rightarrow \begin{matrix}\langle 1234\rangle \\ \langle 1324\rangle\end{matrix} \Rightarrow 1\to 1,\ 2\to 3,\ 3\to 2,\ 4\to 4,$$

$$\begin{matrix}\langle 1342\rangle \\ (234)\end{matrix} \Rightarrow \begin{matrix}\langle 1234\rangle \\ \langle 1342\rangle\end{matrix} \Rightarrow 1\to 1,\ 2\to 3,\ 3\to 4,\ 4\to 2,$$

$$\begin{matrix}\langle 2143\rangle \\ (12)(34)\end{matrix} \Rightarrow \begin{matrix}\langle 1234\rangle \\ \langle 2143\rangle\end{matrix} \Rightarrow 1\to 2,\ 2\to 1,\ 3\to 4,\ 4\to 3,$$

$$\{\mathbb{S}_4\} = \{\mathcal{P}_{[1]}\} \cup \{\mathcal{P}_{[2]}\} \cup \{\mathcal{P}_{[3]}\} \cup \{\mathcal{P}_{[4]}\} \cup \{\mathcal{P}_{[5]}\} \cup \{\mathcal{P}_{[6]}\} \quad ,$$

$$\{\mathcal{P}_{[1]}\} \equiv \{<1423>, <2314>, <3241>, <4132>\}$$

$$\{\mathcal{P}_{[2]}\} \equiv \{<1342>, <2431>, <3124>, <4213>\}$$

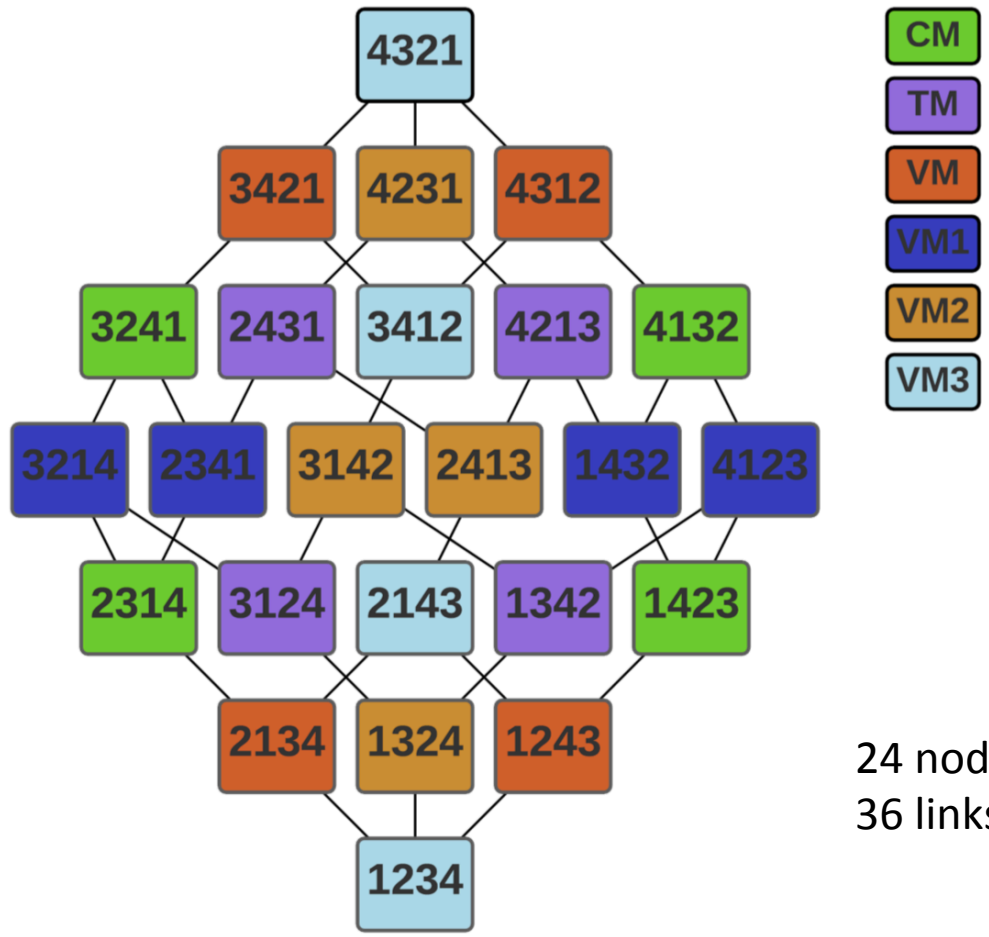
$$\{\mathcal{P}_{[3]}\} \equiv \{<2413>, <1324>, <3142>, <4231>\}$$

$$\{\mathcal{P}_{[4]}\} \equiv \{<1432>, <2341>, <3214>, <4123>\}$$

$$\{\mathcal{P}_{[5]}\} \equiv \{<1243>, <2134>, <3421>, <4312>\}$$

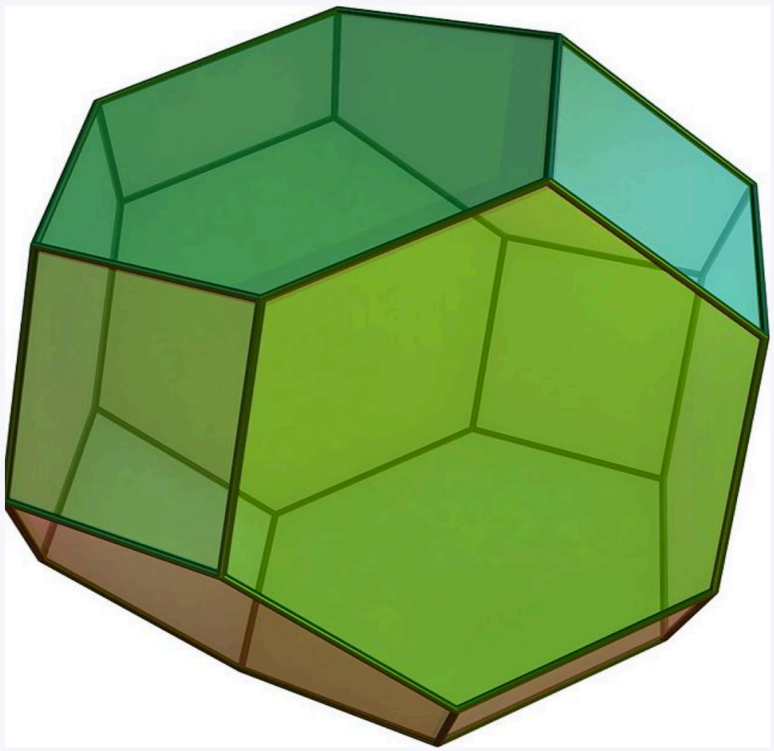
$$\{\mathcal{P}_{[6]}\} \equiv \{<1234>, <2143>, <3412>, <4321>\}$$

Bruhat Weak Ordering in $\mathbb{BC}_4$

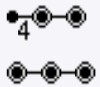


24 nodes
36 links

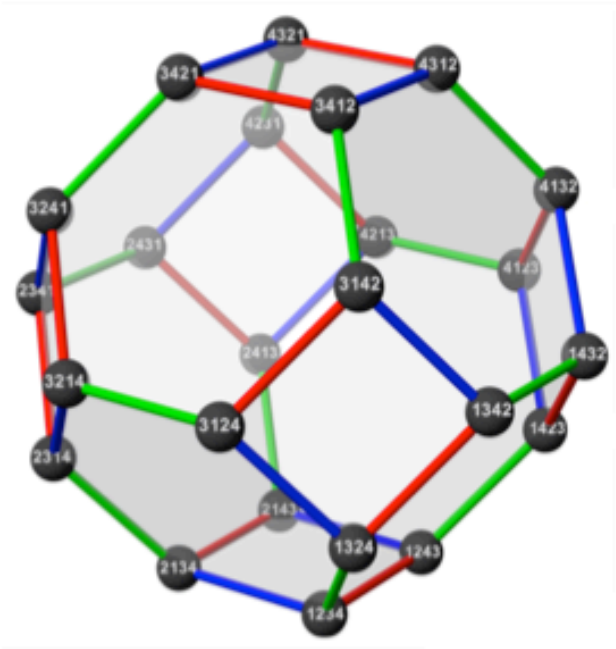
Truncated octahedron



Type	Archimedean solid Uniform polyhedron
Elements	$F = 14$, $E = 36$, $V = 24$ ($\chi = 2$)

Faces by sides	6{4}+8{6}
Conway notation	tO bT
Schläfli symbols	t{3,4} tr{3,3} or $t\begin{Bmatrix} 3 \\ 3 \end{Bmatrix}$
	$t_{0,1}\{3,4\}$ or $t_{0,1,2}\{3,3\}$
Wythoff symbol	2 4 | 3 3 3 2 |
Coxeter diagram	
Symmetry group	O_h, B_3, [4,3], (*432), order 48 T_h, [3,3] and (*332), order 24
Rotation group	O, $[4,3]^+$, (432), order 24
Dihedral angle	4-6: $\arccos(-\frac{1}{\sqrt{3}}) = 125°15'51''$ 6-6: $\arccos(-\frac{1}{3}) = 109°28'16''$

$$A = (6 + 12\sqrt{3})\, a^2 \qquad \approx 26.784\,6097 a^2$$
$$V = 8\sqrt{2} a^3 \qquad \approx 11.313\,7085 a^3 .$$



Permutahedron

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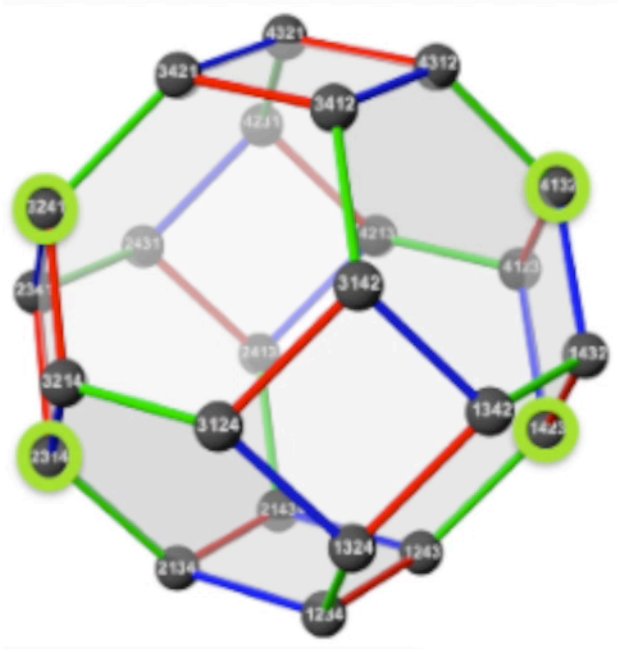

Permutahedron & CM Elements

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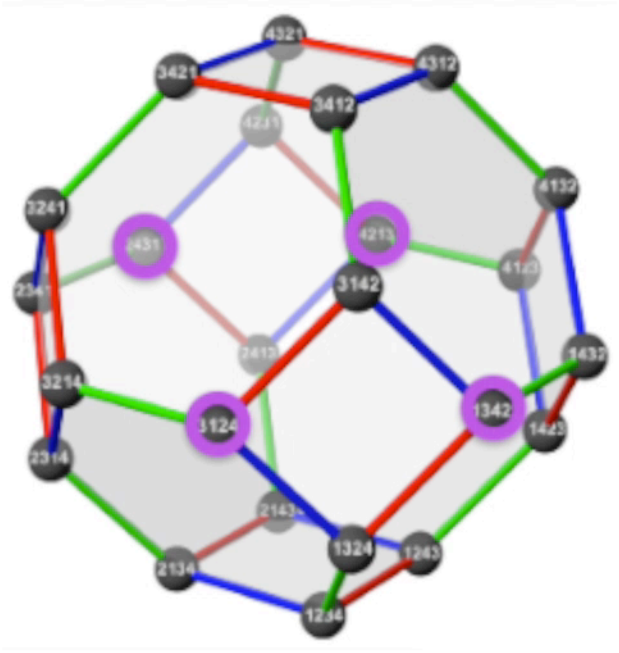

Permutahedron & TM Elements

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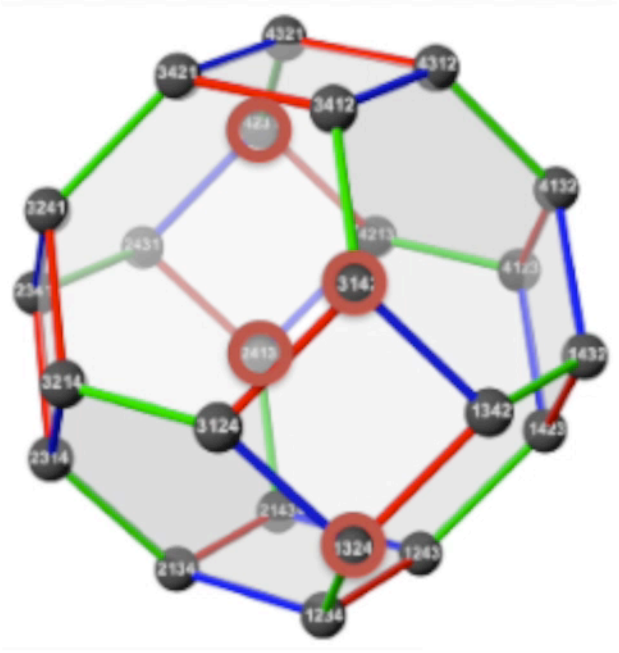

Permutahedron & VM Elements

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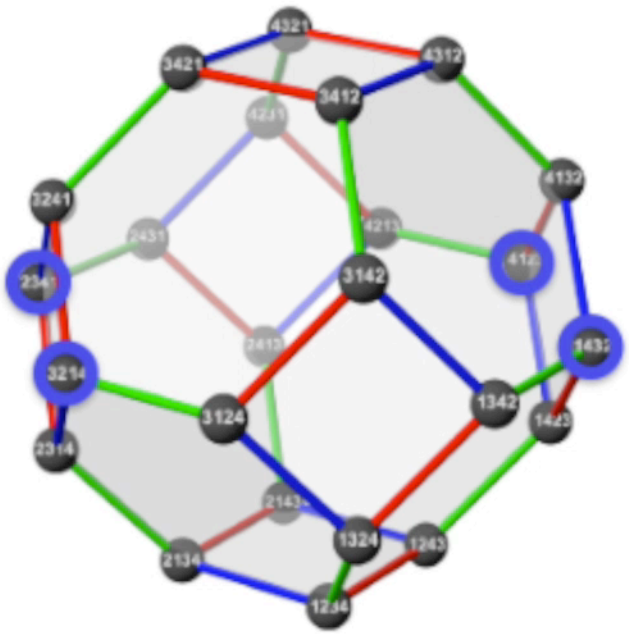

Permutahedron & VM_1 Elements

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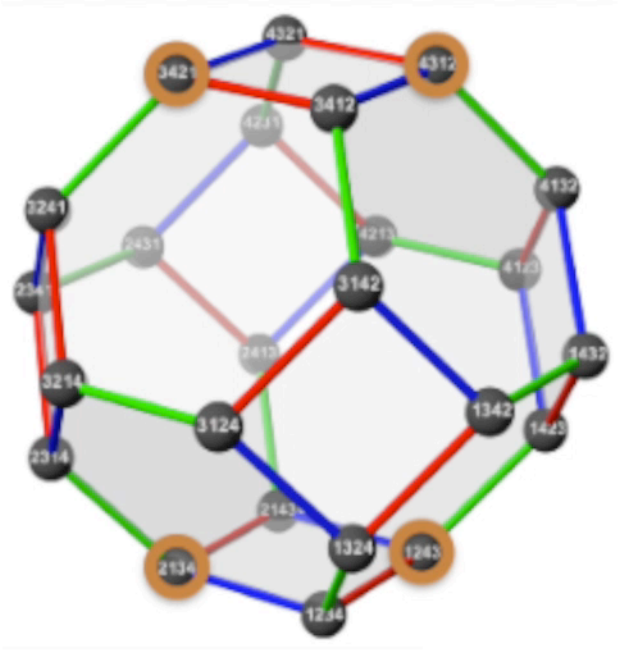

Permutahedron & VM_2 Elements

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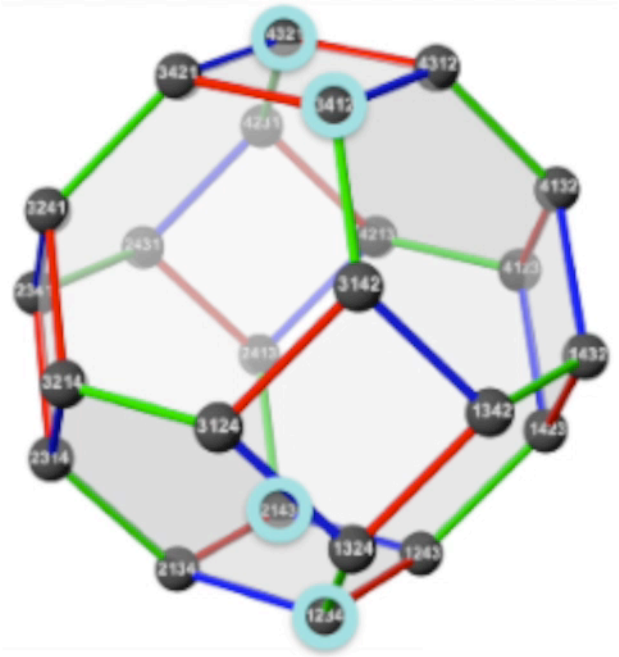

Permutahedron & VM_3 Elements

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Teaching Apps To 'See' Using Eigenvalues & Traces

The 300 "Correlators" of the $\mathbb{S}_4$ Permutahedron

$$\mathcal{A}_{\ell x}\left[\mathcal{P}_{[a|A]},\, \mathcal{P}_{[b|B]}\right]$$

For this purpose, there is introduced a function denoted by $\mathcal{A}_{\ell x}[\mathcal{P}_{[a|A]}, \mathcal{P}_{[b|B]}]$ that we will call a "two-point correlator." This function assigns a number of the minimal Bruhat distance between the elements $\mathcal{P}_{[a|A]}$ and $\mathcal{P}_{[b|B]}$ contained in $\mathbb{S}_4$. This means this is a symmetric 24×24 matrix with 300 possible entries. The symbol $\mathcal{P}_{[a|A]}$ is meant to denote the a-th element in the dissected subset A. If the two subsets are such that $A = B$, we will call these "intra-quartet correlators", and If the two subsets are such that $A \neq B$, we then call these "inter-quartet correlators."

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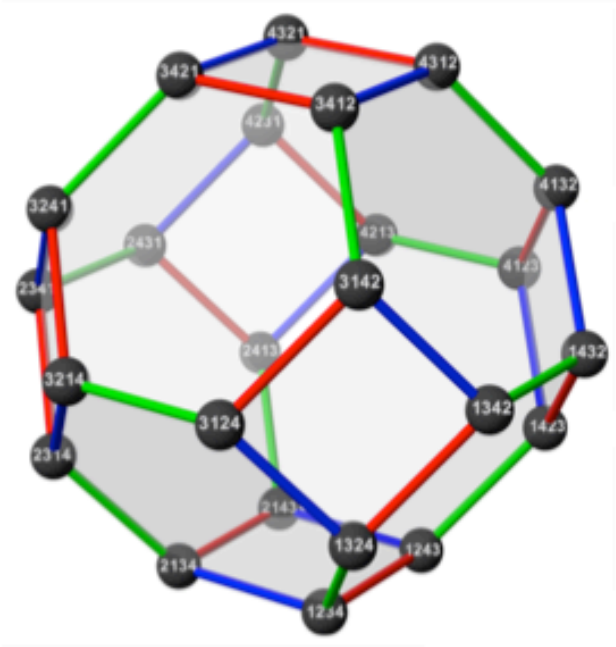


Permutahedron & Correlator Calculations:
Counting Links

The 300 "Correlators" of the $\mathbb{S}_4$ Permutahedron

$$\mathcal{A}_{\ell x}\left[\mathcal{P}_{[a|A]},\ \mathcal{P}_{[b|B]}\right]$$

Eigenvalues \ Traces	4	8	12
$(12,0,0,0)$	-	-	$(\mathcal{P}_{[1]}\|\mathcal{P}_{[3]})(\mathcal{P}_{[2]}\|\mathcal{P}_{[5]})$ $(\mathcal{P}_{[4]}\|\mathcal{P}_{[6]})$
$(12,-4,0,0)$	$(\mathcal{P}_{[2]}\|\mathcal{P}_{[3]})$	$(\mathcal{P}_{[1]}\|\mathcal{P}_{[2]})(\mathcal{P}_{[1]}\|\mathcal{P}_{[6]})$ $(\mathcal{P}_{[2]}\|\mathcal{P}_{[6]})(\mathcal{P}_{[3]}\|\mathcal{P}_{[4]})$ $(\mathcal{P}_{[3]}\|\mathcal{P}_{[5]})(\mathcal{P}_{[4]}\|\mathcal{P}_{[5]})$	-
$(12,-4,-4,0)$	$(\mathcal{P}_{[1]}\|\mathcal{P}_{[5]})(\mathcal{P}_{[2]}\|\mathcal{P}_{[4]})$ $(\mathcal{P}_{[3]}\|\mathcal{P}_{[6]})$	-	-
$(12,-8,0,0)$	$(\mathcal{P}_{[1]}\|\mathcal{P}_{[4]})(\mathcal{P}_{[5]}\|\mathcal{P}_{[6]})$	-	-

The 300 "Correlators" of the $\mathbb{S}_4$ Permutahedron

$\mathcal{P}_{[1]}$ \ $\mathcal{P}_{[1]}$	$\langle 1423 \rangle$	$\langle 2314 \rangle$	$\langle 3241 \rangle$	$\langle 4132 \rangle$
$\langle 1423 \rangle$	0	4	6	2
$\langle 2314 \rangle$	4	0	2	6
$\langle 3241 \rangle$	6	2	0	4
$\langle 4132 \rangle$	2	6	4	0

Table 1: $\{CM\} - \{CM\}$ Two – Point Correlator Values

$\mathcal{P}_{[2]}$ \ $\mathcal{P}_{[2]}$	$\langle 1342 \rangle$	$\langle 2431 \rangle$	$\langle 3124 \rangle$	$\langle 4213 \rangle$
$\langle 1342 \rangle$	0	6	2	4
$\langle 2431 \rangle$	6	0	4	2
$\langle 3124 \rangle$	2	4	0	6
$\langle 4213 \rangle$	4	2	6	0

Table 2: $\{TM\} - \{TM\}$ Two – Point Correlator Values

The 300 "Correlators" of the $\mathbb{S}_4$ Permutahedron

$\mathcal{P}_{[3]}$ \ $\mathcal{P}_{[3]}$	$\langle 1324 \rangle$	$\langle 2413 \rangle$	$\langle 3142 \rangle$	$\langle 4231 \rangle$
$\langle 1324 \rangle$	0	4	2	6
$\langle 2413 \rangle$	4	0	6	2
$\langle 3142 \rangle$	2	6	0	4
$\langle 4231 \rangle$	6	2	4	0

Table 1: $\{CM\} - \{CM\}$ Two – Point Correlator Values

$\mathcal{P}_{[2]}$ \ $\mathcal{P}_{[2]}$	$\langle 1342 \rangle$	$\langle 2431 \rangle$	$\langle 3124 \rangle$	$\langle 4213 \rangle$
$\langle 1342 \rangle$	0	6	2	4
$\langle 2431 \rangle$	6	0	4	2
$\langle 3124 \rangle$	2	4	0	6
$\langle 4213 \rangle$	4	2	6	0

Table 2: $\{TM\} - \{TM\}$ Two – Point Correlator Values

The 300 "Correlators" of the $\mathbb{S}_4$ Permutahedron

$\mathcal{P}_{[3]}$ \ $\mathcal{P}_{[3]}$	$\langle 1324 \rangle$	$\langle 2413 \rangle$	$\langle 3142 \rangle$	$\langle 4231 \rangle$
$\langle 1324 \rangle$	0	4	2	6
$\langle 2413 \rangle$	4	0	6	2
$\langle 3142 \rangle$	2	6	0	4
$\langle 4231 \rangle$	6	2	4	0

Table 3: $\{VM\} - \{VM\}$ Two – Point Correlator Values

$\mathcal{P}_{[4]}$ \ $\mathcal{P}_{[4]}$	$\langle 1432 \rangle$	$\langle 2341 \rangle$	$\langle 3214 \rangle$	$\langle 4123 \rangle$
$\langle 1432 \rangle$	0	6	4	2
$\langle 2341 \rangle$	6	0	2	4
$\langle 3214 \rangle$	4	2	0	6
$\langle 4123 \rangle$	2	4	6	0

Table 4: $\{VM\}_1 - \{VM\}_1$ Two – Point Correlator Values

The 300 "Correlators" of the $\mathbb{S}_4$ Permutahedron

$\mathcal{P}_{[5]}$ \ $\mathcal{P}_{[5]}$	$\langle 1243 \rangle$	$\langle 2134 \rangle$	$\langle 3421 \rangle$	$\langle 4312 \rangle$
$\langle 1243 \rangle$	0	2	6	4
$\langle 2134 \rangle$	2	0	4	6
$\langle 3421 \rangle$	6	4	0	2
$\langle 4312 \rangle$	4	6	2	0

Table 5: $\{VM\}_2 - \{VM\}_2$ Two – Point Correlator Values

$\mathcal{P}_{[6]}$ \ $\mathcal{P}_{[6]}$	$\langle 1234 \rangle$	$\langle 2143 \rangle$	$\langle 3412 \rangle$	$\langle 4321 \rangle$
$\langle 1234 \rangle$	0	2	4	6
$\langle 2143 \rangle$	2	0	6	4
$\langle 3412 \rangle$	4	6	0	2
$\langle 4321 \rangle$	6	4	2	0

Table 6: $\{VM\}_3 - \{VM\}_3$ Two – Point Correlator Values

The 300 "Correlators" of the $\mathbb{S}_4$ Permutahedron

$$\mathcal{A}_{\ell x}\left[\mathcal{P}_{[a|1]}, \mathcal{P}_{[b|1]}\right] = \begin{bmatrix} 0 & 4 & 6 & 2 \\ 4 & 0 & 2 & 6 \\ 6 & 2 & 0 & 4 \\ 2 & 6 & 4 & 0 \end{bmatrix}$$

It is clear that the trace of this matrix vanishes and a direct calculation of its eigenvalues yield $\{12, 0, -4, -8.\}$ expressed in descending order.

The 300 "Correlators" of the $\mathbb{S}_4$ Permutahedron

$$\mathcal{A}_{\ell x}\left[\mathcal{P}_{[a|A]},\ \mathcal{P}_{[b|B]}\right]$$

There are interesting symmetries of the permutahedron that lead to the results of the SUSY quartets 2-point 'correlators' forms to resemble a **Sudoku**-like game!

This is surprisingly true for the inter-quartet 'correlators' as well as for The intra-quartet 'correlators.'

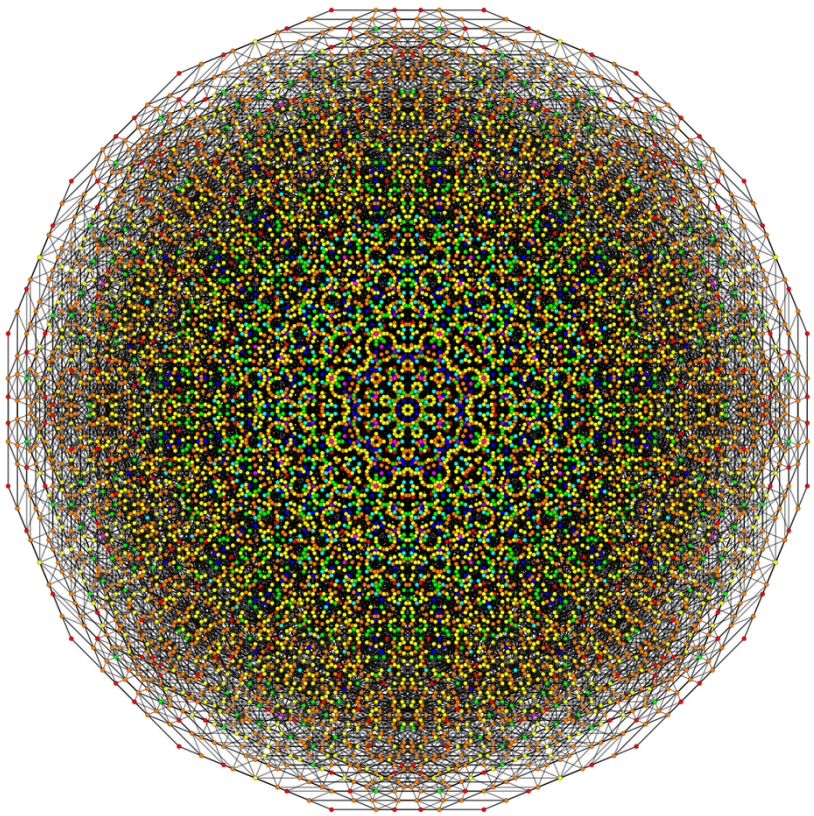
Next Stop: 4D, N = 2 SUSY & The 40,320 Nodes & 141,120 Edges Of The "Hexipentiruncitruncated 7-simplex"

Omnitruncated 7-simplex	
Type	uniform 7-polytope
Schläfli symbol	$t_{0,1,2,3,4,5,6}\{3^6\}$
Coxeter-Dynkin diagrams	
6-faces	
5-faces	
4-faces	
Cells	
Faces	
Edges	141120
Vertices	40320
Vertex figure	Irr. 6-simplex
Coxeter group	A_7, $[[3^6]]$, order 80640
Properties	convex

Hexipentiruncitruncated 7-simplex

Omnitruncated 7-simplex, also known as the

Hexipentiruncitruncated 7-simplex, aka the ***8-permutahedron***



Hexipentiruncitruncated 7-simplex

What's Teaching Got To Do With It?

arXiv.org > hep-th > arXiv:hep-th/0408004

High Energy Physics - Theory

[Submitted on 2 Aug 2004]

Adinkras: A Graphical Technology for Supersymmetric Representation Theory

Michael Faux, S. J. Gates Jr

We present a symbolic method for organizing the representation theory of one-dimensional superalgebras. This relies on special objects, which we have called adinkra symbols, which supply tangible geometric forms to the still-emerging mathematical basis underlying supersymmetry.

arXiv.org > hep-th > arXiv:2012.13308

High Energy Physics - Theory

[Submitted on 24 Dec 2020 (v1), last revised 13 Apr 2021 (this version, v6)]

The 300 "Correlators" Suggests 4D, $\mathcal{N}$ = 1 SUSY Is a Solution to a Set of Sudoku Puzzles

Aleksander J. Cianciara, S. James Gates Jr, Yangrui Hu, Renee Kirk

A conjecture is made that the weight space for 4D, $\mathcal{N}$-extended supersymmetrical representations is embedded within the permutahedra associated with permutation groups $\mathbb{S}_d$. Adinkras and Coxeter Groups associated with minimal representations of 4D, $\mathcal{N}$ = 1 supersymmetry provide evidence supporting this conjecture. It is shown the appearance of the mathematics of 4D, $\mathcal{N}$ = 1 minimal off-shell supersymmetry representations is equivalent to solving a four color problem on the truncated octahedron. This observation suggest an entirely new way to approach the off-shell SUSY auxiliary field problem based on IT algorithms probing the properties of $\mathbb{S}_d$.

arXiv.org > hep-th > arXiv:2012.14015

High Energy Physics - Theory

[Submitted on 27 Dec 2020 (v1), last revised 19 Jan 2021 (this version, v5)]

A Note On Exemplary Off-Shell Constructions Of 4D, $\mathcal{N}$ = 2 Supersymmetry Representations

Devin D. Bristow, John H. Caporaletti, Aleksander J. Cianciara, S. James Gates Jr., Delina Levine, Gabriel Yerger

We continue the search for rules that govern when off-shell 4D, $\mathcal{N}$ = 1 supermultiplets can be combined to form off-shell 4D, $\mathcal{N}$ = 2 supermultiplets. We study the $\mathbb{S}_8$ permutations and Height Yielding Matrix Numbers (HYMN) embedded within the adinkras that correspond to these putative 4D, $\mathcal{N}$ = 2 supermultiplets off-shell supermultiplets. Even though the HYMN definition was designed to distinguish between the raising and lowering of nodes in one dimensional valises supermultiplets, they are shown to accurately select out which combinations of off-shell 4D, $\mathcal{N}$ = 1 supermultiplets correspond to off-shell 4D, $\mathcal{N}$ = 2 supermultiplets. Only the combinations of the chiral + vector and chiral + tensor are found to have valises in the same class. This is consistent with the well known structure of 4D, $\mathcal{N}$ = 2 supermultiplets.

ArXiv-al Bibliographical Links

(1.) https://arxiv.org/abs/hep-th/0408004

(2.) https://arxiv.org/abs/2012.13308

(3.) https://arxiv.org/abs/2012.14015

Adinkra Links

2010 Non-technical Article On Adinkras
https://onbeing.org/blog/symbols-of-power-adinkras-and-the-nature-of-reality/

2021 Non-technical Lecture Using Adinkras Solving 40 Year Old/
4.2+ Billion Unknowns In 11D Supergravity Problem
https://www.youtube.com/watch?v=TA7h_XrDqd4

2021 Technical Lecture Using Adinkras Solving 40 Year Old/
4.2+ Billion Unknowns In 11D Supergravity Problem
https://www.youtube.com/watch?v=Qnx22BCGc9Q

2021 Gemant Award Links

https://www.eurekalert.org/news-releases/925909

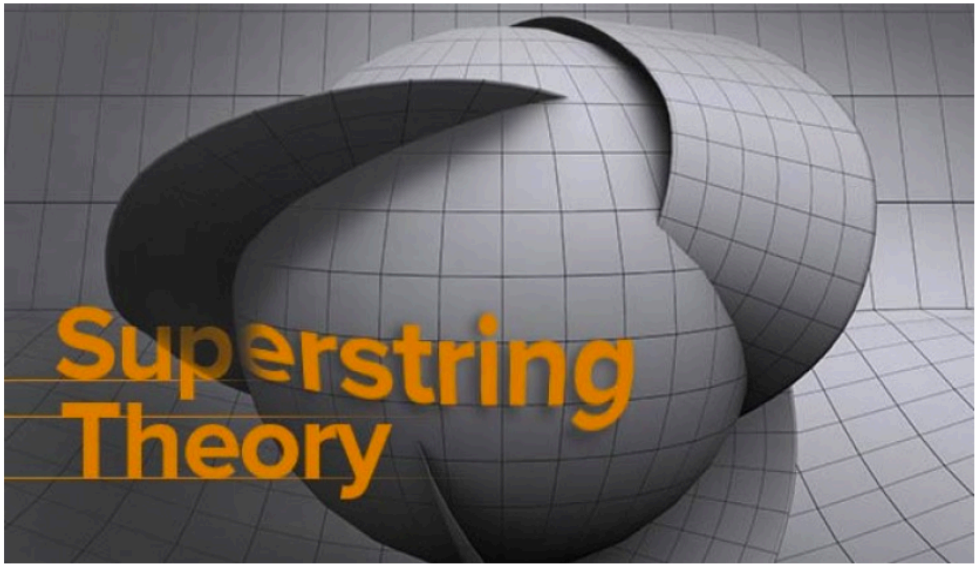
https://mailchi.mp/16cd339bca72/
nithecs-sab-member-honoured-with-2021-andrew-gemant-award

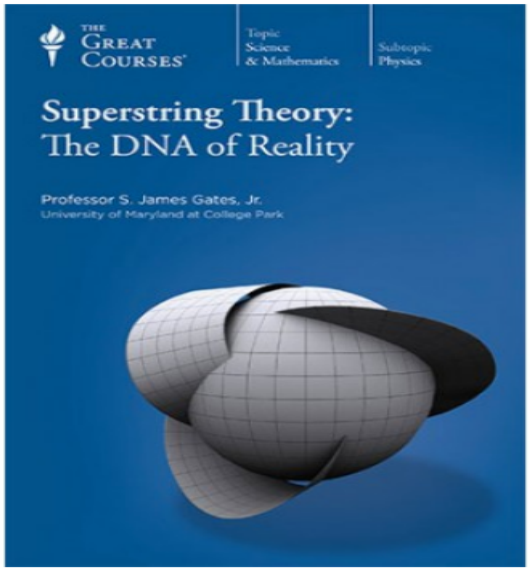
Acknowledgments

The Great Course (formerly The Teaching Company) played a role in my conceptualization of the CGI content in *Superstring Theory: The DNA of Reality* and Mr. Kenneth Griggs animated them in

http://www.teach12.com/ttcx/coursedesclong2.aspx?cid=1284

and most of the CGI media in this presentation uses these.

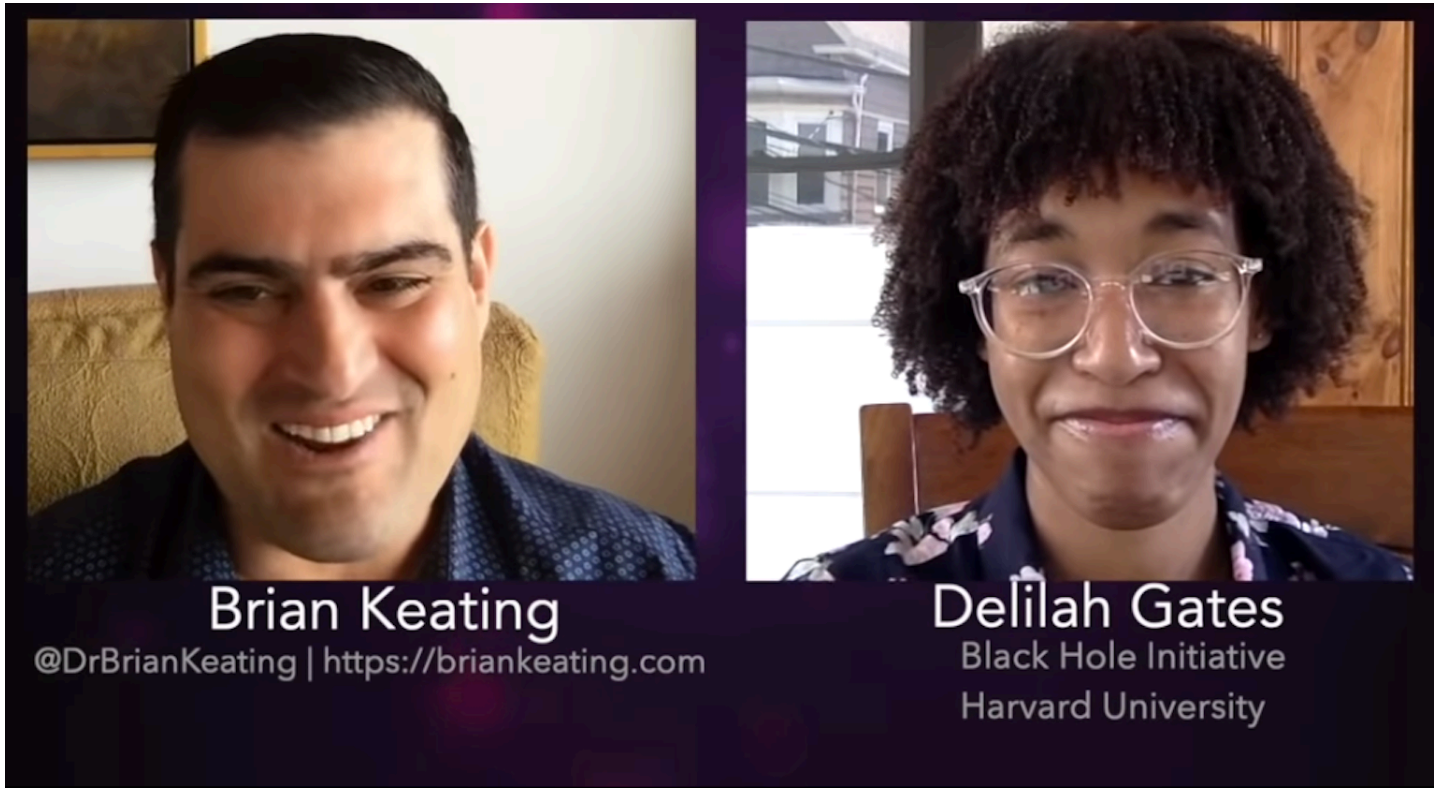




Some SSTPRS Alums

It is my pleasure to inform you that you have been selected to receive the annual Edward Dolan Award by the Department of Mathematical Sciences. This award is given to a graduate student who exhibits exceptional mathematical talent.

Some SSTPRS Alums



Delilah Gates: Black Hole Basics!

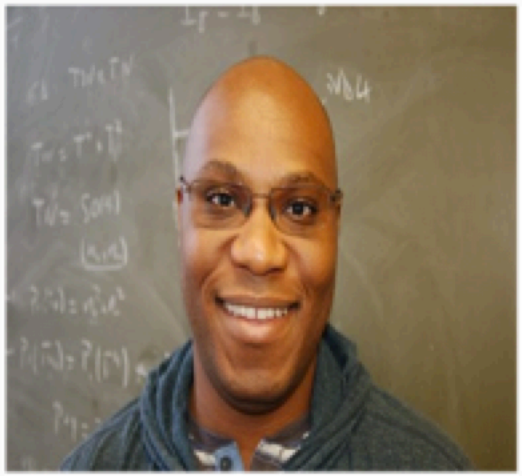
https://www.youtube.com/watch?v=HyYrV9PK8p8

Some SSTPRS Alums

JOHNS HOPKINS
KRIEGER SCHOOL
of ARTS & SCIENCES

Henry A. Rowland Department of
Physics & Astronomy

Ibrahima Bah



Assistant Professor

PhD, University of Michigan

Bloomberg 463

410-516-4122

Iboubah@jhu.edu

Some SSTPRS Alums

SSTPRS
Research Session - 2005

Standing: Xiaolong Liu, Prof. Leopoldo Pando Zayas, Prof. Vincent Rodgers, Chris Negron, Jeff Hansen, Stephen Colodner, Brislin Thomas, Osaro Harriott, Quentin A Collier, Nichole Kiefer, Stephen Gliske, Prof. Jim Gates
Seated: Ninad Jog, Ibrahima Bah, Leo Rodriguez, Nick Romano

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Some SSTPRS Alums

2016 Participants. Bottom row (l-r): Daniel Shin, Mayowa Omokanwaye, Daniel Lay, Shaina Rudman, Karina Martirosova and Isaac Friend. Middle row (l-r): Victor Meszaros, Wes Caldwell, David Kessler, Tamar Lambert-Brown, Alejandro Diaz and Forrest Guyton. Top row (l-r): Jim Gates, Mathew Calkins and Vincent Rodgers. Credit: Faye Levine

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Some SSTPRS Alums

Quantamagazine

Her Key to Modeling Brains: Ignore the Right Details

https://www.quantamagazine.org/mathematician-carina-curto-thinks-like-a-physicist-to-solve-neuroscience-problems-20180619/

SSTPRS Links

https://cmns.umd.edu/news-events/features/3598

https://www.brown.edu/news/2018-07-12/bawabe

https://www.aapt.org/Membership/spotlight_july2020.cfm

https://btpc.brown.edu/2020/08/05/a-virtual-spin-on-a-summer-program-connects-students-from-afar/

https://sites.brown.edu/sjgates/sstprs/

Acknowledgments

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https://sites.brown.edu/sjgates/sstprs/

undergraduate research interns.

Finally, I acknowledge S. J. Gates, III for technical assistance in preparation of the presentation deck.

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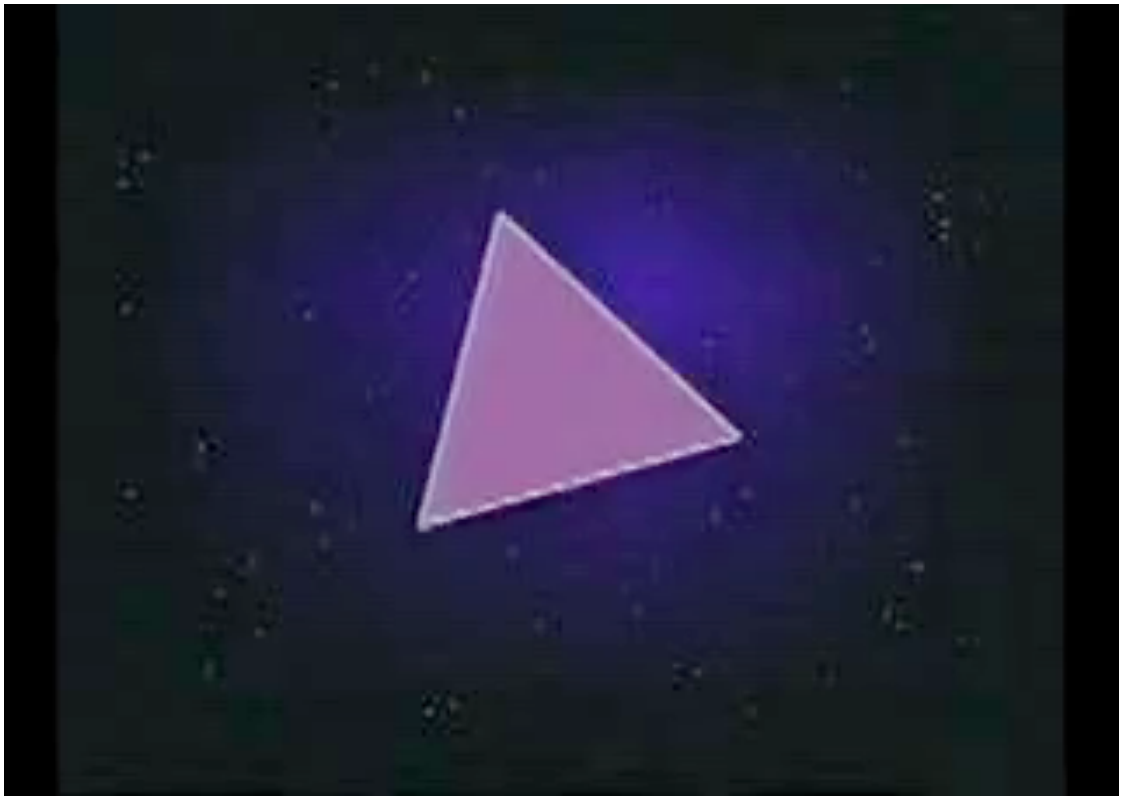
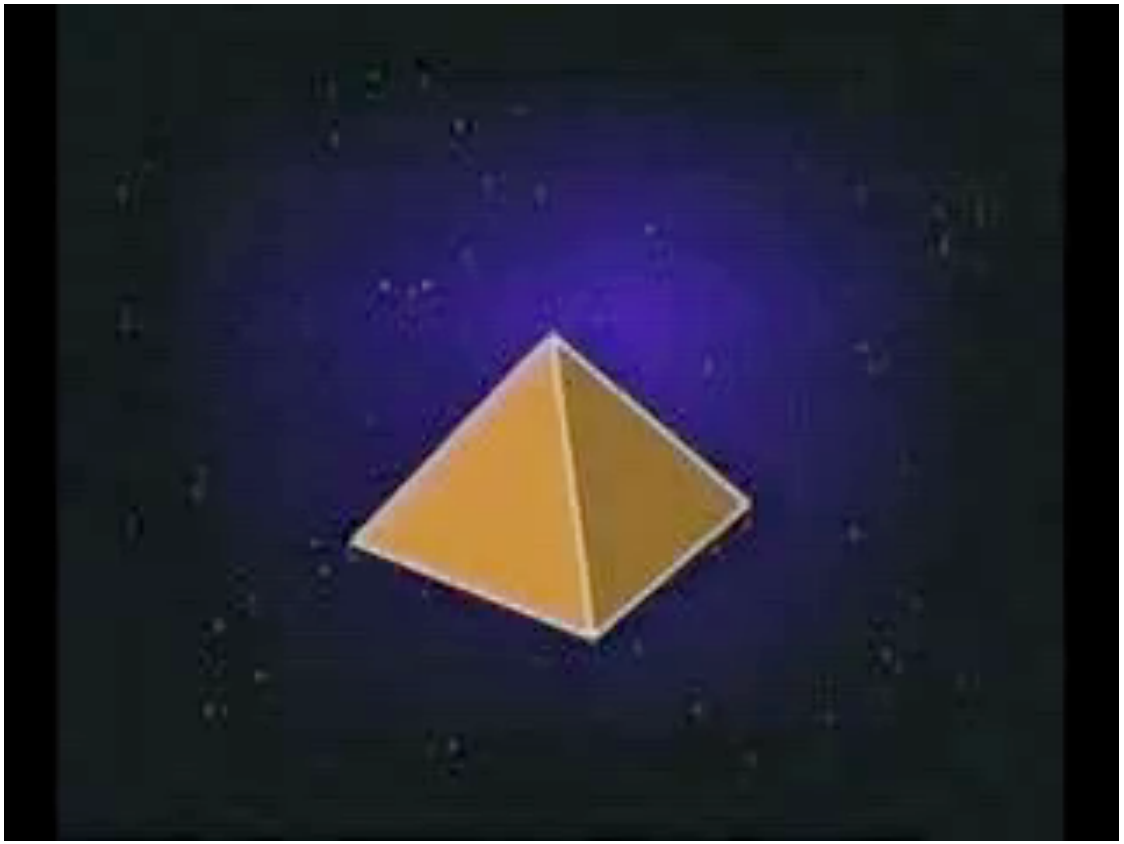


Diamond: 3 Tetrahedra/Unit Cell



Constructing The Octahedron

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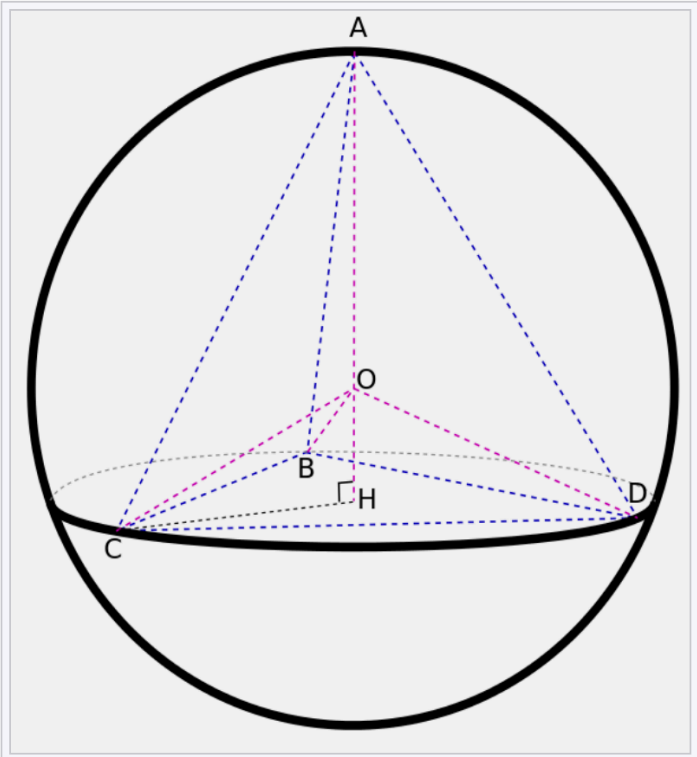
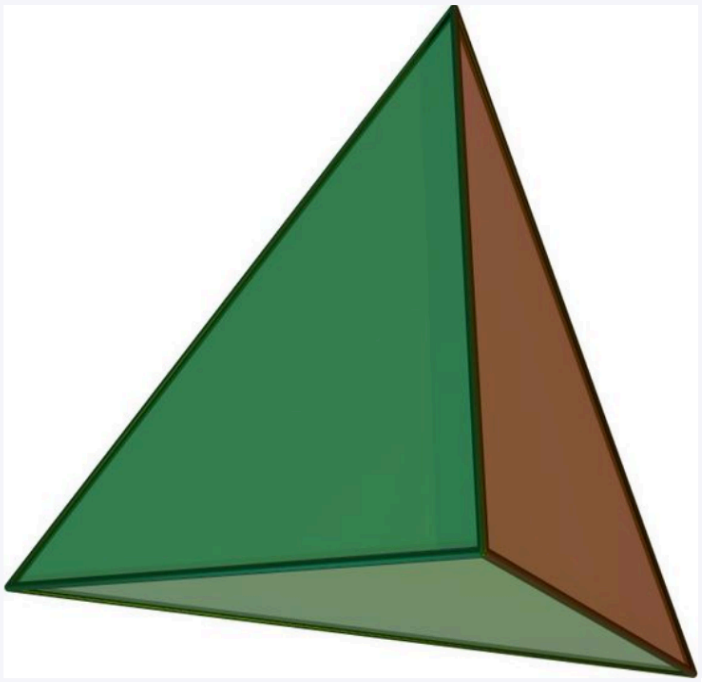
Constructing The Octahedron &
Its Truncation



Face area	$A_0 = \frac{\sqrt{3}}{4}a^2$
Surface area[3]	$A = 4\,A_0 = \sqrt{3}a^2$
Height of pyramid[4]	$h = \frac{\sqrt{6}}{3}a = \sqrt{\frac{2}{3}}\,a$
Centroid to vertex distance	$\frac{3}{4}\,h = \frac{\sqrt{6}}{4}\,a = \sqrt{\frac{3}{8}}\,a$
Edge to opposite edge distance	$l = \frac{1}{\sqrt{2}}\,a$
Volume[3]	$V = \frac{1}{3}A_0 h = \frac{\sqrt{2}}{12}a^3 = \frac{a^3}{6\sqrt{2}}$
Face-vertex-edge angle	$\arccos\left(\frac{1}{\sqrt{3}}\right) = \arctan\left(\sqrt{2}\right)$ (approx. 54.7356°)
Face-edge-face angle, i.e., "dihedral angle"[3]	$\arccos\left(\frac{1}{3}\right) = \arctan\left(2\sqrt{2}\right)$ (approx. 70.5288°)

Properties of the Tetrahedron

Regular tetrahedron



$$v_1 = \left(\sqrt{\frac{8}{9}}, 0, -\frac{1}{3}\right)$$

$$v_2 = \left(-\sqrt{\frac{2}{9}}, \sqrt{\frac{2}{3}}, -\frac{1}{3}\right)$$

$$v_3 = \left(-\sqrt{\frac{2}{9}}, -\sqrt{\frac{2}{3}}, -\frac{1}{3}\right)$$

$$v_4 = (0, 0, 1)$$

Tetrahedron Vertex Unit Vectors