Evgeny Epelbaum, Ruhr-University Bochum

Frontiers in nuclear physics, KITP, Santa Barbara
August 22 - November 4, 2016

RUB

Tutorial on nuclear EFT's

Related tutorials & lecture notes

— general: **EE, Nuclear forces from chiral EFT: A primer, arXiv:1001.3229**

— renormalization:

Lepage, How to renormalize the Schrödinger equation, nucl/th:9706029

— RG analysis:

Birse, The renormalization group and nuclear forces, Phil. Trans. Roy. Soc. Lond. A369 (2011) 2662

— Uncertainty quantification:

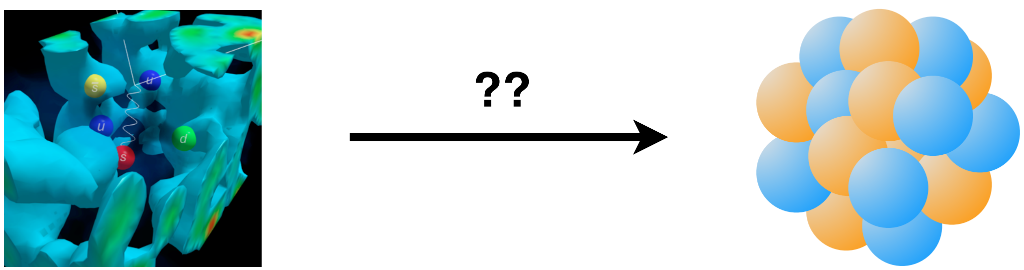
Weselowski et al., Bayesian parameter estimation for effective field theories, J.Phys. G43 (2016) 074001

Grießhammer, Assessing Theory Uncertainties in EFT Power Countings from Residual Cutoff Dependence, arXiv:1511.00490

Why EFT for nuclear physics?

Ultimate goal: predictive, systematically improvable and computationally efficient QCD-based theory for nuclei, nuclear reactions and nuclear matter

Notice: **predictive** requires for a theory to come with **uncertainty estimates**!



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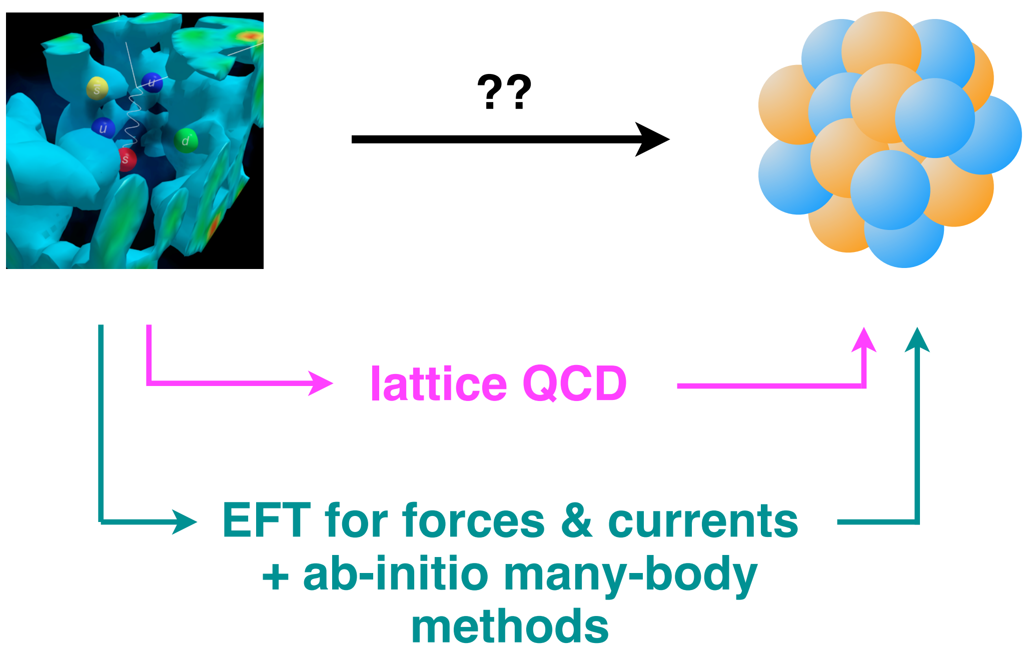

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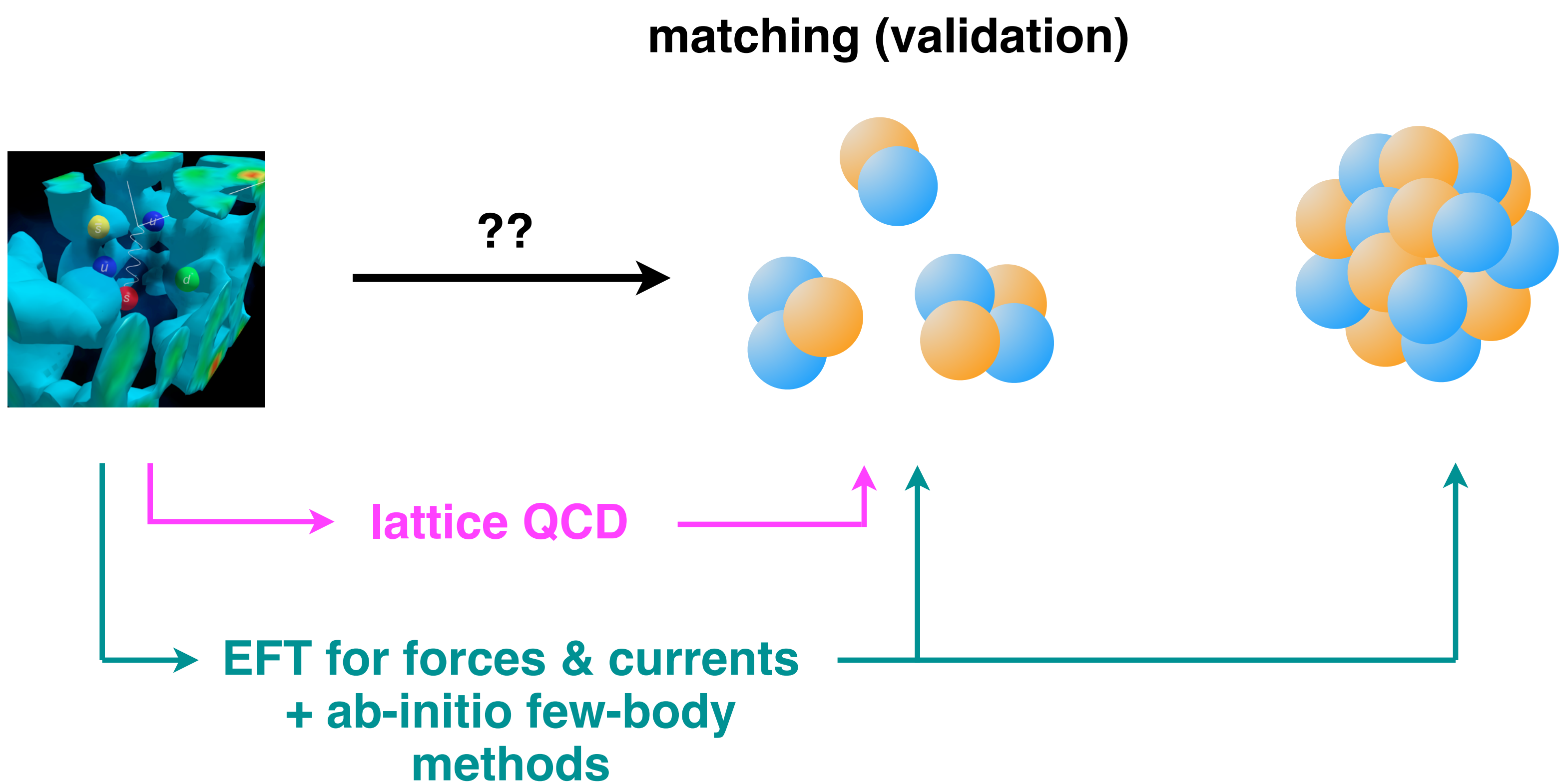


How few is few (& how many is many)?

$$H|\Psi\rangle = E|\Psi\rangle$$

Commonly used ab-initio few- and many-body methods:

> Lippmann-Schwinger & Faddeev-Yakubovski equations, No Core Shell Model, Quantum Monte Carlo, Lorentz Integral Transform, Coupled cluster, nuclear lattice simulations, self-consistent Gorkov Green's functions, many-body perturbation theory, hyperspherical harmonics, …

The applicability range of many-body methods is typically restricted by the size of the model space (convergence). For A > ~ 6, nuclear forces must be softened via appropriate UTs (induced many-body forces…)

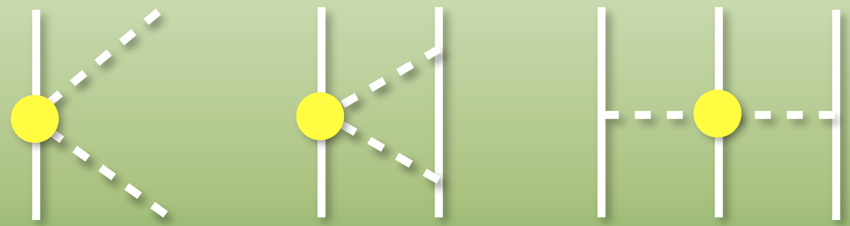
- A = 2: trivial
- A = 3: can be solved on a PC (both discrete & continuum states)
- A = 4: requires supercomputing (scattering so far only at low energies…)
- A > 4: so far, only discrete states (with very few exceptions)…
- A ~ 50: some results available (converged?)

EFTs for nuclear physics

A = 0,1: Chiral perturbation theory

A > 1: Pionless EFT ($Q \ll M_\pi$); chiral EFT ($Q \sim M_\pi$)

A >> 1: In-medium chiral EFT; EFTs using collective DOFs (e.g. to describe deformed nuclei)



Chiral perturbation theory

- **Ideal world [$m_u = m_d = 0$], zero-energy limit:** non-interacting massless GBs
  (+ strongly interacting massive hadrons)

- **Real world [$m_u,\ m_d \ll \Lambda_{QCD}$], low energy:** weakly interacting light GBs
  (+ strongly interacting massive hadrons)

$\longrightarrow$ expand about the ideal world (ChPT)

Chiral Perturbation Theory

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of

Weinberg, Gasser, Leutwyler, Meißner, ...

$$Q = \frac{\text{momenta of pions and nucleons or } M_\pi \sim 140 \text{ MeV}}{\text{hard scales [at best } \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV]}}$$

Manohar, Georgi '84

Tool: Feynman calculus using the effective chiral Lagrangian

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c}_i \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d}_i \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \ldots$$

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Pion-nucleon scattering up to Q^4 in heavy-baryon ChPT

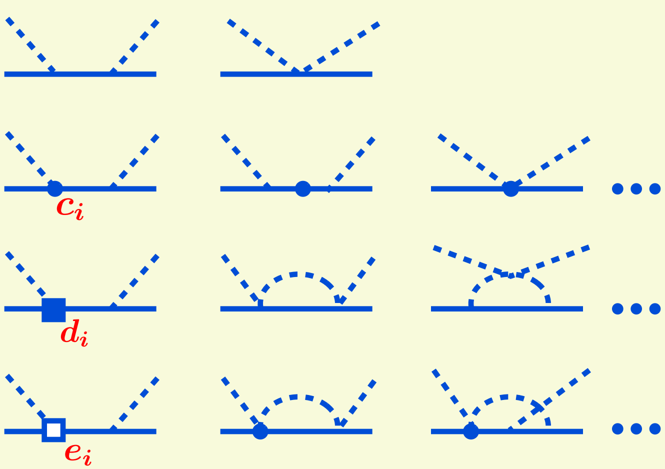
Fettes, Meißner ’00; Krebs, Gasparyan, EE ’12

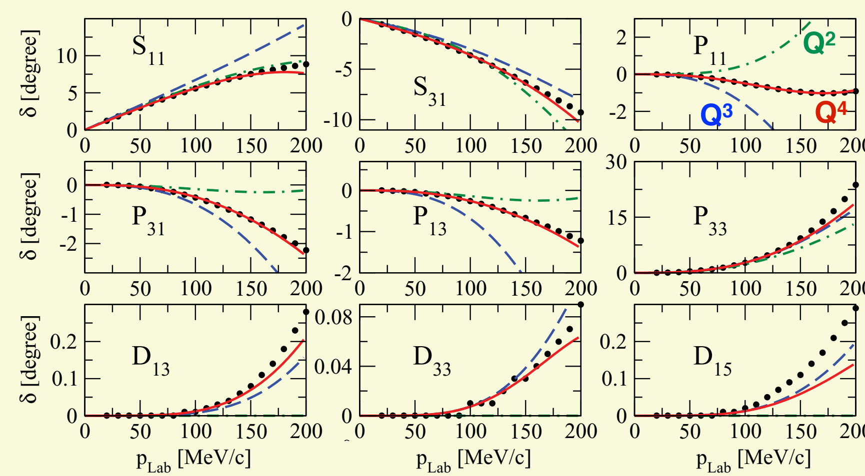
Order Q:

Order Q²:

Order Q³:

Order Q⁴:

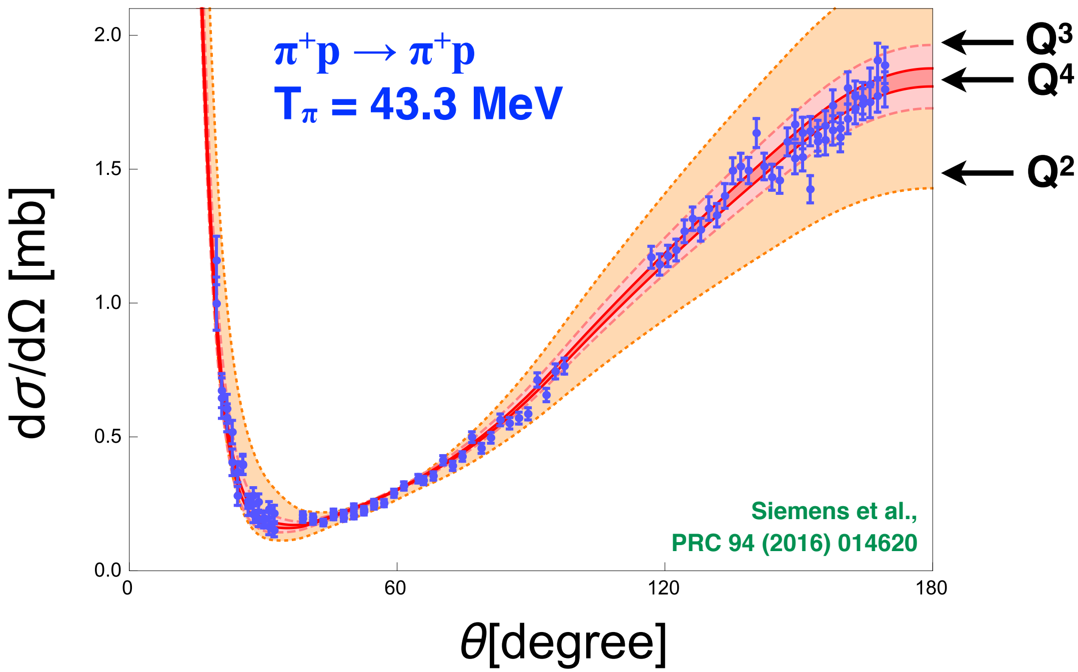




Determination of the πN LECs

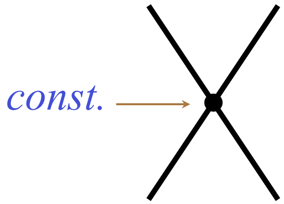
Pion-nucleon LECs can be reliably extracted from:

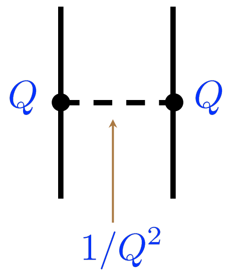
— πN PWA [Fettes, Meißner, Alarcon, Camalich, Gasparyan, EE, Krebs, Deliang, …],
— Roy-Steiner analysis of πN scattering [Hoferichter, Ruiz de Elvira, Kubis, Meißner, Yao, Gegelia, …]
— or directly from πN scattering data [Wendt, Ekström, Siemens, Bernard, EE, Gasparyan, Krebs, Meißner, …]



	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{17}$
$[Q^4]_{\rm HB,\,NN}$, GW PWA	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-0.58
$[Q^4]_{\rm HB,\,NN}$, KH PWA	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-0.37
$[Q^4]_{\rm covariant}$, data	-0.82	3.56	-4.59	3.44	5.43	-4.58	-0.40	-9.94	-0.63	-0.90

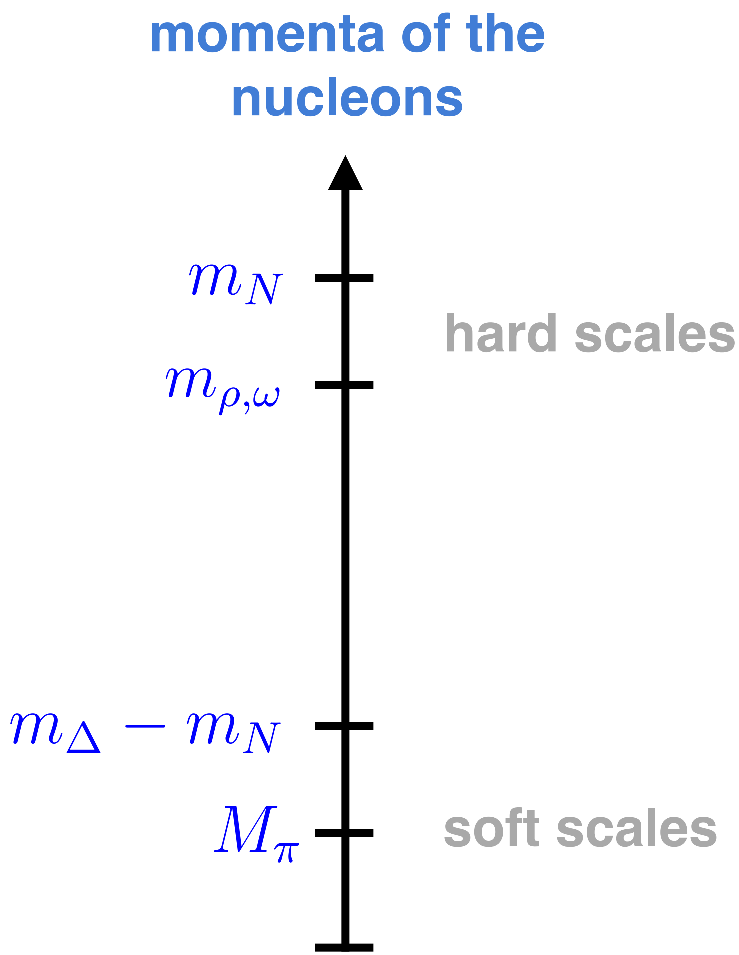
Nuclear EFTs (A > 1)





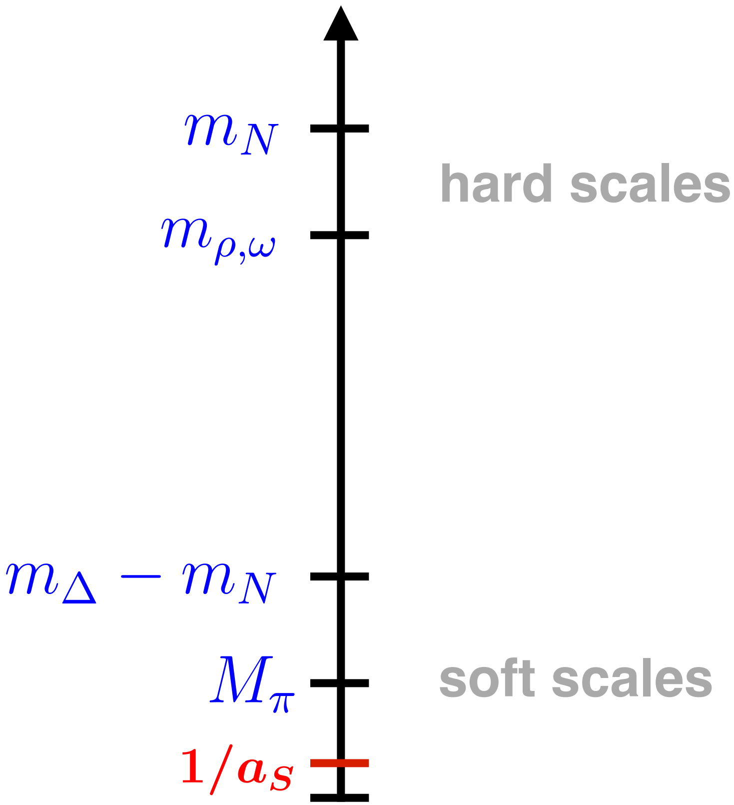
← **Not** suppressed by χ symmetry…

Hierarchy of scales in nuclear physics



Hierarchy of scales in nuclear physics

momenta of the nucleons



A new, soft scale associated with nuclear binding

$Q \sim 1/a_S \simeq 8.5$ MeV (36 MeV) in $^1S_0$ ($^3S_1$)

has to be generated dynamically

(need resummations...)

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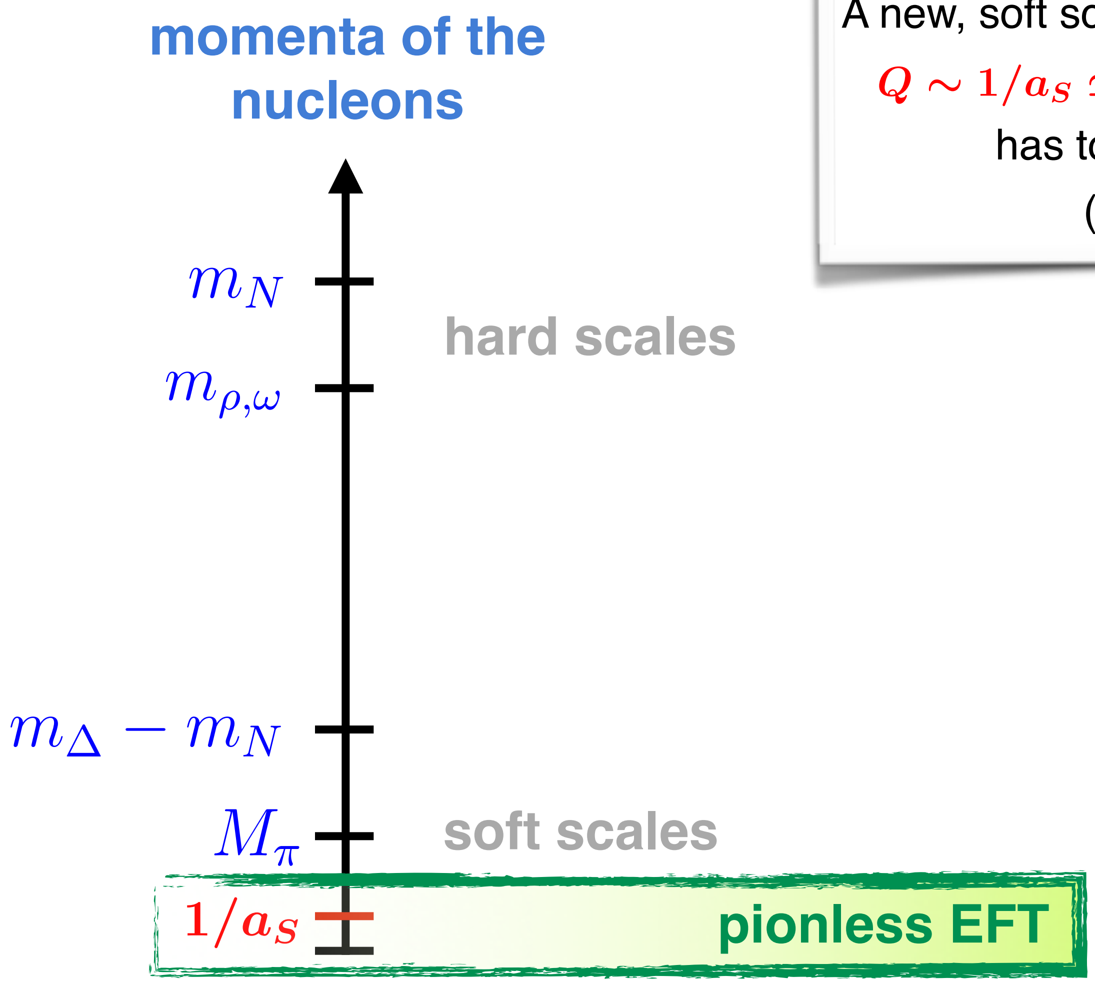

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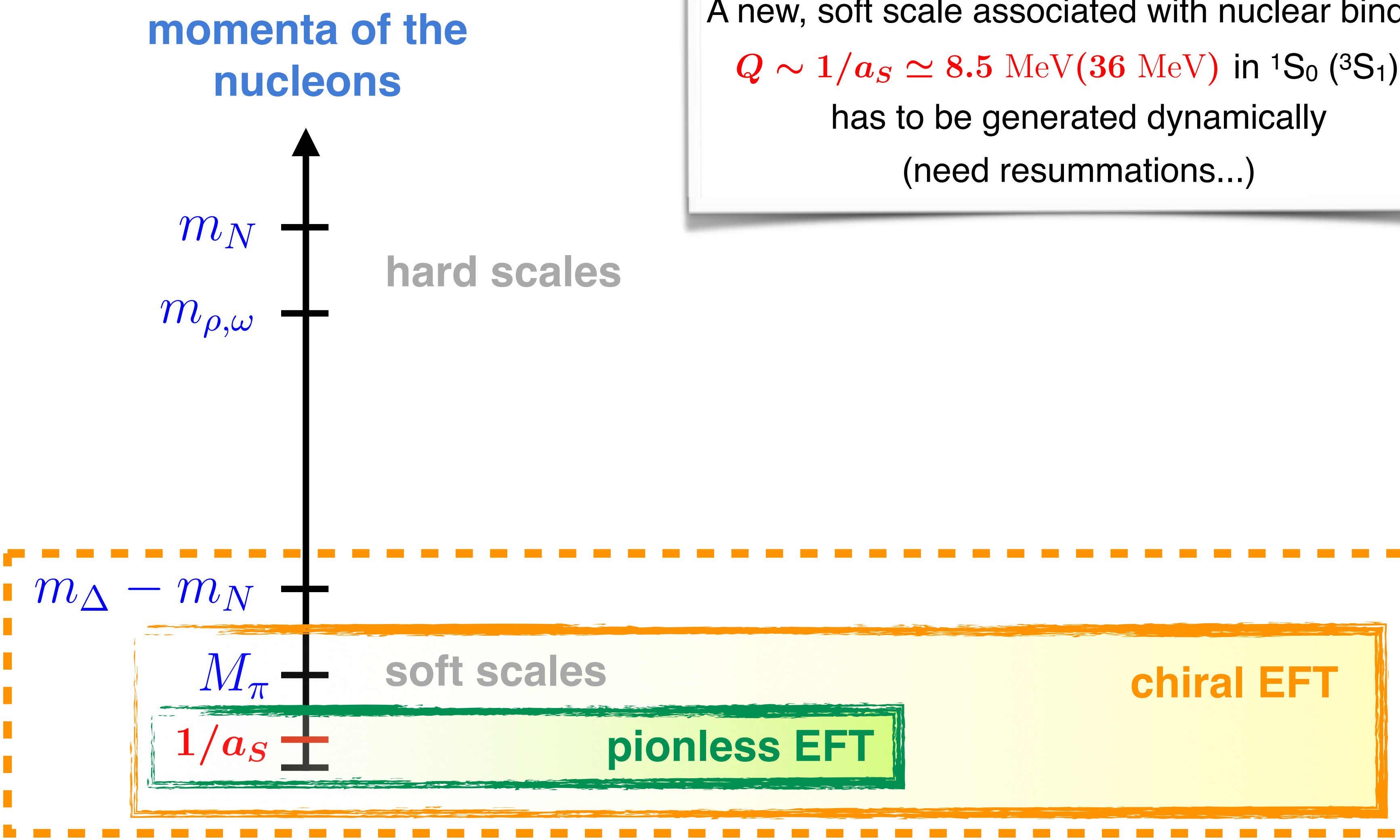


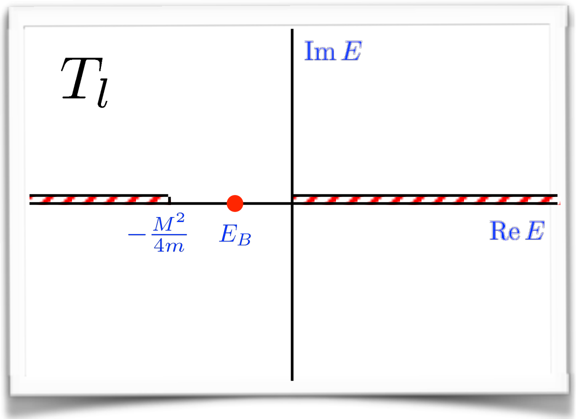
Pionless EFT

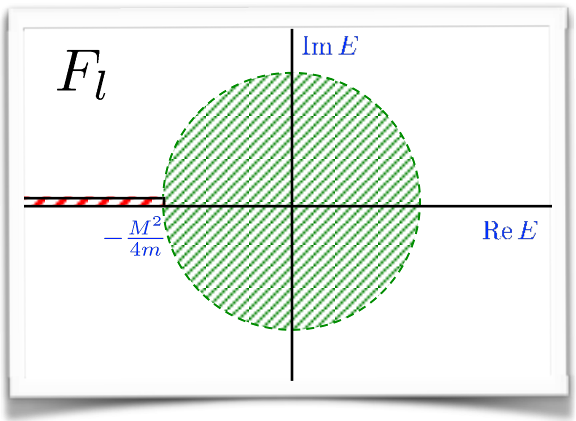
Nonrelativistic nucleon-nucleon scattering (uncoupled case):

effective-range function

$$S_l(k) = e^{2i\delta_l(k)} = 1 + i\frac{mk}{2\pi}T_l(k) \quad \text{where} \quad T_l(k) = \frac{4\pi}{m}\frac{k^{2l}}{F_l(k) - ik^{2l+1}} \quad \text{and} \quad F_l(k) \equiv k^{2l+1}\cot\delta_l(k)$$

If $V(r)$ satisfies certain conditions, F_l is a meromorphic function of k^2 near the origin





→ effective range expansion (ERE): $F_l(k^2) = -\frac{1}{a} + \frac{1}{2}rk^2 + v_2k^4 + v_3k^6 + v_4k^8 + \ldots$

The analyticity domain depends on the range M^{-1} of $V(r)$ defined as $M = \min(\mu)$

such that $\int_{R>0}^{\infty} |V(r)|\, e^{\mu r} dr = \infty$ (for strongly interacting nucleons $M = M_\pi$)

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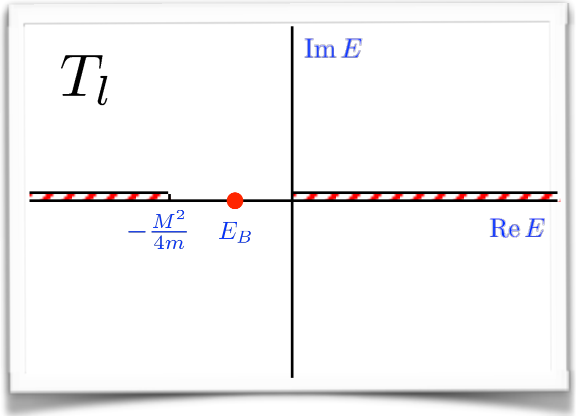

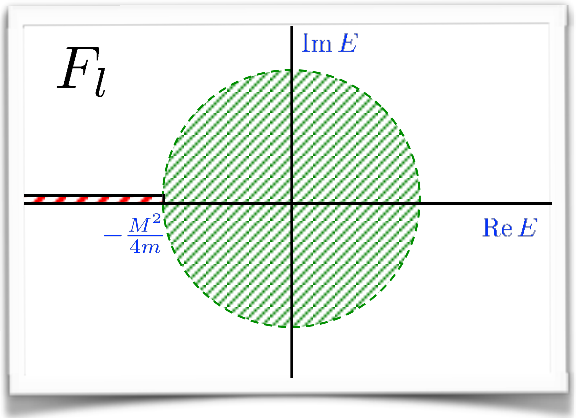

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Both ERE & $\not\pi$-EFT provide an expansion of NN observables in powers of k/M_π, have the same validity range and are based on the same principles ⟶ **ERE ~ $\not\pi$-EFT**

Pionless EFT

Effective Lagrangian: for $Q \ll M_\pi$ only point-like interactions

$$\mathcal{L}_{\rm eff} = N^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2m} \right) N - \frac{1}{2} C_1^0 (N^\dagger N)^2 - \frac{1}{2} C_2^0 (N^\dagger \vec{\sigma} N)^2 - \frac{1}{4} C_1^2 (N^\dagger \vec{\nabla}^2 N)(N^\dagger N) + \text{h.c.} + \ldots$$

Scattering amplitude (S-waves):

$$S = e^{2i\delta} = 1 - i \left(\frac{km}{2\pi} \right) T \,, \qquad T = -\frac{4\pi}{m} \frac{1}{k \cot\delta - ik} = -\frac{4\pi}{m} \frac{1}{\left(-\frac{1}{a} + \frac{1}{2} r_0 k^2 + v_2 k^4 + v_3 k^6 + \ldots\right) - ik}$$

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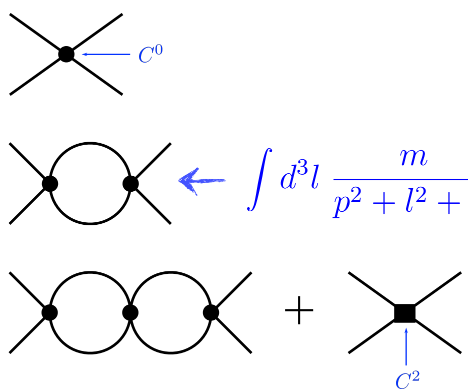
● Natural case

$$|a| \sim M_\pi^{-1}\,,\ |r| \sim M_\pi^{-1}\,,\ \ldots \quad \rightarrow \quad T = T_0 + T_1 + T_2 + \ldots = \frac{4\pi a}{m}\Big[\underbrace{1}_{\sim Q^0} - \underbrace{iak}_{\sim Q^1} + \underbrace{\left(\frac{ar_0}{2} - a^2\right)k^2}_{\sim Q^2} + \ldots\Big]$$

$T_0 =$

$T_1 =$ $\leftarrow \int d^3l\,\frac{m}{p^2 + l^2 + i\epsilon} \sim mQ$

$T_2 =$



$\ldots$

The EFT expansion can be arranged to match the above expansion for *T*.

Using e.g. dimensional or subtractive ragularization yields:

- perturbative expansion for *T*;
- scaling of the LECs: $C^i \sim Q^0$

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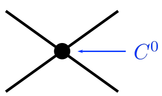
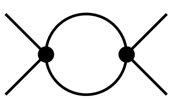
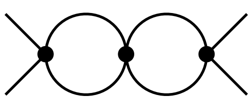
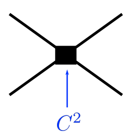
$$|a| \sim M_\pi^{-1}\,, \; |r| \sim M_\pi^{-1}\,, \; \ldots \quad \rightarrow \quad T = T_0 + T_1 + T_2 + \ldots = \frac{4\pi a}{m} \Big[\underset{\substack{\uparrow \\ \sim Q^0}}{1} - \underset{\substack{\uparrow \\ \sim Q^1}}{iak} + \underbrace{\left(\frac{a r_0}{2} - a^2 \right) k^2}_{\sim Q^2} + \ldots \Big]$$

$T_0 =$ 

$T_1 =$  $\leftarrow \int d^3 l \, \frac{m}{p^2 + l^2 + i\epsilon} \sim mQ$

$T_2 =$  $+$ 

...

The EFT expansion can be arranged to match the above expansion for T.

Using e.g. dimensional or subtractive ragularization yields:

- perturbative expansion for T;
- scaling of the LECs: $C^i \sim Q^0$

In reality: $a_{^1S_0} = -23.741\ \text{fm} = -16.6\, M_\pi^{-1} \qquad a_{^3S_1} = -5.42\ \text{fm} = 3.8\, M_\pi^{-1}$

Pionless EFT

- **Large scattering length:** $|a| \gg M_\pi^{-1}$ Kaplan, Savage, Wise '97

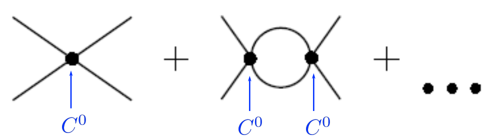
Keep ak fixed, i.e. count $a \sim Q^{-1}$:

$$T = -\frac{4\pi}{m}\frac{1}{\left(-\frac{1}{a}+\frac{1}{2}r_0k^2+v_2k^4+v_3k^6+\ldots\right)-ik} = \frac{4\pi}{m}\frac{1}{(1+iak)}\Bigg[\underset{\sim Q^{-1}}{a} + \underbrace{\frac{ar_0}{2(a^{-1}+ik)}k^2}_{\sim Q^{0}} + \underset{\sim Q^{1}}{\ldots}\Bigg].$$

Notice: perturbation theory for T breaks down as it has a pole at $|k| \sim |a|^{-1} \ll M_\pi$

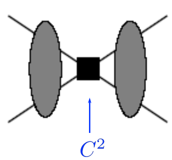
KSW expansion (DR+PDS or subtractive renormalization $C^0 \sim 1/Q,\ C^2 \sim 1/Q^2,\ \ldots$ **)**

$T^{(-1)} =$



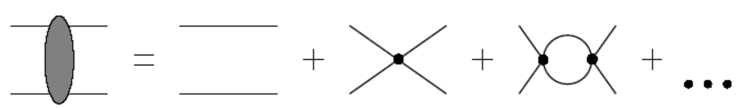
$$= \frac{-C^0(\mu)}{\left[1+\frac{C^0(\mu)\,m}{4\pi}(\mu+ik)\right]},$$

$T^{(0)} =$



$$= \frac{-C^2(\mu)\,k^2}{\left[1+\frac{C^0(\mu)\,m}{4\pi}(\mu+ik)\right]^2}$$

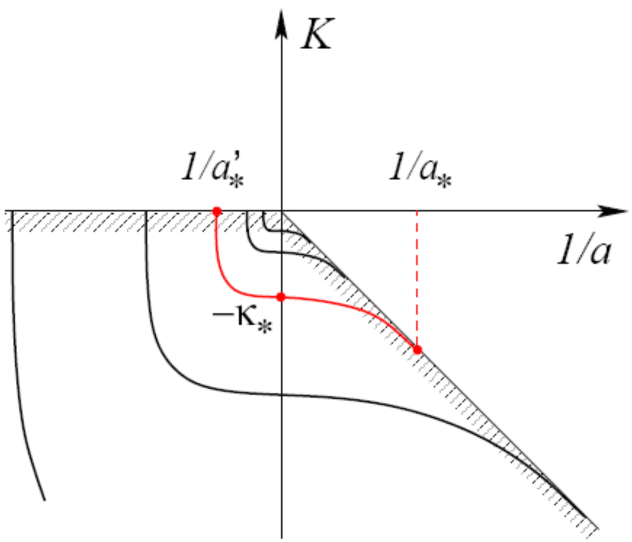
where:



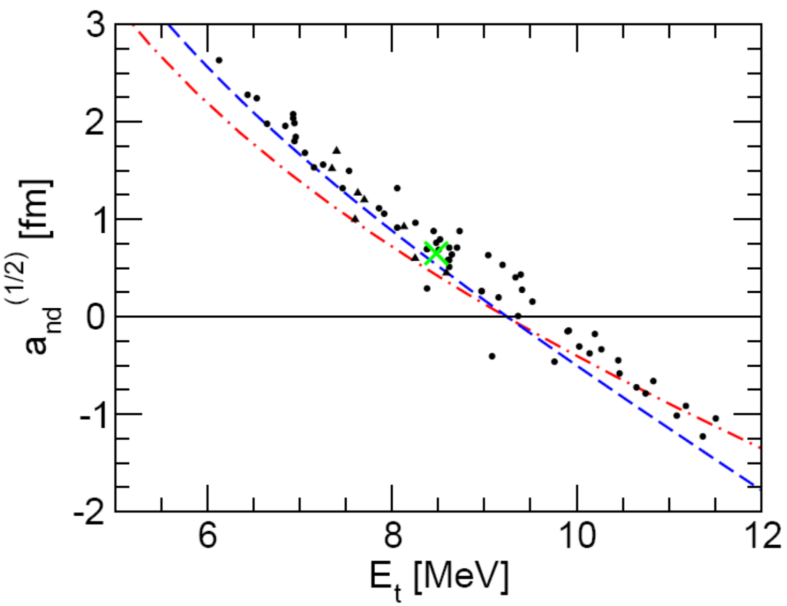
Pionless EFT: (some) applications

- Astrophysical reactions Butler, Chen, Kong, Ravndal, Rupak, Savage, …
- Efimov physics and universality in few-body systems with large 2-body scatt. length (e.g. Phillips/Tjon „lines“) Braaten, Hammer, Meißner, Platter, von Stecher, Schmidt, Moroz, …
- Halo-nuclei Bedaque, Bertulani, Hammer, Higa, van Kolck, Phillips, …
- Parity violation Schindler, Springer … any many other topics…

Efimov effect (3-body spectrum)



Phillips line



Braaten, Hammer, Phys. Rept 428 (06) 259

How to go beyond ERE?

Goal: EFT for NN scattering at typical momenta $Q \sim M_\pi$

Are pions perturbative?

How to test whether or not pion dynamics is treated properly?

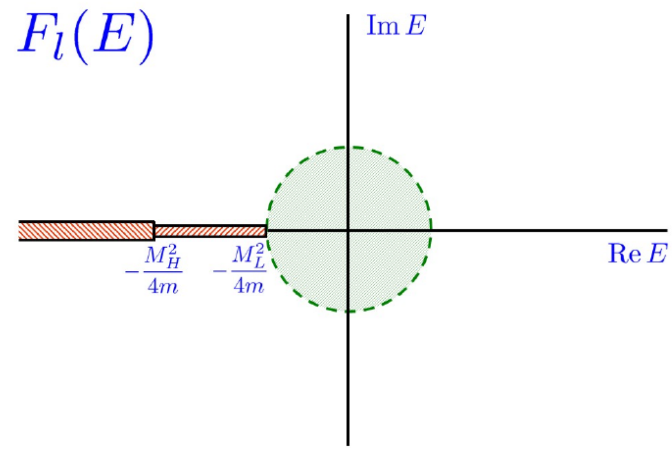
Modified Effective Range Expansion (MERE)

Beyond π-less EFT: higher energies, low-energy theorems (LETs)…

What are low-energy theorems?

Two-range potential: $V(r) = V_L(r) + V_S(r)$

with $M_L^{-1} \gg M_H^{-1}$



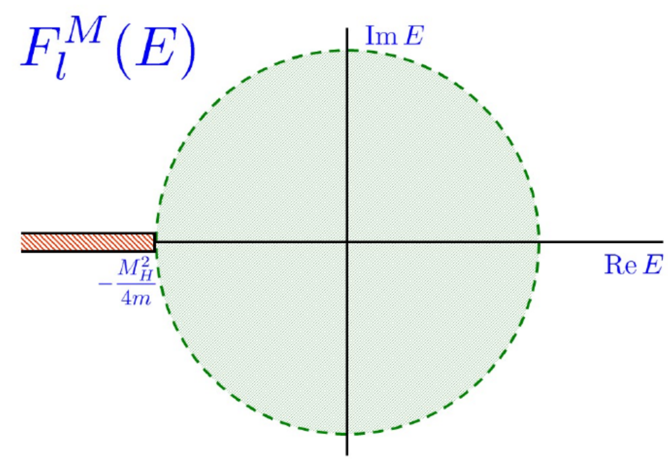
- $F_l(k^2)$ is meromorphic in $|k| < M_L/2$
- $$\boxed{F_l^M(k^2) \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2} \cot\left[\delta_l(k) - \delta_l^L(k)\right]}$$

modified effective range function
Haeringen, Kok '82

$$\underbrace{f_l^L(k)}_{\text{Jost function for } V_L(r)} = \lim_{r\to 0}\left(\frac{l!}{(2l)!}(-2ikr)^l \underbrace{f_l^L(k,r)}_{\text{Jost solution for } V_L(r)}\right)$$

$$M_l^L(k) = Re\left[\frac{(-ik/2)^l}{l!}\lim_{r\to 0}\left(\frac{d^{2l+1}}{dr^{2l+1}}\frac{r^l f_l^L(k,r)}{f_l^L(k)}\right)\right]$$

Per construction, F_l^M reduces to F_l for $V_L = 0$ and is meromorphic in $|k| < M_H/2$



MERE and low-energy theorems

Example: proton-proton scattering

$$F_C(k^2) = C_0^2(\eta)\, k\, \cot[\delta(k) - \delta^C(k)] + 2k\,\eta\, h(\eta) = -\frac{1}{a^M} + \frac{1}{2} r^M k^2 + v_2^M k^4 + \ldots$$

where $\underbrace{\delta^C \equiv \arg\Gamma(1+i\eta)}_{\text{Coulomb phase shift}}$, $\eta = \dfrac{m}{2k}\alpha$, $\underbrace{C_0^2(\eta) = \dfrac{2\pi\eta}{e^{2\pi\eta}-1}}_{\text{Sommerfeld factor}}$, $h(\eta) = \mathrm{Re}\Big[\underbrace{\Psi(i\eta)}_{\text{Digamma function}}\Big] - \ln(\eta)$

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MERE and low-energy theorems

Long-range forces impose correlations between the ER coefficients (low-energy theorems)

Cohen, Hansen '99; Steele, Furnstahl '00

The emergence of the LETs can be understood in the framework of MERE:

$$\underbrace{F_l^M(k^2)}_{\text{meromorphic for } k^2 < (M_H/2)^2} \equiv M_l^L(k) + \frac{k^{2l+1}}{|f_l^L(k)|^2} \cot\left[\delta_l(k) - \delta_l^L(k)\right]$$

can be computed if the long-range force is known

– approximate $F_l^M(k^2)$ by first 1,2,3,… terms in the Taylor expansion in k^2

– calculate all "light" quantities

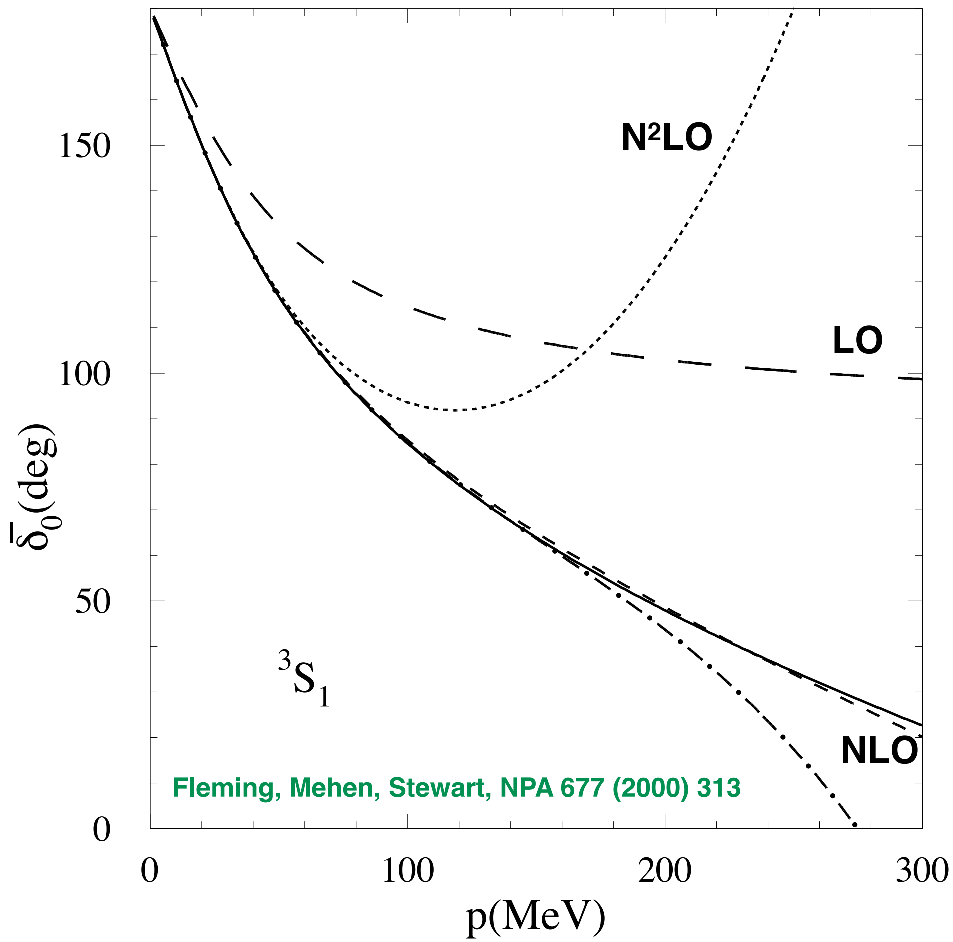
– reconstruct $\delta_l^L(k)$ and predict all coefficients in the ERE

LETs for NN S-waves

$^1S_0$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
NLO KSW Cohen, Hansen '99	fit	fit	-3.3	18	-108
LO Weinberg	fit	1.50	-1.9	8.6(8)	$-37(10)$
Nijmegen PWA	-23.7	2.67	-0.5	4.0	-20

$^3S_1$ partial wave	a [fm]	r [fm]	v_2 [fm^3]	v_3 [fm^5]	v_4 [fm^7]
NLO KSW Cohen, Hansen '99	fit	fit	-0.95	4.6	-25
LO Weinberg	fit	1.60	-0.05	0.8(1)	$-4(1)$
Nijmegen PWA	5.42	1.75	0.04	0.67	-4.0

NN scattering with perturbative pions



see also: Birse, Phys. Rev. C74 (2006) 014003

⟶ pion exchange seems to require a non-perturbative treatment!

Chiral EFT for nuclei

How to include pions non-perturbatively?

For $p \sim M_\pi << m_N$, nucleons are non-relativistic $\longrightarrow$ nuclear dynamics can be efficiently treated within the conventional Schrödinger theory (QM) **Weinberg '90**

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\text{derived in ChPT}}\right] |\Psi\rangle = E|\Psi\rangle$$

$\longrightarrow$ coupled with ab-initio few-body methods, provides access to nuclei

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What is the applicability range of the potential approach?

Strictly speaking, below π production threshold, i.e. $p \sim \sqrt{M_\pi m_N} \sim 400\ \mathrm{MeV}$
(if desired, radiative pions can be included perturbatively…)

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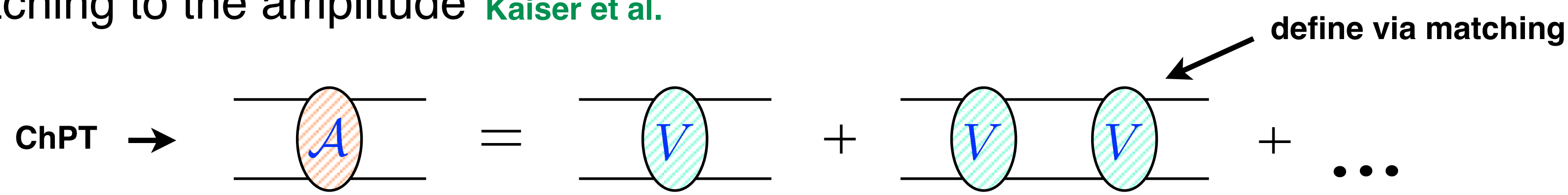
How to derive nuclear forces from the effective chiral Lagrangian?

- Irreducible time-ordered diagrams Weinberg '90; van Kolck et al. '93; Pastori, Piarulli et al. 07-16
- Decouple pion states via a suitable UT in the Fock space EE, Glöckle, Krebs, Meißner

Chiral EFT for nuclei

- Matching to the amplitude **Kaiser et al.**



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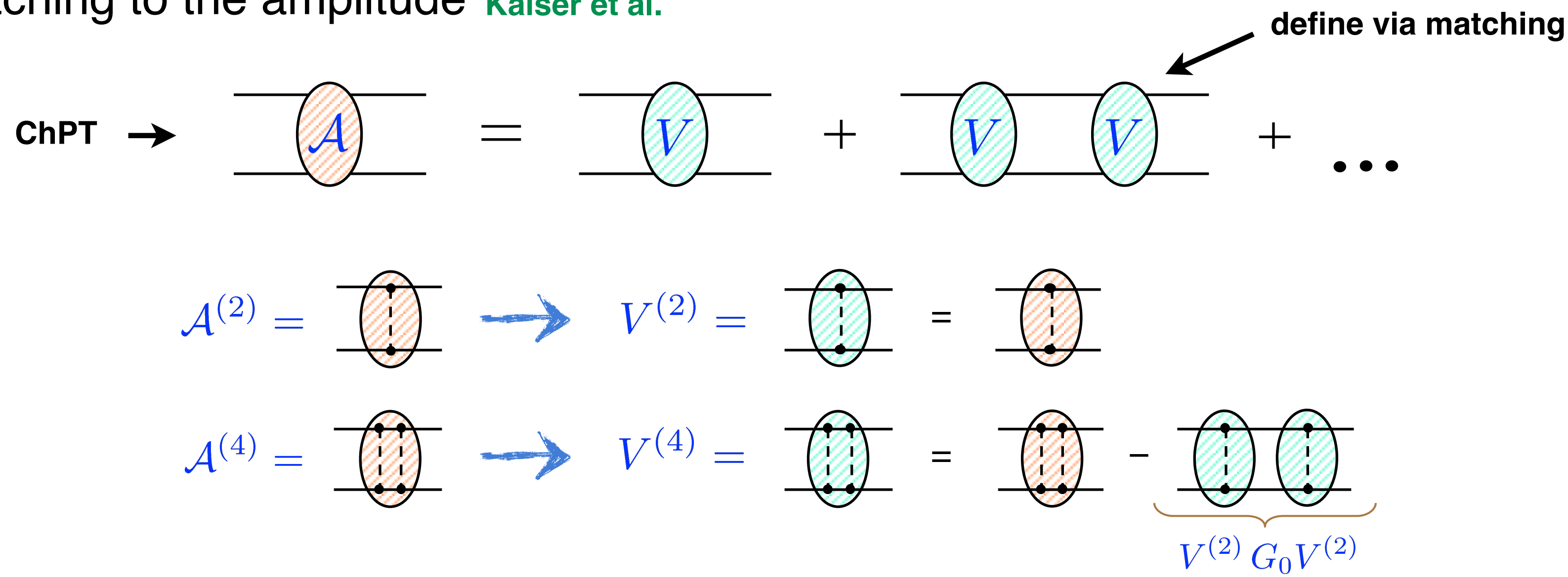

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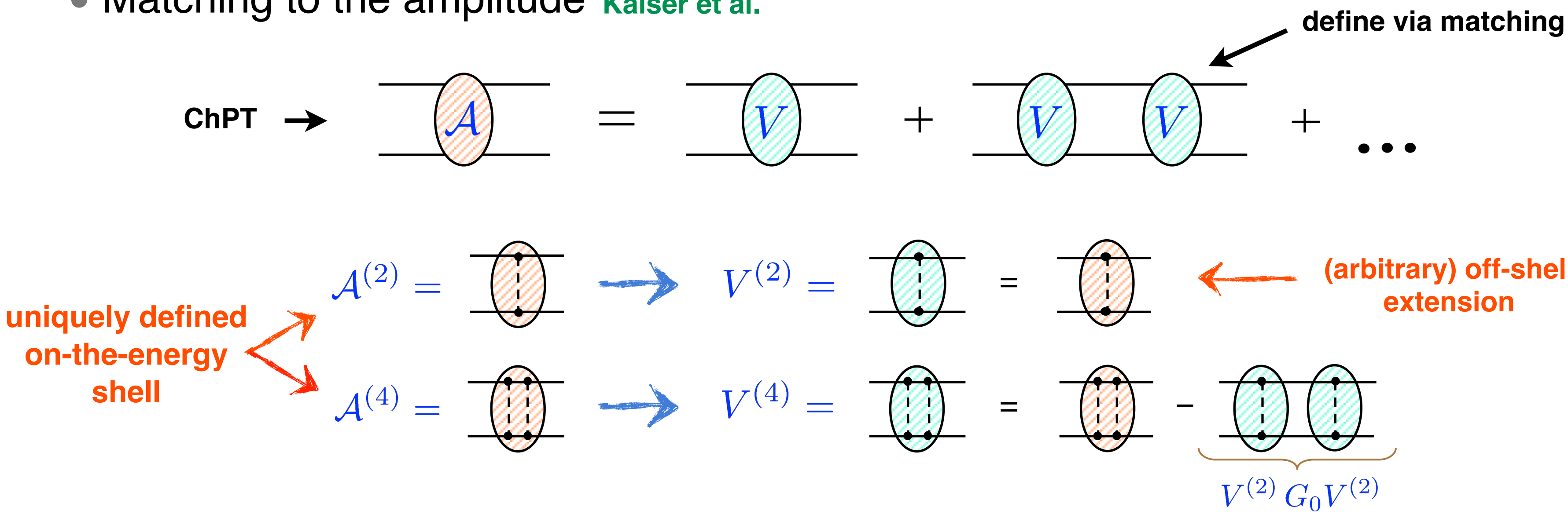

Are nuclear forces directly observable?

No. Contrary to the on-shell amplitude, nuclear forces are scheme-dependent.

Chiral EFT for nuclei

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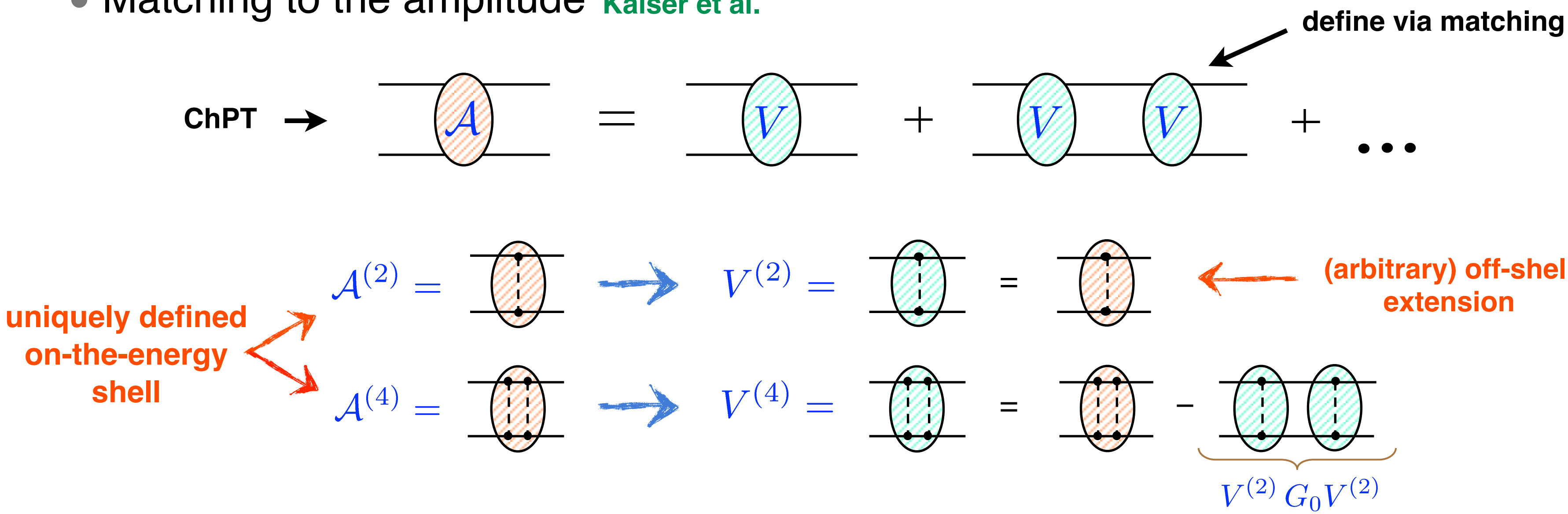

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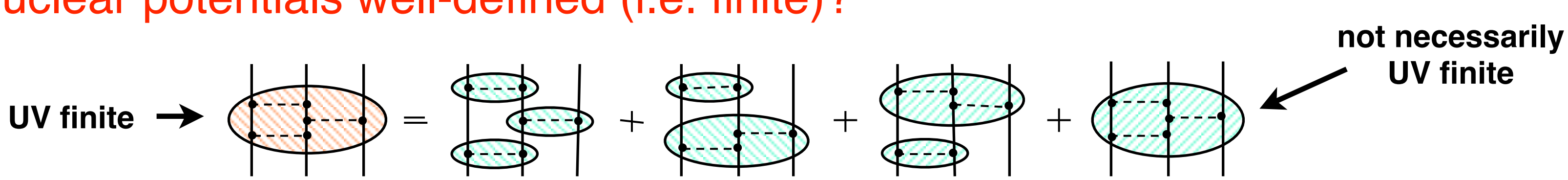
Are nuclear potentials well-defined (i.e. finite)?



So far, it was always possible to renormalize nuclear forces by systematically exploiting their unitary ambiguity…

Chiral EFT for nuclei

How about current operators?

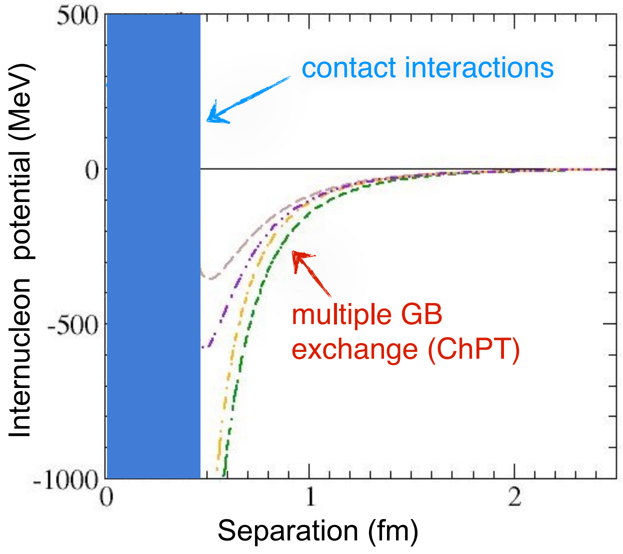
Include coupling to external sources (local χ-symmetry) $H[a, v, s, p]$, eliminate pion fields and read off the current operators **[see Krebs, EE, Meißner, to appear]**

Can one combine currents calculated by the JLab-Pisa group with Bochum-Bonn nuclear forces?

This would be inconsistent (currents & forces correspond to different choices of the basis states)

Where does chiral physics come into play?

The potential is expected to converge at large distances ($r \sim M_\pi$ and beyond). The long-range tail of the force controls the energy behavior of the amplitude in the near-threshold region.

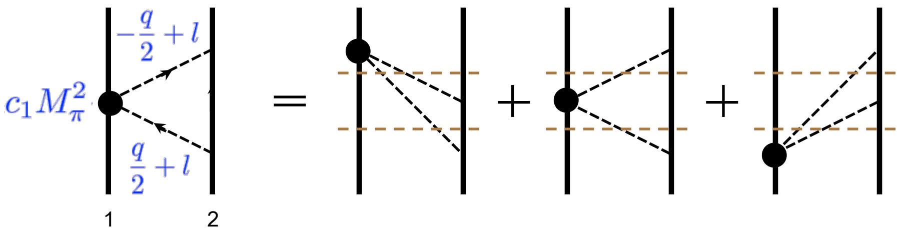


What is the breakdown distance of the chiral expansion of the long-range potential?

Naively (just NDA): $r \sim (4\pi F_\pi)^{-1} \sim 0.2$ fm
However, this seems too optimistic…

Chiral EFT for nuclei

Pion loops in multiple-scattering-like diagrams are enhanced by one power of $\boldsymbol{\pi}$!



$$\frac{1}{2\omega_1\, 2\omega_2}\left[\frac{1}{\omega_1(\omega_1+\omega_2)}+\frac{1}{\omega_1\omega_2}+\frac{1}{\omega_2(\omega_1+\omega_2)}\right]=\frac{1}{2\omega_1^2\omega_2^2}$$

$$V_{2\pi}^{(3)} = \frac{3g_A^2}{2F_\pi^4}c_1 M_\pi^2 \int \frac{d^3 l}{(2\pi)^3}\frac{l^{\prime 2}-\vec{q}^{\,2}}{\omega_+^2\omega_-^2} \quad \text{with} \quad \omega_\pm \equiv \sqrt{(\vec{l}\pm\vec{q})^2+4M_\pi^2}$$

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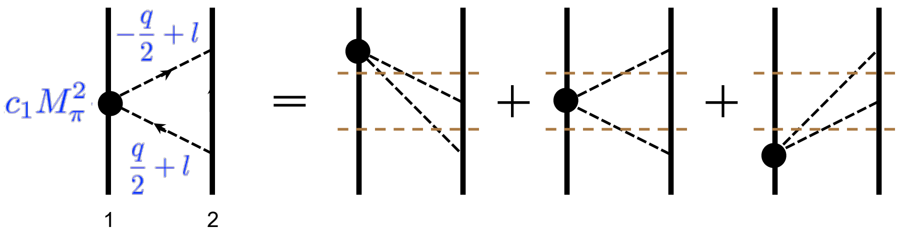

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$$V_{2\pi}^{(3)} \;=\; \frac{3g_A^2}{2F_\pi^4}c_1 M_\pi^2\int\frac{d^3l}{(2\pi)^3}\,\frac{l^{\prime 2}-\vec{q}^{\,2}}{\omega_+^2\omega_-^2} \quad \text{with} \quad \omega_\pm\equiv\sqrt{(\vec{l}\pm\vec{q})^2+4M_\pi^2}$$

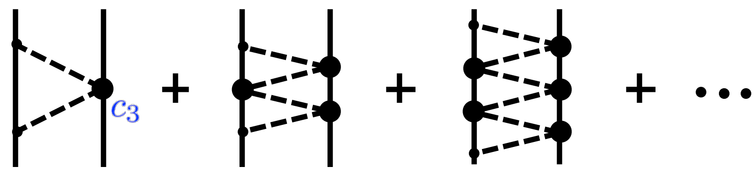
Fourier transformation:

$$\begin{aligned}
V_{2\pi}^{(3)}(\vec{r}) &= \int\frac{d^3q}{(2\pi)^3}\,e^{i\vec{q}\cdot\vec{r}}\,V_{2\pi}^{(3)}(\vec{q}) \\
&= \frac{3g_A^2}{2F_\pi^4}\,c_1M_\pi^2\int\frac{d^3q}{(2\pi)^3}e^{i\vec{q}\cdot\vec{r}}\int\frac{d^3l_1}{(2\pi)^3}\frac{d^3l_2}{(2\pi)^3}\,(2\pi)^3\,\delta(\vec{l}_1+\vec{l}_2-\vec{q})\,\frac{(-2\vec{l}_1\cdot\vec{l}_2)}{[\vec{l}_1^{\,2}+M_\pi^2][\vec{l}_2^{\,2}+M_\pi^2]} \\
&= -\frac{3g_A^2}{F_\pi^4}\,c_1M_\pi^2\left[\int\frac{d^3l}{(2\pi)^3}\,e^{i\vec{l}\cdot\vec{r}}\frac{\vec{l}}{\vec{l}^{\,2}+M_\pi^2}\right]^2 \\
&\longrightarrow\; -\frac{3g_A^2}{F_\pi^4}\,c_1M_\pi^2\left[-i\vec{\nabla}\left(\frac{1}{4\pi}\,\frac{e^{-M_\pi r}}{r}\right)\right]^2=\underbrace{\frac{3g_A^2}{16\pi^2F_\pi^4}\,c_1M_\pi^2\frac{e^{-2x}}{r^4}(1+x)^2}
\end{aligned}$$

only $c_1M_\pi^3/(4\pi F_\pi^2)$ times suppressed compared to $V_{1\pi}^{(0)}$ at $x\sim 1$

Chiral EFT for nuclei

Same arguments apply to all MS-like graphs: enhanced & analytically calculable (in the static approximation), e.g.:



$$V^{c_3}_{\rm resummed}(r) = \frac{3g_A^2 c_3}{32\pi^2 F_\pi^4}\,\frac{e^{-2x}}{r^6}\left[\frac{(2+2x+x^2)^2}{1-\frac{c_3^2}{(4\pi F_\pi^2)^2}\frac{e^{-2x}}{r^6}(2+2x+x^2)^2} + \frac{2(1+x)^2}{1-\frac{c_3^2}{(4\pi F_\pi^2)^2}\frac{e^{-2x}}{r^6}(1+x)^2}\right]$$

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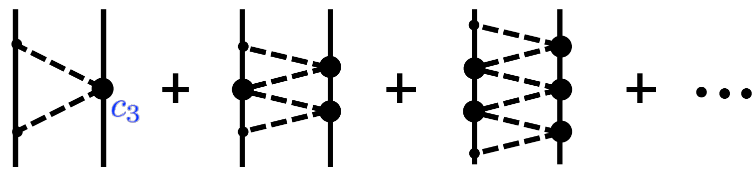

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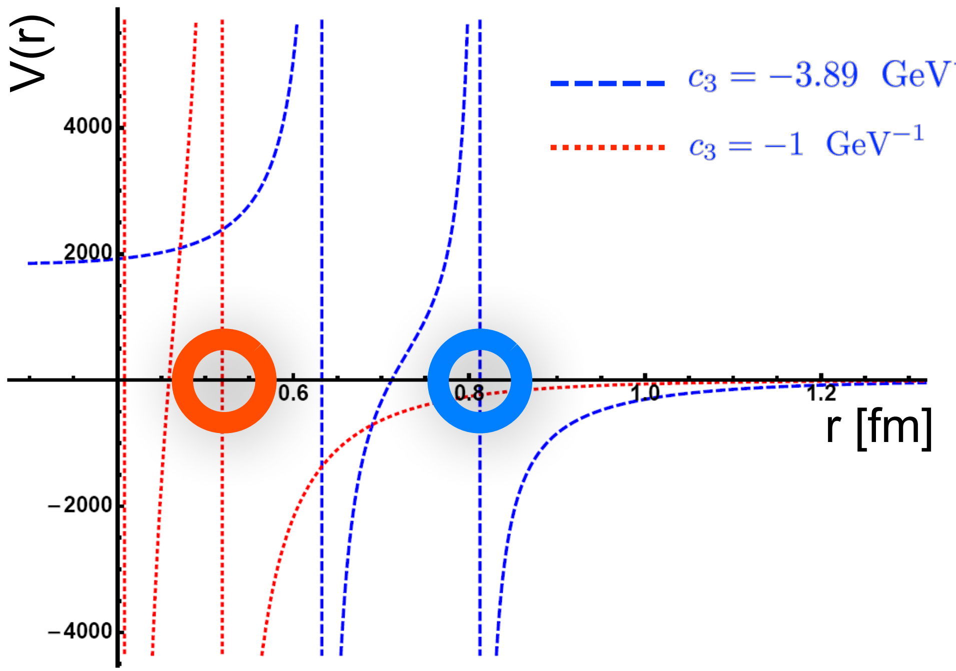

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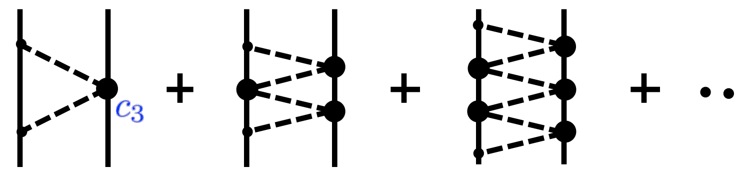

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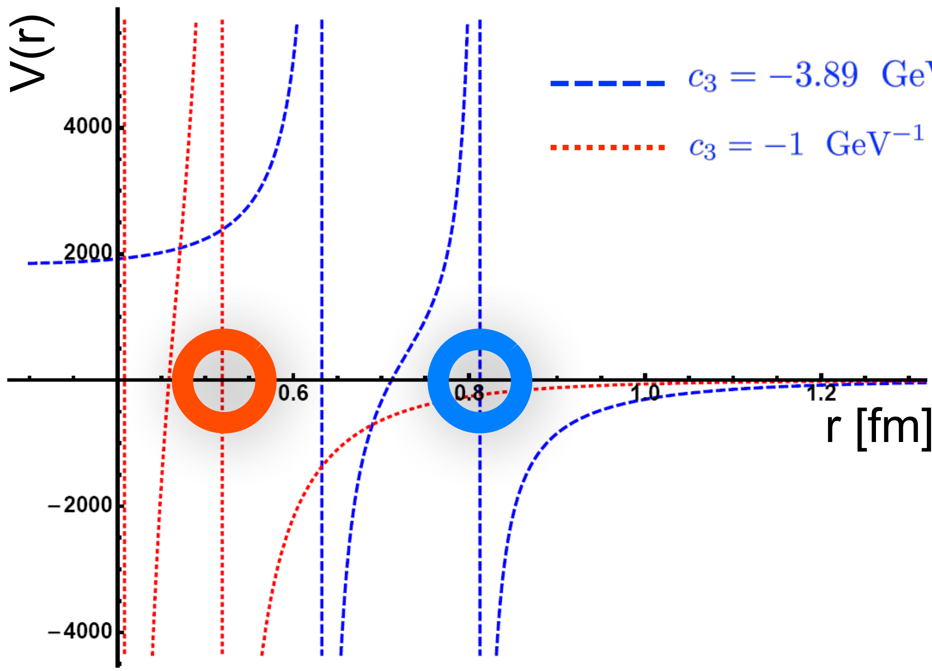

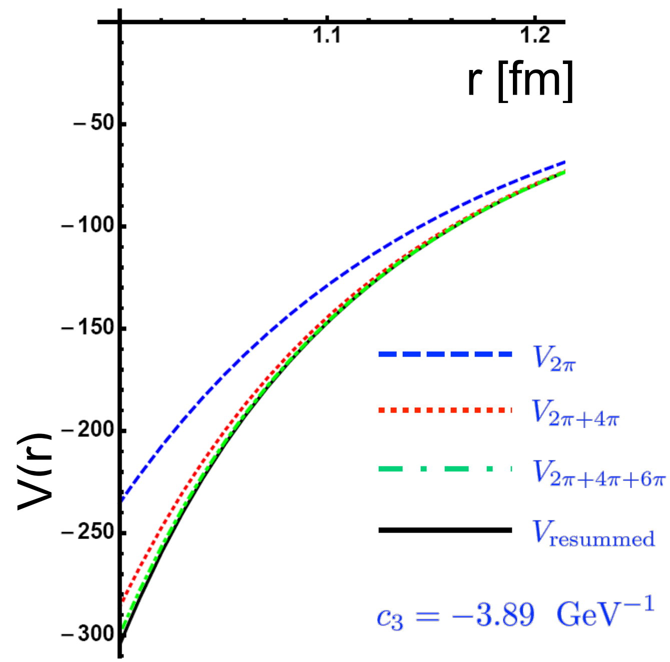

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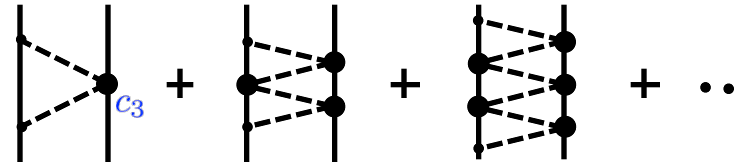

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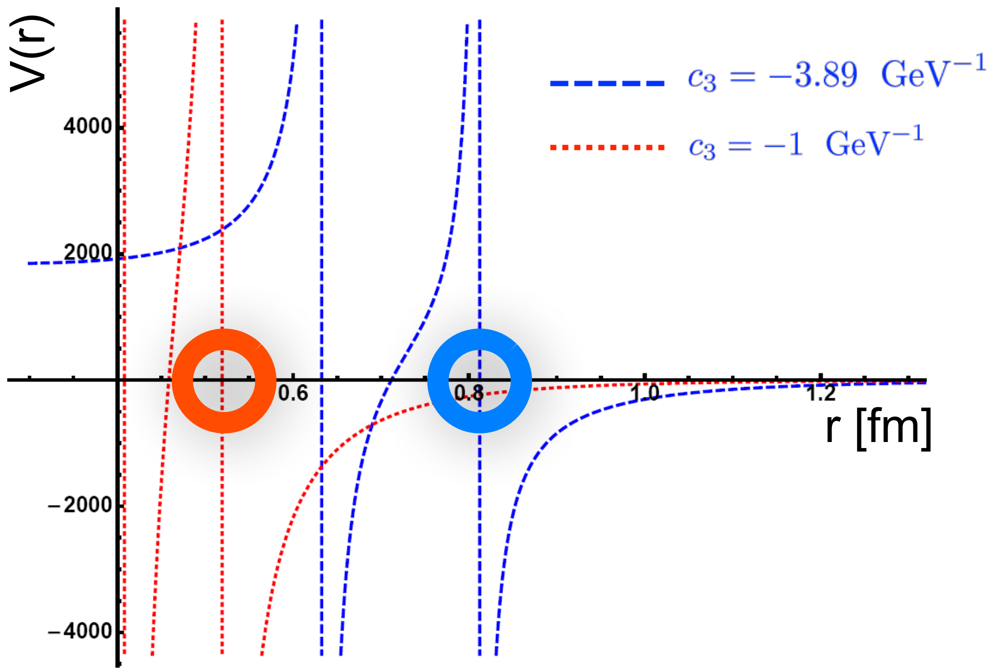
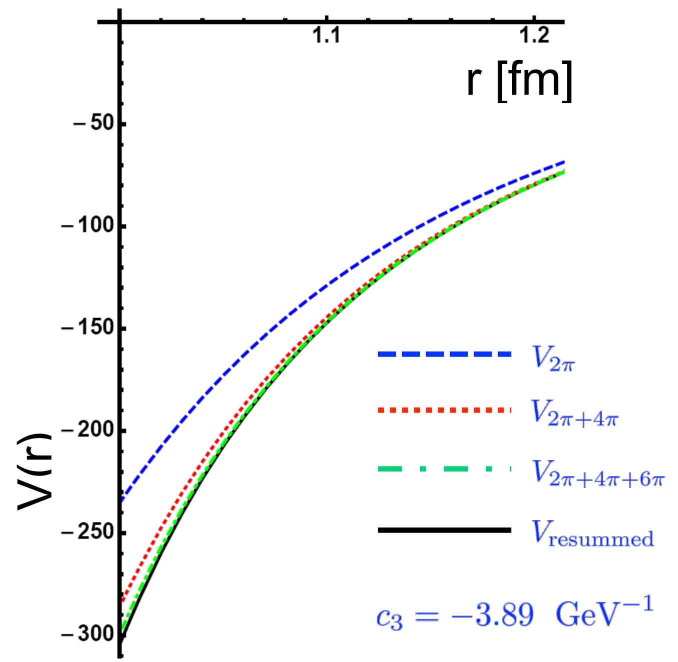




→ $R_b \sim 0.8$ fm (but good convergence of the χ expansion for r > 1fm)

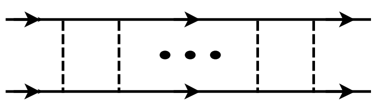
Chiral EFT for nuclei

How to renormalize the Schrödinger equation?

Lepage, Kaplan, Savage, Wise, Fleming, Mehen, Stewart, Phillips, Beane, Cohen, Frederico, Timoteo, Tomio, Birse, Beane, Bedaque, van Kolck, Pavon Valderrama, Ruiz Arriola, Nogga, Timmermanns, EE, Meißner, Entem, Machleidt, Yang, Elster, Long, Gegelia, ...

$$T(\vec{p}\,', \vec{p}) = V_{\rm 2N}(\vec{p}\,', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{\rm 2N}(\vec{p}\,', \vec{k})\, T(\vec{k}, \vec{p})}{p^2 - k^2 + i\epsilon} \quad \text{with} \quad V_{\rm 2N} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \ldots$$

→ Lippmann-Schwinger eq. is linearly divergent, need infinitely many CTs to absorb UV divergences from iterations!



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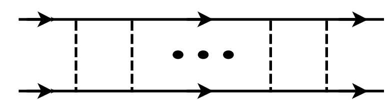

Commonly used approach

EGM, EM, EKM, Gezerlis et al.'14, Piarulli et al.'15, Carlsson et al.'16, …

- Include short-range operators in the potential according to NDA
- Introduce a finite UV regulator $R \sim R_b$ ($\Lambda \sim 500$ MeV)
- Solve the LS equation & tune the **bare** LECs $C_i(R)$ to NN data (implicit renormalization)
- (Numerical) self-consistency checks via error analysis and R_b variation

See: Lepage, „How to renormalize the Schrödinger equation“, nucl-th/9607029 and talk@INT in 2000

Chiral EFT for nuclei

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$$T(\vec{p}\,', \vec{p}) = V_{2\mathrm{N}}(\vec{p}\,', \vec{p}) + m \int \frac{d^3k}{(2\pi)^3} \frac{V_{2\mathrm{N}}(\vec{p}\,', \vec{k})\, T(\vec{k}, \vec{p})}{p^2 - k^2 + i\epsilon} \quad \text{with} \quad V_{2\mathrm{N}} = \alpha \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \ldots$$

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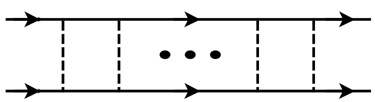

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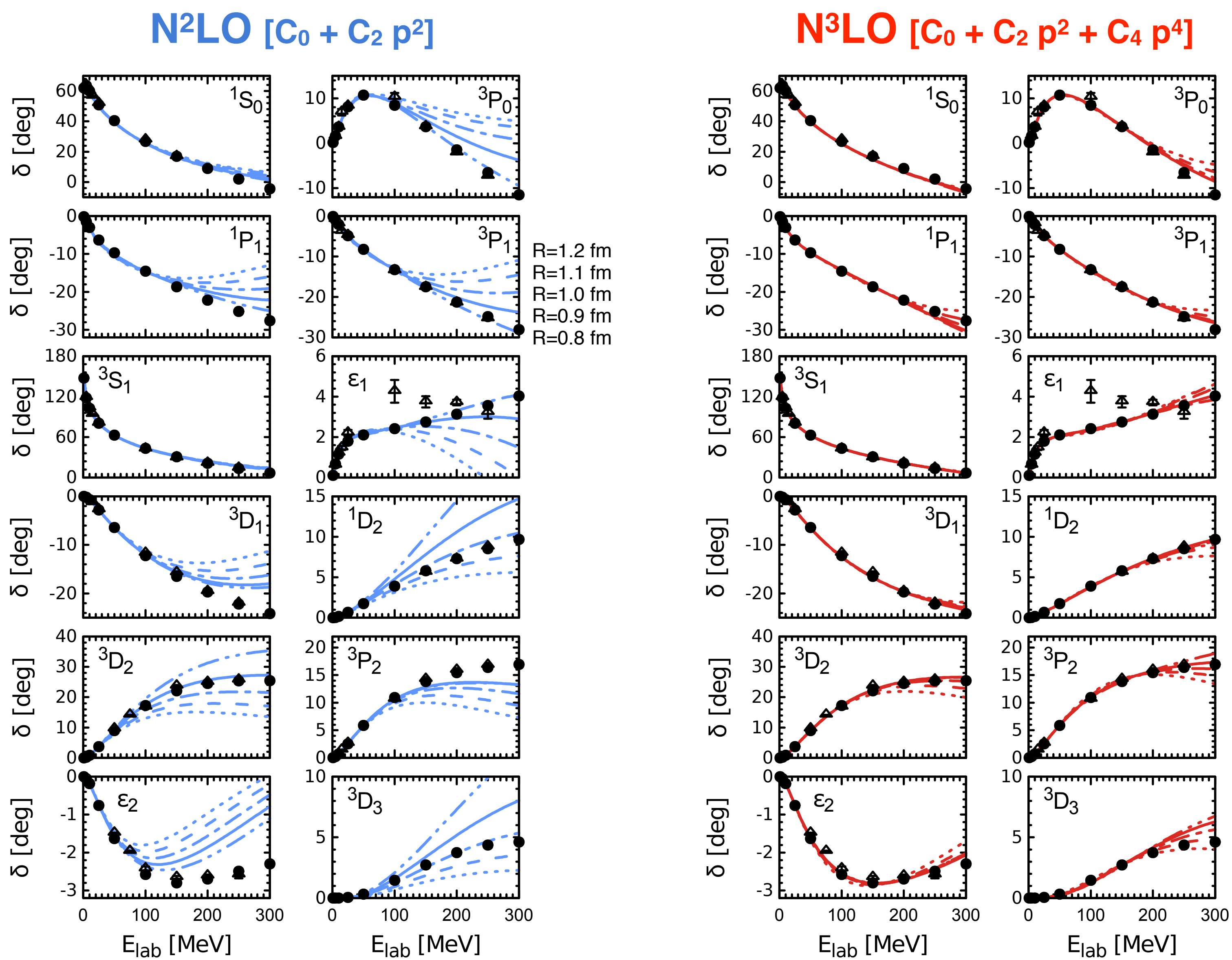
See: Lepage, „How to renormalize the Schrödinger equation“, nucl-th/9607029 and talk@INT in 2000

Do calculated observables show residual regulator dependence?

Yes, this is an unavoidable feature of this approach. The residual cutoff dependence measures the impact of (neglected) higher-order contact terms and can be systematically eliminated by going to higher orders.

Chiral EFT for nuclei



Chiral EFT for nuclei

What expansion of the amplitude does this approach correspond to?

For π-less case/theory with known long-range forces, the expansion corresponds to ERE/MERE (regardless of the size of the scattering length). More generally, RG analysis? [see: Birse, Phil. Trans. Roy. Soc. Lond. A369 (2011) 2662]

Are there alternative approaches?

Yes! In particular, the RG analysis by Birse, studies by Pavon-Valderrama and Yang/Long suggest different specific pattern for contact operators…

Can these scenarios be tested/discriminated?

Yes, possibly by looking at the convergence pattern (requires high orders + uncertainty estimation) [for a related discussion, see: Grießhammer, arXiv:1511.00490]

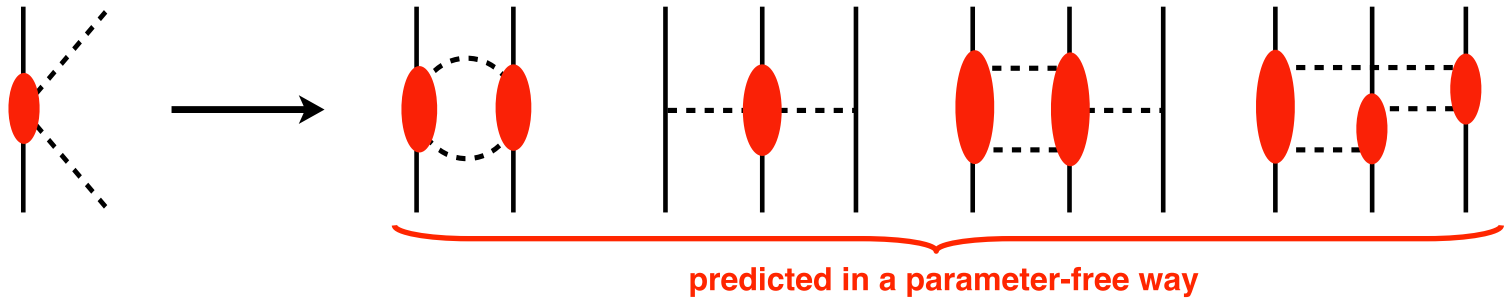
How to assess the theoretical uncertainty?

- Simple estimation of **truncation errors** via cutoff variation (not reliable…) or based on the available lower-order contributions [EE, Krebs, Meißner, EPJA 51 (2015) 53]. More rigorous treatment within a Bayesian approach [Furnstahl et al., PRC 92 (15) 024005].
- **Statistical uncertainties** in $C_i(R)$ have little impact [Ekström et al., J. Phys. G42 (15) 034003].
- Systematic error due to **uncertainties in πN LECs** needs to be analyzed

Chiral EFT for nuclei

Predictive power?

Long-range interactions are completely determined by the chiral symmetry & experimental information on πN scattering



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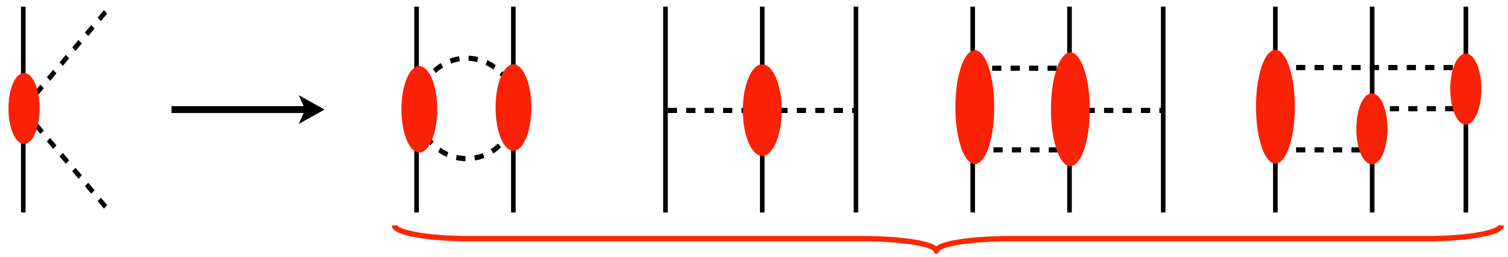

Energy bin	LO	NLO	N^2LO	N^3LO	N^4LO
neutron-proton data					
0 – 100 MeV	130.11	3.79	1.46	1.08	1.08
0 – 200 MeV	104.71	19.88	3.21	1.14	1.09
0 – 300 MeV	111.24	52.03	8.78	1.51	1.15
proton-proton data					
0 – 100 MeV	2046.58	33.68	6.67	0.86	0.84
0 – 200 MeV	1649.58	115.60	81.11	1.95	1.34
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	2 LECs	+ 7 + 2 IB LECs		+ 15 LECs	+ 1 IB LEC

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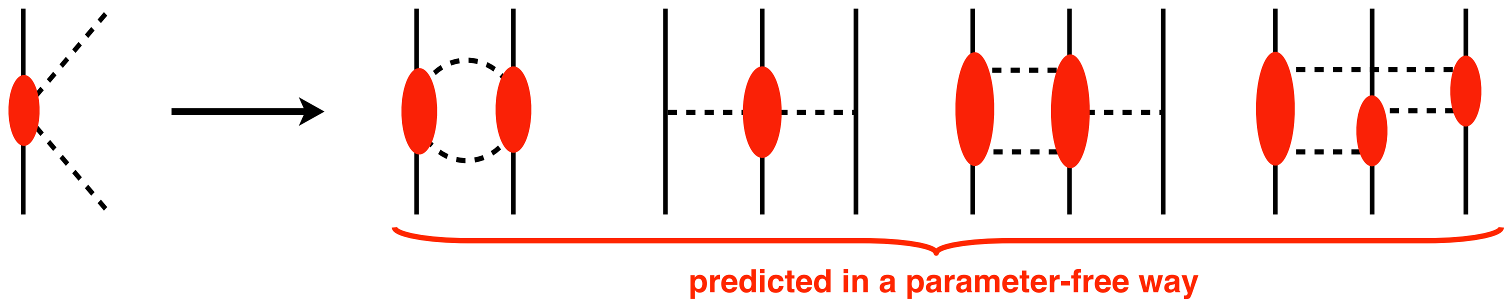

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no new LECs (NLO → N^2LO)

+1 LEC ($^1S_0$) (neutron-proton, N^3LO → N^4LO)

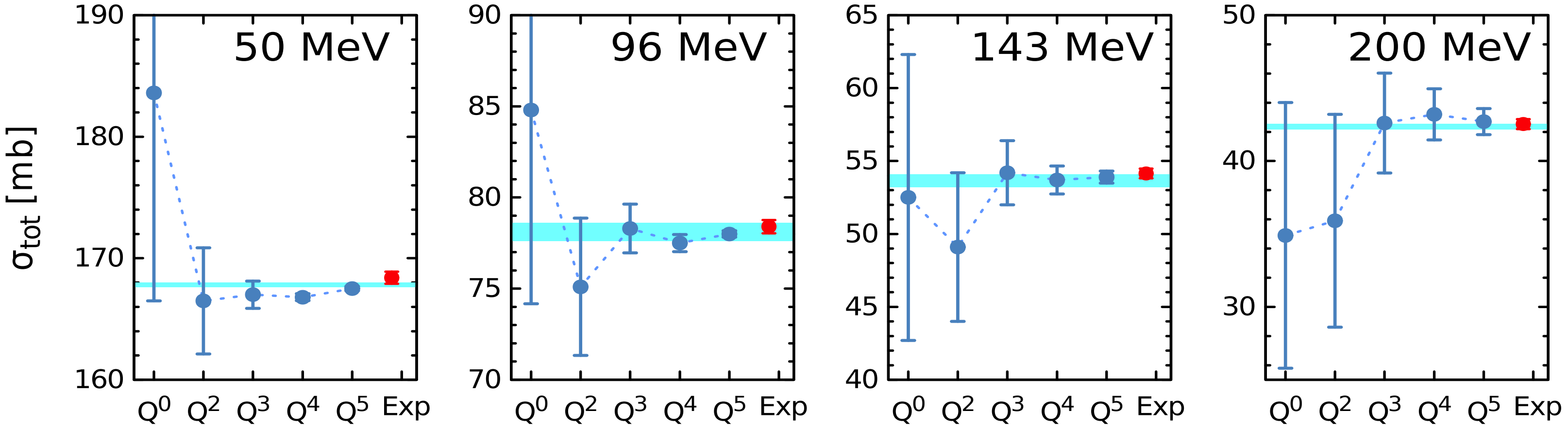
no new LECs (proton-proton, N^3LO → N^4LO)

Clear evidence of the (parameter-free) chiral 2π-exchange!

Chiral EFT for nuclei

What is the currently achievable accuracy?

In the 2N system, the results at N⁴LO (order Q^5, two loops!) are available.



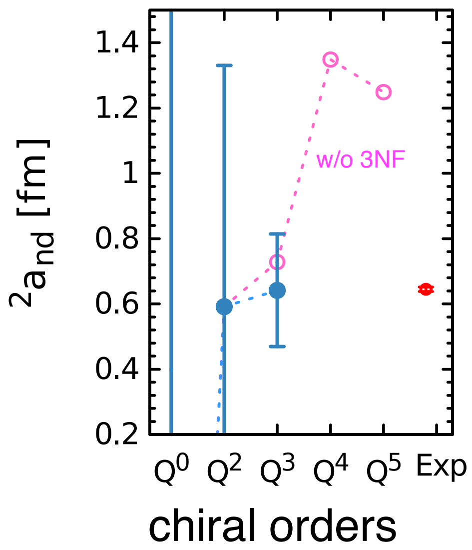
Scattering lengths and effective range parameters extracted from the data

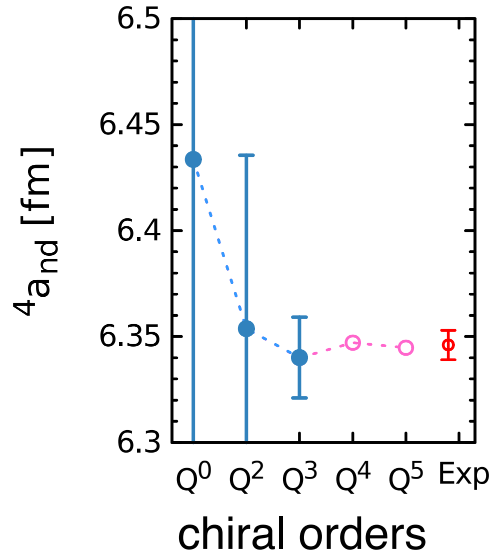
	predictions at N^4LO	Experimental/Empirical values
neutron-proton		
$a_{^1S_0}$ [fm]	$-23.733(6)$	$-23.740(20)$
$r_{^1S_0}$ [fm]	2.677(7)	2.77(5)
$a_{^3S_1}$ [fm]	5.419(1)	5.419(7)
$r_{^3S_1}$ [fm]	1.752(0)	1.753(8)
proton-proton		
$a_{^1S_0}$ [fm]	$-7.816(1)$	$-7.817(4)$
$r_{^1S_0}$ [fm]	2.773(2)	2.78(2)

Chiral EFT for nuclei

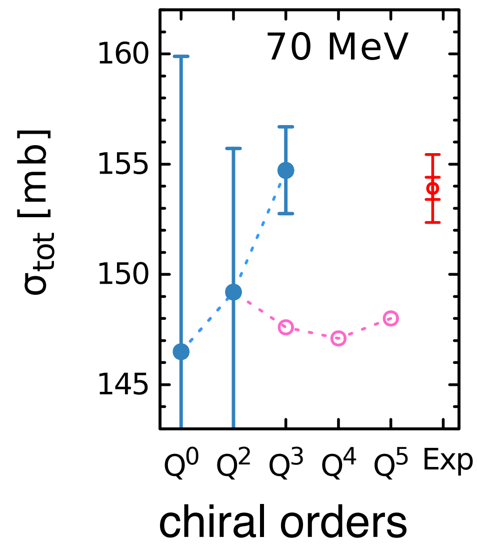
3NF so far only up to N²LO (N³LO in progress by the LENPIC Collaboration…)

nd scattering lengths **[R = 1.0 fm]**





nd σ_{tot} at 70 MeV **[R = 1.0 fm]**



Is chiral EFT always more efficient than pionless EFT?

Not necessarily… For low enough momenta p, the expansion in p/M_π is expected to converge faster than the chiral expansion in $\max(p/M_\pi, M_\pi/m_N)$.

Chiral EFT for hyper-nuclei?

Yes, see **Meißner, Haidenbauer, arXiv:1603.06429** for a review. Need input from lattice QCD!

The future

What are the frontiers/challenges for the near future?

Precision physics beyond the 2N system: challenge the theory

- Lots of predictive power (N^3LO contributions to the 3NF and 4NF are parameter-free, ^3H β-decay & μ-capture reactions are parameter-free up to N^3LO once the short-range 3NF@N^2LO is fixed, …)
- 3NF & long-standing puzzles in 3N continuum
- Push theory to heavier nuclei (underbinding? radii?)
- More reliable error analysis
- Test different power counting schemes

Chiral EFT as a tool to deal with nuclear effects when looking at physics of/beyond the SM (parity violation, EDM, 0νββ, proton charge radius,…)

EFT for lattice QCD (extrapolations), lattice QCD for EFT (quark mass dependence, „data“, …)

EFT for DFT