Predicting Dust Distributions in Protoplanetary Discs

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Abstract

We present the results of three-dimensional numerical simulations that include the effects of hydrodynamical forces and gas drag upon an evolving dusty gas disk. We briefly describe a new parallel, two phase numerical code based upon the smoothed particle hydrodynamics (SPH) technique in which the gas and dust phases are represented by two distinct types of particles. We use the code to follow the dynamical evolution of a population of grains in a gaseous protoplanetary disk in order to understand the distribution of grains of different sizes within the disk. Our "grains" range from metre to submillimetre in size and we discuss the implication for planet formation.

Introduction

Up until recently we had only one observation to test our theories of planet formation - our own Solar System. Now however with new planets and solar systems being identified at a rate approaching one a month, the observational constraints are much tighter and our lack of understanding of many aspects of the planet formation process is all too obvious.

At the most basic level we know that micron size grains of dust in the pre-solar nebula clump and coagulate together to form planets, objects $10^{13} - 10^{14}$ times larger. Planet formation is a multi-stage process, taking us from dust grain to boulder to planetesimal to planetary embryo. Analytical arguments[1] have given us constraints on the time scales for each stage but little more.

Here we are primarily looking at the initial phase - from microns to metres.

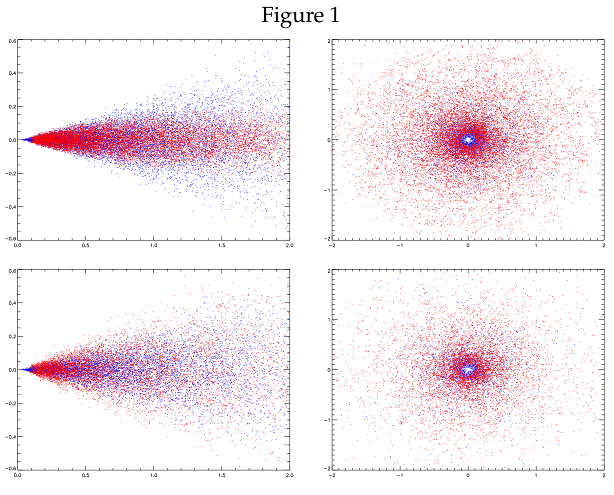
Theoretical models have changed much in recent years and the simple picture[1, 2] of a thin dust layer accumulating at the disk midplane, becoming gravitationally unstable, and breaking into planetesimals now seems unlikely. Assumptions of laminar flow with little turbulence are especially fragile[3, 4, 5].

Recently, Goodman & Pindor[6] showed that globally acting turbulent drag may cause radial instabilities in a dust layer, even if the disk self-gravity is negligible. In their perturbed disks, they predict that over-dense rings form within an orbital period, with the ring thickness similar to the thickness of the dust layer. These rings eventually collapse into planetesimals in the kilometre size range.

Ultimately, the equations governing the evolution of a dusty disk are non-linear and need to account for a staggering range of physics and chemistry (gas pressure, dust drag, surface chemistry and collision physics, radiative transfer and magnetic fields). Hence it is to numerical simulations that we turn to in order to learn more about planet formation.

In this paper we present the first three-dimensional numerical simulations that include the effects of hydrodynamical forces, self-gravity and gas drag upon an evolving dusty gas disk. We use the Smoothed Particle Hydrodynamics (SPH)[7, 8, 9] technique which uses a collection of particles to approximate a fluid. Because of the underlying simplicity of the SPH technique it is easy to incorporate a spectrum of dust sizes with time varying properties. However, at this early stage of investigation we include single classes of dust of various sizes.

Figure 1



Top to bottom 100 μm and 1 μm dust. Left: disc seen from the side. Right: disc seen from above.Red is dust, blue is gas

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Parallel Tree and Multi-Phase SPH

We have written two slightly different SPH codes describing these multiphase fluids. One was developed by Robin Humble and includes self gravity and dust. It is written in C and parallelised using MPI. The other one was developed by Laure Barrière-Fouchet starting from James Murray's code[10] and includes dust but not self gravity. It is written in fortran and parallelised using OpenMP. Gas and dust phases are represented by two distinct types of particles. Only gas particles feel a pressure force and are affected by viscosity. The gas-dust combination feel the drag force and also a mixed term seen in both the dust and gas particle momentum equations. The only dust-dust interaction is through gravity.

The equation of motion is given by[11, 12]:

$$\frac{d\mathbf{v}}{dt} = -\frac{1}{\rho}\nabla P \pm \frac{K}{\rho}(\mathbf{v}_{\rm dust} - \mathbf{v}_{\rm gas}) - \nabla\phi$$

K can take the form of (for example) Epstein (the drag is due to thermal agitation of the particles), Stokes (the drag is due to the wake made by the particle into the gas flow) or turbulent drag. Epstein drag is appropriate for the gas mean free path found in protoplanetary disks outside about 1 AU:

$$K = \frac{\rho\theta C_{\rm drag} c}{r_{\rm d}}$$

where $C_{\rm drag} \approx 1$, c is the local sound speed, $r_{\rm d}$ is the radius of the dust grains, and θ is a volumetric "void fraction" which is a measure of the proportions of gas and dust present.

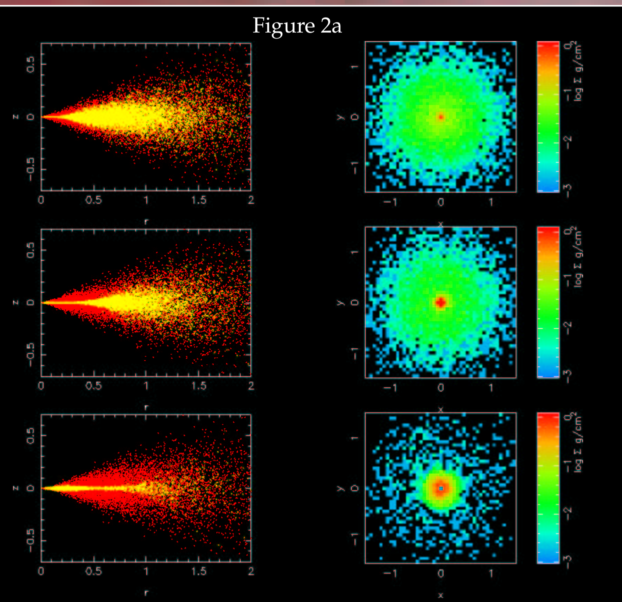
For simplicity we have assumed that the dust grains do not evaporate or coagulate, and that the gas does not condense. We take the dust grains to be incompressible.

Humble's code[14] relies upon a tree-code[15] that uses the tree for neighbour searching in SPH, and for disk self-gravity, whereas Barrière-Fouchet's code is based on a less time-consuming linked-list that only finds close neighbours.

The codes have been extensively compared and the main differences arise from the fact that Humble's code allows higher resolution simulations and Barrière-Fouchet's code seems to be able to simulate smaller size particles.

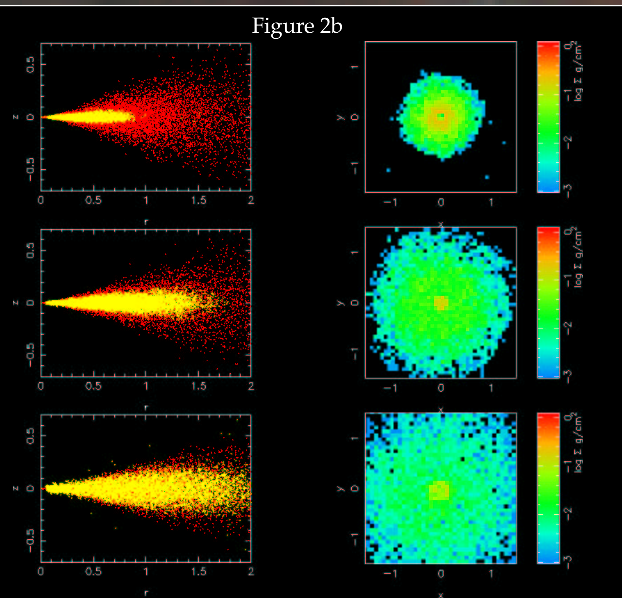
Despite the lack of self gravity in Barrière-Fouchet's code, they both give the same results for the simulations of particles sizes ranging from 10m to 10 μm. Figures 2a and 2b show Humble's results for these sizes, and Figure 1 shows Barrière-Fouchet's results for 100 μm (for the sake of comparison) and 1 μm particles.

Figure 2a



top to bottom - 1m, 0.1m, 1cm dust. $t = t_{\rm final} = 70$. 2x12.5k particles
yellow is dust, red is gas

Figure 2b



top to bottom - 1mm, 0.1mm, 10μm dust. $t = t_{\rm final} = 70$. 2x12.5k particles
yellow is dust, red is gas

The Simulations

In Humble's code, initial conditions mimic a gravitationally bound region in a giant molecular cloud. A prolate spheroid of rotating, self-gravitating gas with a star at the centre is spun down until the gas reaches near equilibrium. The dust is then added (overlaid on the 3D flared gas disk) and the simulations are evolved for 70 time units (about 11(70) orbits at 100(30)AU) which corresponds to about 10^4 years.

In Barrière-Fouchet's code the initial state is a Keplerian gaseous disc already in equilibrium. It relaxes over 10000 timesteps (about 8000 years), then dust is added. Each dust particle is put on the top of a gas particle with the same velocity. To allow precise comparisons, the parameters are the same in both codes.

Model parameters
$M_\star = 1.0M_\odot$
$M_{\rm disk} = 0.01M_\star$
$M_{\rm dust} = 0.01M_{\rm disk}$
$R_{\rm disk} = 100AU$

Grain parameters
$\rho_{\rm d} = 2.4g/cm^3$
$r_{\rm d} = $ 1m, 10^{-1}m, 10^{-2}m, 10^{-3}m, 10^{-4}m, 10^{-5}m

α Disk parameters
$c(R) = c_0(R/100AU)^{-3/8}$
$T(R) \propto (R/100AU)^{-3/4}$
$H/R = 0.1$ at $100AU$
$\gamma = 5/3$
isothermal EOS
no evaporation or coagulation

Code parameters
Low res 2x12500 particles
High res 2x125000 particles
$\varepsilon = 1.0AU$ (self-gravity)
$\alpha_{\rm SPH} = 0.1$
$\beta_{\rm SPH} = 0.0$

Discussion

For large (10m) and small (μm) dust sizes, the dust distribution is expected to stay close to the initial flared disk. Large grains (boulders) are weakly coupled to the gas, and if started in Keplerian motion, they will remain there. Conversely, tiny grains are so strongly coupled to the gas that they are essentially co-moving (on the timescales we are examining here). In both extremes, little dust distribution evolution occurs. The 0.1mm to 10cm interval is where all the (short timescale) interesting dynamics takes place.

In the r-z plots of figures 2a and 2b, significant deviation from the initially flared disk occurs in the inner regions of the 1m and 10cm plots, in the mid regions (from $r = 0.6$ to $r = 0.9$) of the 1cm plot, and in the outer regions of the 1mm and smaller plots. These regions are where the Epstein drag is sufficiently strong that it is able to efficiently remove energy from the dust, and yet not so strong that the dust is tightly coupled to the gas.

The 10cm and 1cm dust exhibits the highest surface density (red in the x-y plots of figures 1a and 1b) and so are probably the most interesting from a planet formation viewpoint. However, the most striking feature of these surface density maps is seen in the 1cm and 1mm images which have a small disk radius and in the case of 1mm dust, abrupt truncation at the outer edge. The explanation for this is merely that the outer edges of these disks is where the efficient drag dissipation occurs. The dust in these outer regions is thus rapidly driven to the midplane and fed inwards to the co-moving regime, and hence the disk truncates.

Summary

We have taken the initial steps to understanding dusty disk dynamics with full 3D hydrodynamics. The Lagrangian nature of the code means that it is computationally trivial to add empirical grain growth models, to change equations of state, and to follow the grain temperature and density histories. This means it is potentially feasible to determine not just the location and mass of planetesimals, but also their likely composition.

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