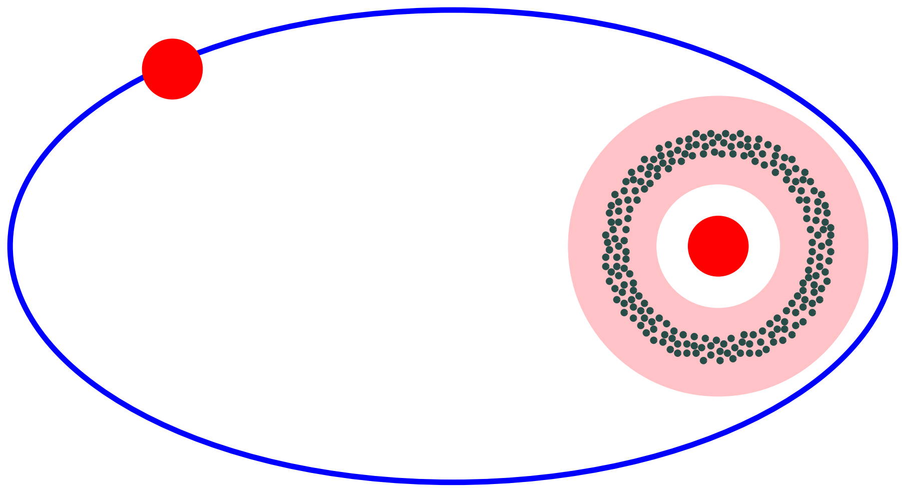
Planetesimal Dynamics in Binary Stellar Systems



Eiichiro Kokubo
National Astronomical Observatory
Shigeru Ida
Tokyo Institute of Technology

A 100% Unix-Based Presentation

Motivation

Observation

Binaries: more common than single stars

Disks: common among young binaries

Planets: At least 5 extrasolar planets in binaries

Motivation

Observation

Binaries: more common than single stars

Disks: common among young binaries

Planets: At least 5 extrasolar planets in binaries

$\Updownarrow$

Standard Scenario of Planet Formation

Single Star + Protoplanetary Disk $\Rightarrow$ Planetary System

Motivation

Observation

Binaries: more common than single stars

Disks: common among young binaries

Planets: At least 5 extrasolar planets in binaries

$$\Updownarrow$$

Standard Scenario of Planet Formation

Single Star + Protoplanetary Disk $\Rightarrow$ Planetary System

$$\Downarrow$$

A Theory for Planet Formation in Binaries

Binary + Protoplanetary Disk $\Rightarrow$ Planetary System

Low-Mass PMS Binaries

Frequency

$f_{\rm binary} \gtrsim 0.5$ for systems with $M_1 + M_2 \lesssim 3M_\odot$

Properties

- Mass ratio of secondary to primary $0 \lesssim M_2/M_1 \lesssim 1$
- Orbital characteristics:
  - $1{\rm day} \lesssim P \lesssim 10^8{\rm days}$ with $\langle P \rangle \simeq 10^5{\rm days}$ $(10^{-2}{\rm AU} \lesssim a \lesssim 10^4{\rm AU}$ with $\langle a \rangle \simeq 10^2{\rm AU})$
  - $0 \lesssim e \lesssim 1$ and $e_{\rm max}$ increases with P
- Disk frequency $f_{\rm disk} \simeq 0.5$
  - circumstellar disk for $a \gtrsim$ a few AU
  - circumbinary disk for $a \lesssim$ a few AU

Previous Studies

Planetesimal Dynamics

Heppenheimer (1978)

- perturbation by secondary and gas drag
- 2-D, no self-gravity

Marzari & Scholl (2000)

- perturbation by secondary, gas drag, and collision
- 2-D, no self-gravity

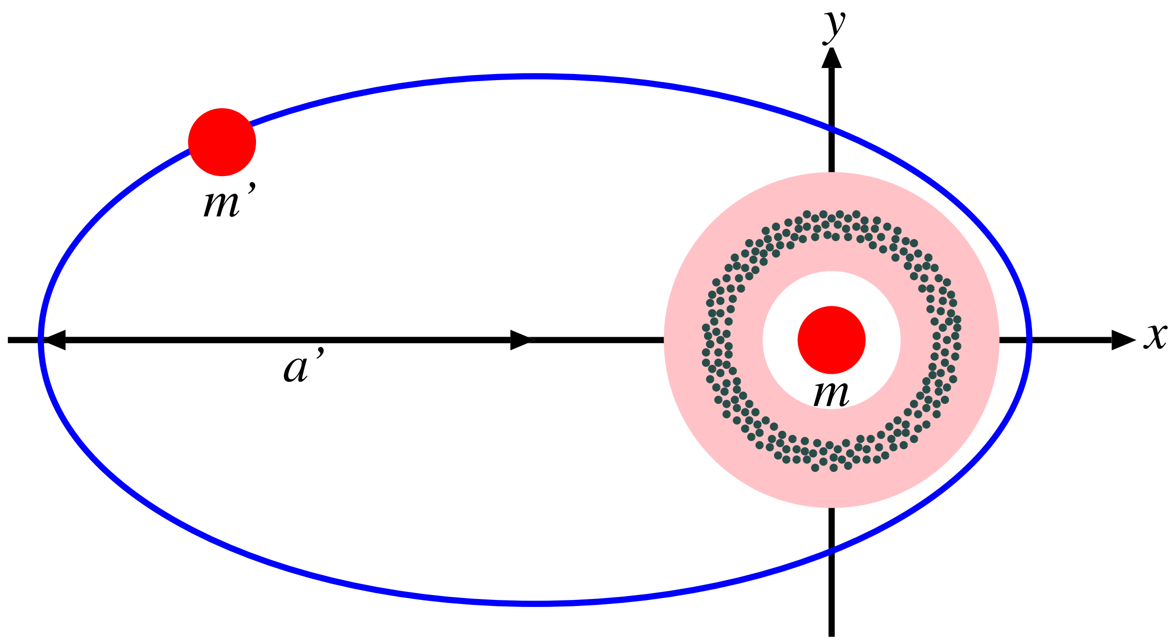
Planetesimal Accretion

Barbieri, Marzari, & Scholl (2002)

Quintana et al. (2002)

- late accretion stage from protoplanets to planets

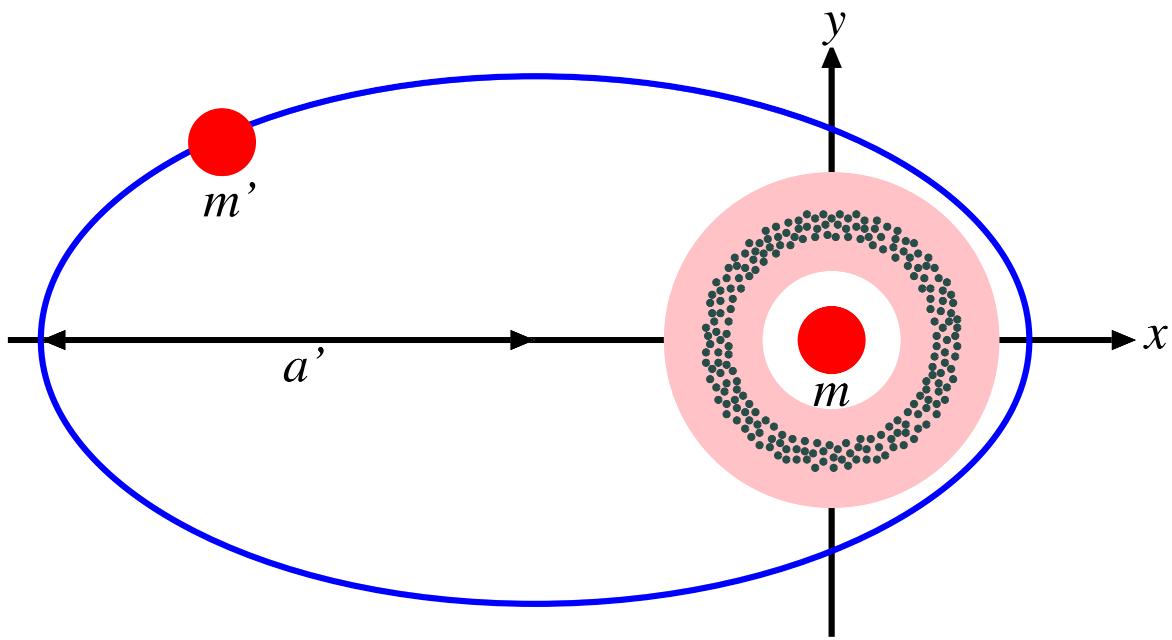
Equation of Motion for Planetesimals



$$\frac{d\boldsymbol{v}_i}{dt} = \underbrace{-Gm\frac{\boldsymbol{x}_i}{|\boldsymbol{x}_i|^3}}_{\text{gravity of primary}} + \underbrace{Gm'\frac{\boldsymbol{x}'-\boldsymbol{x}_i}{|\boldsymbol{x}'-\boldsymbol{x}_i|^3}}_{\text{gravity of secondary}} + \underbrace{\sum_{j\neq i}^{N} Gm_j\frac{\boldsymbol{x}_j-\boldsymbol{x}_i}{|\boldsymbol{x}_j-\boldsymbol{x}_i|^3}}_{\text{mutual gravity}}$$

$$+ \underbrace{\boldsymbol{f}_{\rm gas}}_{\text{gas drag}} + \underbrace{\boldsymbol{f}_{\rm col}}_{\text{collision effect}} \underbrace{-Gm'\frac{\boldsymbol{x}'}{|\boldsymbol{x}'|^3}}_{\text{indirect term}}$$

Equation of Motion for Planetesimals



$$\frac{d\boldsymbol{v}_i}{dt} = \underbrace{-Gm\frac{\boldsymbol{x}_i}{|\boldsymbol{x}_i|^3}}_{\text{gravity of primary}} + \underbrace{Gm'\frac{\boldsymbol{x}'-\boldsymbol{x}_i}{|\boldsymbol{x}'-\boldsymbol{x}_i|^3}}_{\text{gravity of secondary}} + \underbrace{\sum_{j\neq i}^{N} Gm_j\frac{\boldsymbol{x}_j-\boldsymbol{x}_i}{|\boldsymbol{x}_j-\boldsymbol{x}_i|^3}}_{\text{mutual gravity}}$$

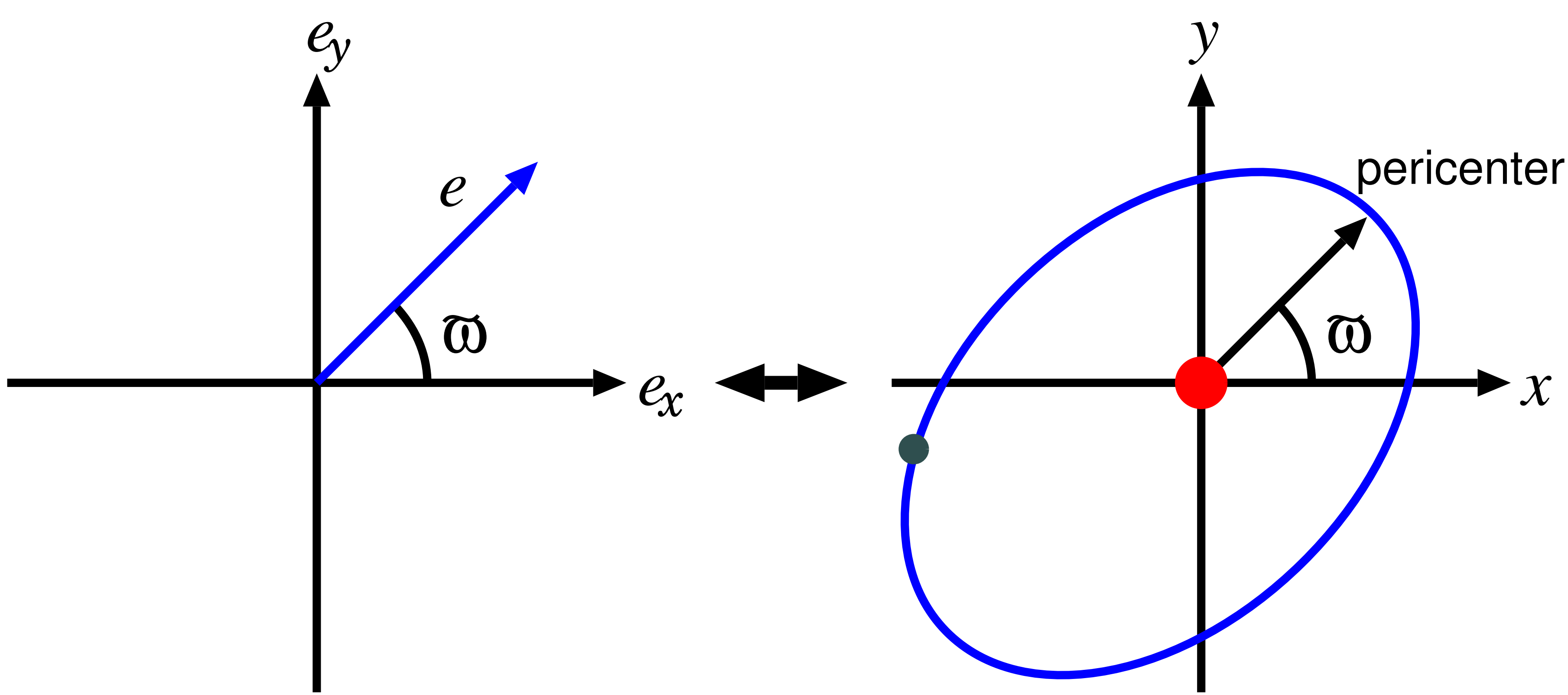
$$\underbrace{-Gm'\frac{\boldsymbol{x}'}{|\boldsymbol{x}'|^3}}_{\text{indirect term}}$$

Eccentricity Vector

$$\boldsymbol{e} = (e_x, e_y) = (e\cos\varpi, e\sin\varpi) = (k, h)$$

e : eccentricity

ϖ : longitude of pericenter



Relative Velocity and Eccentricity

Relative Velocity

$$|\boldsymbol{v}_{ij}| = |\boldsymbol{v}_j - \boldsymbol{v}_i| \simeq |\boldsymbol{e}_j - \boldsymbol{e}_i| v_{\mathrm{K}} = |\boldsymbol{e}_{ij}| v_{\mathrm{K}} \simeq \sigma_e v_{\mathrm{K}}$$

Eccentricity Dispersion

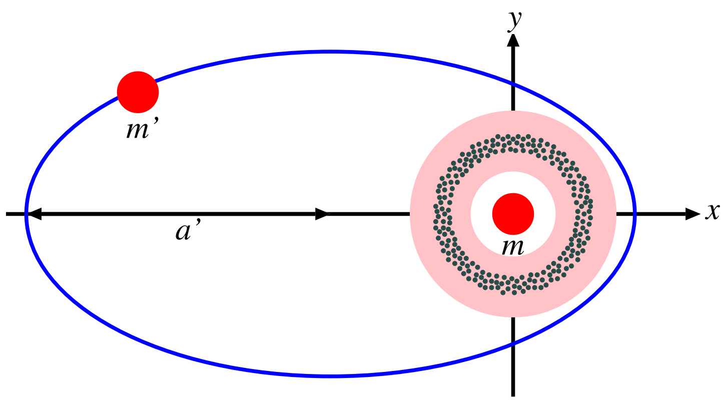
$$\sigma_e = \sqrt{\langle |\boldsymbol{e}_i|^2 \rangle - |\langle \boldsymbol{e}_i \rangle|^2}$$

controls

- the growth mode
- the growth time scale

Numerical Experiments

Model



primary	m
secondary	m', a', e', $i' = 0$, ϖ', n'
planetesimal	$m_{\rm p}$, a, e, i, ϖ, Ω, n

$m = m' = M_\odot$, $a' = 25[\mathrm{AU}]$, $e' = 0.5$
(α Cen model)

Initial Conditions

$N = 1000$, $m_{\rm p} = 10^{24}$g

$a = 1\mathrm{AU}(\Delta a = 0.1\mathrm{AU})$, $\sigma_e = 2\sigma_i$

(minimum-mass disk model: $\Sigma = 10\mathrm{gcm}^{-2}$)

Method of Calculation

4th-order Hermite integrator with GRAPE-6

Disturbing Function of Secondary

Assumptions

$$a/a' \ll 1 \text{ ,} m_{\rm p} \ll m + m' \text{, } e, i \ll 1$$

Disturbing Function

$$R \simeq \frac{m'}{m+m'} n'^2 a^2 \left[\frac{1}{4(1-e'^2)^{3/2}} \left(1 + \frac{3}{2}e^2\right) - \frac{3}{2(1-e'^2)^{3/2}} \sin^2 \frac{i}{2} - \frac{15}{16}\frac{a}{a'}\frac{e'}{(1-e'^2)^{5/2}} e \cos(\varpi - \varpi') \right]$$

Lagrange's Planetary Equation

New Variables

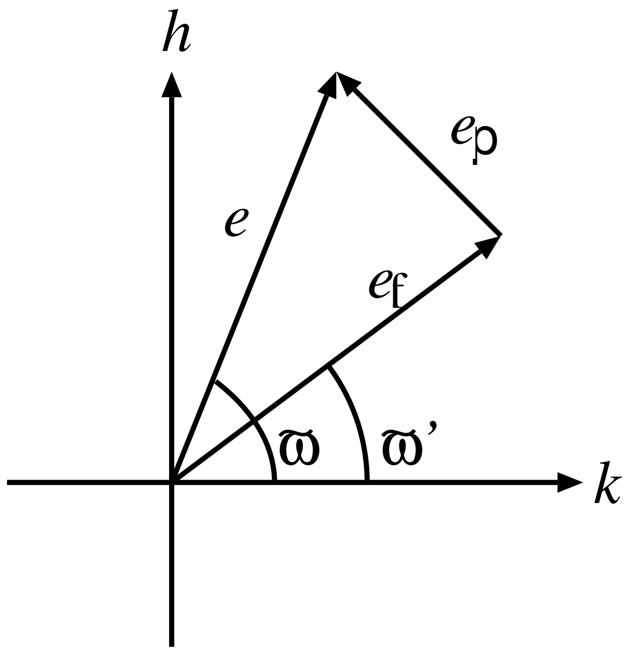
$$h = e \sin \varpi, \qquad k = e \cos \varpi$$
$$p = \sin \frac{i}{2} \sin \Omega, \quad q = \sin \frac{i}{2} \cos \Omega$$

Linearized Planetary Equation

$$\begin{cases} \dfrac{dh}{dt} = \dfrac{1}{na^2} \dfrac{\partial R}{\partial k} = Ak - B \cos \varpi' \\ \dfrac{dk}{dt} = -\dfrac{1}{na^2} \dfrac{\partial R}{\partial h} = -Ah - B \sin \varpi' \end{cases} \qquad \begin{cases} \dfrac{dp}{dt} = \dfrac{1}{4na^2} \dfrac{\partial R}{\partial q} = -Aq \\ \dfrac{dq}{dt} = -\dfrac{1}{4na^2} \dfrac{\partial R}{\partial p} = Ap \end{cases}$$

$$A = \frac{3}{4} \frac{m'}{m + m'} \frac{n'^2}{n} \frac{1}{(1 - e'^2)^{3/2}}$$
$$B = \frac{15}{16} \frac{m'}{m + m'} \frac{n'^2}{n} \frac{a}{a'} \frac{e'}{(1 - e'^2)^{5/2}}$$

Solution to Planetary Equation



$e_{\rm p}$	proper eccentricity
$e_{\rm f}$	forced eccentricity
$i_{\rm p}$	proper inclination
$i_{\rm f} = 0$	forced inclination

$$\begin{cases} h = e_{\rm p} \sin(At + \varpi_{\rm p}) + e_{\rm f} \sin \varpi' \\ k = e_{\rm p} \cos(At + \varpi_{\rm p}) + e_{\rm f} \cos \varpi' \end{cases}$$

$$\begin{cases} p = \sin \dfrac{i_{\rm p}}{2} \sin(-At + \Omega_{\rm p}) \\ q = \sin \dfrac{i_{\rm p}}{2} \cos(-At + \Omega_{\rm p}) \end{cases}$$

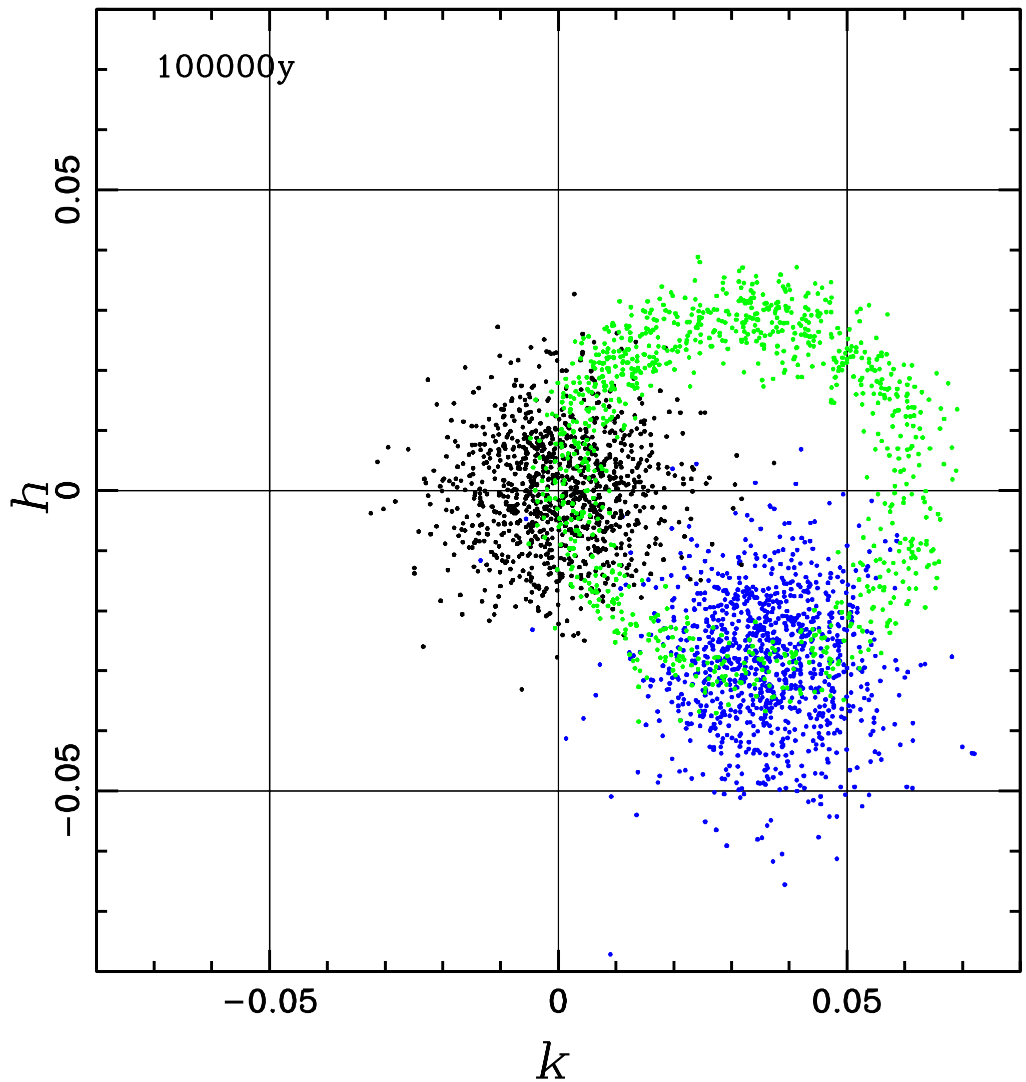
$$e_{\rm f} = \frac{B}{A} = \frac{5}{4} \frac{a}{a'} \frac{e'}{(1 - e'^2)}$$

Animations

No Self-Gravity Case

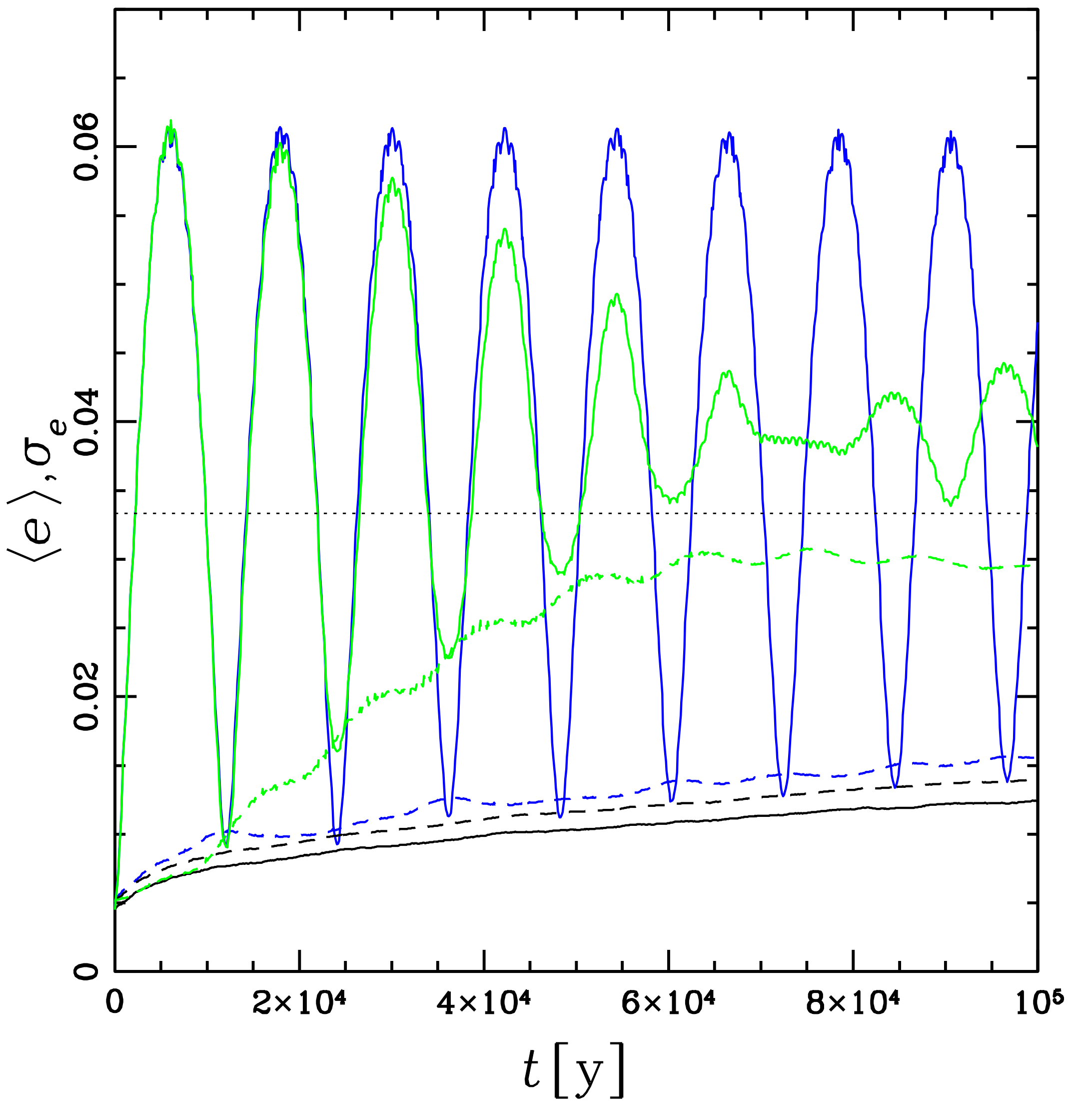
Self-Gravity Case

Evolution of Eccentricity



	blue	green	bla
secondary	○	○	
self-gravity	○	×	

Evolution of Eccentricity

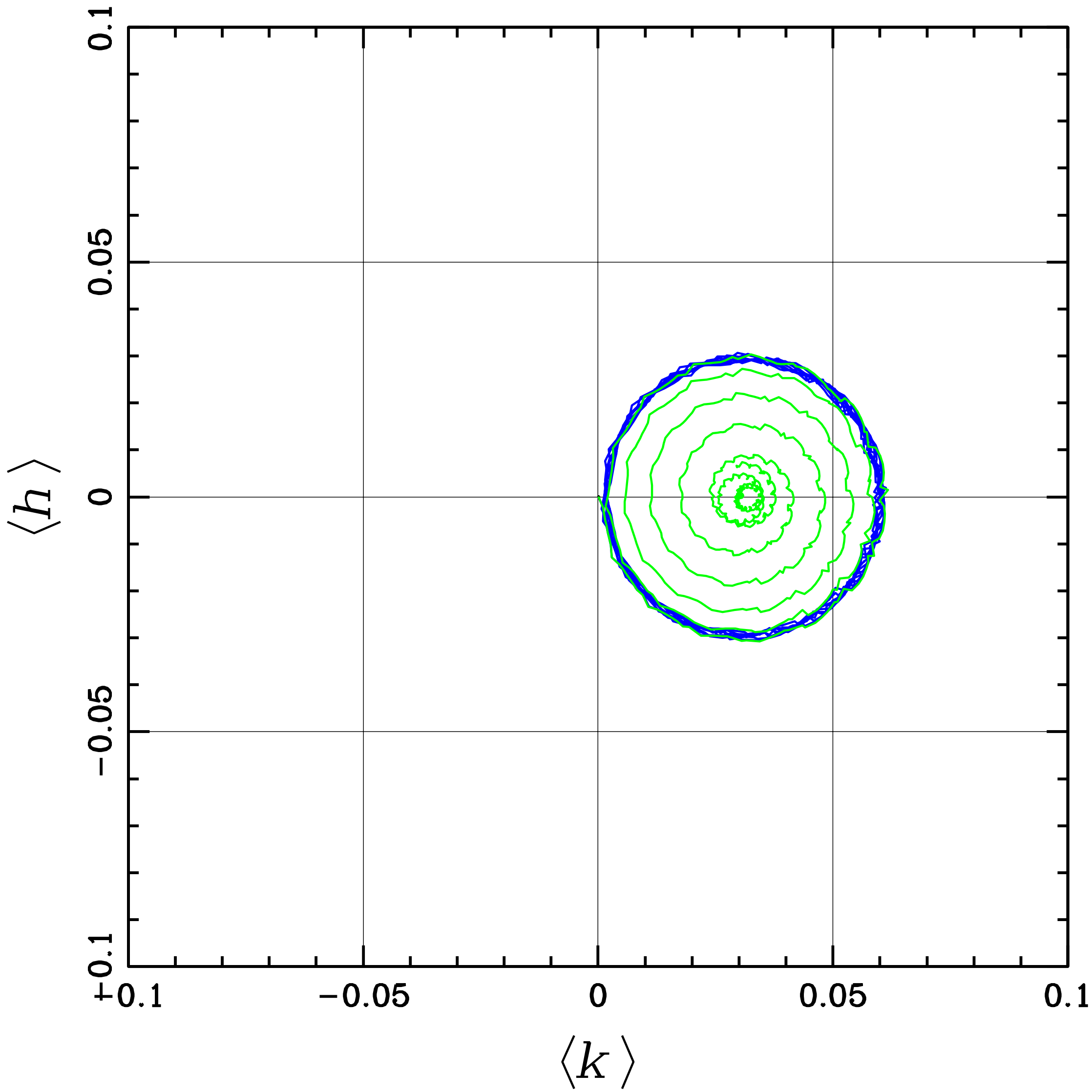


	blue	green	bl
secondary	○	○	
self-gravity	○	×	○

solid: $\langle e \rangle$

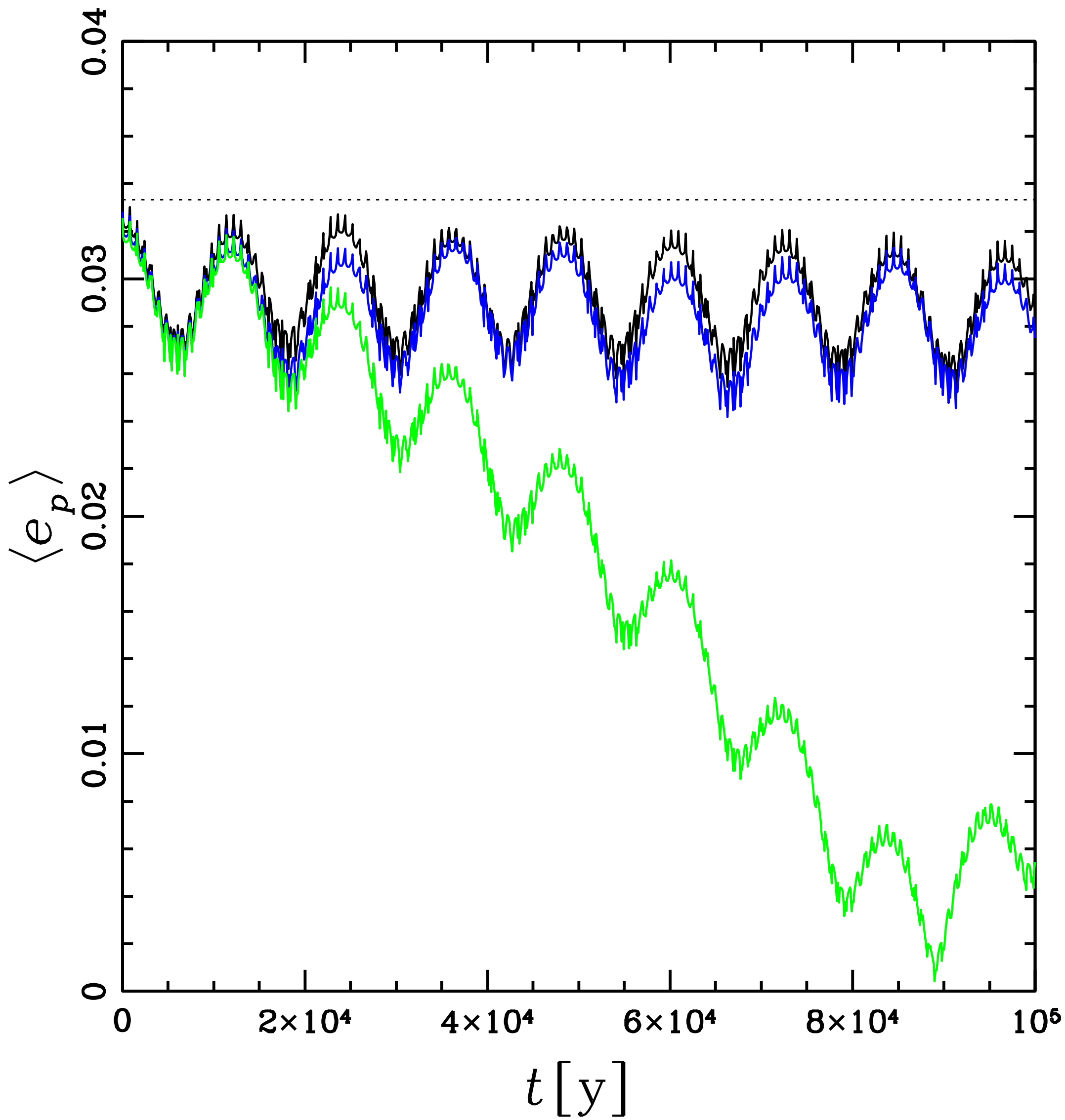
dashed: σ_e

Evolution of Eccentricity



	blue	green	bl
secondary	○	○	
self-gravity	○	×	

Evolution of Eccentricity



	black	blue	green
σ_e	0.005	0.02	0.03

Time Scales

Time Scale of Shear

$$T_{\mathrm{shear}} \simeq \frac{2\pi}{\dfrac{dA}{da}\Delta a} = \frac{16\pi}{9} G^{-1/2} m'^{-1} a'^{3} (1 - e'^{2})^{3/2} m^{1/2} a^{-1/2} \Delta a^{-1}$$

Time Scale of Two-Body Relaxation

$$T_{\mathrm{relax}} \simeq \frac{\sigma^3}{\sqrt{2}\pi G^2 n_m m_{\mathrm{p}}^2 \ln \Lambda} \simeq \frac{m^{3/2} \sigma_e^4}{\sqrt{2}\pi G^{1/2} \Sigma m_{\mathrm{p}} a^{1/2} \ln \Lambda}$$

n_m number density

Σ surface mass desnsity

Condition for Orbital Alignment

Preliminary

Mean Orbital Separation

$$\Delta a = \frac{1}{2\pi a n_{\rm s}} = \frac{m_{\rm p}}{2\pi a \Sigma}$$

Condition

$$T_{\rm relax} \ll T_{\rm shear}$$

$$\Downarrow$$

$$\sigma_e < \sigma_e^{\rm crit} \propto m^{-1/4} m'^{-1/4} a'^{3/4} (1 - e'^2)^{3/8} \Sigma^{1/2} a^{1/4}$$

Summary

Towards a Theory for Planet Formation in Binaries

⇓

Planetesimal Dynamics

Planetesimal orbits perturbed by
secondary and self-gravity

⇓

Orbital alignment of planetesimals

⇓

Small relative velocity between planetesimals
(same relative velocity as the single star case)

⇓

Same growth mode and time scale
as the single star case

Things to Do

Near Future (While at KITP?)

- To perform more simulations with other parameters
- To derive the condition for orbital alignment
- To include gas drag

Next Step

- To include accretion

Application

- Satellite-ring interaction
- Eccentric ring