

KITP , Feb.12, 2004

Criteria for apsidal and nodal resonances in multiple planetary systems

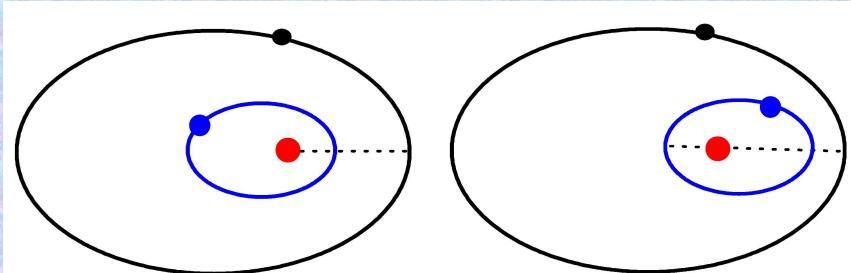
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Apsidal Resonance □ $(\dot{\omega}_1 \approx \dot{\omega}_2)$



aligned apsidal resonance

antialigned apsidal resonance

For the observed about 13 multiple planetary systems □ at least 6 pairs are in apsidal resonance □ Especially HD12661 system is believed in anti-aligned apsidal resonance.

Question:

(1) A easy criterion? (2) relation between apsidal resonance and stability of the system □

What we did

- Obtained analytical criteria for apsidal resonance
- Applied to HD12661 and 47Uma system
- Studied the stability of orbits in apsidal resonance for the HD12661 system
- Extended to the nodal resonance case

1□Criteria for Apsidal resonance

Notations□

the host star, the inner and outer planets

masses m_0, m_1, m_2

orbital elements $(a_1, e_1, \varpi_1, M_1)$
 $(a_2, e_2, \varpi_2, M_2)$

Apsidal Resonances

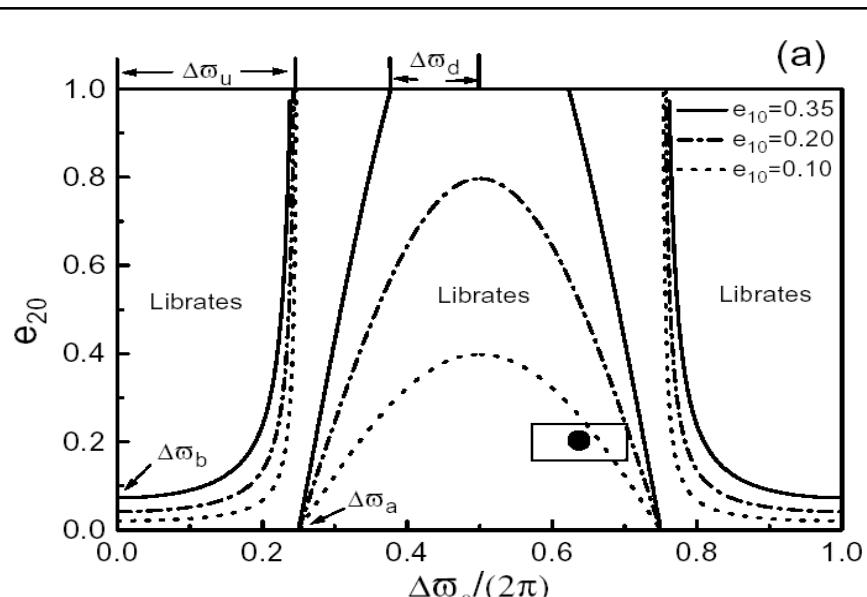
According to the Laplace-Larange
secular perturbation theory:

$$\frac{e_{20}}{e_{10}} < -\frac{5}{2} \frac{q\alpha^{3/2}(1 - \frac{1}{8}\alpha^2)}{1 - q\alpha^{1/2}} \cos \Delta\varpi_0,$$

$$\frac{e_{20}}{e_{10}} > \frac{2}{5} \frac{1 - q\alpha^{1/2}}{\alpha(1 - \frac{1}{8}\alpha^2)} \frac{1}{\cos \Delta\varpi_0} > 0.$$

$$q = m_1/m_2 \quad \alpha = a_1/a_2$$

These are the criterion for the occurrence of
apsidal resonance \square



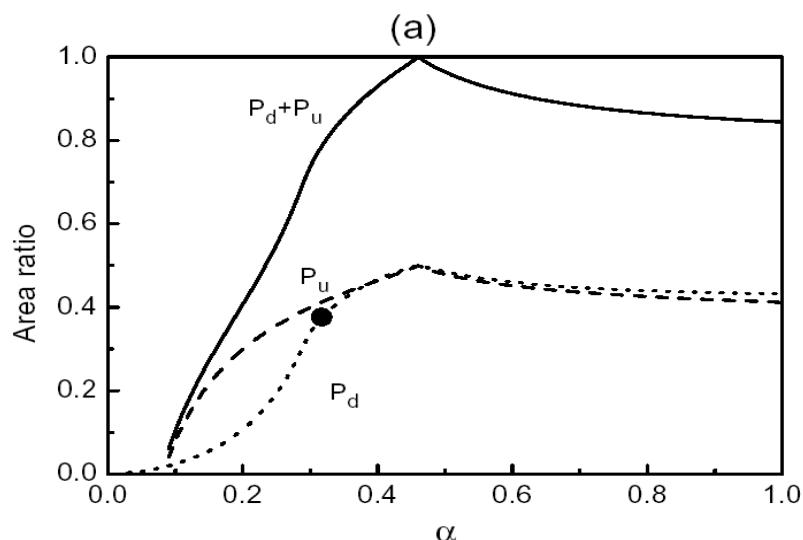
Librating regions for HD12661

Apsidal Resonances

Area ratio (libration region to the total)□

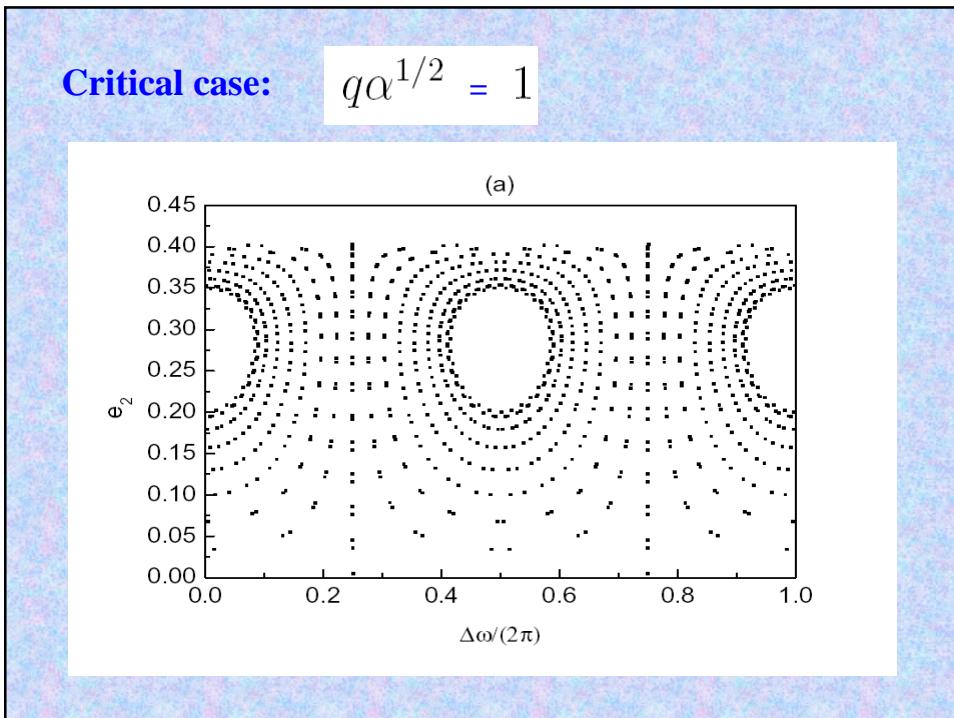
$$\begin{aligned} P_d &= \frac{1}{\pi} [\Delta\varpi_d + \int_{\Delta\varpi_a}^{\Delta\varpi_a + \frac{\pi}{2} - \Delta\varpi_d} Q_d \cos \Delta\varpi \, d\Delta\varpi] \\ &= \frac{1}{\pi} [\Delta\varpi_d + Q_d(1 - \sin \Delta\varpi_d)] \end{aligned}$$

$$\begin{aligned} P_u &= \frac{1}{\pi} [\Delta\varpi_u - \int_{\Delta\varpi_b}^{\Delta\varpi_b + \Delta\varpi_u} Q_u \frac{1}{\cos \Delta\varpi} \, d\Delta\varpi] \\ &= \frac{1}{\pi} [\Delta\varpi_u - Q_u \ln(\frac{1+\sin \Delta\varpi_u}{\cos \Delta\varpi_u})] \end{aligned}$$



**Variation of area ration with alpha fom
HD12661 system**

Apsidal Resonances

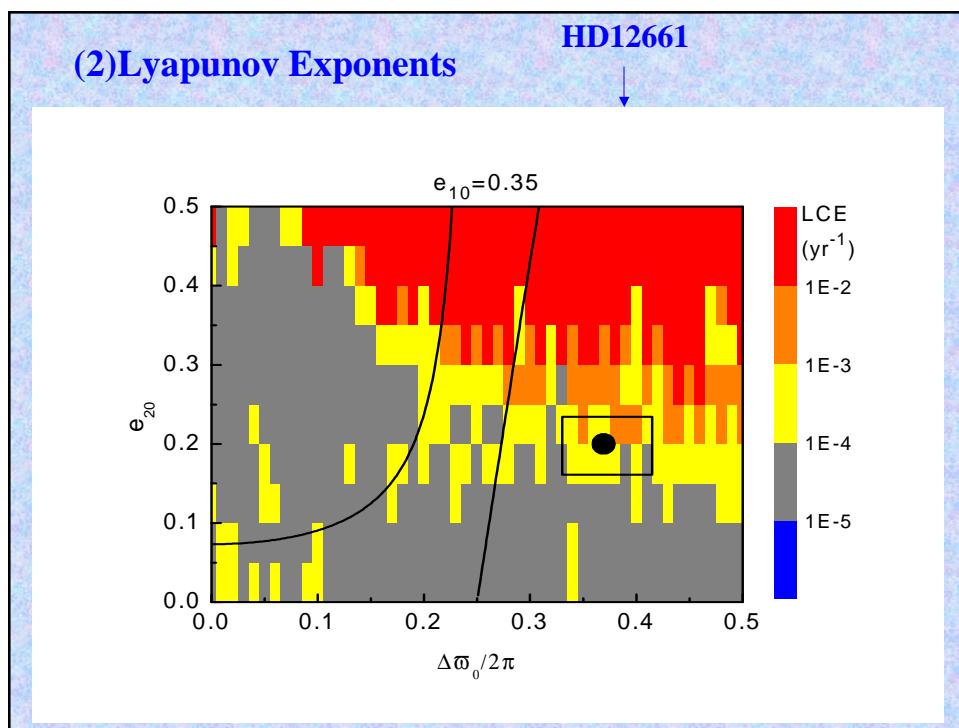
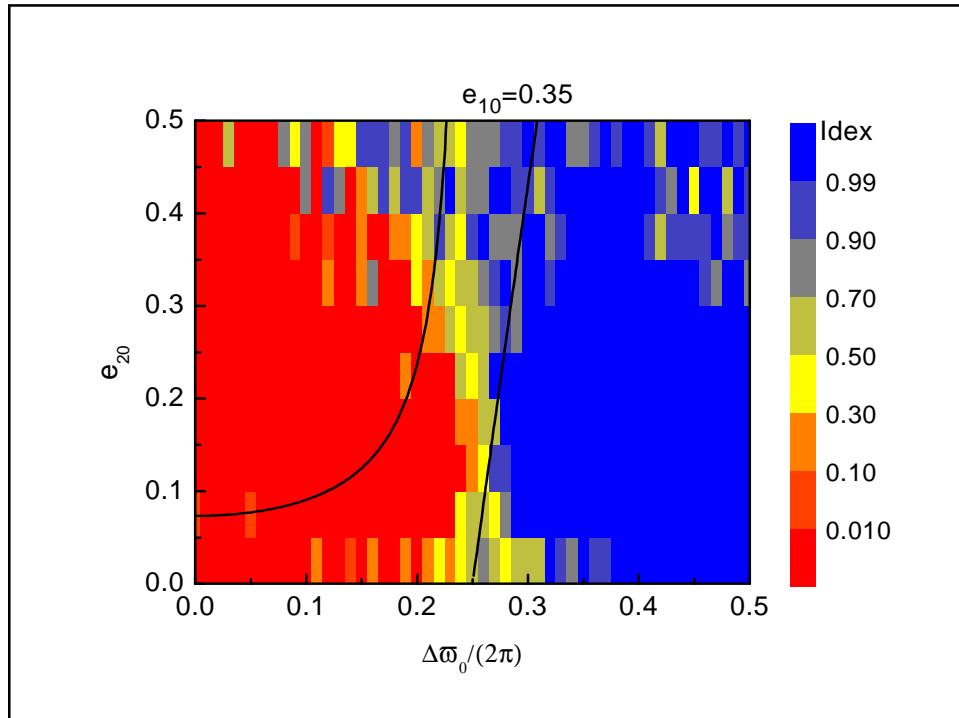


To see the stabilities of orbits in apsidal resonance, We integrate orbits in a general three body problem with RKF(8) code. Runge-Kutta-Fehlberg

(1) define an index to see whether an orbit is in libration region:

$$\text{Index} = \langle I_n \rangle \approx \begin{cases} 0 & \text{aligned libration} \\ 0.5 & \text{circulation} \\ 1 & \text{anti-aligned libration} \\ \text{others} & \text{mixed} \end{cases}$$

Apsidal Resonances



Apsidal Resonances

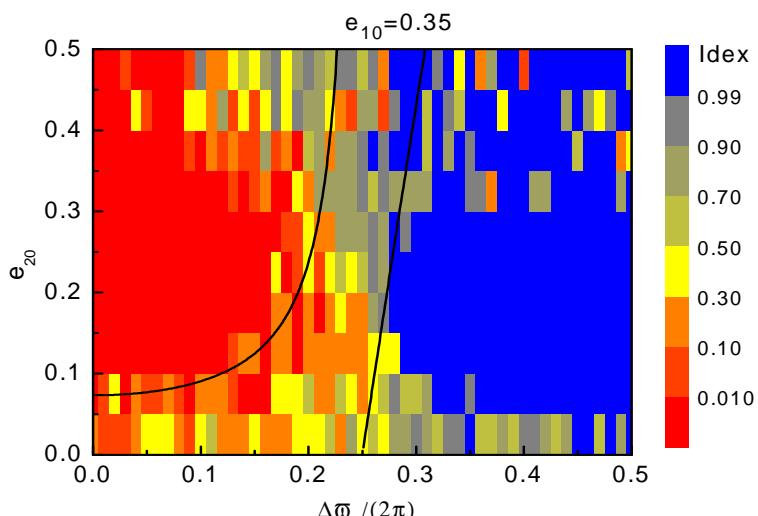
(2) Planets migration under disc tide □

Azimuthal torque to m_2

$$f_2 = -2 \times 10^{-6} \text{ AU}^2 \text{ yr}^{-1}$$

To see whether orbits, initially in apsidal resonances, could remain during migrations.

The evolution time span is 50000 years.



T= - 50000 yr

Apsidal Resonances

Application of the criteria in other systems (6 pairs in apsidal resonance)

Table 3. Extensions of $\Delta\varpi_0$ in apsidal resonance for the observed systems

Planet Pair	$q = \frac{m_1}{m_2}$	$\alpha = \frac{a_1}{a_2}$	$\frac{e_{10}}{e_{20}}$	aligned $\Delta\varpi_0$	anti-aligned $\Delta\varpi_0$
Ups And b-c	0.358	0.0720	0.037	(-79.3°, 79.3°)	- ^c
Ups And b-d	0.181	0.0228	0.040	(-47.0°, 47.0°)	-
Ups And c-d	0.507	0.317	1.08	(-9.0°, 9.0°)	-
55 Cnc b-c	4.15	0.477	0.073	-	(96.8°, 263.2°)
55 Cnc b-d	0.225	0.021	0.107	-	-
55 Cnc c-d	0.054	0.044	1.46	-	-
GJ876 c-b	0.296	0.628	2.70	-	(144.0°, 216.0°)
47 UMa ^d b-c	3.34	0.560	12.2	(-87.9°, 87.9°)	-
HD37124 b-c	0.860	0.184	0.250	(-69.8°, 69.8°)	-
HD12661 b-c	1.46	0.320	1.75	(-67.8°, 67.8°)	(98.6°, 261.4°)
HD82943 c-b	0.540	0.628	1.32	(-59.7°, 59.7°)	(132.8°, 227.2°)
HD168443 b-c	0.450	0.103	2.65	-	-
HD38529 b-c	0.061	0.035	0.806	-	-
HD74156 ^e b-c	0.208	0.080	1.625	-	-

^cHere - means no possible libration $\Delta\varpi_0$.

^dData from Fischer et al. (2002)

^eData from California and Carnegie Planet Search (2003)

Because of mean motion resonance

Conclusion:

- 1□we obtained an effective criteria□
- 2□apsidal resonance would stabilize the orbits;
- 3□when planets in large eccentric orbits, aligned apsidal resonance have larger stable region than anti-aligned one.

Published in ApJ 598, 1290-1300, Dec 1, 2003

Apsidal Resonances

Extend to nodal resonance:
Criteria

$$\frac{I_{20}}{I_{10}} < \frac{2\rho_2}{1 + \rho_2} \cos \Delta\Omega_0,$$

$$\frac{I_{20}}{I_{10}} > \frac{1 + \rho_2}{2} \frac{1}{\cos \Delta\Omega_0} > 0.$$

IAU Symposium. 219: Sydney, 2003

Thanks!