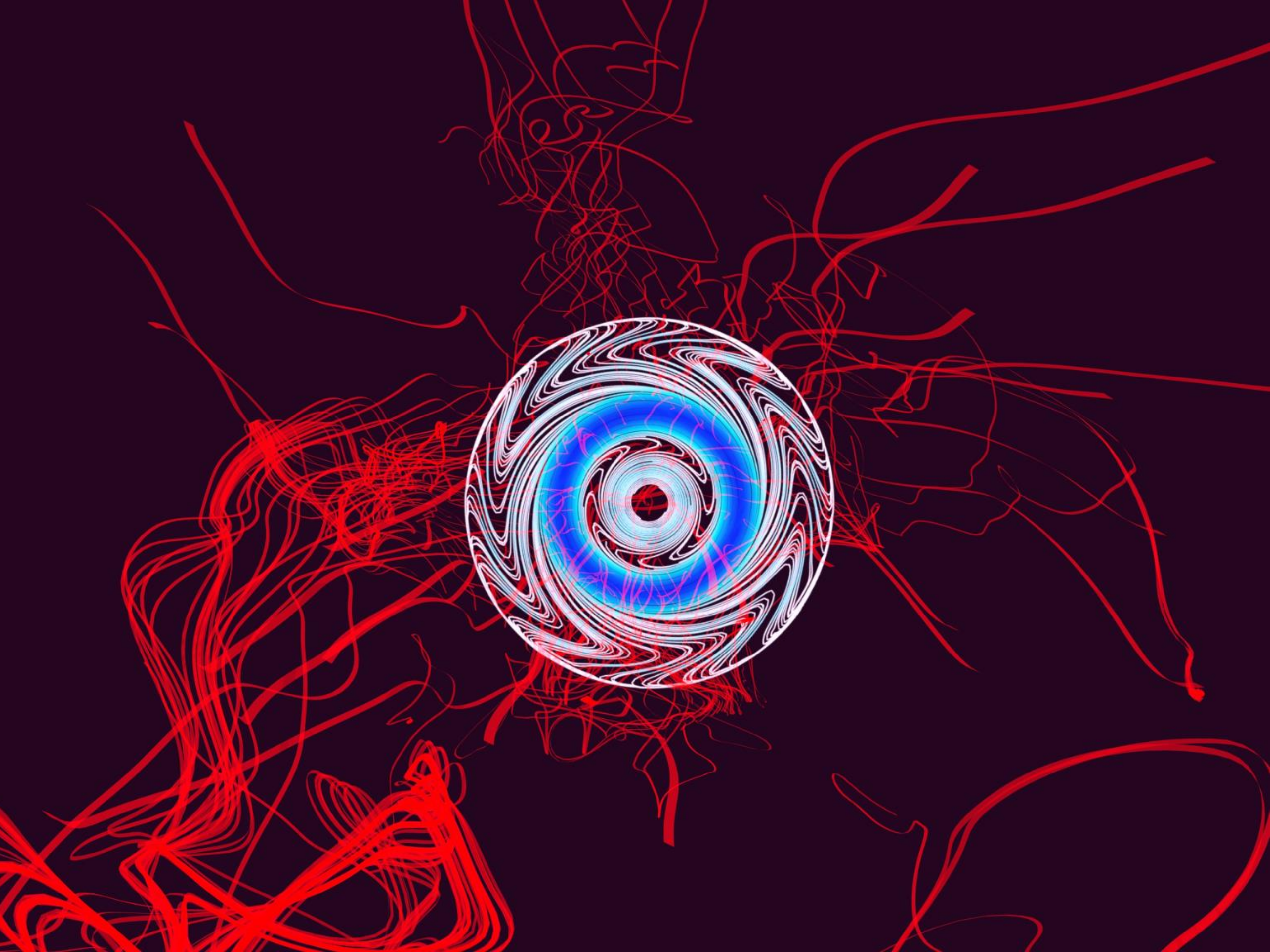
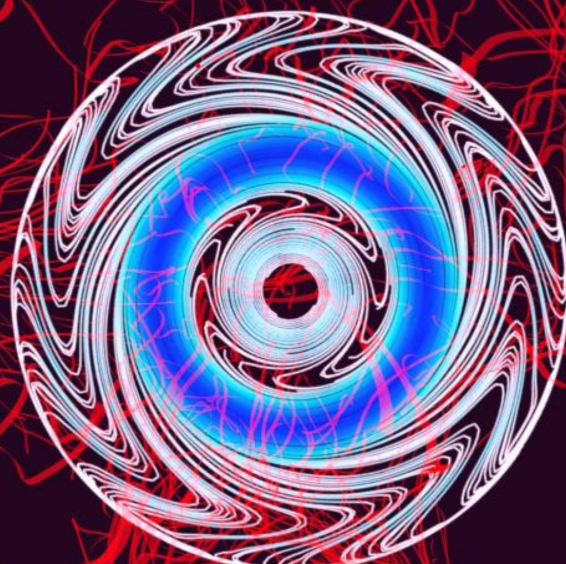


Beyond chaos: the continuing enigma of turbulence



Nigel Goldenfeld
University of Illinois at Urbana-Champaign

Original work with Hong-Yan Shih and Tsung-Lin Hsieh, ~~grudgingly~~ partially supported by NSF

Kavli Institute for Theoretical Physics
University of California, Santa Barbara

What can theoretical physics tell us about turbulence?

Nothing … according to Feynman

- There is a physical problem that is common to many fields, that is very old, and that has not been solved. **It is not the problem of finding new fundamental particles, but something left over from a long time ago—over a hundred years.** Nobody in physics has really been able to analyze it mathematically satisfactorily in spite of its importance to the sister sciences. It is the analysis of *circulating or turbulent fluids*.

Feynman Lectures on Physics, vol 1 (1963).

Nothing … according to Feynman

- There is a physical problem that is common to many fields, that is very old, and that has not been solved. **It is not the problem of finding new fundamental particles, but something left over from a long time ago—over a hundred years.** Nobody in physics has really been able to analyze it mathematically satisfactorily in spite of its importance to the sister sciences. It is the analysis of *circulating or turbulent fluids*.

- The simplest form of the problem is to take a pipe that is very long and push water through it at high speed. **We ask: to push a given amount of water through that pipe, how much pressure is needed? No one can analyze it from first principles and the properties of water.** If the water flows very slowly, or if we use a thick goo like honey, then we can do it nicely.

Feynman Lectures on Physics, vol 1 (1963).

Richard Feynman

That is the central problem which we ought to solve some day, and we have not.

Photo from https://blogs.scientificamerican.com/oscillator/feynman-on-biology/

What can theoretical physics tell us about anything?

What can theoretical physics tell us about anything?

And actually … what is theoretical physics?

Nobel Prize 2016: condensed matter theory

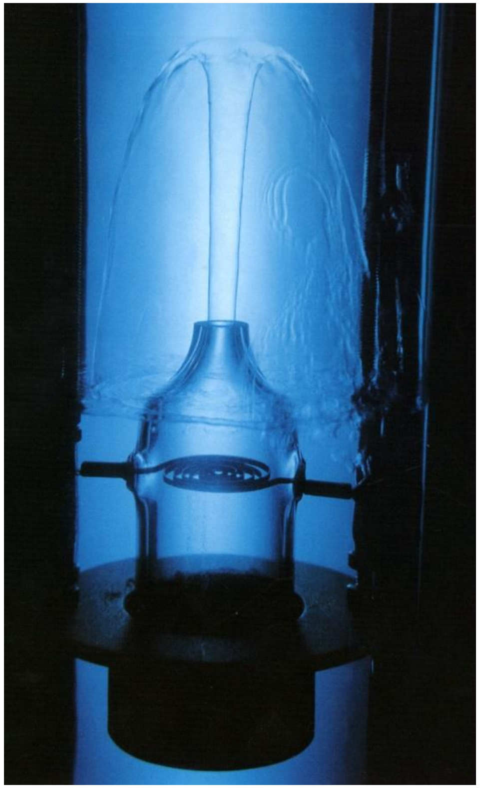
David Thouless　　　Mike Kosterlitz　　　Duncan Haldane

https://magaratimes.com/en/2016/10/04/
nobel-prize-physics-winners-elevate-hopes-for-brand-spanking-new-gen-of-electronics/

Superfluids

Honey – normal fluid with viscosity at room temperature

http://www.honeyassociation.com/



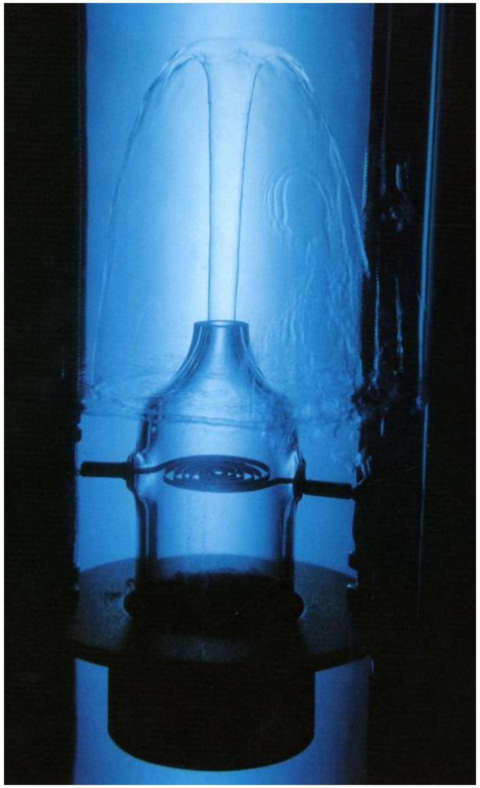
Superfluid helium – no viscosity at one degree above absolute zero

http://pitp.physics.ubc.ca/archives/CWSS/showcase/topics/He4-fountain.jpg

Superfluids

Classical fluid: velocity can “swirl” as slow as you like

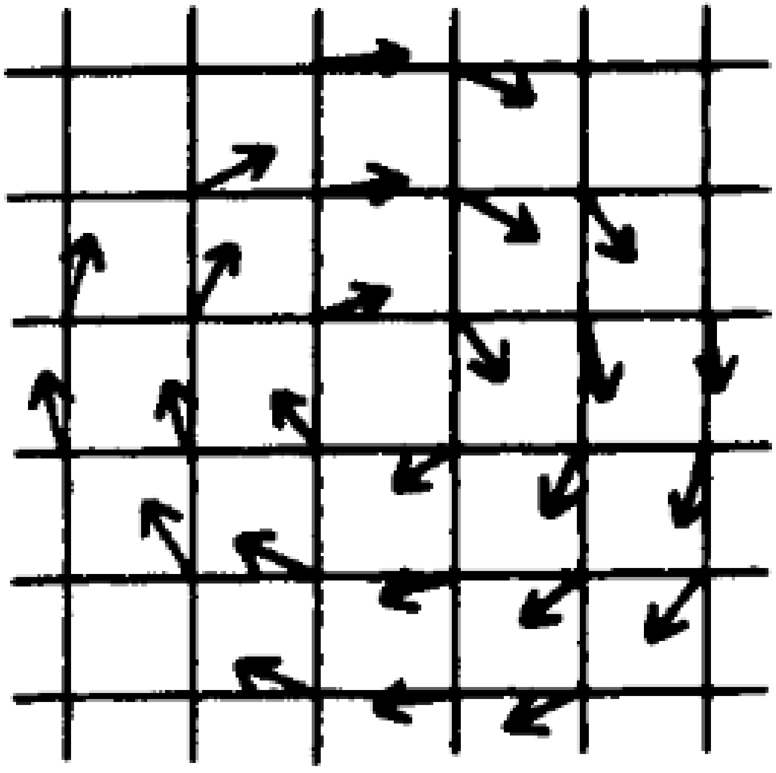
http://www.honeyassociation.com/



Quantum fluid: velocity cannot “swirl” slower than a certain amount

http://pitp.physics.ubc.ca/archives/CWSS/showcase/topics/He4-fountain.jpg

Arrows on a plane



Arrows represent the flow velocity of a quantum fluid.

Arrows on a plane – predict superfluid film phase transitions

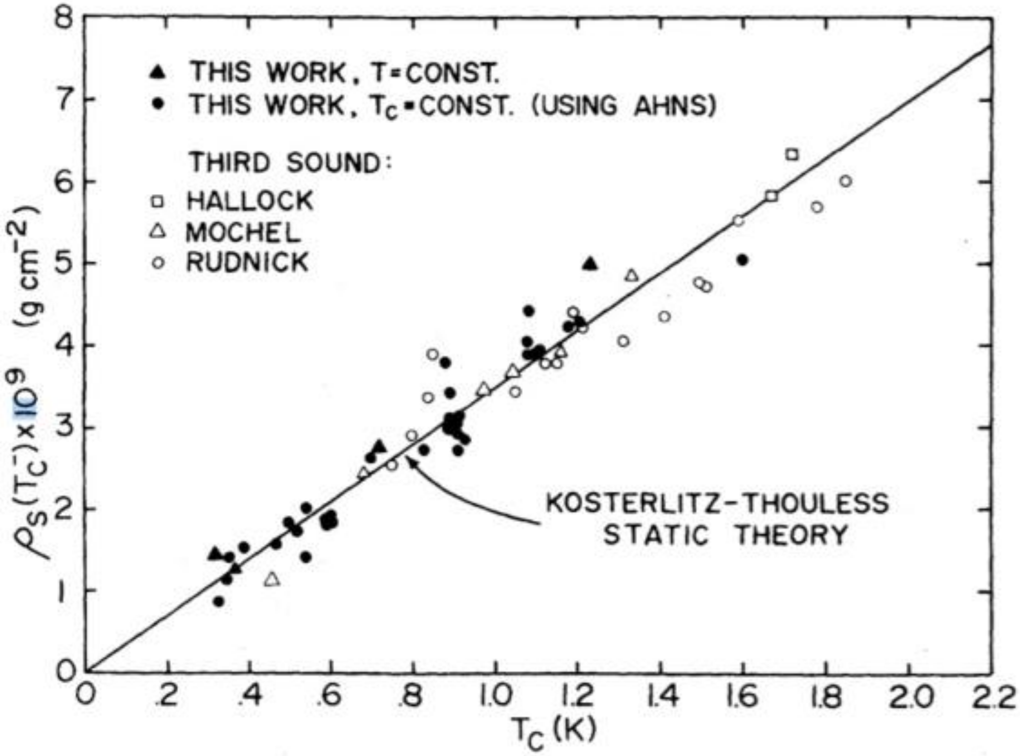
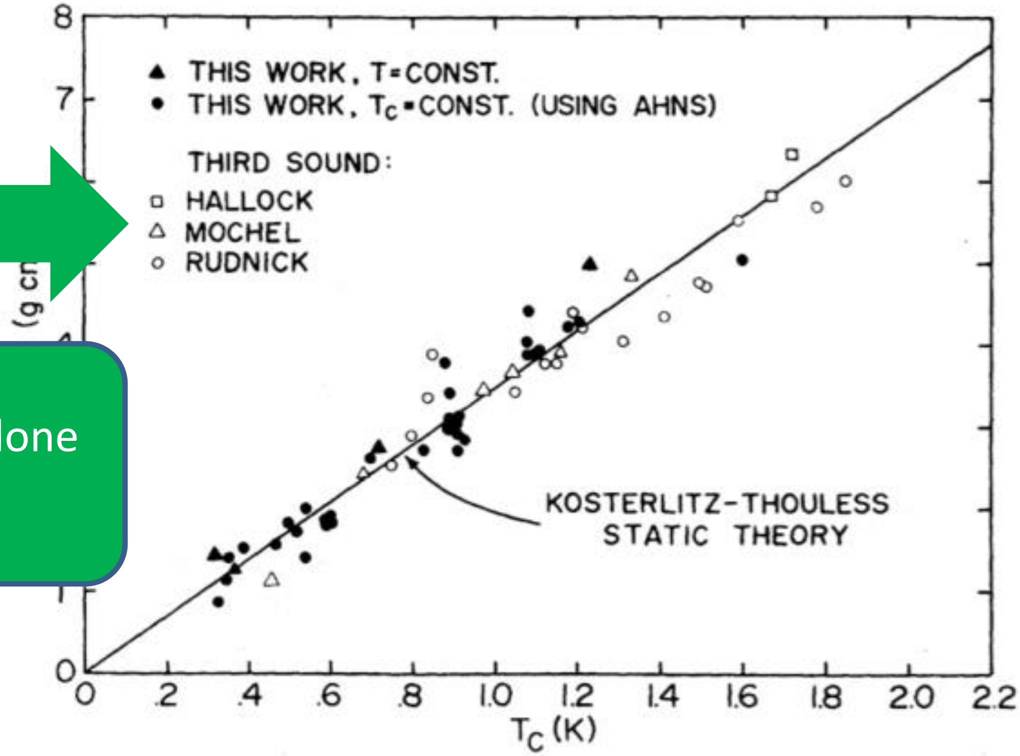


Figure 4: *At the Kosterlitz-Thouless transition the superfluid density and critical temperature are predicted to have a linear relation depending only on fundamental constants* $\rho(T_c) = T_c \frac{2}{\pi} \frac{m^2 k_B}{\hbar^2}$. *(Figure from Ref. [10].)*

Arrows on a plane – predict superfluid film phase transitions



Measurements done at UIUC

Figure 4: *At the Kosterlitz-Thouless transition the superfluid density and critical temperature are predicted to have a linear relation depending only on fundamental constants* $\rho(T_c) = T_c \frac{2}{\pi} \frac{m^2 k_B}{\hbar^2}$. *(Figure from Ref. [10].)*

Amazing and surprising fact

- Superfluids in two dimensions are pretty complicated
- Kosterlitz and Thouless' Nobel Prize work predicted how helium goes from behaving like a normal liquid to a superfluid just by …
- **Looking at the simplest way that arrows on a plane could be described energetically**

Amazing and surprising fact

- Superfl
  comp

- Koste
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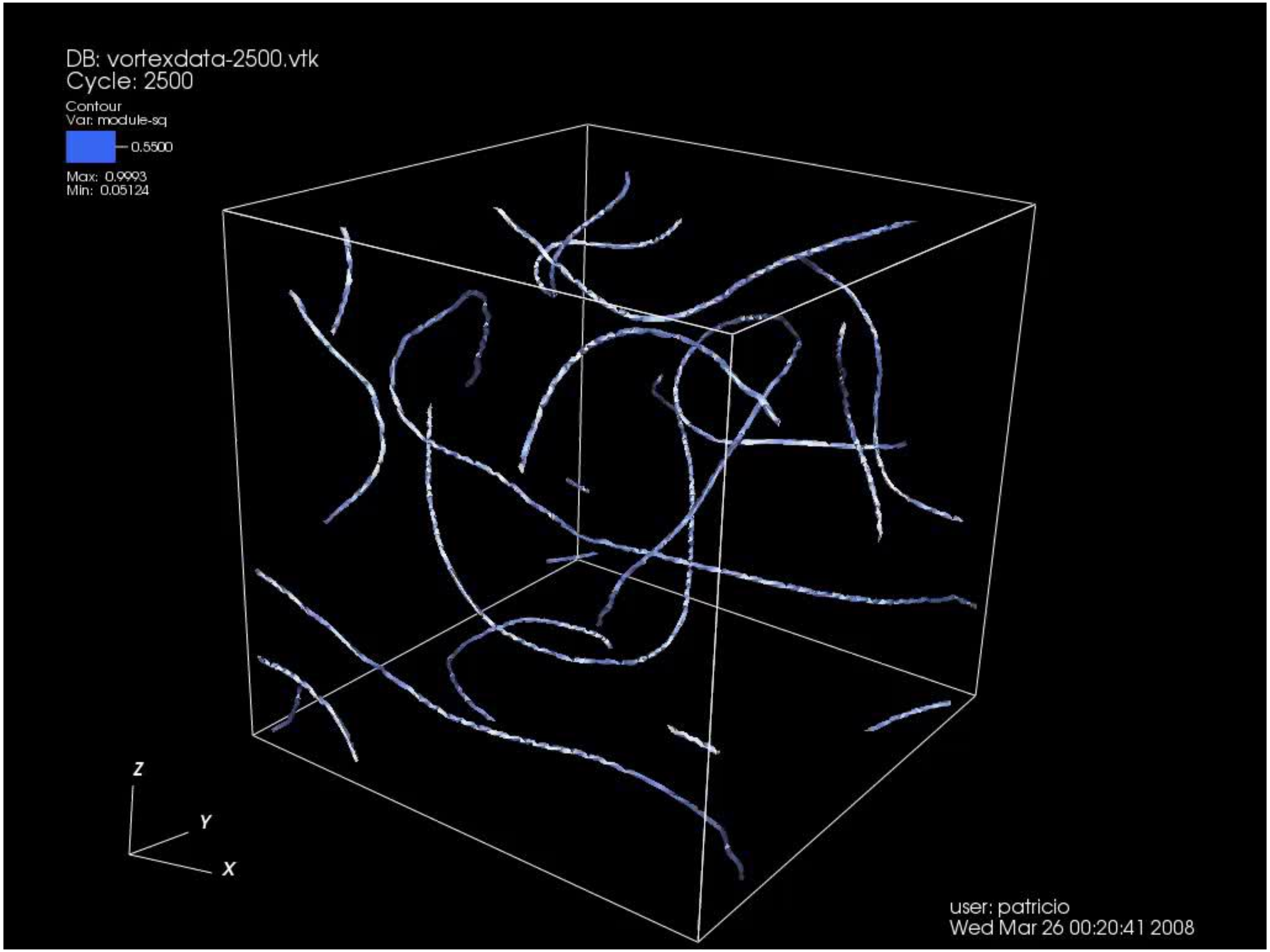
The moral of the story is that simple models can predict very complex phenomena. In fact everything we know about the physical world comes from looking for the simplest mathematical models (that have the right symmetry and topology).

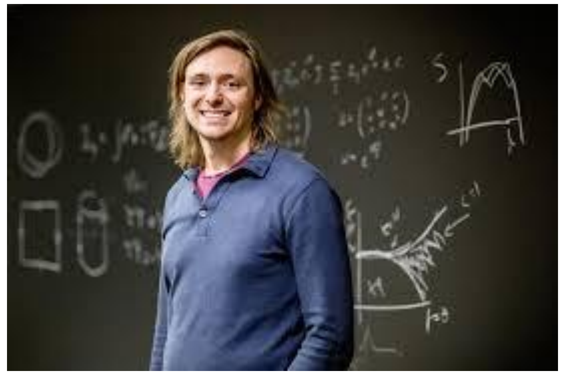
- **Looking at the simplest way that arrows on a plane could be described energetically**

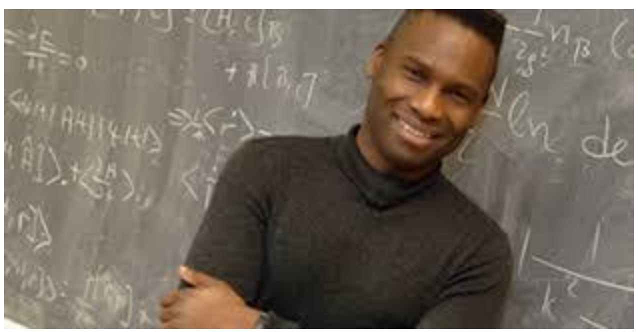
Superfluid turbulence in 3D



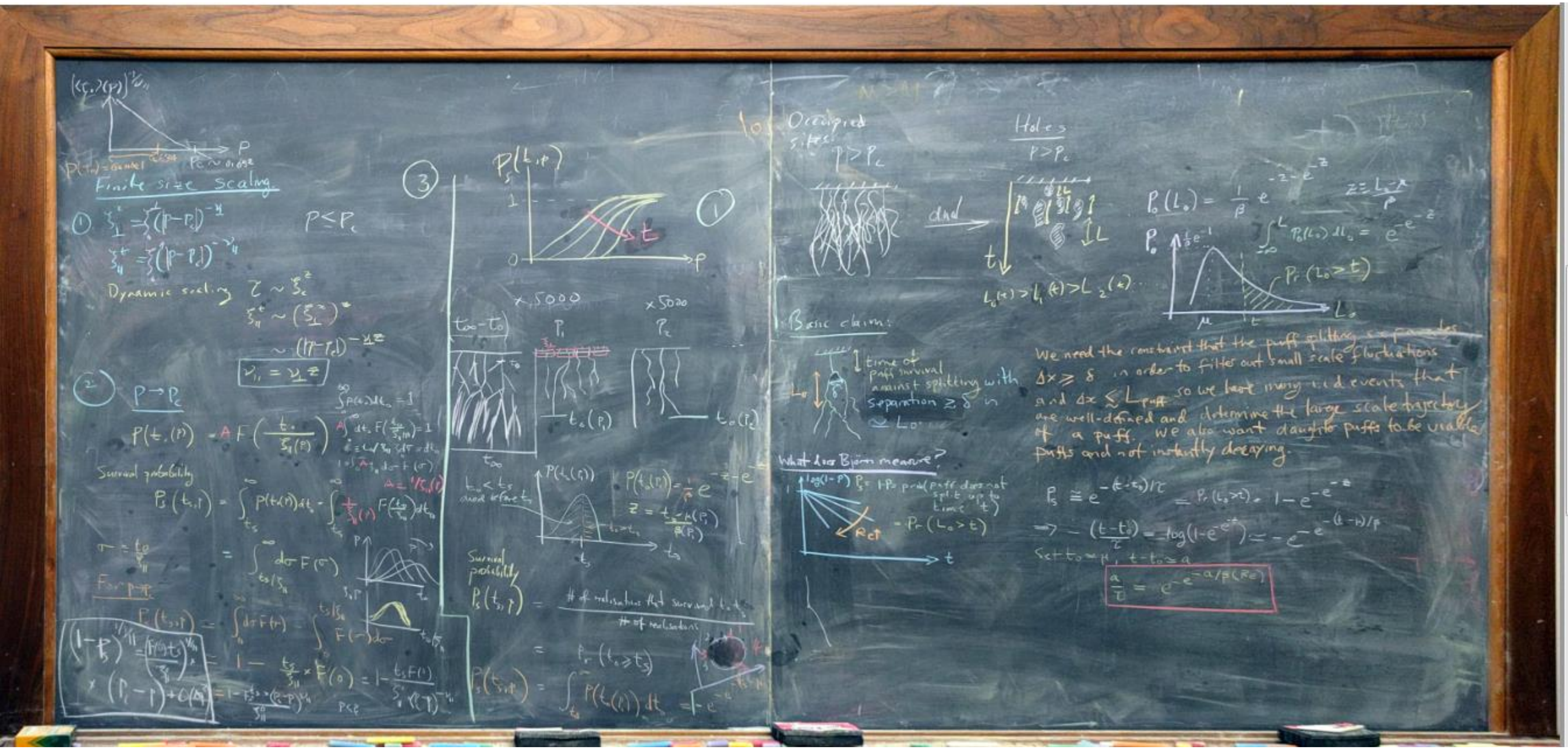
Patricio Jeraldo & NG

Theoretical condensed matter physicists at UIUC

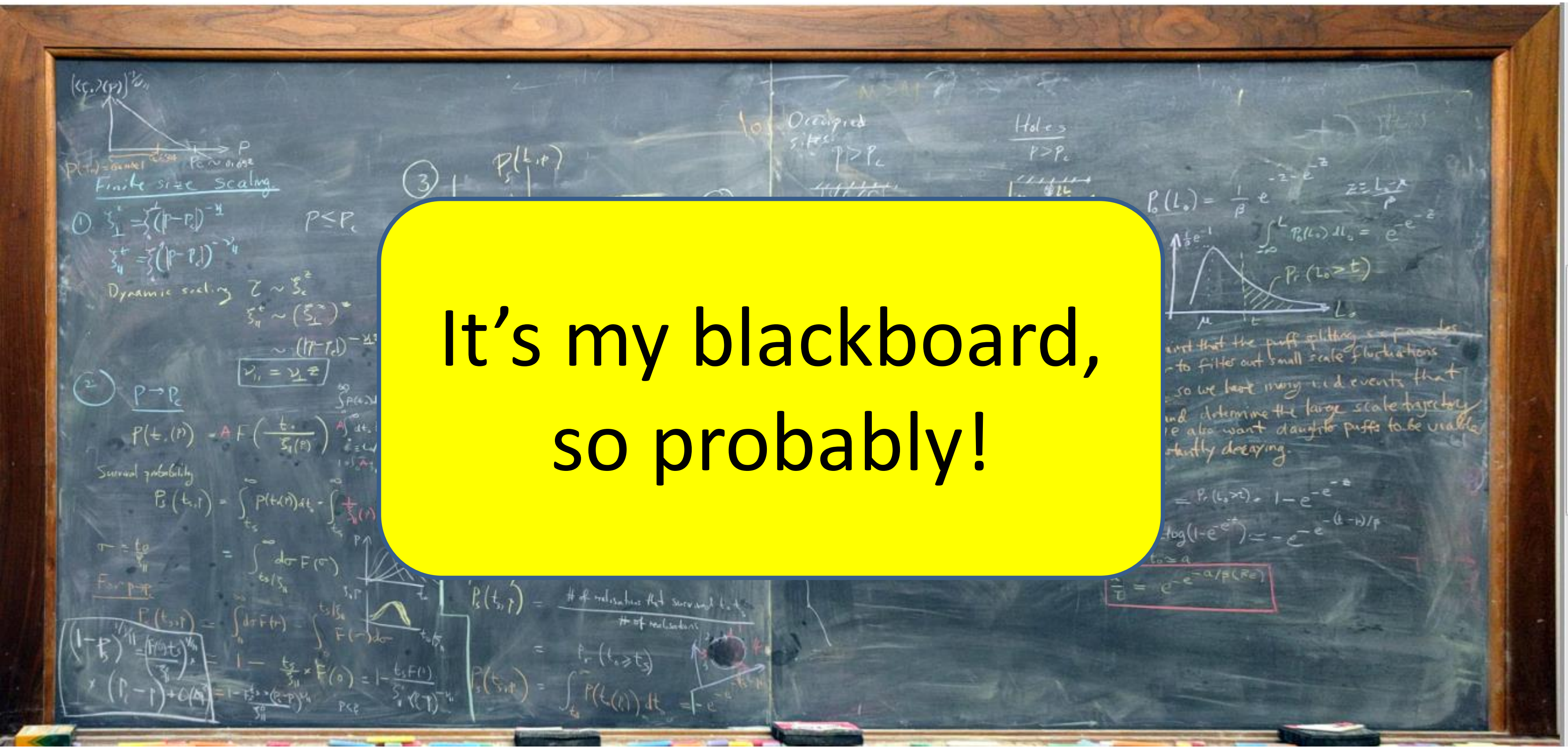




Is this theoretical physics?

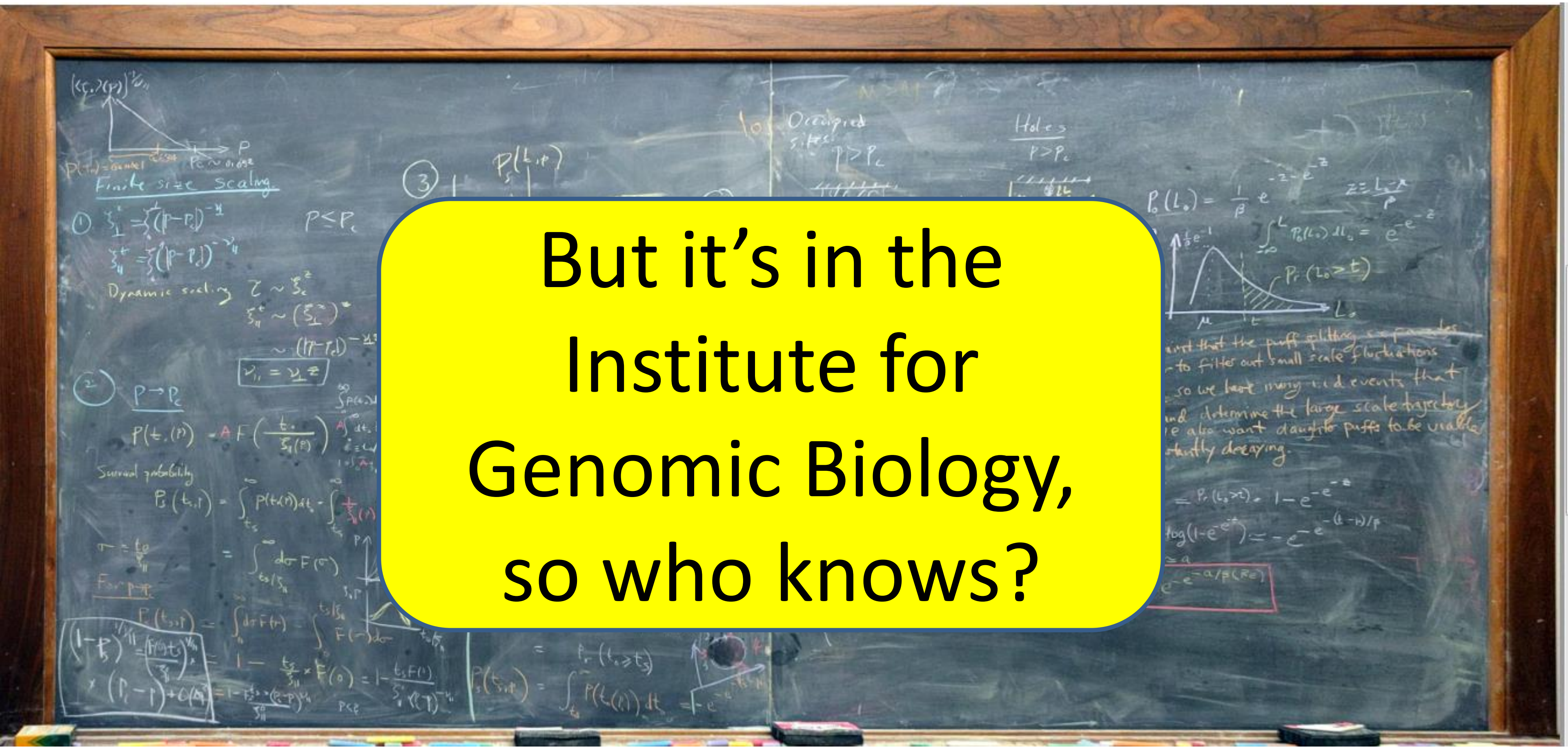


Is this theoretical physics?



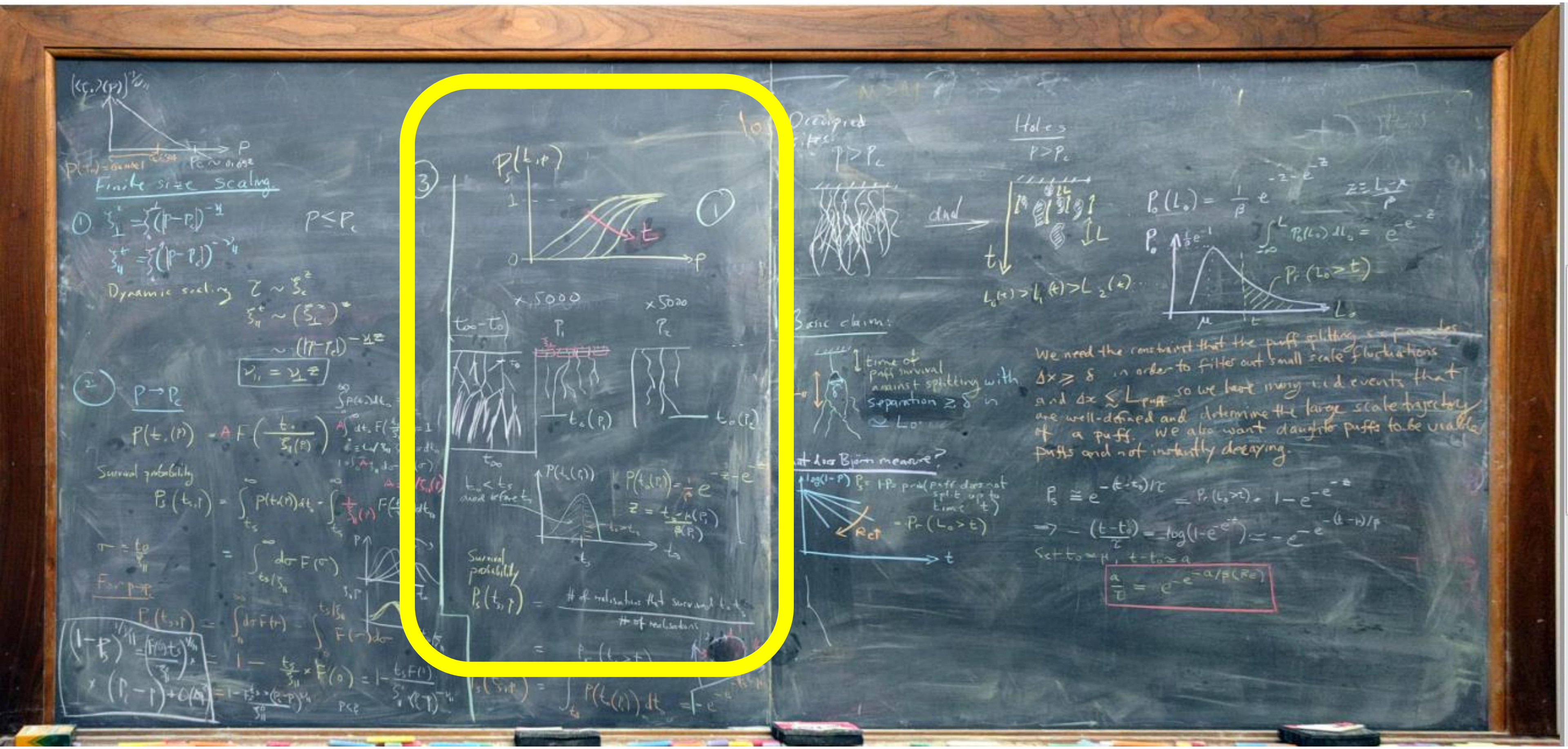
It’s my blackboard, so probably!

Is this theoretical physics?

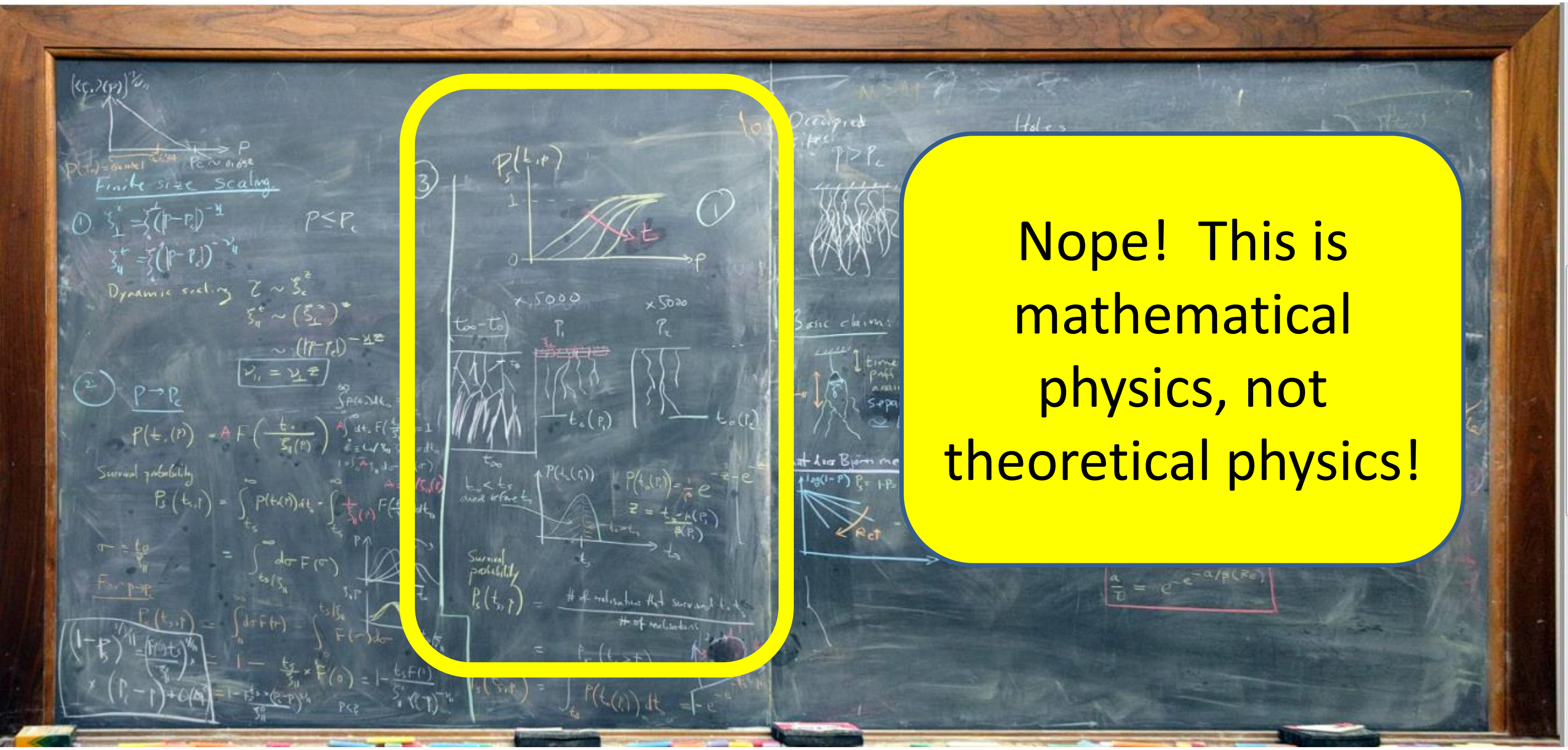


But it's in the Institute for Genomic Biology, so who knows?

Is this theoretical physics?

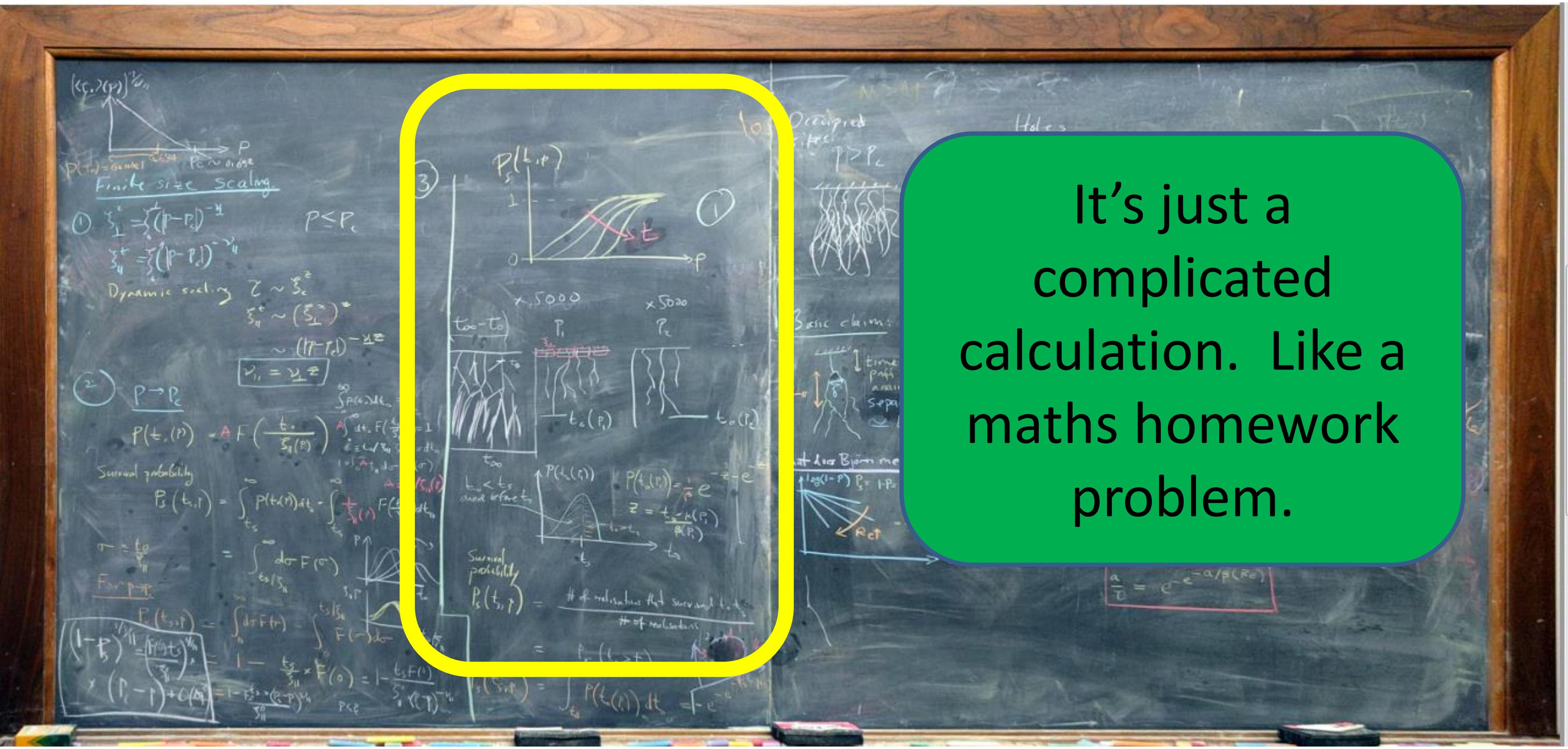


Is this theoretical physics?



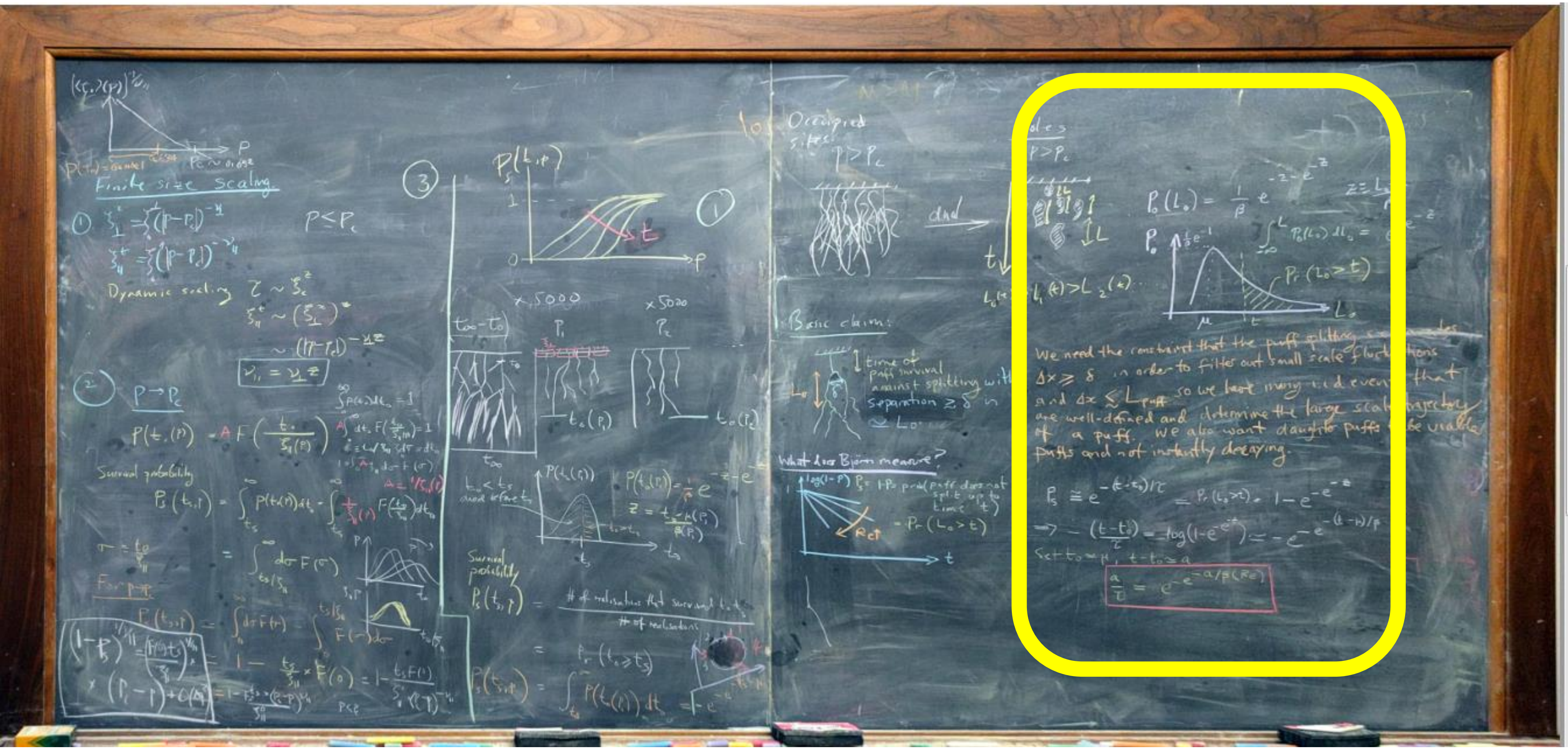
Nope! This is mathematical physics, not theoretical physics!

Is this theoretical physics?

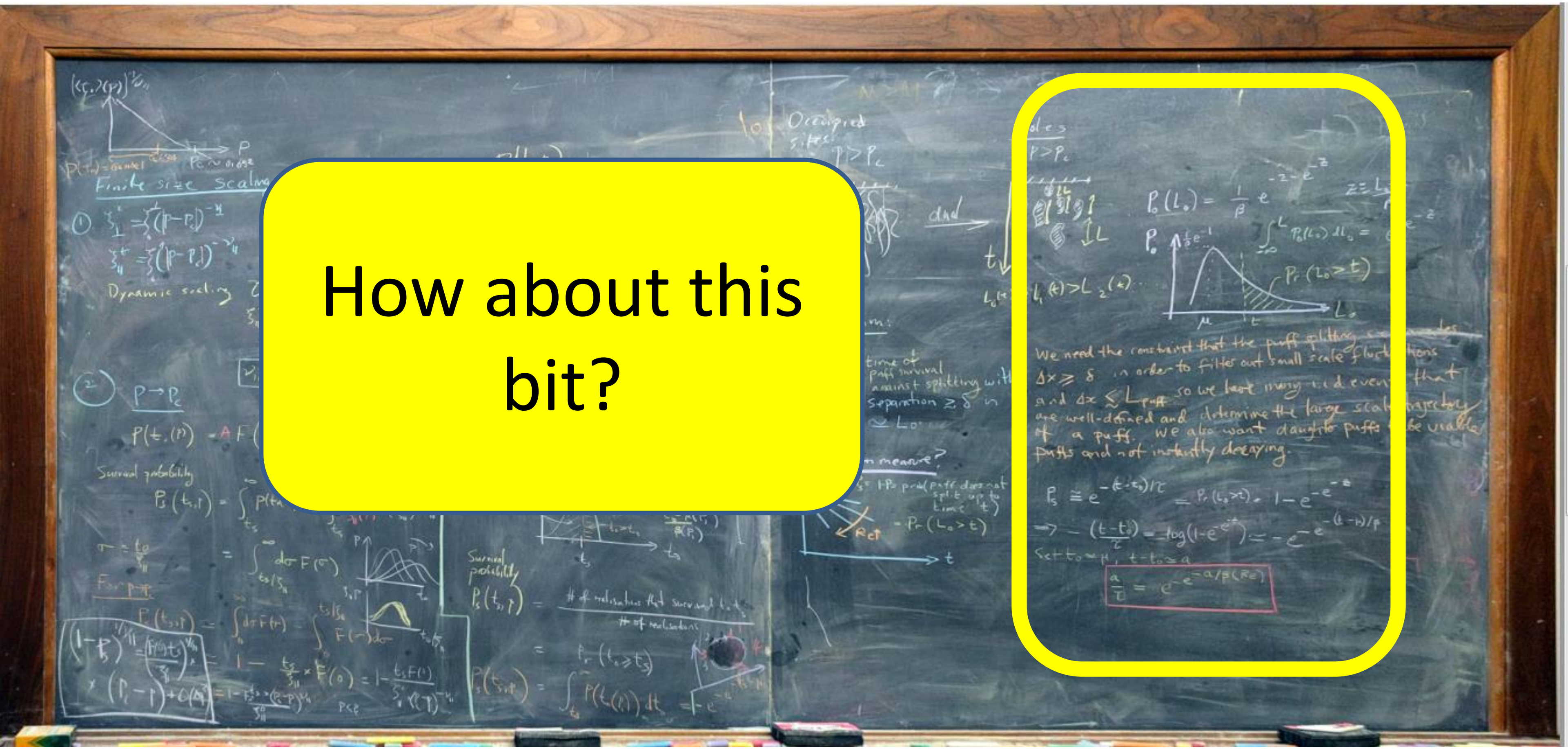


It's just a complicated calculation. Like a maths homework problem.

Is this theoretical physics?

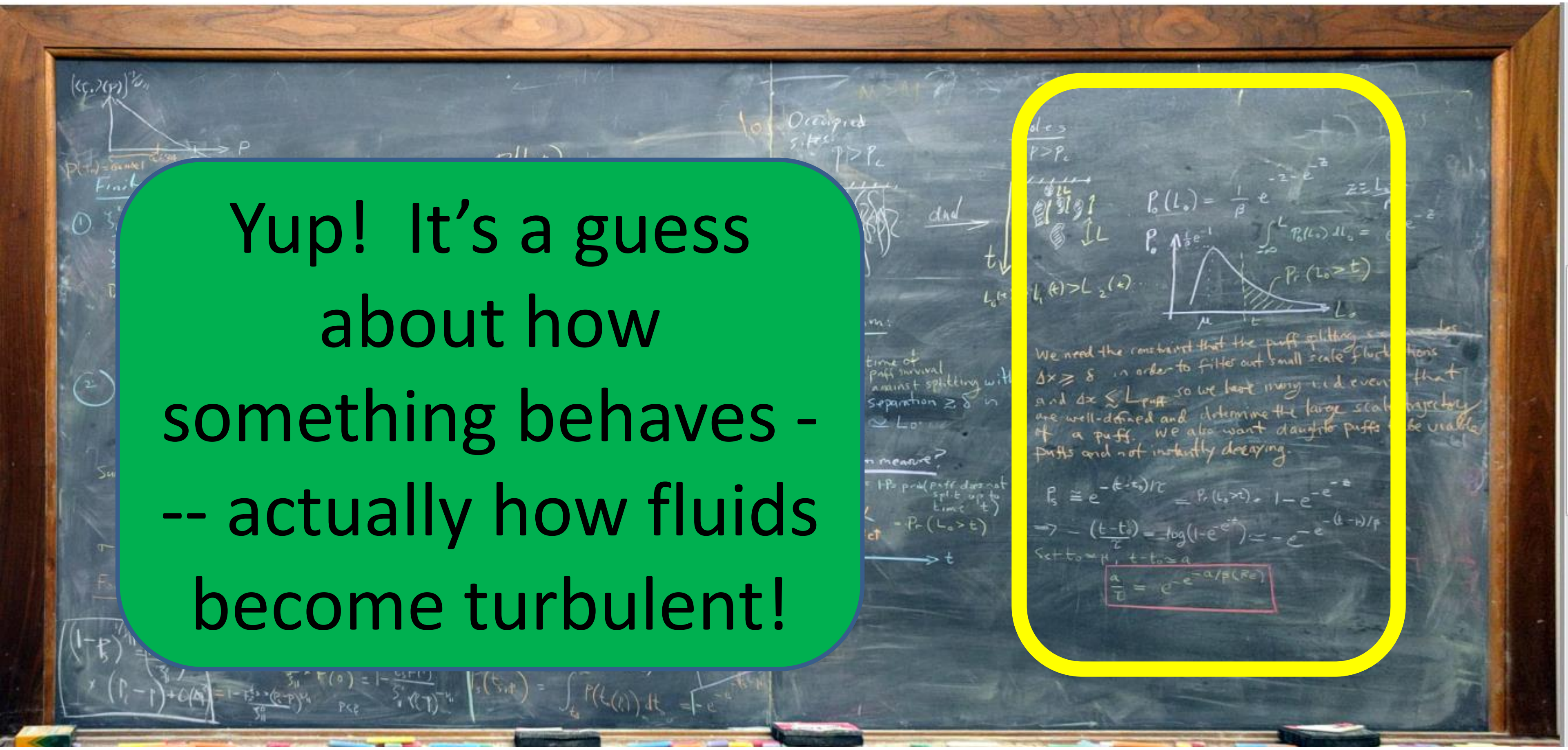


Is this theoretical physics?



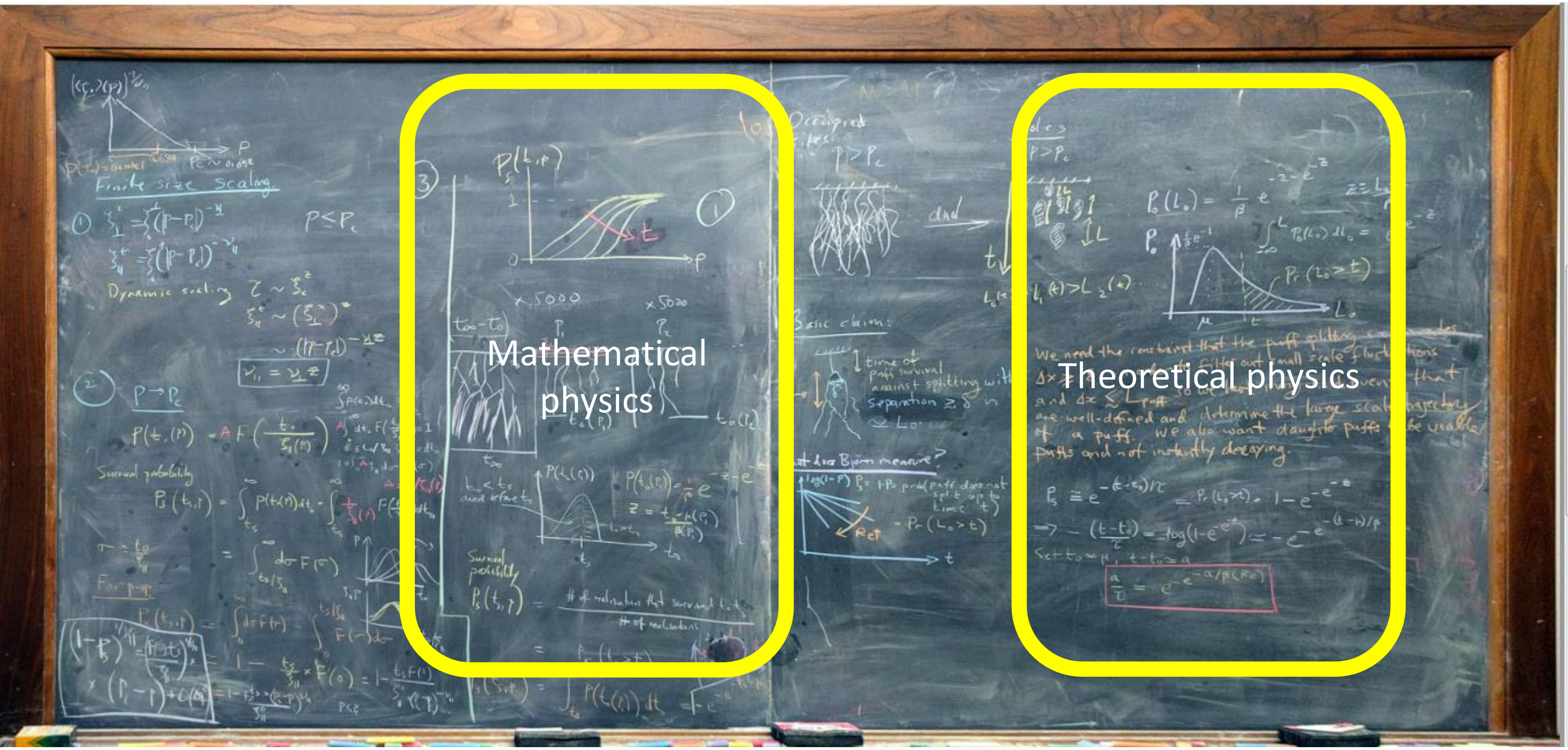
How about this bit?

Is this theoretical physics?



Yup! It’s a guess about how something behaves --- actually how fluids become turbulent!

Is this theoretical physics?



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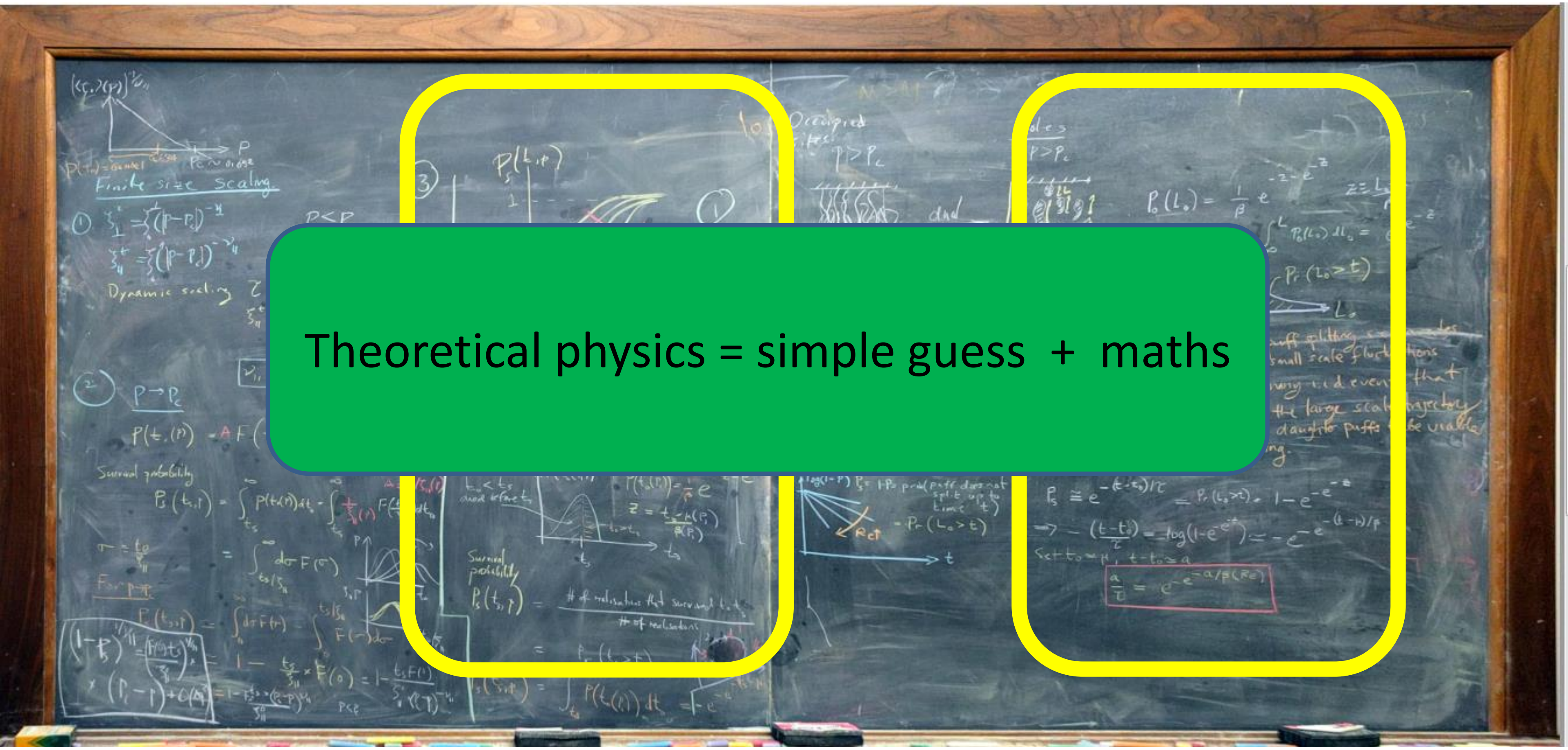

Theoretical physics = simple guess + maths

Why do we need to guess in turbulence?

Why do we need to guess in turbulence?

$$F = m\,a$$

Why do we need to guess in turbulence?

$$F = m\,a$$

Force

mass

acceleration

Why do we need to guess in turbulence?

$$a = F/m$$

acceleration

Force

mass

Why do we need to guess in turbulence?

$$a = F/m$$

The force in fluid dynamics is usually gravity, pressure, viscosity …

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The force in fluid dynamics is usually gravity, pressure, viscosity … but it could be magnetic fields in stars or a plasma fusion experiment …

Why do we need to guess in turbulence?

$$a = F/m$$

The force in fluid dynamics is usually gravity, pressure, viscosity … but it could be magnetic fields in stars or a plasma fusion experiment … or even come from quarks and gluons in a particle physics experiment

If we know acceleration, we can calculate speed

Acceleration means “how much your speed changes every second”

Mid-term exam

Question: A car accelerates from rest at 9 meters per second per second for three seconds.

Please tell me:

Mid-term exam

Question: A car accelerates from rest at 9 meters per second per second for three seconds.

Please tell me: **What is the brand of the car?**

Mid-term exam

Question: A car accelerates from rest at 9 meters per second per second for three seconds.

Please tell me: **What is the brand of the car?**

Answer: After 1 second it is going 9 meters/second. After 2 seconds it is going 18 meters/second. After 3 seconds it is going 27 meters/second.

Mid-term exam

Question: A car accelerates from rest at 9 meters per second per second for three seconds.

Please tell me: **What is the brand of the car?**

Answer: After 1 second it is going 9 meters/second. After 2 seconds it is going 18 meters/second. After 3 seconds it is going 27 meters/second.

27 meters/second = 60 miles per hour

0-60 mph in 3 seconds. Must be a Porsche!

Car and Driver (2014)

Acceleration of a fluid

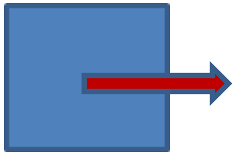
When you try to apply

$$a = F/m$$

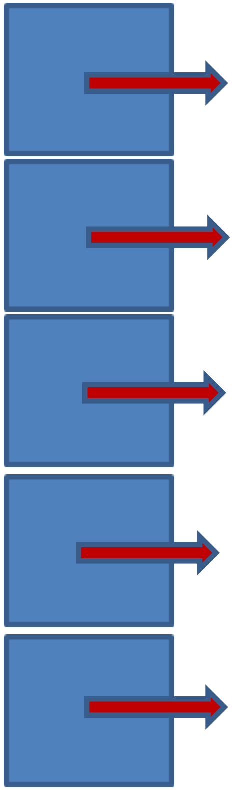
to every point in a fluid, not just a single particle, it gets very complex.

Acceleration of a fluid

PARTICLE

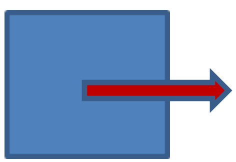


FLUID



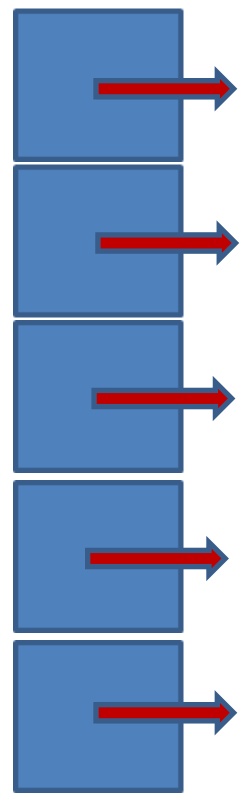
Acceleration of a fluid

PARTICLE



Predictable

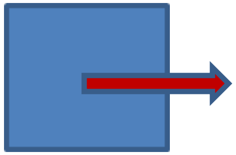
FLUID



Predictable

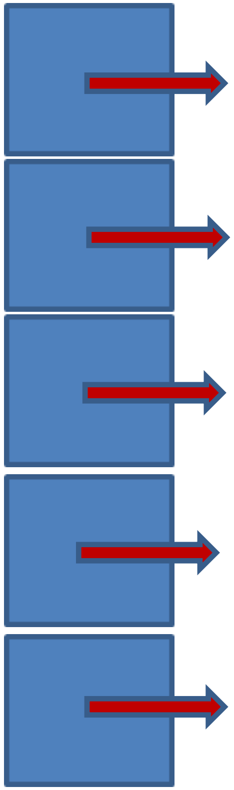
Acceleration of a fluid

PARTICLE



Predictable

FLUID

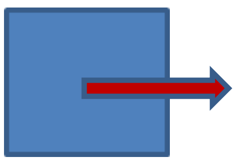


Predictable

Laminar

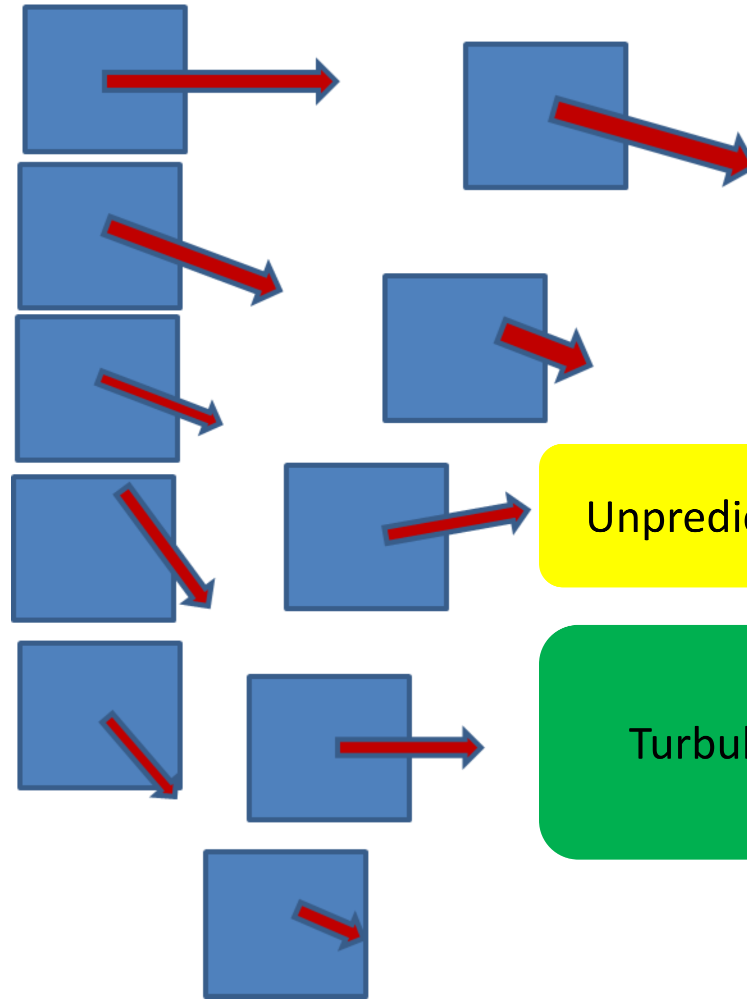
Acceleration of a fluid

PARTICLE



Predictable

FLUID

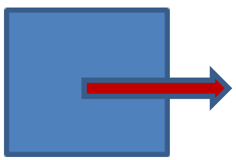


Unpredictable

Turbulent

Chaos vs. Turbulence

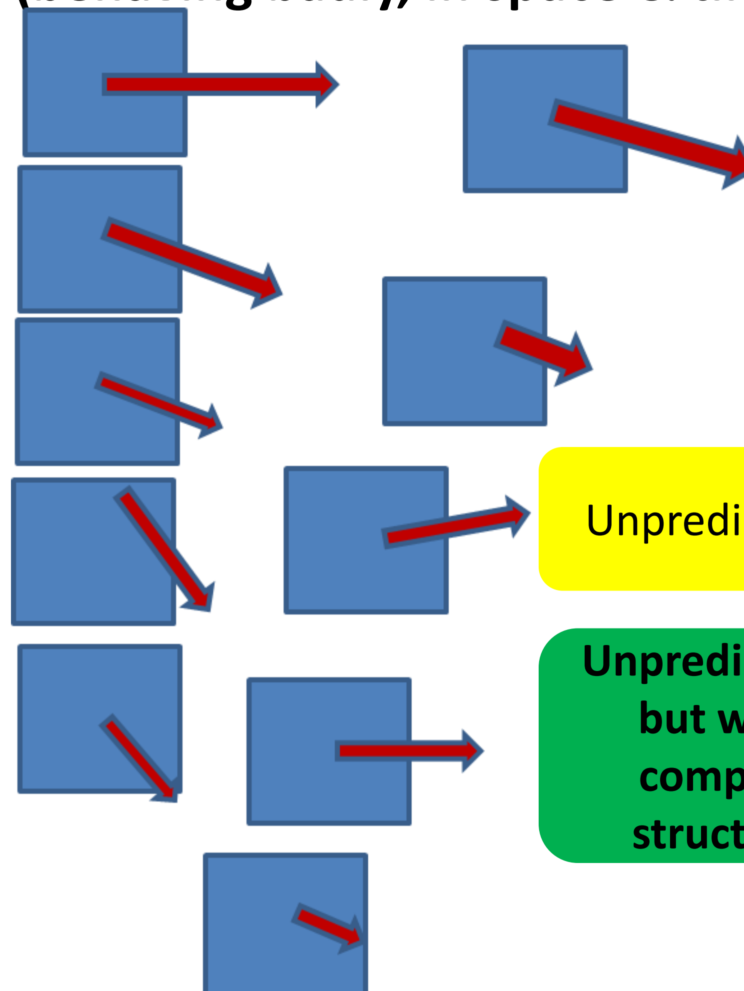
Chaos – one particle
(behaving badly, in time)



Unpredictable

Unpredictable but with “simple” structure

Turbulence – many particles
(behaving badly, in space & time)

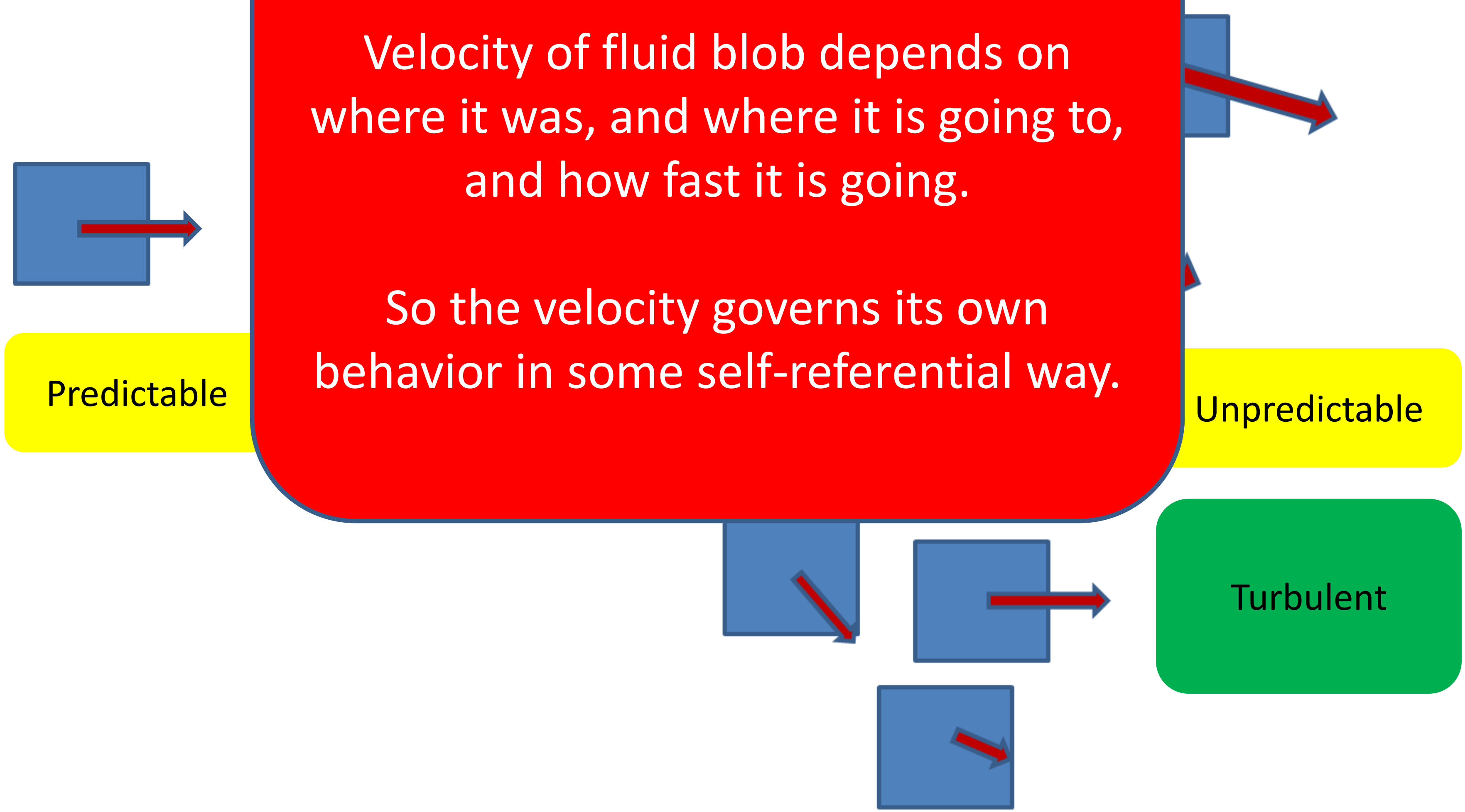


Unpredictable

Unpredictable but with complex structure

Acceleration of a fluid

PARTICLE



Velocity of fluid blob depends on where it was, and where it is going to, and how fast it is going.

So the velocity governs its own behavior in some self-referential way.

What is turbulence?

Turbulence is stochastic and wildly fluctuating

Soap film experiment

M. A. Rutgers, X-l. Wu, and W. I. Goldburg.
"The Onset of 2-D Grid Generated Turbulence in Flowing Soap Films,"
Phys. Fluids 8, S7, (Sep. 1996).

Turbulence generates structure at many scales

Soap film experiment

M. A. Rutgers, X-l. Wu, and W. I. Goldburg.
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The big guess in turbulence

A.N. Kolmogorov

- Eddies spin off other eddies without losing energy.
  - Does not involve friction!
  - Hypothesis due to Richardson, Kolmogorov, …
- Turbulence is a scale-invariant cascade of energy.

http://scienceworld.wolfram.com/biography/Kolmogorov.html

Scale-invariant cascade

Biology

Swift & de Morgan poem

Great fleas have little fleas

Upon their backs to bite 'em,

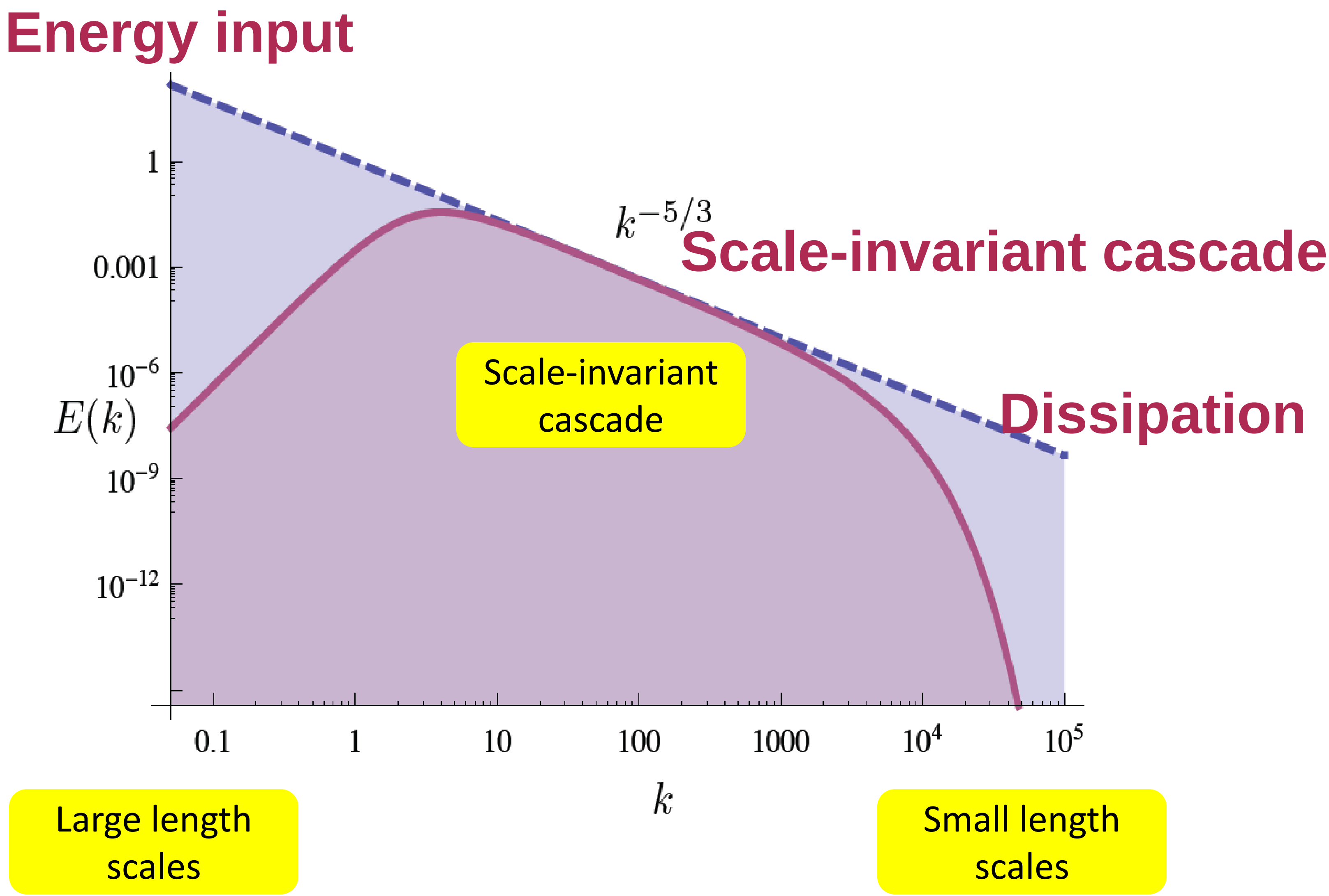
And little fleas have lesser
fleas,

And so *ad infinitum*.

Scale-invariant cascade

Biology	Turbulence
Swift & de Morgan poem	**Richardson poem**
Great fleas have little fleas	Big whorls have little whorls
Upon their backs to bite 'em,	That feed on their velocity,
And little fleas have lesser fleas,	And little whorls have lesser whorls
And so *ad infinitum*.	And so on to viscosity.

The big guess in turbulence



Turbulent cascades

3D forward cascade

2D inverse cascade

Energy flows to small scales

Energy flows to large scales

Turbulent cascades

$\boxed{\text{3D forward cascade}}$

$\boxed{\text{2D inverse cascade}}$

$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

Turbulent cascades

2D forward cascade

2D inverse cascade

Vorticity flows to small scales

Energy flows to large scales

Turbulent cascades

2D forward cascade

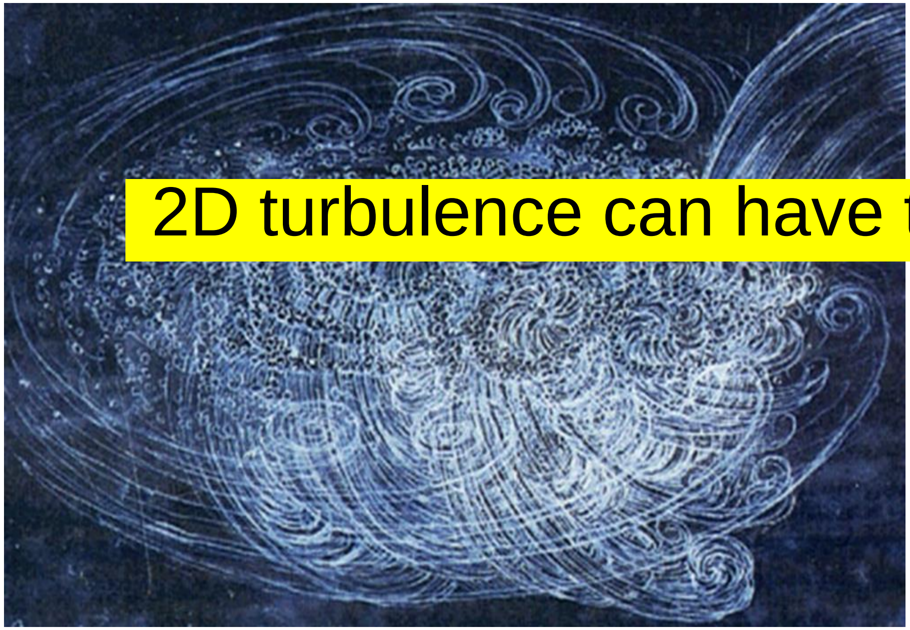
2D inverse cascade

$$E(k) \propto \lambda^{2/3} k^{-3}$$

$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

Turbulent cascades

2D turbulence can have two separate cascades



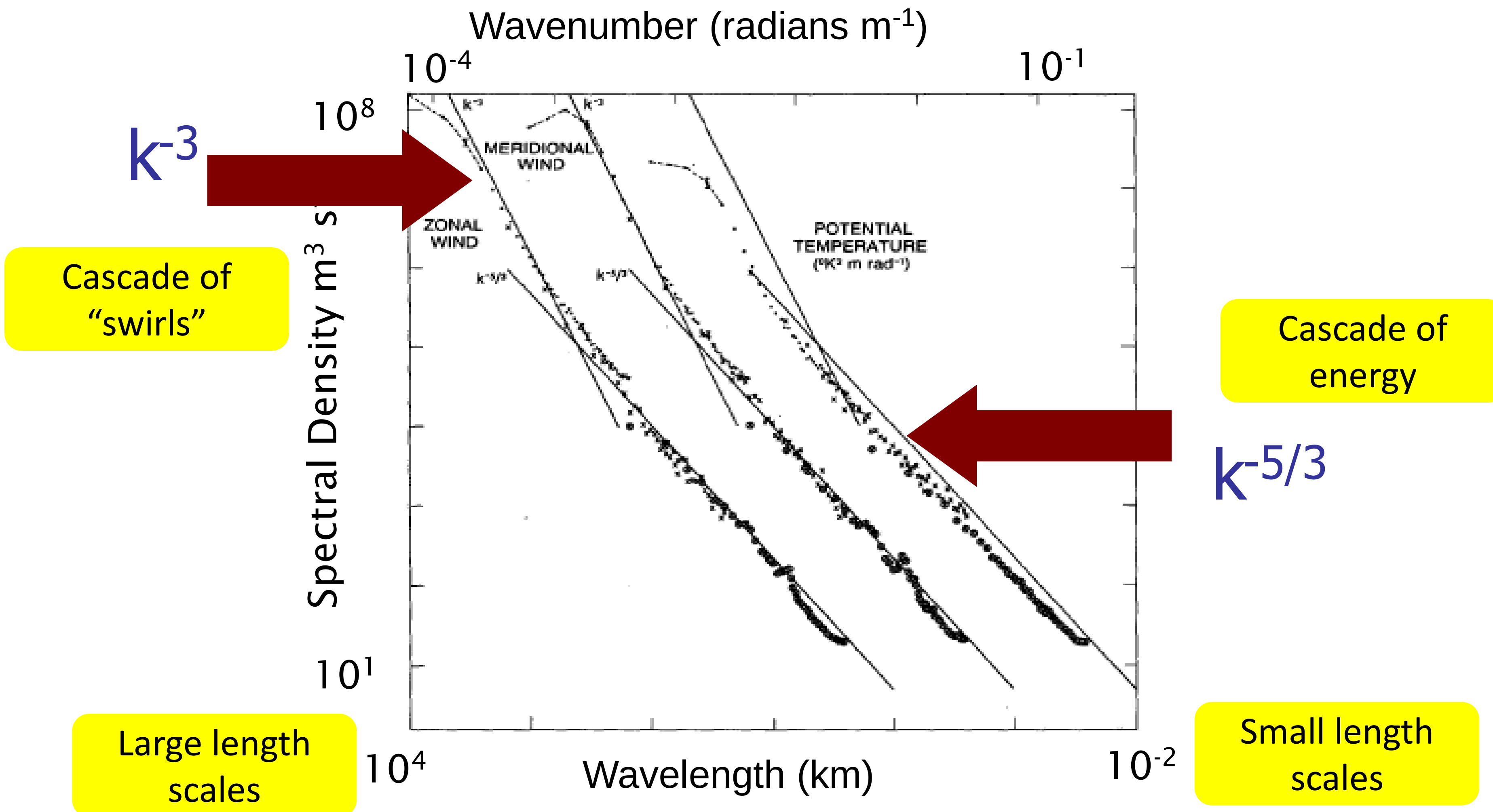
2D forward cascade

$$E(k) \propto \lambda^{2/3} k^{-3}$$

2D inverse cascade

$$E(k) \propto \epsilon^{2/3} k^{-5/3}$$

Scale-invariant cascades in the atmosphere



G. D. Nastrom and K. S. Gage, "A Climatology of Atmospheric Wavenumber Spectra of Wind and Temperature Observed by Commercial Aircraft", Jour. Atmos. Sci. vol 42, 1985 p953

The story so far …

The flow tells the flow how to change the flow

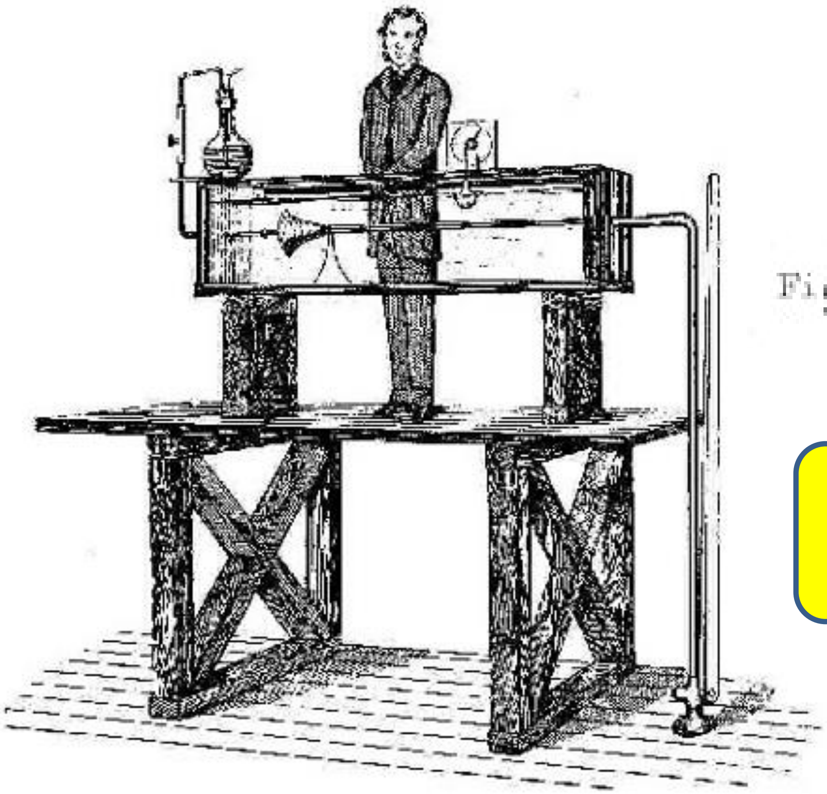
We can guess the outcome: out of complexity comes a simple picture of scale-invariant cascades

… sounds nice

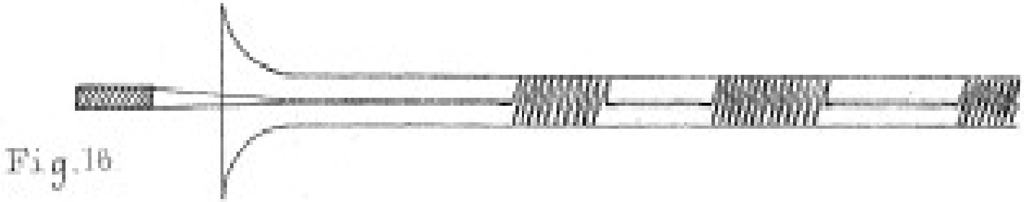
So is it solved? What’s new?

Reynolds & Turbulence

- Reynolds (1883) first to study turbulence systematically



Sees "Flashes" of turbulence

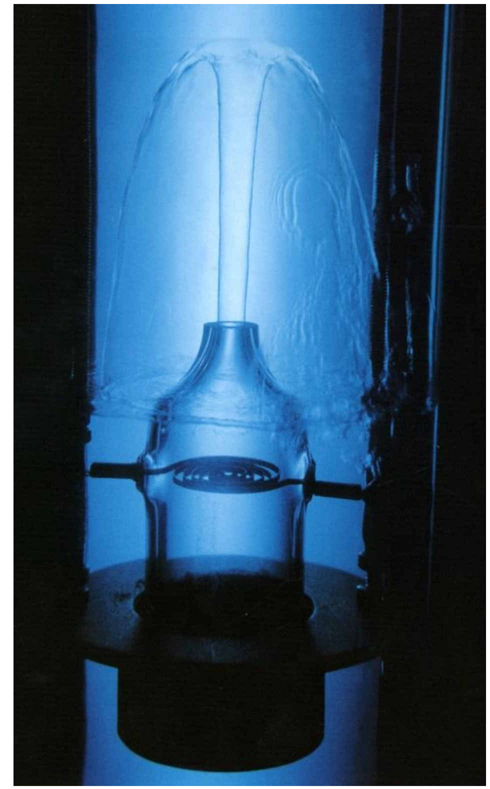


Introduces "Reynolds number"

Re = flow speed * size/(viscosity/density)

Superfluids

Classical fluid: velocity can “swirl” as slow as you like



Quantum fluid: velocity cannot “swirl” slower than a certain amount

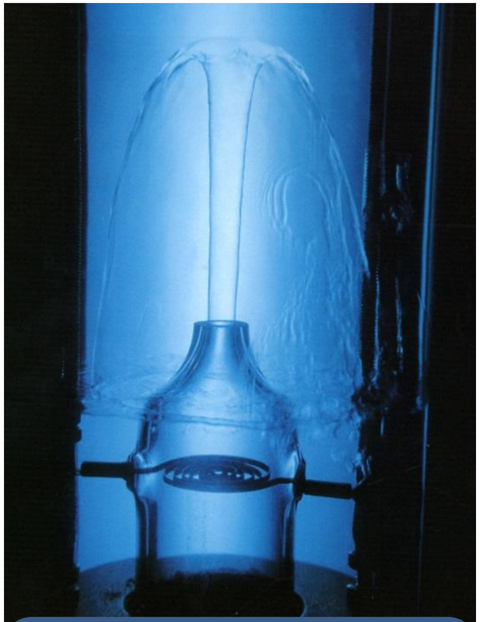
http://www.honeyassociation.com/

http://pitp.physics.ubc.ca/archives/CWSS/showcase/topics/He4-fountain.jpg

Superfluids

LAMINAR:
Re much less than 1

Classical fluid: velocity can “swirl” as slow as you like



TURBULENT:
Re ~ 1 million

Quantum fluid: velocity cannot “swirl” slower than a certain amount

http://www.honeyassociation.com/

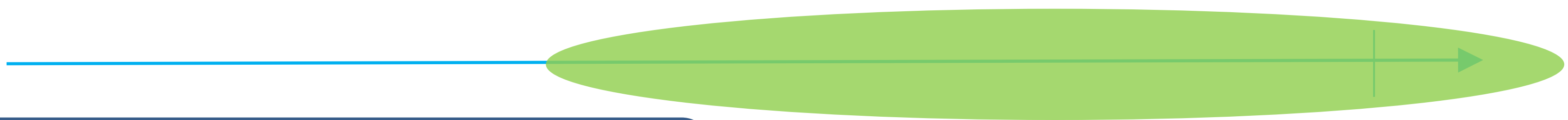
http://pitp.physics.ubc.ca/archives/CWSS/showcase/topics/He4-fountain.jpg

Scale-invariant cascade in fully-developed turbulence?

Re ~ 1800 - 2400

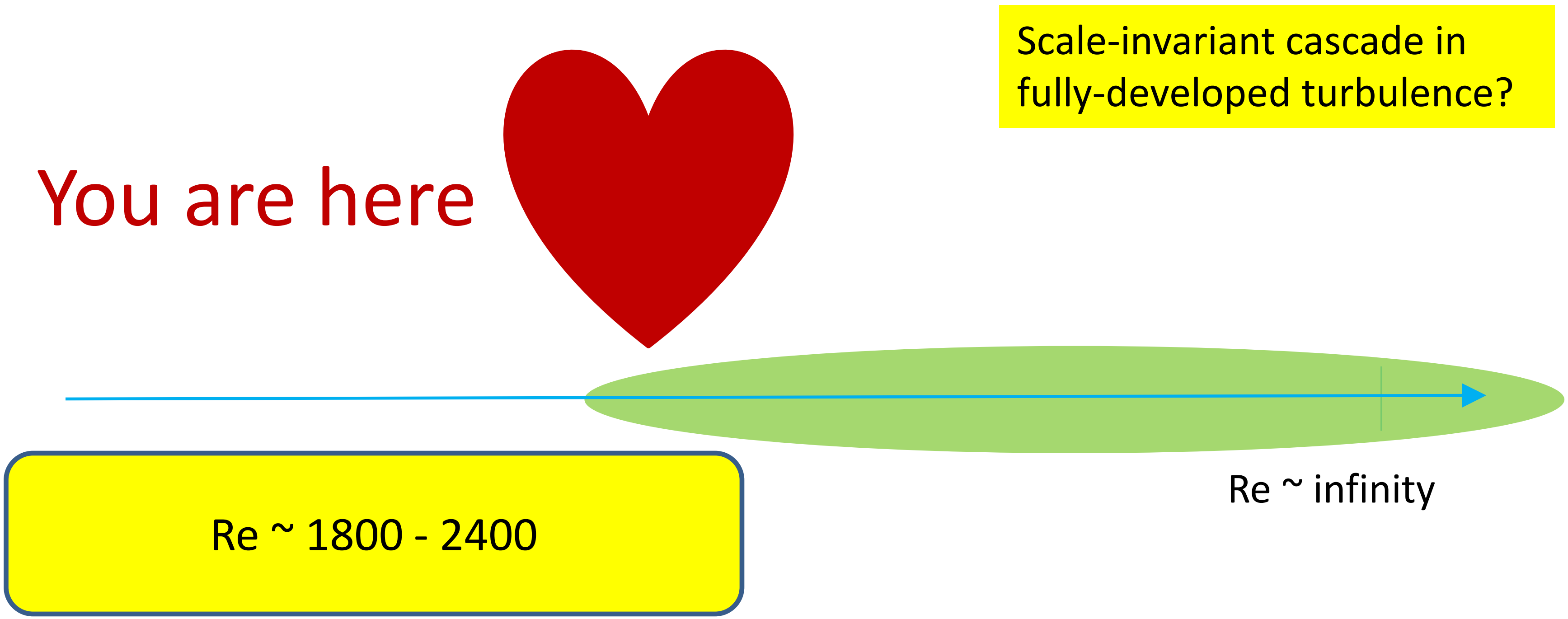
Re ~ infinity

Scale-invariant cascade in fully-developed turbulence?



Re ~ infinity

Re ~ 1800 - 2400


Scale-invariant cascade in fully-developed turbulence?
You are here
Re ~ infinity
Re ~ 1800 - 2400

Scale-invariance, but no cascade at laminar-turbulence transition

Scale-invariant cascade in fully-developed turbulence?

Re ~ 1800 - 2400

Re ~ infinity

Q. Is there universal scaling behaviour in fully-developed turbulence?

Q. How do fluids become turbulent as you increase their flow speed?

Scale-invariance, but no cascade at laminar-turbulence transition

Scale-invariant cascade in fully-developed turbulence?

Re ~ 1800 - 2400

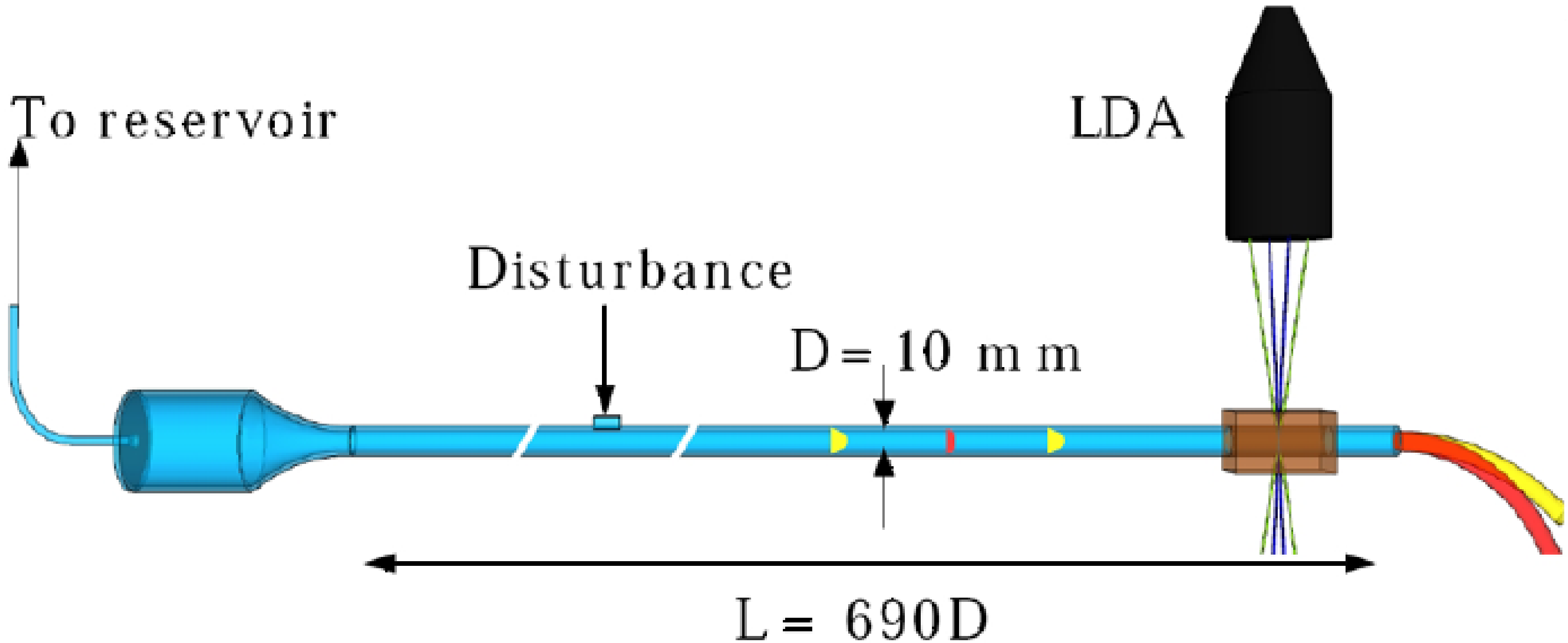
Re ~ infinity

Q. Is there universal scaling behaviour in fully-developed turbulence?

Q. How do fluids become turbulent as you increase their flow speed?

Precision measurement of turbulent transition

Q: will a puff survive to the end of the pipe?

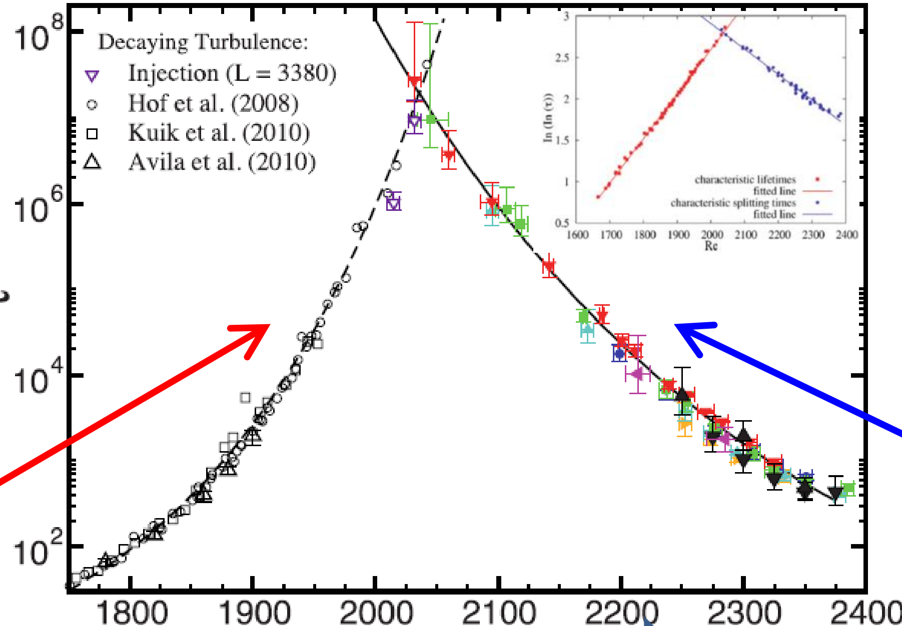


Many repetitions ➔ survival probability P(Re, t)

Hof et al., *PRL* **101**, 214501 (2008)

Fluid in a pipe near onset of turbulence

Log [lifetime]



Turbulent puff lifetime

Mean time between puff split events

Reynolds number

Avila *et al.*, *Science* **333**, 192 (2011)   Song *et al.*, *J. Stat. Mech.* 2014(2**)**, P020010   Hof *et al.*, *PRL* **101**, 214501 (2008)

Predator-prey ecosystem near extinction

https://interstices.info/jcms/n_49876/des-especes-en-nombre

Predator-prey ecosystem near extinction

Prey birth rate too small

➔ Predator eats all prey and starves and goes extinct

Prey birth rate high enough

➔ Predator population supported by prey: coexistence

Extinction

Coexistence

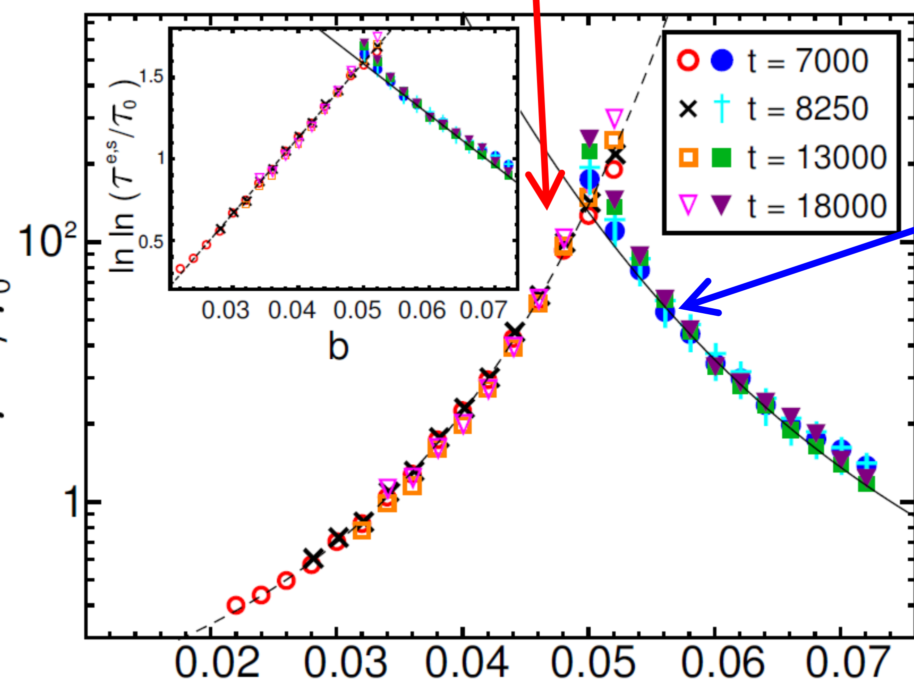
Birth rate

https://interstices.info/jcms/n_49876/des-especes-en-nombre

Predator-prey ecosystem near extinction

Prey lifetime

Log (lifetime)



Mean time between population split events

Prey birth rate (b)

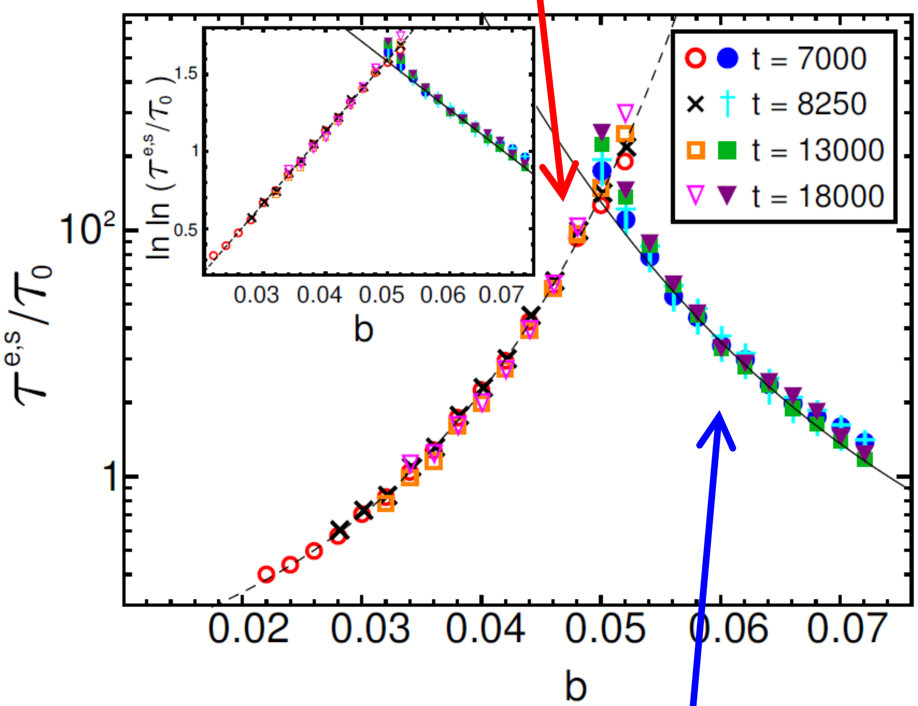
Super-exponential scaling: $\frac{\tau}{\tau_0} \sim \exp(\exp \mathrm{b})$

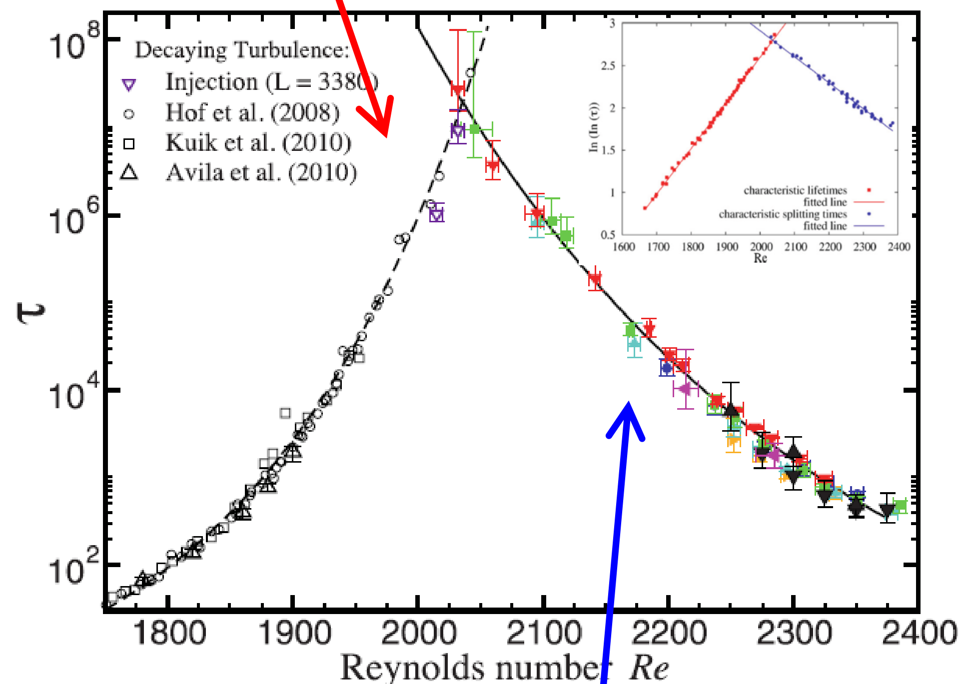
Shih, NG, Tsieh, Nature Physics (2016)

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime





Mean time between population split events

Mean time between puff split events

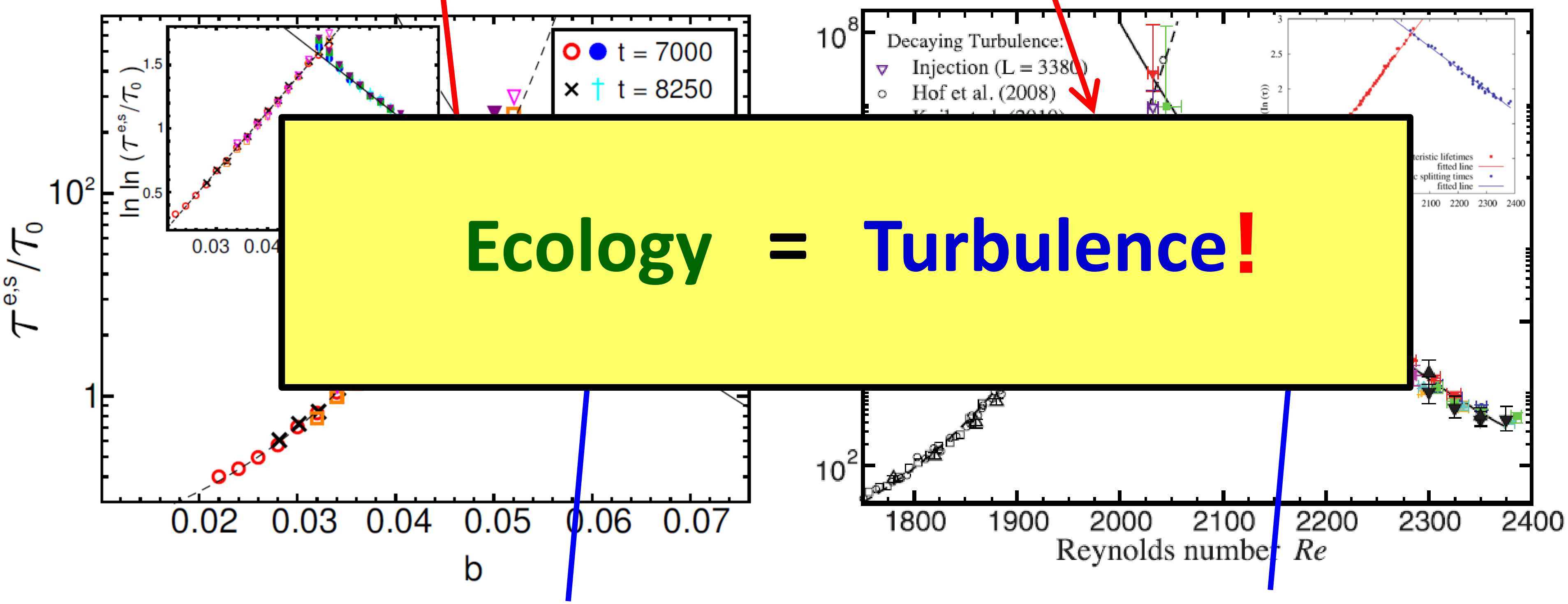
Shih, NG, Tsieh, Nature Physics (2016)

Avila *et al.*, *Science* **333**, 192 (2011)

Predator-prey vs. transitional turbulence

Prey lifetime

Turbulent puff lifetime



Ecology = Turbulence!

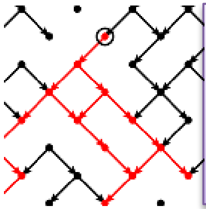
Mean time between population split events

Mean time between puff split events

Shih, NG, Tsieh, Nature Physics (2016)

Avila *et al.*, *Science* **333**, 192 (2011)

Turbulence transition – highly connected!

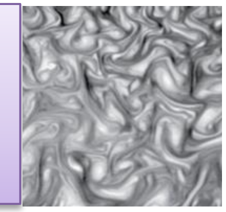


Directed Percolation

(Wikimedia Commons)

(Boffetta and Ecke, 2012)

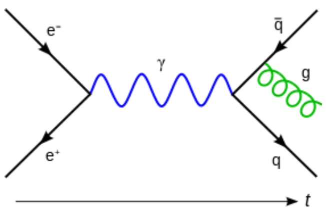
(Classical) Turbulence



Reggeon field theory
(Janssen, 1981)

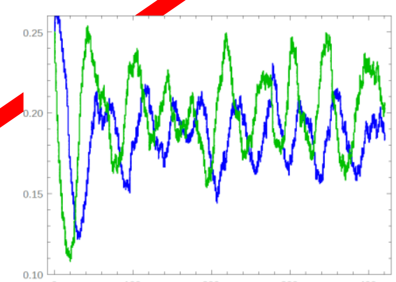
Direct Numerical Simulations
of Navier-Stokes

High energy physics



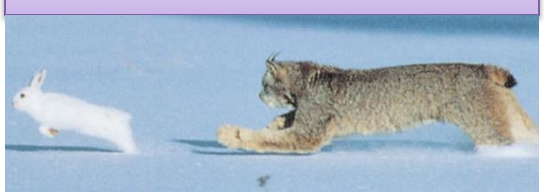
(Wikimedia Commons)

Two-fluid model



Extinction transition
(Mobilia et al., 2007)

Ecology

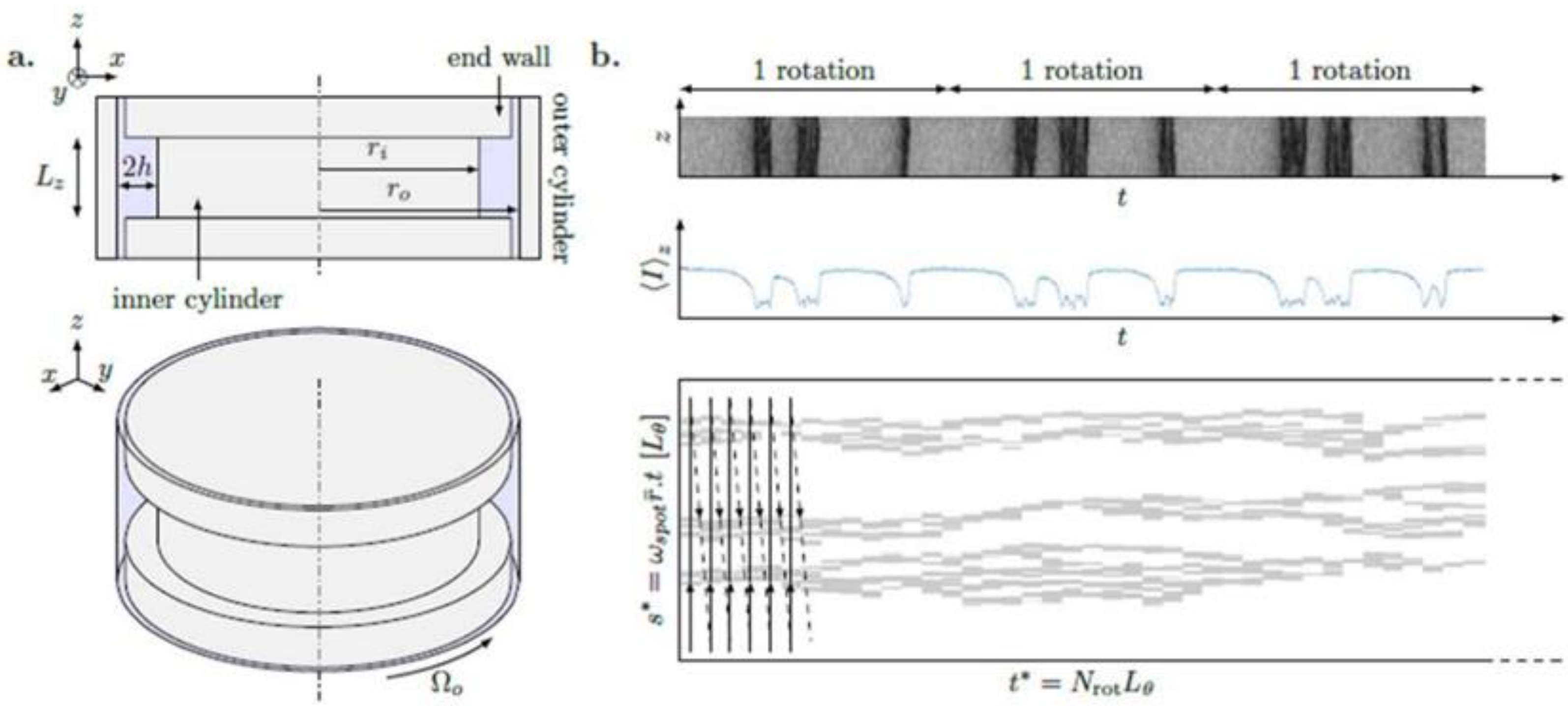


(Pearson Education, Inc., 2009)

Turbulence and “directed percolation”

Fluid between concentric cylinders, outer one rotating

Turbulent patches

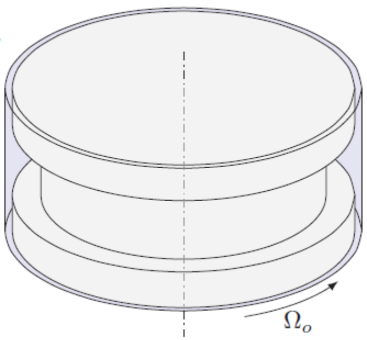


Position of turbulent patches changes in time

Lemoult et al., Nature Physics (2016)

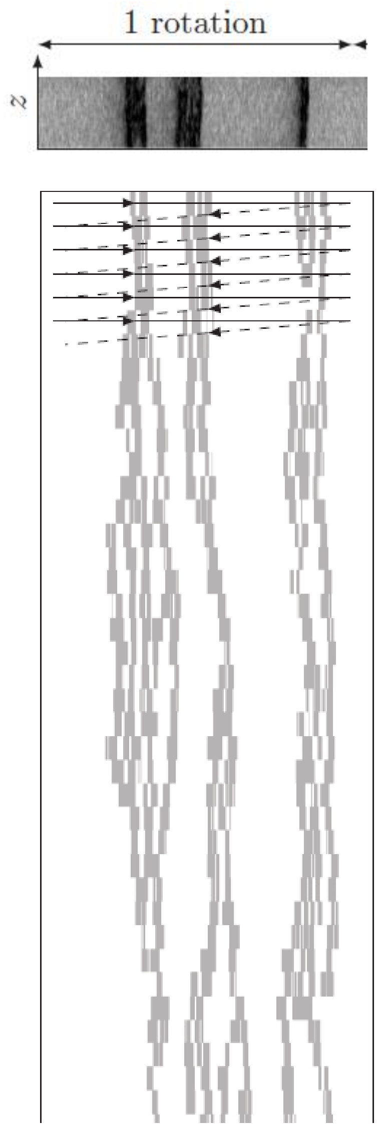
Turbulence and “directed percolation”

Couette



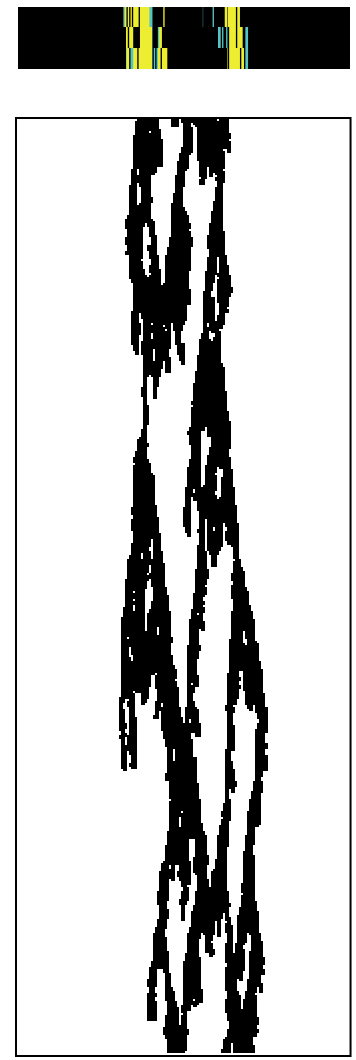
space

time



space

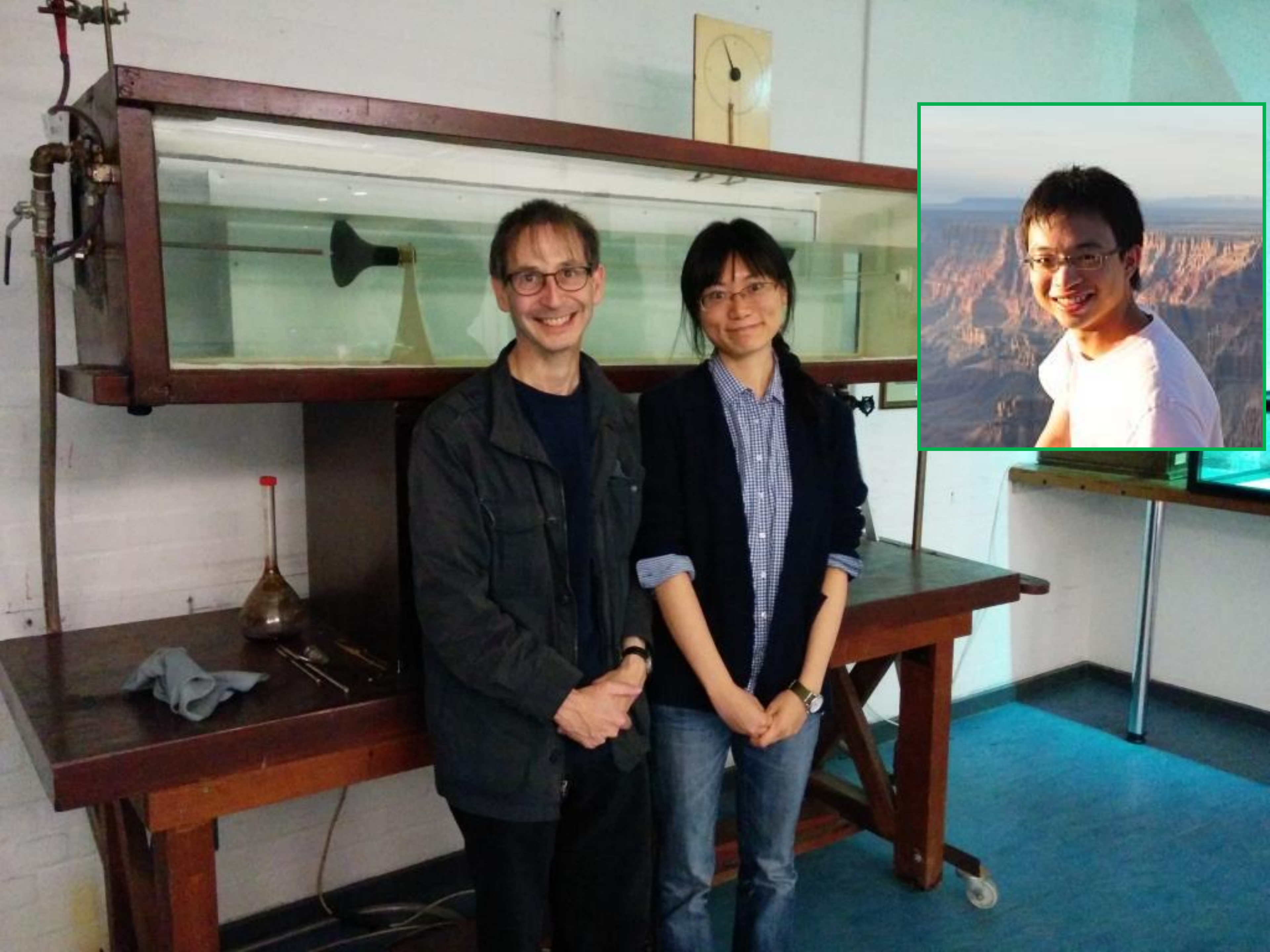
time



Ecology

What did you learn today?

- Turbulence is an unpredictable complex flow with structure at a wide range of length scales
- In your body, the flow of blood is at the edge of turbulence, and crossing that transition can be fatal
- Through great ingenuity by many people we now can measure the transition very accurately and quantify its behavior.
- The transition to turbulence is mathematically like the extinction transition in an ecosystem, directed percolation and even the scattering of high energy elementary particles.



Take-home messages

- Theoretical physics starts with a simple guess, because the exact equations are too hard to solve
  - Compare the consequences of that guess with experiment
  - And the experiments suggest new guesses etc…

- Simple models --- but not too simple --- can describe very complicated phenomena
  - Physics is successful because we only work on simple problems
  - We do not really know to what extent this approach can be used in more complex sciences like biology

- The problem of turbulence is not solved --- but we understand a lot more than we did in Feynman's day
  - Thanks to brilliant and controlled experiments
  - Healthy interactions between experimenters, theorists and computational physicists

Turbulence is a life force. It is opportunity.
Let's love turbulence and use it for change.

Lucky Numbers 34, 15, 28, 4, 19, 20