Wormholes and Entanglement

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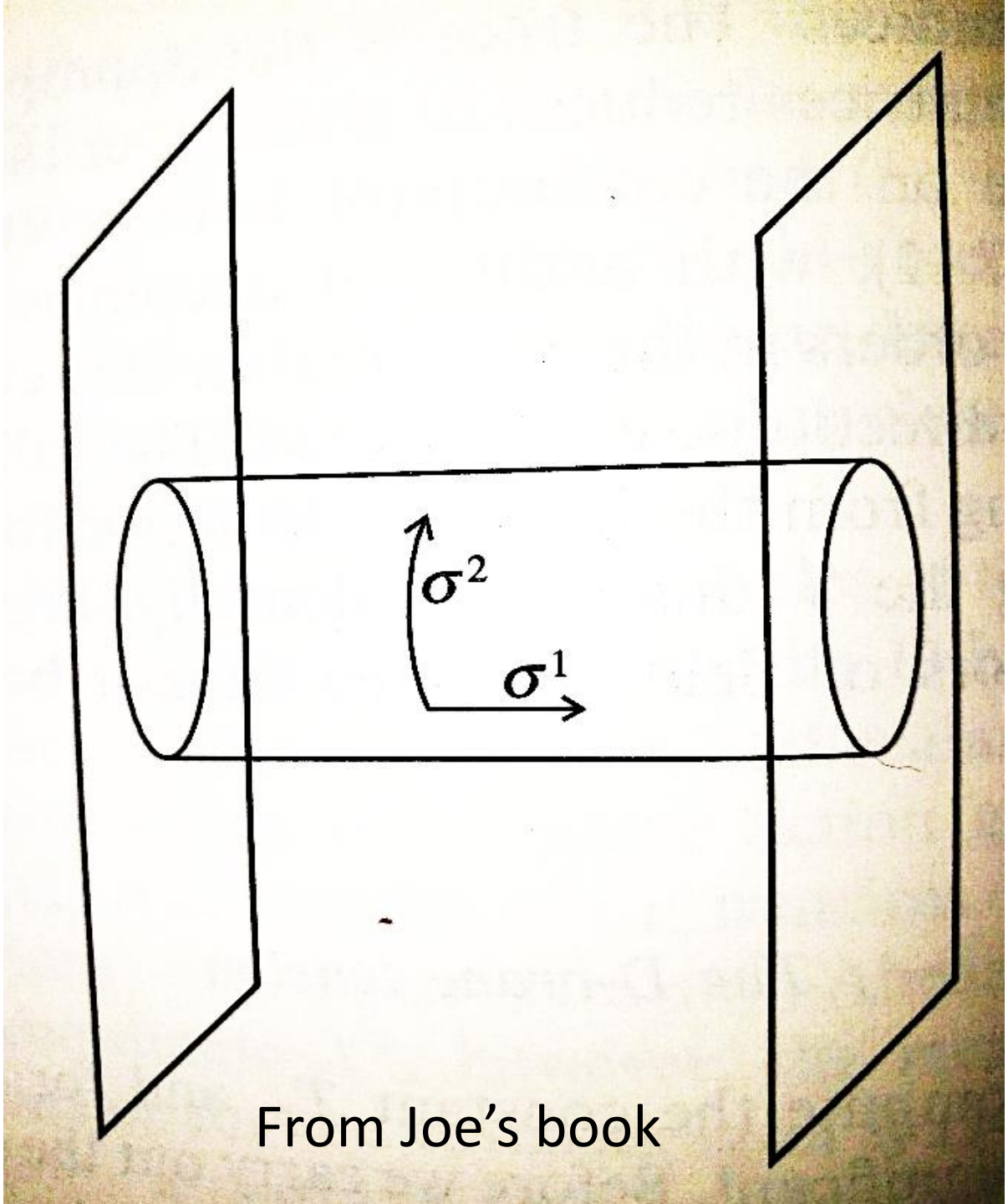
Joe Polchinski Memorial Symposium

Joe had a long fascination with wormholes

- Mostly with Euclidean wormholes.
- Several papers analyzing Coleman's idea.
- One proposing them as an ingredient for solving the information paradox:

3. **A Possible resolution of the black hole information puzzle**
Joseph Polchinski (Santa Barbara, KITP), Andrew Strominger (UC, Santa Barbara). Jul 7, 1994. 15 pp.
Published in **Phys.Rev. D50 (1994) 7403-7409**
UCSB-TH-94-20
DOI: 10.1103/PhysRevD.50.7403
e-Print: **hep-th/9407008** | **PDF**

His most famous ``wormhole'' was…



From Joe's book

Dai, Leigh, Polchinski, 89
Polchinski, 95

D-branes

- Put string dualities under a firmer footing.
- Black holes.
- Non perturbative formulations: BFSS matrix model, AdS/CFT.
- ….

- We are made of D-branes…

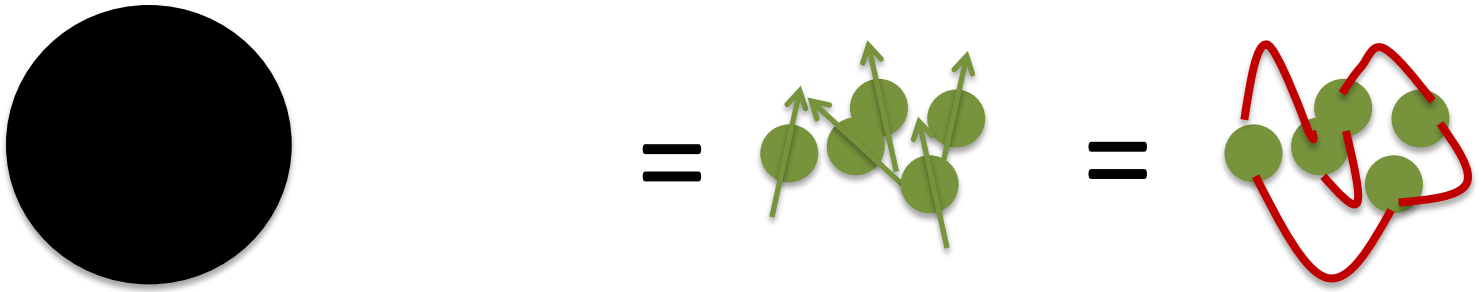
See Strassler talk

Outline

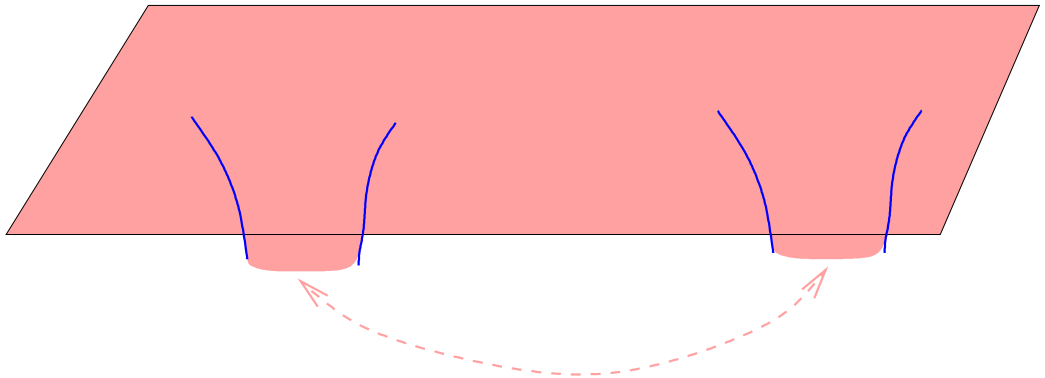
- Non traversable wormholes
- Traversable wormholes.
- Simple dynamics in Nearly-AdS_2
- The Hayden Preskill problem revisited.
- Traversable wormhole solution in 4d.

Black holes as quantum systems

- A black hole seen from the outside can be described as a quantum system with S degrees of freedom (qubits).

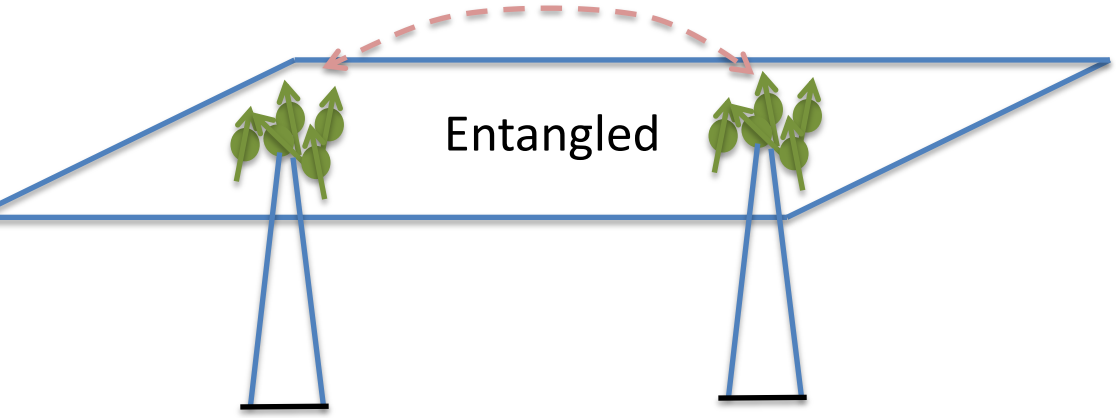


Lorentzian non-traversable wormholes



Connected through the interior

=

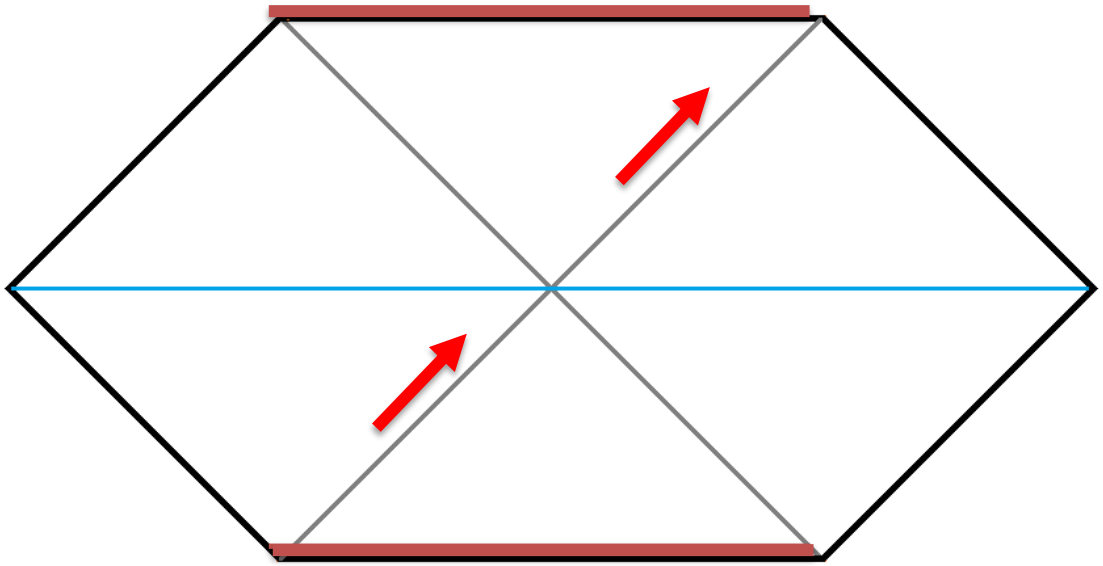


W. Israel, JM
ER =EPR , JM. Susskind

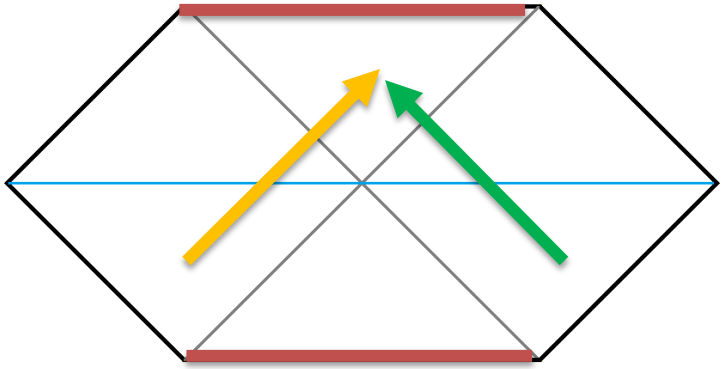
In a particular entangled state $|TFD\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L |E_n\rangle_R$

- In this set up the horizon is “exact”, it does not come from coarse graining.

Left observer sends a signal at early times



The signal is just behind
The horizon, but cannot be extracted by the right observer

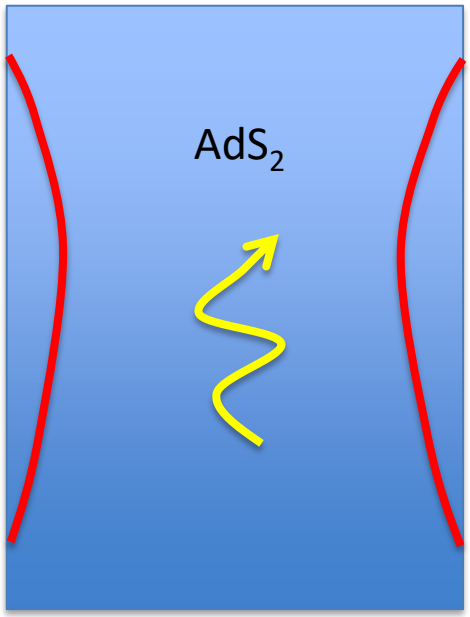


"What happens in the interior stays in the interior"

Including gravitational dynamics

.. in a simple case

The surprisingly simple gravitational dynamics of N-AdS_2



$NAdS_2$ = AdS_2 + location of boundary

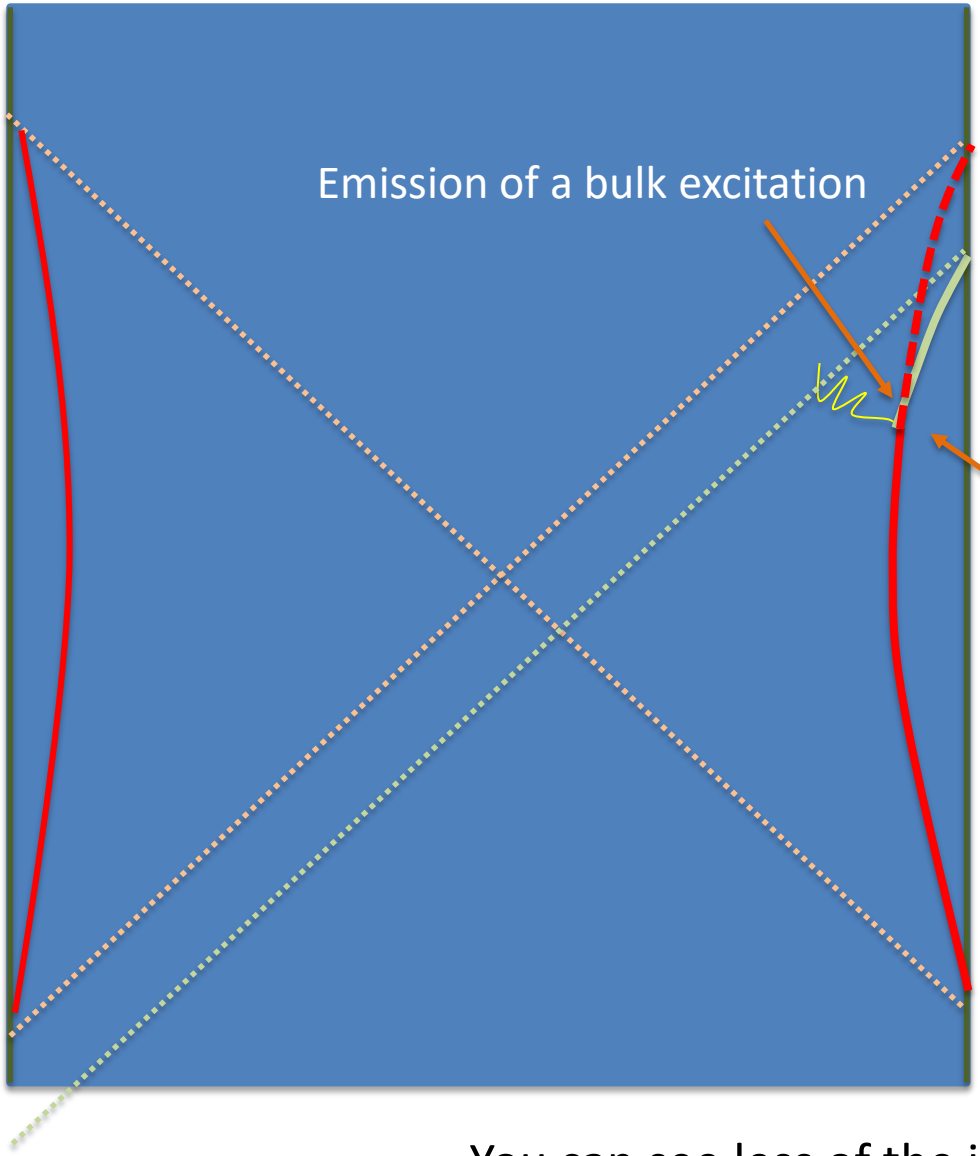
Dynamics of the boundary is SL(2) invariant.

Proper time along the boundary = time of the dual quantum system. Motion of the boundary = relation between the two times.

$$(H_{L\,Bdy} \times H_{\rm bulk} \times H_{R\,Bdy})/SL(2,R)$$

Kitaev Suh
JM Stanford Yang
…

Dynamics



New position of the horizon

The boundary trajectory gets a "kick" determined by local energy momentum conservation.

You can see less of the inside

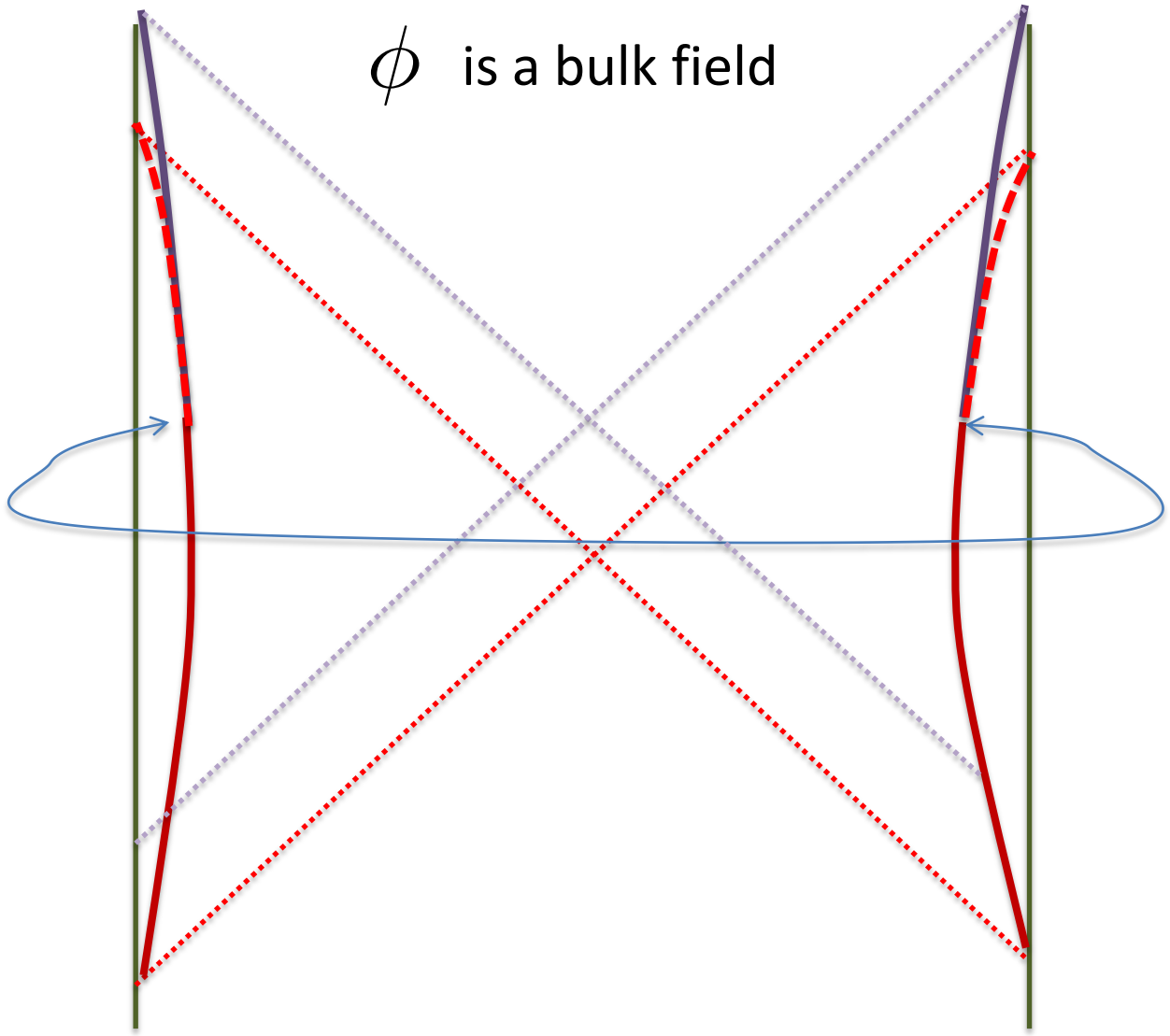
Could you see more?

- Yes!
- Introducing an interaction between the two sides.

Gao Jafferis Wall

Interaction between the two boundaries

Gao Jafferis Wall



Insert this in the path integral

$$e^{ig\phi_L(t_L)\phi_R(t_R)}$$

approximate

$$e^{ig\langle\phi_L(t_L)\phi_R(t_R)\rangle}$$

Force between the two boundaries.
(Can be attractive for the right sign of g).
Kicks the trajectories inwards.
Creates negative null energy.

More precisely: we have N bulk fields to enhance the effect.

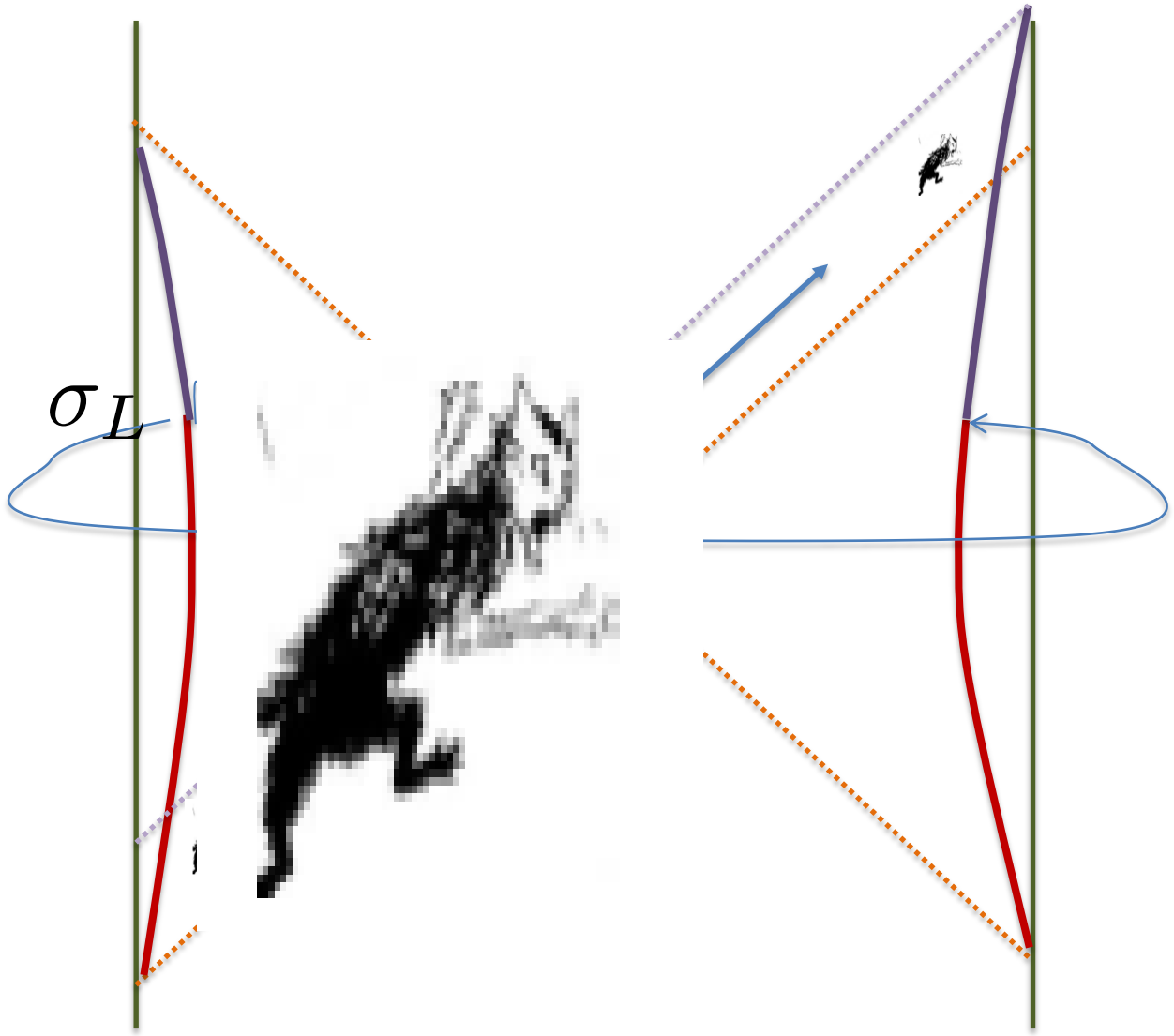
Does this interaction violate causality?

- If this is the bulk dual of two separate quantum mechanical systems, then we are free to introduce an interaction between the two quantum mechanical systems.

- We could have two entangled black holes that are close to each other at some instant.

- The full causality structure depends on how this geometry is embedded in a bigger space.

Quantum Teleportation interpretation



Measure

$$\phi_L \longrightarrow \sigma_L$$

Act on the right with

$$e^{ig\sigma_L\phi_R(t_R)}$$

From the point of view of the right we get the same, whether we measure or not.

Bennett, Brassard, Crepeau, Jozsa, Peres, Woetters

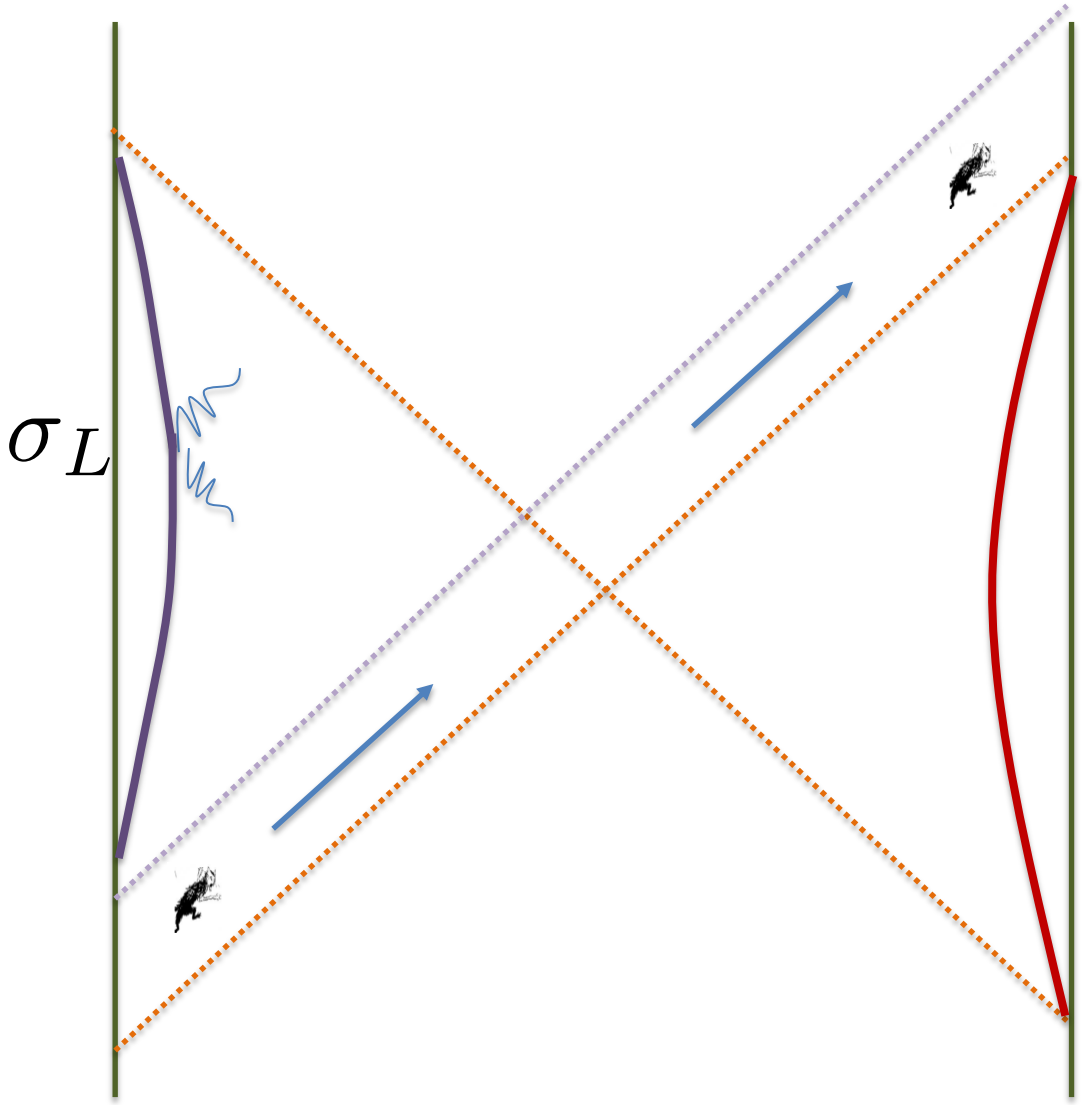
- The interesting fact is how the teleportee feels.
- It feels that it went through empty space.

- It offers an opportunity to explore a bigger region of AdS_2 than the Rindler wedge. Or to explore the “interior”.

Comments on teleportation

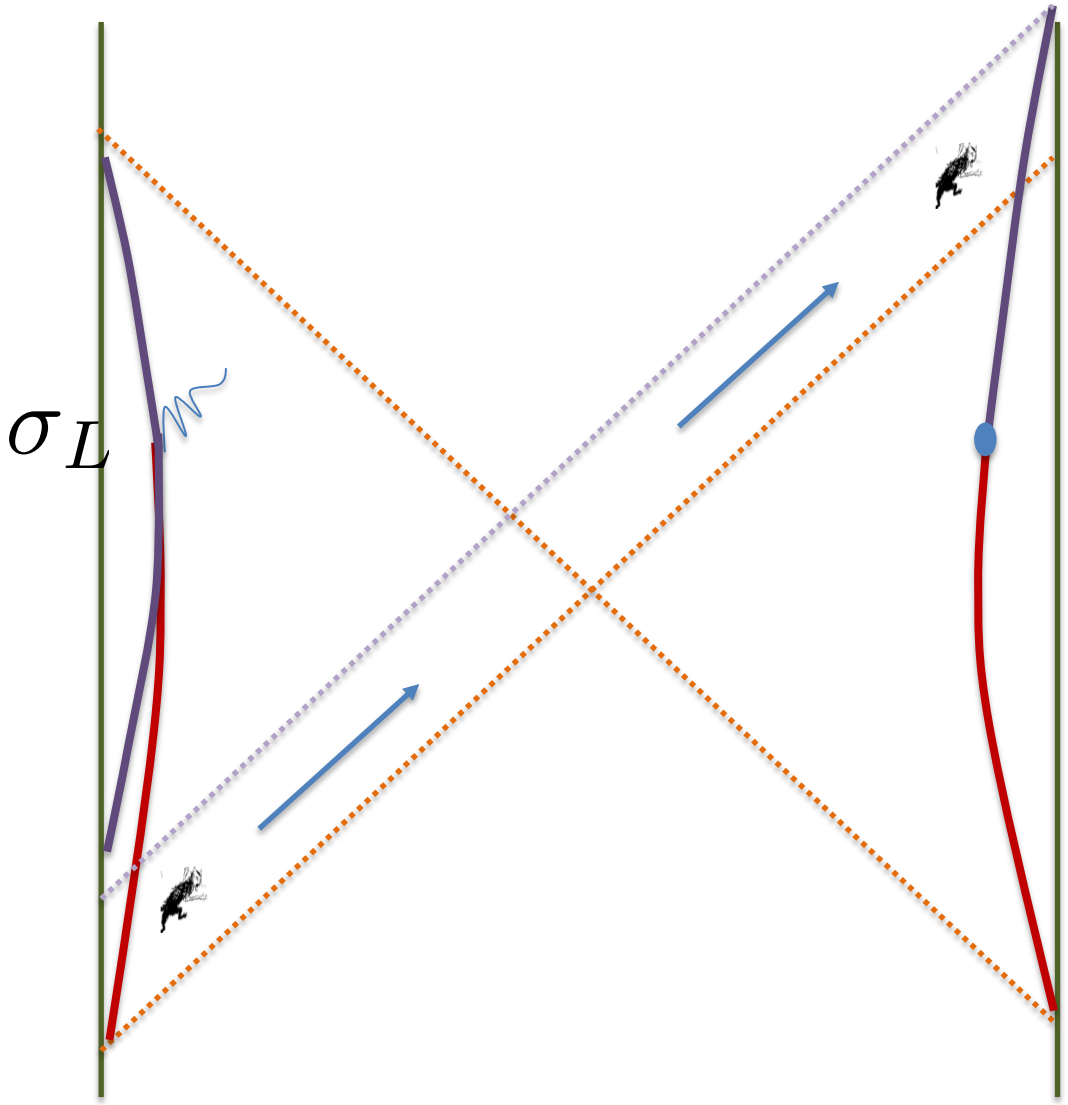
- The message is “secret” because it goes through a wormhole!.
- Where is the message after you do the measurement on the left, but before you extract it ?

Message is lost to the left after the measurement



It is also not accessible to the right if the right observer does not know the result of the measurement !

Message is lost to the left after the measurement



It is accessible to the right after she learns the result of the measurement.

Shift in the HRT surface.

See Alhmeiri's talk

New knowledge, if used wisely, can expand your horizons!

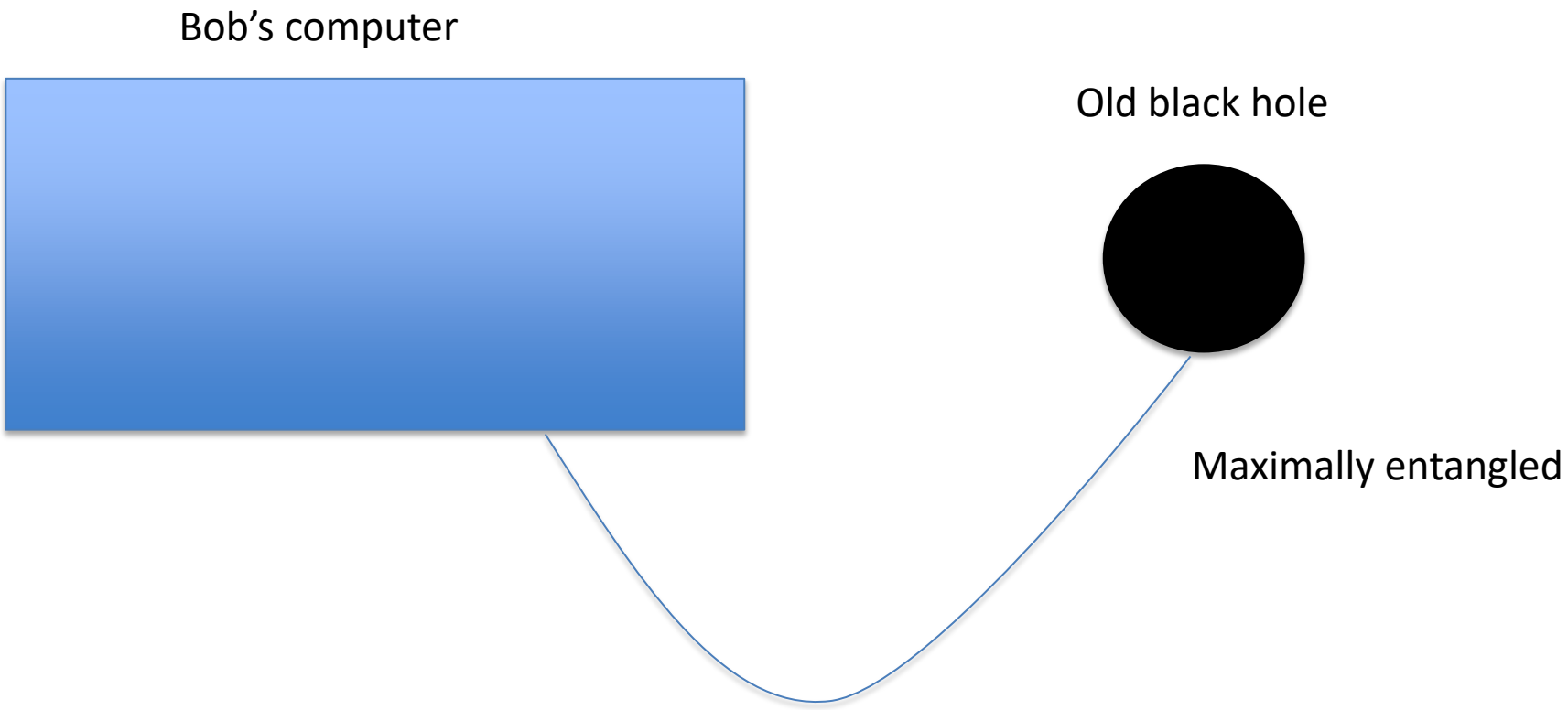
Geometry seems to encode these properties “better” than the standard QM description!

Application to the Hayden Preskill problem

Closely related to the AMPS paradox

JM, Stanford, Z. Yang

Starting assumption



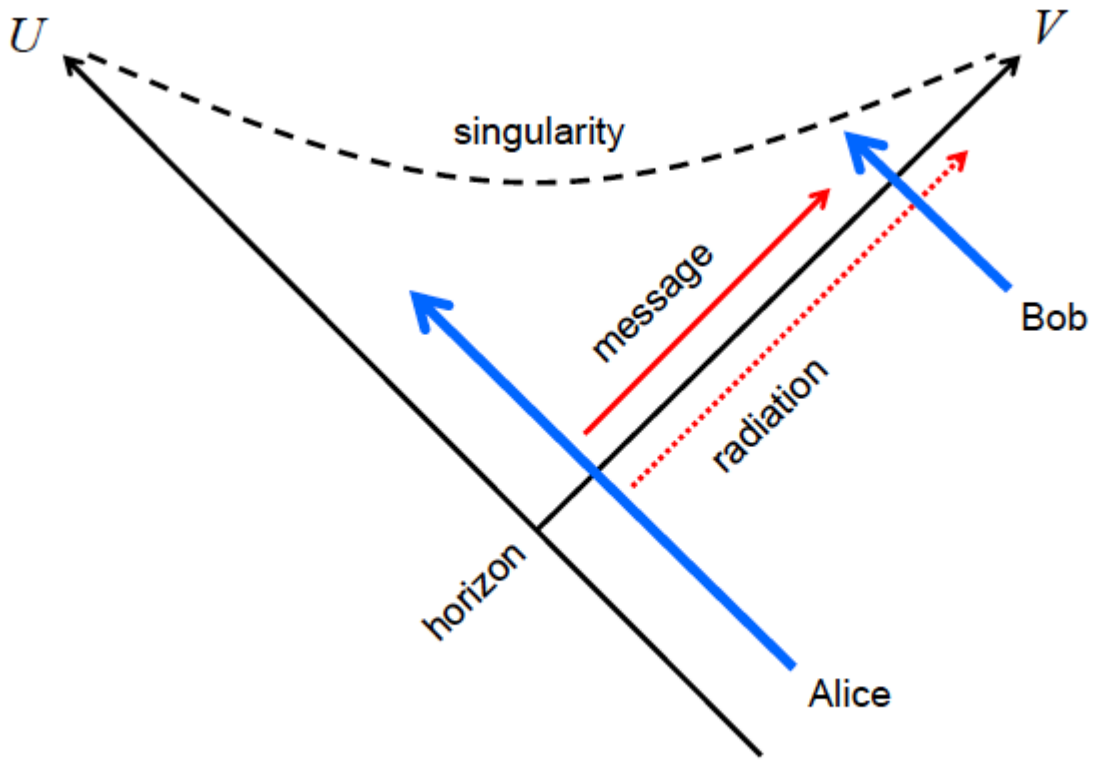
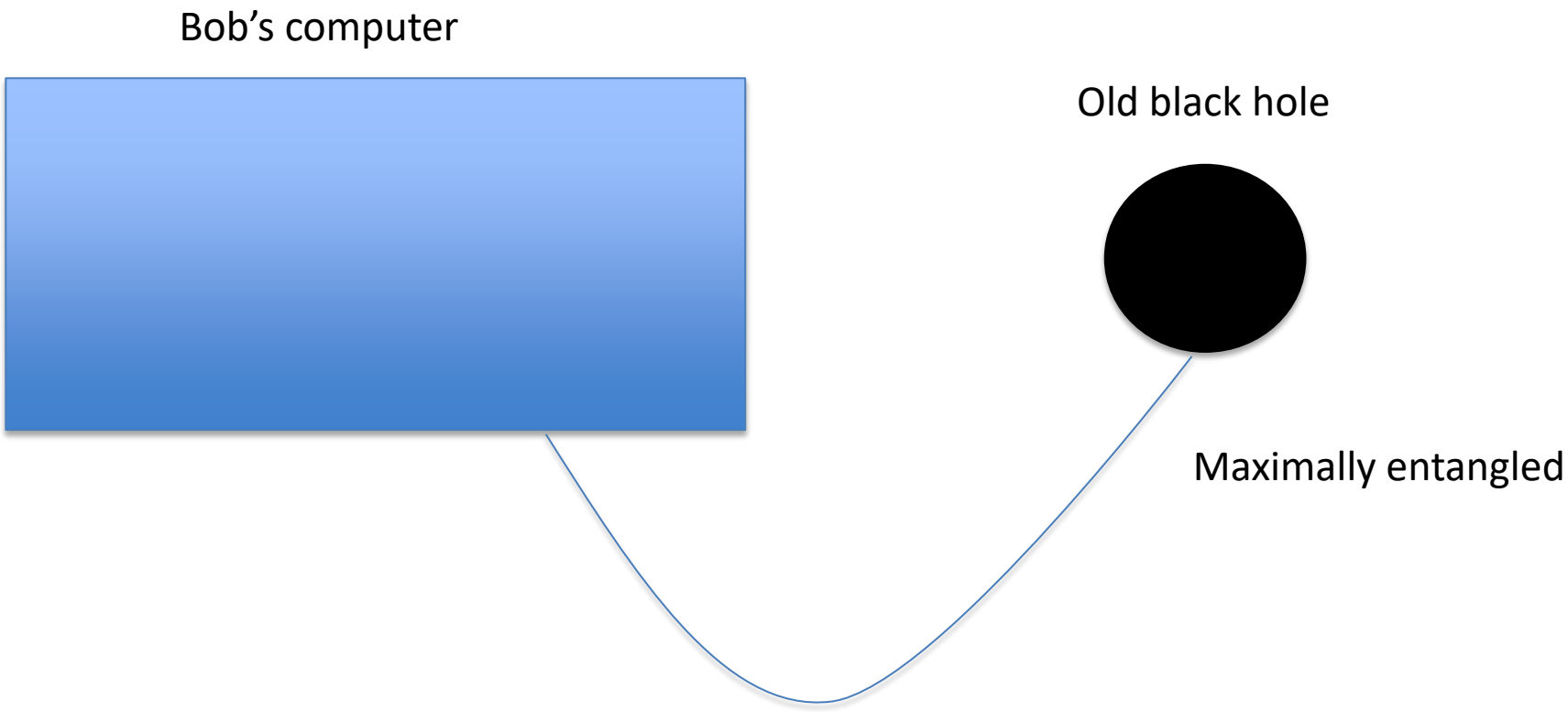


Figure from the paper of Preskill and Hayden.

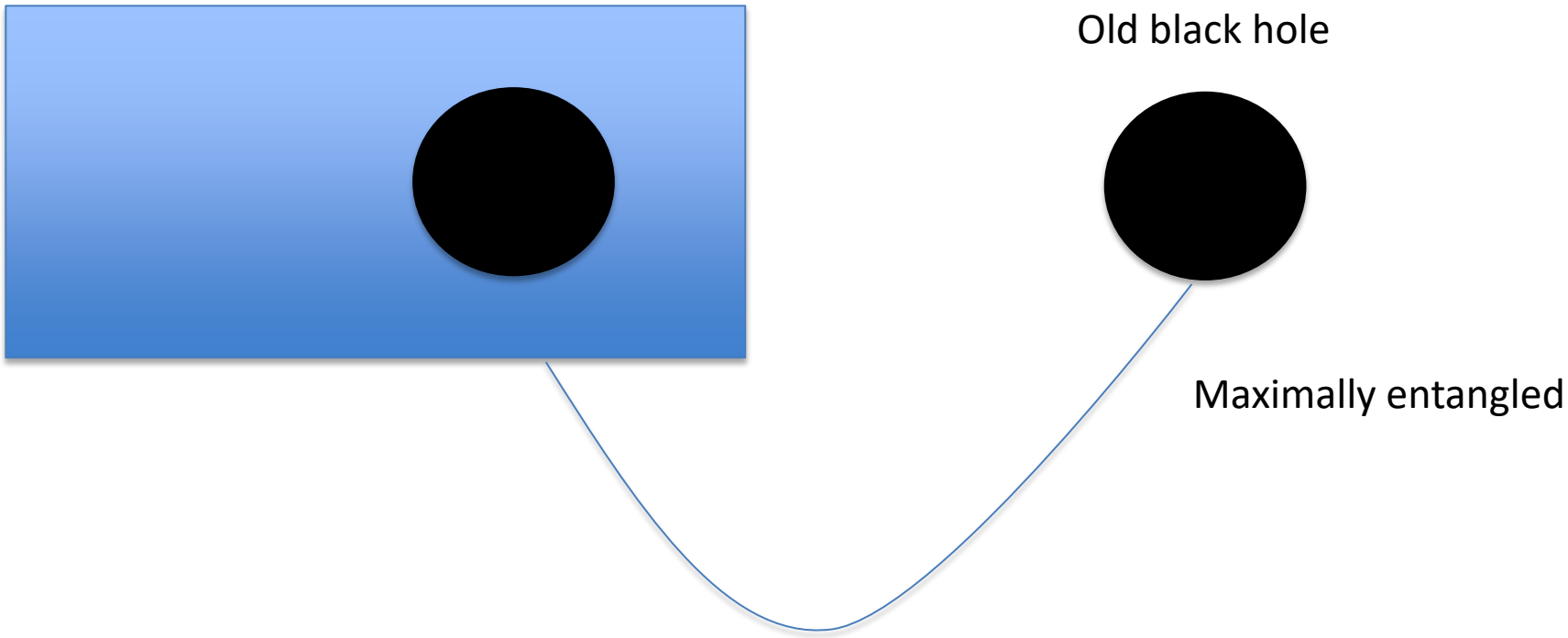
Duplication of information in the geometry. But it cannot be checked.

Something illegal has happened, but beyond the reach of the ``police''


Bob's computer
Old black hole
Maximally entangled

Bob → produces a second black hole,
maximally entangled with the first.

(This is hard to do
Harlow Hayden)



Bob → produces a second black hole,
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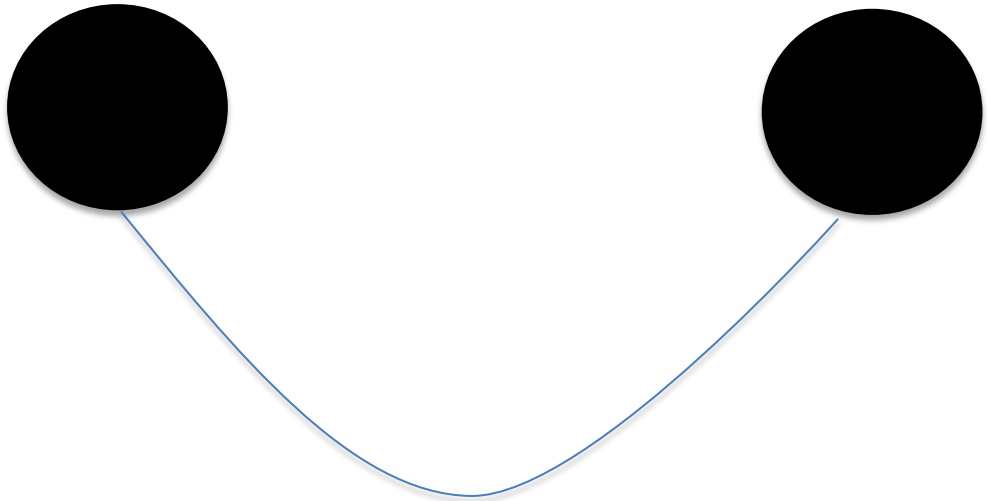
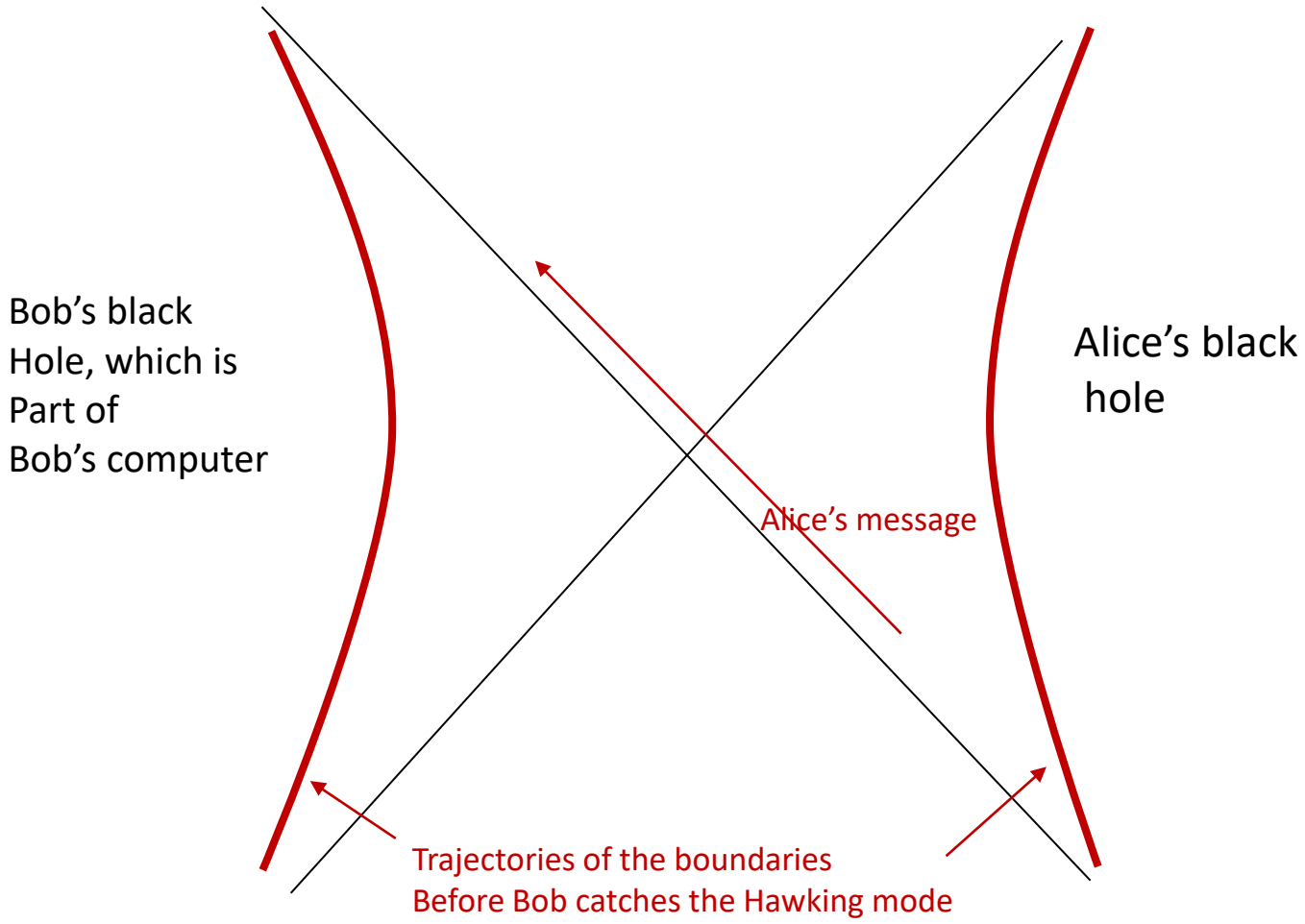
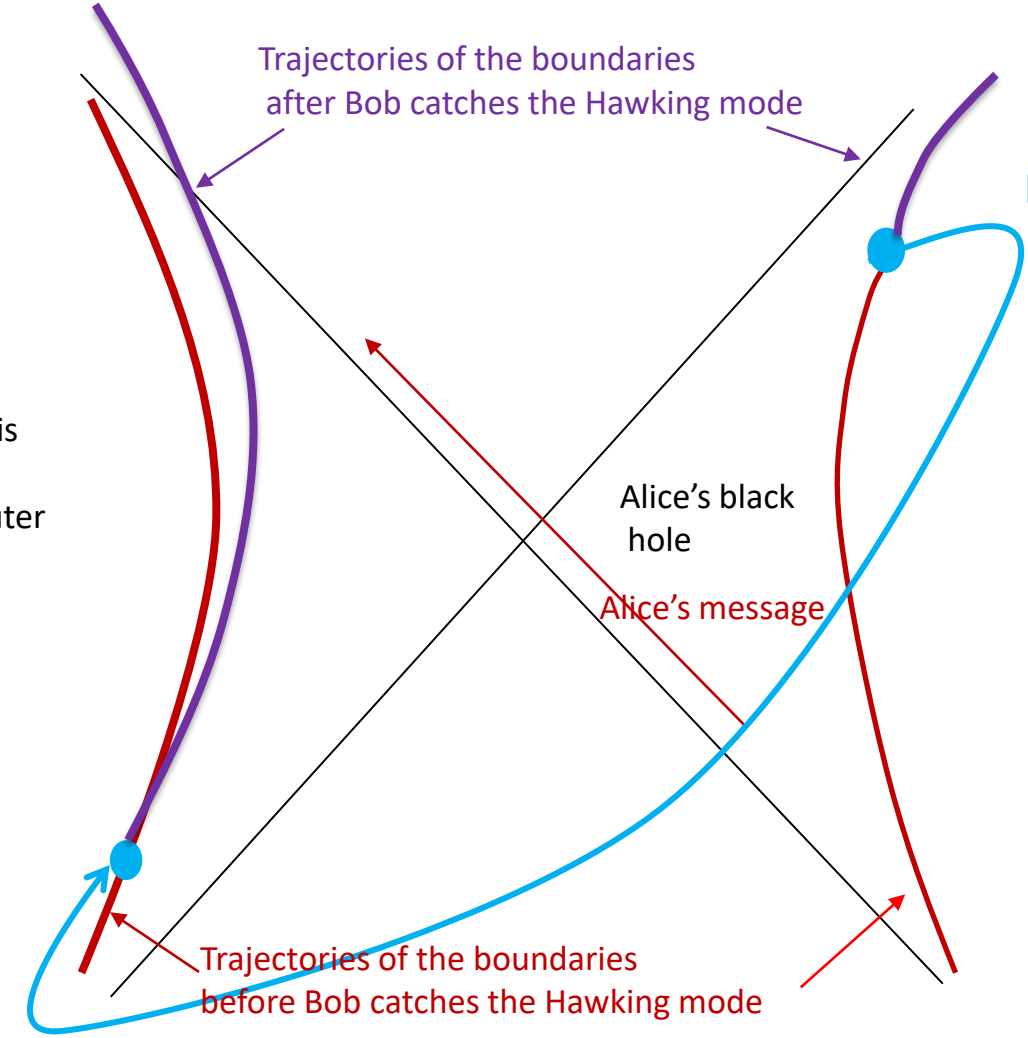


Maximally entangled

Say they are nearly AdS_2 black holes…

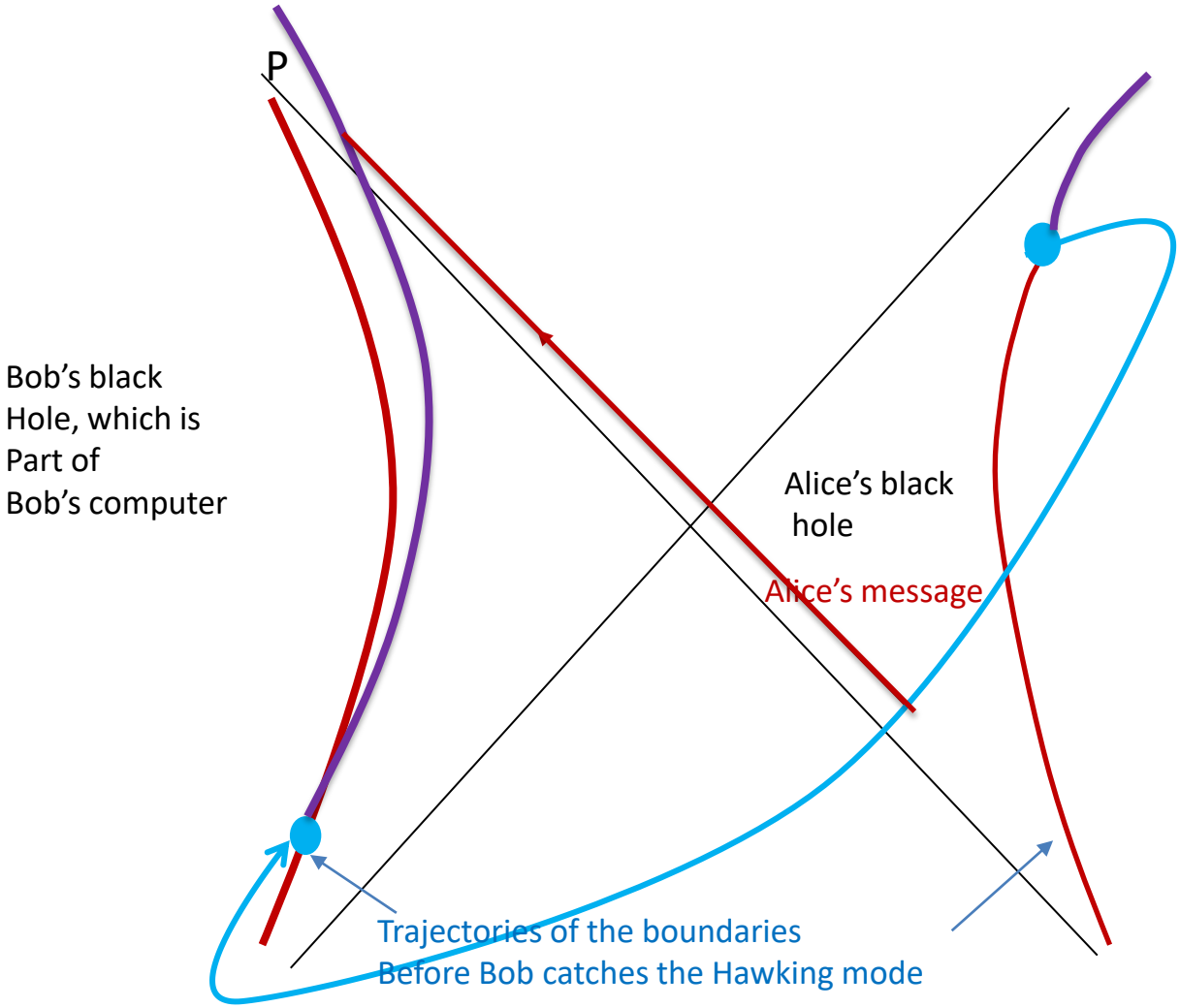

Bob's black
Hole, which is
Part of
Bob's computer
Alice's black
hole
Alice's message
Trajectories of the boundaries
Before Bob catches the Hawking mode

Bob gets some radiation and feeds it to his computer.



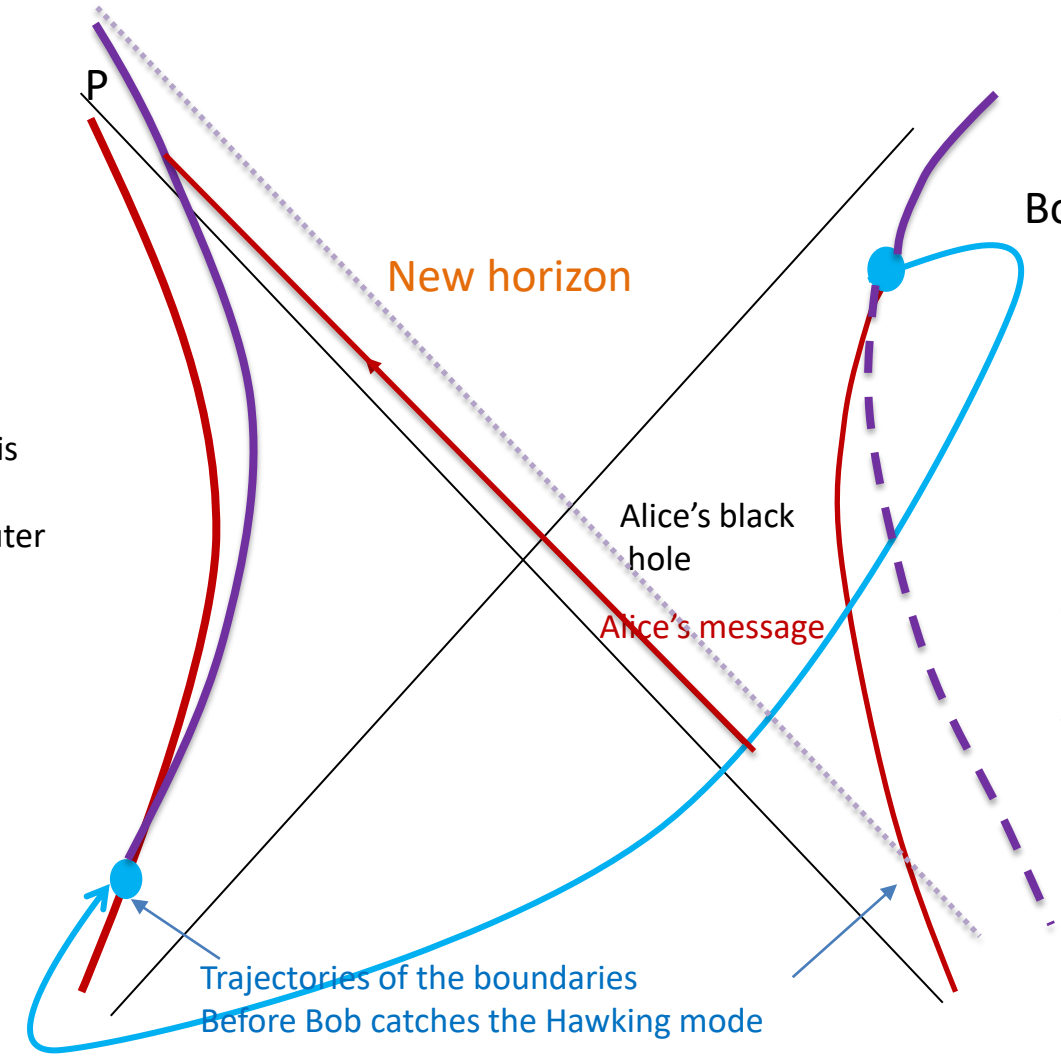
Feeding
his computer =
black hole.

Bob now gets the message at P



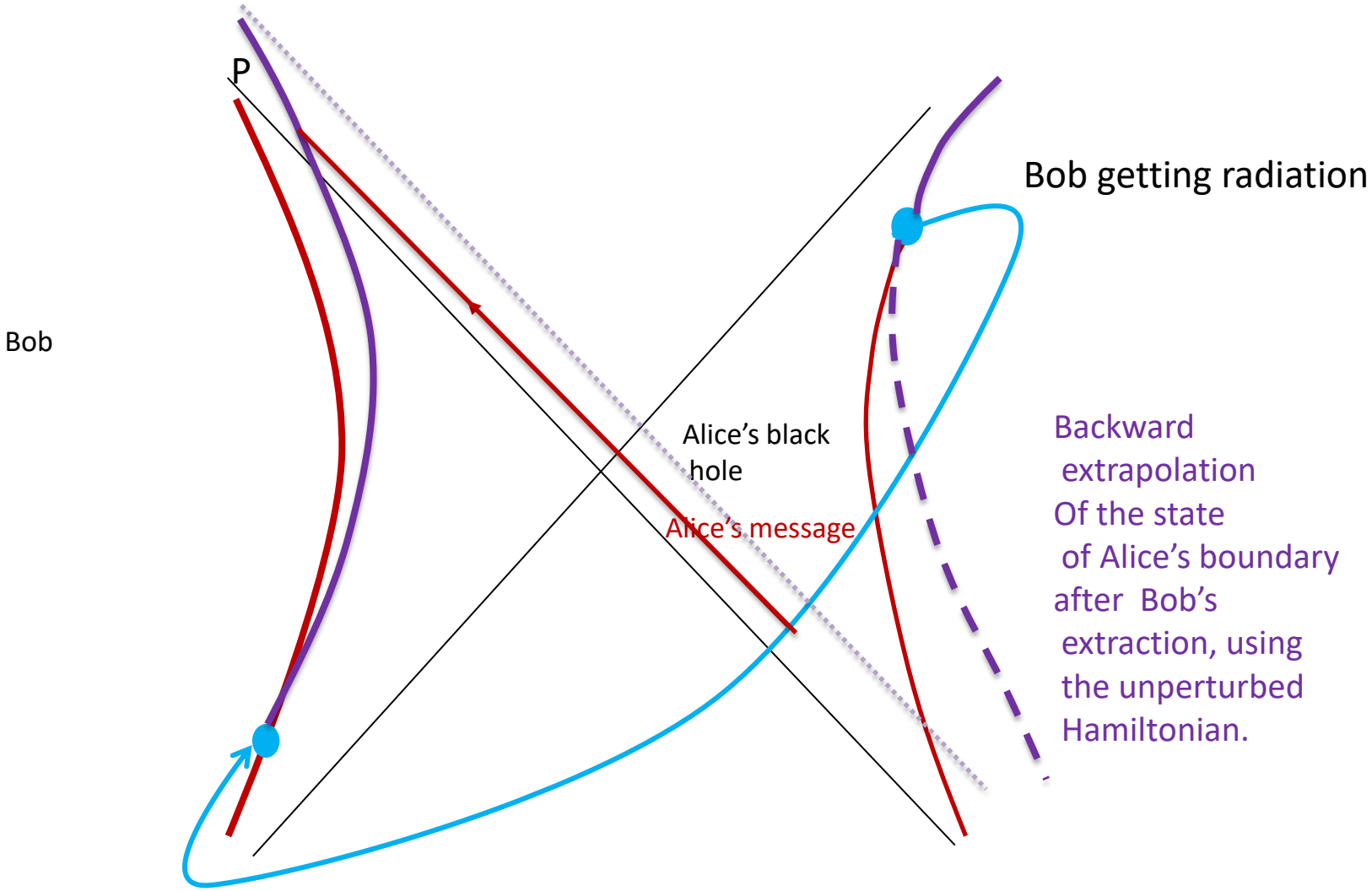
Message was obtained from the inside!

The message switched sides !



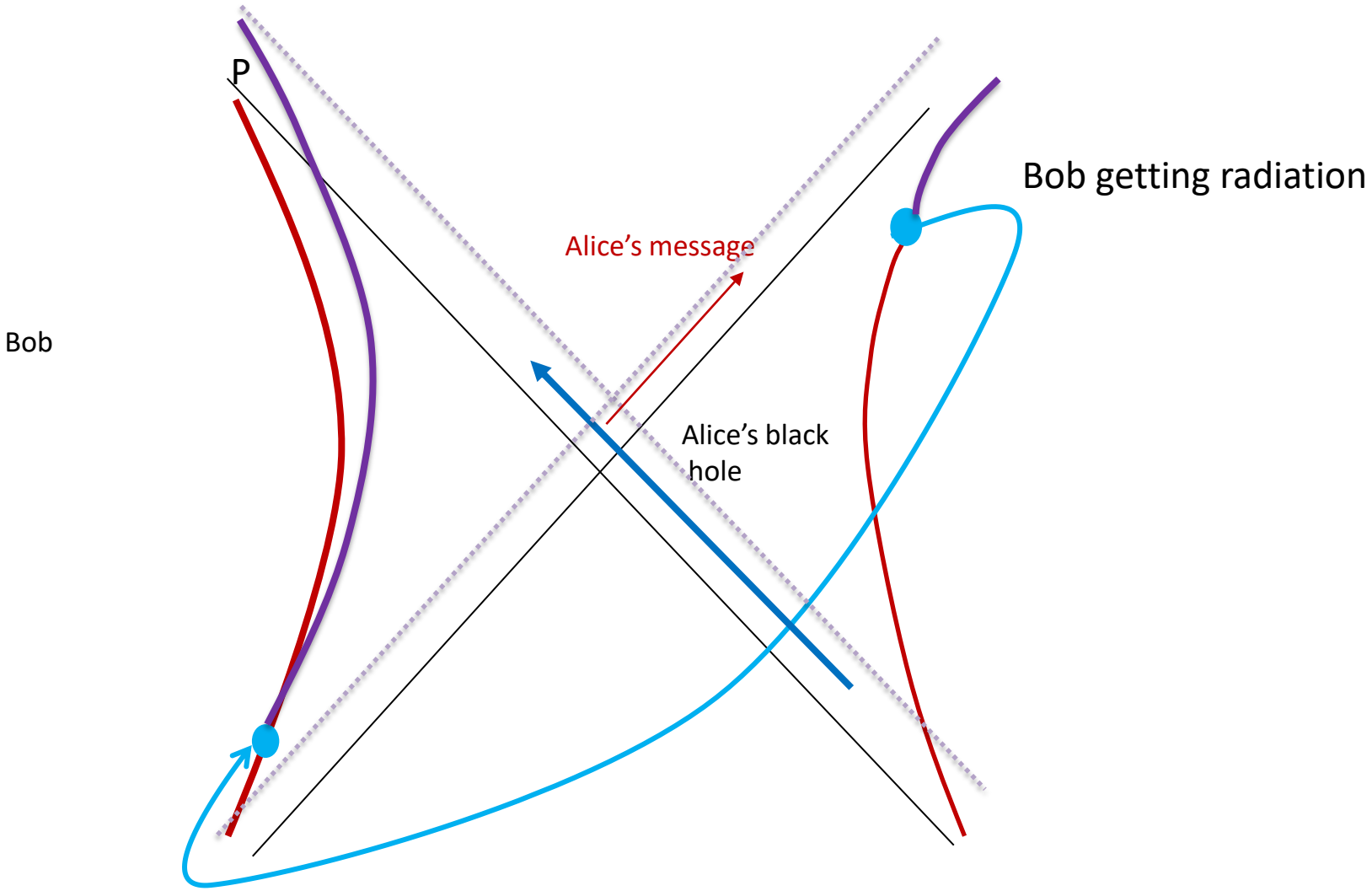
Before transfer: Alice has the message but Bob does not

After transfer: Bob has it but Alice does not !



Only one copy of the message throughout!

More like the HP figure

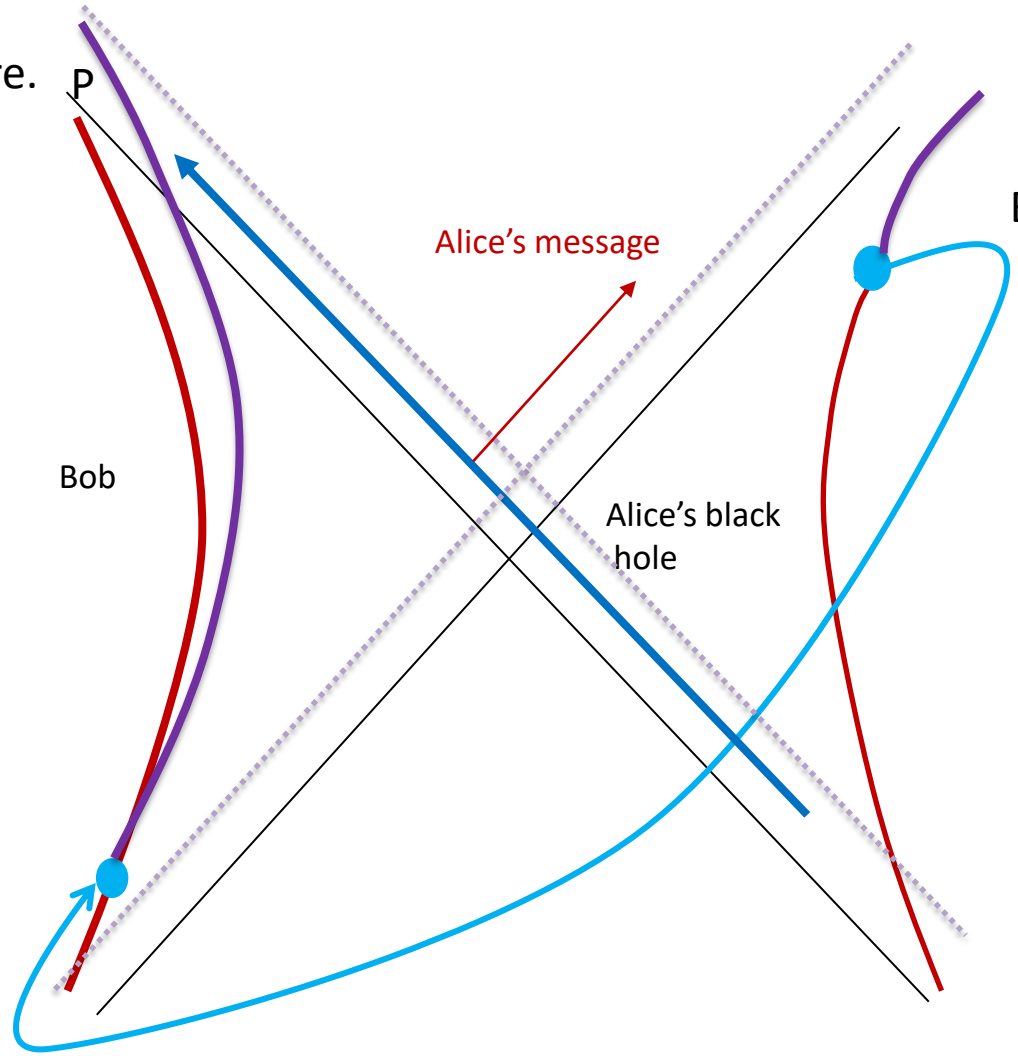


Now Bob cannot get the message !, it is still in Alice's posesion.

Now Alice send is later…

Bob will not get it here.

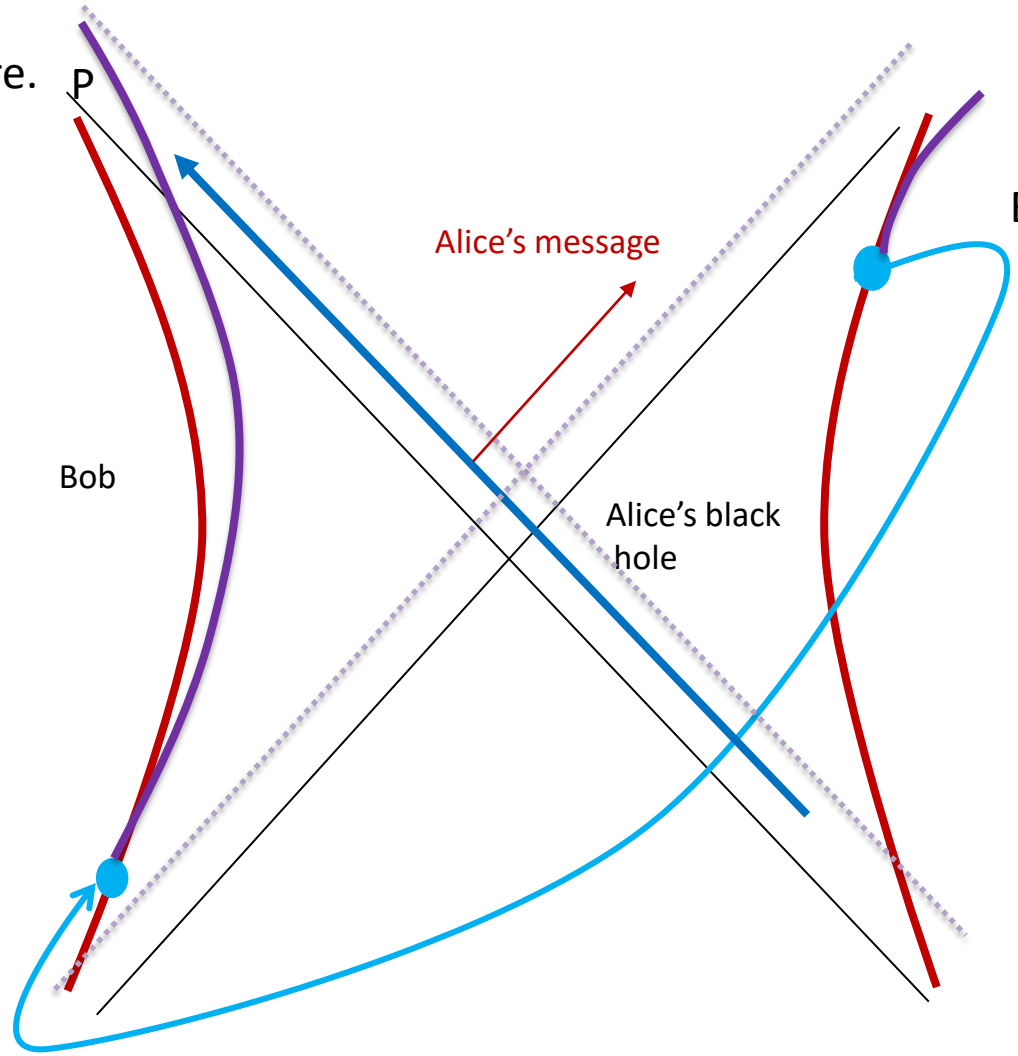
Bob gets the machinery that sent Alice's message, but with no message.



Now Alice send is later…

Bob will not get it here.

Bob can extract that machinery



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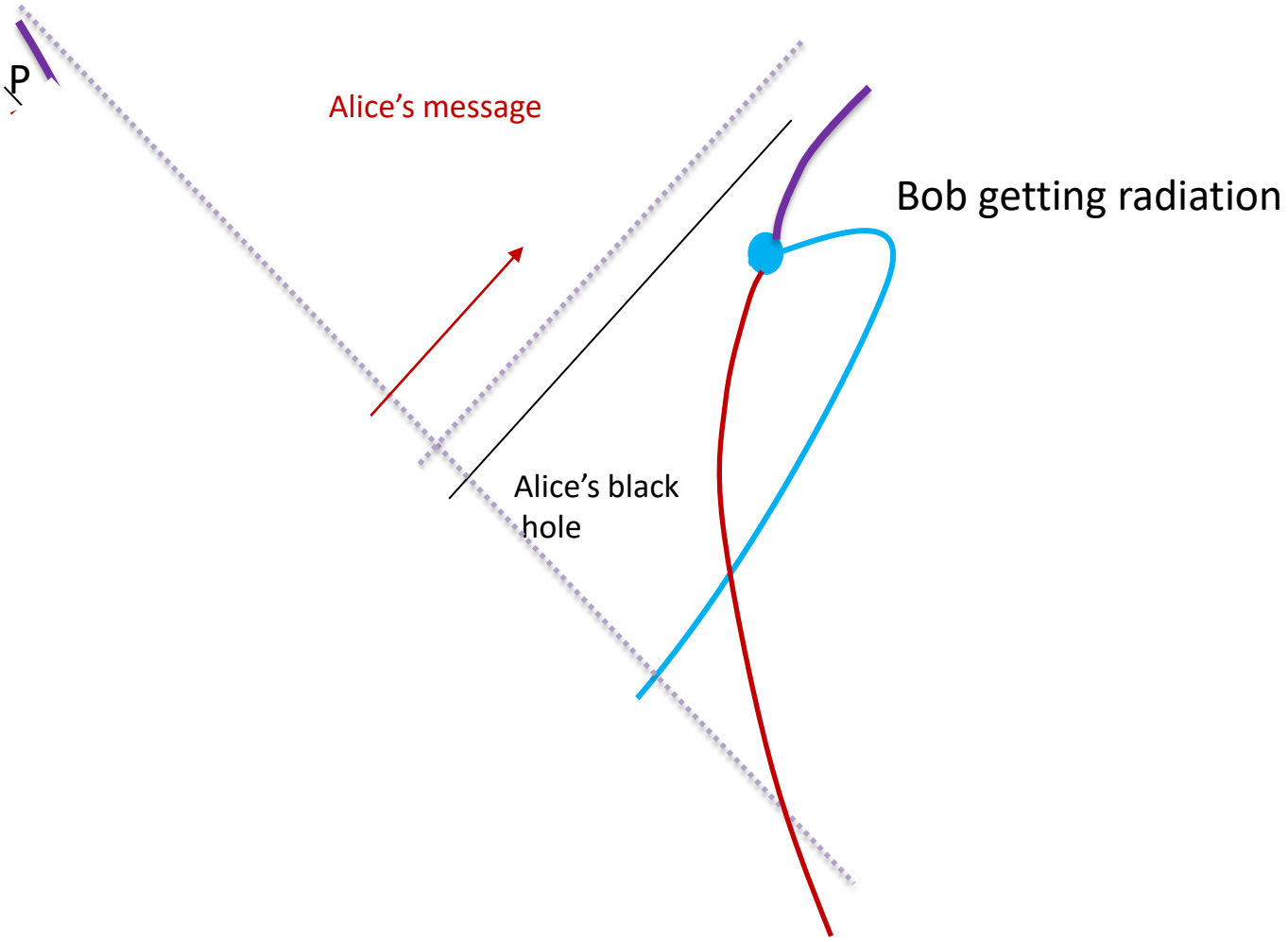

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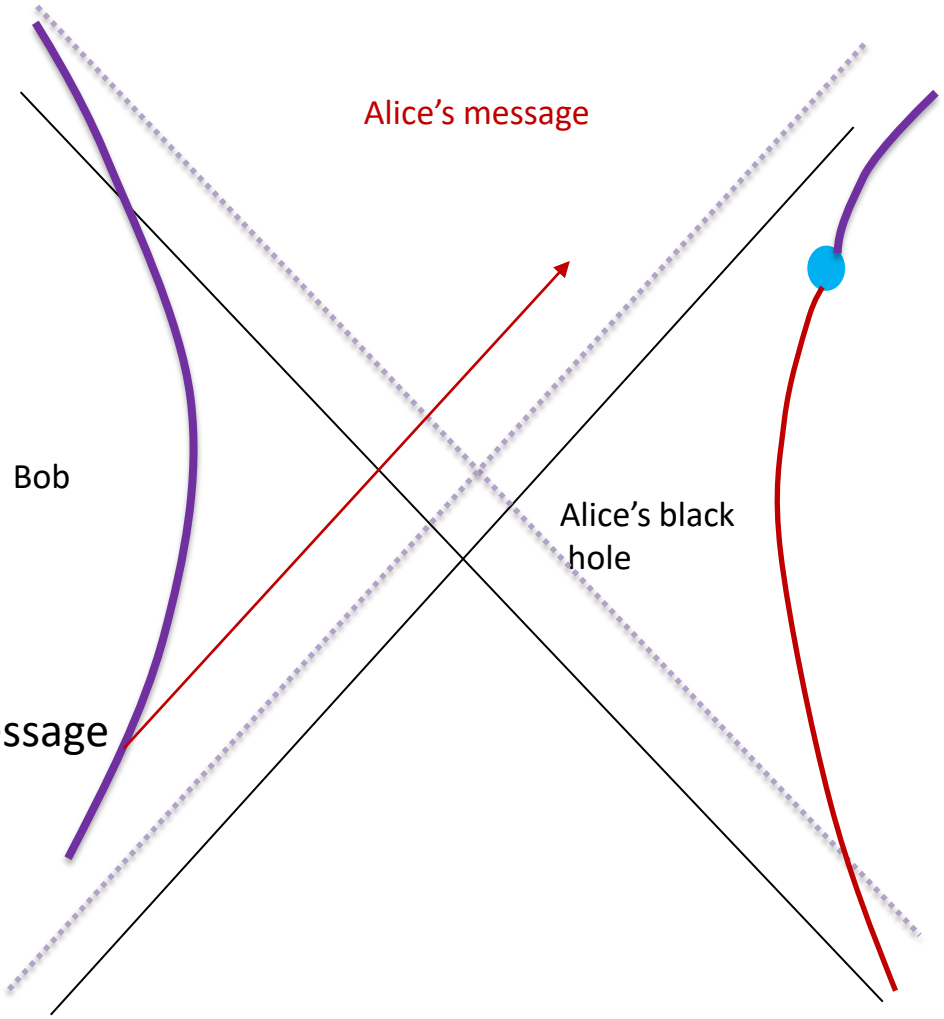


Bob getting radiation

Bob can evolve his side backwards in time and recover the message here

Summary

- The process of extracting the message puts it out of reach from Alice.
- The message is never duplicated in the bulk picture.
- No need to invoke unknown new transplanckian physics to solve the no-cloning problem.
- All understandable from standard rules of gravity on the wormhole geometry.
- What is the geometric picture of the process leading up to the two entangled black holes ?

When doing the complex quantum computation that extracts the message:

It is important to include the spacetime generated by the quantum computation!

Like the Maxwell demon:

The quantum computer also has a gravity dual, which is connected to the interior of the original black hole

We gave a geometric interpretation of when the decoding is done in a particular way.

What if it is done in different ways ?
Do they all have a geometric interpretation?

How “realistic” is the TFD state ?

- Is this state simple to make ?
- Isn’t it hopelessly fine tuned and unstable ?
- Is is just an unphysical mathematical idealization?

- All of these were said of the single sided Schwarzschild solution…

Method to prepare the TFD

JM Qi

TDF for NAdS$_2$ as the ground state of a coupled system

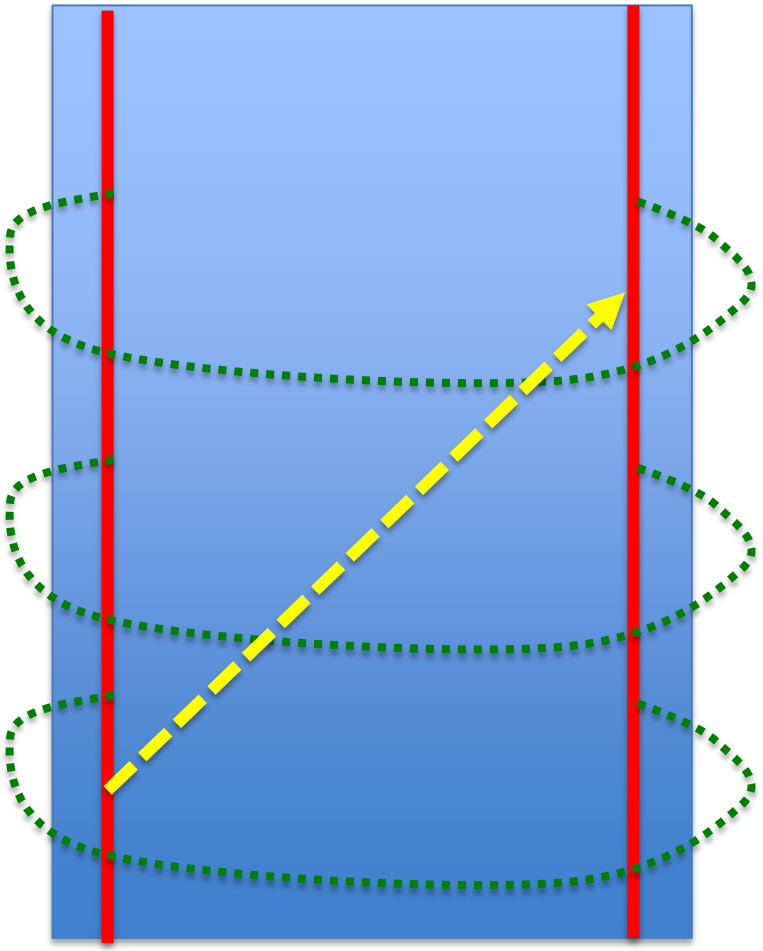
JM , Qi. See also: Cottrell, Freivogel, Hofman, Lokhande Martyn , Swingle Wu, Hsieh

- Keep the interaction for ever.

$$S_{int} = \mu \int du \phi_L(u) \phi_R(u)$$

- Get an "eternally" traversable wormhole → similar to the whole global AdS$_2$ spacetime.

NAdS$_2$ gravity + Interaction $H_L + H_R + H_{int}$



Interactions

Boundaries now move “straight up”

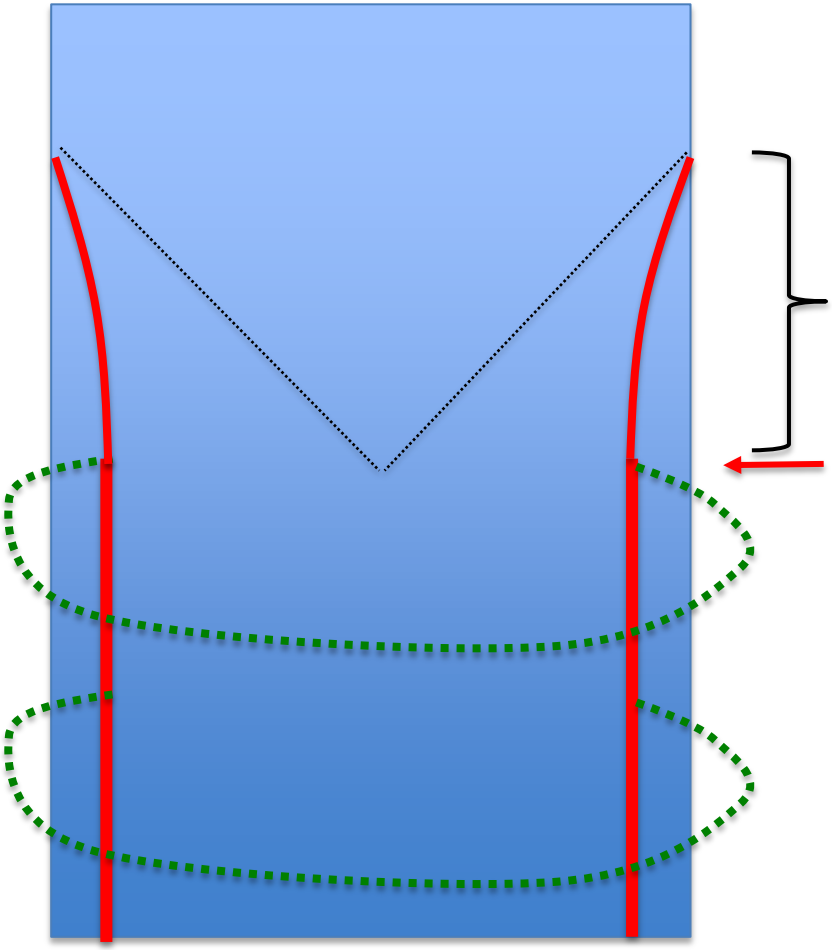
Signals can now propagate from one boundary to the other.

“Practical” method to make the TFD double

- Couple the two systems.
- Couple them weakly to a heat sink.
- Wait.
- Get to the ground state.

$$|G\rangle \sim |TFD\rangle$$

- If we turn off the interaction, it will evolve as before.



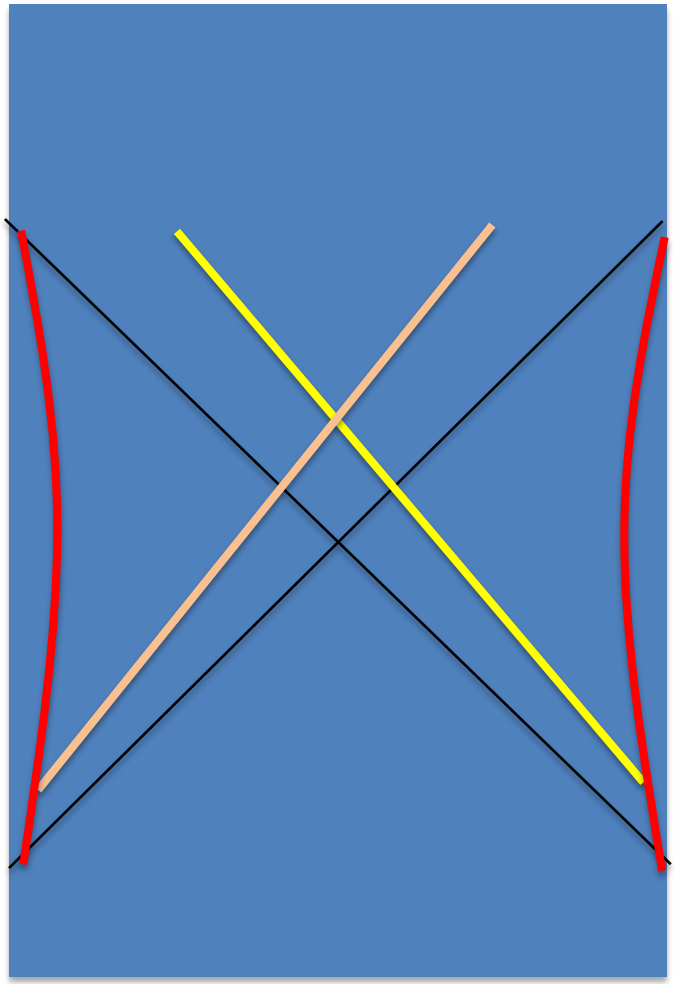
This part of the diagram represents the evolution of the TFD state with the decoupled Hamiltonian

Turn off the interaction at t=0

The full spacetime diagram represents the full evolution of the state.

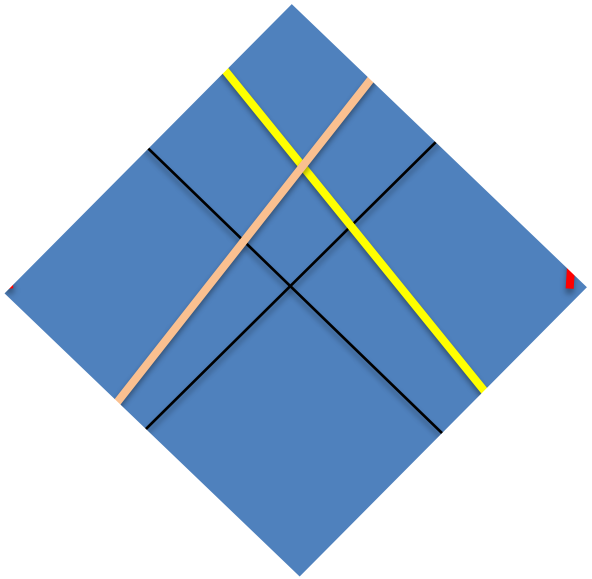
Interactions behind the horizon

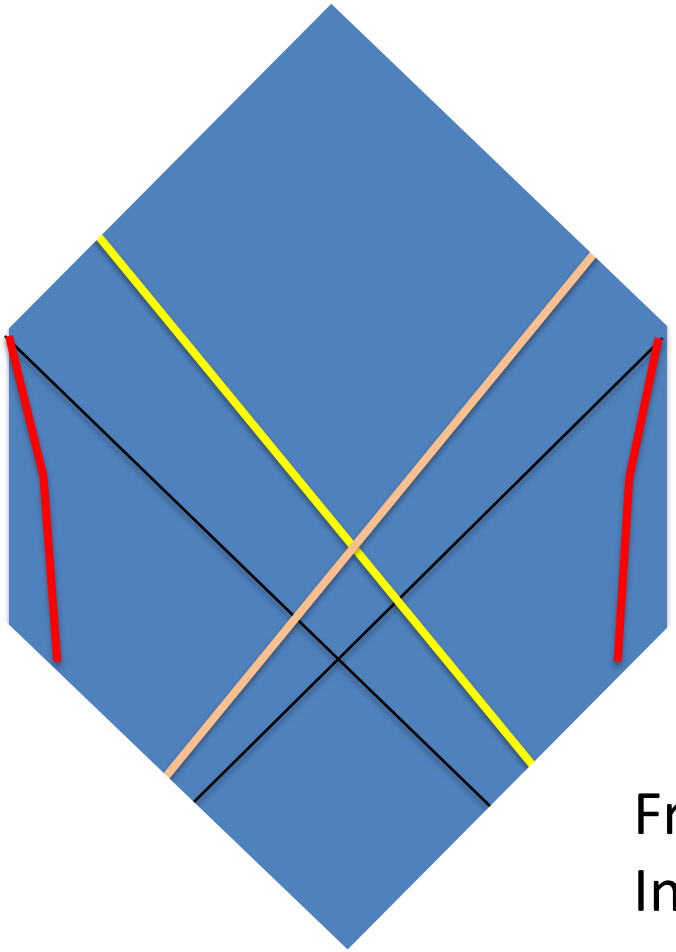
$$H = H_L + H_R$$



Evolve the TFD, backwards, insert some excitations with unitary operators.

We expect that the initial state is describing the Wheeler de Wit patch



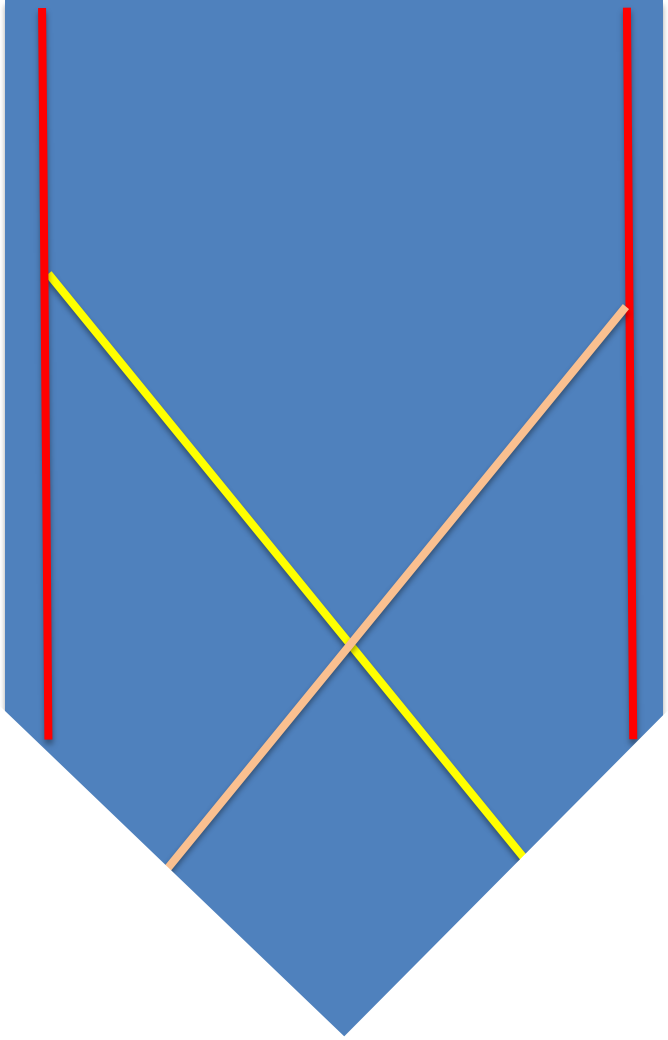


We can evolve it with the decoupled hamiltonian

$$H = H_L + H_R$$

From the boundary theory they should NOT Interact in any way.

Fortunately, their interaction is behind the horizon so we cannot see it from either boundary.



We can evolve it with the coupled hamiltonian

$$H = H_L + H_R + H_{int}$$

We can now see the interaction.

It is OK because the underlying Hamiltonian has an interaction between the two sides.

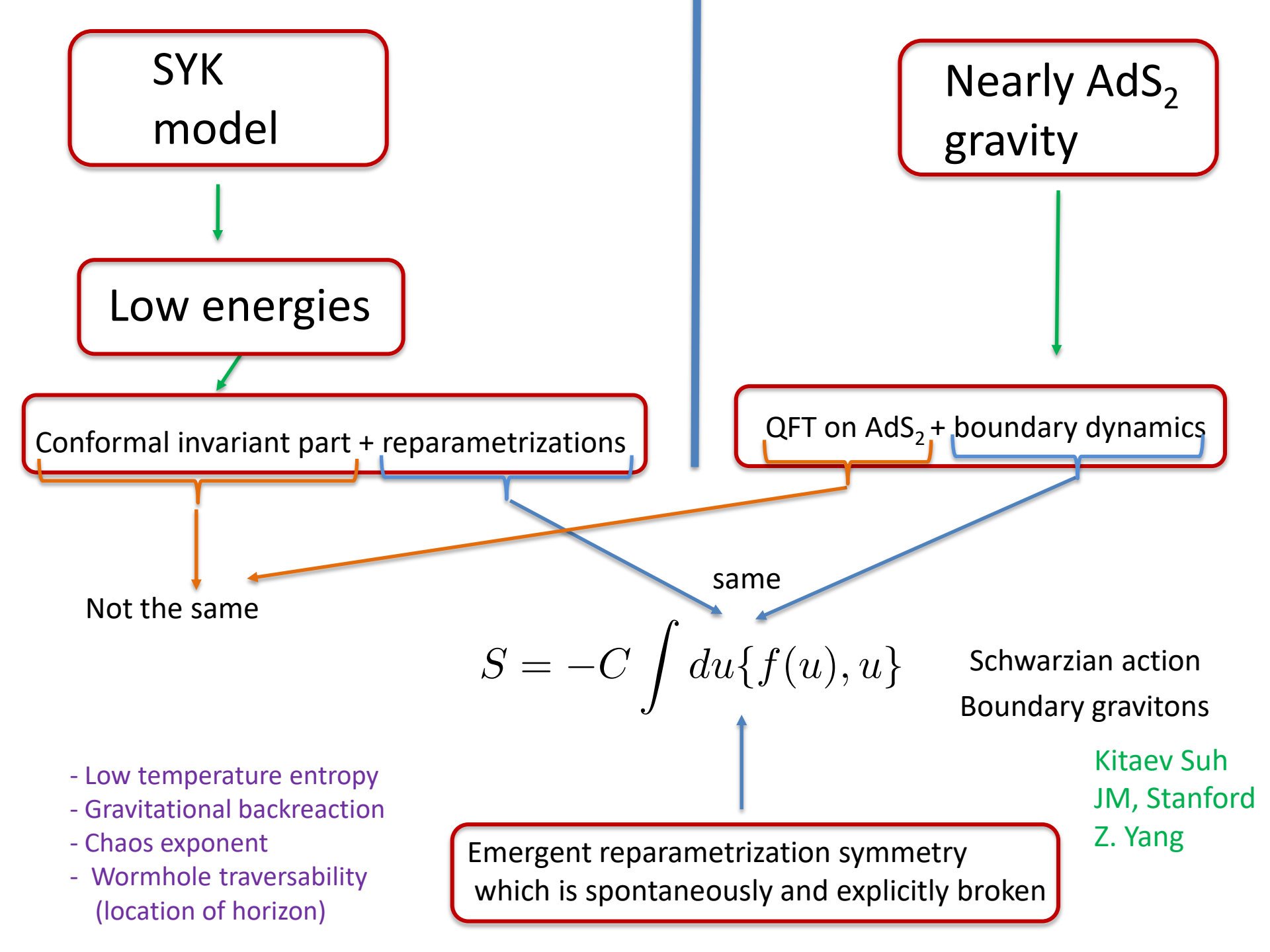
Makes that interaction behind the horizon more real. But always through the lens of a particular evolution.

Acting with two sided operators we can ~~see~~ create the interior

- The above discussion used gravity.
- Can the same process happen in a simple quantum mechanical model ?

- The SYK model !

What is the relation between SYK and AdS_2 ?

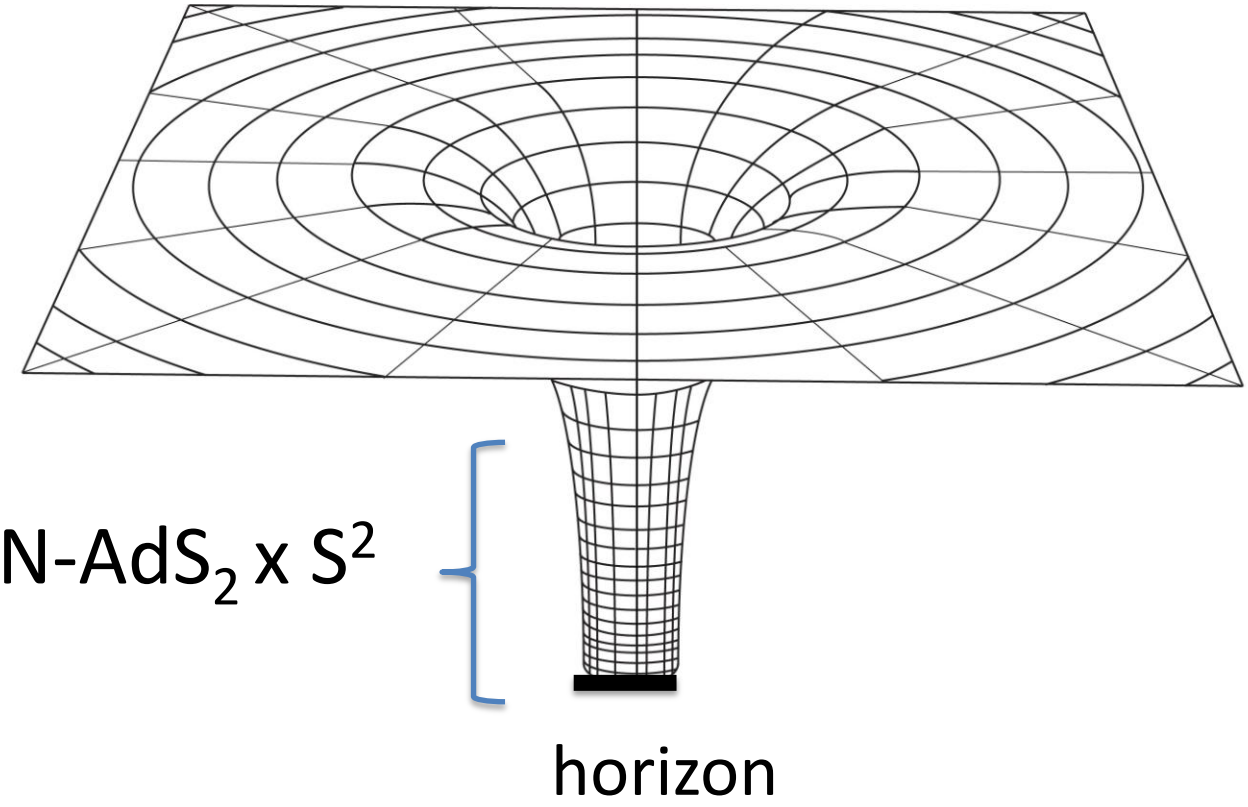

SYK model
Low energies
Conformal invariant part + reparametrizations
Nearly AdS2 gravity
QFT on AdS2 + boundary dynamics
Not the same
same
$S = -C \int du\{f(u), u\}$
Schwarzian action
Boundary gravitons
Kitaev Suh
JM, Stanford
Z. Yang
- Low temperature entropy
- Gravitational backreaction
- Chaos exponent
- Wormhole traversability (location of horizon)
Emergent reparametrization symmetry which is spontaneously and explicitly broken

What do these two dimensional black holes and wormholes have to do with our real four dimensional world ?

Near extremal black holes

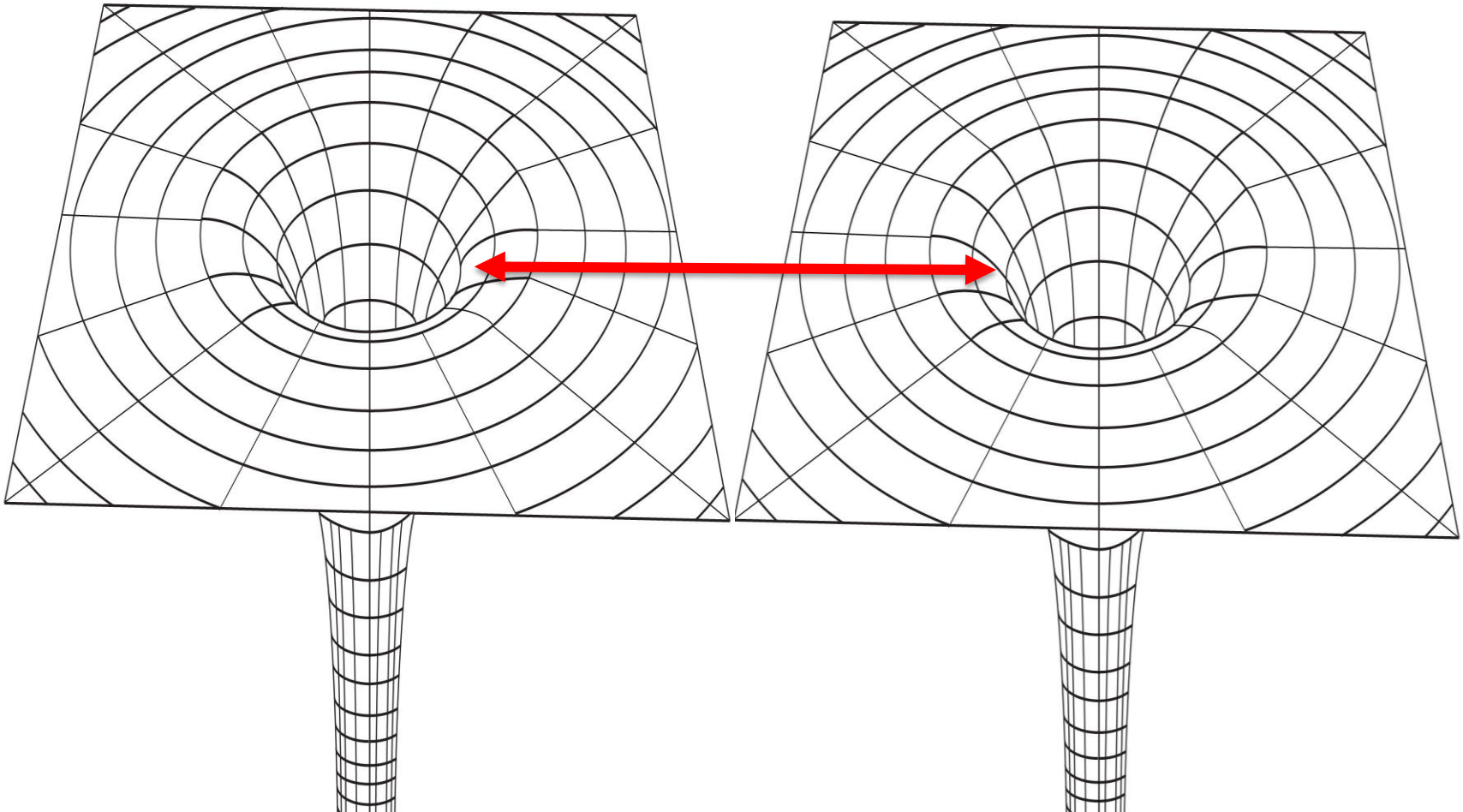
$$M \geq Q$$

$$M \sim Q$$



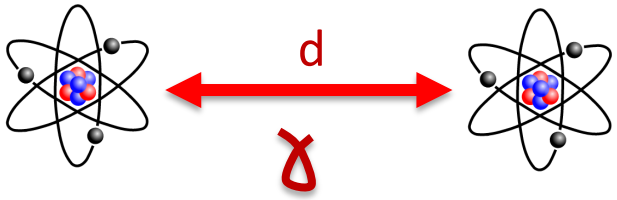
Interaction between two black holes ?

→ Get them relatively close together



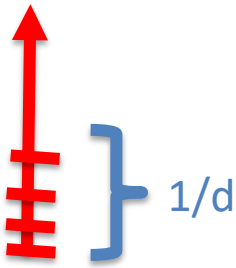
Analogy: Van der Waals interaction

Two neutral atoms exchanging photons.



$$H_{int} \propto \frac{\vec{d}_L . \vec{d}_R}{d^3}$$

d small enough so that 1/d is larger than the gap between the ground state and the next states.



Entangle the two atoms.

Here we get the following traversable wormhole geometry

JM, Milekhin, Popov

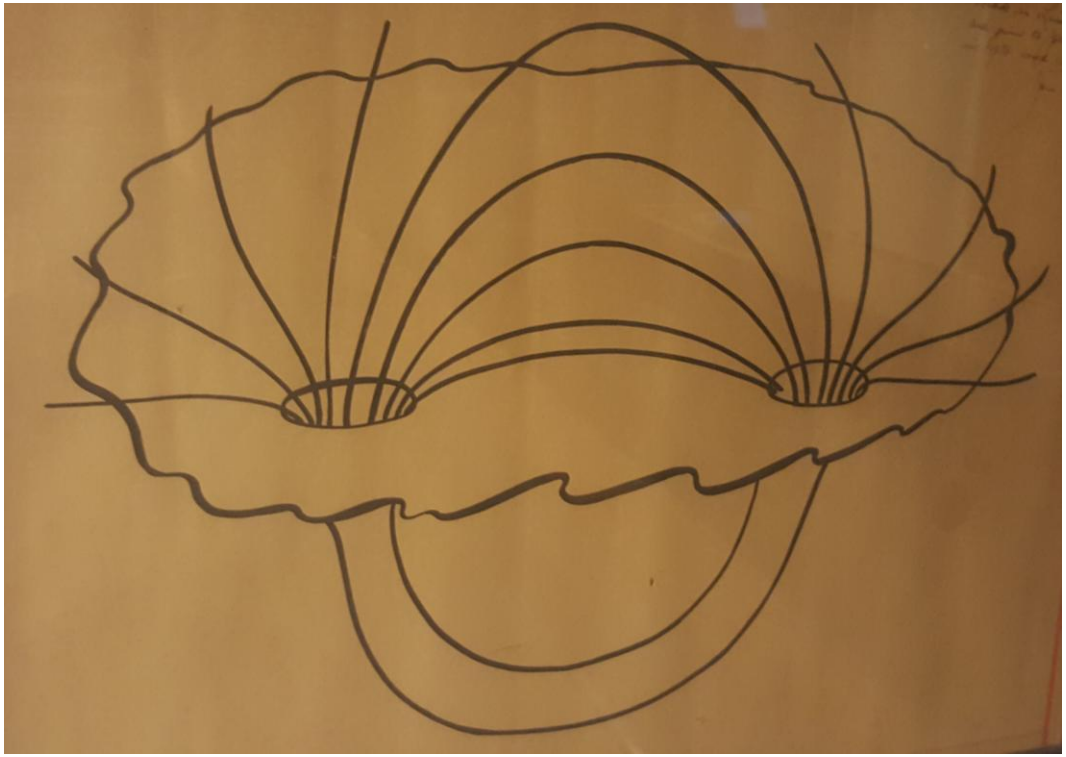


Figure: Wheeler 1966

The theory

$$S = \int d^4x \left[R - F^2 + i\bar{\psi} \not{D} \psi \right]$$

Einstein + U(1) gauge field + massless charged fermion

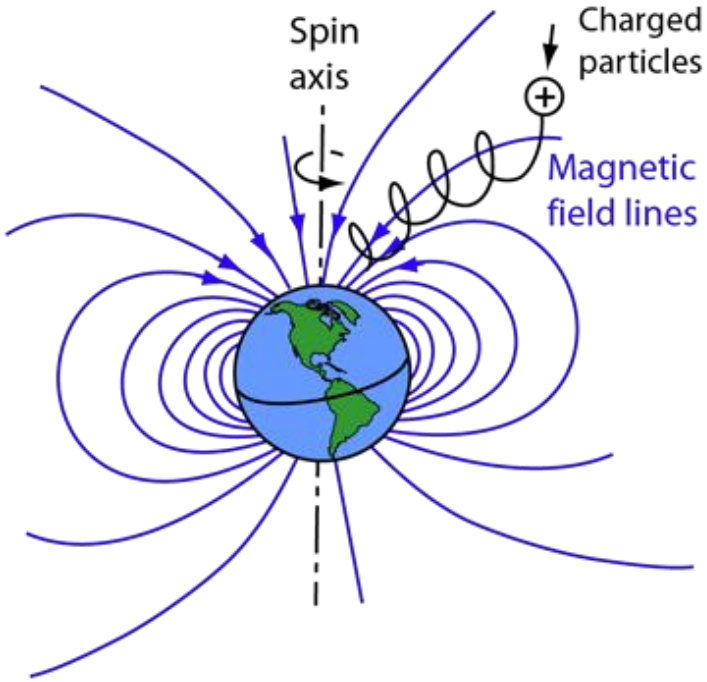
Could be the Standard Model at very small distances, with the fermions effectively massless. The U(1) is the hypercharge. SU(3) x SU(2) x U(1).

$l_{\rm Planck} = 1$

Black holes with opposite magnetic charge

4 d massless fermion →
set of two dimensional massless fermions along the field lines

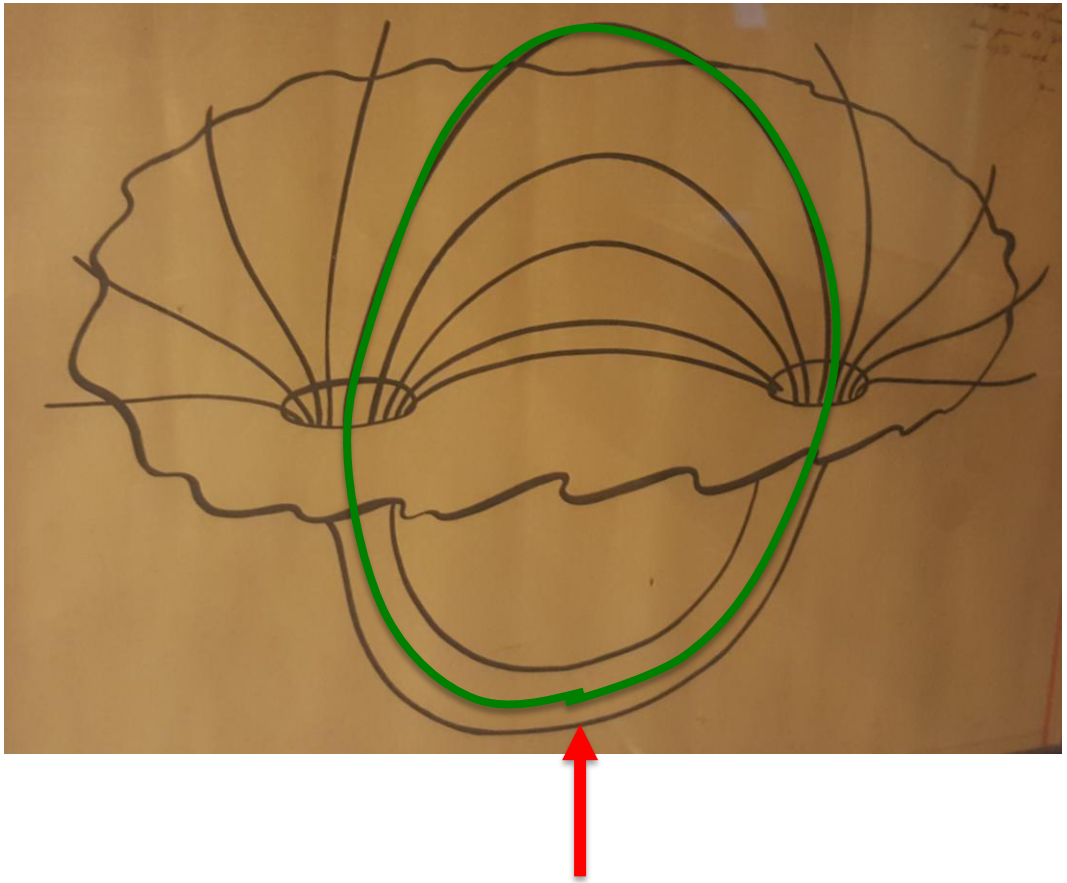
Similar to:



Joe's paper on strongly coupled systems in magnetic fields. (w/Almheiri)

Fermion trajectories

Positive magnetic charge



Negative magnetic charge

Charged fermion moves along this closed circle.

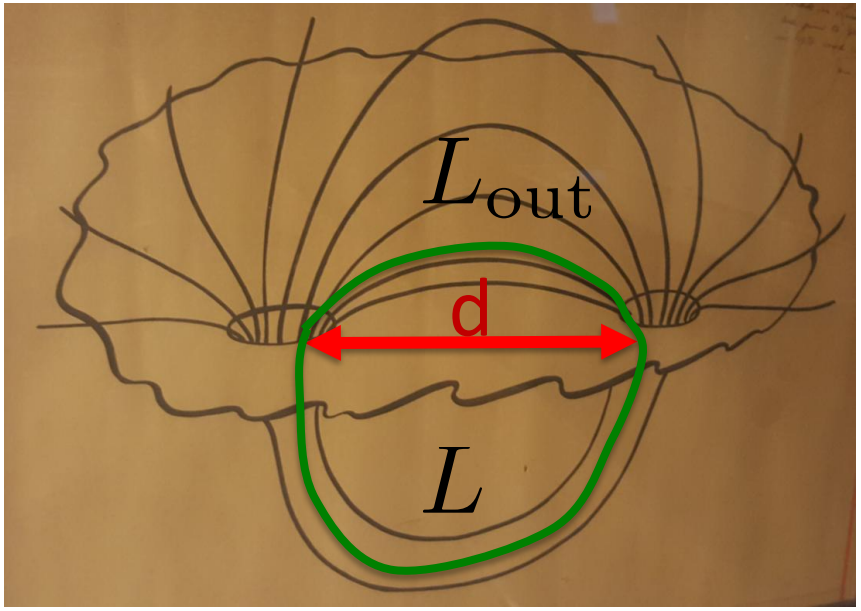
Casimir energy

Assume: "Length of the throat" is larger than the distance.

$$L \gg L_{out} > d$$

Casimir energy is of the order of

$$E \propto -\frac{q}{L}$$



Full energy also need to take into
Account the conformal anomaly because
AdS_2 has a warp factor.
That just changes the numerical factor.

- Including this energy in Einstein's equation → we find a solution with the wormhole shape.

- Has no horizon.

- Represents a pair of entangled black holes in a state similar to the TFD.

- To prevent the two mouths from falling into each other → make them orbit around each other.

- Gives a long lived state.

Question

- How quickly does the wormhole if we started from two separate black holes ?

- In a similar SYK model? → it seems that it happens in a time of order N^0 .

- In 4 dimensions → We don't know.

Entropy and entanglement

- Total spacetime has no entropy and no horizon.
- If we only look at one object → entanglement entropy = extremal black hole entropy

- Wormhole = two entangled black holes

- Total Hamiltonian $H = H_L + H_R + H_{\rm int}$

Generated by fermions in exterior

What is a black hole ?

- State ?
- State and some particular evolution law, which does permit us to access some region.

- Same state can be evolved in two ways:
  - Decoupled evolution → two black holes
  - Coupled evolution → traversable wormhole, no horizon.

Conclusions

- The traversable wormhole construction lets us explore the connected spacetime produced by an entangled state: the TFD.
- New light on the Hayden Preskill process.
- Inspiration to build traversable wormhole solutions in four dimensions.