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Kavli Institute for
Theoretical Physics

Adventures in the ALPs

Effective Lagrangians and Flavor Observables with Axions and Axion-Like Particles

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based on:
Martin Bauer, MN, Sophie Renner, Marvin Schnubel & Andrea Thamm
2012.12272 (Xmas paper → JHEP) and 2102.13112

Virtual KITP Program *New Physics from Precision at High Energies*
(March 8 — May 21, 2021)

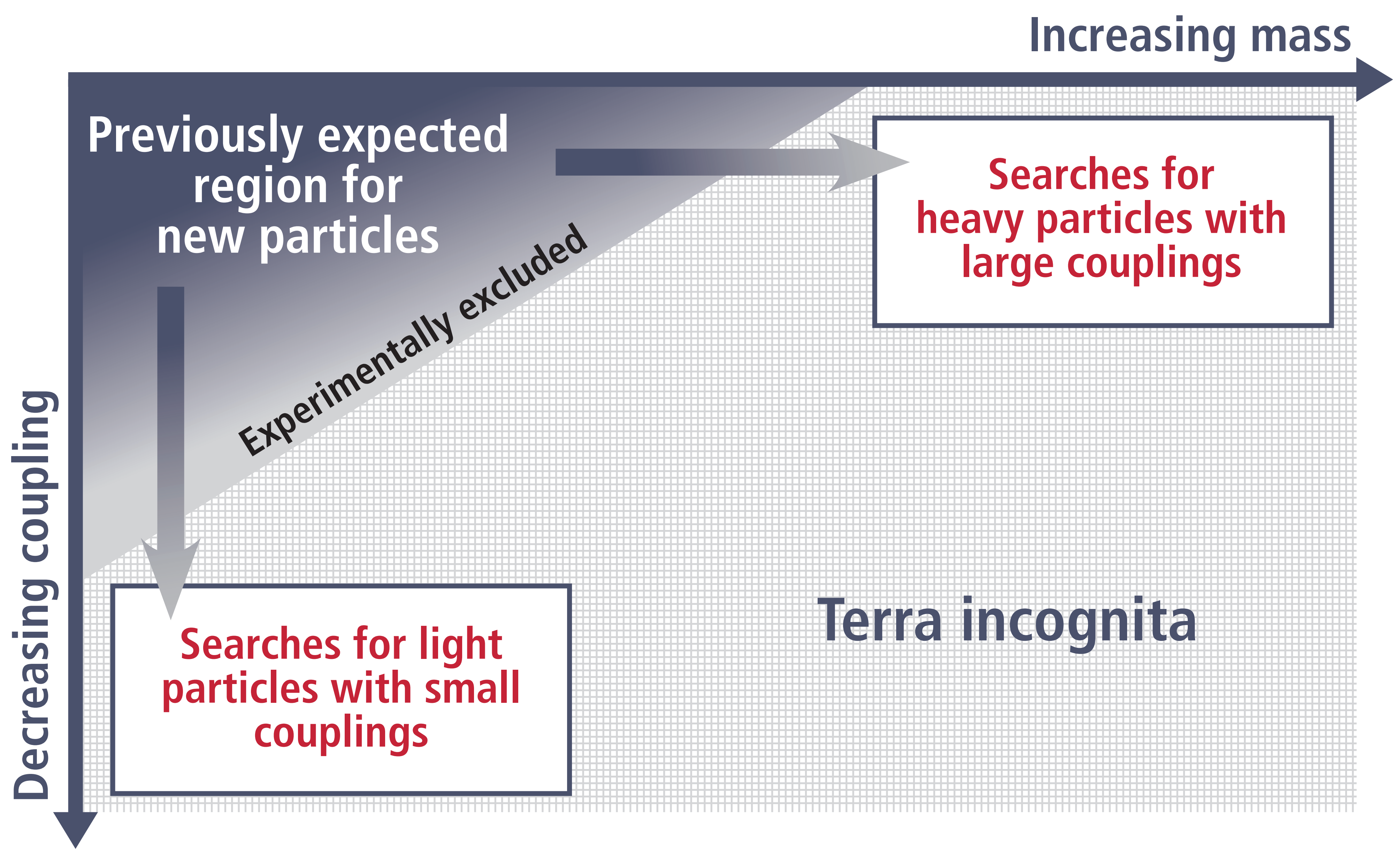
Outline

- M. Neubert — Effective ALP Lagrangians
  - Effective Lagrangian in the UV
  - Running to the weak scale and weak-scale matching
  - Running below the weak scale and matching to the chiral Lagrangian
  - Weak decays involving ALPs in the chiral Lagrangian
- Marvin Schnubel — Lepton flavor observables
- Andrea Thamm (March 25) — Quark flavor observables

Motivation

Axions and axion-like particles (ALPs) are well motivated theoretically:

- Peccei-Quinn solution to strong CP problem
  [Peccei, Quinn (1977); Weinberg (1978); Wilczek (1978)]
- ALPs as pseudo Nambu-Goldstone bosons
- Importance of low-energy processes in constraining ALP couplings
- Light but weakly-coupled new particles are an interesting alternative to heavy new particles and might provide hints about physics at energies scales out of the reach for direct searches at the LHC



Effective Lagrangian in the UV

Assume the scale of global symmetry breaking $\Lambda = 4\pi f$ is above the weak scale, and consider the most general effective Lagrangian for a pseudoscalar boson a coupled to the SM via classically shift-invariant interactions, broken only by a soft mass term: [Georgi, Kaplan, Randall (1986)]

hermitian matrices

$$\mathcal{L}_{\text{eff}}^{D\le 5} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + \frac{\partial^\mu a}{f}\sum_F \bar\psi_F \boldsymbol{c}_F \gamma_\mu \psi_F$$

$$+ c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G_{\mu\nu}^a\tilde G^{\mu\nu,a} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{f}W_{\mu\nu}^A\tilde W^{\mu\nu,A} + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{f}B_{\mu\nu}\tilde B^{\mu\nu}$$

Couplings to Higgs bosons only arise in higher orders: [Dobrescu, Landsberg, Matchev (2000); Bauer, MN, Thamm (2017)]

$$\mathcal{L}_{\text{eff}}^{D\ge 6} = \frac{C_{ah}}{f^2}(\partial_\mu a)(\partial^\mu a)\,\phi^\dagger\phi + \frac{C'_{ah}}{f^2}m_{a,0}^2\,a^2\phi^\dagger\phi + \frac{C_{Zh}}{f^3}(\partial^\mu a)\left(\phi^\dagger\, iD_\mu\,\phi + \text{h.c.}\right)\phi^\dagger\phi + \dots$$

A redundant operator

- The only possible dimension-5 coupling to the Higgs doublet

$$\mathcal{L}_{\rm eff}^{D\le 5} \supset c_\phi\, O_\phi = c_\phi\, \frac{\partial^\mu a}{f} \left(\phi^\dagger iD_\mu \phi + \text{h.c.} \right)$$

is a redundant operator, which can be removed by means of the field redefinitions $\phi \to e^{ic_\phi\, a/f}\, \phi$ and $F \to e^{-i\beta_F\, c_\phi\, a/f}\, F$ as long as:

$$\beta_u - \beta_Q = -1\,, \qquad \beta_d - \beta_Q = 1\,, \qquad \beta_e - \beta_L = 1$$

- This adds $\boldsymbol{c}_F \to \boldsymbol{c}_F + \beta_F\, c_\phi\, \mathbb{1}$ to the ALP-fermion couplings, i.e.:

$$O_\phi = \mathcal{O}_\phi + \sum_F \beta_F\, O_F\,, \qquad \text{with} \quad O_F = \frac{\partial^\mu a}{f}\, \bar\psi_F^i \gamma_\mu \psi_F^i$$

vanishes by the EOMs

Alternative operator basis

A useful alternative form of the Lagrangian involves non-derivative couplings:

$$\mathcal{L}_{\text{eff}}^{D\le 5} = \frac{1}{2}\,(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}\,a^2 - \frac{a}{f}\left(\bar{Q}\,\phi\,\tilde{\boldsymbol{Y}}_d\,d_R + \bar{Q}\,\tilde{\phi}\,\tilde{\boldsymbol{Y}}_u\,u_R + \bar{L}\,\phi\,\tilde{\boldsymbol{Y}}_e\,e_R + \text{h.c.}\right)$$

$$+\,\tilde{c}_{GG}\,\frac{\alpha_s}{4\pi}\,\frac{a}{f}\,G_{\mu\nu}^a\,\tilde{G}^{\mu\nu,a} + \tilde{c}_{WW}\,\frac{\alpha_2}{4\pi}\,\frac{a}{f}\,W_{\mu\nu}^A\,\tilde{W}^{\mu\nu,A} + \tilde{c}_{BB}\,\frac{\alpha_1}{4\pi}\,\frac{a}{f}\,B_{\mu\nu}\,\tilde{B}^{\mu\nu}$$

where:

[Bauer, MN, Renner, Schnubel, Thamm (2020)]

$$\tilde{\boldsymbol{Y}}_d = i\left(\boldsymbol{Y}_d\,\boldsymbol{c}_d - \boldsymbol{c}_Q\boldsymbol{Y}_d\right), \qquad \tilde{\boldsymbol{Y}}_u = i\left(\boldsymbol{Y}_u\,\boldsymbol{c}_u - \boldsymbol{c}_Q\boldsymbol{Y}_u\right), \qquad \tilde{\boldsymbol{Y}}_e = i\left(\boldsymbol{Y}_e\,\boldsymbol{c}_e - \boldsymbol{c}_L\boldsymbol{Y}_e\right)$$

$$\tilde{c}_{GG} = c_{GG} + T_F\,\text{Tr}\left(\boldsymbol{c}_u + \boldsymbol{c}_d - N_L\,\boldsymbol{c}_Q\right)$$

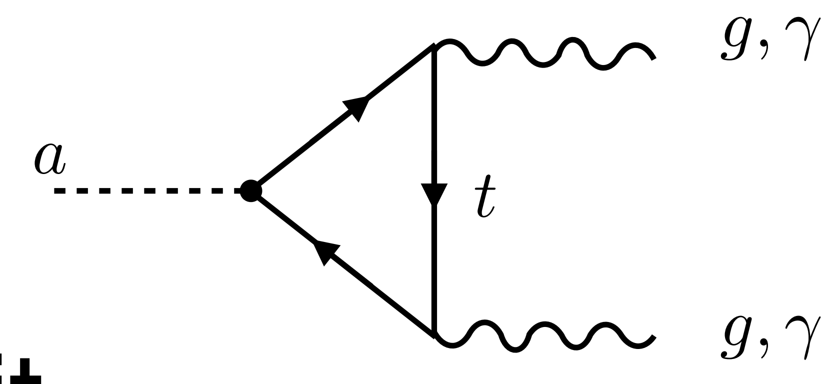
$$\tilde{c}_{WW} = c_{WW} - T_F\,\text{Tr}\left(N_c\,\boldsymbol{c}_Q + \boldsymbol{c}_L\right)$$

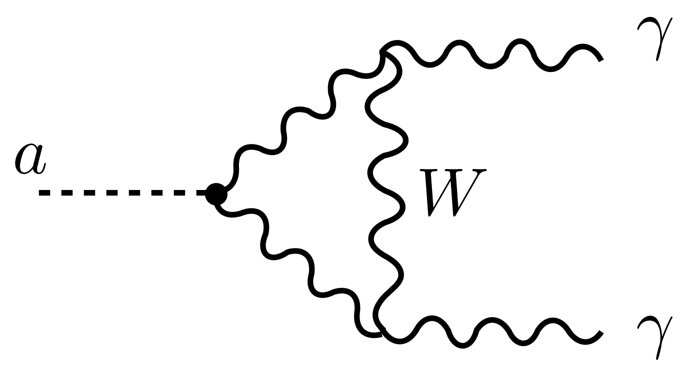
$$\tilde{c}_{BB} = c_{BB} + \text{Tr}\left[N_c\left(\mathcal{Y}_u^2\,\boldsymbol{c}_u + \mathcal{Y}_d^2\,\boldsymbol{c}_d - N_L\,\mathcal{Y}_Q^2\,\boldsymbol{c}_Q\right) + \mathcal{Y}_e^2\,\boldsymbol{c}_e - N_L\,\mathcal{Y}_L^2\,\boldsymbol{c}_L\right]$$

Alternative operator basis

Advantages of the original basis:

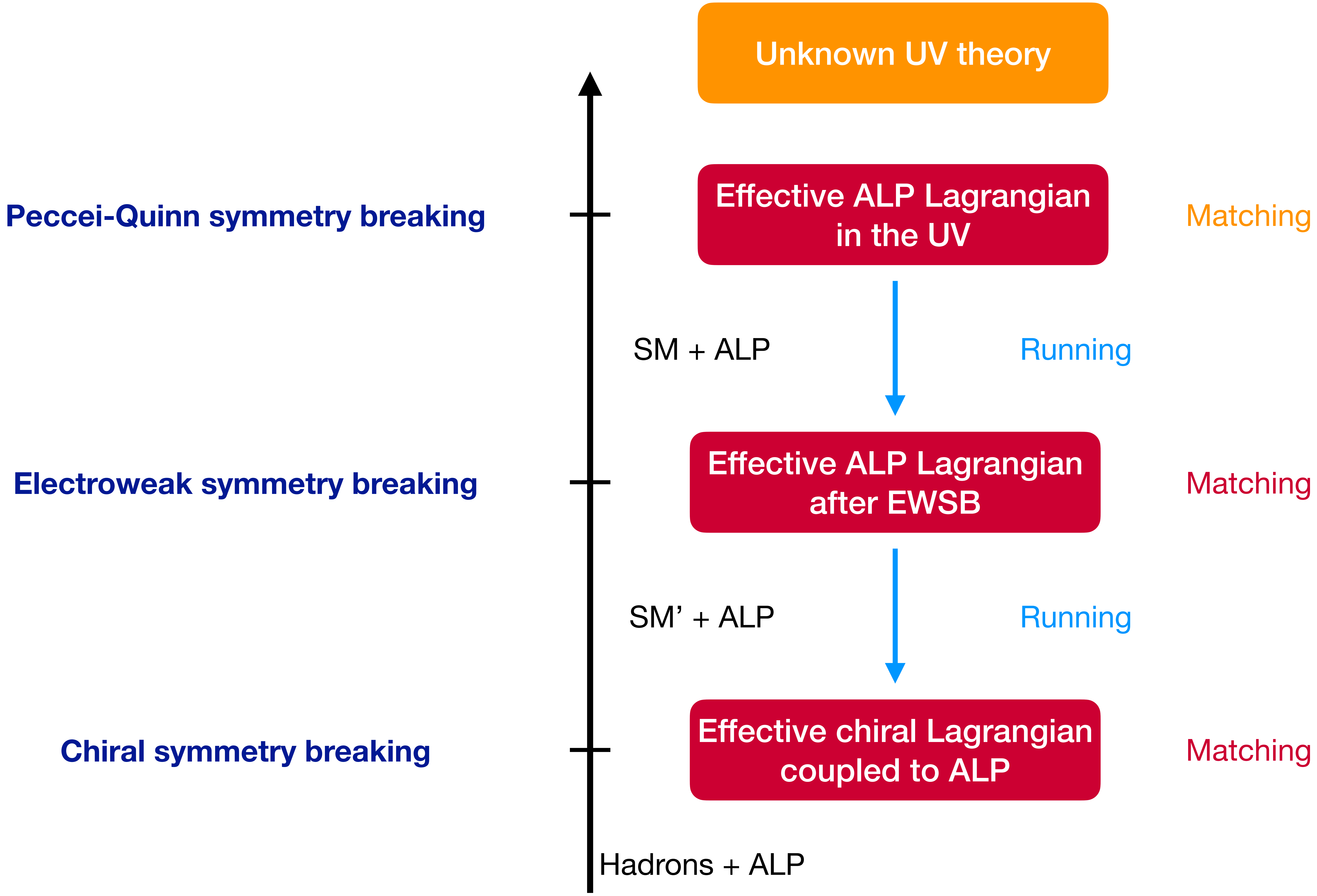
- shift symmetry is explicit
- heavy particles decouple from ALPs in loops, e.g.





- Yukawa suppression of fermion couplings is explicit
- fewer parameters in ALP-fermion couplings: 5 x 9 = 45 compared with 3 x 18 = 54 for generic matrices $\tilde{\boldsymbol{Y}}_d$, $\tilde{\boldsymbol{Y}}_u$ and $\tilde{\boldsymbol{Y}}_e$

Yet there are some redundancies, because the derivative ALP couplings are only defined modulo generators of exact global symmetries of SM (baryon and three lepton flavor numbers), which allows us to set e.g. $c_L^{ii} = 0$ and $c_Q^{33} = 0$ or $c_{WW} = 0$ (removes 4 parameters)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Peccei-Quinn symmetry breaking
Effective ALP Lagrangian in the UV
Matching
SM + ALP
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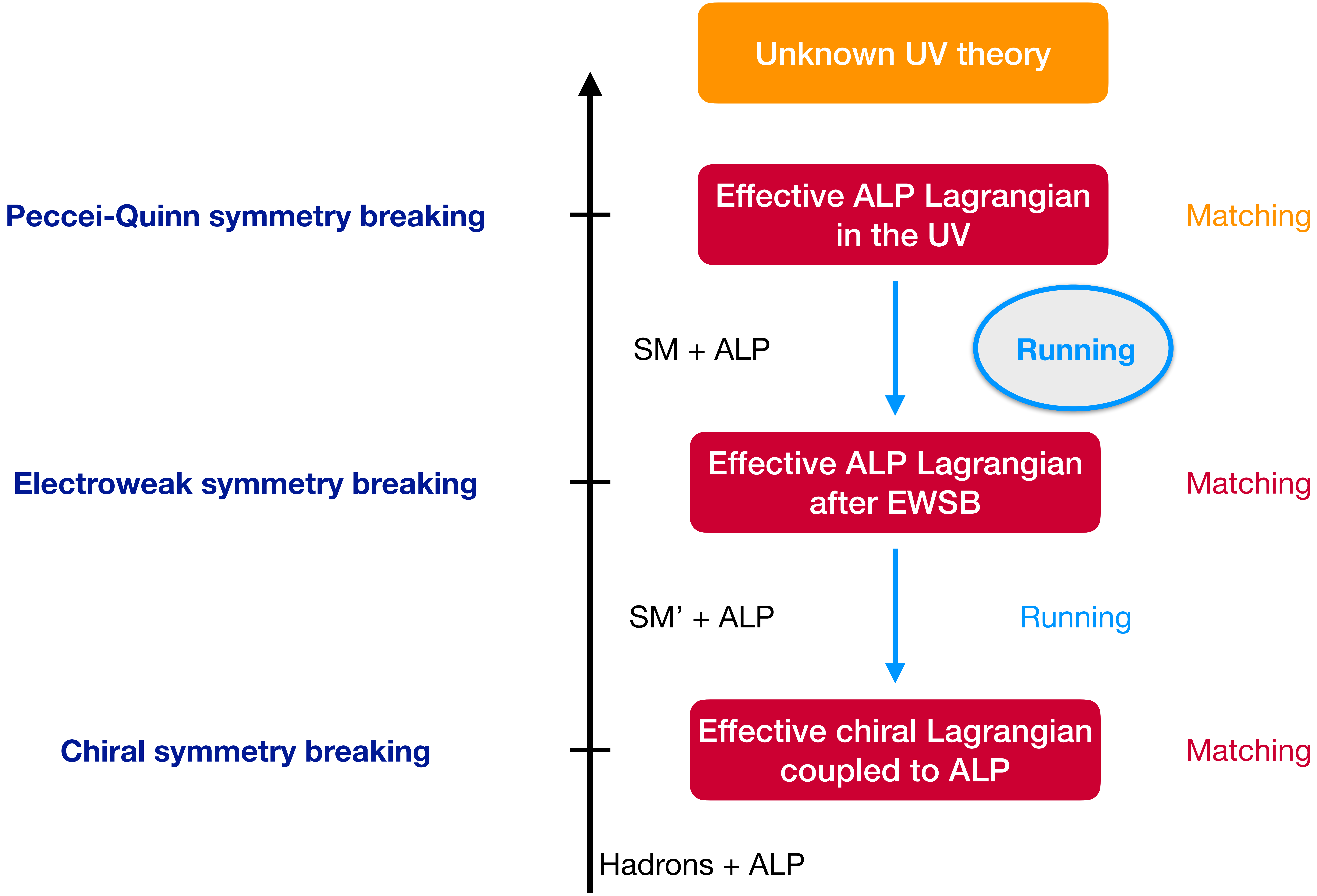
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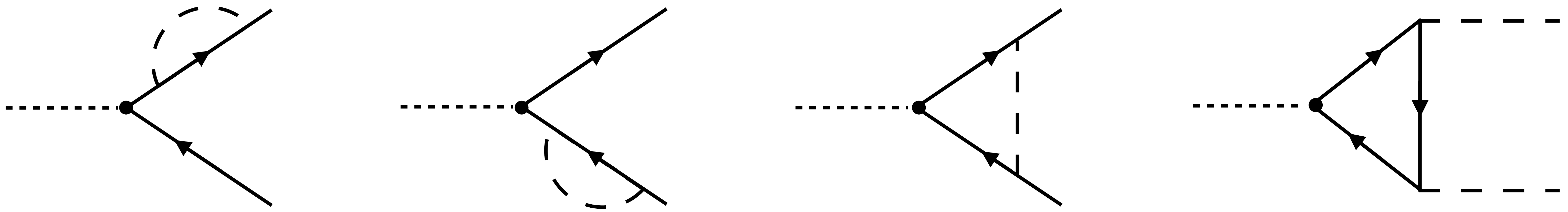
Evolution to the weak scale

Factoring out the gauge couplings from c_{VV} ensures that (at least to 2 loops):

$$\boxed{\frac{d}{d\ln\mu}\, c_{VV}(\mu) = 0\,; \quad V = G, W, B}$$

[Chetyrkin, Kniehl, Steinhauser, Bardeen (1998)]

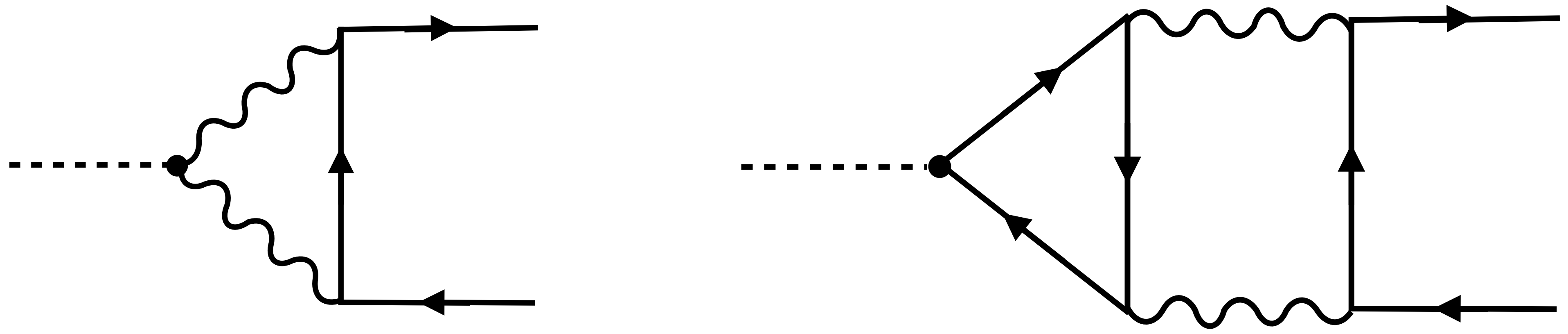
For the ALP-fermion couplings, we have computed:



1-loop Yukawa int.

[Choi, Im, Park, Yun (2017);
Martin Camalich, Pospelov, Vuong, Ziegler, Zupan (2020);
Heiles, König, MN (2020)]

requires the redundant Higgs operator as counterterm



2-loop gauge int.

[Altarelli, Ross (1988);
Chetyrkin, Kniehl, Steinhauser, Bardeen (1998)]

[Kodaira (1980); Larin (1993)]

Evolution to the weak scale

We find: [Bauer, MN, Renner, Schnubel, Thamm (2020); see also: Chala, Guedes, Ramos, Santiago (2020)]

$$\begin{aligned}\frac{d}{d\ln\mu}\,\boldsymbol{c}_Q(\mu) &= \frac{1}{32\pi^2}\left\{\boldsymbol{Y}_u\boldsymbol{Y}_u^\dagger+\boldsymbol{Y}_d\boldsymbol{Y}_d^\dagger,\boldsymbol{c}_Q\right\}-\frac{1}{16\pi^2}\left(\boldsymbol{Y}_u\,\boldsymbol{c}_u\boldsymbol{Y}_u^\dagger+\boldsymbol{Y}_d\,\boldsymbol{c}_d\boldsymbol{Y}_d^\dagger\right)\\ &\quad+\left[\boxed{\frac{\beta_Q}{8\pi^2}\,X}-\frac{3\alpha_s^2}{4\pi^2}\,C_F^{(3)}\,\tilde{c}_{GG}-\frac{3\alpha_2^2}{4\pi^2}\,C_F^{(2)}\,\tilde{c}_{WW}-\frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_Q^2\,\tilde{c}_{BB}\right]\mathbb{1}\end{aligned}$$

$$\frac{d}{d\ln\mu}\,\boldsymbol{c}_q(\mu)=\frac{1}{16\pi^2}\left\{\boldsymbol{Y}_q^\dagger\boldsymbol{Y}_q,\boldsymbol{c}_q\right\}-\frac{1}{8\pi^2}\,\boldsymbol{Y}_q^\dagger\boldsymbol{c}_Q\boldsymbol{Y}_q+\left[\boxed{\frac{\beta_q}{8\pi^2}\,X}+\frac{3\alpha_s^2}{4\pi^2}\,C_F^{(3)}\,\tilde{c}_{GG}+\frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_q^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

$$\frac{d}{d\ln\mu}\,\boldsymbol{c}_L(\mu)=\frac{1}{32\pi^2}\left\{\boldsymbol{Y}_e\boldsymbol{Y}_e^\dagger,\boldsymbol{c}_L\right\}-\frac{1}{16\pi^2}\,\boldsymbol{Y}_e\,\boldsymbol{c}_e\boldsymbol{Y}_e^\dagger+\left[\boxed{\frac{\beta_L}{8\pi^2}\,X}-\frac{3\alpha_2^2}{4\pi^2}\,C_F^{(2)}\,\tilde{c}_{WW}-\frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_L^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

$$\frac{d}{d\ln\mu}\,\boldsymbol{c}_e(\mu)=\frac{1}{16\pi^2}\left\{\boldsymbol{Y}_e^\dagger\boldsymbol{Y}_e,\boldsymbol{c}_e\right\}-\frac{1}{8\pi^2}\,\boldsymbol{Y}_e^\dagger\boldsymbol{c}_L\boldsymbol{Y}_e+\left[\boxed{\frac{\beta_e}{8\pi^2}\,X}+\frac{3\alpha_1^2}{4\pi^2}\,\mathcal{Y}_e^2\,\tilde{c}_{BB}\right]\mathbb{1}$$

with:

$$X=\mathrm{Tr}\left[3\boldsymbol{c}_Q\left(\boldsymbol{Y}_u\boldsymbol{Y}_u^\dagger-\boldsymbol{Y}_d\boldsymbol{Y}_d^\dagger\right)-3\boldsymbol{c}_u\boldsymbol{Y}_u^\dagger\boldsymbol{Y}_u+3\boldsymbol{c}_d\boldsymbol{Y}_d^\dagger\boldsymbol{Y}_d-\boldsymbol{c}_L\boldsymbol{Y}_e\boldsymbol{Y}_e^\dagger+\boldsymbol{c}_e\boldsymbol{Y}_e^\dagger\boldsymbol{Y}_e\right]$$

Lagrangian at the weak scale

Effective Lagrangian in the broken phase:

$$\mathcal{L}_{\text{eff}}(\mu_w) = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + \mathcal{L}_{\text{ferm}}(\mu_w) + c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G_{\mu\nu}^a\tilde{G}^{\mu\nu,a} + c_{\gamma\gamma}\frac{\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$+ c_{\gamma Z}\frac{\alpha}{2\pi s_w c_w}\frac{a}{f}F_{\mu\nu}\tilde{Z}^{\mu\nu} + c_{ZZ}\frac{\alpha}{4\pi s_w^2 c_w^2}\frac{a}{f}Z_{\mu\nu}\tilde{Z}^{\mu\nu} + c_{WW}\frac{\alpha}{2\pi s_w^2}\frac{a}{f}W_{\mu\nu}^+\tilde{W}^{-\mu\nu}$$

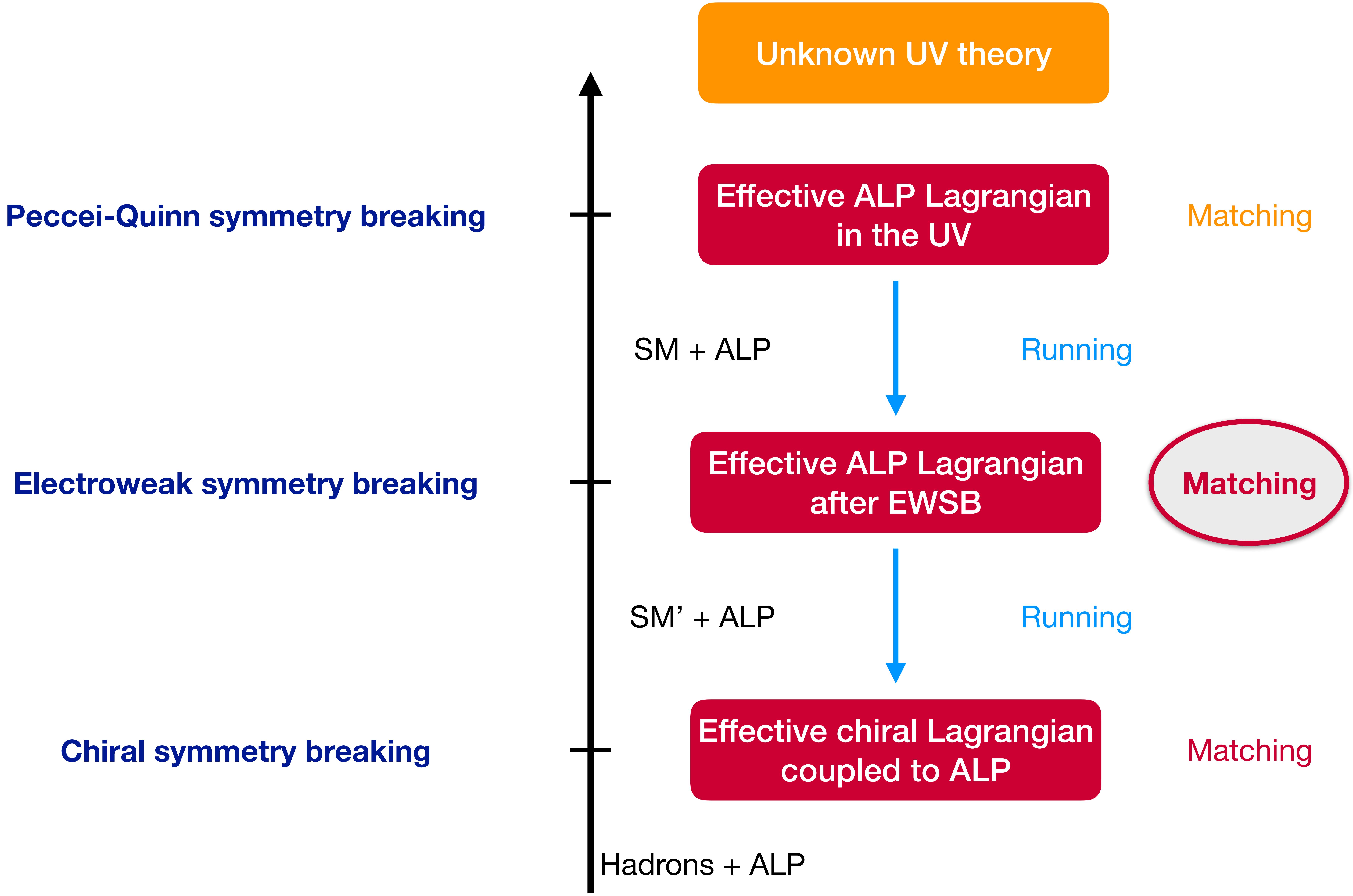
with:

matrices $\boldsymbol{c}_Q$, $\boldsymbol{c}_u$ etc. rotated to the mass basis

$$\mathcal{L}_{\text{ferm}}(\mu_w) = \frac{\partial^\mu a}{f}\Big[\bar{u}_L\boldsymbol{k}_U\gamma_\mu u_L + \bar{u}_R\boldsymbol{k}_u\gamma_\mu u_R + \bar{d}_L\boldsymbol{k}_D\gamma_\mu d_L + \bar{d}_R\boldsymbol{k}_d\gamma_\mu d_R$$

$$+ \bar{\nu}_L\boldsymbol{k}_\nu\gamma_\mu\nu_L + \bar{e}_L\boldsymbol{k}_E\gamma_\mu e_L + \bar{e}_R\boldsymbol{k}_e\gamma_\mu e_R\Big]$$

In the next step, we integrate out the heavy particles *t*, *W*, *Z* and *h*.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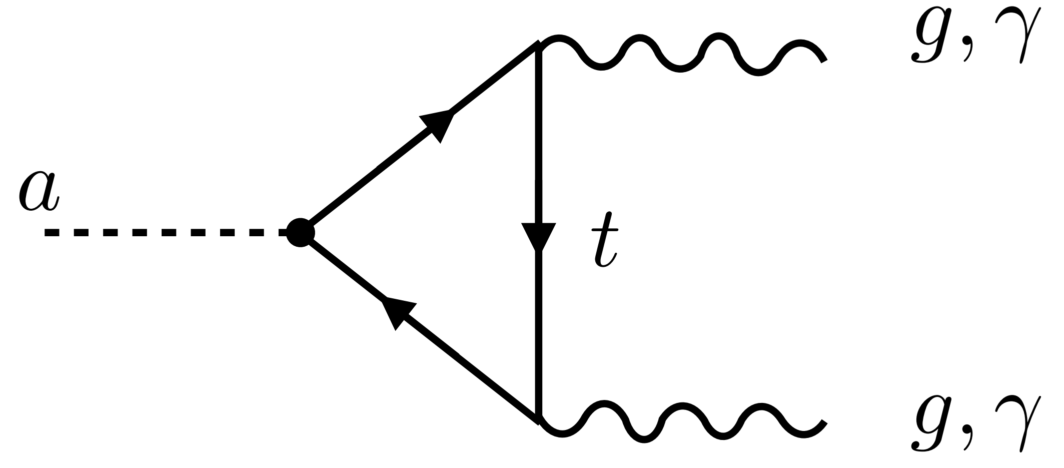
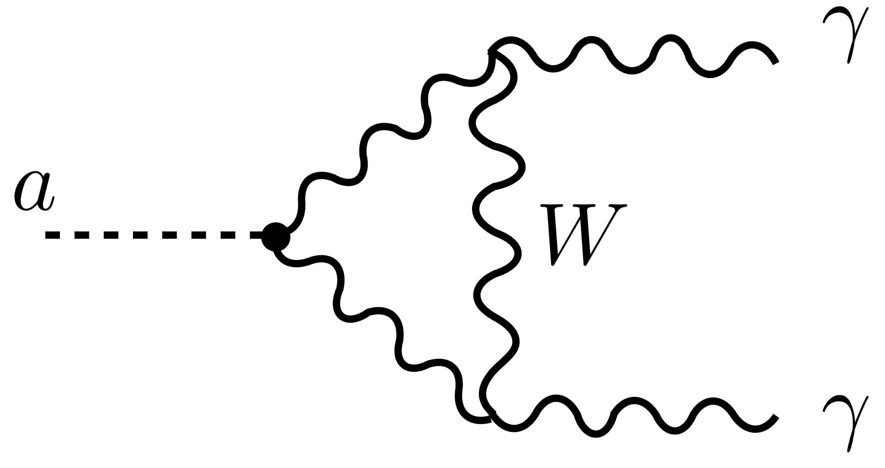
Weak-scale matching

Matching contributions to the ALP-boson couplings are absent in the standard basis:

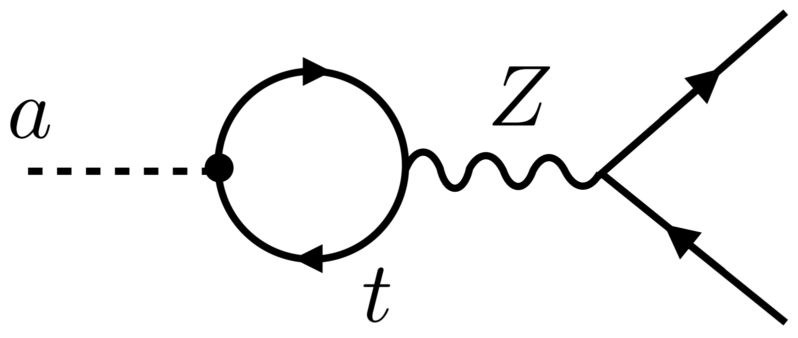


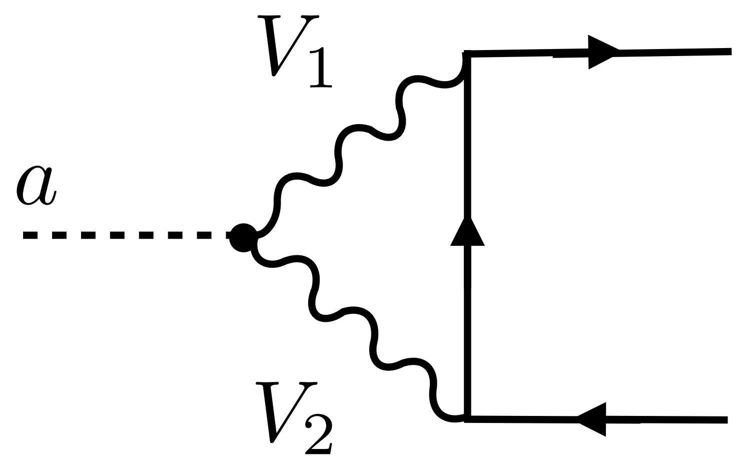


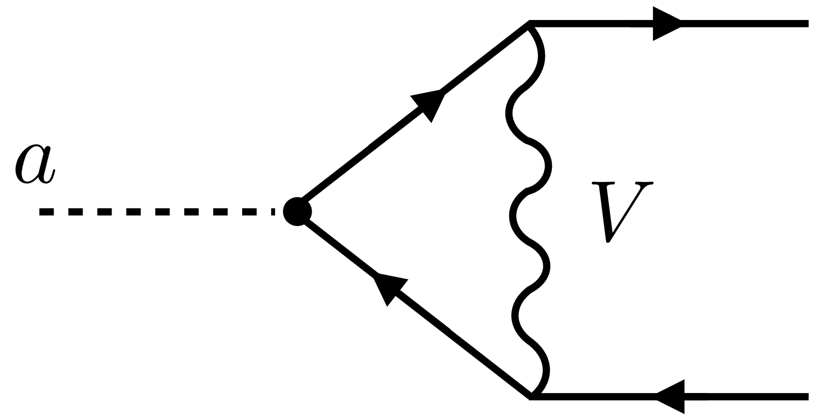
$$\sim \frac{m_a^2}{m_t^2}\,, \quad \frac{m_a^2}{m_W^2}$$

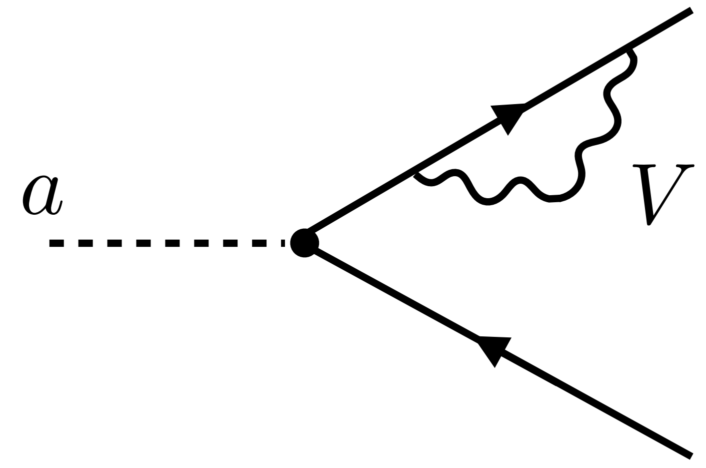
[Bauer, MN, Thamm (2017)]

but there are non-trivial matching conditions to the ALP-fermion couplings:





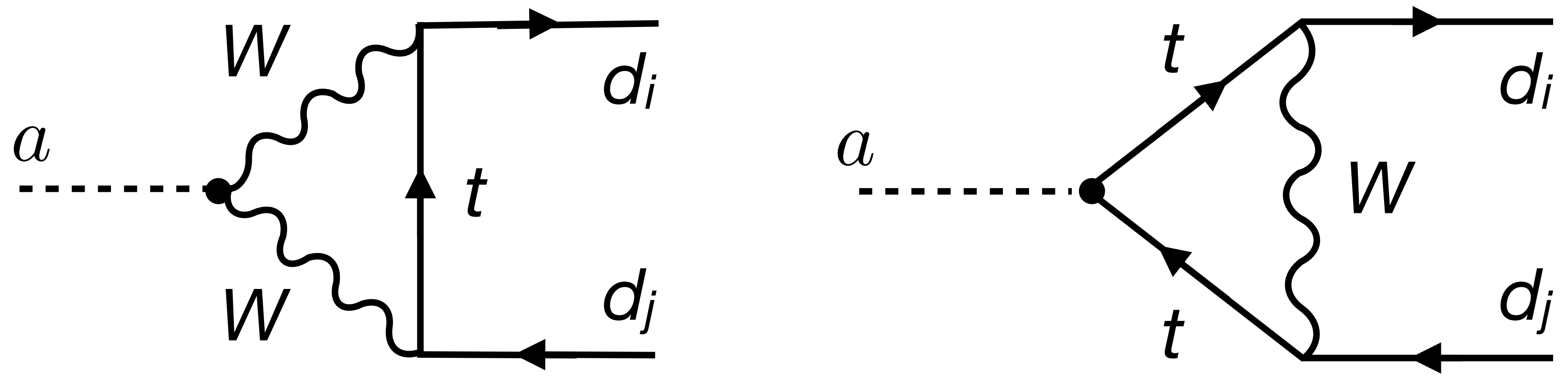




[Bauer, MN, Thamm (2017);
Bauer, MN, Renner, Schnubel, Thamm (2020)]

Weak-scale matching

These include, in particular, flavor-violating contributions to k_D:



$$\left[\hat{\Delta} k_D(\mu_w)\right]_{ij} = \frac{y_t^2}{16\pi^2} \bigg\{ V_{mi}^* V_{nj} \left[k_U(\mu_w)\right]_{mn} (\delta_{m3} + \delta_{n3}) \left[-\frac{1}{4} \ln \frac{\mu_w^2}{m_t^2} - \frac{3}{8} + \frac{3}{4} \frac{1 - x_t + \ln x_t}{(1-x_t)^2} \right]$$

$$+ V_{3i}^* V_{3j} \left[k_U(\mu_w)\right]_{33} + V_{3i}^* V_{3j} \left[k_u(\mu_w)\right]_{33} \left[\frac{1}{2} \ln \frac{\mu_w^2}{m_t^2} - \frac{1}{4} - \frac{3}{2} \frac{1 - x_t + \ln x_t}{(1-x_t)^2} \right]$$

$$- \frac{3\alpha}{2\pi s_w^2} c_{WW} V_{3i}^* V_{3j} \frac{1 - x_t + x_t \ln x_t}{(1-x_t)^2} \bigg\}$$

[Bauer, MN, Renner, Schnubel, Thamm (2020)]

ALP couplings at the weak scale

Results for the flavor-diagonal couplings with $f = 1$ TeV and $\mu_w = m_t$:

$$\boxed{\mathcal{L}_{\text{ferm}}^{\text{diag}}(\mu) = \sum_{f\neq t} \frac{c_{ff}(\mu)}{2} \frac{\partial^\mu a}{f} \bar{f}\gamma_\mu\gamma_5 f}$$

with $\quad c_{f_if_i}(\mu) = [k_f(\mu)]_{ii} - [k_F(\mu)]_{ii}$

We find:

$$c_{uu,cc}(\mu_w) \simeq c_{uu,cc}(\Lambda) - 0.116\, c_{tt}(\Lambda) - \left[6.35\, \tilde{c}_{GG}(\Lambda) + 0.19\, \tilde{c}_{WW}(\Lambda) + 0.02\, \tilde{c}_{BB}(\Lambda) \right] \cdot 10^{-3}$$

$$c_{dd,ss}(\mu_w) \simeq c_{dd,ss}(\Lambda) + 0.116\, c_{tt}(\Lambda) - \left[7.08\, \tilde{c}_{GG}(\Lambda) + 0.22\, \tilde{c}_{WW}(\Lambda) + 0.005\, \tilde{c}_{BB}(\Lambda) \right] \cdot 10^{-3}$$

$$c_{bb}(\mu_w) \simeq c_{bb}(\Lambda) + 0.097\, c_{tt}(\Lambda) - \left[7.02\, \tilde{c}_{GG}(\Lambda) + 0.19\, \tilde{c}_{WW}(\Lambda) + 0.005\, \tilde{c}_{BB}(\Lambda) \right] \cdot 10^{-3}$$

$$c_{e_ie_i}(\mu_w) \simeq c_{e_ie_i}(\Lambda) + 0.116\, c_{tt}(\Lambda) - \left[0.37\, \tilde{c}_{GG}(\Lambda) + 0.22\, \tilde{c}_{WW}(\Lambda) + 0.05\, \tilde{c}_{BB}(\Lambda) \right] \cdot 10^{-3}$$

ALP couplings at the weak scale

Corresponding results with $f = 10^9$ TeV:

$$c_{uu,cc}(m_t) \simeq c_{uu,cc}(\Lambda) - 0.350\, c_{tt}(\Lambda) - \Big[12.6\,\tilde{c}_{GG}(\Lambda) + 0.84\,\tilde{c}_{WW}(\Lambda) + 0.10\,\tilde{c}_{BB}(\Lambda)\Big] \cdot 10^{-3}$$

$$c_{dd,ss}(m_t) \simeq c_{dd,ss}(\Lambda) + 0.353\, c_{tt}(\Lambda) - \Big[16.8\,\tilde{c}_{GG}(\Lambda) + 1.30\,\tilde{c}_{WW}(\Lambda) + 0.07\,\tilde{c}_{BB}(\Lambda)\Big] \cdot 10^{-3}$$

$$c_{bb}(m_t) \simeq c_{bb}(\Lambda) + 0.294\, c_{tt}(\Lambda) - \Big[16.5\,\tilde{c}_{GG}(\Lambda) + 1.23\,\tilde{c}_{WW}(\Lambda) + 0.06\,\tilde{c}_{BB}(\Lambda)\Big] \cdot 10^{-3}$$

$$c_{e_ie_i}(m_t) \simeq c_{e_ie_i}(\Lambda) + 0.352\, c_{tt}(\Lambda) - \Big[2.09\,\tilde{c}_{GG}(\Lambda) + 1.30\,\tilde{c}_{WW}(\Lambda) + 0.38\,\tilde{c}_{BB}(\Lambda)\Big] \cdot 10^{-3}$$

Note that **all ALP couplings** enter via the matching conditions:

$$\boxed{\begin{aligned} \tilde{c}_{GG} &= c_{GG} + T_F\,\mathrm{Tr}\,(\boldsymbol{c}_u + \boldsymbol{c}_d - N_L\,\boldsymbol{c}_Q)\,, \\ \tilde{c}_{WW} &= c_{WW} - T_F\,\mathrm{Tr}\,(N_c\,\boldsymbol{c}_Q + \boldsymbol{c}_L)\,, \\ \tilde{c}_{BB} &= c_{BB} + \mathrm{Tr}\Big[N_c\,(\mathcal{Y}_u^2\,\boldsymbol{c}_u + \mathcal{Y}_d^2\,\boldsymbol{c}_d - N_L\,\mathcal{Y}_Q^2\,\boldsymbol{c}_Q) + \mathcal{Y}_e^2\,\boldsymbol{c}_e - N_L\,\mathcal{Y}_L^2\,\boldsymbol{c}_L\Big] \end{aligned}}$$

ALP couplings at the weak scale

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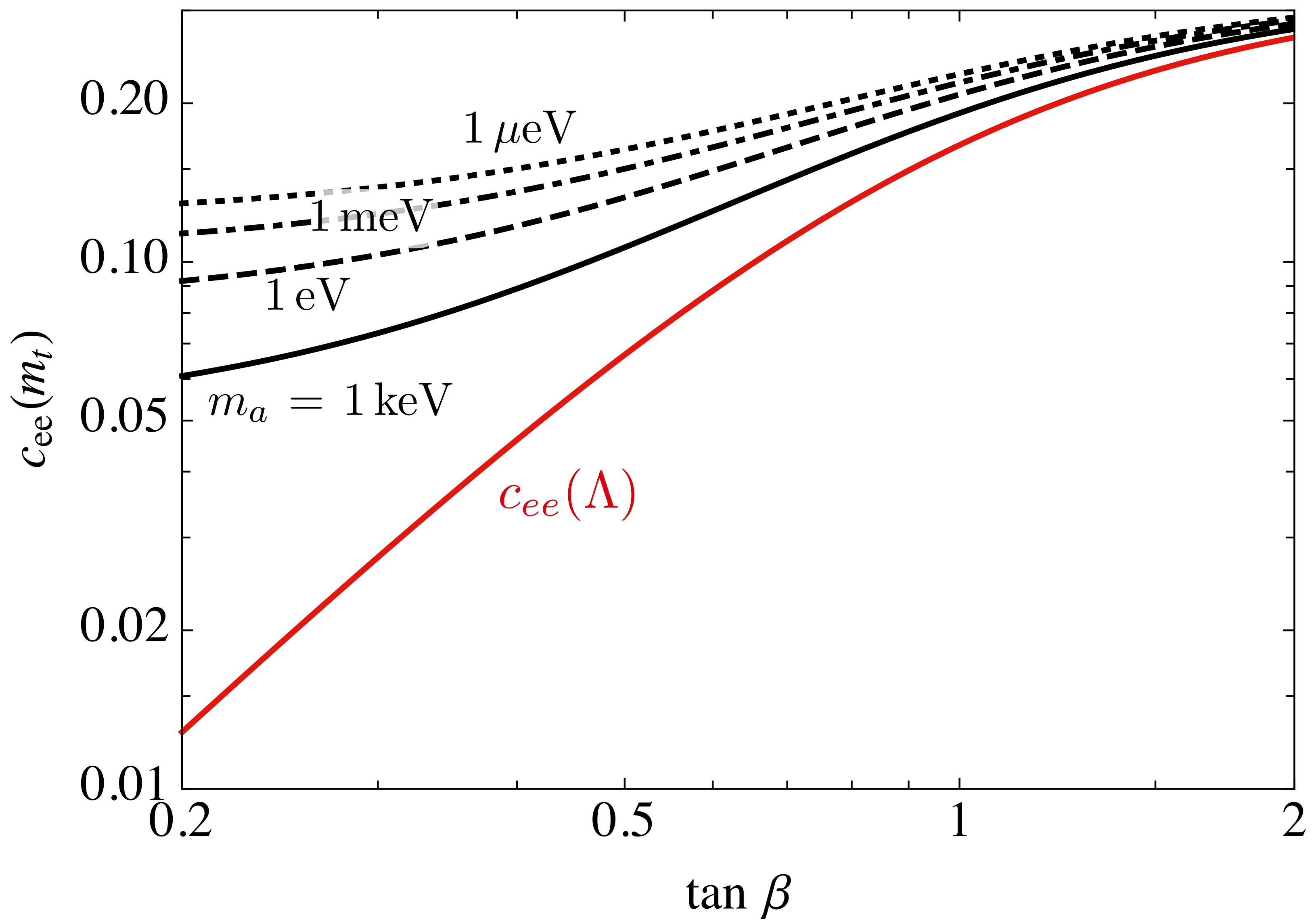
The one-loop admixture of c_{tt} into all ALP-fermion couplings can have a very important effect, since it induces an ALP-lepton coupling even in lepto-phobic ALP models



ALP-electron coupling in the DFSZ model for different values of $\tan\beta = v_u/v_d$

[Bauer, MN, Renner, Schnubel, Thamm (2020)]

ALP couplings at the weak scale

Flavor off-diagonal coefficients with $f = 1$ TeV and $\mu_w = m_t$:

$$\boxed{\mathcal{L}_{\text{ferm}}^{\text{FCNC}}(\mu) = -\frac{ia}{2f} \sum_f \Big[(m_{f_i} - m_{f_j})\,(k_f + k_F)_{ij}\, \bar{f}_i f_j + (m_{f_i} + m_{f_j})\,(k_f - k_F)_{ij}\, \bar{f}_i \gamma_5 f_j \Big]}$$

with:

$$[k_u(\mu_w)]_{ij} = [k_u(\Lambda)]_{ij}\,; \quad i,j \neq 3\,,$$

$$[k_U(\mu_w)]_{ij} = [k_U(\Lambda)]_{ij}\,; \quad i,j \neq 3\,,$$

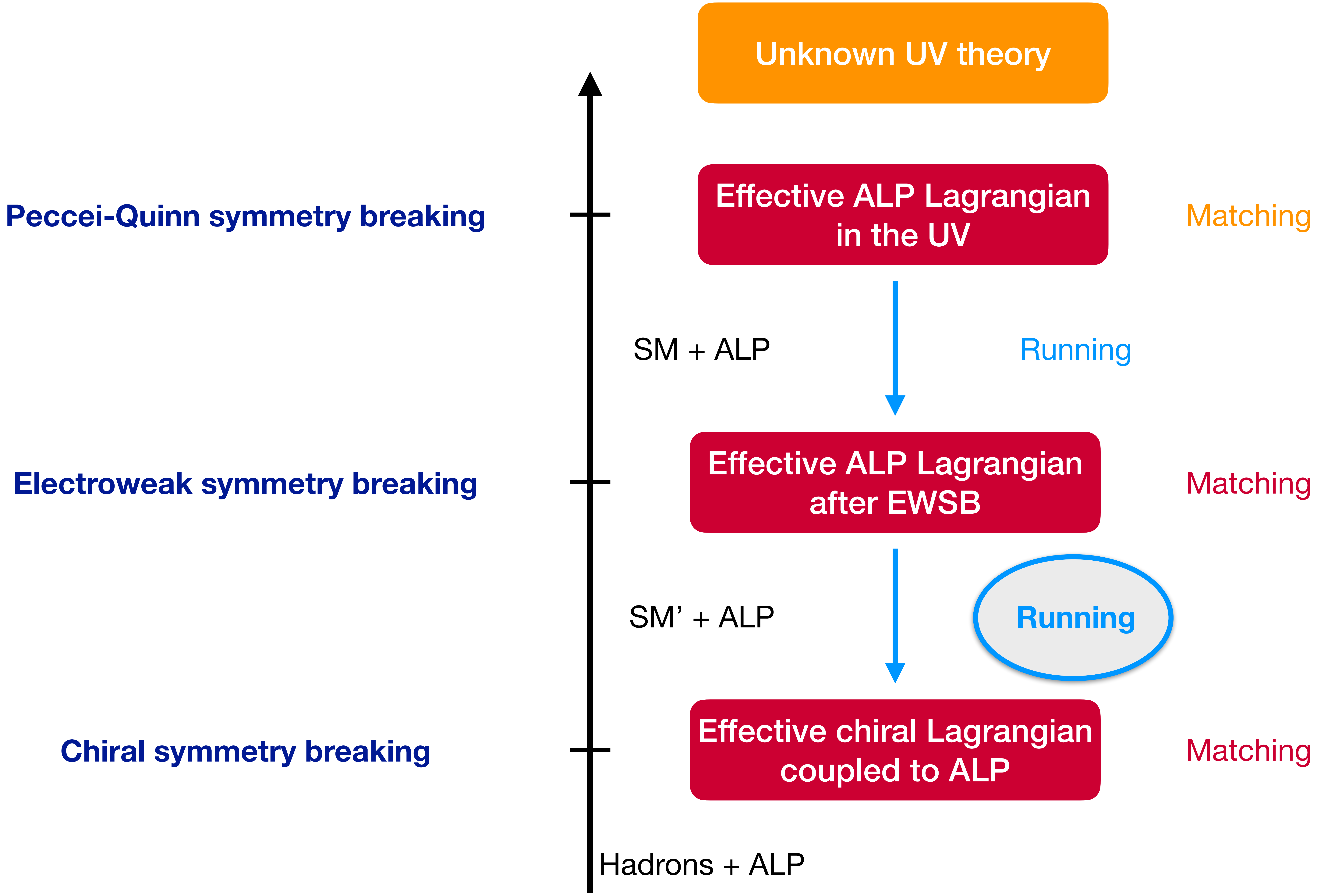
(top quark has been integrated out)

$$[k_d(\mu_w)]_{ij} = [k_d(\Lambda)]_{ij}\,,$$

$$[k_e(\mu_w)]_{ij} = [k_e(\Lambda)]_{ij}\,,$$

$$[k_L(\mu_w)]_{ij} = [k_L(\Lambda)]_{ij}\,.$$

$$[k_D(m_t)]_{ij} \simeq [k_D(\Lambda)]_{ij} + 0.019\, V_{ti}^* V_{tj} \Big[c_{tt}(\Lambda) - 0.0032\, \tilde{c}_{GG}(\Lambda) - 0.0057\, \tilde{c}_{WW}(\Lambda) \Big]$$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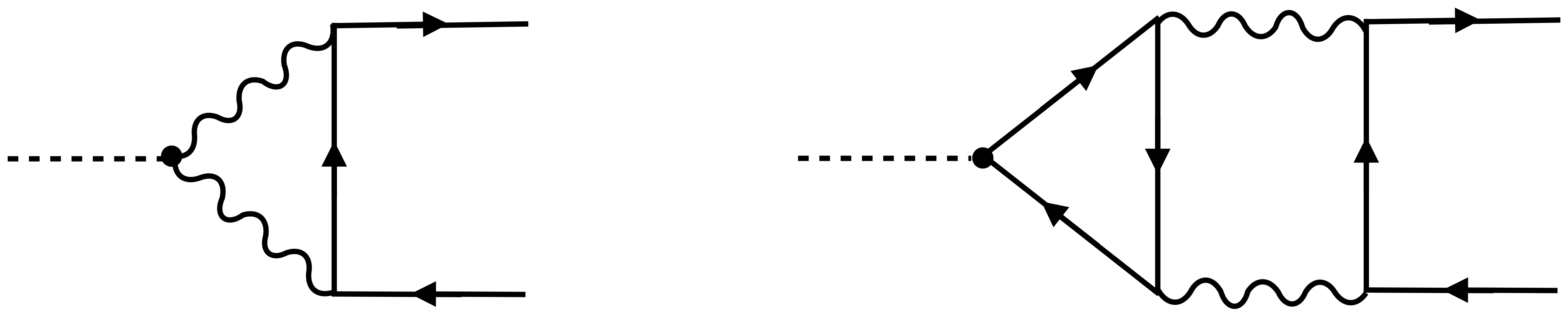
Electroweak symmetry breaking
Effective ALP Lagrangian after EWSB
Matching
SM' + ALP
Running
Chiral symmetry breaking
Effective chiral Lagrangian coupled to ALP
Matching
Hadrons + ALP

Evolution below the weak scale

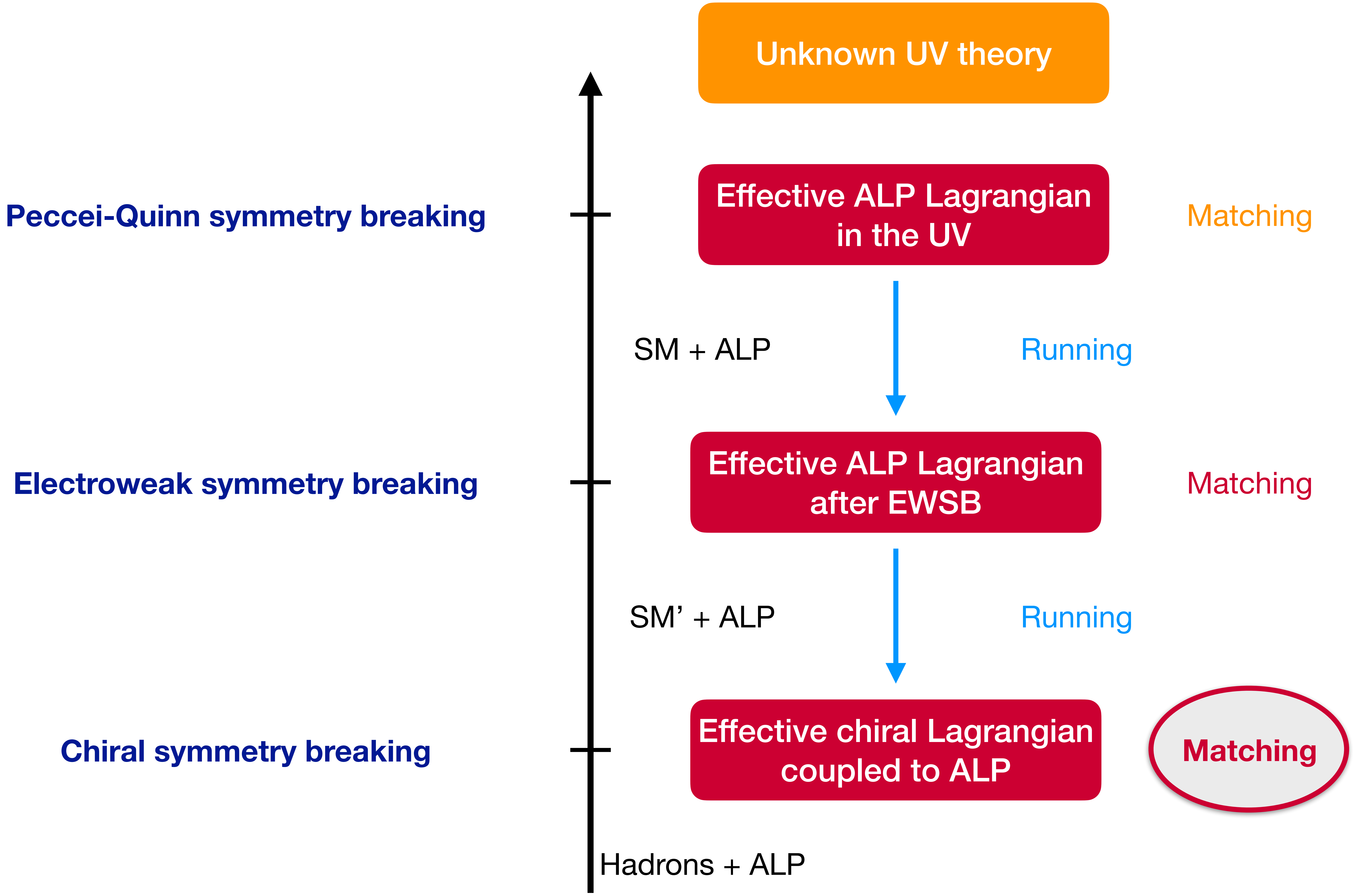
In this case only gluon and photon loops contribute:



We find numerically with $\mu_0 = 2$ GeV:

$$c_{qq}(\mu_0) = c_{qq}(m_t) + \Big[3.0\,\tilde{c}_{GG}(\Lambda) - 1.4\,c_{tt}(\Lambda) - 0.6\,c_{bb}(\Lambda)\Big]\cdot 10^{-2}$$
$$+\,Q_q^2\Big[3.9\,\tilde{c}_{\gamma\gamma}(\Lambda) - 4.7\,c_{tt}(\Lambda) - 0.2\,c_{bb}(\Lambda)\Big]\cdot 10^{-5}\,,$$
$$c_{\ell\ell}(\mu_0) = c_{\ell\ell}(m_t) + \Big[3.9\,\tilde{c}_{\gamma\gamma}(\Lambda) - 4.7\,c_{tt}(\Lambda) - 0.2\,c_{bb}(\Lambda)\Big]\cdot 10^{-5}\,.$$

[Bauer, MN, Renner, Schnubel, Thamm (2020)]


Unknown UV theory
Peccei-Quinn symmetry breaking
Effective ALP Lagrangian in the UV
Matching
SM + ALP
Running
Electroweak symmetry breaking
Effective ALP Lagrangian after EWSB
Matching
SM’ + ALP
Running
Chiral symmetry breaking
Effective chiral Lagrangian coupled to ALP
Matching
Hadrons + ALP

Matching to the chiral Lagrangian

Georgi, Kaplan, Randall (1986) have developed a model-independent chiral Lagrangian approach valid for any ALP model

In the quark mass basis, the starting point is (at $\mu_\chi \approx 4\pi f_\pi$):

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm QCD} + \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2}a^2 + c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f}G_{\mu\nu}^a\tilde{G}^{\mu\nu,a} + c_{\gamma\gamma}\frac{\alpha}{4\pi}\frac{a}{f}F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{\partial^\mu a}{f}\left(\bar{q}_L\boldsymbol{k}_Q\gamma_\mu q_L + \bar{q}_R\boldsymbol{k}_q\gamma_\mu q_R + \dots\right)$$

three light quarks *u*, *d*, *s*

Matching to the chiral Lagrangian

To bosonize this theory, one first eliminates the ALP-gluon coupling using the chiral rotation: [Srednicki (1985); Georgi, Kaplan, Randall (1986); Krauss, Wise (1986); Bardeen, Peccei, Yanagida (1987)]

$$q(x) \to \exp\left[-i\left(\boldsymbol{\delta}_q + \boldsymbol{\kappa}_q\gamma_5\right)c_{GG}\,\frac{a(x)}{f}\right]q(x) \qquad \text{with} \qquad \mathrm{Tr}\,\boldsymbol{\kappa}_q = \kappa_u + \kappa_d + \kappa_s = 1$$

diagonal in the quark mass basis

Modified quark mass matrix and ALP couplings:

$$\hat{\boldsymbol{m}}_q(a) = \exp\left(-2i\boldsymbol{\kappa}_q\,c_{GG}\,\frac{a}{f}\right)\boldsymbol{m}_q$$

$$\hat{c}_{\gamma\gamma} = c_{\gamma\gamma} - 2N_c\,c_{GG}\,\mathrm{Tr}\,\boldsymbol{Q}^2\,\boldsymbol{\kappa}_q$$

$$\left.\begin{aligned} \hat{\boldsymbol{k}}_Q(a) &= e^{i\boldsymbol{\phi}_q^-\,a/f}\left(\boldsymbol{k}_Q + \boldsymbol{\phi}_q^-\right)e^{-i\boldsymbol{\phi}_q^-\,a/f} \\ \hat{\boldsymbol{k}}_q(a) &= e^{i\boldsymbol{\phi}_q^+\,a/f}\left(\boldsymbol{k}_q + \boldsymbol{\phi}_q^+\right)e^{-i\boldsymbol{\phi}_q^+\,a/f} \end{aligned}\right\} \quad \text{with} \quad \boldsymbol{\phi}_q^\pm = c_{GG}\left(\boldsymbol{\delta}_q \pm \boldsymbol{\kappa}_q\right)$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]

Matching to the chiral Lagrangian

- The light pseudoscalar mesons are described by $\boldsymbol{\Sigma}(x) = \exp\left[\frac{i\sqrt{2}}{f_\pi}\,\lambda^a\,\pi^a(x)\right]$

- Derivative ALP couplings to fermions are included in the covariant derivative:

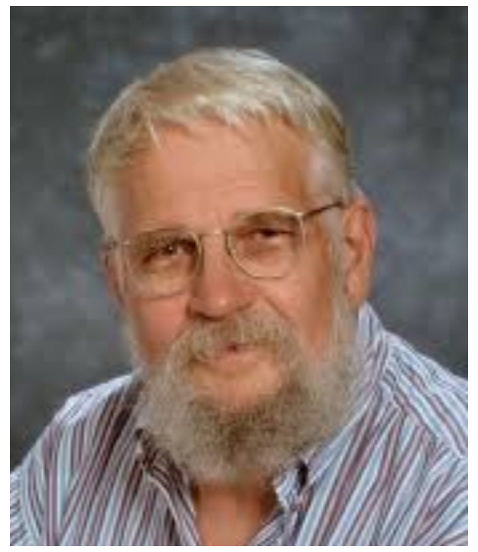
$$iD_\mu\boldsymbol{\Sigma} = i\partial_\mu\boldsymbol{\Sigma} + e\,A_\mu\,[\boldsymbol{Q},\boldsymbol{\Sigma}] + \frac{\partial_\mu a}{f}\left(\hat{\boldsymbol{k}}_Q\,\boldsymbol{\Sigma} - \boldsymbol{\Sigma}\,\hat{\boldsymbol{k}}_q\right)$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]

- Leading-order effective chiral Lagrangian:

$$\mathcal{L}_{\text{eff}}^{\chi} = \frac{f_\pi^2}{8}\,\text{Tr}\big[\boldsymbol{D}^\mu\boldsymbol{\Sigma}\,(\boldsymbol{D}_\mu\boldsymbol{\Sigma})^\dagger\big] + \frac{f_\pi^2}{4}\,B_0\,\text{Tr}\big[\hat{\boldsymbol{m}}_q(a)\,\boldsymbol{\Sigma}^\dagger + \text{h.c.}\big]$$
$$+ \frac{1}{2}\,\partial^\mu a\,\partial_\mu a - \frac{m_{a,0}^2}{2}\,a^2 + \hat{c}_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}$$

[Gasser, Leutwyler (1985)]

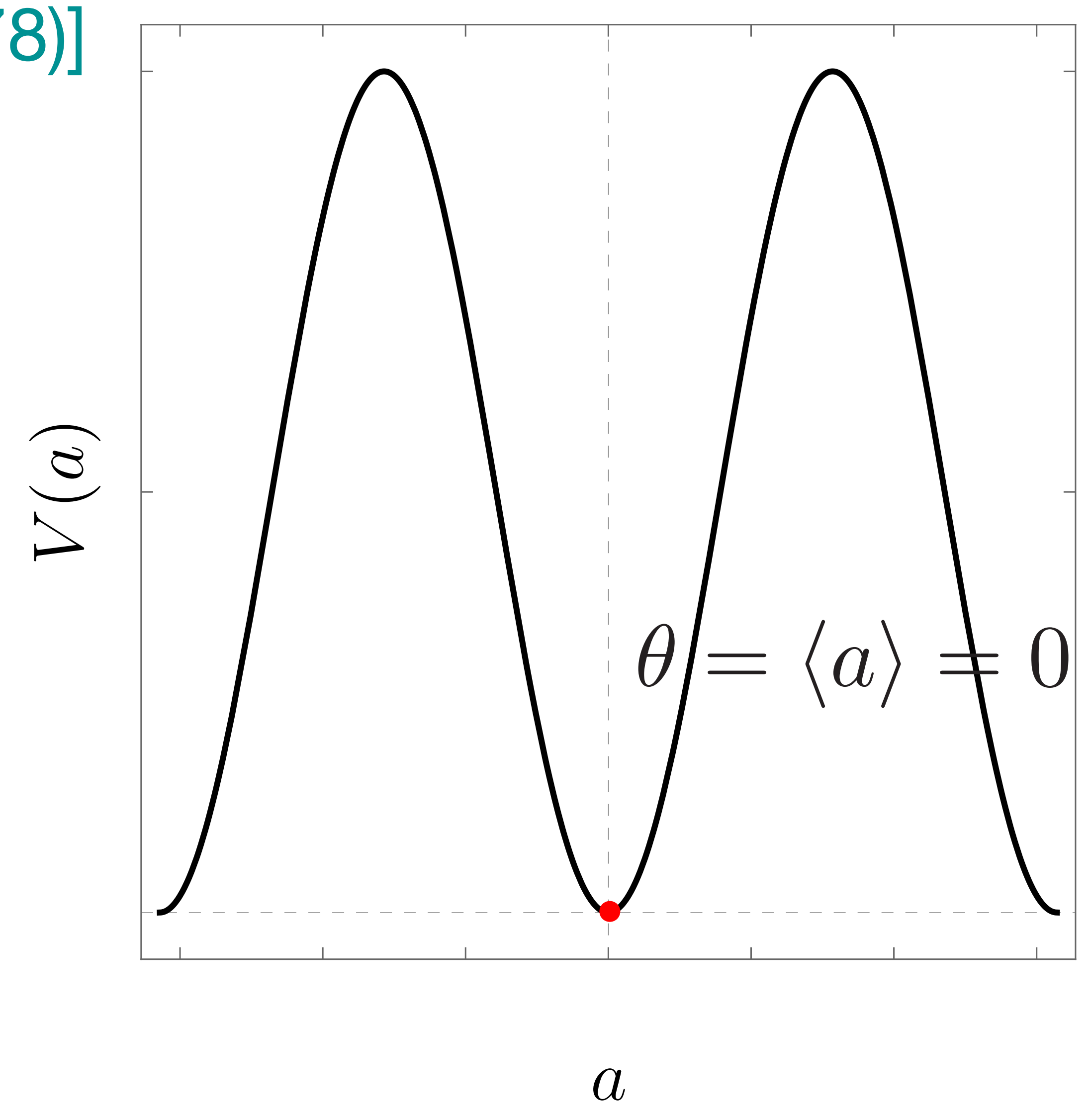
Matching to the chiral Lagrangian

- Quadratic terms in the Lagrangian yield the QCD instanton contribution to the ALP mass in addition to the bare mass term: [Bardeen, Tye, Vermaseren (1978); Shifman, Vainshtein, Zakharov (1980); Di Veccia, Veneziano (1980)]

$$m_a^2 = c_{GG}^2 \frac{f_\pi^2 m_\pi^2}{f^2} \frac{2 m_u m_d}{(m_u + m_d)^2} + m_{a,0}^2 \left[1 + \mathcal{O}\left(\frac{f_\pi^2}{f^2} \right) \right]$$

- The ALP potential is periodic in the field and breaks the classical shift symmetry to the discrete subgroup: [Weinberg (1978); Wilczek (1978)]

$$a(x) \to a(x) + n\pi \frac{f}{c_{GG}}$$



Matching to the chiral Lagrangian

- In addition, there is mass mixing and kinetic mixing between the ALP and the neutral pseudoscalar mesons, e.g.:

$$\pi^0 = \pi^0_{\text{phys}} + \frac{f_\pi}{2\sqrt{2}\,f}\left(\frac{m_a^2}{m_\pi^2 - m_a^2}\,\Delta c_{ud} - \delta_\kappa\right) a_{\text{phys}} + \mathcal{O}\!\left(\frac{f_\pi^2}{f^2}\right)$$

where:

$$\boxed{\Delta c_{ud} = c_{uu}(\mu_\chi) - c_{dd}(\mu_\chi) + 2c_{GG}\,\frac{m_d - m_u}{m_d + m_u}}\,, \qquad \delta_\kappa = 4c_{GG}\,\frac{m_u\kappa_u - m_d\kappa_d}{m_d + m_u}$$

- Often authors eliminate the mass mixing by the "default choice" $\boldsymbol{\kappa}_q = \boldsymbol{m}_q^{-1}/\text{Tr}\,(\boldsymbol{m}_q^{-1})$, which is adequate for the QCD axion

- For ALPs, the mixing can be eliminated by choosing:

$$\kappa_u = \frac{m_d}{m_u + m_d} + \frac{m_a^2}{m_\pi^2 - m_a^2}\,\frac{\Delta c_{ud}}{4c_{GG}}\,, \qquad \kappa_d = \frac{m_u}{m_u + m_d} - \frac{m_a^2}{m_\pi^2 - m_a^2}\,\frac{\Delta c_{ud}}{4c_{GG}}$$

[Bauer, MN, Renner, Schnubel, Thamm (2020)]

Matching to the chiral Lagrangian

- In many paper the mixing angles are treated as “physical” quantities, which is misleading, because they are κ_q dependent quantities
- We keep the auxiliary parameters κ_q and δ_q arbitrary and explicitly check that physical quantities are independent of their choice
- In terms of the physical mass eigenstates, the SU(2) chiral Lagrangian is found to be:

now only QED

$$
\begin{aligned}
\mathcal{L}_{\chi PT}^{\rm ALP} &= \frac{1}{2}\,\partial^\mu a\,\partial_\mu a - \frac{m_a^2}{2}\,a^2 + \frac{1}{2}\,\partial^\mu\pi^0\,\partial_\mu\pi^0 - \frac{m_\pi^2}{2}\,(\pi^0)^2 + D^\mu\pi^+ D_\mu\pi^- - m_\pi^2\,\pi^+\pi^- + \mathcal{O}\left(\frac{\pi^4}{f_\pi^2}\right) \\
&\quad - \frac{\boxed{\Delta c_{ud}}}{6\sqrt{2}f_\pi f}\,\frac{m_\pi^2}{m_a^2 - m_\pi^2}\Big[4\partial^\mu a\left(\pi^0\pi^+ D_\mu\pi^- + \pi^0\pi^- D_\mu\pi^+ - 2\pi^+\pi^-\partial_\mu\pi^0\right) \\
&\qquad + m_a^2\,a\left(2\pi^+\pi^-\pi^0 + (\pi^0)^3\right)\Big] + \mathcal{O}\left(\frac{a\pi^5}{f_\pi^3 f}\right)
\end{aligned}
$$

[Bauer, MN, Renner, Schnubel, Thamm (2020)]

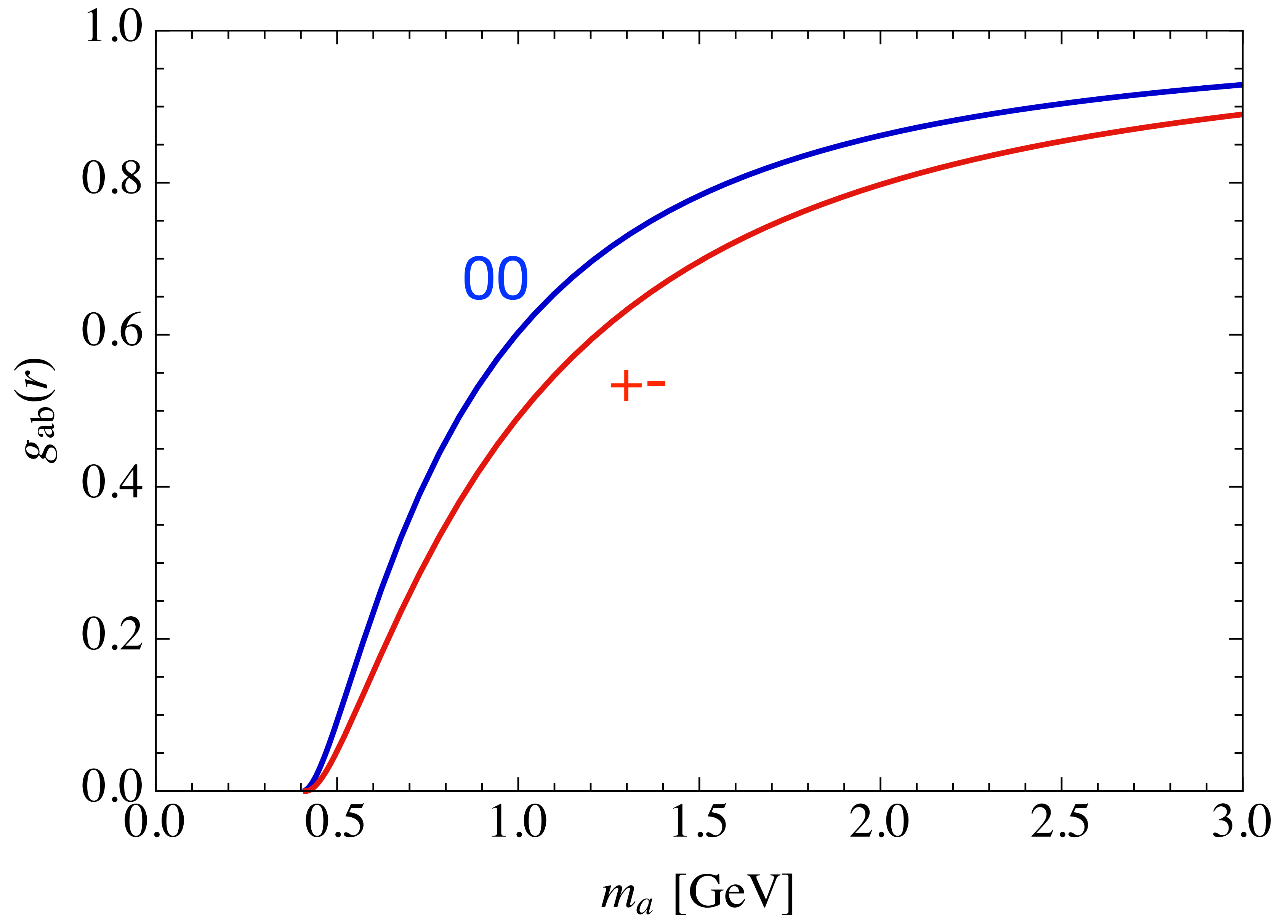
Matching to the chiral Lagrangian

For the hadronic decay $a \to 3\pi$ we find:

$$\mathcal{M}(a \to \pi^0\pi^0\pi^0) = -\frac{\Delta c_{ud}}{\sqrt{2} f_\pi f}\, \frac{m_\pi^2 m_a^2}{m_a^2 - m_\pi^2}$$

$$\mathcal{M}(a \to \pi^+\pi^-\pi^0) = -\frac{\Delta c_{ud}}{\sqrt{2} f_\pi f}\, \frac{m_\pi^2 (m_{+-}^2 - m_\pi^2)}{m_a^2 - m_\pi^2}$$

$$\Gamma(a \to \pi^a\pi^b\pi^0) = \frac{m_a m_\pi^4}{6144\pi^3 f_\pi^2 f^2} (\Delta c_{ud})^2\, g_{ab}\!\left(\frac{m_\pi^2}{m_a^2}\right)$$



These are the dominant hadronic decay channels for an ALP with mass above $3\, m_\pi$ and below 2 GeV [Bauer, MN, Thamm (2017)]

Weak decay $K \to \pi a$

- Strongest particle-physics constraint on ALP couplings for mass range $m_a < m_K - m_\pi \approx 354$ MeV

- Despite a 35-year history, we find that even nowadays most papers on this process are based on inconsistent equations

- The chiral implementation of the leading SU(3)-octet weak-interaction operator is: [Bernard, Draper, Soni, Politzer, Wise (1985); Crewther (1986); Kambor, Missimer, Wyler (1990)]

$$\mathcal{L}_{\text{weak}} = -\frac{4G_F}{\sqrt{2}}\, V_{ud}^* V_{us}\, g_8 \left[L_\mu L^\mu\right]^{32}$$

where L_μ^{ij} is the chiral representation of the left-handed current $\bar{q}_L^i \gamma_\mu q_L^j$

Weak decay $K \to \pi a$

Georgi, Kaplan, Randall used:

$$L_\mu^{ij} = -\frac{if_\pi^2}{4}\, e^{i(\phi_{q_i}^- - \phi_{q_j}^-)\, a/f} \left[\mathbf{\Sigma}\, \partial_\mu \mathbf{\Sigma}^\dagger\right]^{ij}$$

where the phase factor results from the chiral rotation, but the Noether theorem gives instead: [Bauer, MN, Renner, Schnubel, Thamm (2021)]

$$\boxed{\begin{aligned} L_\mu^{ji} &= -\frac{if_\pi^2}{4}\, e^{i(\phi_{q_i}^- - \phi_{q_j}^-)\, a/f} \left[\mathbf{\Sigma}\, (\boldsymbol{D}_\mu \mathbf{\Sigma})^\dagger\right]^{ji} \\ &\ni -\frac{if_\pi^2}{4} \left[1 + i(\delta_{q_i} - \delta_{q_j} - \kappa_{q_i} + \kappa_{q_j})\, c_{GG}\, \frac{a}{f}\right] \left[\mathbf{\Sigma}\, \partial_\mu \mathbf{\Sigma}^\dagger\right]^{ji} \\ &\quad + \frac{f_\pi^2}{4}\, \frac{\partial^\mu a}{f} \left[\hat{\boldsymbol{k}}_Q - \mathbf{\Sigma}\, \hat{\boldsymbol{k}}_q\, \mathbf{\Sigma}^\dagger\right]^{ji} \qquad \leftarrow \text{crucial extra terms!} \end{aligned}}$$

Weak decay $K \to \pi a$

Cancellation of auxiliary parameters:

$$D_1 \ni \frac{N_8}{2f}\, c_{GG}\, (\kappa_u - \kappa_d)\, (m_\pi^2 - m_a^2)$$

$$D_2 \ni -\frac{N_8}{6f}\, c_{GG}\, (2m_K^2 + m_\pi^2 - 3m_a^2)\, (\kappa_u + \kappa_d - 2\kappa_s)$$

$$D_3 \ni \frac{N_8}{2f}\, c_{GG} \Big[-(\delta_d - \delta_s - \kappa_d + \kappa_s)\, (m_K^2 + m_\pi^2 - m_a^2) + (\delta_u - \delta_d + \kappa_u + \kappa_s)\, (m_K^2 - m_\pi^2 + m_a^2) + (\delta_u - \delta_s + \kappa_u + \kappa_d)\, (m_K^2 - m_\pi^2 - m_a^2) \Big]$$

previously omitted contributions

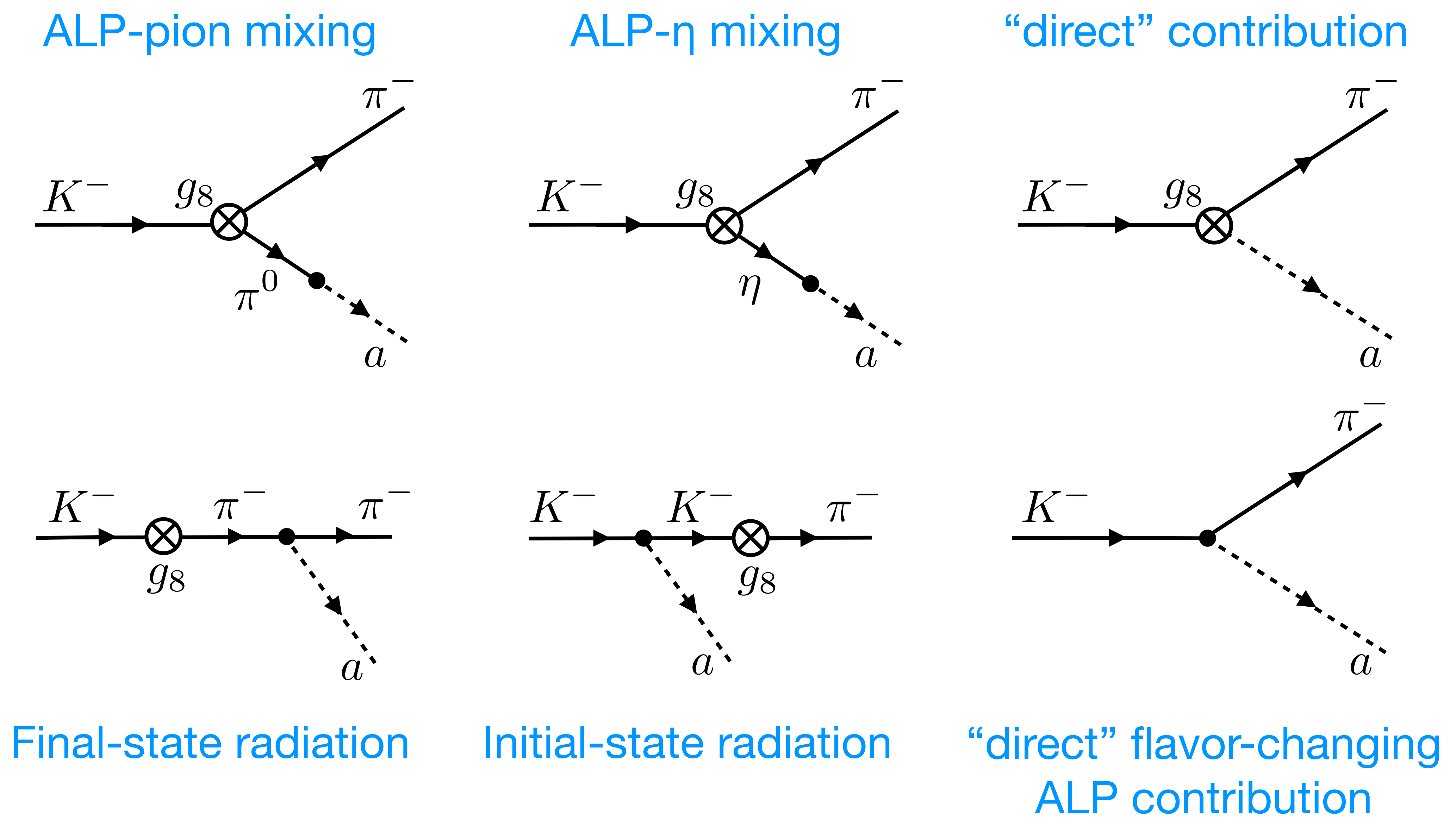
$$D_4 \ni -\frac{N_8}{f}\, c_{GG}\, m_K^2\, (\delta_u - \delta_d)$$

$$D_5 \ni \frac{N_8}{f}\, c_{GG}\, m_\pi^2\, (\delta_u - \delta_s)$$

with:

$$N_8 = -\frac{G_F}{\sqrt{2}}\, V_{ud}^* V_{us}\, g_8\, f_\pi^2$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]



- Find that omitted contributions have a huge effect (parametrically dominant terms)
- Including only the first two diagrams (ALP-meson mixing) gives an uncontrolled approximation (except in very special cases)

Weak decay $K \to \pi a$

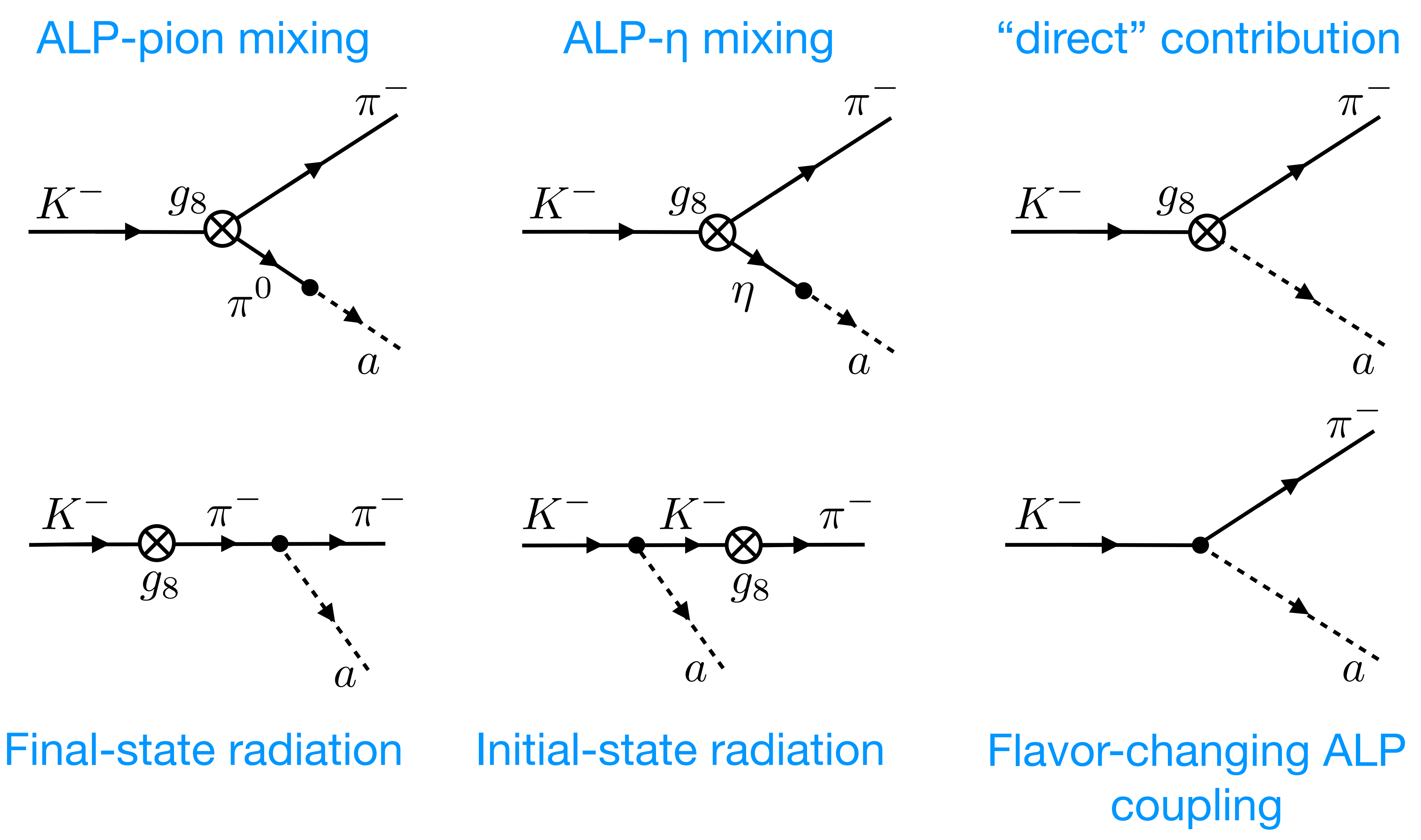
Decay amplitude:

$$
\begin{aligned}
i\mathcal{A}_{K^-\to\pi^-a} = \frac{N_8}{4f}\bigg[& 16\,c_{GG}\,\frac{(m_K^2-m_\pi^2)(m_K^2-m_a^2)}{4m_K^2-m_\pi^2-3m_a^2} \\
& + 6\,(c_{uu}+c_{dd}-2c_{ss})\,m_a^2\,\frac{m_K^2-m_a^2}{4m_K^2-m_\pi^2-3m_a^2} \\
& + (2c_{uu}+c_{dd}+c_{ss})\,(m_K^2-m_\pi^2-m_a^2) + 4c_{ss}m_a^2 \\
& + (k_d+k_D-k_s-k_S)\,(m_K^2+m_\pi^2-m_a^2)\bigg] \\
& - \frac{m_K^2-m_\pi^2}{2f}\,[k_q+k_Q]^{23}
\end{aligned}
$$

with:

$$N_8 = -\frac{G_F}{\sqrt{2}}\,V_{ud}^*V_{us}\,g_8\,f_\pi^2$$

[Bauer, MN, Renner, Schnubel, Thamm (2021)]



Georgi, Kaplan and Randall have only considered the axion-gluon coupling c_{GG} and find a result smaller by a factor

$$\frac{m_u}{2(m_u+m_d)} \approx 0.16$$

$K \to \pi a$ phenomenology

Expressing the ALP couplings in terms of the couplings at the scale $\Lambda = 4\pi f$ with $f = 1$ TeV, and assuming MFV, we find:

strong-interaction phase of g_8

$$|\mathcal{A}_{K^-\to\pi^- a}| \simeq 10^{-11}\,\text{GeV}\left[\frac{1\,\text{TeV}}{f}\right] \times \Big[e^{i\delta_8}\left(3.58\,c_{GG} + 1.79\,c_{uu}(\Lambda) + 1.81\,c_{dd}(\Lambda)\right)$$
$$+\, e^{i\alpha}\left(-\,65.8\,c_{uu}(\Lambda) + 0.32\,c_{dd}(\Lambda) + 0.21\,c_{GG} + 0.38\,c_{WW}\right)$$
$$-\,1.12\cdot 10^7\,k_D^{12}(\Lambda)\Big]$$

weak phase of V_{td}^*

← proportional to $V_{td}^* V_{ts}$ in MFV

The coefficients refer to $m_a = 0$, but they vary by less than 10% over the entire allowed mass range. Two "benchmarks": [see e.g.: Gori, Perez, Tobioka (2020)]

- only $c_{GG} \neq 0$: "indirect" contribution (g_8) dominates
- only $c_{WW} \neq 0$: "direct" contribution (from RG running) dominates

$K \to \pi a$ phenomenology

More generally, one can derive bounds $|c_{ii}|/f < \left[\Lambda_{ii}^{\text{eff}}\right]^{-1}$ for all relevant ALP couplings using the NA62 upper limit $\text{Br}(K^- \to \pi^- X) < 2.0 \cdot 10^{-10}$ (90% CL), which implies:

c_{ii}	c_{GG}	c_{WW}	c_{uu}	c_{dd}	k_D^{12}	$k_D^{12}/\lvert V_{td}V_{ts}\rvert$
$\Lambda_{ii}^{\text{eff}}$ [TeV]	61.3	6.5	1126	31.0	$1.9 \cdot 10^8$	60 000

- very strong bounds on flavor-changing ALP couplings in the UV
- strong bounds on ALP couplings to fermions (c_u or c_Q)
- relatively strong bounds on ALP-boson couplings

Summary

- Axions and axion-like particles appear in well-motivated extensions of the SM, in particular those addressing the strong CP problem
- They are an interesting target for searches in high-energy physics (using flavor and collider probes), astroparticle physics and cosmology
- If the scale of Peccei-Quinn symmetry breaking is far above the weak scale, it is important to connect the low-energy ALP couplings in a systematic way with the couplings in the UV theory
- A correct implementation of the left-handed quark currents in the chiral Lagrangian is required to correctly obtain the $K \to \pi a$ decay amplitude

UC SANTA BARBARA
Kavli Institute for
Theoretical Physics

Adventures in the ALPs Part II: Lepton Flavour Violation

Marvin Schnubel, Matthias Neubert

Prisma+ Cluster of Excellence

Johannes Gutenberg-University, Mainz

New Physics from Precision at High Energies

KITP, UC Santa Barbara

16.03.2021

M. Bauer, M. Neubert, S. Renner, MS, A. Thamm (arXiv: 1908.00008, 2012.12272, 2102.13112 and work in preparation)

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Precision Physics, Fundamental Interactions and Structure of Matter

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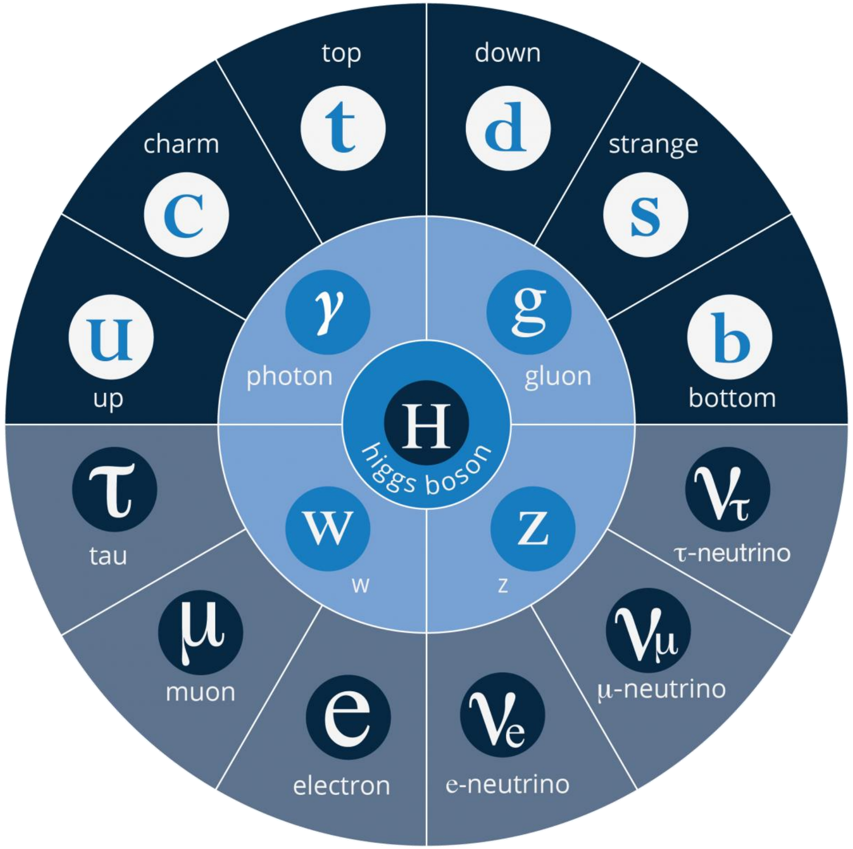
Motivation



- Lepton flavor number is accidental symmetry of Standard Model

➢ No Standard Model (SM) background in absence of neutrino masses

- With neutrino masses and oscillations:

$$\mathrm{Br}^{\mathrm{SM}}(\mu \to e\gamma) \approx 10^{-55}$$

[Petkov (1977), Hernandéz-Tomé, Lopéz Castro, Roig (2019)]

- In low mass region best bounds from cosmology → focus on mass region

$$0.1\,\mathrm{MeV} < m_a < 10\,\mathrm{GeV}$$

[Kim (1987)]

The ALP-Model

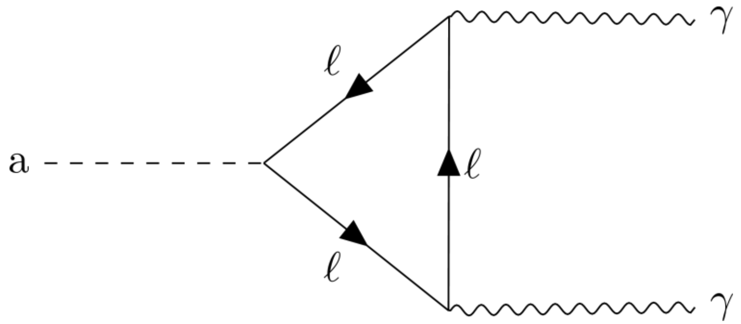
- Assume the existence of a new spin-0 resonance a, which is a gauge singlet under the SM and whose mass is much lighter than the electroweak scale
- The low energy Lagrangian reads

$$\mathscr{L}_{\rm eff}^{D\leq 5} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{\partial_\mu a}{\Lambda} \sum_\ell \bar{\ell}(k_E P_L + k_e P_R)\gamma^\mu \ell + e^2 c_{\gamma\gamma}^{\rm eff} \frac{a}{\Lambda} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Allow ALP-lepton coupling to be off-diagonal
- Neglect ALP-neutrino couplings

The ALP-Model

- Photon coupling loop-induced:



- with $c_{\gamma\gamma}^{\text{eff}} = c_{\gamma\gamma} + \sum_\ell c_{\ell\ell}\ B_1\left(\frac{m_\ell^2}{m_a^2}\right)$, where $B_1(x) \approx 1, x \ll 1$ and $B_1(x) \approx -\frac{1}{3x}, x \gg 1$
- where we have defined $c_{\ell_i \ell_j} = \sqrt{|k_e|_{ij}^2 + |k_E|_{ij}^2}$ [Bauer, Neubert, Thamm (2017)]
- Assume universal ALP-lepton coupling $\frac{c_{ee}}{f} = \frac{c_{\mu\mu}}{f} = \frac{c_{\tau\tau}}{f} = 1\,\text{TeV}^{-1}, \Lambda = 4\pi f$

The ALP lifetime effect

- Effective branching ratios often depend on ALP decay length, such that
  - $\mathrm{Br}^{\mathrm{eff}} = \mathrm{Br} \times \mathfrak{f}$ for decays
  - $\mathrm{Br}^{\mathrm{eff}} = \mathrm{Br} \times (1 - \mathfrak{f})$ for invisible ALPs

with $\mathfrak{f} = \int_0^{\frac{\pi}{2}} \sin\theta \, \mathrm{d}\theta \exp\left(-\frac{m_a R_{\max}}{\tau_0 |p^T_{\mathrm{Lab}}|}\right)$ the fraction of ALPs that decay in a detector of radius $R_{\max}$

➢ These effects can alter the shapes of excluded regions drastically.

Constraints of LFV ALP couplings

- In each sector assume one coupling to be dominant
- Apply experimental cuts and take event selection criteria into account, e.g. time difference between decay products and detector geometry
- Give prospects of future experiments where available
- Physics of muon sector can be easily transferred to tau sector

Constraints in the Muon Sector

Overview over Branching Ratios and Projections

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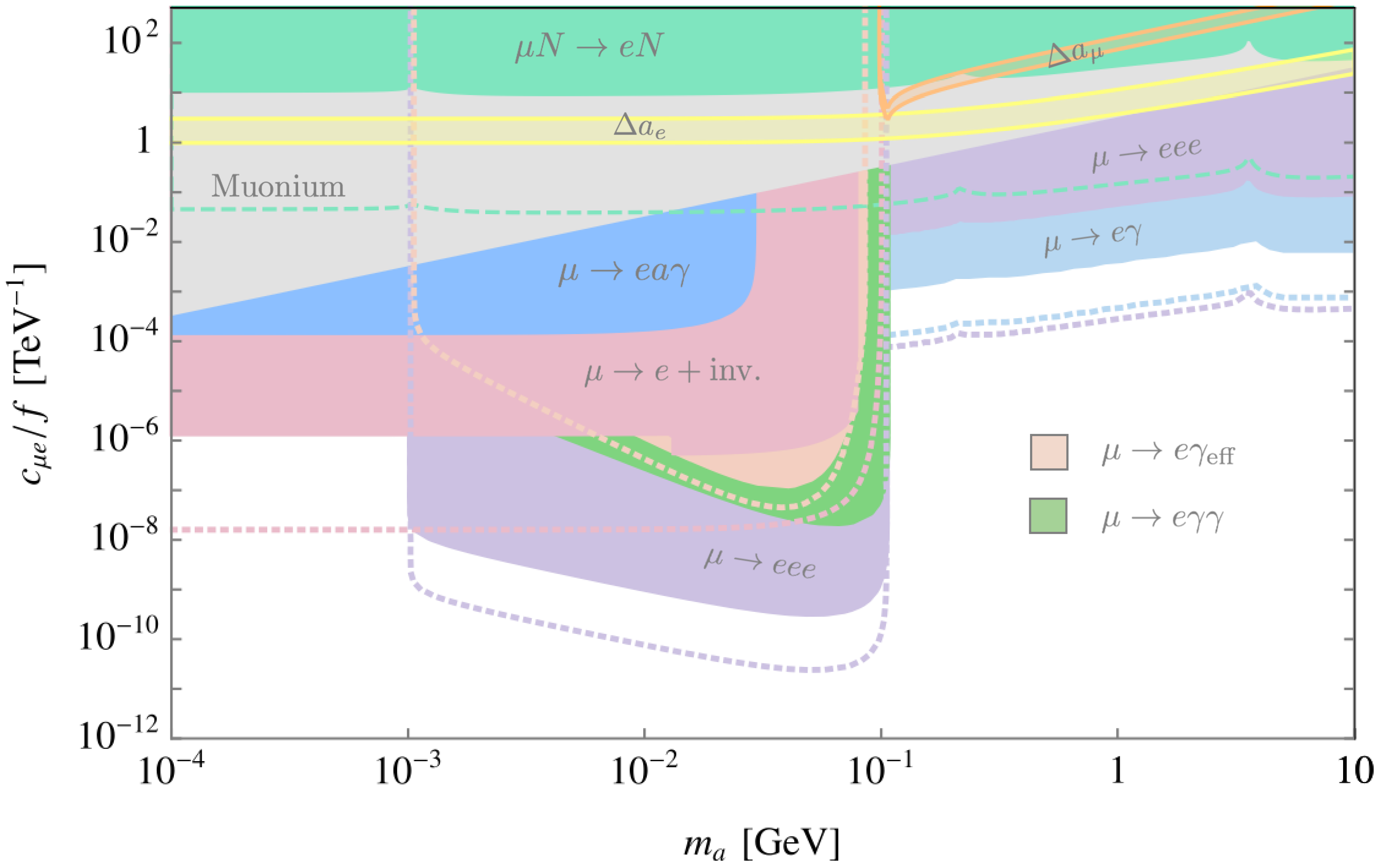
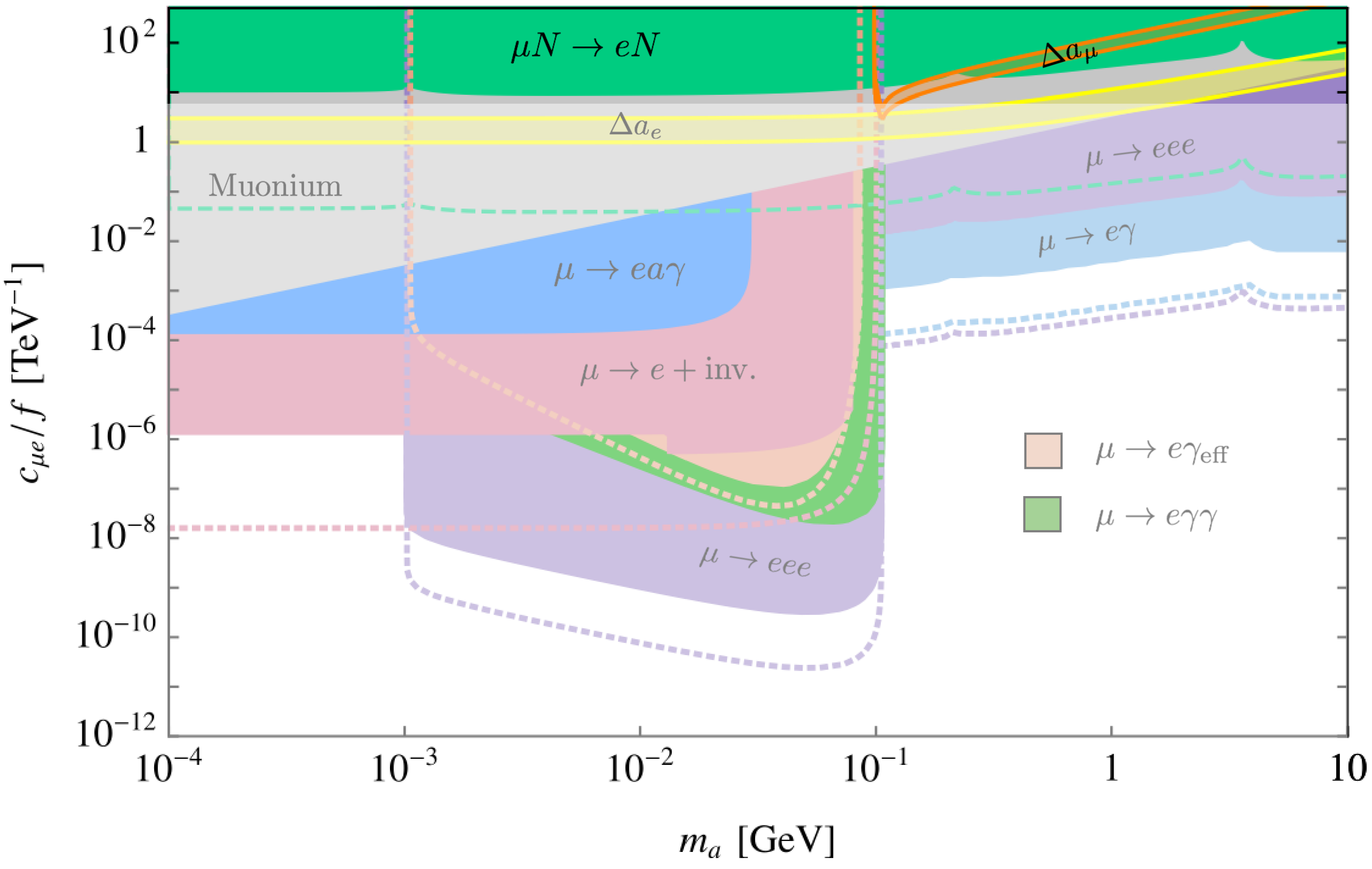
LFV Channel	Current limit	Projection
$\mu \to e\gamma$	4.2×10^{-13} [MEG Coll. (2016)]	6×10^{-14} [MEGII Coll. (2018)]
$\mu \to 3e$	1.0×10^{-12} [SINDRUM Coll. (1988)]	1×10^{-16} [Perrevoort, Mu3e (2018)]
$\mu \to ea, m_a < 13\,\mathrm{MeV}$	5.8×10^{-5} [Bayes *et al* (2014)]	1×10^{-8} [Perrevoort, Mu3e (2018)]
$\mu \to ea, m_a > 13\,\mathrm{MeV}$	9.0×10^{-6}	
$\mu \to ea\gamma$	1.1×10^{-9} [Bolton *et al* (1988)]	
$\mu \to e\gamma\gamma$	7.2×10^{-11} [LAMPF Coll (1986)]	
$\mu N \to eN$	7.0×10^{-13} [SINDRUM-II (2006)]	1×10^{-17} [Mu2e (2014)] [COMET (2020)]

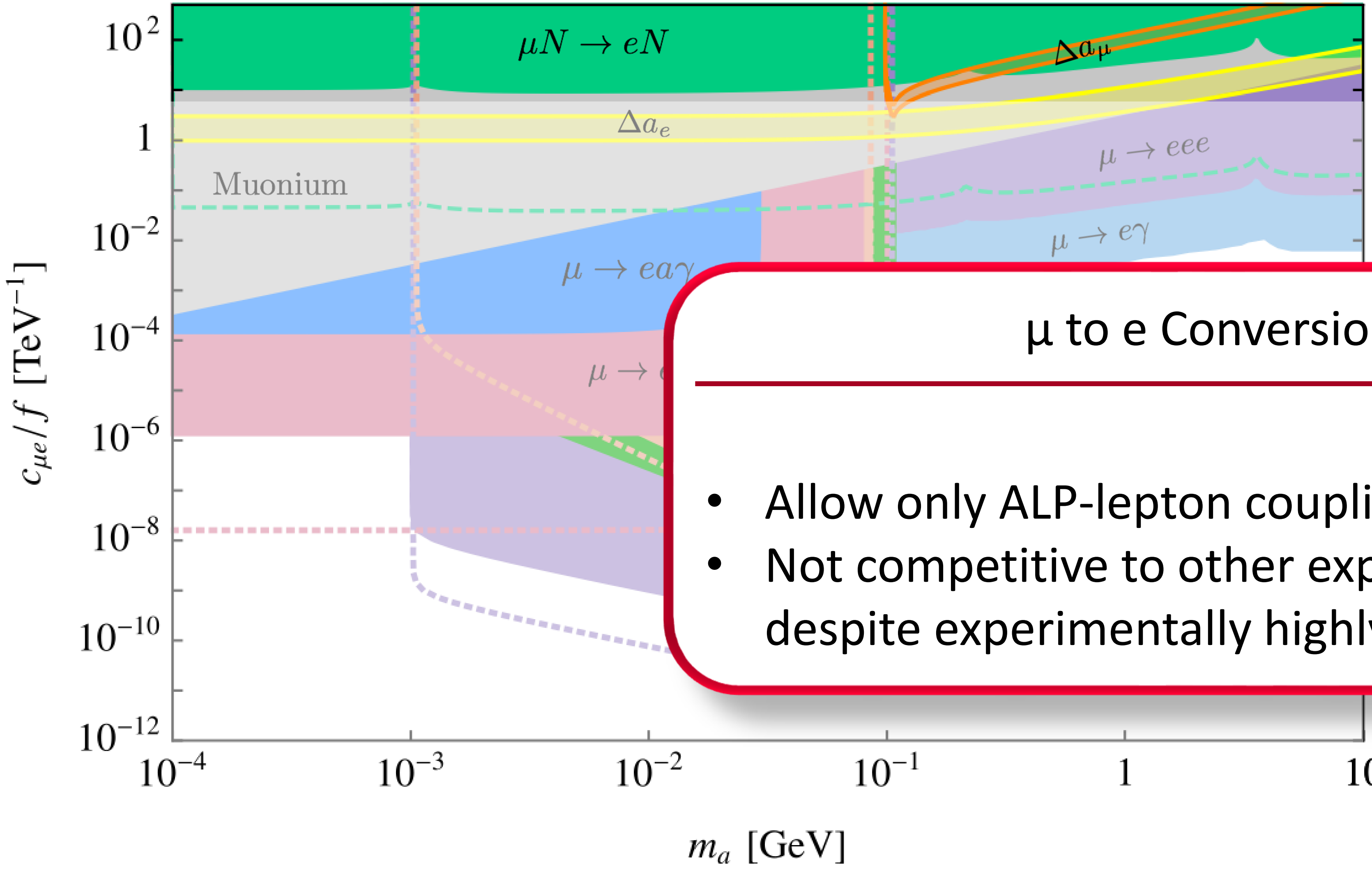
Constraints on LFV ALP-e-μ Coupling



Constraints on LFV ALP-e-μ Coupling



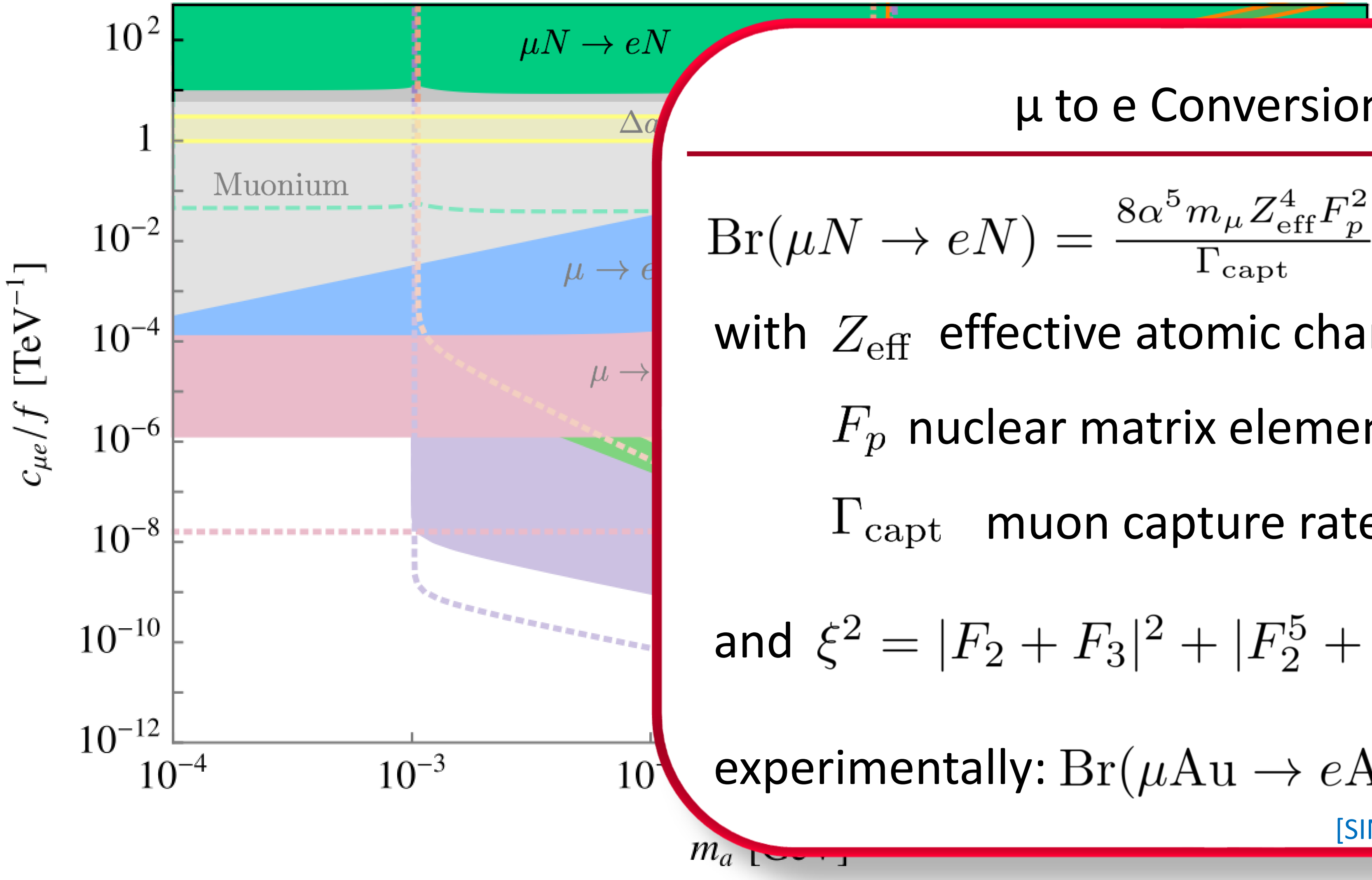
Constraints on LFV ALP-e-μ Coupling



μ to e Conversion

- Allow only ALP-lepton couplings
- Not competitive to other experiments despite experimentally highly constraining

Constraints on LFV ALP-e-μ Coupling



μ to e Conversion

$$\mathrm{Br}(\mu N \to eN) = \frac{8\alpha^5 m_\mu Z_{\text{eff}}^4 F_p^2}{\Gamma_{\text{capt}}} \xi^2$$

[Kuno, Okada (2001)]

with Z_{eff} effective atomic charge

F_p nuclear matrix element

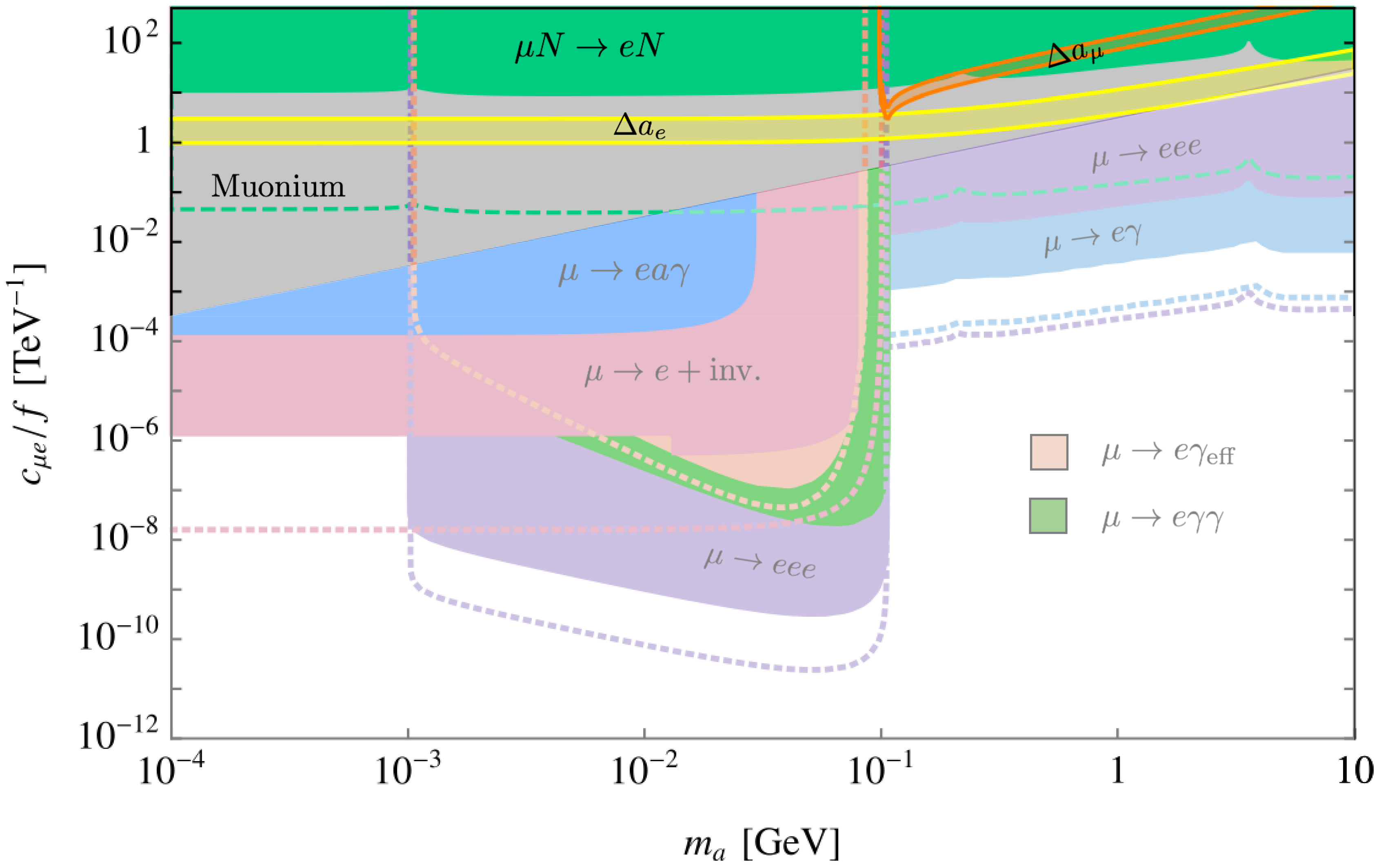
Γ_{capt} muon capture rate

and $\xi^2 = |F_2 + F_3|^2 + |F_2^5 + F_3^5|^2 \Big|_{q^2 = -m_\mu^2}$

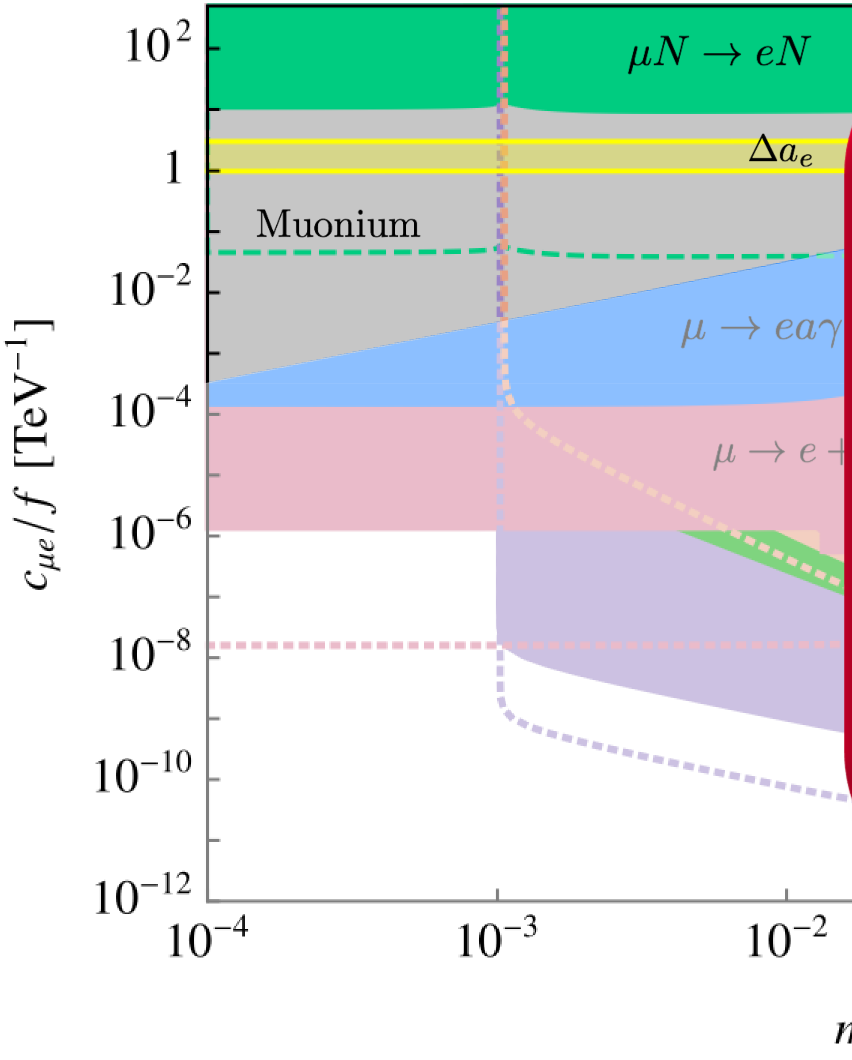
experimentally: $\mathrm{Br}(\mu\mathrm{Au} \to e\mathrm{Au}) < 7 \times 10^{-13}$

[SINDRUM-II (2006)]

Constraints on LFV ALP-e-μ Coupling

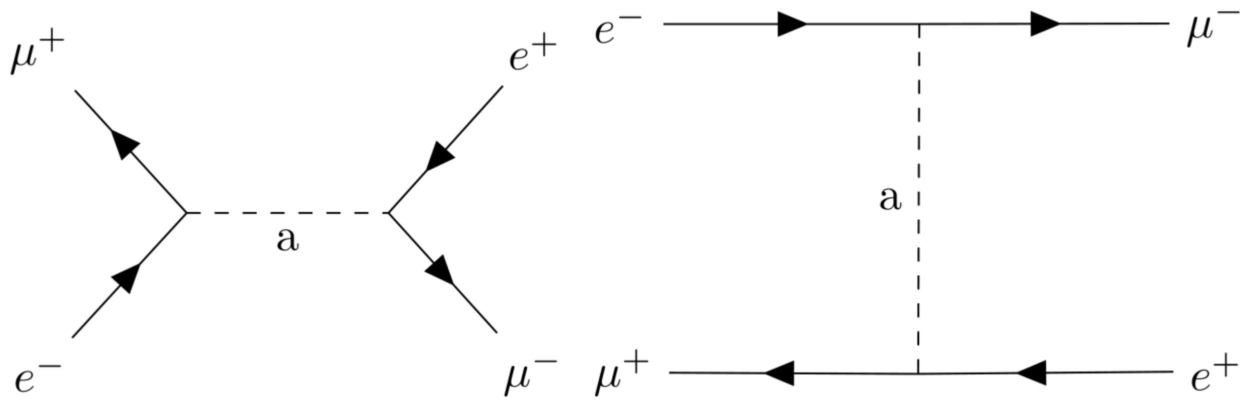


Constraints on LFV ALP-e-μ Coupling



Muonium-Antimuonium oscillation

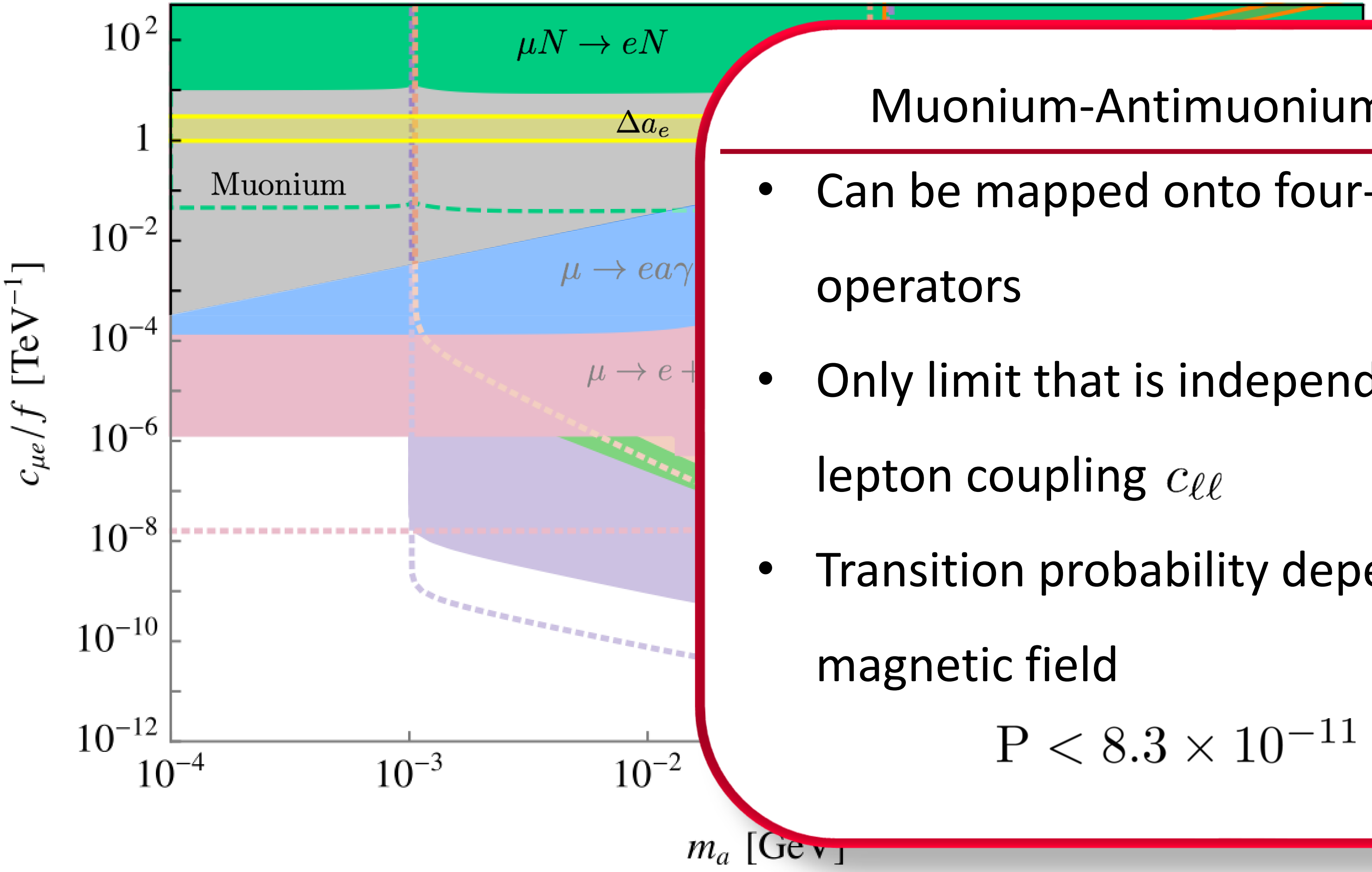
- Contributes via s- and t-channel diagrams



- In both cases $s \approx t \approx m_\mu^2$

[Hou, Wong (1996)]

Constraints on LFV ALP-e-μ Coupling



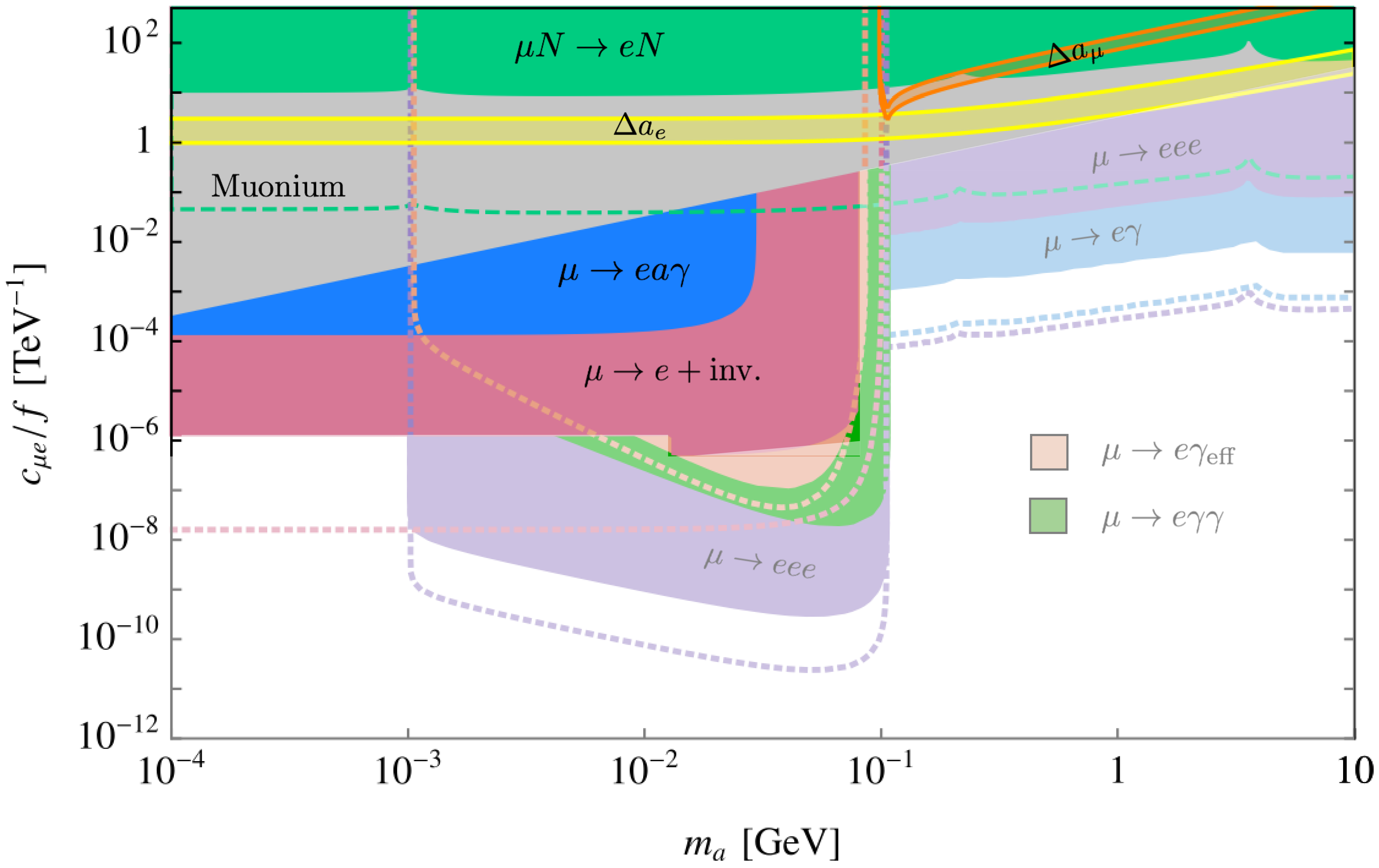
Muonium-Antimuonium oscillation

- Can be mapped onto four-fermion operators
- Only limit that is independent of diagonal lepton coupling $c_{\ell\ell}$
- Transition probability depends on magnetic field

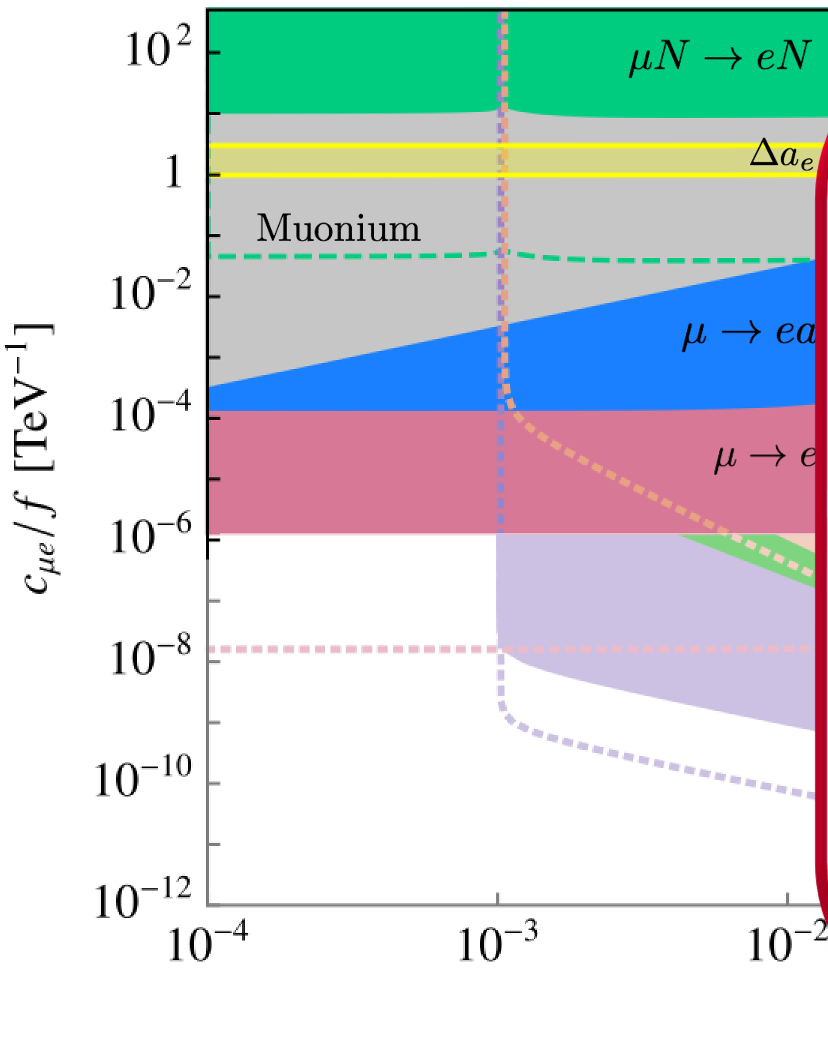
$$\mathrm{P} < 8.3 \times 10^{-11}$$

[Willmann et al. (1999)]

Constraints on LFV ALP-e-μ Coupling



Constraints on LFV ALP-e-μ Coupling

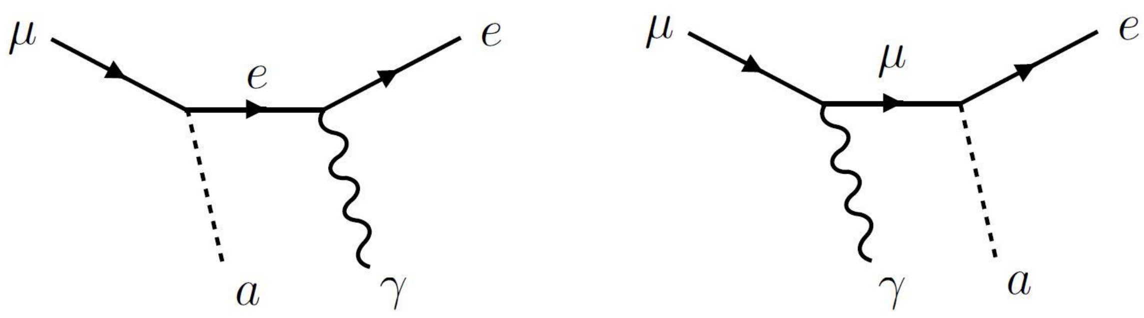


μ to ea and μ to eaγ

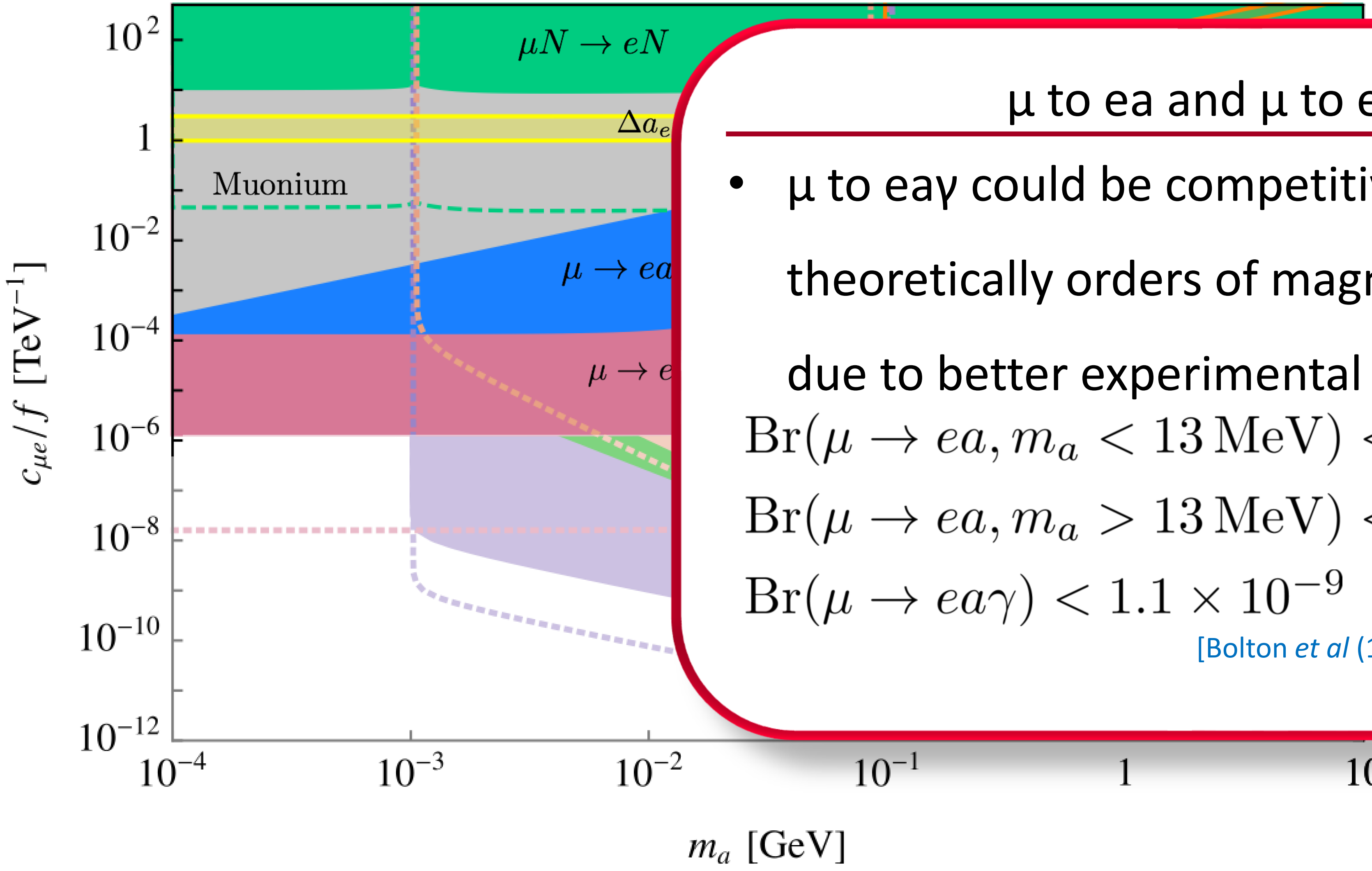
- Strongest constraints in small mass region

$$\Gamma(\mu \to ea) \approx \frac{m_\mu^3}{32\pi f^2}\left(1 - \frac{m_a^2}{m_\mu^2}\right)|c_{e\mu}|^2$$

- Can mediate μ to 3e, eγγ
- μ to eaγ is μ to ea with ISR/FSR



Constraints on LFV ALP-e-μ Coupling



μ to ea and μ to eaγ

- μ to eaγ could be competitive despite theoretically orders of magnitude smaller due to better experimental handling

$$\mathrm{Br}(\mu \to ea, m_a < 13\,\mathrm{MeV}) < 1.22 \times 10^{-6}$$

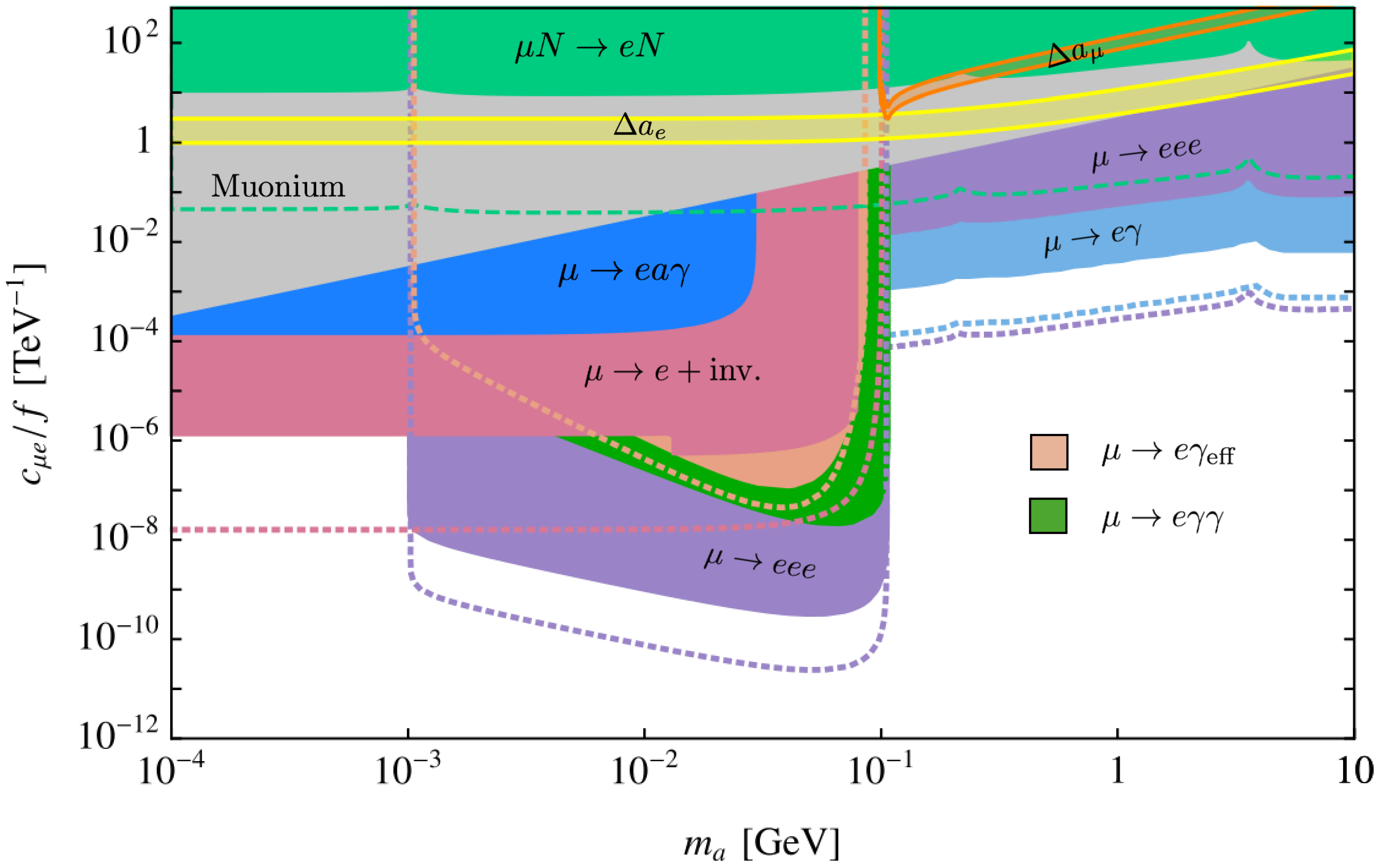
$$\mathrm{Br}(\mu \to ea, m_a > 13\,\mathrm{MeV}) < 5.02 \times 10^{-7}$$

[Bayes *et al* (2014)]

$$\mathrm{Br}(\mu \to ea\gamma) < 1.1 \times 10^{-9}$$

[Bolton *et al* (1988)]

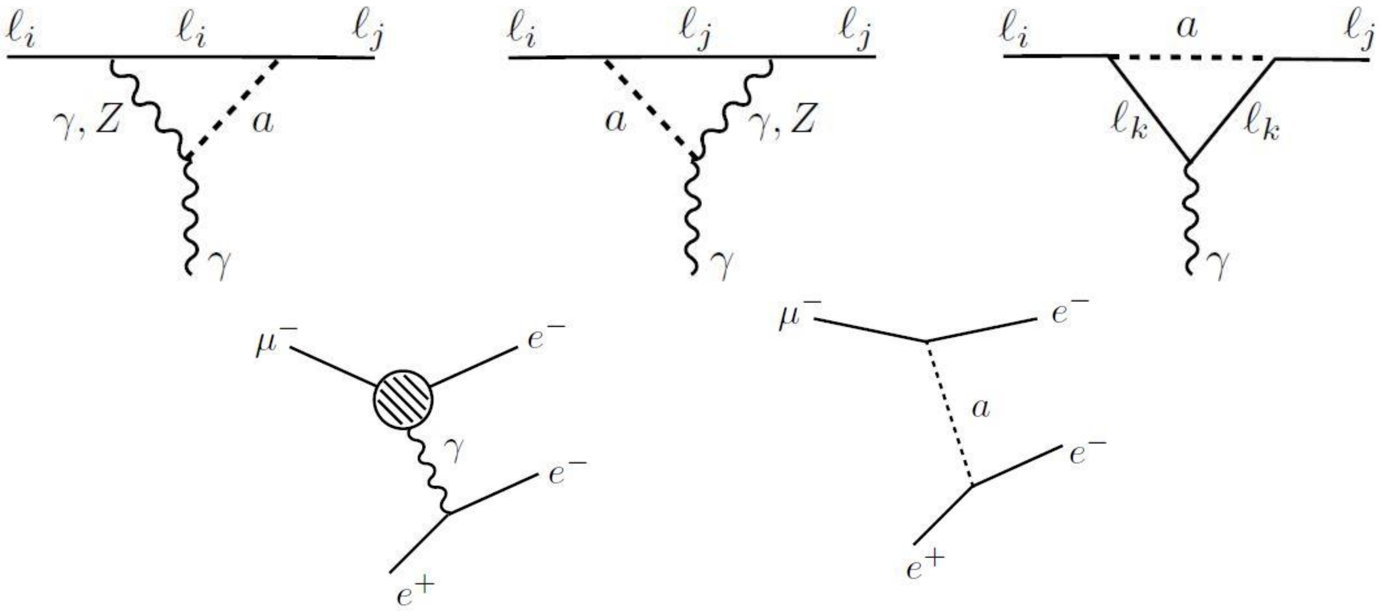
Constraints on LFV ALP-e-μ Coupling



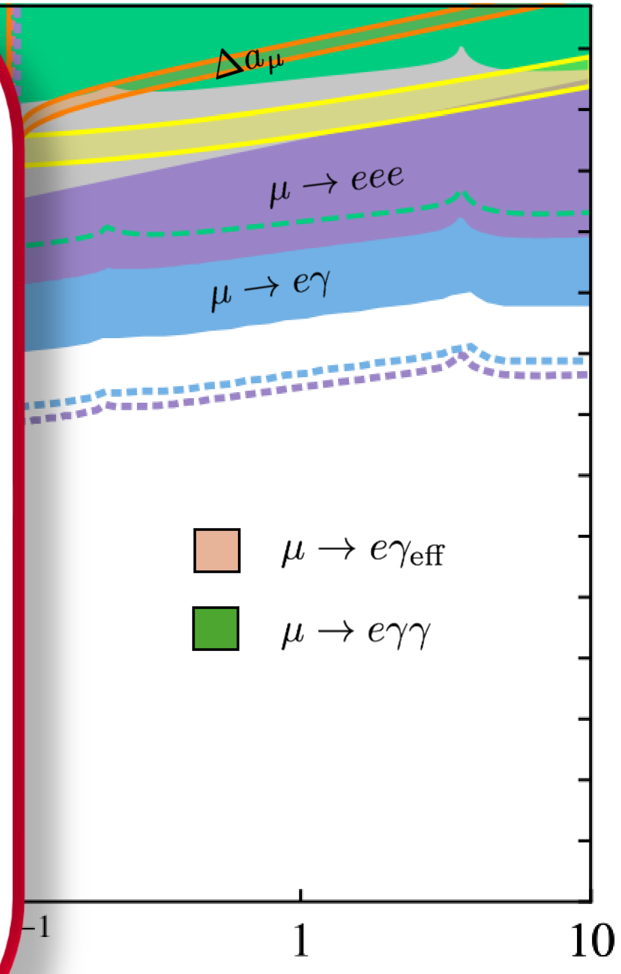
Constraints on LFV ALP-e-μ Coupling

Muon decays

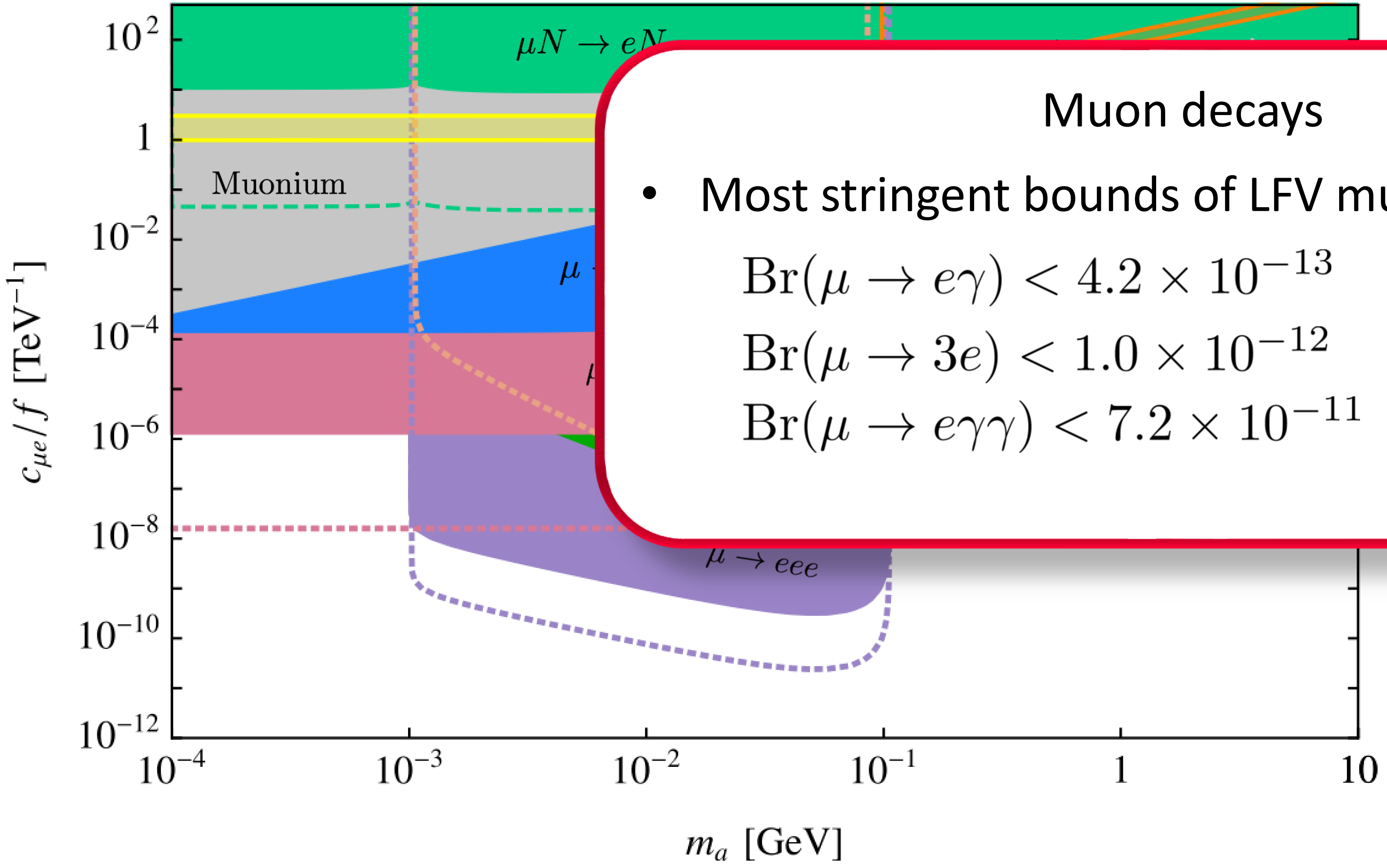
- Strongest bounds for $m_a > 2m_e$



- Strongest limit achieved with on-shell ALP



Constraints on LFV ALP-e-μ Coupling



Muon decays

- Most stringent bounds of LFV muon decays:

$$\mathrm{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}$$ [MEG Coll. (2016)]

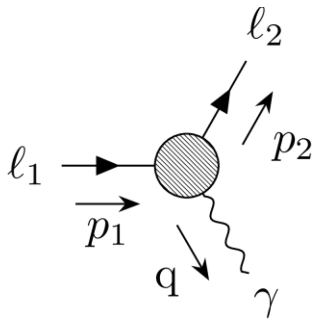
$$\mathrm{Br}(\mu \to 3e) < 1.0 \times 10^{-12}$$ [SINDRUM Coll. (1988)]

$$\mathrm{Br}(\mu \to e\gamma\gamma) < 7.2 \times 10^{-11}$$ [LAMPF Coll (1986)]

Details on Muon decays

- For μ to eγ use form–factor decomposition:

$$\Gamma^\mu(p_1, p_2) =$$



$$= \bar{\ell}_2(p_2)[F_1(q^2)\gamma^\mu + F_2(q^2)(p_1 + p_2)^\mu + F_3(q^2)q^\mu$$
$$+ F_1^5(q^2)\gamma^\mu\gamma_5 + F_2^5(q^2)(p_1 + p_2)^\mu\gamma_5 + F_3^5(q^2)q^\mu\gamma_5]\ell_1(p_1)$$

- By using Ward identity can get rid of F_1 and F_1^5

Details on Muon decays

- We find $\Gamma(\mu \to e\gamma) = \frac{m_\mu^3}{8\pi}\left(|F_2(q^2=0)|^2 + |F_2^5(q^2=0)|^2\right)$
$$= \frac{\alpha m_\mu^5 |c_{e\mu}|^2}{4096\pi^4 f^4}\left| c_{\mu\mu} g_1(x) + \frac{\alpha}{\pi} c_{\gamma\gamma}^{\text{eff}} g_2(x)\right|^2$$
with $x = m_a^2/m_\mu^2$

- For simplicity set m_e=0

- Calculate form-factors at arbitrary q^2 because diagrams appear as sub-diagrams in µ to 3e

Details on Muon decays

- For $2m_e < m_a < m_\mu$ can have subsequent $\mu \to ea,\ a \to ee$ decay
- $\mathrm{Br}(\mu \to 3e) \approx \mathrm{Br}(\mu \to ea) \times \mathrm{Br}(a \to ee)$

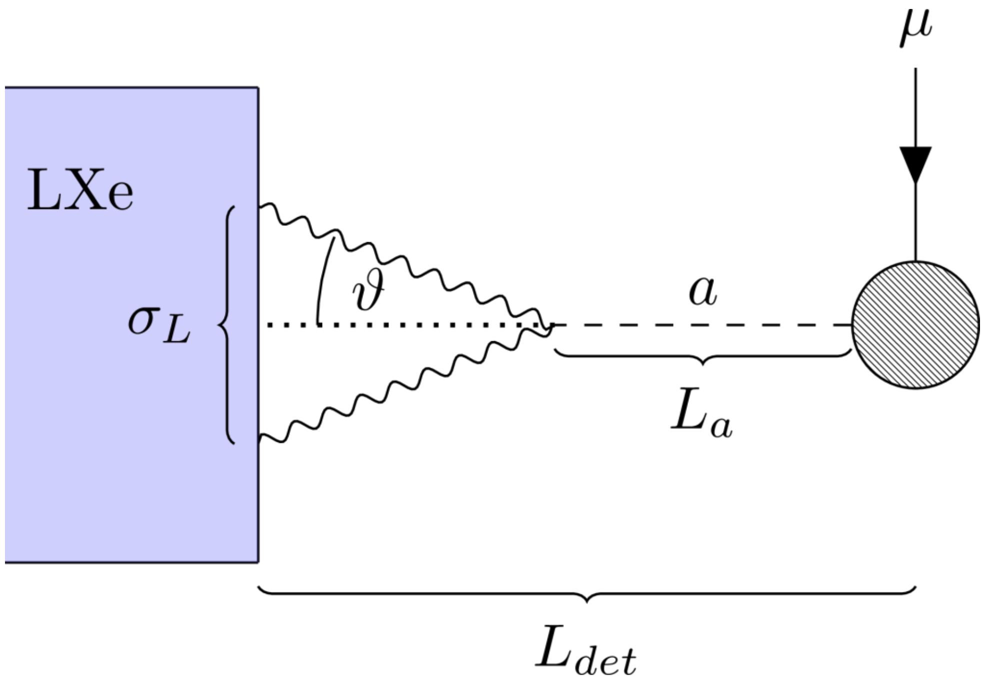
➢ Many orders of magnitude more sensitive to LFV couplings than $\mu \to e\gamma$

➢ Overcomes phase-space suppression of 3-body decay

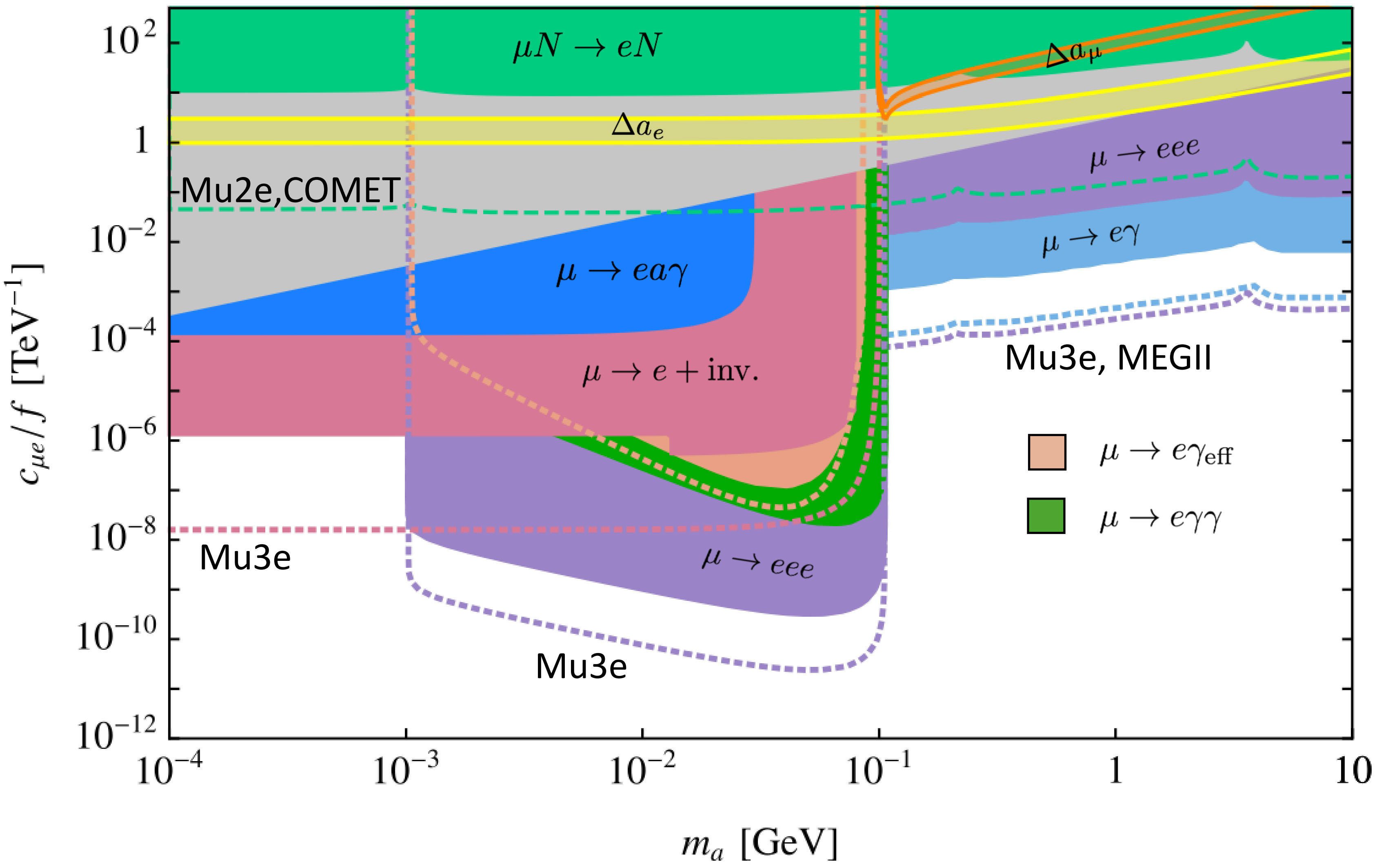
- Use same technique for $\mu \to e\gamma\gamma$
- Without strong time cuts the search for $\mu \to e\gamma\gamma$ would be sensitive to lighter ALPs

Details on Muon Decays

- If ALP is boosted and decay photons hit the detector closer than its spatial resolution, can mimic $\mu \to e\gamma$, $\mathrm{Br}(\mu \to e\gamma^{\mathrm{eff}}) = \mathrm{Br}(\mu \to e\gamma\gamma) \times \mathfrak{f}^{\mathrm{decay\ length}} \times \mathfrak{f}^{\mathrm{mimic}}$



Constraints on LFV ALP-e-μ Coupling



Anomalous magnetic Moments

- Current tension of experiment and theory prediction of 3.7σ (a_μ) and 2.4σ (a_e)

[Bennet *et al* (2006), Kesharvarzi *et al* (2018), Davier *et al* (2020)]
[Hanneke, Fogwell, Gabrielse (2008) and (2011)]

- Similar diagrams as $\mu \to e\gamma$

- a_μ receives contribution from flavor-preserving couplings:

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{theo}} = \frac{2m_\mu}{e} F_2(0) = -\frac{m_\mu^2 c_{\mu\mu}^2}{16\pi^2 f^2}\left[h_1(x_\mu) + \frac{2\alpha}{\pi}\frac{c_{\gamma\gamma}}{c_{\mu\mu}}\left(\log\frac{\mu^2}{m_\mu^2} - h_2(x_\mu)\right)\right]$$

- And flavor-violating ones:

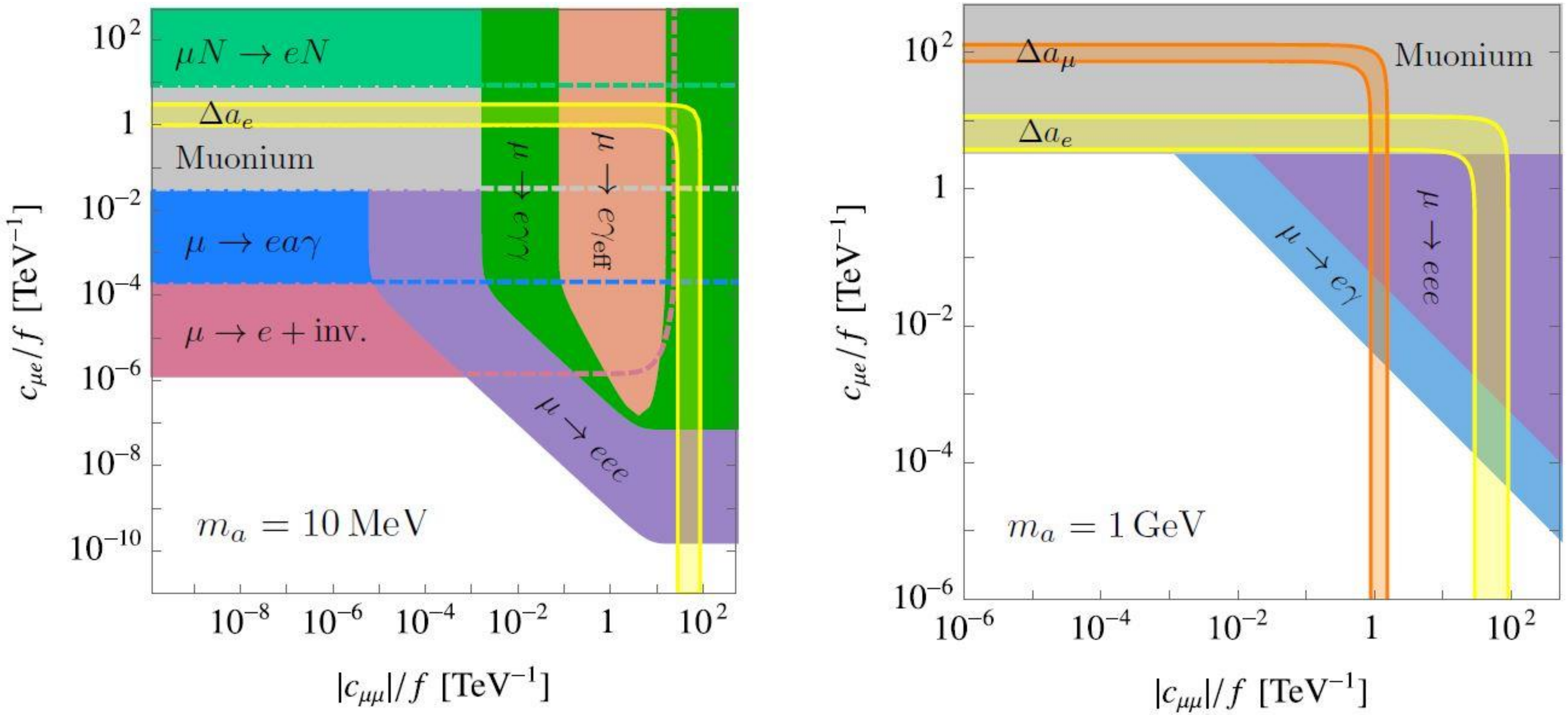
[Bauer, Neubert, Thamm (2017), Chang *et al* (2001), Marciano *et al* (2016)]

$$\Delta a_\mu = \frac{m_\mu m_\tau}{16\pi^2 f^2}\,\mathrm{Re}\big[(k_e)_{23}(k_E)_{23}^*\big]\,h(x_\tau) + \mathcal{O}\Big(\frac{m_\mu}{m_\tau}\Big)$$
$$- \frac{m_\mu^2}{32\pi^2 f^2}\Big(\big|(k_E)_{12}\big|^2 + \big|(k_e)_{12}\big|^2\Big)\,j(x_\mu) + \mathcal{O}\Big(\frac{m_e}{m_\mu}\Big)$$

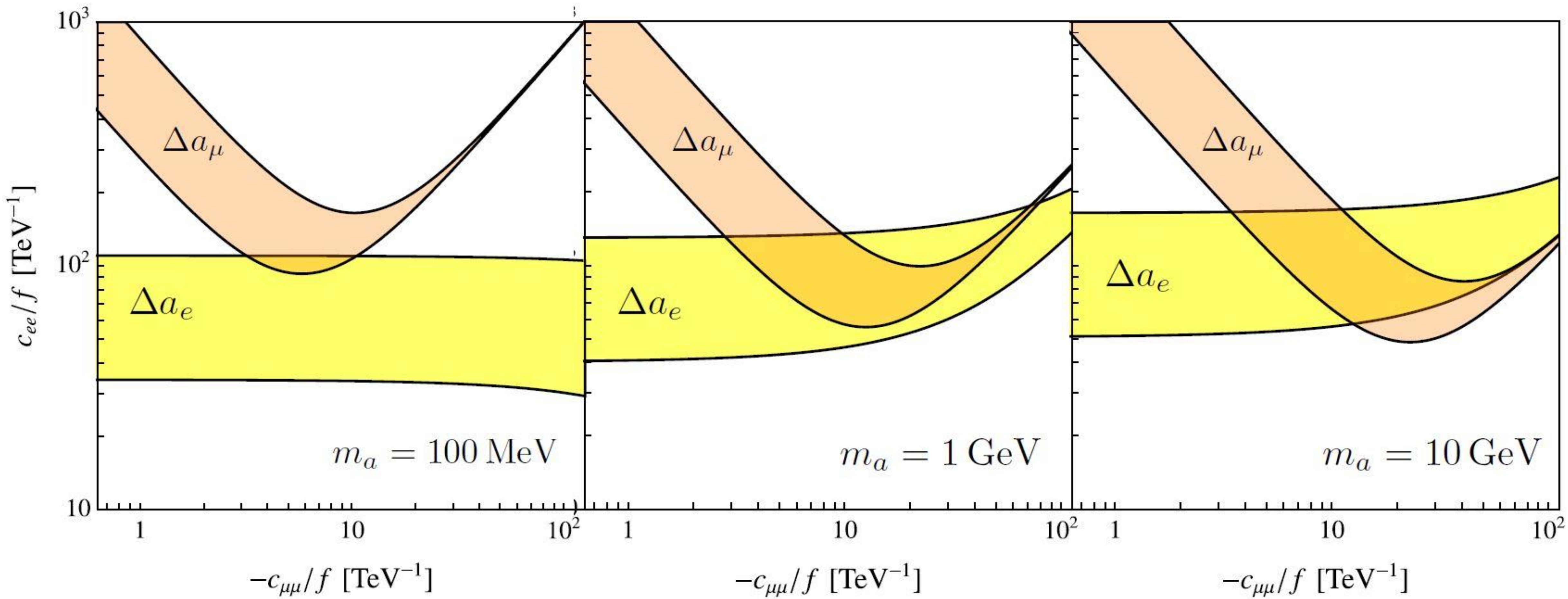
Anomalous magnetic Moments

- Explanation of both Δa_μ and Δa_e based on flavor-violating couplings to e and μ is ruled out by muonium oscillations for all masses m_a [Endo, Iguro, Kitahara (2020)]
- Explanation of both Δa_μ or Δa_e based on τ-couplings is ruled out by $\mu \to e\gamma$
- Both anomalies can be explained simultaneously if
  - $c_{\gamma\gamma}$ is present at tree level and $-c_{\gamma\gamma}^{\text{eff}}/c_{\mu\mu} \sim 10 - 30$
  - Non-universal lepton couplings $-c_{ee}/c_{\mu\mu} \sim 10 - 30$ and $m_a > 2m_e$

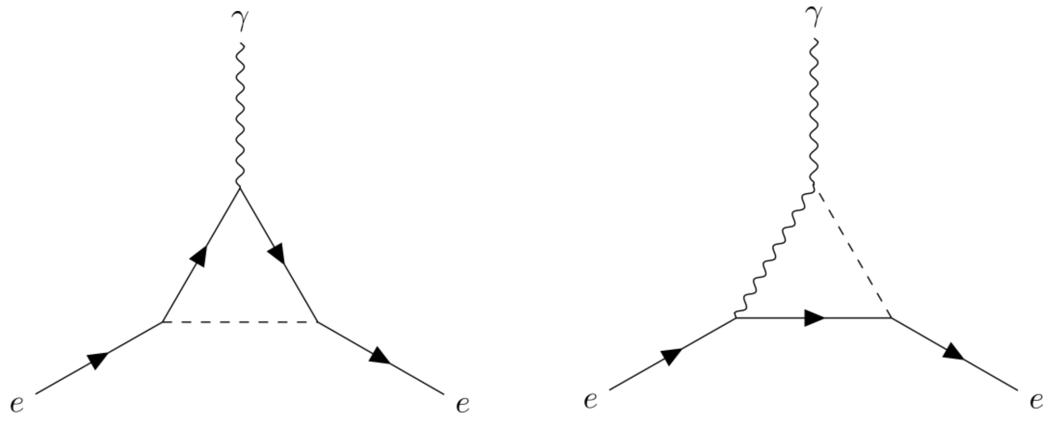
Anomalous magnetic Moments



Anomalous magnetic Moments



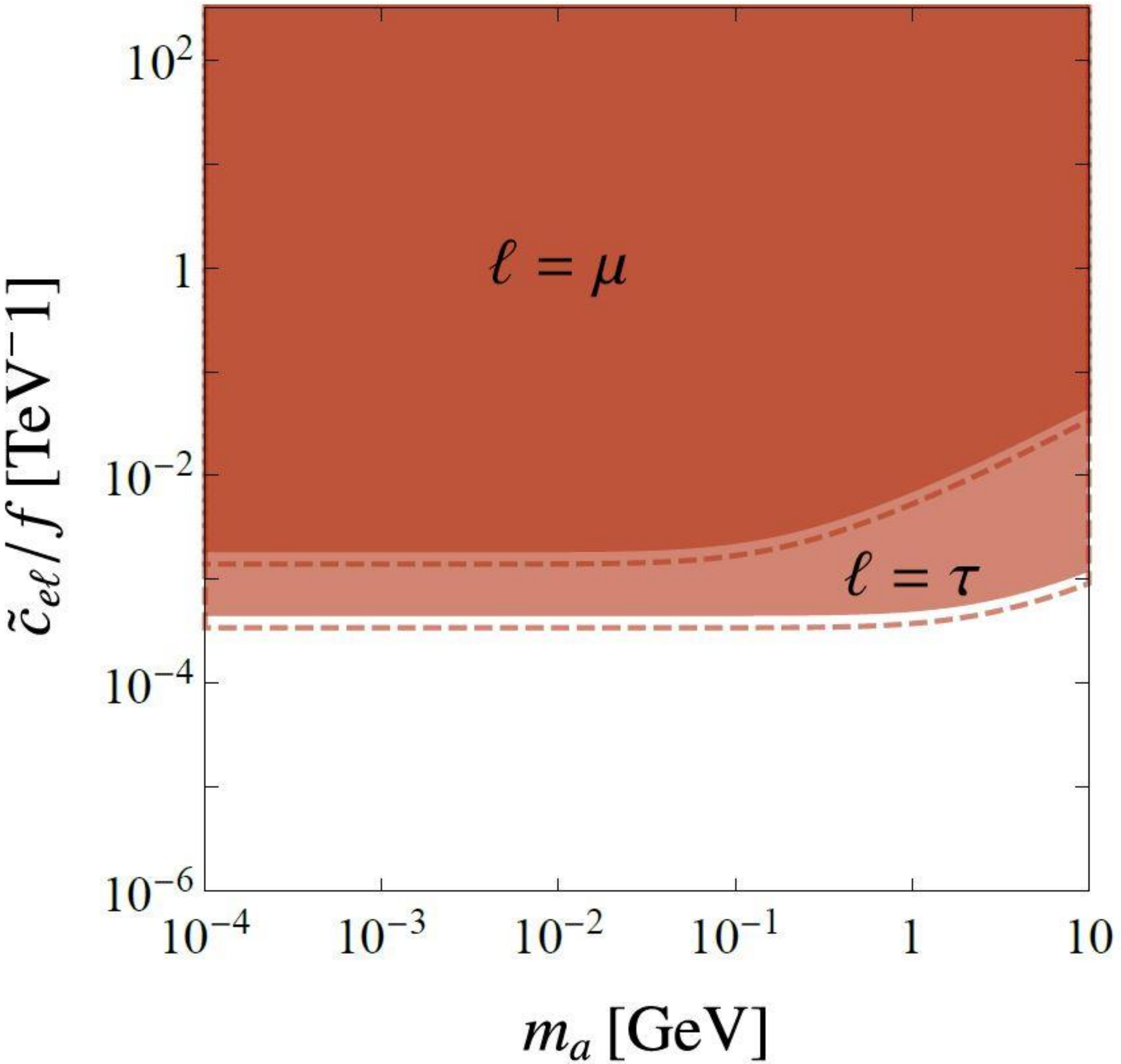
Anomalous electric Moments



- $|d_e| = \frac{|F_2^5(q^2{=}0)|}{2}$
- Low SM background, $|d_e^{\mathrm{SM}}| \sim \mathcal{O}(10^{-37}\,\mathrm{ecm}) \leftrightarrow |d_e^{\mathrm{exp}}| < 1.1 \times 10^{-29}\,\mathrm{ecm}$

  [Bernreuther, Suzuki (1991), Booth (1993), ACME Collaboration (2018)]
- SM contribution arises at 4-loop order
- Only constrains imaginary part $\mathrm{Im}\,((k_E)^*_{e\ell}(k_e)_{e\ell})$

Anomalous electric Moments

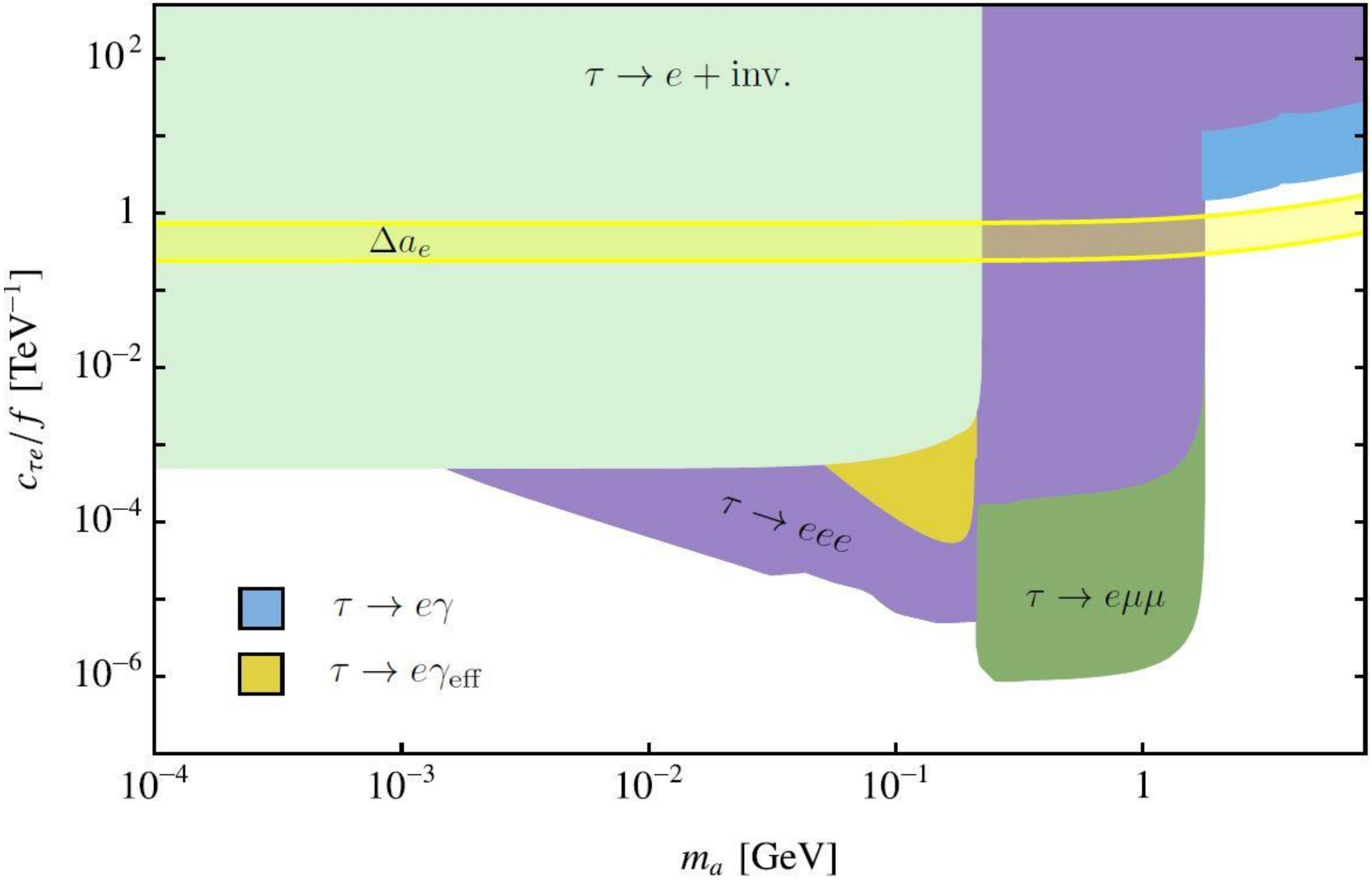


Constraints in the Tau Sector

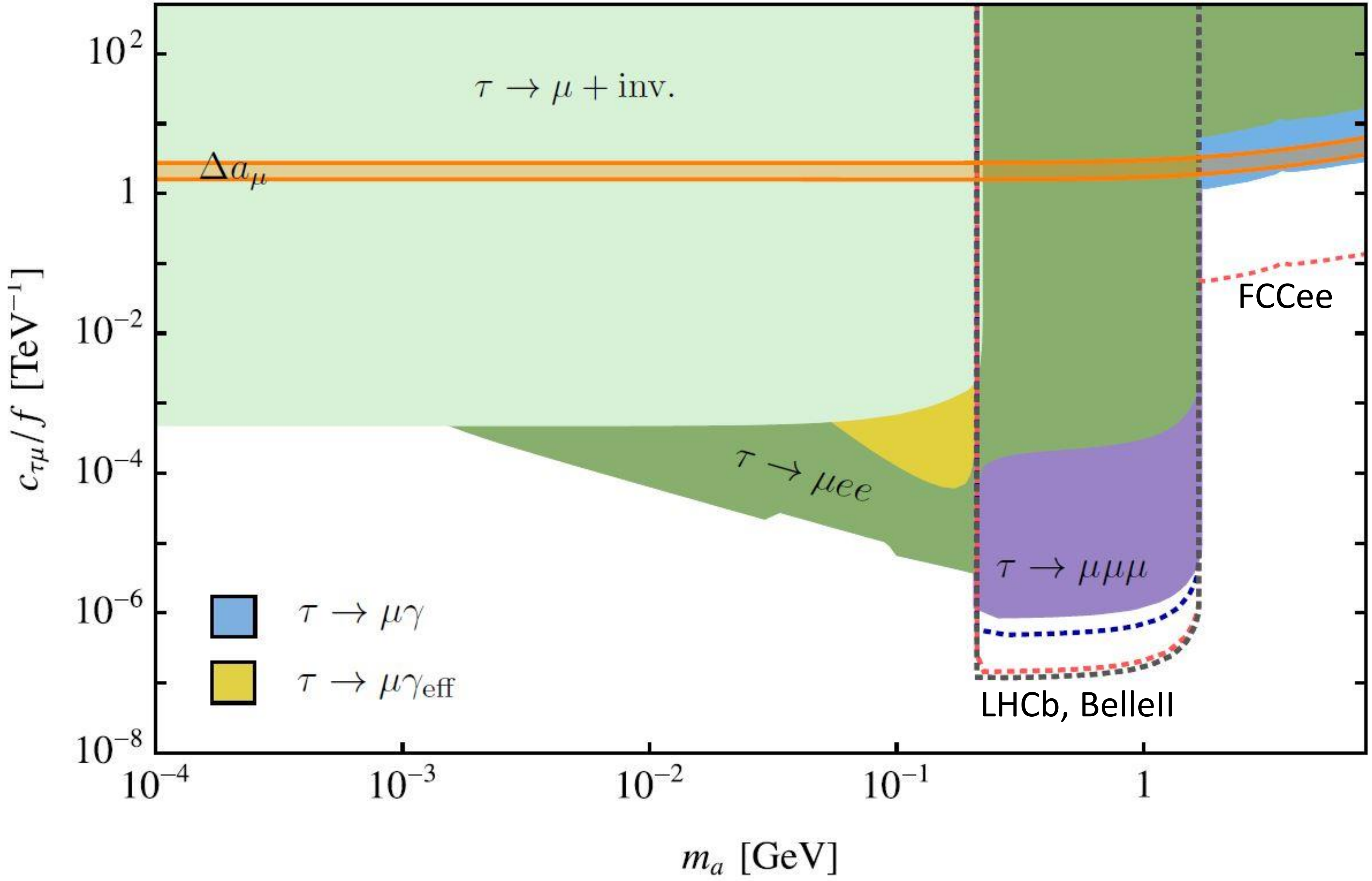
Overview over Branching Ratios and Projections

LFV Channel	Current limit
$\tau \to ea$	6.6×10^{-4} [ARGUS]
$\tau \to e\gamma$	1.3×10^{-6} [BaBar]
$\tau \to 3e$	1.1×10^{-6} [BaBar]
$\tau \to \mu a$	4.9×10^{-4} [ARGUS]
$\tau \to \mu\gamma$	1.5×10^{-4} [BaBar]
$\tau^- \to \mu^- e^+ e^-$	9.3×10^{-7} [BaBar]
$\tau \to 3\mu$	1.0×10^{-6} [BaBar]

Constraints on LFV ALP-τ-e Coupling



Constraints on LFV ALP-τ-μ Coupling



Summary and Conclusion

- Studied lepton-flavor violating ALP couplings and their constraints from decay and non-decay experiments
- Transferred the results from muon to tau sector
- Largest constraints arise in mass range $2m_e < m_a < m_\mu$ due to resonant ALP decays
- We have shown that searches for LFV transitions provide highly complementary constraints on ALP couplings to photons and leptons, strengthening the case for a broad program of experiments hunting LFV decays.

Summary and Conclusion

- W… lepton-flavor violating ALP couplings and their constraintss from decay and non…
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- We have shown, that searches for LFV transitions pro… ntary constraints on ALP couplings to photons and leptons, strengthening … d program of experiments hunting LFV decays.

Thank you for your attention!

Backup Slides

Technical Details

- Partial decay rate of μ to 3e: $\mathrm{d}\Gamma = \frac{1}{(2\pi)^3}\frac{1}{32m_\mu^3}|\overline{\mathcal{M}}|^2\mathrm{d}s_{12}\mathrm{d}s_{23}$

$$
\begin{aligned}
|\overline{\mathcal{M}}|^2 = & |c_{e\mu}|^2\,|c_{ee}|^2\,\frac{m_e^2 m_\mu^2}{f^4}\left\{\frac{2s_{23}\,(s_{12}+s_{13})}{|s_{23}-m_a^2+im_a\Gamma_a|^2} - \frac{s_{13}s_{23}}{\mathrm{Re}[(s_{23}-m_a^2+im_a\Gamma_a)(s_{13}-m_a^2-im_a\Gamma_a)]}\right\} \\
& + 4e^2\Bigg[2\,(s_{12}+s_{13})\;\mathrm{Re}\left[F_2^*(s_{23})F_3(s_{23}) + F_2^{5*}(s_{23})F_3^5(s_{23})\right] \\
& + \frac{1}{s_{23}}(m_\mu^2\,(s_{12}+s_{13}) - 2s_{12}s_{13})(|F_2(s_{23})|^2 + |F_2^5(s_{23})|^2) \\
& + \frac{1}{m_\mu^2}(s_{23}\,(s_{12}+s_{13}) + 2s_{12}s_{13})(|F_3(s_{23})|^2 + |F_3^5(s_{23})|^2) \\
& + s_{12}\Big(F_2^*(s_{23})F_2(s_{13}) + F_2^{5\,*}(s_{23})F_2^5(s_{13}) + F_2^*(s_{23})F_3(s_{13}) \\
& + F_2^{5\,*}(s_{23})F_3(s_{13}) + F_2^5(s_{13})F_3^{5\,*}(s_{23}) + F_2(s_{13})F_3^*(s_{23})\Big) \\
& + \frac{s_{12}\,(s_{13}+s_{23})}{m_\mu^2}\;\left(F_3(s_{23})F_3^*(s_{13}) + F_3^5(s_{23})F_3^{5\,*}(s_{13})\right)\Bigg] \\
& + \frac{2es_{23}m_e}{f^2}c_{ee}\,\mathrm{Re}\Bigg[\frac{(k_e)_{21}+(k_E)_{21}}{s_{23}-m_a^2-im_a\Gamma_a}(m_\mu^2F_2^5(s_{13}) + (s_{12}+s_{13})F_3^5(s_{13})) \\
& + \frac{(k_e)_{21}-(k_E)_{21}}{s_{23}-m_a^2-im_a\Gamma_a}(m_\mu^2F_2(s_{13}) + (s_{12}+s_{13})F_3(s_{13}))\Bigg] + (1\leftrightarrow 2)
\end{aligned}
$$

$$s_{ij} = (p_i+p_j)^2$$

p_1 and p_2 electron momenta
p_3 positron momentum

Analytic Expressions of the Form Factors

- The $\ell_1 \to \ell_2\gamma$ form factors via ALP read (for arbitrary q^2):

$$F_2(q^2) = -\frac{m_i e Q_i}{16\pi^2 f^2}\Big((k_F)_{ij} - (k_f)_{ij}\Big)\left(\frac{\alpha}{4\pi}c_{\gamma\gamma}^{\text{eff}} g_2(q^2, m_i, m_a) + \frac{1}{4}c_{ii} g_1(q^2, m_i, m_a)\right)$$

$$F_2^5(q^2) = -\frac{m_i e Q_i}{16\pi^2 f^2}\Big((k_F)_{ij} + (k_f)_{ij}\Big)\left(\frac{\alpha}{4\pi}c_{\gamma\gamma}^{\text{eff}} g_2(q^2, m_i, m_a) + \frac{1}{4}c_{ii} g_1(q^2, m_i, m_a)\right)$$

$$F_3(q^2) = -\frac{m_i e Q_i}{16\pi^2 f^2}\Big((k_F)_{ij} - (k_f)_{ij}\Big)\left(\frac{\alpha}{4\pi}c_{\gamma\gamma}^{\text{eff}} h_2(q^2, m_i, m_a) + \frac{1}{4}c_{ii} h_1(q^2, m_i, m_a)\right)$$

$$F_3^5(q^2) = -\frac{m_i e Q_i}{16\pi^2 f^2}\Big((k_F)_{ij} + (k_f)_{ij}\Big)\left(\frac{\alpha}{4\pi}c_{\gamma\gamma}^{\text{eff}} h_2(q^2, m_i, m_a) + \frac{1}{4}c_{ii} h_1(q^2, m_i, m_a)\right)$$

Analytic Expressions of the Form Factors

- With the loop functions:

$$g_1(q^2, m_i, m_a) = \int_0^1 \mathrm{d}x \left[\frac{1-x^2}{1-r_a+x(1-r_q)} + \frac{r_a + x^3 r_q}{(1-r_a+x(1-r_q))^2} \ln\left(\frac{(1-x)r_a + x^2}{1-x(1-x)r_q} \right) \right]$$

$$g_2(q^2, m_i, m_a) = -\left(I_1(r_a, r_q) + I_2(r_a, r_q) - 2\delta_2 - 2\ln\left(\frac{\mu^2}{m_\mu^2}\right) \right)$$

$$h_1(q^2, m_i, m_a) = \int_0^1 \mathrm{d}x \left[\frac{1-x^2}{1-r_a+x(1-r_q)} + \frac{r_a + 2x^2(1-r_a) + x^3(2-r_q)}{(1-r_a+x(1-r_q))^2} \ln\left(\frac{(1-x)r_a + x^2}{1-x(1-x)r_q} \right) \right]$$

$$h_2(q^2, m_i, m_a) = -I_1(r_a, r_q)$$

Analytic Expressions of the Form Factors

- With the functions:

$$
\begin{aligned}
I_1(r_a, r_q) =& \frac{2+2r_a-5r_q+r_a r_q}{2(1-r_q)^2} + r_a\left[\frac{8+r_a}{2(1-r_q)^2} - \frac{3(1-r_a)}{(1-r_q)^3} - \frac{2-r_a+r_a^2}{2(1-r_a)(1-r_q)}\right]\ln(r_a) \\
&+ \frac{(1-r_a)(2r_a-r_q-r_a r_q)}{2(1-r_q)^2}\ln(r_a-1) - \frac{3(r_a-r_q)^2}{(1-r_q)^3}\ln(r_a-r_q) \\
&+ \frac{(r_a-r_q)^2(2+r_q)}{(1-r_q)^4}\left[\mathrm{Li}_2\left(1-\frac{1}{r_a}\right) - \mathrm{Li}_2\left(1-\frac{r_q}{r_a}\right) - \ln(r_q)\ln\left(1-\frac{r_q}{r_a}\right)\right] \\
I_2(r_a, r_q) =& -\frac{5+r_a-6r_q}{1-r_q} - \frac{r_a(3r_a-2r_q-r_a r_q)}{(1-r_q)^2}\ln(r_a) \\
&+ \frac{(1-r_a)^2}{1-r_q}\ln(r_a-1) + \frac{2(r_a-r_q)^2}{(1-r_q)^2}\ln(r_a-r_q) \\
&- \frac{2(r_a-r_q)^2}{(1-r_q)^3}\left[\mathrm{Li}_2\left(1-\frac{1}{r_a}\right) - \mathrm{Li}_2\left(1-\frac{r_q}{r_a}\right) - \ln(r_q)\ln\left(1-\frac{r_q}{r_a}\right)\right]
\end{aligned}
$$

Form Factors used in Δa_μ

- Used loop functions are

$$h_{1,2}(x_\mu) = h_{1,2}(0, m_\mu, m_a)$$
$$h(x) = \frac{2x^2}{(x-1)^3}\log x - \frac{3x-1}{(x-1)^2}$$
$$j(x) = 1 + 2x - 2x^2 \log \frac{x}{x-1}$$

- Where $h_{1,2}$ are the same as in the decay form factors

Bounds from Muonium Oscillations

- Dependence on magnetic field encoded in parameter $\delta_B = (1 + X^2)^{-\frac{1}{2}}$

  where $X = \frac{\mu_B B}{a}\left(g_e + \frac{me}{m_\mu} g_\mu\right) \approx 6.24 \frac{B}{\mathrm{Tesla}}$

  and the Muonium 1S hyperfine splitting $a \approx 1.864 \times 10^{-5}\,\mathrm{eV}$