S. Brodsky
SLAC

$\boxed{\text{Conformal Aspects of QCD}}$

* AdS/CFT : duality $\quad AdS_5 \times S^5 \Leftrightarrow$ SQCD ($N=4$)
  SO(4,2)

* $\lim_{\beta \to 0,\, m_q \to 0}$ PQCD $\Rightarrow$ Conformal QCD (fixed α_s) Parisi

* "Conformal Template"

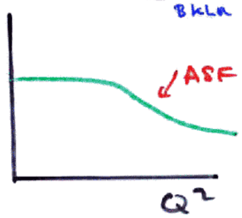
$$\mathcal{O} = \sum_{n=0} C_n \, \alpha_s^n (Q_n^*)$$

conformal coeff. $\quad$ absorbs $\beta \neq 0$

* Conformal Expansions OPE, Dist. Ampl. BLFS, BL
* Commensurate Scale relations BL
* Generalized Crewther Relation BKLR

* Evidence for IR fixed point

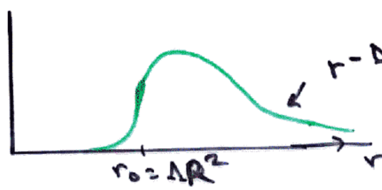
Dyson-Schwinger Eqns
Lattice
Effective charges: $R_{e^+e^-}(s) \to \alpha_R(s)$, $\alpha_\tau(m_\tau^2)$



* Use AdS/CFT + (bnd. condit) to derive conformal predictions

$$z = \frac{R^2}{r} : \quad \left[z^2 \frac{d^2}{dz^2} - (d-1)\, z \frac{d}{dz} - (\mu R)^2 + z^2 M^2 \right] \psi(z) = 0$$

$d=4$ $\quad$ $\ell(\ell+d)$ $\quad$ eigenmodes $\Rightarrow$ spectra



Polchinski Strassler
de Teramond SJB

$F_H(Q^2) \sim \left(\frac{1}{Q^2}\right)^{n_H - 1}$

$(1-x)^{2n_s - 1}$

dimensional counting

P. Rossi
CLAS
hep-ph/0405207

Scaling in deuteron photodisintegration

$\frac{d\sigma}{dt}(\gamma d \to n p)$

5

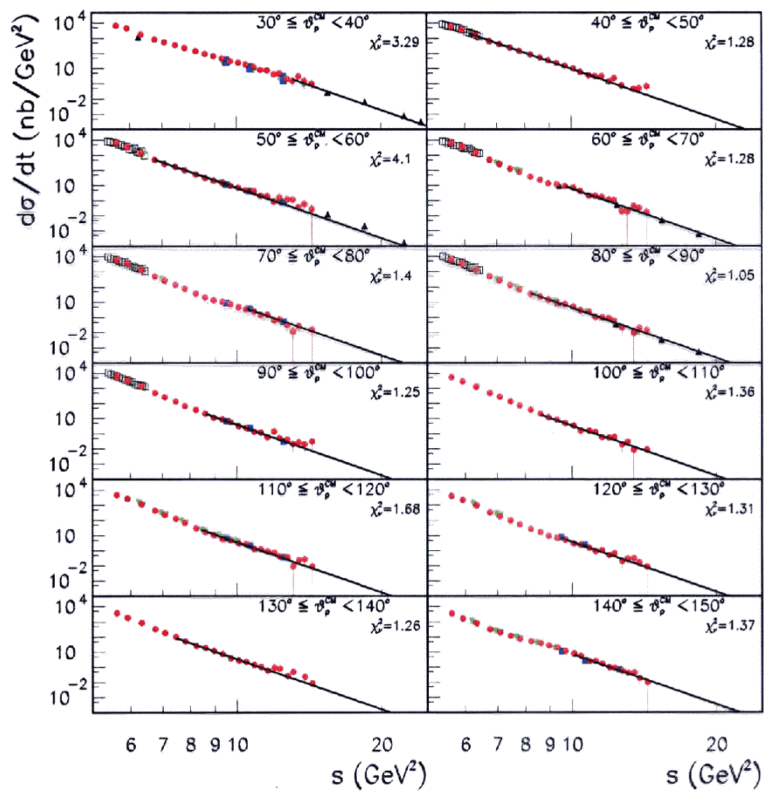
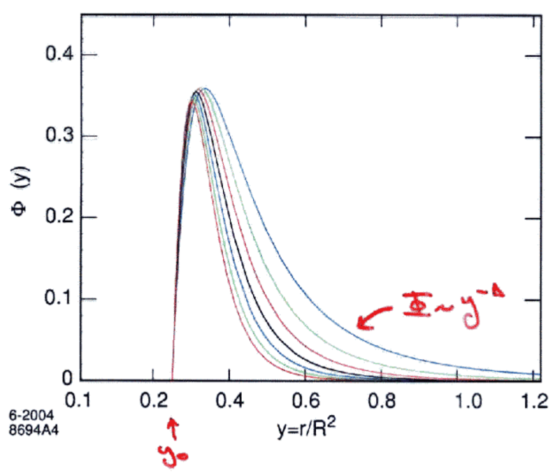


FIG. 2: Fits of the cross sections $d\sigma/dt$ to s^{-11} for $P_T \geq P_T^{th}$ and proton angles between 30° and 150° (solid lines). Data are from CLAS [12] (full/red circles), Mainz [15] (open/black squares), SLAC [5, 6, 7] (full-down/green triangles), JLab Hall A [11] (full/blue squares) and Hall C [8, 9] (full-up/black triangles). Also shown in each panel is the χ^2_ν value of the fit.

$$\boxed{\frac{d\sigma}{dt} = \frac{1}{s^{11}} F(\theta_{cm})}$$

Normalized Modes in the Bulk

Exact Solution $L = 0, 1, 2, \cdots$



Boundary cond.t $\Rightarrow$ $\Phi(r_0) = 0$ $\qquad r_0 = \Lambda_{QCD} R^2$

$y_0 = \Lambda_{QCD}$

$\Phi(r) \sim r^{-\Delta}$ at large r

Can we relate hadron wavefunctions in 3+1 to the $AdS_5 \otimes S^5$ solutions ?

Use light-front Fock expansion at fixed $\tau = t + z/c$ $\qquad H_{LF}|\Psi_h\rangle = m_h^2 |\Psi_h\rangle$

$x_i = k_i^+/P^+$

n-1 integrations

$$|\Psi_h(P^+, \vec{P}_\perp)\rangle = \sum_{n,\lambda_i} \int [dx_i\, d^2k_{\perp i}]\, \psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda)$$

$$|n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda\rangle$$

$$\sum_{i=1}^{n} x_i = 1 \;,\quad \sum_i \vec{k}_{\perp i} = 0 \;,\quad \lambda_i = S_z^i$$

Introduce UV regulator: $\quad k_{\perp i}^2 < \Lambda^2 = Q^2$

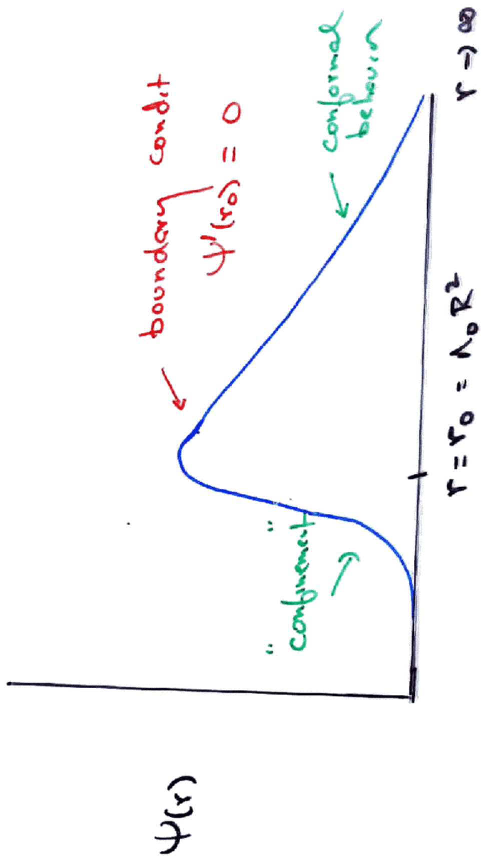
$$|\Psi_h(Q)\rangle \;,\quad |\Psi_{n/h}(Q)\rangle$$

$$\Psi_{n/h}(Q) \sim \int^{Q} [d^2k_\perp]^{n-1} [a^+(k_\perp)]^n\, \psi_n(\vec{k}_\perp)$$

Identify Q-dep. of $\Psi_{n/h}(Q)$

with $\Psi(Q) \sim Q^{-\Delta_n}$

from AdS_5 with $m^2 \Rightarrow m_n^2 = \sum_{i=1}^{n} \frac{k_\perp^2 + m^2}{x}$

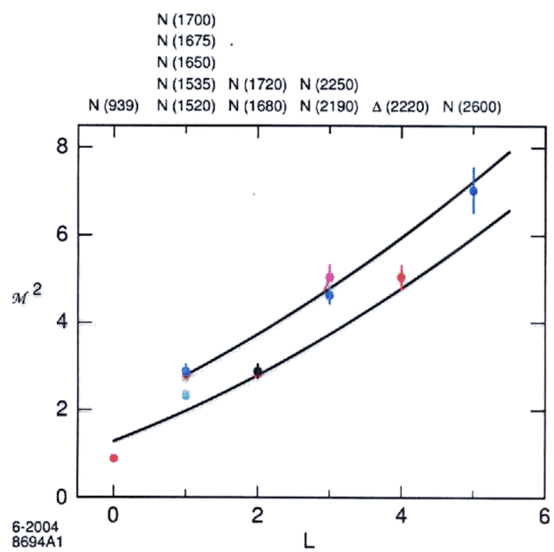


Produces eigenvalues $M^2 = M^2(J)$

Similar to Chew-Frautschi $M^2 \propto J$

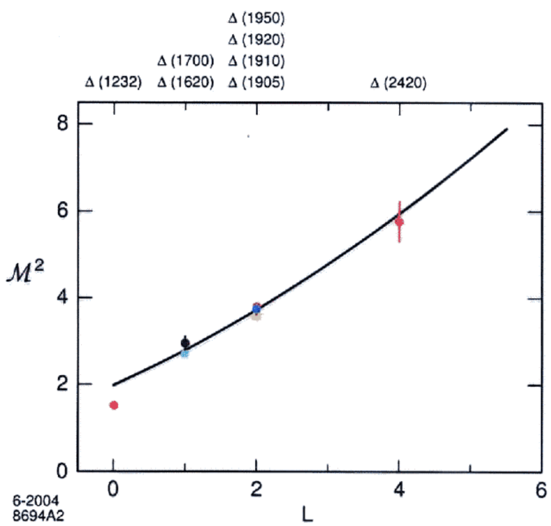
One Parameter Λ_0

N Spectrum ($\Lambda_o = 0.22$ GeV)



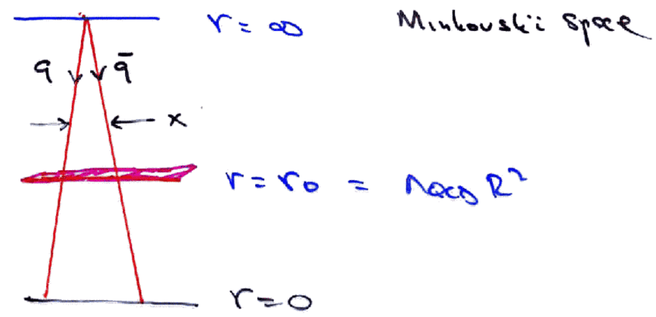
The **56** $S = \frac{1}{2}$ irrep for the $J = L+S$ family: $N\frac{1}{2}^+(939)$, $N\frac{5}{2}^+(1680)$ and $N\frac{9}{2}^+(2220)$ states (red) and the $J = L-S$ state: the $N\frac{3}{2}^+(1720)$ (black). The **70**, $S = \frac{1}{2}$ irrep for the $J = L+S$ and $J = L-S$ families are respectively by the $N\frac{3}{2}^-(1520)$ (green) and the $N\frac{1}{2}^-(1535)$ (aqua). The **70**, $S = \frac{3}{2}$ irrep for the $J = L+S$, $J = L+S-1$ and $J = |L-S|$ families are the $N\frac{5}{2}^-(1675)$ and $N\frac{9}{2}^-(2250)$ states (mauve), the $N\frac{3}{2}^-(1700)$, $N\frac{7}{2}^-(2190)$ and $N\frac{11}{2}^-(2600)$ states (blue) and the $N\frac{1}{2}^-(1650)$ (yellow) respectivley.

Δ Spectrum ($\Lambda_{o} = 0.22$ GeV).

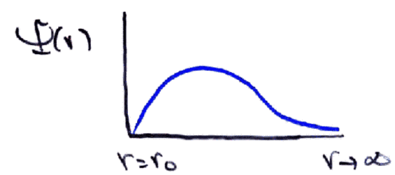


The **56** $S = \frac{3}{2}$ irrep for the $J = L + S$ family consists of the $\Delta\frac{3}{2}^{+}(1232)$, $\Delta\frac{7}{2}^{+}(1950)$ and $\Delta\frac{11}{2}^{+}(2420)$ states (red); the $J = L + S - 1$ state $\Delta\frac{5}{2}^{+}(1905)$ (green); the $J = L + S - 2$ state $\Delta\frac{3}{2}^{+}(1920)$ (blue) and the $J = |L - S|$ state $\Delta\frac{1}{2}^{+}(1910)$ (yellow). The **70** $S = \frac{1}{2}$ irrep for the $J = L + S$ and $J = L - S$ families is constituted respectively by the $\Delta\frac{3}{2}^{-}(1700)$ (black) and the $\Delta\frac{1}{2}^{-}(1620)$ (aqua).

Gravitational analog



Falling objects in AdS Fifth dimension



Light Unflavored Baryons under SU(6)

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56} \oplus \mathbf{70} \oplus \mathbf{70} \oplus \mathbf{20}$$

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

Table: $SU(6)$ multiplet structure for N and Δ unflavored baryon resoances. Only confirmed states corresponding to 3 and 4 stars in PDG are included: K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66**, 010001 (2002).

- It is remakable that confirmed states correspond to L even for the $SU(6)$ **56** representation and to L odd for the **70**. Radial excitations are not listed.

Form of ψ_{LF} from QCD + Conf. Sym.

(anom dim ignored)

$$\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i, l_{z_i})$$

GJT
S13

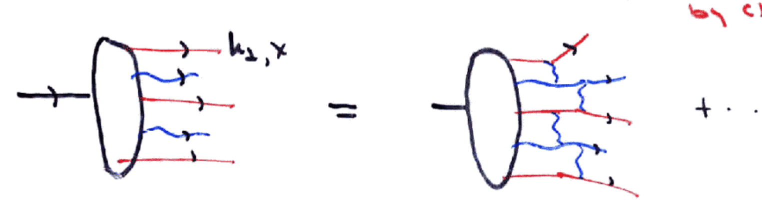
$$\sim \frac{(g_s N_c)^{\frac{1}{2}(n-1)}}{\sqrt{N_c}} \prod_{i=1}^{n-1} (k_{\perp i}^{\pm})^{|l_{z_i}|}$$

$k_\perp^\pm = k_x \pm i k_y$

$g_s = g_{YM}^2$

$$\times \left[\frac{\Lambda_0}{\mathcal{M}_h^2 - \sum_{i=1}^{n} \frac{(k_\perp^2 + m^2)_i}{x_i} + \Lambda_0^2} \right]^{n+|l_z|-1}$$

large $k_\perp$ dot by CF



AdS:

$(g_s N_c)^{\frac{1}{2}(n-1)}$ instead of $(g_{YM}^2 N_c)^{n-1}$

S. J. Rey, J. T. Lee (2001)
J. Maldacena (1998)

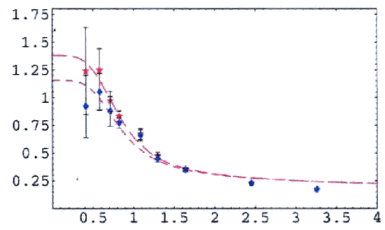
Light-Front hadron dynamics and AdS/CFT correspondence

G. F. de Téramond + SJB

Phys. Lett. B 582 (2004) 211

Effective coupling Landau gauge

$\alpha_s(q^2)$



agrees with DSE - - - and α_R

FIG. 5: The running coupling $\alpha_s(q)$ as a function of momentum q(GeV) of the $\beta = 6.4$, 56^4 lattice using the ghost propagator of the l_A copy (star) and that of the average (diamond). The DSE approach with $\kappa = 0.24$(long dashed line and that with $\kappa = 0.20$(short dashed line) are also plotted.

DSE: Alkofer von Smekal

$\alpha_s(q^2)$

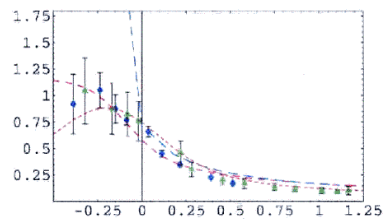
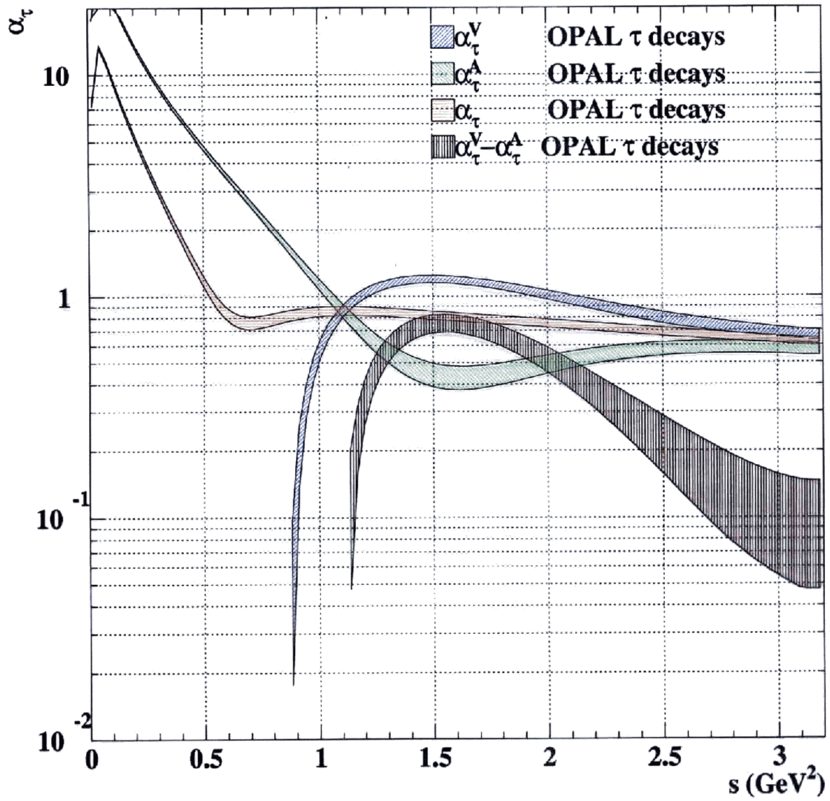


FIG. 6: The running coupling $\alpha_s(q)$ as a function of logarithm of momentum $\log_{10} q$(GeV) of the $\beta = 6.4$, 56^4(diamond) and 48^4(triangle) lattice using the ghost propagator of the sample average. The DSE approach with $\kappa = 0.20$ (dashed line), contour improved perturbation method using $e^{70/6\beta_0}\tilde\Lambda_{\overline{MS}}$ (dotted line) are also plotted. The infrared divergent dashed line is the result of $\widetilde{MOM}$ scheme.

* S. Furui + H. Nakajima hep-lat/0403021

N. Brown + M. Pennington

BMMR
hep-ph/0212078



$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0}\left\{\frac{1}{2} - \frac{1}{\pi}\tan^{-1}\left(\frac{1}{\pi}\ln\frac{Q^2}{\Lambda^2}\right)\right\}$$

$$\Rightarrow \frac{4\pi}{\beta_0} \quad \text{at} \quad Q^2 = 0$$

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln\frac{Q^2}{\Lambda^2}}\left\{1 - \frac{1}{3}\frac{\pi^2}{\ln^2 Q^2/\Lambda^2} + \frac{1}{5}\frac{\pi^4}{\ln^4\frac{Q^2}{\Lambda^2}} \cdots\right\}$$

Frozen coupling in IR

Obeys standard RGE.

Hadron Distribution Amplitudes

{ Lepage SJB

$$\phi_M(x,\tilde{Q}) = \int \frac{d^2k_\perp}{16\pi^3}\, \psi^{(\tilde{Q})}_{q\bar{q}/M}(x,\vec{k}_\perp)$$

gauge-invariant $\qquad \mathcal{M}^2 = \sum_i \left(\frac{k_\perp^2+m^2}{x}\right)_i < \tilde{Q}^2$

valence
n=2
LF WF
$(A^+=0)$

key input to hard exclusive amplitudes

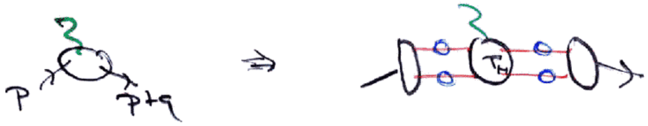
leading twist

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\, \phi_\pi(y,\tilde{Q})\, T_H(x,y,\tilde{Q})\, \phi_\pi(x,\tilde{Q})$$

Factorization Theorems, Hard Scott Expansion

Evolution Equations

$$\frac{\partial}{\partial \ln \tilde{Q}} \phi_M(x,\tilde{Q}) = \int V(x,y)\, \phi_M(y,Q)\, dy$$



{ Lepage, SJB
Efremov Radyushkin
Chernyak

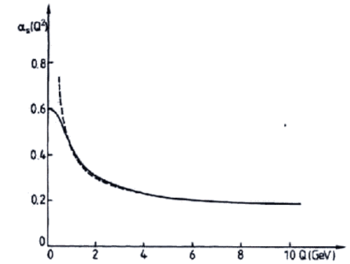


FIG. 2. The saturating $\alpha_s(Q^2)$ [Eq. (12), solid curve] compared to lowest-order QCD with $\Lambda=200$ MeV [Eq. (11), dashed curve]; to allow for thresholds we let N_f in (11) be the number of flavors with $4m_f^2<Q^2$ but demanded that α_s be continuous (Λ refers to the $N_f=2$ regime); the fitted function is $\alpha_s(Q^2)=0.25\exp(-Q^2)+0.15\exp(-Q^2/10)+0.20\exp(-Q^2/1000)$, with Q in GeV.

We have solved for mesons with the Hamiltonian (1) in three stages. In the first two of these stages we treat the Hamiltonian

$$\tilde{H}_1=(p^2+m_1^2)^{1/2}+(p^2+m_2^2)^{1/2}+\tilde{H}_{12}^{\text{conf}} +\tilde{H}_{12}^{\text{hyp}}+\tilde{H}_{12}^{\text{so}} \tag{14}$$

($\tilde{H}$ denotes an operator that has been modified by the relativistic effects described above and detailed in Appendix A) by directly diagonalizing in a large harmonic-oscillator sectors. This diagonalization is first performed in $|jm;ls\rangle$ sectors where $\mathbf{L}=\mathbf{r}\times\mathbf{p}$, $\mathbf{S}=\mathbf{S}_1+\mathbf{S}_2$, and $\mathbf{J}=\mathbf{L}+\mathbf{S}$. The off-diagonal effects of $\tilde{H}_{12}^{\text{tensor}}$ [the tensor part of (4) which can cause ${}^3L_J\leftrightarrow{}^3L'_J$ mixing] and of $\tilde{H}_{[12]}^{\text{so}}$ (the antisymmetric piece of the spin-orbit interaction which arises only if the quark masses are unequal, in which circumstance it can cause ${}^3L_J\leftrightarrow{}^1L_J$ mixing) are then treated perturbatively by diagonalizing the mass matrix in the basis of eigenvectors of the $|jm;ls\rangle$ sectors. At both stages the basis used is expanded until we find convergence.

For most states the solution of our Hamiltonian problem is complete at this point, but for self-conjugate isoscalar mesons we must also consider the effects[5] of H_A.

In mesons, single-gluon annihilation is forbidden by color conservation, but annihilation via multiple gluons is expected. For heavy quarks where the annihilation is controlled by a small α_s, this process will (at least in the absence of anomalies) be dominated by the minimum number of gluons allowed: two for even and three for odd charge-conjugation states. On general grounds we expect this effect to lead to a contribution to the mass matrix with diagonal entries of the form

$$A_{Q\bar{Q}\to Q\bar{Q}} \approx \alpha_s^{\,n} \frac{|\Psi_{Q\bar{Q}}(0)|^2}{M_Q^{\,2}}\,, \tag{15}$$

where $n=2$ or 3 as $C=+$ or $-$. Since $\Psi_{Q\bar{Q}}(0)\simeq 0$ if $L>0$, we may further expect this effect to be very small in heavy-quark systems unless $L=0$ [it will not be exactly zero both because the annihilation actually occurs over a region of size m_Q^{-1} and because there will be relativistic smearing of the quarks over a region of size m_Q^{-1}. Even in S waves, however, this effect should be quite small in the triplet states as can be seen by comparing to H^{hyp} and noting that $\alpha_s^3 \ll \alpha_s$ even for $c\bar{c}$. Thus in heavy quark systems we can anticipate that the only place where annihilation might be noticeable is in the states $n\,{}^1S_0$.

In light-quark systems we must, on the other hand, expect H_A to play a more important role. In the absence of a calculation of the annihilation amplitudes we must then treat H_A phenomenologically and consequently the predictive power of our model is reduced for such light-isoscalar mesons. This weakness is somewhat alleviated by two factors: (1) Even in the light mesons H_A is usually small, and there is considerable phenomenological evidence to reinforce one's expectation that it becomes weaker as a meson system becomes more excited. Thus in practice H_A can often simply be ignored. (2) All of the self-conjugate isoscalar mesons in a given ${}^{2S+1}L_J$ sector can, if it is necessary to consider annihilation at all, be described by the introduction of a single new annihilation parameter $A({}^{2S+1}L_J)$, and, with the exception of the pseudoscalar mesons which we will discuss extensively below, this description is insensitive to uncertainties in how the effects of H_A should be implemented.

With the exception of the pseudoscalar mesons, our prescription for gluon annihilation mixing is adapted directly from Eq. (15) above with relativistic modifications motivated by the observations of Appendix A: for the annihilation amplitude from $q_i\bar{q}_i\to q_j\bar{q}_j$ in the channel ${}^{2S+1}L_J$ we take

$$A({}^{2S+1}L_J)_{ji}=4\pi(2L+1)\left\{A({}^{2S+1}L_J)\left|\frac{\alpha_s(M_j^2)\alpha_s(M_i^2)}{\pi^2}\right|^{n/2}\right\}\frac{S_L(\Psi_i)S_L(\Psi_j)}{m_im_j}\,, \tag{16}$$

where $A({}^{2S+1}L_J)_{ji}$ depends on the unperturbed annihilation channel masses M_j and M_i, n is as above, and where $S_L(\Psi_i)$ is a smearing of the $q_i\bar{q}_i$ wave function at the origin:

$$S_L(\Psi_i)\equiv\frac{1}{(2\pi)^{3/2}}\int d^3p\,\frac{1}{\sqrt{4\pi}}\Phi_i(p)\left[\frac{p}{E_i}\right]^L\frac{m_i}{E_i}\,. \tag{17}$$

Here $\phi_i(\mathbf{p})=\Phi_i(p)Y_{LM}(\theta_p\phi_p)$ is the full normalized

$\alpha_R(Q=\sqrt{s})/\pi$

freeze? →

$\frac{\alpha_R}{\pi}$

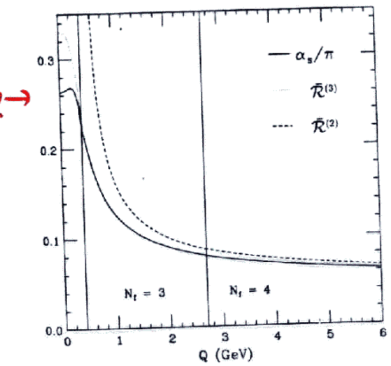
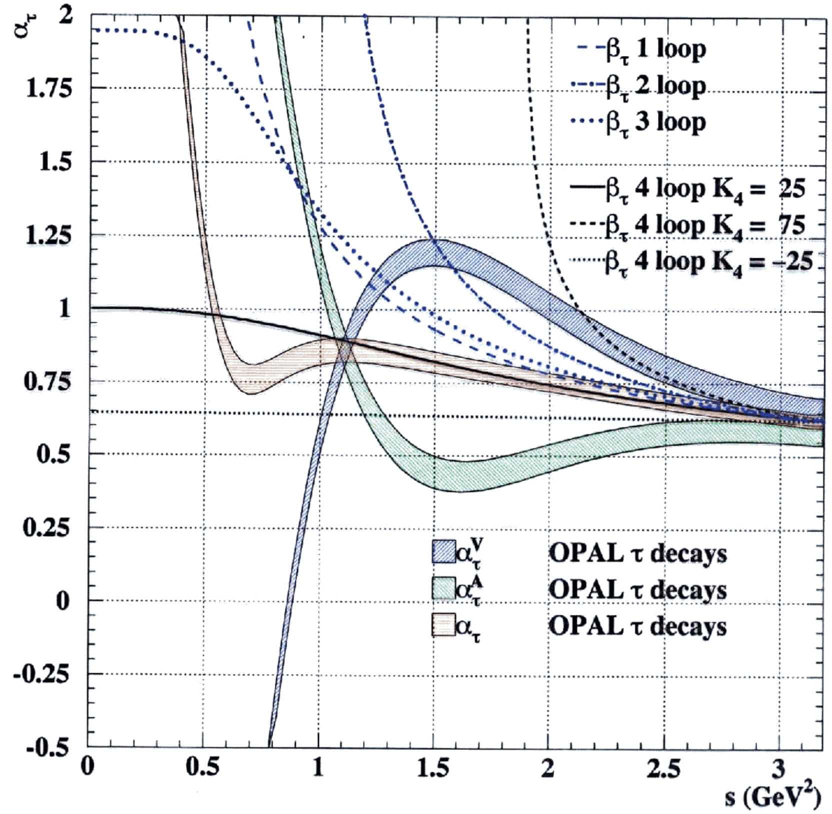


FIG. 2. The optimized third-order results for $\bar{a} = \alpha_s/\pi$ and $\tilde{\mathcal{R}}^{(3)}$. Also shown is the second-order result, $\tilde{\mathcal{R}}^{(2)}$. Quark thresholds are indicated by the vertical lines.

$\frac{\alpha_R(Q)}{\pi}$

Mattingly + Stevenson



Data for $\alpha_\tau(s)$ shows:

* Fixed-point behavior for $\alpha_\tau(s)$ at small scales

{ Gribov, Shirkov, Maxwell + Howe ...
Hoyer + Rothsman (gluon condensate)

* $\alpha_\tau^V(s) - \alpha_\tau^A(s) \sim \frac{1}{s}$ as predicted by OPE

Canonical Form:

Timelike eff charge

$$\alpha_{eff}(s) = \frac{4\pi}{\beta_0}\left\{\frac{1}{2} - \frac{1}{\pi}\tan^{-1}\left(\frac{1}{\pi}\ln s/\Lambda^2\right)\right\}$$

Radyushkin 1982!

$$\Rightarrow \frac{4\pi}{\beta_0} \quad \text{at} \quad s \to 0$$

$$\alpha_{eff}(s) = \frac{4\pi}{\beta_0 \ln s/\Lambda^2}\left\{1 - \frac{1}{3}\frac{\pi^2}{\ln^2 s/\Lambda^2} + \frac{1}{5}\frac{\pi^4}{\ln^4 s/\Lambda^2} + \cdots\right\}$$

$\Rightarrow$ Shirkov form for spacelike

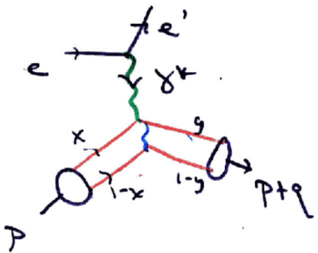
Zwanziger; Alkofer etal

* Frozen coupling in infrared – Conformal

* Dimensional counting rules

SJB, C. Ji:
D. Robertson, Pang

Why do quark counting rules work in QCD?



Need $\alpha_V(\ell^2)$ at low scales

SJB Lepage
Radyushkin
Efremov

$$F_\pi(Q^2) = \int_0^1 dx \int_0^1 dy\, \phi_M(x,\tilde{Q})\, T_H(x,y,Q^2)\, \phi_M(y,\tilde{Q})$$

$$T_H(x,y,Q^2) = \frac{16\pi C_F \alpha_s(\mu)}{(1-x)(1-y)Q^2}\left[1 + \mathcal{O}\left(\frac{\alpha_s}{\pi}\right)\right]$$

$$\alpha_s(\mu) \to \alpha_V\left((1-x)(1-y)Q^2\right)$$

$\sim \frac{1}{20}Q^2$!

Petronzio + Parisi
Gribov
Cornwall
Zerwas
Shirkov
Radyushkin

"Frozen" coupling at low scales

e.g.: $\alpha_V(Q) = \dfrac{4\pi}{\beta_0 \ln\left(\dfrac{Q^2 + 4m_g^2}{\Lambda_V^2}\right)}$

$m_g^2 = 0.3\ \text{GeV}^2$
$\Lambda_V = 0.16\ \text{GeV}$

$\therefore\ \alpha_s \sim$ const at low scales.

References:

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* R.C. Brower + C.I. Tan hep-th/0207071
* S.J. Rey + J.T. Lee hep-th/9803001
* S.J. Brodsky + G.F. de Teramond hep-ph/0310227
* O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri + Y. Oz hep-th/9905111

G. dT + S.J.B

Example of conformal string theory calculation

$AdS_5 \otimes S^5$: $z = R^2/r$

$$ds^2 = \frac{r^2}{R^2}\left(dt^2 - d\vec{x}^2\right) - \frac{R^2}{r^2}dr^2 - R^2\, d\Omega_5^2(y)$$

$$= \frac{R^2}{z^2}\left(dx_\mu^2 - dz^2\right) - R^2 d\Omega_5^2$$

10-dimensional string amplitude $\Phi(x, z, y)$

Laplace Eqn $S=0$

$$\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^A}\left(\sqrt{g}\, g^{AB}\frac{\partial}{\partial x^B}\Phi\right) = 0$$

$$\sqrt{g} = \left(R^2/z\right)^{d+1}\sqrt{\Omega(y)} \qquad d=4$$

$$\Phi(x, z, y) = \sum_\ell \Psi_\ell(x, z)\, \phi_\ell(y)$$

spherical harmonics

hadronic plane wave in 3+1: $\Psi_\ell(x, z) = e^{-iP\cdot x} f(z)$

"dilaton" amplitude

$$* \quad \left[z^2\frac{d}{dz^2} - (d-1)\, z\frac{\partial}{\partial z} - (\lambda R)^2 + z^2 M^2\right] F(z) = 0$$

$$* \quad (\lambda R)^2 = \ell(\ell+d)$$

Dirac Equation in 10-dimension

$$\chi(x,r,y) = \sum_\ell \Psi_\ell(x,r)\,\eta_\ell(y)$$

$$\Psi(x,r) = C\, e^{-iP\cdot x}\, r^{-\frac{d+1}{2}}$$

$$\times\left[J_\alpha\left(\frac{MR^2}{r}\right) u_+(P) + J_{\alpha+1}\left(\frac{MR^2}{r}\right) u_-(P)\right]$$

$$\alpha = \lambda R - \tfrac{1}{2} \qquad M^2 = P^2$$

$$\hat{\Gamma}\, u_\pm = \pm u_\pm$$

$$\hat{\Gamma} = i\,\Gamma_1 \cdots \Gamma_d \qquad \text{chirality } d \text{ even}$$

$$\lambda R = \pm\left(\ell + \tfrac{d}{2} + \tfrac{1}{2}\right)$$

$$\Psi \sim r^{-\Delta}, \qquad \Delta = \tfrac{d}{2} + |\lambda R|$$

Towards realistic QCD

break conformal symmetry : $h(r) \neq 1$

introduce $\Lambda_0 = \Lambda_{QCD}$

physics at $r \sim r_0 = \Lambda_0 R^2$

Modify AdS_5 metric : $z = R^2/r$

$$ds^2 = \frac{r^2}{R^2}\, h(r)\, dx_\mu^2 - \frac{R^2}{r^2} h^{-1}(r)\, dr^2$$

$$- R^2 h^{-1}(r)\, d\Omega_5^2(y)$$

$$\Phi(x,z,y) = \sum_\ell \Psi_\ell(x,z)\,\phi_\ell(y)$$

$$* \quad \left[h^2(z)\left(z^2 \frac{d^2}{dz^2} - (d-1)\, z\,\partial_z - (\lambda R)^2\right) + z^2 M^2\right] f(z) = 0$$

$$\Psi_\ell(x,z) = C\, e^{-iP\cdot x} f(z)$$

GDT
SDB

$$(\lambda R)^2 = \ell(\ell+d) \qquad d=4$$

$h(z)$ subst.

10-dim Dirac Wave Equation

M. Henningson and K.Sfetsos, Phys. Lett. B **431**, 63 (1998);
W. Muck and K. S. Viswanathan, Phys. Rev. D **58**, 106006 (1998).

- We solve the 10-dim Dirac Equation

$$\Gamma^A D_A \hat{\Psi} = 0$$

subject to boundary conditions at z_o and $z \to 0$.

- $\hat{\Psi}$ is expanded in terms of eigenfunctions $\eta_l(y)$ of the Dirac operator on the compact space X with eigenvalues λ_l:

$$\hat{\Psi}(x,r,y) = \sum_l \Psi_l(x,r)\eta_l(y)$$

- From the 10-dim Dirac equation:

$$\left[z^2\partial_z^2 - dz\partial_z + z^2\mathcal{M}^2 - \left(\lambda^2R^2 - \frac{d^2}{4} - \frac{d}{2} - \hat{\Gamma}\lambda R\right)\right] f(z) = 0,$$

$$i \not{D}_X \; \eta(y) = \lambda \; \eta(y),$$

where $\Psi(x,z) = e^{-iP\cdot x} f(z)$, $P_\mu P^\mu = \mathcal{M}^2$ and

$\hat{\Gamma} u_\pm = \pm u_\pm$.

For AdS_5, $\hat{\Gamma}$ is the four dim chirality operator γ_5.

- For each eigenvalue λ_l of the Dirac equation in the compact manifold X, Ψ_l is

$$\Psi(r,x) = Ce^{-iP\cdot x} z^{\frac{d+1}{2}} \left[J_\alpha(z\mathcal{M}) \; \mu_+(P) + J_{\alpha+1}(z\mathcal{M}) \; \mu_-(P)\right],$$

where $\alpha = \lambda R - \frac{1}{2}$, . and

$$\lambda R = \pm\left(l + \frac{d}{2} + \frac{1}{2}\right) \qquad l = 0, 1, 2...,$$

the eigenvalues of the Laplace equation on S^{d+1}.

R. Camporesi and A. Higuchi, arXiv:gr-qc/9505009.

- Mass eigenvalues are determined by the roots of $J_\alpha(z\mathcal{M}) = 0$. and $J_{\alpha+1}(z\mathcal{M}) = 0$.

Solution: Cylindrical Bessel Functions

$$\Psi(x,r) = C_i e^{-iP\cdot x} r^{-d/2} \begin{cases} J_\alpha\left(\frac{mR^2}{r}\right) & \text{ok} \\ N_\alpha\left(\frac{mR^2}{r}\right) \end{cases}$$

where $\alpha^2 = \left(\frac{d}{2}\right)^2 + (\lambda R)^2$ $\quad d=4$

$(\lambda R)^2 = \ell(\ell+d)$ $\quad$ eigenvalues on S^{d+1}

At large r:

$$\Phi(r) \sim r^{-\Delta}$$

$$\Delta = \frac{1}{2}\left(d + \sqrt{d^2 + 4\lambda^2 R^2}\right)$$

$$= d + \ell$$

Thus for hard scattering $r \sim Q R^2$

$$\Phi(Q) \sim Q^{-\Delta}$$

The large $k_\perp$ behavior of LFWFs is predicted from OPE, conformal QCD

Belitsky, Ji, Yuan

$$\Psi_{n/h}(\vec{k}_\perp) \sim (k_\perp)^\ell \left[\frac{1}{k_\perp^2}\right]^{n+\ell-1}$$

ie: $\Delta_{n,\ell} = n + \ell$

Compare with result from $AdS_5 \times S^5$: ($SU(4)$)

$$\Delta = d + \ell(S^5)$$

These match for meson $(n=2)$

if $\boxed{\ell = \ell(S^5) + 2}$ $\quad$ Orbital angular momentum from S^5

Minimum: $\ell = 0 \Rightarrow$

$$\ell(S^5) = -2$$

and $(\lambda R)^2 = -4$. $\quad$ Allowed! See Maldacena, Tesi 2003

$\Psi_h(r,x) \sim r^{-\Delta} e^{ip\cdot x}$ de Teramond SJB

$r \sim QR^2$

$$\Phi_h^{(n)}(Q) \sim \int^{Q^2} [d^2k_\perp]^{n-1} [a^+(k)]^n \, \psi_{n/h}(\vec{k}_\perp) \sim Q^{-\Delta}$$

interpolating operator in AdS_5

$\Delta = n + \ell$ LFWF

$$\Rightarrow \psi_{n/h}(\vec{k}_\perp) \sim \left(\frac{1}{k_\perp^2}\right)^{n-1} \qquad \ell = 0$$

$$\sim k_\perp^\ell \left(\frac{1}{k_\perp^2}\right)^{n+\ell-1} \qquad \ell \neq 0$$

Model:

$\psi_{n/h}(x_i, \vec{k}_{\perp i}, \ell_z)$

see also Belitsky, Ji, Yuan

here: no perturbation theory

$$\sim (g_s N_c)^{\frac{1}{2}(n-1)} \prod_{i=1}^{n-1} (k_{\perp i}^{\pm})^{|\ell_{zi}|} \left[\frac{\Lambda_0}{M_h^2 - \sum_i \left(\frac{k_\perp^2 + m^2}{x}\right)_i + \Lambda_0^2}\right]^{n+|\ell| - 1}$$

+ soft

$$k_{\perp i}^{\pm} = k_i^1 \pm i k_i^2$$

$$\Lambda_0^2 = 1/\alpha' = \text{string const} = M^2_{\text{glueball}}$$

$$g_s \Rightarrow g^2_{QCD}$$

$g_s N_c$ large. Strong coupling, conformal

Analytic Limits of QCD

"Abelian Correspondence Principle" Huet + SJB

$$\lim_{N_c \to 0} QCD\left(\text{fixed } \hat{\alpha} = C_F \alpha_s,\ \hat{n} = \frac{n_F}{T C_F}\right)$$

$$= \text{Abelian Theory } (\hat{\alpha}, \hat{n})$$

$$C_F = \frac{N_c^2 - 1}{2N_c}, \quad T = 1/2$$

* hadron, nuclei $\Rightarrow$ atoms, molecules

"Conformal Correspondence Principle"

$$\lim_{\substack{\beta \to 0 \\ m_q \to 0}} QCD = \text{Conformal QCD} \ (\text{fixed } \bar{\alpha}_s)$$

Parisi; Frishman, Lepage, SJB; Sachrajda

* Remarkable constraints on QCD predictions

* Conformal expansion of distribution amplitude and OPE — V. Braun et al

* Commensurate Scale Relations — Lu, Rathsman, Gardi, Grunberg, Kataev, SJB

No renormalization scale

Conformal symmetry acts as template

* $$R(s) = \sum_{n=0} C_n \alpha_s^n (Q_n^*)$$

observable — conformal coefficient — absorbs $\beta \neq 0$.

Q_n^* determined by Banks-Zaks + BLM
Gard:, Grunberg, Rathsman, SJB

C_n : no n! growth ; no renormalons

* Hadron distribution amplitudes

conformal

- $$\phi_M(x_i, Q) = x_1 x_2 \sum_n a_n C_n^{3/2}(x_1 - x_2) e^{-\gamma_n \zeta(Q)}$$

$$\zeta(Q) = \frac{1}{2\pi} \int^{Q^2} \frac{d\ell^2}{\ell^2} \alpha_s(\ell^2)$$

Lepage SJB

- $$\phi_B(x_i, Q) = x_1 x_2 x_3 \sum_n a_n P_n(x_i) e^{-\gamma_n \zeta(Q)}$$

conformal Jacobi polynomials
Braun, Derkachov, Manashov, Korchemsky

* $C_n^{3/2}$, P_n determined from conf. symmetry

* Deviations from conf. symmetry in $\alpha_s(\ell^2)$

Conformal symmetry : initial approximation

* then compute corrections from $\beta \neq 0$, $m \neq 0$.

Example Generalized Crewther Relation

$$\boxed{\left[1 - \frac{\alpha_{g_1}}{\pi}\right]\left[1 + \frac{\alpha_R}{\pi}\right] = 1}$$

exact to all orders in conformal QCD

Crewther

If $\beta \neq 0$, $\alpha_{g_1} = \alpha_{g_1}(Q^2)$

$\alpha_R = \alpha_R(s)$

Conformal Symmetry maintained

$$\boxed{\left[1 - \frac{\alpha_{g_1}(Q^{*2})}{\pi}\right]\left[1 + \frac{\alpha_R(s)}{\pi}\right] = 1}$$

Q^{*2}/s computed : absorbs
Fixed $\beta \neq 0$

* thresholds at commensurate scales

BLM

* No scale ambiguity physical coupling

H. Lu, G. Gabadadze
A. Kataev
SJB

* Commensurate Scale Relations
relate observable to observable

Linear vs. NON-linear trajectories

Linear
- Originally in the Veneziano Model
- Linear confining potential
- String model

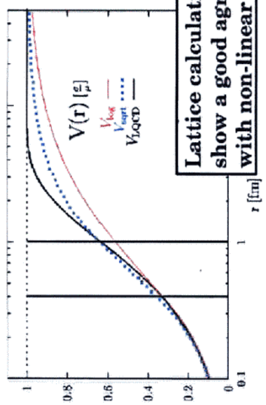
NON-Linear
- From data analysis — M.Brisudova et al. Phys.Rev. D61 (2000) 054013; L.Burakovsky hep-ph/9904322

$\alpha_N(t) = -0.4 + 0.9\,t + 0.125\,t^2$

$\alpha_P(t) = 1.1 + 0.25\,t + 0.5\,(0.16\pm0.02)\,t^2$

- From theory (Froissart bound)
- String model with variable tension + flux tube braking

- Different shapes: $\alpha(t) \sim \mathrm{sqrt}(-t)$ $\alpha(t) \sim -\log(-t)$ $\alpha(t) \sim$ fixed value
- From trajectory -> qq potential
- Smooth interp. between f/b peaks



Lattice calculations show a good agreement with non-linear traject.

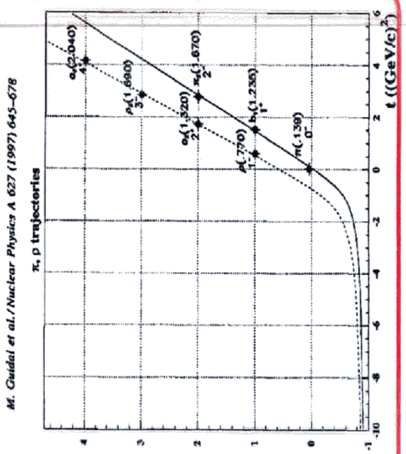
Saturating Regge trajectories: -1 when $-t \to \infty$

M.Sergeenko Z.Phys. C64 (1994) 315 ($\alpha(t) = -1$ $-t > 3$ GeV^2)

QCD motivated q-q potential

$V(r) = -4/3\,\alpha_s\,1/r + \kappa r + V_0$

- Large $-t$ -> 'hard' processes
- $d\sigma/dt(90^0) \sim$ independent on t
- Scaling according to parton scattering
- s behavior given by counting rules
- **Linear confining potential: linear long range traj. (2q strongly interacting)**
- **One Gluon Exchange: saturating traj (2q interacting via a hard gluon)**



QNP2004 - Exclusive reactions at large momentum transfer - Marco Battaglieri - INFN Genova

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Some consequence of conformal scaling of ψ_n^{LF}

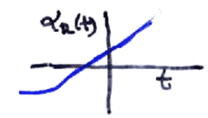
* $F_{\Delta S_z=0}(Q^2) = \left[\frac{1}{Q^2}\right]^{n-1}$ (mod anom. dim.)

* $\frac{Q^2 F_2(Q^2)}{F_1(Q^2)} \sim$ const (mod logs)

* N_c large $\Rightarrow$ dominance of quark interchange; suppression of pinch contribs

* $\alpha_{Reggeon}(t) \Rightarrow$ neg. integer at $t \to -\infty$

"Saturation"



Sauit, Gunion, Blankenbecler, SJB, Coon

* Non-perturbative normalization $\sum c_n \alpha_s^n \Rightarrow \sqrt{\alpha_s}$

* Orbital dep. of LFWFs   Karmanov, Smirnov; Hiller, Hwang, SJB

* x-dep of distribution amplitudes   Müller, Braun, Ball, Korchemsky

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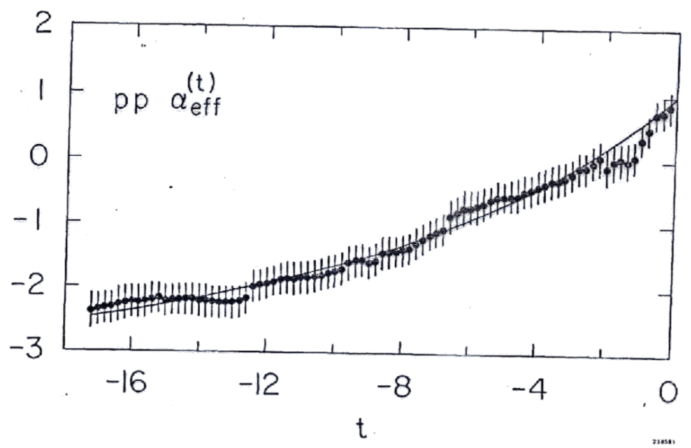

FIG. IIIA.4a