

Regge trajectories
revisited
from the gauge/string correspondence

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Outline

Introduction / Motivation

Classical Regge trajectories { flat space
confining backgrounds

- Quantum Based on:
• "Regge trajectories revisited" in the KS, MN
Non-linear gauge-string correspondence hep-th/0311190 (NPB)
with L. Pando Zayas J. Sonnenschein

Meson Regge trajectories

- work in progress on Regge trajectories for mesons

Conclusion

M. Kruckenski
L. Pando Zayas
J. Sonnenschein

Introduction / Motivation

- String theory has come full circle:

(late 60's) Old days \leftrightarrow dual models: bosonic string theories in d-dim were describing strong interactions in d-dim.

(after 97) Recent years \leftrightarrow AdS/CFT duality (string/gauge):

string theories in d non-compact dim. are hole dual to gauge theories in d-1 dim.

- Then (early days of strong interaction physics)

- Regge trajectories (order in multitude)



$$J = \alpha' E^2 + \alpha_0$$

α' : Regge slope $\sim 0.9 \text{ GeV}^{-2}$

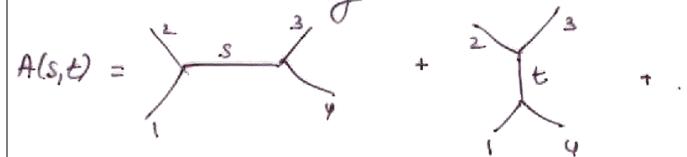
α_0 : Regge intercept

- seemed to continue indefinitely ($J \approx 11/2$)

R.b = first hint at string theory

(more on this later)

- S-t duality



$$A_t = - \sum_J \frac{g_J(-s) J}{t - M_J^2} \stackrel{s-t \text{ duality}}{=} A_s = - \sum_J \frac{g_J(-t) J}{s - M_J^2}$$

was experimentally observed in the Regge region (large s , small $|t|$)

- Veneziano postulated

$$A(s,t) = \frac{P(-\alpha(s)) P(-\alpha(t))}{P(-\alpha(s) - \alpha(t))}, \quad \alpha(s) = \alpha_0 + \alpha' s$$

$$\left. \begin{array}{l} \sim_{s \rightarrow \infty} \alpha(t) \\ |t| \text{ fixed} \\ \sim_{s \nearrow} \alpha(t) \end{array} \right\} \Rightarrow J = \alpha(t)$$

- On the other hand at fixed scattering angle ($|t|, s \rightarrow \infty$ fixed)

$$A(s,t) \sim e^{-s} \quad (\text{softness of strong scattering})$$

- contradictory to data.

- To explain $\sigma^{\text{tot}} \sim s^{\alpha(\phi)-1}$ a new trajectory was introduced, additional to the observed p.w..

Pomeron Regge trajectory $\alpha(\phi) \approx 1.08$, $\alpha' \approx 0.25 \text{ GeV}^{-2}$

- To summarize, the dual models

- explained Regge physics
- predicted S-duality
- failed to explain DIS (strings were behaving softly)
- had a massless spin 2 particle
- required higher dimensions, had a tachyon etc...

 string theory became a theory of quantum gravity

until ...

 QCD became the theory of strong interactions

NOW

- '97 Maldacena puts forward the AdS/CFT conjecture:
IIB strings on $\text{AdS}_5 \times S_5$ are holographically dual to $N=4$ $(1+4)$, large N , $SU(N)$ SYM gauge theory

$$g_s N = \text{fixed} \quad (\text{but large for sugra validity})$$

$$g_m^2 \sim g_s ; \quad R_{\text{AdS}} = R_{S^5} \sim (g_s N)^{1/4} ; \quad \int_{S^5} F_S = N$$

$\text{AdS}_5 : SO(4,2) \rightarrow$ conformal group

$S^5 : SO(6)/SO(5) \rightarrow SO(6) : R$ -symm. group

AdS / CFT dictionary

- states on AdS \leftrightarrow operators in the CFT
- single particle \leftrightarrow single trace
- multi- \leftrightarrow multi-
- sugra modes \leftrightarrow (BPS) chiral primaries

2002 BMN : explore sectors with large charge

- exploit the semiclassical regime and go beyond sugra
- BMN: large R-charge ops \leftrightarrow string shrunk to a point orbiting in S^5
- GKP twist-two operators \leftrightarrow folded spinning strings in AdS_5

Tseytlin ... integrable models \leftrightarrow states with 2,3 ang. mom. in S^5

 Use the idea of intro. large quantum numbers to simplify the problem of quantizing the string in backgrounds dual to compacting theories

- glueballs \leftrightarrow spinning folded closed strings
- mesons of heavy quarks \leftrightarrow spinning Wilson loops ending on the boundary
- baryons of $\psi \bar{\psi}$ \leftrightarrow strings attached to a baryonic vertex
- mesons of light quarks \leftrightarrow spinning Wilson loops ending on probe branes

Classical Regge trajectories

A. Flat space

Spinning open string:

- fermionic dof play no rôle here
- Polyakov action

$$S = \frac{T_S}{2} \int d\sigma \int dt \sqrt{g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\mu$$

- Neumann boundary conditions: $\partial_\sigma X^\mu \Big|_{\sigma=0, \pi} = 0$
- Virasoro constraints: $\begin{cases} \dot{X}^\mu \dot{X}'_\mu = 0 \\ \dot{X}^\mu \dot{X}_\mu + \dot{X}'^\mu \dot{X}'_\mu = 0 \end{cases}$

The ansatz: $X^0 = e\tau$

$$X^1 = e \cos \sigma \cos \theta$$

$$X^2 = e \cos \sigma \sin \theta$$



Energy

$$E = T_S \int_0^\pi \dot{X}^0 = \pi T_S e$$

Angular momentum

$$J = T_S \int_0^\pi (\dot{X}^2 \dot{X}^1 - \dot{X}^1 \dot{X}^2)$$

$$= \frac{\pi}{2} T_S e^2$$

Regge trajectory: $\gamma = \gamma(\epsilon^2)$

$$\gamma = \alpha' E^2$$

Closed spinning strings

$$S = \int d\sigma \int dt \sqrt{g} g^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\mu$$

Periodic b.c.

Virasoro constraints

Ansatz: $X^0 = e\tau$; $X^1 = e \sin \sigma \cos \theta$; $X^2 = e \sin \sigma \sin \theta$

$$E = 2\pi T_S e; J = \pi T_S e^2$$

$$\Rightarrow \text{Regge trajectory}$$

$$\gamma = \frac{\alpha'}{2} E^2$$

B. Confining backgrounds

(= holographic duals of confining theories)

- deformations of the AdS_5 geometry with the property that Wilson loops show confining behavior: $E = T_g \cdot L + \dots$
- J.Sonnenschein et al (99) proved that the following represent sufficient conditions for confinement:

$$ds^2 = g_{00}^{(r)} dt^2 + g_{ii}^{(r)} dx^i dx^i + g_{rr}^{(r)} dr^2 + \dots$$

$$f^2 = g_{00} \cdot g_{rr}$$

$$g^2 = g_{00} \cdot g_{rr}$$

- $f(r)$ has a minimum at r_0 s.t. $f'(r_0) \neq 0$
or
- $g(r)$ diverges at some r_{div} s.t. $f(r_{div}) \neq 0$.



$f(r_0)T_s$ is the quark-antiquark string tension

The set-up:

$$ds^2 = h^{-1/2}(r) dx^\mu dx^\nu \eta_{\mu\nu} + h^{1/2}(r) dr^2 + \dots$$

compact directions

Assume r_0 s.t. $h'(r_0) = 0$
 $h(r_0) \neq 0$.

The ansatz:

$$x^0 = e\tau; \quad x^1 = \underline{x(r)} \cos(\omega\tau); \quad x^2 = \sin(\omega\tau) \underline{x(r)}$$

$$r(\tau) = \text{constant}$$

The eqns. of motion:

$$-\partial_\alpha (h^{-1/2} \gamma^{\alpha\beta} \partial_\beta X^0) = 0 \quad \leftarrow \text{satisfied at } r(\tau) = r_0$$

- the radial eqn of motion:

$$0 = \partial_r (h^{1/2} \partial_r r) - \frac{1}{2} \partial_r h^{-1/2} (e^2 - (\omega)^2 \underline{x}^2 + \underline{\partial_r x}^2)$$

\checkmark

- the Virasoro constraint

$$0 = h^{1/2} (\partial_\tau r)^2 + h^{-1/2} (-e^2 + (\omega)^2 \underline{x}^2 + \underline{\partial_r x}^2)$$

+ periodic boundary conditions $\tau \approx \tau + 2\pi$
 $\Rightarrow \omega = 1$

$$\Rightarrow x^0 = e\tau; \quad x^1 = e \cos \tau \sin \sigma; \quad x^2 = e \sin \tau \sin \sigma$$

$$r = r_0$$

The Regge trajectory:

$$\left. \begin{aligned} E &= 2\pi g_{\text{ee}}(r_0) T_S \\ J &= \pi^2 g_{\text{ee}}(r_0) T_S \end{aligned} \right\} \quad J = \frac{E^2}{4\pi T_{S,\text{eff}}}$$

$$\text{where } T_{S,\text{eff}} = \frac{g_{\text{ee}}(r_0)}{2\pi\alpha'}$$

⇒ Classically, this is the same qualitative Regge behavior as for a closed spinning string in flat space:

- it is strictly linear
- it has zero intercept

Quantum corrected Regge trajectories

- Semiclassical quantization:

$$X^{\mu}(r, \varepsilon) = \bar{X}^{\mu}(r, \varepsilon) + \delta X(r, \varepsilon)$$

\uparrow
background field

quantum fluctuation

- Virasoro constraint $\hat{J}(\{X^\mu\}) = 0$

$$\Rightarrow \epsilon(E_{\text{qu}} - E_{\text{cl}}) = \int d\sigma \langle \delta J | \hat{J}(\{\delta X^\mu\}) | \delta \rangle$$

$$\delta \langle \delta J \rangle = J_{\text{cl}} \langle \delta \rangle$$

\uparrow
ground state

⇒ The quantum correction to the energy of the ground state is given by the sum of the zero-point energies of the quantum dof

⇒ For the Regge trajectories of spinning strings in flat space, the effect of the quantum corrections is rather trivial: it will generate a non-vanishing intercept (at best).

⇒ In confining backgrounds we will derive nonlinear Regge trajectories.

A. Flat space

Bosonic fluctuations:

$$X^\mu = \bar{X}^\mu + \delta X^\mu$$

$$S_\delta = S_{\text{cls}} + S_{B, \text{qu}} = S_{\text{cls}} + \frac{T_S}{2} \int \sqrt{g} g^{\mu\rho} \partial_\mu \delta X \cdot \partial_\rho \delta X$$

choose the conformal gauge

$$\eta_{\mu\rho} = (\star g)_{\mu\rho} = \partial_\mu \bar{X} \cdot \partial_\rho \bar{X} = e^2 \cos^2 \sigma \eta_{\mu\rho} \equiv \sqrt{g} \eta_{\mu\rho}$$

measure (world-sheet scalars)

$$\|\delta X\| = \sqrt{\delta X \cdot (\delta X)^2}$$

path integral:

$$\int dX e^{S_{\text{cls}} + S_{\text{qu}}} = e^{S_{\text{cls}}} \cdot \det(\Delta_g)^{-d/2}$$

in d space-time dimensions

$$\begin{aligned} \text{ghosts: } & \int d\theta d\bar{\theta} \exp \left[T_S \int \sqrt{g} b_{\mu\rho} \gamma^\mu \gamma^\rho \bar{P}_\theta C_\theta \right] \\ &= (\det P_\theta)^{1/2} \\ &= \det D_g \end{aligned}$$

The quantum correction to the energy of the ground state

$$e\Delta E = -\pi \frac{d-2}{24}$$

Fermionic fluctuations

- Polyakov \rightarrow Green-Schwarz action

$$\begin{aligned} \text{Flat space: } & S_{\text{flat}} = T_S \int \sqrt{g} g^{\mu\rho} \Pi_\alpha \Pi_\rho - i \epsilon^{\mu\rho} (\beta_3) \bar{\theta} \bar{P}^\alpha \partial_\rho \theta^\beta \\ & \cdot (\partial_\alpha X^\mu - \frac{1}{2} \epsilon^{\alpha\beta} \bar{\theta}^\kappa P^\mu \partial_\kappa \theta^\beta) \\ & \Pi_\alpha = \partial_\alpha X^\mu - i \bar{\theta}^\kappa P^\mu \partial_\kappa \theta^\alpha \\ & \kappa, I, J = 1, 2 \end{aligned}$$

- Drukker, Gross, Tseytlin (2000)

- work in a kappa-fixed gauge $\theta^1 = \theta^2 \equiv \theta$
(OK for IIB string theory)

- GS is manageable in semiclassical quantization

- cancellation of conformal anomaly works
 $10 - 26 + 8 \cdot 4 \cdot \frac{1}{2} = 0$

\uparrow
θ's are world-sheet scalars

- expand around the spinning string config:

$$\begin{aligned} S_F &= \frac{i}{2} T_S \int \underbrace{\sqrt{g} g^{\mu\rho} \partial_\mu \bar{X}^\mu}_{S^0} \bar{\theta} P_\rho \partial_\mu \theta \\ &= \frac{i}{2} T_S \int e \bar{\theta} \left[P^0 + (P^1 \cos \sigma - P^2 \sin \sigma) \sin \sigma \right] \dot{\theta} \\ &\quad + e \bar{\theta} \left[P^1 \sin \sigma + P^2 \cos \sigma \right] \cos \sigma \dot{\theta}' \end{aligned}$$

S^0, S^1 related to 10-d Dirac matrices by unitary transforms

$$U_{01} = \exp\left(-\frac{1}{2} \operatorname{arccosh} \frac{1}{\cos\sigma} P^0 P^1\right)$$

$$U_{12} = \exp\left(\frac{1}{2} \sigma P^1 P^2\right)$$

$$\Rightarrow g^0 = \cos\sigma U_{12} U_{01} P^0 U_{01}^{-1} U_{12}^{-1}$$

$$g^1 = \cos\sigma U_{12} U_{01} P^2 U_{01}^{-1} U_{12}^{-1}$$

and with the field redefinition

$$\sqrt{\epsilon} \cos\sigma \Phi = U_{12} U_{01} \Psi$$

the fermionic fluctuations action becomes free

$$S_F = \int \bar{\Psi} (P^0 \partial_\tau + P^2 \partial_\sigma) \Psi$$

\Rightarrow The ground state energy receives no quantum corrections for the supersymmetric string spinning in a flat space background.

\Rightarrow The Regge trajectory remains linear with zero intercept at one loop level.

B. Quantum corrections for spinning string configurations in confining backgrounds

- We investigated both Klebanov-Strassler and Maldacena-Nunez backgrounds which represent embeddings of W_2 SYM in string theory (large N limit).
- The classical spinning string configuration is located at $r=0$, which corresponds to the IR of the dual gauge theory.

$$\underline{r=\infty}$$

⊗ 5 compact directions

~~\mathbb{R}^4~~ $\rightarrow r=0$

- The states dual to these folded back closed spinning strings are glueballs.
- Both confining backgrounds yield the same spectrum and the same Regge trajectory (at one loop in our semiclassical analysis).

$$J(t) = \alpha(t) = \alpha' t + \alpha_0 + \beta \sqrt{t}$$

$t \approx E^2$

- Klebanov-Strassler background

(N D3 and M fractional D3-branes on the deformed conifold)

- History:

Klebanov-Witten: Add N D3-branes at the singularity of the conifold



$$ds_{10}^2 = h^{-1/2} dx^M dx_M + h^{1/2} ds_6^2$$

$$ds_6^2 = dr^2 + r^2 ds_{T^{1,1}}^2 \Leftrightarrow \sum_{n=1}^4 \tilde{g}_{nn}^2 = 0, \tilde{g}_{nn} \in \mathbb{C}$$

$$ds_{T^{1,1}}^2 = \frac{1}{g} (dx + \sum_i \cos \theta_i d\phi_i)^2 + \frac{1}{6} \sum_{i=1}^3 (\sin^2 \theta_i)^{\frac{2}{3}}$$

- the dual gauge theory has $N_f^2/4$ susy, $SU(N) \times SU(N)$ gauge group and is conf.

Klebanov-Tseytlin: Add on top M fractional D3-branes

- break conformal inv.

$$\frac{1}{g_1^2} - \frac{1}{g_2^2} \sim \int_B \sim \text{logarithmic running}$$

- $SU(N+M) \times SU(N)$ gauge symmetry

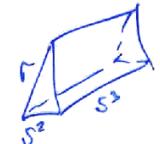
- the warp factor has a log sing (naked) in a region where the D3-brane charge becomes neg.

- it was conjectured that the singularity should be smoothed/excised by nonperturbative effects

Klebanov-Strassler :

$$ds_{10}^2 = h^{-1/2}(r) dx^M dx_M + h^{1/2}(r) ds_6^2$$

$$ds_6^2 = \frac{1}{2} \varepsilon^{4/3} K(r) \left[\frac{1}{3K^2(r)} (dr^2 + g_5^2) \right]$$



$$+ \cosh^2 \frac{r}{2} (g_3^2 + g_4^2) + \sinh^2 \frac{r}{2} (g_1^2 + g_2^2) \right]$$

where $g_1 \dots g_5$ are 1-forms s.t.

$$ds_{T^{1,1}}^2 = \frac{1}{g} g_5^2 + \frac{1}{6} \sum_{i=1}^4 g_i^2$$

$h(r)$ is known only as an integral

$$h(r) = (g_5 M \omega^4)^2 2^{2/3} \varepsilon^{-8/3} I(r) \approx a_0 - a_1 r^2 + \mathcal{O}(r^4)$$

$$a_0 \approx 0.71805, \quad a_1 = \frac{2^{2/3} 3^{2/3}}{18}$$

$$K(r) = \frac{(\sinh(2r) - 2r)^{1/3}}{2^{1/3} \sinh(r)} \approx 2^{4/3} 3^{4/3} + \mathcal{O}(r^2)$$

In addition, 3-form fields $\frac{F_2}{H_3}$ ($g_5^2 F_3^2 = H_3^2$)

$$F_5 + * F_5; \quad F_5 = B_2 \wedge F_3$$

ε is the deformation parameter

$$\sum_{m=1}^4 \tilde{g}_{mm}^2 = \varepsilon^2$$

The K-T singularity was resolved in K-S through chiral symmetry breaking

Quadratic fluctuations in the KS background

Begin with the GS action

$$\mathcal{S} = \frac{T_5}{2} \int \sqrt{Y} \left(g^{\alpha\beta} \partial_\mu X^\alpha \partial_\nu X^\beta + \epsilon^{\alpha\beta} B_{\mu\nu}(X) \partial_\mu X^\alpha \partial_\nu X^\beta + i (g^{\alpha\beta} \delta^{ij} - \epsilon^{\alpha\beta} (\rho_3)^{ij}) \partial_\mu X^\alpha e_\mu^m(X) \bar{\theta}_{mp}^j \theta_{np}^i \right) + \dots$$

Pauli: $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, i, j = 1, 2$

$$\mathcal{D}_\alpha = \partial_\alpha + \frac{i}{4} \partial_\alpha X^\mu \left[(w_{mn\mu} - \frac{1}{2} H_{mn\mu} \rho_3) R^{mn} + \frac{1}{3!} F_{mnp} R^{mnp} \bar{s}_1 + \frac{i}{2 \cdot 5!} F_{mnpqr} R^{mnpqr} \bar{s}_2 \right]$$

bosonic fluctuations:

$$ds_6^2 = \frac{e^{4/3}}{2^{2/3} 3^{1/3}} \left[\underbrace{\frac{1}{2} g_S^2 + g_3^2 + g_4^2}_{\text{round } S^3} + \frac{1}{2} d\tau^2 + \underbrace{\frac{1}{2} (g_1^2 + g_2^2)}_{\text{round } S^2} \right]$$

$$\sim d\theta^2 + d\phi^2 + d\psi^2 + 2 \cos\theta d\theta d\phi$$

$$\text{around } \theta = \frac{\pi}{2}$$

$$\sim dy_1^2 + dy_2^2 + dy_3^2$$

$$\underbrace{d\tau_1^2 + d\tau_2^2 + d\tau_3^2}_{\sim d\tau^2 + d\tau_2^2 + d\tau_3^2}$$

Expand the warp factor

$$\frac{1}{2} \int \sqrt{Y} g_{rr}(X) \Big|_{r=0} \partial_\alpha \tau^i \partial_\beta \tau^i + \frac{1}{2} \frac{\partial^2 g_{\mu\nu}}{\partial r^2} \Big|_{r=0} r^2 \partial_\mu \bar{x}^\mu \partial_\nu x^\nu$$

will become massive

The quadratic action for the 3 τ^i fluctuations is

$$\frac{T_5}{2} \int \sqrt{Y} \left(g^{\alpha\beta} g_S M \alpha' \frac{a_0^{1/2}}{2^{4/3} 3^{1/3}} \partial_\alpha \tau^i \partial_\beta \tau^i + \frac{a_0}{2a_0} \tau^i \tau^i \right)$$

- we identified 3 massive fluctuations with a σ -dependent mass

$$m_B^2 = 2 m_0^2 e^2 \cos^2 \sigma$$

- the corresponding eqn of motion

$$* [\partial_\tau^2 - \partial_\sigma^2 + 2 m_0^2 e^2 \cos^2 \sigma] \tau^i(\sigma, \tau) = 0$$

Fourier-expand $e^{in\tau} \tau_i(\sigma)$

$$\Rightarrow \left[\frac{\partial}{\partial \sigma^2} + \left(n^2 - \frac{2 m_0^2 e^2}{2} \right) - m_0^2 e^2 \cos 2\sigma \right] \tau^i(\sigma) = 0$$

Mathieu eqn.

(in general, only numerical soln. are known)

but a series expansion is also available

- the spectral problem of $*$ may be solved perturbatively in m_0

$$\lambda_{n,r} = n^2 + r^2 + m_0^2 e^2 + \frac{1}{2(r^2-1)} \frac{m_0^4 e^4}{4} + \dots$$

Keeping only the leading term in the $(m_0\epsilon)$ expansion we find

$$\epsilon \Delta E = \frac{\pi}{2} \left(-\frac{8}{12} + 3 m_0 \epsilon \right) + \dots$$

\uparrow
3 massive θ^i dof
5 massless dof

- fermionic fluctuations

$$S_F \propto \frac{i}{2} T_S \int \sqrt{g_{\theta\theta}} \left[\bar{\theta} (P^0 - (P^1 \cos\theta - P^2 \sin\theta) \sin\theta) \dot{\theta} - \bar{\theta} (P^1 \sin\theta + P^2 \cos\theta) \cos\theta \right] - \frac{1}{2} \sqrt{g} \partial^2 \theta$$

- use similar unitary transformations as in flat space to simplify S_F
- identify 8 massive fermionic (2d) dof with mass

$$m_P^2 = 2 l^2 e^2 \cos^2 \theta$$

gathering all contrib., the quantum corr.

$$\Rightarrow \boxed{\epsilon \Delta E = \left(\frac{3}{2} m_0 - 4l \right) \pi e}$$

Maldacena-Nunez background

- N D5 branes wrapped on S^2
- IR: $U=1$ SYM $(SU(N))$ + KK modes

The spectrum of quadratic fluctuations revealed the same features as before (KS):

- 5 massless + 3 massive ($m^2 = 2 m_0^2 \cos^2 \theta$) bosonic fluct.
- 8 massive 2d fermionic fluct ($m_F^2 = 2 l^2 e^2 \cos^2 \theta$)

	K-S	M-N
m_0	$\frac{3^{1/6} a_0^{1/2}}{a_0} \frac{\epsilon^{2/3}}{g_s M \alpha'^1}$	$\frac{2}{3} \frac{1}{\sqrt{g_s N \alpha'^1}}$
l	$\frac{3^{1/2}}{2^{7/6} a_0} g_s^{-1} \frac{\epsilon^{2/3}}{g_s M \alpha'^1}$	$\frac{2^{1/2}}{g_s N \sqrt{g_s N \alpha'^1}}$
	$\frac{1}{g_s M \alpha'^1} \sim M g_b^2$ $\frac{\epsilon^{4/3}}{g_s M} \sim T_S$	$\frac{1}{g_s N \alpha'^1} \sim M g_b^2$ $\frac{\sqrt{g_s N}}{\alpha'^1} \sim T_c$

- The Regge trajectory becomes non-linear due to

$$-\Delta E = -\left(\frac{3}{2} m_0 - 4l \right) > 0$$

The non-linearity of the Regge trajectory takes the form:

$$\bar{J} = \frac{1}{2} \alpha'_{\text{eff}} E^2 - \alpha'_{\text{eff}} \left(\frac{3}{2} m_0 - 4l \right) E + \frac{1}{2} \alpha'_{\text{eff}} \left(\frac{3}{2} m_0 - 4l \right)^2$$

leap of faith?

Comparison with experimental data:

- H Meyer and M.J. Teper (03) : lattice QCD glueball spectra are compatible with the soft Pomeron
- UA8 Collaboration (97) : the Pomeron Regge trajectory

$$\alpha(t) = 1.1 + 0.25 \text{ GeV}^{-2} t + \alpha'' t^2$$

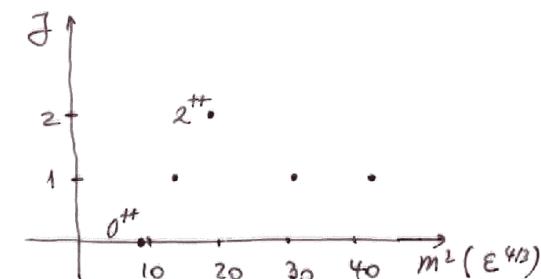
with $\alpha'' = 0.079 \pm 0.012 \text{ GeV}^{-4}$

We obtain a qualitative agreement:

- positive intercept
- positive curvature ($4l - 3/2 m_0 > 0$)

- attempts to match the glueball spectra from sugra: Csaki, Ooguri, Oz, Terning (98)
(non-sugra pure YM from non-extremal D4 on the thermal circle)

- for the KS background, the glueball masses were identified (from sugra fluct.) by Caceres et al.



→ get a prediction for the Regge slope

$$J = 0.234 t + \alpha_0$$

- Burakowski, Goldman, Britudove, (03) and Szczepaniak

proposed a dual string model with a string tension similar to the one they used to model mesons

Dual models with Mandelstam analyticity

$$\alpha(t) = \alpha(0) + \delta_{1/2} (\sqrt{T} - \sqrt{T-t})$$

identified $\alpha(0) = 1.079 \pm 0.007$

$$\sqrt{T} = 11.57 \pm 1.1 \text{ GeV}$$

from a fit that combined scattering data and lattice data

Meson Regge trajectories

- abundance of data
- trajectories are approx. linear, with the degree of non-linearity flavor dep.
- Set-up:

(Witten) N_c non-extremal D4 branes, compactified on a thermal circle of radius R
= holo dual to $d=4$, $SU(N_c)$ Yang-Mills

$$ds^2 = U^{3/2} \left(-(dx^0)^2 + dR^2 + R^2 d\phi^2 + dy^2 \right) + f(U) U^{3/2} d\psi^2 + K(U) (dg^2 + g^2 d\Omega_4^2)$$

$$U(g) = \left(g^{3/2} + \frac{U_N^3}{4g^{3/2}} \right)^{2/3} \quad \begin{array}{c} \text{---} \\ p=\infty \end{array}$$

$$K(U) = U^{1/2} g^{-2} \quad \begin{array}{c} \text{---} \\ p_f \end{array}$$

$$p_A = \left(\frac{1}{2}\right)^{2/3} U_N \quad \begin{array}{c} \text{---} \\ p_A \end{array}$$

$$p = \begin{cases} \text{---} & g=0 \\ \dots & \dots \\ \text{---} & g=\infty \end{cases}$$

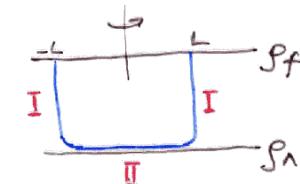
$$f(U) = 1 - \frac{U_N^3}{U^2}$$

$$\int_{S^4} F_4 = N_c$$

Add a D6 brane probe that terminates at some finite distance p_f

(see also Kruczenski et al.
0311270)

Consider next a spinning Wilson loop



The ansatz: $X^0 = c t$

$$\phi = \omega t$$

$$R = R(\sigma); \rho = \rho(\sigma)$$

with boundary conditions: $\frac{d\rho}{dR} = \frac{d\rho}{d\sigma} \cdot \frac{d\sigma}{dR} = \infty$ at $R = \pm L$

For $L \gg 1$ the shape of the Wilson loop is well approx. by regions I and II.

Region I: $\rho \rightarrow \infty$

- the equation of motion is satisfied in this limit

$$\frac{d}{dR} \left(\frac{U(\rho) \sqrt{1-\omega^2 R^2}}{\rho} \right) = \frac{3}{4} \frac{dU}{dp} \frac{dp}{dR} \frac{\sqrt{1-\omega^2 R^2}}{\rho} + \frac{1}{2} \sqrt{1-\omega^2 R^2} \left(\frac{1}{2} \frac{du}{dp} - \frac{2u}{p} \right) \frac{dp}{dR}$$

$$\text{rhs} = \text{lhs} = \sqrt{1-\omega^2 R^2} \left(\frac{du}{dp} - \frac{u}{p} \right) \frac{dp}{dR}$$

$$E_I = T \int_{p_A}^{p_f} dp \sqrt{(1-\omega^2 R^2)^{-1}} \sqrt{U^{3/2} \left(\frac{dp}{dR} \right)^2 + K} U^{3/4} \Big|_{R=\pm L}$$

$$= \frac{1}{\sqrt{1-\omega^2 R^2}} \Big|_{R=\pm L} \int_{p_A}^{p_f} dp \cdot \frac{U}{\rho} \quad = \text{energy of relativistic quarks}$$

$$J_I = \frac{m g \omega R^2}{\sqrt{1-\omega^2 R^2}}$$

Summarizing, we have recovered an old dual model from the past: that of an open spinning string, with massive endpoints.

Burekowsky-Goldman model:

$$S = \int d\sigma d\tau \gamma(R) \sqrt{1 - \dot{\phi}^2 R(\tau)} (dR/d\tau)^2 + \sum_{i=1,2} m_i \int d\tau \sqrt{1 - \dot{\phi}^2 L^2}$$

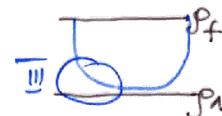
effective string tension

Fit with non-linear meson Regge trajectories identifies

$$\gamma(R) = \frac{r}{1 + (\pi \mu R)^2} \quad \text{which leads } (m_i \approx 0) \text{ to the square-root trajectory}$$

$$J = \frac{1}{\pi \mu} \left(\frac{r}{R} - \sqrt{(\pi \mu)^2 - E^2} \right).$$

A perturbative Wilson loop solution including corrections coming from region III would naturally generate an R -dependent effective string tension.



Region II: $\phi \approx 0$ (the loop flattens)

- the equation of motion is satisfied only for $\phi = \text{constant} = \phi_A$

$$\frac{d}{dR} \left(\sqrt{U} \frac{\sqrt{1 - \omega^2 R^2}}{\phi^2} \frac{d\phi}{dR} \right) = \frac{3}{2} \sqrt{U} \frac{dU}{d\phi} \cdot \sqrt{1 - \omega^2 R^2}$$

$$= 0 \text{ for } \phi = \phi_A. \left(\frac{dU}{d\phi} \Big|_{\phi_A} \approx 0 \right)$$

Energy and angular momentum contributions:

$$E_{II} = T_s \int_{-L_2}^{L_2} dR \frac{\sqrt{U^{3/2} + K(\phi/dR)^2}}{\sqrt{1 - \omega^2 R^2}} U^{3/4}$$

$$\approx T_s \int_{-L_2}^{L_2} dR U_A^{3/2} \frac{1}{\sqrt{1 - \omega^2 R^2}}$$

$$J_{II} = T_s U_A^{3/2} \int_{\text{gauge dual}} dR \cdot \frac{\omega R^2}{\sqrt{1 - \omega^2 R^2}}$$

$q\bar{q}$ string tension

Same expressions as for a spinning string at $\phi = \text{const.} = \phi_A$

In particular, ignoring for the moment the region I,

$$E_{II} \approx T_{eff} \cdot \frac{2}{\omega} \arcsin \omega L \xrightarrow{\omega L \gg 1} \frac{\pi T_{eff}}{\omega}$$

$$J_{II} \approx T_{eff} \cdot \frac{2}{\omega^2} \left[\arcsin(\omega L) - \omega L \sqrt{1 - (\omega L)^2} \right] \xrightarrow{\omega L \gg 1} \frac{\pi T_{eff}}{\omega^2}$$

\Rightarrow Regge behaviour $J = \text{eff. } E^2$

Conclusions:

- We derived glueball Regge trajectories from the gauge/string correspondence
- $\mathcal{F} = \frac{1}{2} \alpha' E^2 - \alpha'^1 Z_0 E + \frac{1}{2} \alpha'^1 Z_0^2$
- we found qualitative agreement with experiment
- we addressed in a simplified manner the issue of Regge trajectories for dynamical mesons
- future directions :
 - refine the picture of meson Regge trajectories
 - (2+1) : better lattice data
 - revisit Veneziano and Virasoro-Shapiro
 - baryon Regge trajct? ^{simpl.}
 - :