New Challenges to Theory in the Description of LHC Collisions

Multi-Leg Amplitudes at Work

Jeppe R. Andersen

CP3-Origins, University of Southern Denmark

KITP
May 17, 2011

Overview of Talk

Elements of Proton Collisions

Hard scattering, **shower**, **matching to fixed order**
multiple interactions, underlying event...

Jets to the rescue!

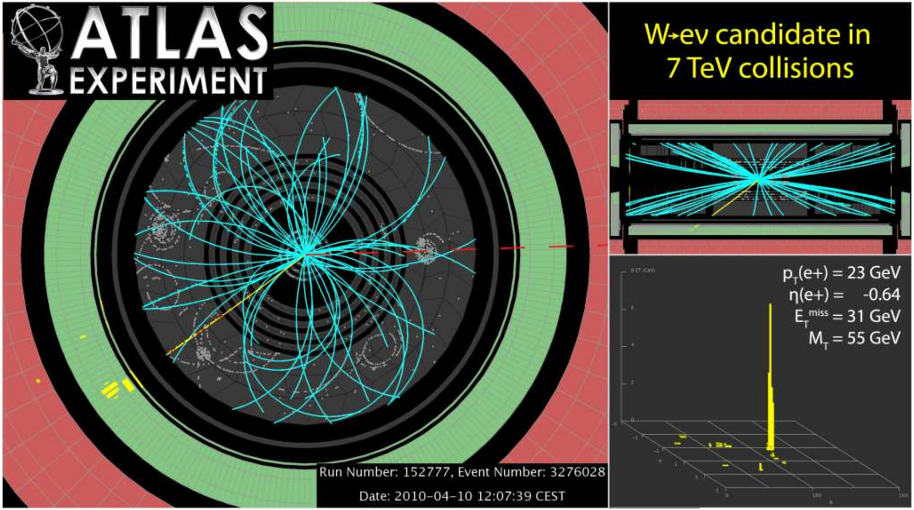
Multi-Jet Predictions

Why we **must** care about HO corrections (in some situations)...
A new approach to multiple, wide-angle emissions from the **hard scattering**
Predictions for dijets, W+jets, H+jets,...Results of **first data**

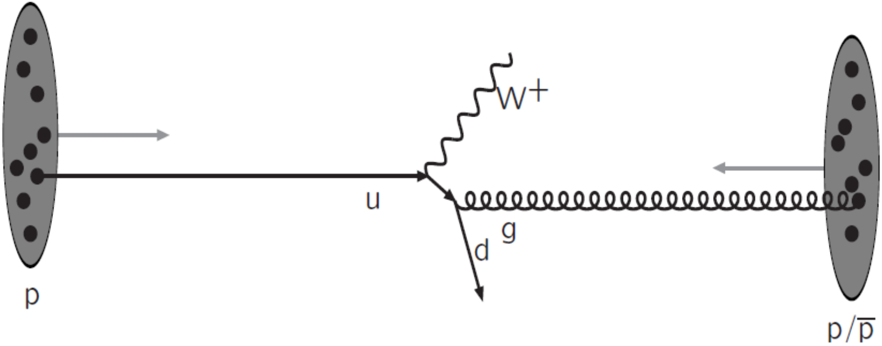
Measuring *CP*-Properties of the gg-Higgs Boson Coupling

Measure couplings before proclaiming discovery of the **SM Higgs Boson**. **Perturbative corrections are large**. **Define observable** which is **stable against these corrections**.

A Real pp-Collision

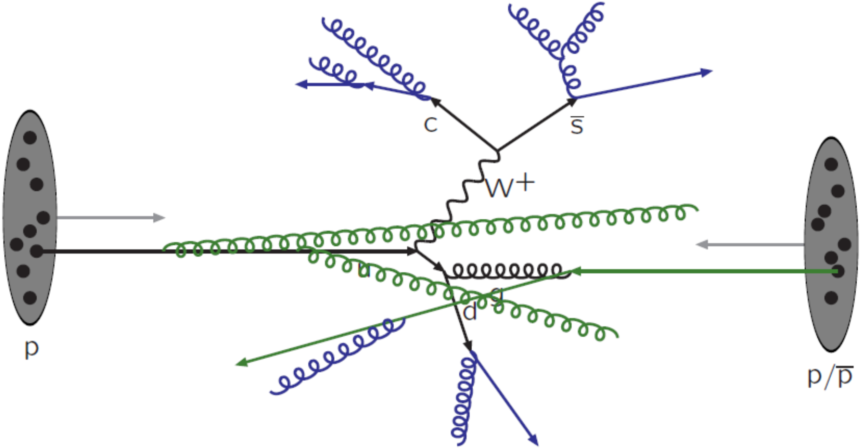


The Theoretical Description, I*)



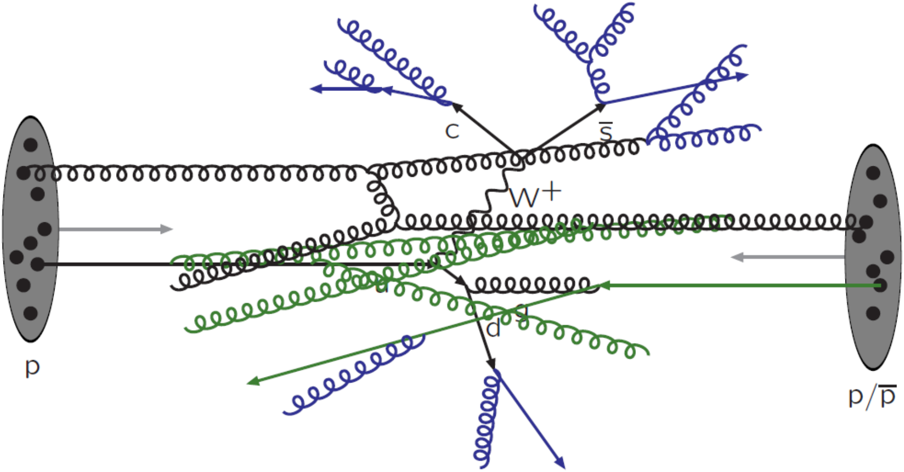
*)Drawing by R. Corke

The Theoretical Description, II*)



*)Drawing by R. Corke

The Theoretical Description, III*)



Drawing by R. Corke

Jet (algorithms) to the Rescue!

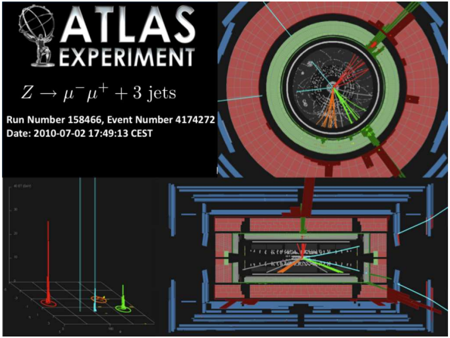
Depending on the question we want to answer, we **may not need** to describe all the **stages of the collision**.

The notion of jets allow us to **compare pure perturbation theory** (few partons) to **experimental observation** (many hadrons)

Transverse Momentum

Rapidity: $y = \ln \frac{E+p_z}{E-p_z}$

still need to ensure (relative) insensitivity to underlying event, multiple interactions. . . ask questions only about relatively hard jets ($p_\perp > 30$ GeV?)



Jet (algorithms) to the Rescue!

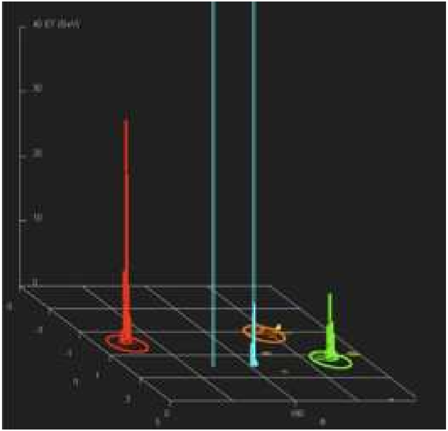
Depending on the question we want to answer, we **may not need** to describe all the **stages of the collision**.

The notion of jets allow us to **compare pure perturbation theory** (few partons) to **experimental observation** (many hadrons)

Transverse Momentum

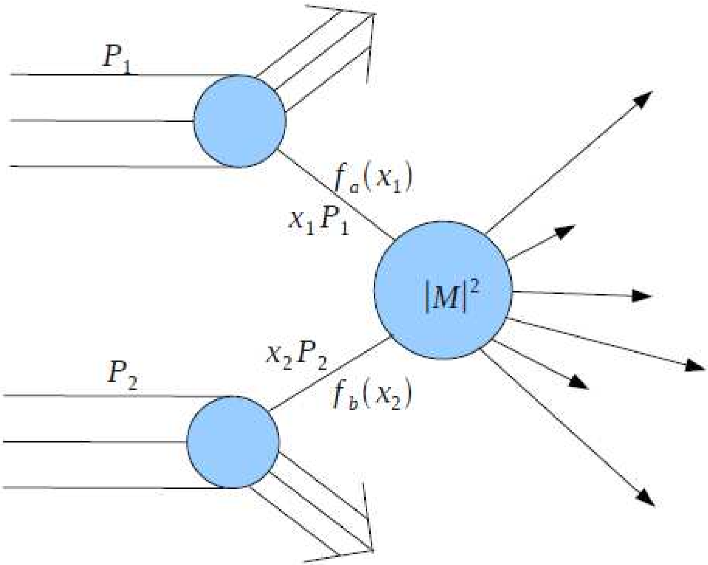
Rapidity: $y = \ln \frac{E+p_z}{E-p_z}$

still need to ensure (relative) insensitivity to underlying event, multiple interactions. . . ask questions only about relatively hard jets ($p_\perp > 30$ GeV?)



Obviously need the jet algorithms to be well-defined both experimentally (many discrete hits) and theoretically (probing singularity structure). Use fastjet!

The Perturbative Description



Why Study Multi-jet Observables?

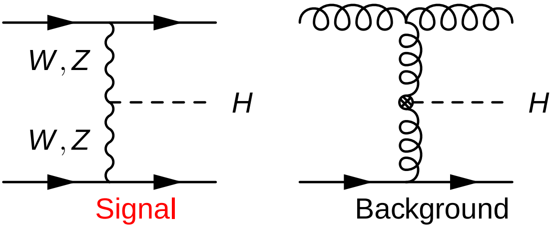
We don't have a choice!

1. Many BSM (e.g. SUSY) particles will have *decay chains* involving the production of jets (e.g. 4 jets + $\not{p}_T$). Calculation of signal is easy (one process), SM contribution is very hard (several processes).
2. All LHC processes involves QCD-charged particles; sometimes the (n+1)-jet cross section is as large as the n-jet cross section!
3. It is a challenge we cannot ignore !

Why Study Multi-jet Observables?

$H + (n \geq 2)$ jets

1. When(!) a fundamental scalar has been found at the LHC we need to determine whether this one is responsible for the observed EWSB
2. Determine the couplings to Z or W by studying the angular distribution of the jets

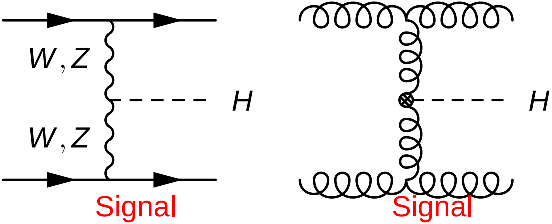


Important to understand the behaviour of the **QCD process** in order to separate **the two channels**

Why Study Multi-jet Observables?

$H + (n \geq 2)$ jets

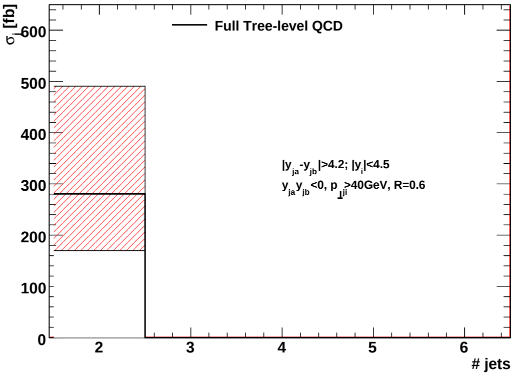
1. Relax standard WBF cuts slightly
2. Can also determine the **Higgs Boson** coupling to gluons by measuring the angular distribution of the jets
3. Can determine if a new particle really is the SM Higgs boson



Important to understand the behaviour of the **QCD process** in order to **extract** *CP*-**properties.**

Are fixed order calculation always sufficient?

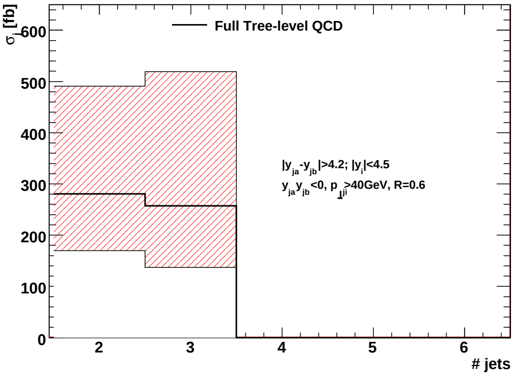
Sometimes the $(n+1)$-jet rate is as large as the n-jet rate
Higgs Boson plus n jets at the LHC at leading order



C.D. White, JRA

Are fixed order calculation always sufficient?

Sometimes the $(n+1)$-jet rate is as large as the n-jet rate
Higgs Boson plus n jets at the LHC at leading order

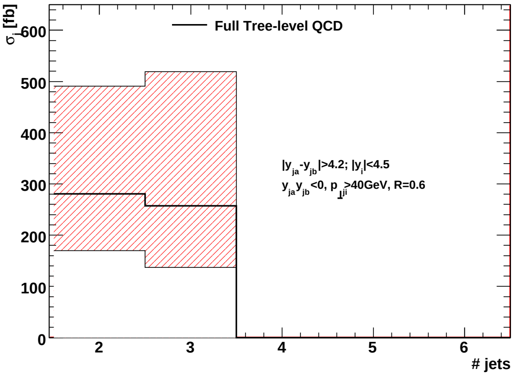


C.D. White, JRA

Need NLO not just to stabilise ren./fac. scale dependence - but also to investigate the final state

Are fixed order calculation always sufficient?

Sometimes the $(n+1)$-jet rate is as large as the n-jet rate
Higgs Boson plus n jets at the LHC at leading order

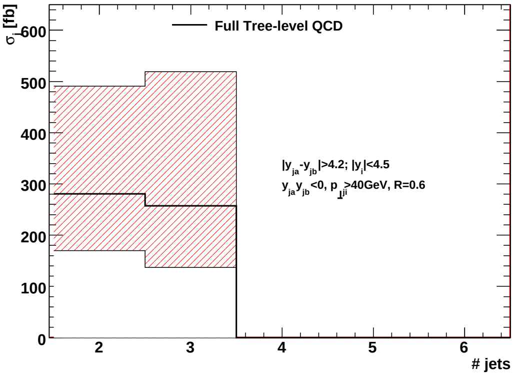


C.D. White, JRA

Indication that we need to go further! However, full fixed order tools **exhausted** (full $2 \to 3$ with a massive leg at two loops **untenable!**).

Are fixed order calculation always sufficient?

Sometimes the $(n+1)$-jet rate is as large as the n-jet rate
Higgs Boson plus n jets at the LHC at leading order



C.D. White, JRA

The method we develop will allow a **stabilisation** of the perturbative series by **resummation**

Multiple ($\geq$ 2) hard jets. . .

Smaller number of jets solved satisfactory (?) already. . . (POWHEG, MC@NLO, NNLO,. . .)
Special radiation pattern from **current-current** scattering
Look into **higher order corrections beyond** “inclusive K-factor”
Concentrate on the **hard, perturbative corrections** relevant for a description of the final state **in terms of jets**?

Goal

Build framework for **all-order summation** (virtual+real emissions). Exact in another limit than the usual soft&collinear. Better suited for describing **radiation relevant for multi-jet** production.

Insight

Can use the insight gained from studying the relevant limit to **guide and improve** analyses: CP-properties of the Higgs-boson couplings

The Challenge, A Solution [?], Status

The Challenge (fka Problem), (in trivial statements)

Hard emission is less suppressed at **increasing collider energies**.
New problem for the LHC-era (W+jets, H+jets, . . .)
NLO gets the **one** hard emission right, but **one may not be sufficient**.
Parton shower does **many emissions**, but **not the hard ones**.
PS+matching is **good at Tevatron**, but **sufficient at LHC?**

A Solution [?]

High Energy Jets (HEJ): The Brief Introduction

Status

What data can already tell us about our perturbative tools

The Challenge, A Solution [?], Status

The Challenge (fka Problem), (in trivial statements)

Hard emission is less suppressed at **increasing collider energies**.
New problem for the LHC-era (W+jets, H+jets, . . .)
NLO gets the **one** hard emission right, but **one may not be sufficient**.
Parton shower does **many emissions**, but **not the hard ones**.
PS+matching is **good at Tevatron**, but **sufficient at LHC?**

A Solution [?]

High Energy Jets (HEJ): The Brief Introduction

Status

What data can already tell us about our perturbative tools

The Challenge, A Solution [?], Status

The Challenge (fka Problem), (in trivial statements)

Hard emission is less suppressed at **increasing collider energies**.
New problem for the LHC-era (W+jets, H+jets, . . .)
NLO gets the **one** hard emission right, but **one may not be sufficient**.
Parton shower does **many emissions**, but **not the hard ones**.
PS+matching is **good at Tevatron**, but **sufficient at LHC?**

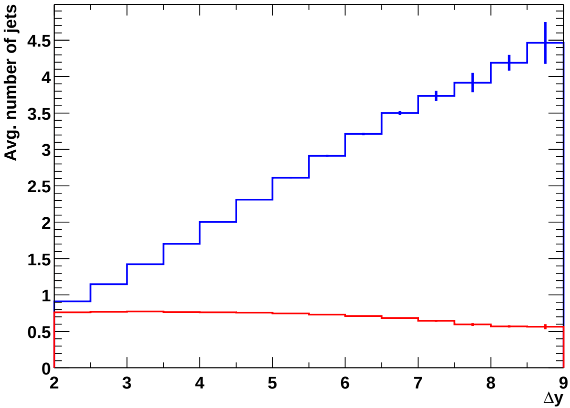
A Solution [?]

High Energy Jets (HEJ): The Brief Introduction

Status

What data can already tell us about our perturbative tools

The Challenge: $\langle \#\text{jets} \rangle$ vs. Δy



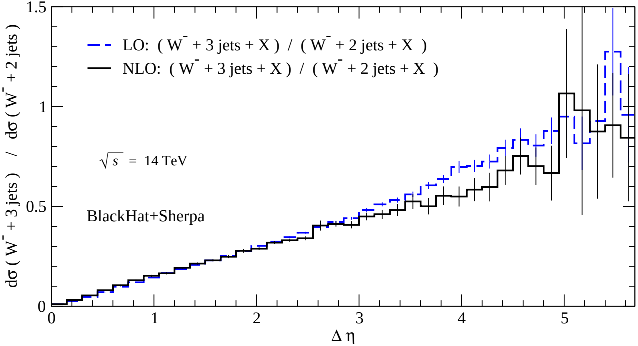
Red: Average number of central ($|y| < 1$) jets.

JRA, V. Del Duca, F. Maltoni, W.J. Stirling, hep-ph/0105146

Basic observation of increasing phase space for hard emissions with increasing Δy is the motivation for e.g. BFKL resummation.

However, don't just take *my* word for it...

W+Multiple Jets @ NLO

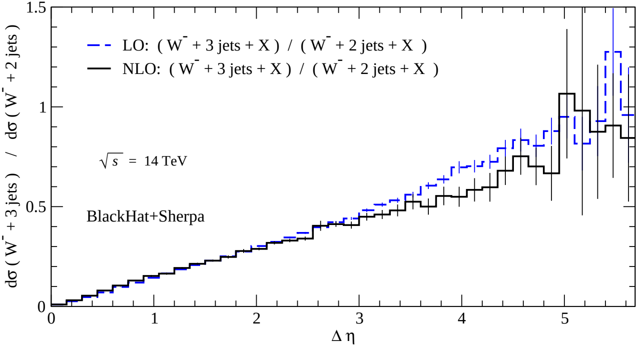


BlackHat, arXiv:0912.4927

The inclusive 3-jet rate is large compared to the inclusive 2-jet rate, even for normal rapidity spans obviously, the inclusive 3-jet rate "ought to" be smaller than the inclusive 2-jet rate.

The large contribution from real radiative corrections to W+dijets is not revealed by the inclusive K-factor (actually less than one)

W+Multiple Jets @ NLO



BlackHat, arXiv:0912.4927

All calculational methods and processes will agree on the opening of phase space as Δy increases

The mechanism for emission differ between processes (WBF vs. GF) and calculational methods (full NLO, shower, . . .). Can be tested against data!

HEJ (High Energy Jets)

Goal (inspired by the great Fadin & Lipatov)

Sufficiently **simple** model for hard radiative corrections that the all-order sum can be evaluated explicitly (completely exclusive)

but. . .

Sufficiently **accurate** that the description is relevant

To boldly go. . .

In a previous episode at a CERN seminar series:
A wise man said. . .

"Use known results to gain deeper insights. . . "

young* ~~postdoc~~ faculty

"Use insight to gain yet unknown results. . . "

*or so I **still** prefer to think

To boldly go…

**In a previous episode at a CERN seminar series:
A wise man said…**

"Use known results to gain deeper insights…"

young* ~~postdoc~~ faculty

"Use insight to gain yet unknown results…"

---

*or so I **still** prefer to think

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Soft limit $\rightarrow$ **eikonal approximation** $\rightarrow$ enters many resummation formalisms.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider. . .

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Soft limit $\rightarrow$ **eikonal approximation** $\rightarrow$ enters many resummation formalisms.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider…

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Soft limit $\longrightarrow$ **eikonal approximation** $\longrightarrow$ enters many resummation formalisms.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider. . .

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Soft limit $\longrightarrow$ **eikonal approximation** $\longrightarrow$ enters many resummation formalisms.

Like all good limits, the eikonal approximation is applied **outside its strict region of validity**.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider. . .

Regge and High Energy Factorisation

In the High Energy Limit, $2 \to 2$ **scattering amplitudes** are **dominated** by the ***t*-channel exchange** of the particle of the **highest spin** allowed by the scattering theory

$$\mathcal{M}^{p_a p_b \to p_1 p_2} \xrightarrow{\text{Regge limit}} \hat{s}^{\hat{\alpha}(\hat{t})} \, \gamma(\hat{t})$$

Regge (1959)

$\hat{s} = (p_a + p_b)^2, \hat{t} = (p_a - p_1)^2$, Regge limit: $\hat{s} \to \infty, \hat{t}$ fixed.

Multi-particle generalisation?

$$\mathcal{M}^{p_a p_c \to p_{a'} p_b p_{c'}} \xrightarrow{\text{Multi Regge limit}}$$

$$\hat{s}_1^{\hat{\alpha}(\hat{t}_1)} \, \hat{s}_2^{\hat{\alpha}(\hat{t}_2)} \, \gamma(\hat{t}_1, \hat{t}_2, \frac{s_{12}}{s_1 s_2})$$

MRK: $\hat{s}_{12}, \hat{s}_1, \hat{s}_2 \to \infty, t_1, t_2$ fixed

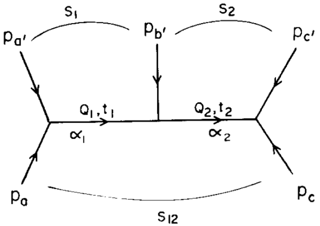
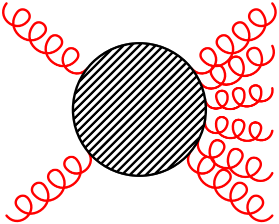


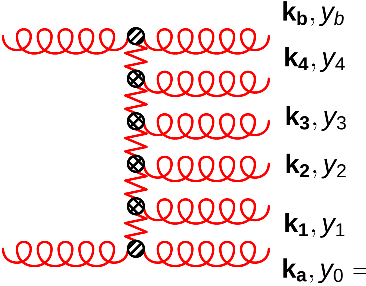
Fig. 2.1. Five-particle diagram showing notation.

Brower, DeTAR, Weis (1974)



High Energy Limit $\longrightarrow$

$|\hat{t}|$ fixed, $\hat{s} \to \infty$



$$\mathcal{A}^R_{2\to 2+n} = \frac{\Gamma_{A'A}}{q_0^2}\left(\prod_{i=1}^{n} e^{\omega(q_i)(y_{i-1}-y_i)}\frac{V^{J_i}(q_i,q_{i+1})}{q_i^2 q_{i+1}^2}\right)e^{\omega(q_{n+1})(y_n-y_{n+1})}\frac{\Gamma_{B'B}}{q_{n+1}^2}$$

$q_i = \mathbf{k}_a + \sum_{l=1}^{i-1}\mathbf{k}_l$

LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

Maintain (at LL) terms of the form

$$\left(\alpha_s \ln \frac{\hat{s}_{ij}}{|\hat{t}_i|}\right)$$

to all orders in α_s.

At LL only gluon production; at NLL also quark–anti-quark pairs produced. Approximation of any-jet rate possible.

Comparison of 3-jet scattering amplitudes

Universal behaviour of scattering amplitudes in the HE limit:

$$\forall i \in \{2,\ldots,n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$
$$\forall i,j : |p_{i\perp}| \approx |p_{j\perp}|$$

$$\left|\overline{\mathcal{M}}^{MRK}_{gg\to g\cdots g}\right|^2 = \frac{4\,s^2}{N_C^2-1}\;\frac{g^2\,C_A}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2}.$$

$$\left|\overline{\mathcal{M}}^{MRK}_{qg\to qg\cdots g}\right|^2 = \frac{4\,s^2}{N_C^2-1}\;\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_A}{|p_{n\perp}|^2},$$

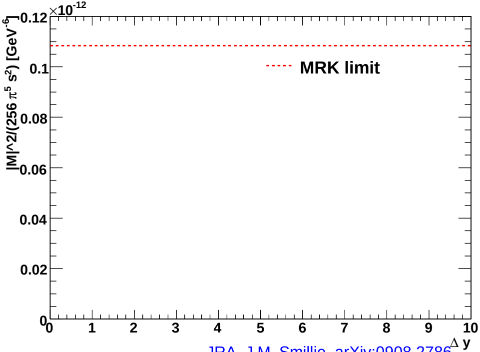
$$\left|\overline{\mathcal{M}}^{MRK}_{qQ\to qg\cdots Q}\right|^2 = \frac{4\,s^2}{N_C^2-1}\;\frac{g^2\,C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\,g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\,C_F}{|p_{n\perp}|^2},$$

Allow for analytic resummation (BFKL equation).
However, how well does this actually approximate the amplitude?

Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.

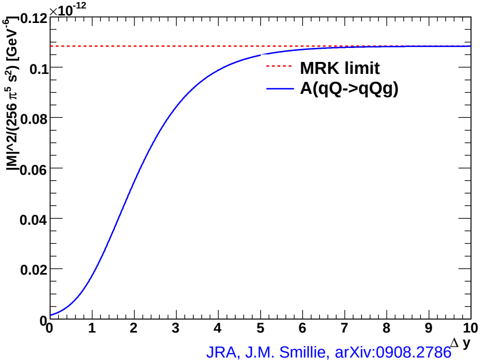


JRA, J.M. Smillie, arXiv:0908.2786

Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:

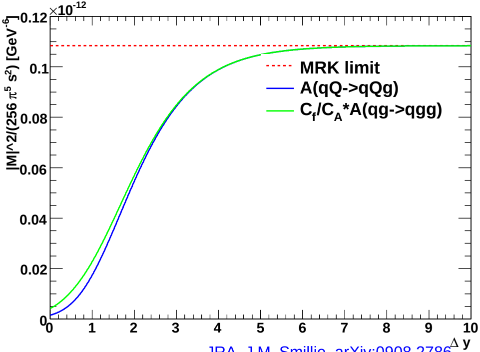
40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.



Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.

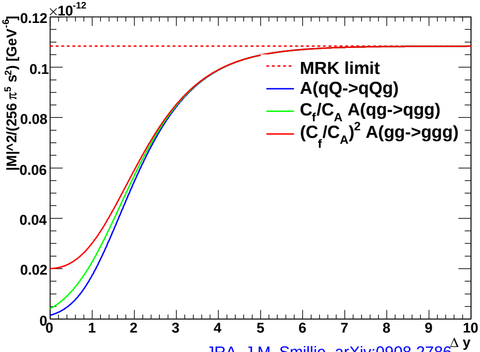


JRA, J.M. Smillie, arXiv:0908.2786

Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.

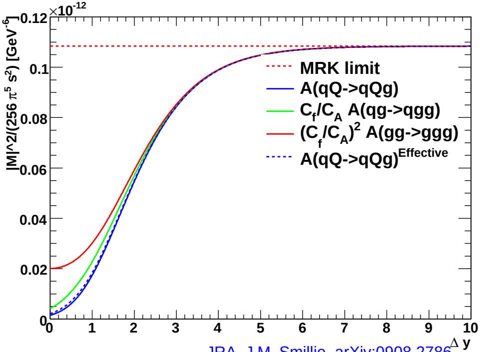


JRA, J.M. Smillie, arXiv:0908.2786

Comparison of 3-jet scattering amplitudes

Study just a slice in phase space:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.



JRA, J.M. Smillie, arXiv:0908.2786

1) Inspiration from Fadin&Lipatov: dominance by t-channel
2) No kinematic approximations in invariants (denominator)
3) Accurate definition of currents (coupling through t-channel exchange)
4) Gauge invariance. Not just asymptotically.

Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \to q(1)Q(2)$:

$$M_{q^-Q^-\to q^-Q^-} = \langle 1|\mu|a\rangle \frac{g^{\mu\nu}}{t} \langle 2|\nu|b\rangle$$

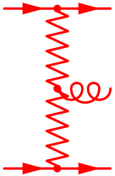
t**-channel factorised**: Contraction of (local) currents across t-channel pole

$$\left|\overline{\mathcal{M}}^t_{qQ\to qQ}\right|^2 = \frac{1}{4\left(N_C^2-1\right)} \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\, C_F\, \frac{1}{t_1}\right) \cdot \left(g^2\, C_F\, \frac{1}{t_2}\right).$$

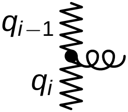
Extend to $2 \to n \ldots$

J.M.Smillie and JRA: arXiv:0908.2786

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles

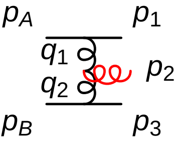


$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

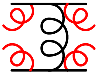


$$\mu \quad V^\mu(q_{i-1}, q_i)$$

$$j^\nu = \overline{\psi}\gamma^\nu\psi$$



$$\begin{aligned} V^\rho(q_1, q_2) = &-(q_1+q_2)^\rho \\ &+\frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_A}+\frac{p_2\cdot p_B}{p_A\cdot p_B}+\frac{p_2\cdot p_n}{p_A\cdot p_n}\right)+p_A\leftrightarrow p_1 \\ &-\frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_B}+\frac{p_2\cdot p_A}{p_B\cdot p_A}+\frac{p_2\cdot p_1}{p_A\cdot p_1}\right)-p_B\leftrightarrow p_3. \end{aligned}$$



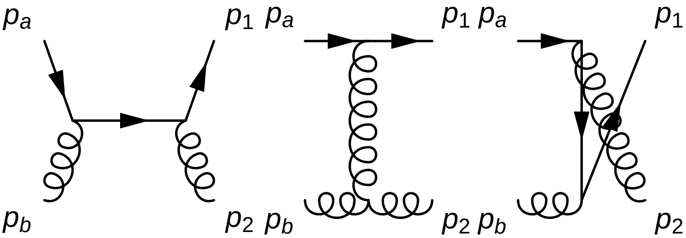
Building Blocks for an Amplitude

$p_g \cdot V = 0$ can easily be checked (gauge invariance)
The approximation for $qQ \to qgQ$ is given by

$$\left|\overline{\mathcal{M}}^t_{qQ\to qgQ}\right|^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{qQ\to qQ}\right\|^2 \cdot \left(g^2\,C_F\,\frac{1}{t_1}\right)\cdot\left(g^2\,C_F\,\frac{1}{t_2}\right) \cdot\left(\frac{-g^2 C_A}{t_1 t_2}\,V^\mu(q_1,q_2)V_\mu(q_1,q_2)\right).$$

Quark-Gluon Scattering

"What happens in 2 $\rightarrow$ 2-processes with gluons? Surely the t-channel factorisation is spoiled!"



Direct calculation ($q^- g^- \rightarrow q^- g^-$):

$$M = \frac{g^2}{\hat{t}} \times \frac{p_{2\perp}^*}{|p_{2\perp}|} \left(t^2_{ae} t^b_{e1} \sqrt{\frac{p_b^-}{p_2^-}} - t^b_{ae} t^2_{e1} \sqrt{\frac{p_2^-}{p_b^-}} \right) \langle b|\sigma|2\rangle \times \langle 1|\sigma|a\rangle.$$

Complete t-channel factorisation!

J.M.Smillie and JRA

Quark-Gluon Scattering

For the helicity choices where a qQ-channel exists, the t-channel current generated by a gluon in qg scattering is that of a quark, but with a colour factor

$$\frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}$$

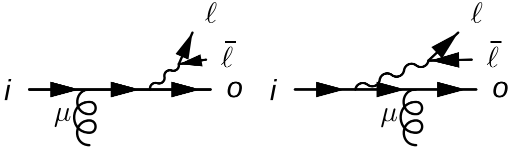
instead of C_F. Tends to C_A in MRK limit.

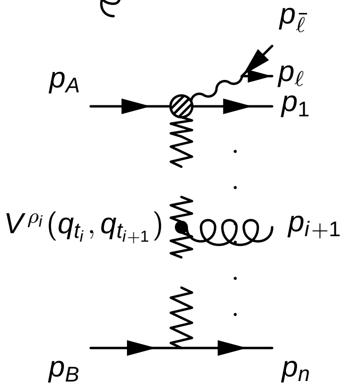
Similar results for e.g. $g^+g^- \to g^+g^-$. **Exact, complete** t**-channel factorisation**.

By using the formalism of **current-current scattering**, we get a better description of the t-channel pole than by using just the BFKL kinematic limit.

W+Jets

Two currents to calculate for $W + jets$:



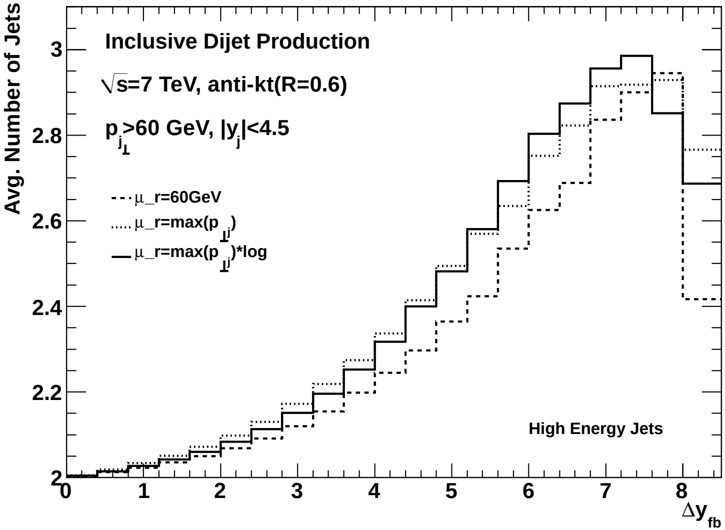


All-Orders and Regularisation

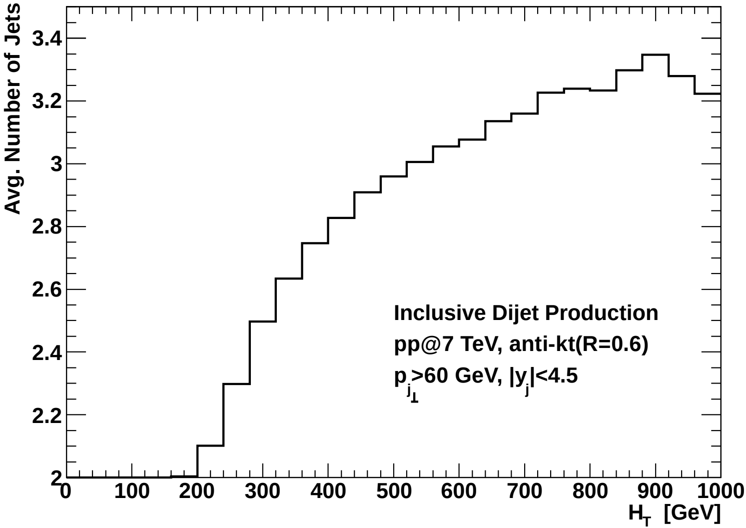
- Have prescription for $2 \to n$ matrix element, including virtual corrections: Lipatov Ansatz $1/t \to 1/t\,\exp(-\omega(t)\Delta y_{ij})$
- Organisation of cancellation of IR (soft) divergences is easy
- Can calculate the sum over the n-particle phase space explicitly ($n \sim 30$) to get the all-order corrections (just as if one had provided all the $N^{30}LO$ matrix elements and a regularisation procedure)
- **Match** to n-jet tree-level where this can be evaluated in reasonable time

J.M. Smillie, JRA arXiv:0908.2786, arXiv: 0910:5113, arXiv:1101.5394

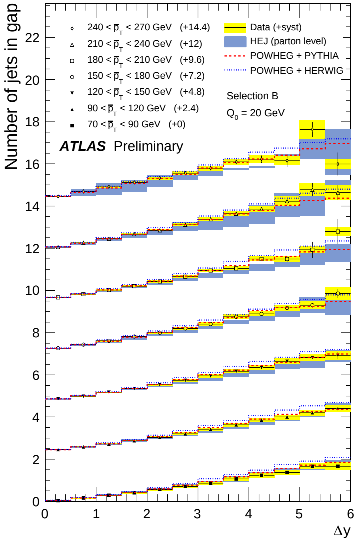
Results for pure dijets



Avg. jets vs H_T



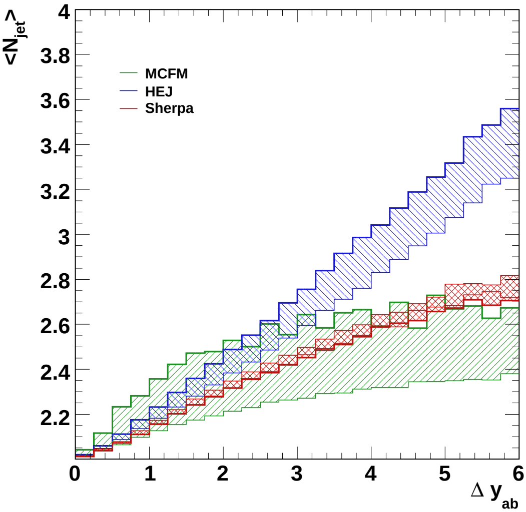
Results from Atlas



ATLAS-CONF-2011-038

Why is $\langle \#\text{jets} \rangle$ in dijets interesting?

Similarities to H+dijets



Dijets can help in investigating jet veto efficiencies, . . .
Relative large differences in perturbative descriptions.

JRA, J. Campbell, S. Höche,

arXiv:1003.1241

CP Properties of Higgs-Boson Couplings from Hjj through Gluon Fusion

Stabilising the Extraction against Higher Order Corrections

Why Hjj, The Problem, The Solution

Why study Higgs Boson production in Association with Dijets?

The distribution in the **azimuthal angle** between the **two** jets in *Hjj* allows for a **clean extraction** of CP properties

The Problem

. . . in a region of phase space where the **perturbative corrections are large**.

How do we deal with events with **three or more** jets?

The Solution

By constructing an azimuthal observable, which takes into account the **information from all the jets** of the event!

Why Hjj, The Problem, The Solution

Why study Higgs Boson production in Association with Dijets?

The distribution in the **azimuthal angle** between the **two** jets in *Hjj* allows for a **clean extraction** of CP properties

The Problem

. . . in a region of phase space where the **perturbative corrections are large**.
How do we deal with events with **three or more** jets?

The Solution

By constructing an azimuthal observable, which takes into account the **information from all the jets** of the event!

Why Hjj, The Problem, The Solution

Why study Higgs Boson production in Association with Dijets?

The distribution in the **azimuthal angle** between the **two** jets in *Hjj* allows for a **clean extraction** of CP properties

The Problem

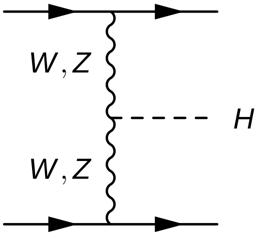
. . . in a region of phase space where the **perturbative corrections are large**.

How do we deal with events with **three or more** jets?

The Solution

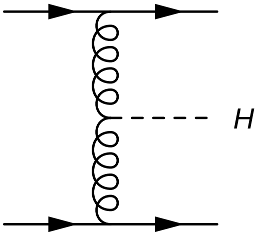
By constructing an azimuthal observable, which takes into account the **information from all the jets** of the event!

Higgs Couplings through Azimuthal Correllations



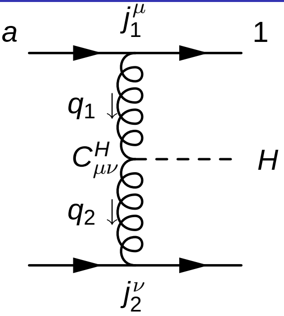
Considerations for Weak Boson Fusion

Higgs Couplings through Azimuthal Correllations



...and gluon fusion (Higgs coupling to gluons through top loop)

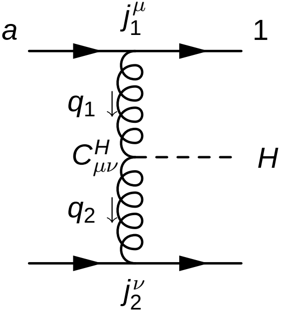
Higgs Couplings through Azimuthal Correllations



$$\mathcal{M} \propto \frac{j_1^\mu\ C^H_{\mu\nu}\ j_2^\nu}{t_1\ t_2}, \qquad j_1^\mu = \overline{\psi}_1 \gamma^\mu \psi_a$$

$$C_H^{\mu\nu} = a_2\ (q_1 q_2 g^{\mu\nu}\ -\ q_1^\nu q_2^\mu) + a_3\ \varepsilon^{\mu\nu\rho\sigma}\ q_{1\rho}\ q_{2\sigma}.$$

Higgs Couplings through Azimuthal Correllations



$$\mathcal{M} \propto \frac{j_1^\mu \, C^H_{\mu\nu} \, j_2^\nu}{t_1 \, t_2}, \qquad j_1^\mu = \overline{\psi}_1 \gamma^\mu \psi_a$$

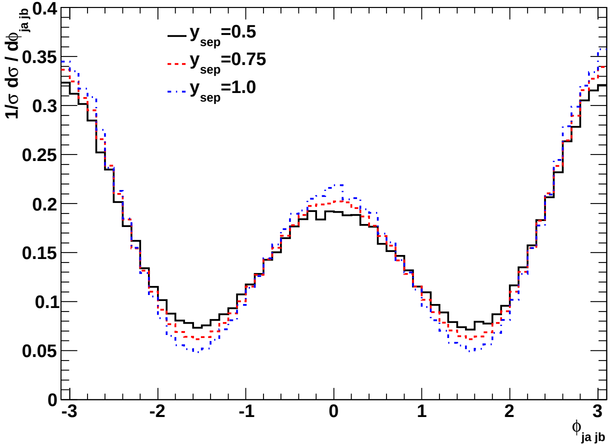
$$C_H^{\mu\nu} = a_2 \, (q_1 q_2 g^{\mu\nu} \; - \; q_1^\nu q_2^\mu) + a_3 \, \varepsilon^{\mu\nu\rho\sigma} \, q_{1\rho} \, q_{2\sigma}.$$

Take e.g. the term $\varepsilon^{\mu\nu\rho\sigma} \, q_{1\rho} \, q_{2\sigma}$: for $|p_{1,z}| \gg |p_{1,x,y}|$ and for small energy loss (i.e. $p_{a,e} \sim p_{1,e}$):

$$\left[j_1^0 \, j_2^3 - j_1^3 \, j_2^0 \right] (\mathbf{q}_{1\perp} \times \mathbf{q}_{2\perp}) \, .$$

In this limit, the azimuthal dependence of the propagators is also suppressed: $|\mathcal{M}|^2$:$\sin^2(\phi)$ **(CP-odd)**, $\cos^2(\phi)$ **(CP-even)**.

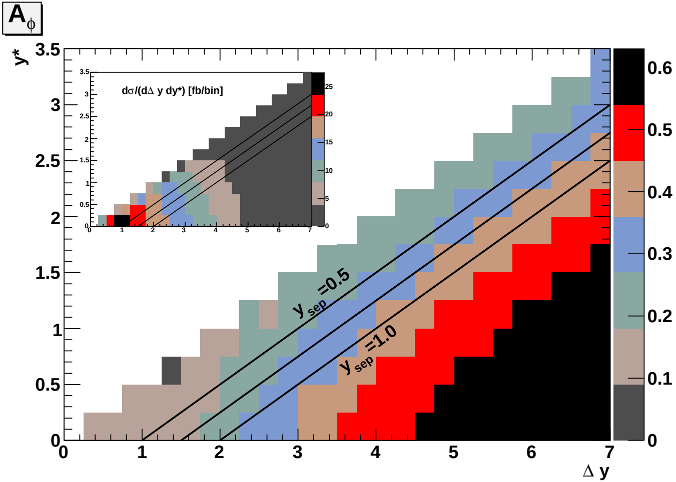
Azimuthal distribution



JRA, K. Arnold, D. Zeppenfeld, arXiv:1001.3822

$$CP\text{-even}, \; p_{j\perp} > 40 \text{ GeV}, \quad y_{ja} < y_h < y_{jb},$$
$$|y_{j_a,j_b}| < 4.5, \min\left(|y_h - y_{j_a}|, |y_h - y_{j_b}|\right) > y_{\text{sep}}.$$

Signature and Cross Section

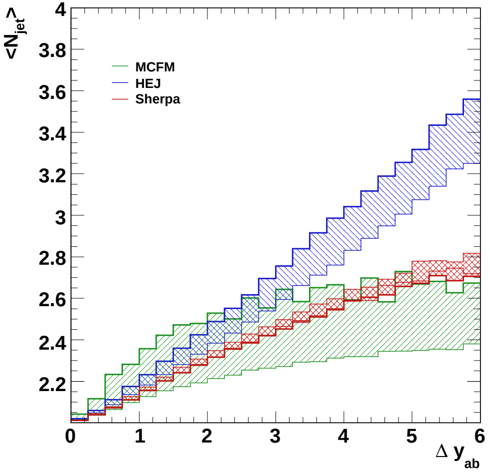


$$\Delta y = |y_{j_a} - y_{j_b}|, \quad y^* = y_h - \frac{y_{j_a}+y_{j_b}}{2}.$$

JRA, K. Arnold, D. Zeppenfeld

Rapidity separation between the jets and the Higgs Boson **enhance the azimuthal correlation**.

Increasing Rapidity Span → Increasing Number of Jets



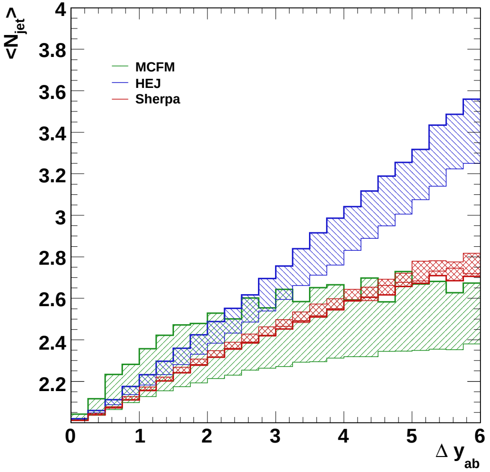
J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

2 hardest jets?

Increasing Rapidity Span → Increasing Number of Jets



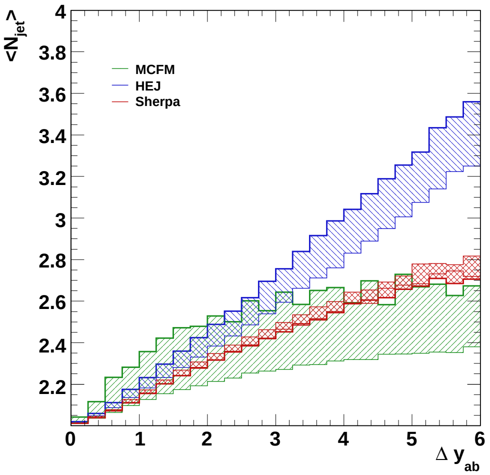
J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

2 hard jets furthest apart in rapidity?

Increasing Rapidity Span → Increasing Number of Jets



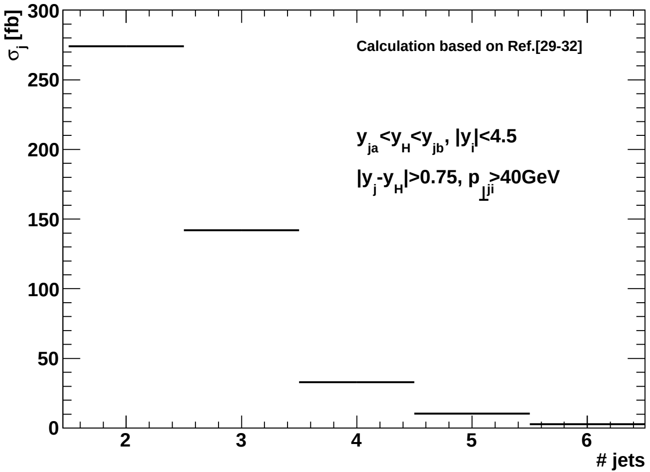
J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.1241

All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

Significant washing out of the azimuthal correlation observed at tree-level *hjj*

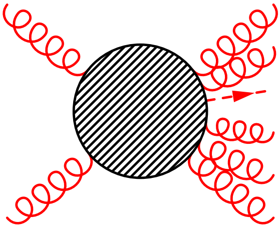
Many Jets!



Calculation based on all-order approximant to the *n*-particle matrix element, which reproduces the exact result in the limit of large invariant mass between all particles.

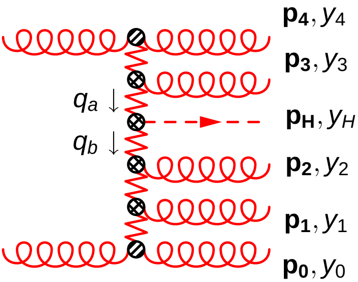
JRA&C.D. White, JRA&J.M. Smillie

Develop Insight Into the Perturbative Corrections



High Energy Limit $\longrightarrow$

$|p_{\perp,i}|$ fixed, $\hat{s}_{ij} \to \infty$

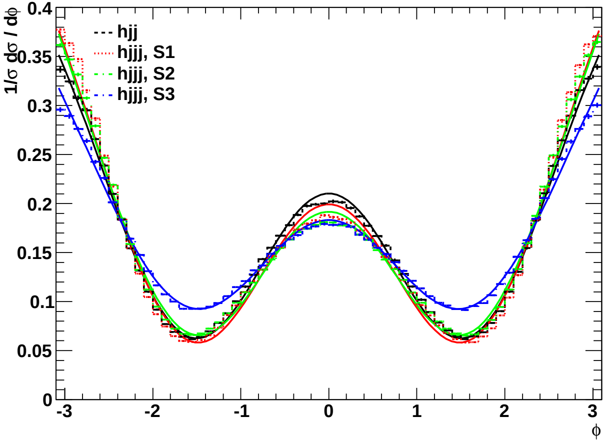


$$\left|\mathcal{M}_{gg\to g\cdots ghg\cdots g}\right|^2 \to \frac{4\hat{s}^2}{N_C^2-1}\left(\prod_{i=1}^{j}\frac{C_A\,g_s^2}{\mathbf{p}_{i\perp}^2}\right)\frac{|C^H(\mathbf{q}_{a\perp},\mathbf{q}_{b\perp})|^2}{\mathbf{q}_{a\perp}^2\ \mathbf{q}_{b\perp}^2}\left(\prod_{i=j+1}^{n}\frac{C_A\,g_s^2}{\mathbf{p}_{i\perp}^2}\right)$$

$$C^H(\mathbf{q}_{a\perp},\mathbf{q}_{b\perp}) = -i\,\frac{\alpha_s}{3\pi v}\,\mathbf{q}_{a\perp}\cdot\mathbf{q}_{b\perp}, \quad y_0 < \cdots < y_j < y_H < y_{j+1} < y_n$$

The **High Energy Limit** tells us to investigate the **azimuthal angle** between the **sum of the jet vectors** either side in rapidity of the Higgs Boson!

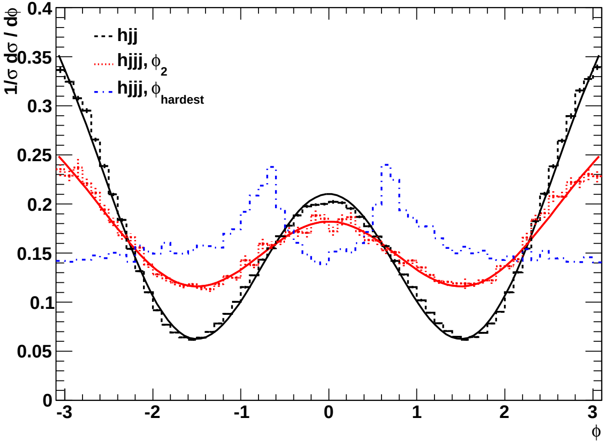
And It Even Works!



JRA, K. Arnold, D. Zeppenfeld, arXiv:1001.3822

Three subsamples of tree-level three-jet events: two jets on same side of the Higgs boson parallel (S1), perpendicular (S2) or anti-parallel (S3). Azimuthal correlation almost unchanged from hjj.

...Much Better Than Any Alternative



JRA, K. Arnold, D. Zeppenfeld, arXiv:1001.3822

Two hardest jets on one side, and the softest on the other (all above 40GeV - $1/3$ of inclusive 3-jet cross section). Using **just the two hardest** jets gives **unsatisfactory** result.

Summary

1. Despite the enormous success of the Unitaritists, there might still be some work left for the rest of us to do in the perturbative description of LHC scatterings
2. HEJ is a new perturbative tool for the description of multi-jet events at high energy colliders
   1. Simplify pert. corrections by concentrating on widely separated emissions
   2. Filling in the details of each jet (soft, collinear) is a job left for a parton shower
3. The insight gained from the study has already improved analysis for the LHC
4. Even the data already collected will shed light on the multi-jet environment in the new high energy domain.