Finiteness patterns and nonrenormalization theorems in extended supergravities

KITP, Santa Barbara

K.S. Stelle

April 27, 2011

Main references for this talk:

G. Bossard, P.S. Howe & K.S.S., 0901.4661, 0908.3883 & 1009.0743

H. Elvang & M. Kiermaier, 1007.4813

N. Beisert, H. Elvang, D. Freedman, M. Kiermaier, A. Morales & S. Stieberger, 1009.1643

General history:
Counterterms: Deser, Kay & KSS; Galperin, Ivanov, Kalitsin, Ogievetsky & Sokatchev; Howe ,Townsend & KSS; Kallosh

Unitarity-based calculations: Bern, Carrasco, Dixon, Johansson & Roiban

Algebraic renormalization: Dixon; Howe, Lindstrom & White; Piguet & Sorella; Hennaux; Stora; Baulieu & Bossard

Ectoplasm: Gates, Grisaru, Knut-Whelau, & Siegel; Berkovits & Howe; Bossard, Howe & KSS; Bossard, Howe, Lindstrom, KSS & Wulff

Superspace cohomology: Bonora, Pasti & Tonin

Spinorial cohomology: Cederwall, Nilsson & Tsimpis; Howe & Tsimpis; Berkovits & Howe

Cohomological non-renormalization: Bossard, Howe & KSS

Manifest E7 duality & SU(8) anomalies: Bossard, Hillman & Nicolai; Marcus

Duality implications for counterterms: Elvang & Kiermaier; Bossard, Howe & KSS; Beisert, Elvang, Freedman, Kiermaier, Morales & Stiebeger; Elvang, Freedman & Kiermaier

D=4 counterterm uniqueness: Drummond, Heslop, Howe & Kerstan

Laplace equations for counterterms: Green & Vanhove

Manifestly E7 invariant counterterms: Howe & Lindstrom; Kallosh

Are there quantum miracles happening in maximal supergravity?

Outline

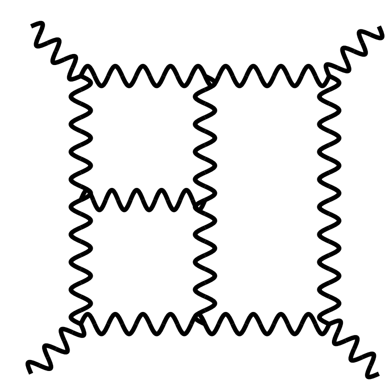
- Nonrenormalization theorems and BPS degree
- Unitarity-based calculations
- Ectoplasm & superspace cohomology
- Duality constraints on counterterms
- Infinities nix infinities: dimensional reduction & duality
- Current outlook

Ultraviolet Divergences in Gravity

- Simple power counting in gravity and supergravity theories leads to a naïve degree of divergence

$$\Delta = (D-2)L+2$$

in D spacetime dimensions. So, for D=4, L=3, one expects $\Delta = 8$. In dimensional regularization, only logarithmic divergences are seen ($\frac{1}{\epsilon}$ poles, $\epsilon = D-4$), so 8 powers of momentum would have to come out onto the external lines of such a diagram.



- Local supersymmetry implies that the pure curvature part of such a D=4, 3-loop divergence candidate must be built from the square of the Bel-Robinson tensor

Deser, Kay & K.S.S 1977

$$\int \sqrt{-g}T_{\mu\nu\rho\sigma}T^{\mu\nu\rho\sigma}\ , \qquad T_{\mu\nu\rho\sigma} = R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}R_{\rho\alpha\sigma\beta} + {}^{*}R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\,{}^{*}R_{\rho\alpha\sigma\beta}$$

- This is directly related to the α'^3 corrections in the superstring effective action, except that in the string context such contributions occur with finite coefficients. In string theory, the corresponding question is how poles might develop in $(\alpha')^{-1}$ as one takes the zero-slope limit $\alpha' \to 0$ and how this bears on the ultraviolet properties of the corresponding field theory.

Berkovits 2007

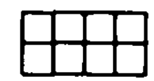
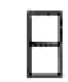
Green, Russo & Vanhove 2007, 2010

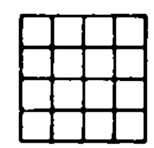
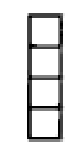
- The consequences of supersymmetry for the ultraviolet structure are not restricted to the requirement that counterterms be supersymmetric invariants.
- There exist more powerful "non-renormalization theorems," the most famous of which excludes infinite renormalization in D=4, N=1 supersymmetry of chiral invariants, given in N=1 superspace by integrals over just half the superspace: $\int d^2\theta W(\phi(x,\theta,\bar{\theta}))\ , \quad \bar{D}\phi = 0$ (c.f. full superspace $\int d^4\theta L(\phi,\bar{\phi})$)
- However, maximally extended SYM and supergravity theories do not have formalisms with all supersymmetries linearly realised "off-shell" in superspace. So the power of such nonrenormalization theorems is restricted to the off-shell linearly realizable subalgebra.

- The degree of "off-shell" supersymmetry is the maximal supersymmetry for which the algebra can close without use of the equations of motion.
- Knowing the extent of this off-shell supersymmetry is tricky, and may involve formulations (*e.g.* harmonic superspace) with infinite numbers of auxiliary fields. Galperin, Ivanov, Kalitsin, Ogievetsky & Sokatchev
- For maximal N=4 Super Yang-Mills and maximal N=8 supergravity, the linearly realizable supersymmetry has been known since the 1980's to be at least *half* the full supersymmetry of the theory. So at that time the first generally allowed counterterms were expected to have "1/2 BPS" structure as compared to the full supersymmetry of the theory.

- The 3-loop R^4 candidate maximal supergravity counterterm has a structure very similar to that of an F^4 N=4 super Yang-Mills invariant. Both of these are 1/2 BPS invariants, involving integration over just half the corresponding full superspaces:

Howe, K.S.S. & Townsend 1981
Kallosh 1981

$$\Delta I_{SYM} = \int (d^4\theta d^4\bar{\theta})_{105} \, \mathrm{tr}(\phi^4)_{105}$$  ***105*** ϕ_{ij}  ***6*** of SU(4)

$$\Delta I_{SG} = \int (d^8\theta d^8\bar{\theta})_{232848} (W^4)_{232848}$$  ***232848*** W_{ijkl}  ***70*** of SU(8)

- Versions of these supergravity and SYM operators do occur as counterterms at one loop in D=8. However, the one-loop level often has special renormalization features, so one needs to be careful not to make unwarranted conclusions about the general acceptability of these counterterms.

- Of course, there are other symmetries in supergravity beside diffeomorphism invariance and supersymmetry. In particular, D=4 N=8 supergravity also has a rigid nonlinearly realised E_7 symmetry. At leading order, this symmetry is realised by constant shifts of the 70 scalars.

- The R^4 candidate satisfies at least the minimal requirement of invariance under such constant shifts of the 70 scalars because, at the leading 4-particle order, the integrand may be written such that every scalar field is covered by a derivative.

Unitarity-based calculations

Bern, Carrasco, Dixon, Johansson & Roiban 2007 ... 2011

- The calculational front has made impressive progress since the late 1990s.
- These have led to unanticipated and surprising cancellations at the 3- and 4-loop orders, yielding new lowest possible orders for the super Yang-Mills and supergravity divergence onsets:

Max. SYM first divergences, current lowest possible orders.

Dimension D	10	8	7	6	5	4
Loop order L	1	1	2	3	6?	∞
BPS degree	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Gen. form	$\partial^2 F^4$	F^4	$\partial^2 F^4$	$\partial^2 F^4$	$\partial^2 F^4$	finite

Blue: known divergences

Max. supergravity first divergences, current lowest possible orders.

Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	6?	5?
BPS degree	0	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	0	$\frac{1}{4}$
Gen. form	$\partial^{12} R^4$	$\partial^{10} R^4$	R^4	$\partial^6 R^4$	$\partial^6 R^4$	$\partial^{12} R^4$	$\partial^4 R^4$

Algebraic Renormalization

Dixon; Howe, Lindstrom & White; Piguet & Sorella; Hennaux; Stora; Baulieu & Bossard

- Another approach to analyzing the divergences in supersymmetric gauge theories, using the full supersymmetry, begins with the Callan-Symanzik equation for the renormalization of the Lagrangian as a operator insertion, governing, *e.g.*, mixing with the half-BPS operator $S^{(4)} = \mathrm{tr}(F^4)$. Letting the classical action be $S^{(2)}$, the C-Z equation in dimension D is $\mu\dfrac{\partial}{\partial\mu}[S^{(2)}\cdot\Gamma] = (4-D)[S^{(2)}\cdot\Gamma] + \gamma_{(4)}g^{2n_{(4)}}[S^{(4)}\cdot\Gamma] + \cdots$, where $n_{(4)} = 4,\ 2,\ 1$ for $D = 5,\ 6,\ 8$.
- From this one learns that $(n_{(4)} - 1)\beta_{(4)} = \gamma_{(4)}$ so the beta function for the $S^{(4)} = \mathrm{tr}(F^4)$ operator is determined by the anomalous dimension $\gamma_{(4)}$.

- Combining the supersymmetry generator with a commuting spinor parameter to make a scalar operator $Q = \bar{\varepsilon} Q$, the expression of SUSY invariance for a D-form density in D-dimensions is $Q\mathcal{L}_D + d\mathcal{L}_{D-1} = 0$. Combining this with the SUSY algebra $Q^2 = -i(\bar{\varepsilon}\gamma^\mu\varepsilon)\partial_\mu$ and using the Poincaré Lemma, one finds $i_{i(\bar{\varepsilon}\gamma\varepsilon)}\mathcal{L}_D + S_{(Q)|\Sigma}\mathcal{L}_{D-1} + d\mathcal{L}_{D-2} = 0$.

- Hence, one can consider cocycles of the extended nilpotent differential $d + S_{(Q)|\Sigma} + i_{i(\bar{\varepsilon}\gamma\varepsilon)}$ acting on formal form-sums $\mathcal{L}_D + \mathcal{L}_{D-1} + \mathcal{L}_{D-2} + \cdots$.

- The supersymmetry Ward identities then imply that the whole cocycle must be renormalized in a coherent way. In order for an operator like $S^{(4)}$ to mix with the classical action $S^{(2)}$, their cocycles need to have the same structure.

Ectoplasm

Gates, Grisaru, Knut-Whelau, & Siegel 1998
Berkovits and Howe 2008
Bossard, Howe & K.S.S. 2009

- The construction of supersymmetric invariants is isomorphic to the construction of cohomologically nontrivial closed forms in superspace: $I = \int_{M_0} \sigma^* \mathcal{L}_D$ is invariant (where σ^* is a pull-back to a section of the projection map to the purely bosonic "body" subspace M_0) if $\mathcal{L}_D$ is a closed form in superspace, and is nonvanishing only if $\mathcal{L}_D$ is nontrivial.
- Using the BRST formalism, now handle all gauge symmetries including space-time diffeomorphisms by the nilpotent BRST operator s. The invariance condition for $\mathcal{L}_D$ is $s\mathcal{L}_D + d_0\mathcal{L}_{D-1} = 0$, where d_0 is the usual bosonic exterior derivative. Since $s^2 = 0$ and s anticommutes with d_0, one obtains $s\mathcal{L}_{D-1} + d_0\mathcal{L}_{D-2} = 0$, etc.

- Solving the BRST Ward identities thus becomes a cohomological problem. Note that the supersymmetry ghost is a commuting field. One needs to study the cohomology of the nilpotent operator $\delta = s + d_0$, whose components $\mathcal{L}_{D-q,q}$ are (D-q) forms with ghost number q, *i.e.* (D-q) forms with q spinor indices. The spinor indices are totally *symmetric* since the supersymmetry ghost is *commuting*.

- For gauge-invariant supersymmetric integrands, this establishes an isomorphism between the cohomology of closed forms in superspace (aka "ectoplasm") and the construction of BRST-invariant counterterms.

Superspace cohomology

Bonora, Pasti & Tonin

- Flat superspace has a standard basis of invariant 1-forms

$$
\begin{aligned}
E^a &= dx^a - \frac{i}{2} d\theta^\alpha (\Gamma^a)_{\alpha\beta}\theta^\beta \\
E^\alpha &= d\theta^\alpha
\end{aligned}
$$

dual to which are the superspace covariant derivatives (∂_a, D_α)

- There is a natural bi-grading of superspace forms into even and odd parts: $\Omega^n = \oplus_{n=p+q}\Omega^{p,q}$

- Correspondingly, the flat superspace exterior derivative splits into three parts with bi-gradings (1,0), (0,1) & (-1,2):

$$
d = \underbrace{d_0(1,0)}_{\text{bosonic der.}} + \underbrace{d_1(0,1)}_{\text{fermionic der.}} + \underbrace{t_0(-1,2)}_{\text{torsion}}
$$

$$
d_0 \leftrightarrow \partial_a \qquad d_1 \leftrightarrow \partial_\alpha
$$

where for a (p,q) form in flat superspace, one has

$$
(t_o\omega)_{a_2\cdots a_p\beta_1\cdots\beta_{q+2}} \sim (\Gamma^{a_1})_{(\beta_1\beta_2}\omega_{a_1\cdots a_p\beta_3\cdots\beta_{q+2})}
$$

- The nilpotence of the total exterior derivative d implies the relations

$$\begin{aligned} t_0^2 &= 0 \\ t_0 d_1 + d_1 t_0 &= 0 \\ d_1^2 + t_0 d_0 + d_0 t_0 &= 0 \end{aligned}$$

- Then, since $d\mathcal{L}_D = 0$, the lowest dimension nonvanishing component (or "generator") $\mathcal{L}_{D-q,q}$ must satisfy $t_0 \mathcal{L}_{D-q,q} = 0$ so $\mathcal{L}_{D-q,q}$ belongs to the t_0 cohomology group $H_t^{D-q,q}$.

- Starting with the t_0 cohomology groups $H_t^{p,q}$, one then defines a spinorial exterior derivative $d_s \ : \ H_t^{p,q} \to H_t^{p,q+1}$ by $d_s[\omega] = [d_1 \omega]$, where the [ ] brackets denote H_t classes.

Cederwall, Nilsson & Tsimpis 2002
Howe & Tsimpis 2003

- One finds that d_s is nilpotent, $d_s^2 = 0$, and so one can define spinorial cohomology groups $H_s^{p,q} = H_{d_s}(H_t^{p,q})$.

The groups $H_s^{0,q}$ give multi *pure spinors.*

- This formalism gives a way to reformulate BRST cohomology in terms of spinorial cohomology. The lowest dimension component, or *generator*, of a counterterm's superform must be d_s closed, *i.e.* it must be an element of $H_s^{D-q,q}$.

- Solving $d_s[\mathcal{L}_{D-q,q}] = 0$ allows one to solve for all the higher components of $\mathcal{L}_D$ in terms of $\mathcal{L}_{D-q,q}$ for normal cocyles.

- To see how this formalism works, consider N=1 supersymmetry in D=10. Corresponding to the κ symmetries of strings and 5-branes, we have the D=10 Gamma matrix identities $t_0\Gamma_{1,2} = 0 \qquad t_0\Gamma_{5,2} = 0$.

- The second of these is relevant to the construction of *d*-closed forms in D=10. One may have a generator
$$L_{5,5} = \Gamma_{5,2}M_{0,3}$$
where $d_s[M_{0,3}] = 0$. The simplest example of such a form corresponds to a full superspace integral over *S*:
$$M_{\alpha\beta\gamma} = T_{\alpha\beta\gamma,\delta_1\cdots\delta_5}(D^{11})^{\delta_1\cdots\delta_5}S$$
where $T_{\alpha\beta\gamma,\delta_1\cdots\delta_5}$ is constructed from the D=10 Gamma matrices; it is totally symmetric in $\alpha\beta\gamma$ and totally antisymmetric in $\delta_1\cdots\delta_5$.

Cohomological non-renormalization

- Spinorial cohomology allows one to derive non-renormalization theorems for counterterms: the cocycle structure of candidate counterterms must match that of the classical action.
  - For example, in maximal SYM, this leads to non-renormalization theorems ruling out the F^4 counterterm otherwise expected at L=4 in D=5.
  - Similar non-renormalization theorems exist in supergravity, but their study is complicated by local supersymmetry and the density character of counterterm integrands.

- Examples of operators that are ruled out by the ectoplasm/algebraic renormalization analysis include half-BPS counterterms such as the $\mathrm{tr}(F^4)$ or $(\mathrm{tr}(F^2))^2$ SYM counterterms. In D dimensions, the generator component of such a 1/2 BPS cocycle is an (0,D) form of dimension 8-D/2. Since the structure of this cocycle is different (i.e. it is longer) from than that of the SYM Lagrangian, the corresponding 1/2 BPS counterterm is *illegal*.

- Similar considerations allow one to rule out the R^4 counterterm in *N=8* supergravity, although the density character of supergravity invariants complicates analysis of their non-leading structure.

Bossard, Howe & K.S.S. 2009

Duality invariance constraints

cf also Broedel & Dixon 2010

- Maximal supergravity has a series of duality symmetries which extend the automatic GL(11-D) symmetry obtained upon dimensional reduction from D=11, e.g. E_7 in the N=8, D=4 theory, with the 70 scalars taking their values in an $E_7/SU(8)$ coset target space.
- The N=8, D=4 theory can be formulated in a manifestly E_7 covariant (but non-manifestly Lorentz covariant) formalism. Anomalies for SU(8), and hence E_7, cancel.

  Bossard, Hillman & Nicolai 2010

  Marcus 1985
- Combining the requirement of continuous duality invariance with the spinorial cohomology requirements gives further restrictions on counterterms.

- In a curved superspace, an invariant is constructed from the top (pure "body") component in a coordinate basis: $I = \frac{1}{D!}\int d^Dx\, \varepsilon^{m_D\ldots m_1}\, E_{m_D}{}^{A_D}\cdots E_{m_1}{}^{A_1}\, L_{A_1\ldots A_D}(x,\theta=0)$.

- Referring this to a preferred "flat" basis and identifying $E_M{}^A$ components with vielbeins and gravitinos, one has in D=4

$$I = \frac{1}{24}\int \left(e^a_\wedge e^b_\wedge e^c_\wedge e^d\; L_{abcd} + 4e^a_\wedge e^b_\wedge e^c_\wedge \psi^{\underline{\alpha}} L_{abc\underline{\alpha}} + 6e^a_\wedge e^b_\wedge \psi^{\underline{\alpha}}_\wedge \psi^{\underline{\beta}}\; L_{ab\underline{\alpha}\,\underline{\beta}}\right.$$
$$\left. +4e^a_\wedge \psi^{\underline{\alpha}}_\wedge \psi^{\underline{\beta}}_\wedge \psi^{\underline{\gamma}} L_{a\underline{\alpha}\,\underline{\beta}\,\underline{\gamma}} + \psi^{\underline{\alpha}}_\wedge \psi^{\underline{\beta}}_\wedge \psi^{\underline{\gamma}}_\wedge \psi^{\underline{\delta}} L_{\underline{\alpha}\,\underline{\beta}\,\underline{\gamma}\underline{\delta}}\right)$$

  - Thus the "soul" components of the cocycle also contribute to the local supersymmetric covariantization.

- Since the gravitinos do not transform under the E_7 duality, the L_{ABCD} form components have to be *separately* duality invariant.

- At leading order, the $E_7/SU(8)$ coset generators of E_7 simply produce *constant shifts* in the 70 scalar fields, as we have seen. This leads to a much easier check of invariance than analysing the full spinorial cohomology problem.
- Although the pure-body (4,0) component L_{abcd} of the R^4 counterterm has long been known to be shift-invariant at lowest order (since all 70 scalar fields are covered by derivatives), it is harder for the fermionic "soul" components to be so, since they are of lower dimension.

  Howe, K.S.S. & Townsend 1981

- Thus, one finds that the maxi-soul (0,4) $L_{\alpha\beta\gamma\delta}$ component is *not* invariant under constant shifts of the 70 scalars. Hence the D=4, N=8, 3 -loop R^4 1/2 BPS counterterm is not E_7 duality invariant, so it is ruled out as an allowed counterterm.

  Bossard, Howe & K.S.S. 2010

N=5, N=6

- Similar analysis of the D=4 3-loop R^4 invariants in N=5 and N=6 supergravities shows them to be likewise ruled out by the analogous requirements of SU(5,1) and SO*(12) duality invariances.
- In N=6 supergravity, there is a 4-loop $\partial^2 R^4$ type invariant. Similar analysis indicates that this also is ruled out.
    - In maximal supergravity, such a $\Delta = 10$ invariant might have been expected at one loop in D=10. However, in maximal supergravity this invariant vanishes subject to the classical field equations. But in D=4, N=6 it does not vanish, so it could have been a threatening counterterm.

Infinities nix infinities: dimensional reduction versus duality

Elvang & Kiermaier 2010 (from IIA string theory)
Bossard, Howe & K.S.S. 2010 (just from supergravity)
Beisert, Elvang, Freedman, Kiermaier, Morales & Stiebeger 2010

- Left out of control so far are some of the most interesting cases: L=5,6 in D=4 maximal supergravity, corresponding to the 1/4 BPS $\partial^4 R^4$ and 1/8 BPS $\partial^6 R^4$ type counterterms.
  - Here, a different kind of duality-based argument comes into play.
- In fact, the *existence* of the 1/2 BPS L=1, D=8 R^4, the 1/4 BPS L=2, D=7 $\partial^4 R^4$ and the 1/8 BPS L=3, D=6 $\partial^6 R^4$ types of divergences together with the *uniqueness* of the corresponding D=4 counterterm structures allows one to rule out the corresponding D=4 candidates.

Drummond, Heslop, Howe & Kerstan 2003

- The existence of these D=8, 7 & 6 divergences indicate that the corresponding forms of the R^4, $\partial^4 R^4$ & $\partial^6 R^4$ counterterms have to be such that the purely gravitational parts of these invariants are not dressed by e^ϕ scalar prefactors – otherwise, they would violate the corresponding $SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$, $SL(5,\mathbb{R})$ & $SO(5,5)$ duality symmetries: lowest-order shift symmetries would then be violated.
- Upon dimensional reduction to D=4, the Einstein-frame classical N=8 action $\int d^4x(R\sqrt{-g} + \ldots)$ is arranged to have no scalar prefactors. But then dimensional reduction of the R^4, $\partial^4 R^4$ & $\partial^6 R^4$ counterterms in general causes such prefactors to appear.

- These dimensional reductions from D=8, 7 & 6 do not have the needed $SU(8)$ symmetry. But they can be rendered $SU(8)$ invariant by averaging, *i.e.* by integrating the dimensionally reduced counterterms over $SU(8)/(SO(3)\times SO(2))$, $SU(8)/SO(5)$ or $SU(8)/(SO(5)\times SO(5))$. Elvang & Kiermaier 2010
  - The action of $SU(8)$ on evident scalar combinations such as the compactification volume modulus $\phi = \vec{\alpha}\cdot\vec{\phi}$ is highly nonlinear, so $SU(8)$ averaging is difficult to do explicitly. Green & Vanhove 2005; Green, Russo & Vanhove 2010
  - However, some ideas from string theory come to the rescue: scalar prefactors need to satisfy certain Laplace equations, even in the pure supergravity limit.

- Starting from a known duality invariant in some higher dimension D, the dimensional reduction to D=4 giving the n-loop candidate $\partial^{2(n-3)}R^4$ counterterm has a scalar prefactor $f_n(\phi)$ satisfying

Bossard, Howe & K.S.S. 2010

$$\left(\Delta + \frac{D-4}{D-2}n(32-D-n)\right) f_n(\phi) = 0$$

- This Laplace equation is SU(8) covariant, and must be satisfied equally by the dimensional reduction of the D-dimensional counterterm and by the SU(8) averaged version of this counterterm.

- Infinitesimal shift invariance for the 70 scalars, and hence E_7 invariance, can only be realised if $f_n(\phi) = 1$.

- Starting from the known infinities at L=1,2&3 loops in D=8,7&6, one deduces yet again the impossibility of E_7 invariance in D=4 for the 1/2 BPS R^4 counterterm, but also for the 1/4 BPS $\partial^4 R^4$ counterterm and the 1/8 BPS $\partial^6 R^4$ counterterm.

Drummond, Heslop, Howe & Kerstan 2003

- Since these D=4 counterterm candidates are *unique*, just based on supersymmetry and the linearly realised SU(8) symmetry, their failure to be invariant rules out the corresponding infinities.

- However, one can then turn around the argument and use the D=4 uniqueness of the 1/2, 1/4 and 1/8 BPS counterterm types to investigate the possibilities for duality invariance in higher dimensions, where the same general Laplace equation needs to be satisfied.

- In this way, one (re)learns that the only dimensions in which the 1/2 BPS R^4 type counterterm can be duality invariant are D=11 and D=8 (duality group $SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$).
- Similarly, the only dimension in which the 1/4 BPS $\partial^4 R^4$ type invariant can be duality invariant is D=7 (duality group $SL(5,\mathbb{R})$). And the only dimension in which the 1/8 BPS $\partial^6 R^4$ type invariant can be duality invariant is D=6 (duality group SO(5,5)).
- One place where this observation is directly relevant is in D=5, L=4, where a 1/8 BPS $\partial^6 R^4$ counterterm would otherwise have been expected. Indeed, this has been found to be absent by explicit calculation.

Bern, Carrasco, Dixon, Johansson & Roiban 2009

Current outlook

- Work to understand the L=7, D=4 counterterm ($\Delta = 16$) is currently underway. Preliminary results indicate that there does exist an E_7 invariant counterterm at this level. It may or may not be expressible as the volume of N=8 superspace $\int d^{32}\theta(\det E)$. So this $\Delta = 16$ order is the current leading candidate.

Bossard, Howe, K.S.S. & Vanhove (w.i.p.)

E_7 invariant counterterms certainly exist for L>7:
Howe & Lindstrom 1981
Kallosh 1981

- Current divergence expectations for maximal supergravity:

Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	6	7
BPS degree	0	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	0	0
Gen. form	$\partial^{12}R^4$	$\partial^{10}R^4$	R^4	$\partial^6 R^4$	$\partial^6 R^4$	$\partial^{12}R^4$	$\partial^8 R^4$

Blue: known divergences   Green: anticipated divergences

Is it time for another bet, Zvi?