# optimal control theory for quantum gates in open quantum systems 

Christiane P. Koch

$$
\begin{aligned}
& \text { U N I K A S S E L } \\
& \text { V E R S I T A' }
\end{aligned}
$$

## some terminology

\&
basics of optimal control

## principle of coherent / quantum control

wave properties of matter (superposition principle)
variation of phase between different, but indistinguishable quantum pathways:
constructive interference in desired channel
destructive interference in all other channels

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'intuitive' approaches
optimal control theory

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Brumer \& Shapiro


Tannor \& Rice


STIRAP


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- bichromatic control
- pump-dump/probe
- STIRAP
$\rightsquigarrow$ DFS \& other
symmetry-adapted control


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'intuitive' approaches

- bichromatic control
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- STIRAP
$\rightsquigarrow$ DFS \& other
symmetry-adapted control
optimal control theory
- theory: iterative solution of control equations
- exp.:



## optimal control theory

time/frequency 'phase space' picture

$$
\begin{aligned}
& t=0 \\
& \left|\varphi_{i}\right\rangle
\end{aligned} \sim \sim \quad \begin{aligned}
& t=T \\
& \left|\varphi_{f}\right\rangle
\end{aligned}
$$

define the objective :

$$
\mathrm{GOAL} \equiv\left\|\left\langle\varphi_{i}\right| \hat{\mathbf{U}}^{+}(T, 0 ; \varepsilon)\left|\varphi_{f}\right\rangle\right\|^{2}=-J_{T}
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as a functional of the field $\varepsilon$

## optimal control theory

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include additional constraints:

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J=J_{T}+\int_{0}^{T} J_{t}(\varepsilon, \varphi) \mathrm{d} t
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optimize $J$ :

$$
\begin{array}{lcl}
\partial_{\epsilon} J=0 & \partial_{\varphi(t)} J=0 & \partial_{\epsilon}^{2} J>0 \\
& |\varphi(t)\rangle=\hat{\mathbf{U}}(t, 0 ; \epsilon)\left|\varphi_{i}\right\rangle & \text { can only be fulfilled locally! }
\end{array}
$$

## control tasks

in an ideal quantum world
(w/o decoherence)

## optimal control for quantum gates



$$
\operatorname{Tr}\{\hat{0}+\hat{\mathrm{p}} \hat{N}(T, 0 ; \varepsilon) \hat{\mathrm{P}} N\}
$$

Palao E Kosloff, PRA 68, 062308 (2003)

- desired gate operation : Ô
- desired fidelity :

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1-\varepsilon \text { where } \varepsilon<10^{-4}
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$\Delta \epsilon(t)=\frac{S(t)}{2 \alpha} \mathfrak{I m}\left[\sum_{k=1}^{N}\left\langle\varphi_{k, i n i}\right| \hat{\mathbf{O}}^{+} \hat{\mathbf{U}}^{+}\left(T, t ; \epsilon^{\text {old }}\right) \hat{\boldsymbol{\mu}} \hat{\mathbf{U}}\left(t, 0 ; \epsilon^{\text {new }}\right)\left|\varphi_{k, i n i}\right\rangle\right]$

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- what gate time $T$ needed ?
- best choice of target Ô ?


## minimum gate time

example: controlled phasegate $\hat{\mathbf{O}}=\operatorname{diag}\left(e^{i \chi}, 1,1,1\right)$ for atoms in an optical lattice

Goerz, Calarco, Koch, J Phys B 44, 154011 (2011)

goal: perform a two-qubit gate on the logical basis
while restoring the motional state of the atoms

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## best choice of target

Müller, Reich, Murphy, Yuan, Vala, Whaley, Calarco, Koch, PRA 84, 042315 (2011) optimize for the entangling content of a two-qubit gate rather than for a specific gate

## local invariants functional

$$
J_{T}=\Delta g_{1}^{2}+\Delta g_{2}^{2}+\Delta g_{3}^{2}
$$

with $\Delta g_{i}{ }^{2}=\left|g_{i}(\hat{\mathbf{O}})-g_{i}(\hat{\mathbf{U}})\right|^{2}$ and $g_{i}(\hat{\mathbf{O}})$ the local invariants of $\hat{\mathbf{O}}$
optimization determines that gate out of a local equivalence class that can best be implemented in an automated fashion

$$
\begin{gathered}
\hat{\mathbf{U}}=\hat{\mathbf{k}}_{1} e^{-\frac{i}{2} \sum_{j=x, y, z} c_{j} \hat{\sigma}_{j}^{1} \hat{\sigma}_{j}^{2} \hat{\mathbf{k}}_{2}} \\
g_{1}, g_{2}, g_{3} \Longleftrightarrow c_{x}, c_{y}, c_{z}
\end{gathered}
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$J_{T}$ 8th degree polynomial in $\left\{\left|\varphi_{k}\right\rangle\right\}$
$\rightsquigarrow$ nonlinear Krotov method


Reich, Ndong, Koch, J. Chem. Phys. 136, 104103 (2012)

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can be generalized to all perfect entanglers
P. Watts $\mathcal{E}$ J. Vala combination of $g_{i}$ which vanishes on boundary of $W_{P E}$, e.g.

$$
J_{T}=g_{3} \sqrt{g_{1}^{2}+g_{2}^{2}-g_{1}}
$$

# how to tackle control tasks 

in the real world ?
(with decoherence)

## OCT for open quantum systems

$$
\rho(T)=\mathcal{D}(\rho(0)) \quad \text { e.g. : } \quad \frac{\partial \rho}{\partial t}=\frac{i}{\hbar}[H, \rho]_{-}+\mathcal{L}_{D}(\rho)
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(1) state-to-state: $\rho(t=0) \rightarrow \rho(t=T)=\rho_{\text {target }}$
$\Rightarrow$ maximize $\operatorname{Tr}\left[\rho(T) \rho_{\text {target }}\right]$
Bartana, Kosloff, Tannor, J. Chem. Phys. 106, 1435 (1997)
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(2) gates: lift $\operatorname{Tr}\left\{O^{+} P_{N} U(T, 0 ; \varepsilon) P_{N}\right\}$ to Liouville space $\Rightarrow$ maximize $\frac{1}{d^{2}} \sum_{j=1}^{d^{2}} \operatorname{Tr}\left[O \rho_{j} O^{+} \rho_{j}(T)\right]$

Kallush \& Kosloff, Phys. Rev. A 73, 032324 (2006)
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\text { but: } U \Leftrightarrow \mathcal{D}(\rho) \text { !? }
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## two propositions:

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how to determine $U$ to calculate $g_{i}(U)$ ?
analytical reconstruction possible based on $2 d-1$ specific $\rho_{j}$
using concepts 'commutant space' and 'total rotation'

## differentiating unitaries: commutant space

 assume unitary time evolutionset of states $\left\{\rho_{i}=\rho_{i}(t=0)\right\}$ and $\left\{\rho_{i}^{U}(T)=U \rho_{i} U^{+}\right\}$

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for a given set $\left\{\rho_{i}\right\}$ consider the map $\mathcal{M}$

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$$
\mathcal{K}\left(\left\{\rho_{i}\right\}\right)=\{\mathbb{1}\}
$$

$\mathbb{1}$ is only time evolution that leaves all $\left\{\rho_{i}\right\}$ unchanged

## minimal set of states: total rotation

given a $\rho$, we cannot distinguish those $U$ with common eigenbasis with $\rho$ from $\mathbb{I}$
(1) fix a basis: basis-complete projectors $\left\{P_{i}\right\}$
$d$ orthonormal one-dimensional projectors
$\Longrightarrow \rho=\sum_{i=1}^{d} \lambda_{i} P_{i}, \lambda_{i} \neq \lambda_{j}$

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$$

(2) construct a $\rho^{\prime}$ guaranteed to have no common eigenspace with any $P_{i}$ : total rotation

$$
\begin{aligned}
& \Longrightarrow \rho^{\prime}=P_{T R} \quad \text { with } \\
& \quad P_{T R} P_{i} \neq 0 \quad \forall P_{i} \in\left\{P_{i}\right\} \\
& \text { (note: } d P_{T R}{ }^{\prime} \mathrm{s}=\text { mutually unbiased basis) }
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$$

(2) construct a $\rho^{\prime}$ guaranteed to have no common eigenspace with any $P_{i}$ : total rotation
$\Longrightarrow \rho^{\prime}=P_{T R} \quad$ with

$$
P_{T R} P_{i} \neq 0 \quad \forall P_{i} \in\left\{P_{i}\right\}
$$

(note: $d P_{T R}$ 's $=$ mutually unbiased basis)
$\Longrightarrow \rho(T), \rho^{\prime}(T)$ are sufficient to distinguish any two unitaries (and thus measure success of control) provided time evolution coherent

## gate optimization

third state sufficient to check whether time evolution is unitary

$$
J_{T}=\sum_{j=1}^{3}\left[1-\operatorname{Tr}\left[O \rho_{j} O^{+} \rho_{j}(T)\right]\right]
$$

$\rho_{1, i j}=\frac{2(d-i+1)}{d(d+1)} \delta_{i j} \quad \rho_{2, i j}=\frac{d^{2}-2}{d^{2}} \delta_{i 1} \delta_{1 j}+\frac{1}{d^{2}} \delta_{1 j}+\frac{1}{d^{2}} \delta_{i 1} \quad \rho_{3, i j}=\frac{1}{d} \delta_{i j}$
fix the basis totally rotated state check unitality on logical subspace

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## $J_{T}$ attains its minimum only if

(1) $\mathcal{D}$ is a unitary dynamical map on the logical subspace
(2) $\mathcal{D}\left(\rho_{1}\right)=O \rho_{1} O^{+}$and $\mathcal{D}\left(\rho_{2}\right)=O \rho_{2} O^{+}$

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$\rightarrow$ propagation of 3 states sufficient, irrespective of $\operatorname{dim} \mathcal{H}$

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$\rightarrow$ propagation of 3 states sufficient, irrespective of $\operatorname{dim} \mathcal{H}$
$\rightarrow$ implications for device characterization

## gate optimization with 3 states

## theorem:

Let $\mathrm{DM}(d)$ be the space of $d \times d$ density matrices and $\mathcal{D}: \mathrm{DM}(d) \mapsto \mathrm{DM}(d)$ a dynamical map.
The following statements are equivalent:
(1) $\mathcal{D}$ is unitary.
(2) $\mathcal{D}$ maps a set $\mathcal{A}$ of $d$ one-dimensional orthogonal projectors onto a set of $d$ one-dimensional orthogonal projectors and a totally rotated projector $P_{T R}$ (w.r.t. $\mathcal{A}$ ) onto a one-dimensional projector.
(3) $\mathcal{D}$ is unital and leaves the spectrum of a complete and totally rotating set of density matrices invariant. important for optimization of complex systems (d large)

## examples:

## can OCT help us find better gates?

## example 1:

## Rydberg C-phase gate for trapped atoms

## Rydberg gate



$$
\begin{aligned}
\hat{\mathbf{H}}_{1 q} & =\left(\begin{array}{cccc}
0 & 0 & \mu \epsilon_{1}(t) & 0 \\
0 & E_{1} & 0 & 0 \\
\mu \epsilon_{1}(t) & 0 & \Delta_{1} & \mu \epsilon_{2}(t) \\
0 & 0 & \mu \epsilon_{2}(t) & \Delta_{1}+\Delta_{2}
\end{array}\right) \\
\hat{\mathbf{H}}_{2 q} & =\hat{\mathbf{H}}_{1 q} \otimes \mathbb{1}+\mathbb{1} \otimes \hat{\mathbf{H}}_{1 q}-U|r r\rangle\langle r r|
\end{aligned}
$$

## Rydberg gate



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\end{gathered}
$$

optimization including spontaneous emission

yes!
fidelity:
$0.7 \rightarrow 0.85$
iteration Schulte-Herbrüggen, Spörl, Khaneja, Glaser, J. Phys. B 44154013 (2011)

## Rydberg gate

optimization including spontaneous emission

fidelity improvement at the price of bang-like solutions

## example 2:

## state preparation of

 superconducting qutrits
## superconducting qubits 1: phase qutrit



Shalibo, Rofe, Barth, Friedland, Bialczak, Martinis, Katz, Phys. Rev. Lett. 108, 037701 (2012)

Shalibo, PhD thesis (2012)

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optimization of state preparation for tomography

$$
T_{1}^{|1\rangle}=170 \mathrm{~ns}, T_{1}^{|2\rangle}=86 \mathrm{~ns} \quad\left(\Gamma_{n, n-1} \sim n\right) \quad T_{2}^{|0\rangle\langle 1|}=75 \mathrm{~ns}, T_{2}^{|1\rangle\langle 2|}=65 \mathrm{~ns}, T_{2}^{|0\rangle\langle 2|}=25 \mathrm{~ns}
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$$

$$
\Phi_{\mathrm{ext}}(\mathrm{t})
$$


experiment sequence of $\pi$ and $\pi / 2$ pulses with $S(t)=(1-\cos [2 \pi t / T]) / 2$

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## experiment

sequence of $\pi$ and $\pi / 2$ pulses with $S(t)=(1-\cos [2 \pi t / T]) / 2$ $T=10 \mathrm{~ns}$


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## experiment

sequence of $\pi$ and $\pi / 2$ pulses with $S(t)=(1-\cos [2 \pi t / T]) / 2$ $T=10 \mathrm{~ns}$

implementing
$\hat{\mathbf{O}}=\exp \left[-i \phi \hat{\lambda}_{i} / 2\right]$

## superconducting qubits 1: phase qutrit

optimization of state preparation for tomography: $\hat{\lambda}_{1}$

$$
T_{1}^{|1\rangle}=170 \mathrm{~ns}, T_{1}^{|2\rangle}=86 \mathrm{~ns} \quad\left(\Gamma_{n, n-1} \sim n\right) \quad T_{2}^{|0\rangle\langle 1|}=75 \mathrm{~ns}, T_{2}^{|1\rangle\langle 2|}=65 \mathrm{~ns}, T_{2}^{|0\rangle\langle 2|}=25 \mathrm{~ns}
$$

fidelity: 99.3\% (no decoherence)


## superconducting qubits 1: phase qutrit

optimization of state preparation for tomography: $\hat{\lambda}_{1}$
$T_{1}^{|1\rangle}=170 \mathrm{~ns}, T_{1}^{|2\rangle}=86 \mathrm{~ns}$
$\left(\Gamma_{n, n-1} \sim n\right)$
$T_{2}^{|0\rangle\langle 1|}=75 \mathrm{~ns}, T_{2}^{|1\rangle\langle 2|}=65 \mathrm{~ns}, T_{2}^{|0\rangle\langle 2|}=25 \mathrm{~ns}$
fidelity: $99.3 \% \rightsquigarrow 91.7 \%$ (with decoherence)


## superconducting qubits 1: phase qutrit

optimization of state preparation for tomography: $\hat{\lambda}_{1}$

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T_{1}^{|1\rangle}=170 \mathrm{~ns}, T_{1}^{|2\rangle}=86 \mathrm{~ns} \quad\left(\Gamma_{n, n-1} \sim n\right) \quad T_{2}^{|0\rangle\langle 1|}=75 \mathrm{~ns}, T_{2}^{|1\rangle\langle 2|}=65 \mathrm{~ns}, T_{2}^{|0\rangle\langle 2|}=25 \mathrm{~ns}
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fidelity: $99.3 \% \rightsquigarrow 91.7 \%$ vs $92.2 \%$ (optimized under decoherence)


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T_{1}^{|1\rangle}=170 \mathrm{~ns}, T_{1}^{|2\rangle}=86 \mathrm{~ns} \quad\left(\Gamma_{n, n-1} \sim n\right) \quad T_{2}^{|0\rangle\langle 1|}=75 \mathrm{~ns}, T_{2}^{|1\rangle\langle(2 \mid}=65 \mathrm{~ns}, T_{2}^{|0\rangle\langle 2|}=25 \mathrm{~ns}
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## example 3:

C-phase gate for transmon qubits

## superconducting qubits 2: transmon qubits



J Koch, Yu, Gambetta, Houck, Schuster, Majer, Blais, Devoret, Girvin,Schoelkopf, Phys. Rev. A 76, 042319 (2007)

DiCarlo, Chow, Gambetta, Bishop, Johnson, Schuster, Majer, Blais, Frunzio, Girvin, Schoelkopf, Nature 460, 240 (2009)
comparatively long decoherence times

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T_{2}=20 \ldots 100 \mu \mathrm{~s}
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comparatively long decoherence times $T_{2}=20 \ldots 100 \mu \mathrm{~s}$
gate times $T<250 \mathrm{~ns}$
will 'beat' decoherence

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$\curvearrowright$ qubit-qubit \& qubit-cavity coupling too weak for fast gates

$$
\begin{aligned}
\hat{\mathbf{H}}= & \sum_{i=1,2}\left[\omega_{i} \hat{\mathbf{b}}_{i}^{+} \hat{\mathbf{b}}_{i}-\alpha_{i} \hat{\mathbf{b}}_{i}^{+} \hat{\mathbf{b}}_{i}^{+} \hat{\mathbf{b}}_{i} \hat{\mathbf{b}}_{i}\right]+J\left(\hat{\mathbf{b}}_{1}^{+} \hat{\mathbf{b}}_{2}+\hat{\mathbf{b}}_{1} \hat{\mathbf{b}}_{2}^{+}\right) \\
& +\omega_{c} \hat{\mathbf{a}}^{+} \hat{\mathbf{a}}+\epsilon^{*}(t) \hat{\mathbf{a}}+\epsilon(t) \hat{\mathbf{a}}^{+}+\sum_{i=1,2} g_{i}\left(\hat{\mathbf{b}}_{i}^{+} \hat{\mathbf{a}}+\hat{\mathbf{b}}_{i} \hat{\mathbf{a}}^{+}\right)
\end{aligned}
$$

## superconducting qubits 2: transmon qubits

 optimization of C-Phase w/o dissipationfast gate: $F_{a v}=99.72 \%$


## superconducting qubits 2: transmon qubits

## optimization of C-Phase w/o dissipation

fast gate: $F_{a v}=99.72 \%$

slow gate: $F_{a v}=99.93 \%$
population dynamics for $|\Psi(t=0)\rangle=|00\rangle$


$$
F_{a v}=99.93 \% \rightsquigarrow 97.68 \%
$$

## superconducting qubits 2: transmon qubits

## optimization of C-Phase w/o dissipation

fast gate: $F_{a v}=99.72 \%$ population dynamics for $|\Psi(t=0)\rangle=|01\rangle$


$$
F_{a v}=99.72 \% \rightsquigarrow 99.06 \%
$$

slow gate: $F_{a v}=99.93 \%$
population dynamics for $|\Psi(t=0)\rangle=|01\rangle$


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## superconducting qubits 2: transmon qubits

## optimization of C-Phase w/o dissipation

fast gate: $F_{a v}=99.72 \%$

slow gate: $F_{a v}=99.93 \%$
population dynamics for $|\Psi(t=0)\rangle=|10\rangle$


$$
F_{a v}=99.93 \% \rightsquigarrow 97.68 \%
$$

## superconducting qubits 2: transmon qubits

## optimization of C-Phase w/o dissipation

fast gate: $F_{a v}=99.72 \%$ population dynamics for $|\Psi(t=0)\rangle=|11\rangle$


$$
F_{a v}=99.72 \% \rightsquigarrow 99.06 \%
$$

slow gate: $F_{a v}=99.93 \%$
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fast gate with additional transitions involving

- qubit $|1\rangle \rightarrow|2\rangle$
- cavity $|0\rangle \rightarrow|2\rangle$ simultaneous with qubit $|1\rangle \rightarrow|0\rangle$


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fast gate with additional transitions involving

- qubit $|1\rangle \rightarrow|2\rangle$
- cavity $|0\rangle \rightarrow|2\rangle$ simultaneous with qubit $|1\rangle \rightarrow|0\rangle$
$\longrightarrow$ OCT using full complexity of $\hat{\mathrm{H}}$ : gates fast enough to beat decoherence


## summary

- OCT can be adapted to QIPC tasks by proper choice of functional
- OCT yields fast high-fidelity gates for complex systems ex: transmons in the non-dispersive regime
- optimizing under dissipation yields improved gates
provided some regions of Hilbert space less effected by decoherence than others ex: Rydberg gate for trapped atoms
- gate optimization requires only 3 (and not $d^{2}$ ) states $\Longrightarrow$ paving the way for OCT for complex systems
- so far: bang-like and adiabatic solutions


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will smarter functionals yield smarter solutions?


## acknowledgments

Daniel Reich


Giulia Gualdi
Michael Goerz

## collaborators

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Nadav Katz \& Co., HU Jerusalem
Tommaso Calarco \& Co., U Ulm
Jiri Vala \& Co., NU Ireland
€€€:
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