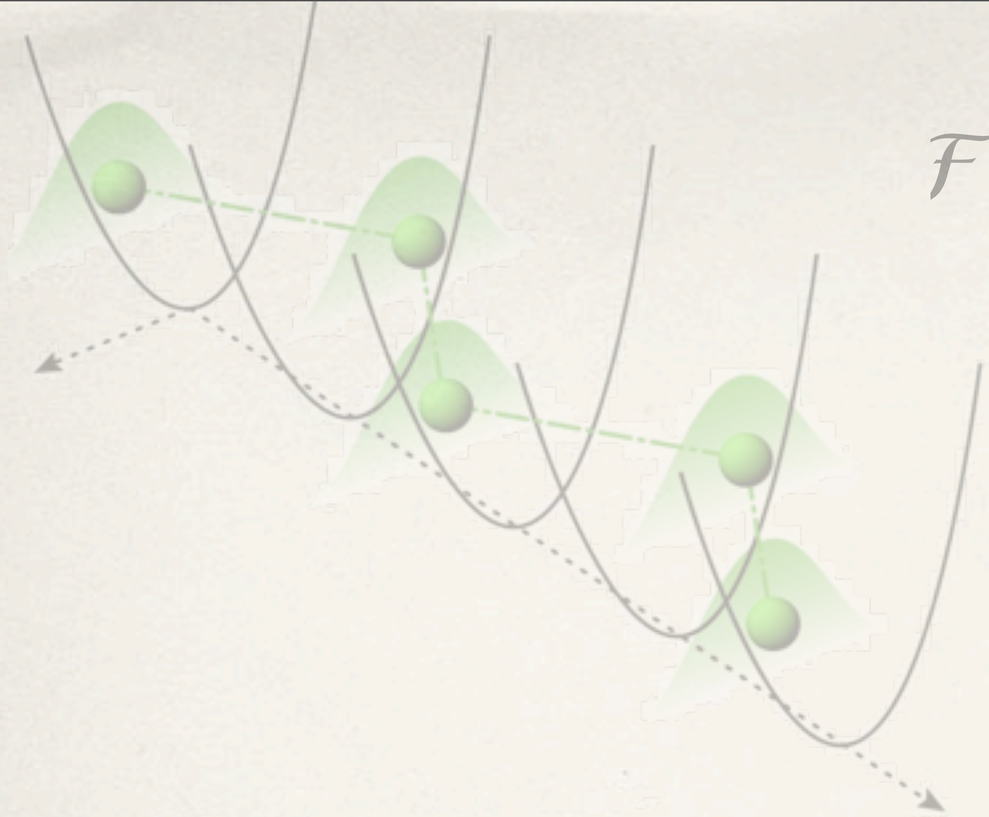
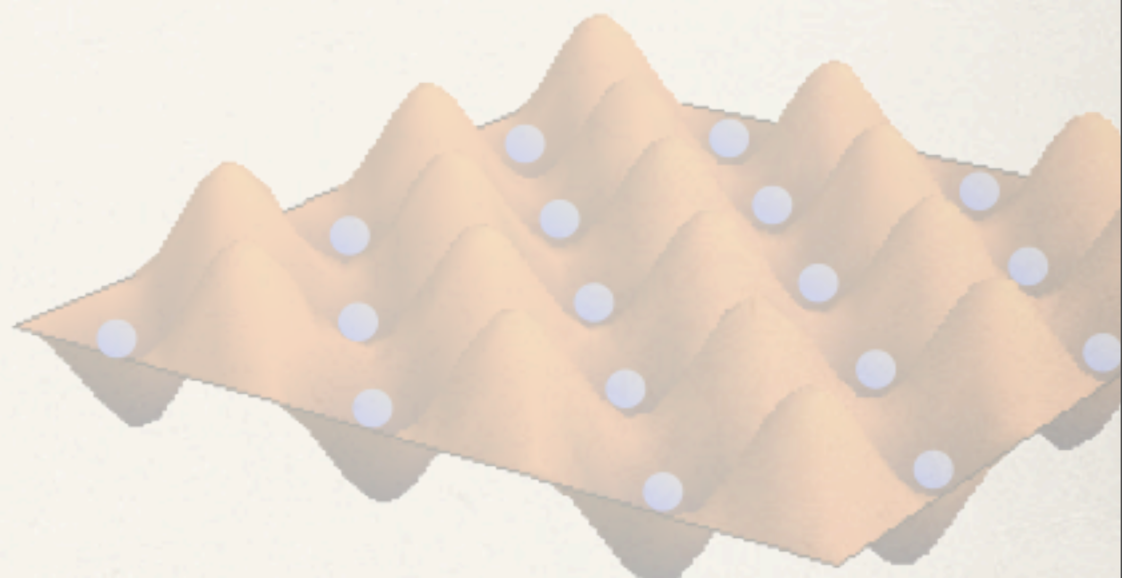
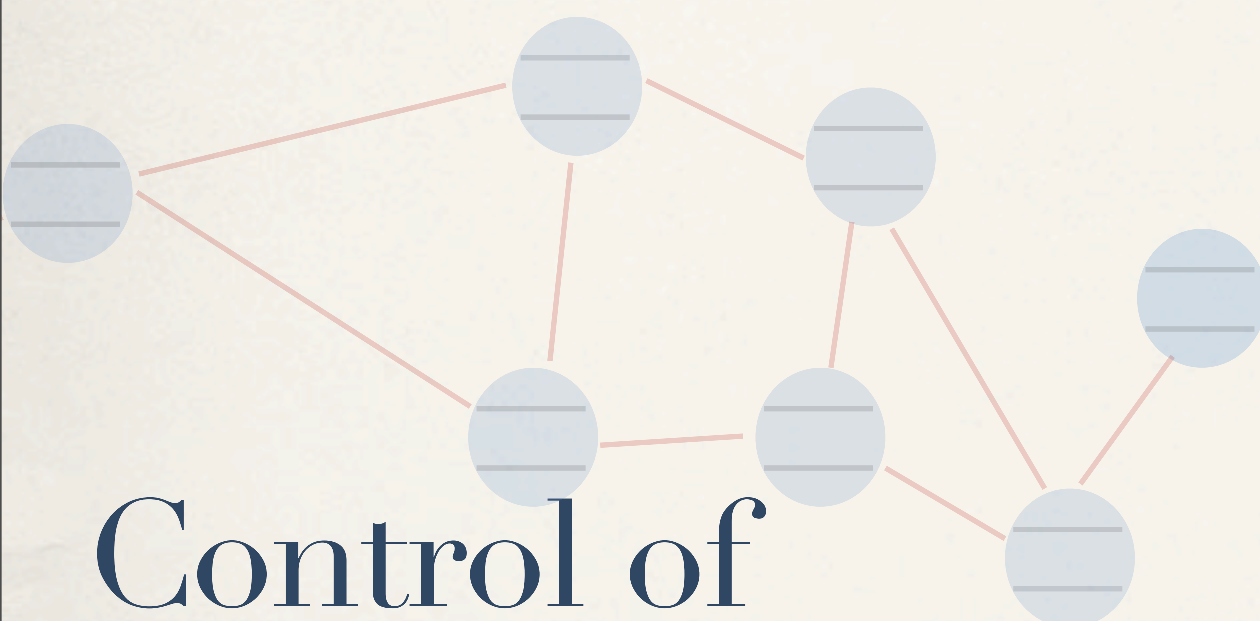
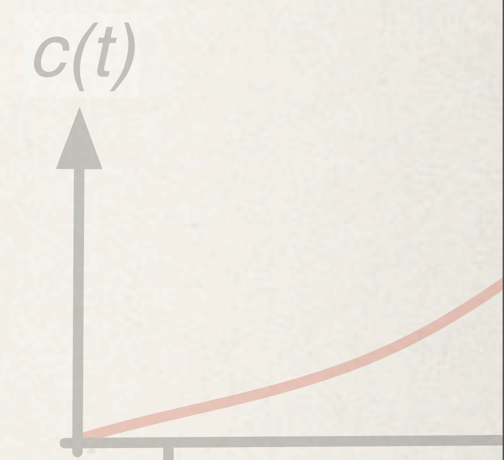








Control of many-body quantum dynamics

Simone Montangero - Ulm University

KITP - 01/03/2013

Optimal control

Optimal control

Optimal control

Optimal control

Optimal control

- Few-body quantum systems: standard optimal control (high-accuracy, complete knowledge, many iterations...)

H. Rabitz, NJP (2009)

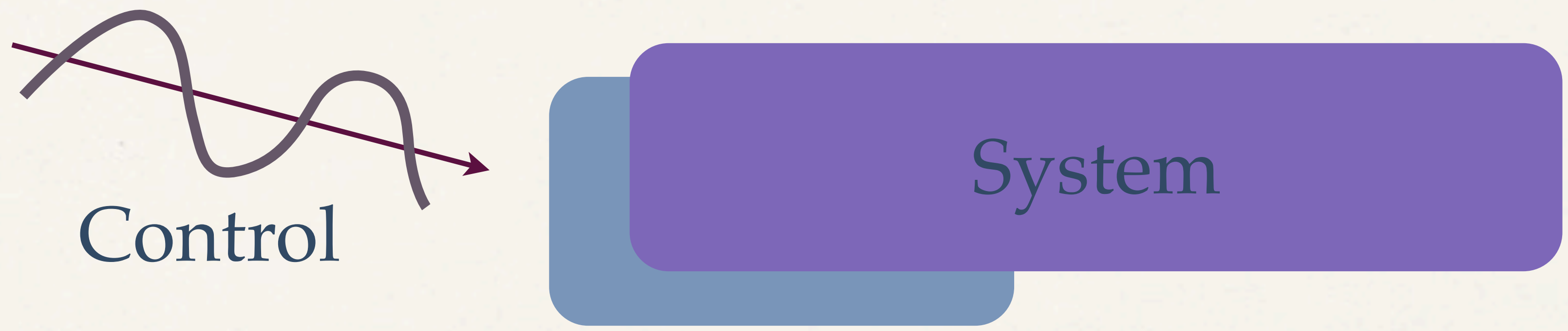
Altafini & Ticozzi IEEE (2012)

Optimal control

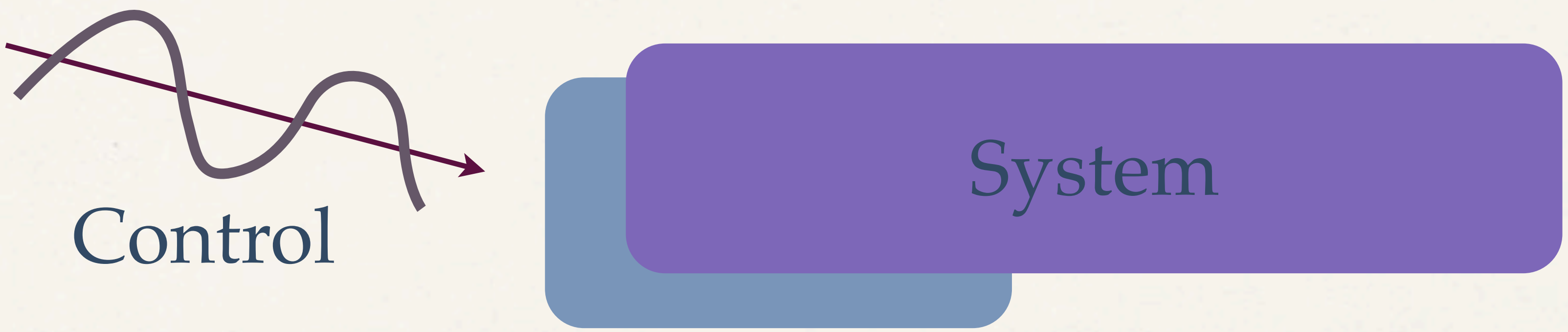
- Few-body quantum systems: standard optimal control (high-accuracy, complete knowledge, many iterations...)
- Many-body?

H. Rabitz, NJP (2009)
Altafini & Ticozzi IEEE (2012)

CRAB optimization

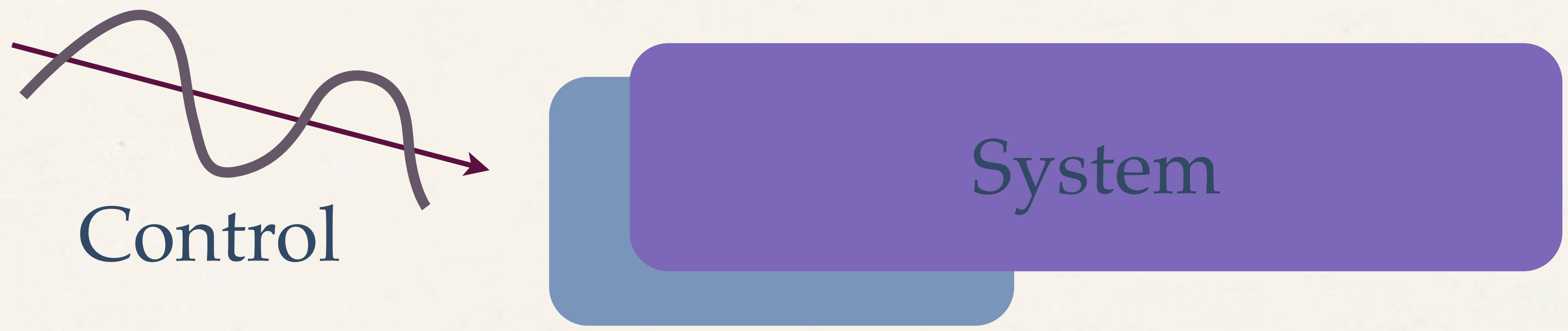


CRAB optimization



- Simple and versatile optimal control technique
- Unique optimal control integrated with tensor network methods (t-DMRG, ...)
- Works for open systems

CRAB optimization



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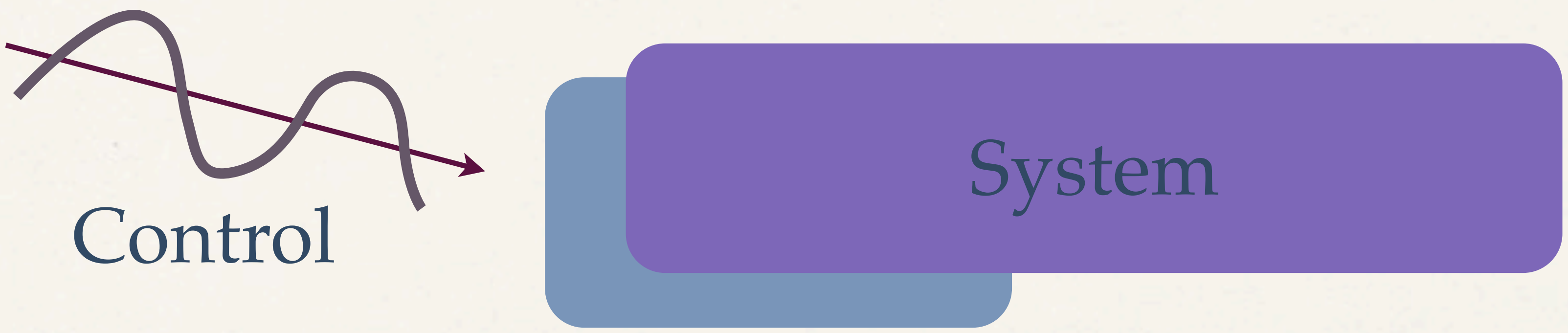
Control

CRAB optimization

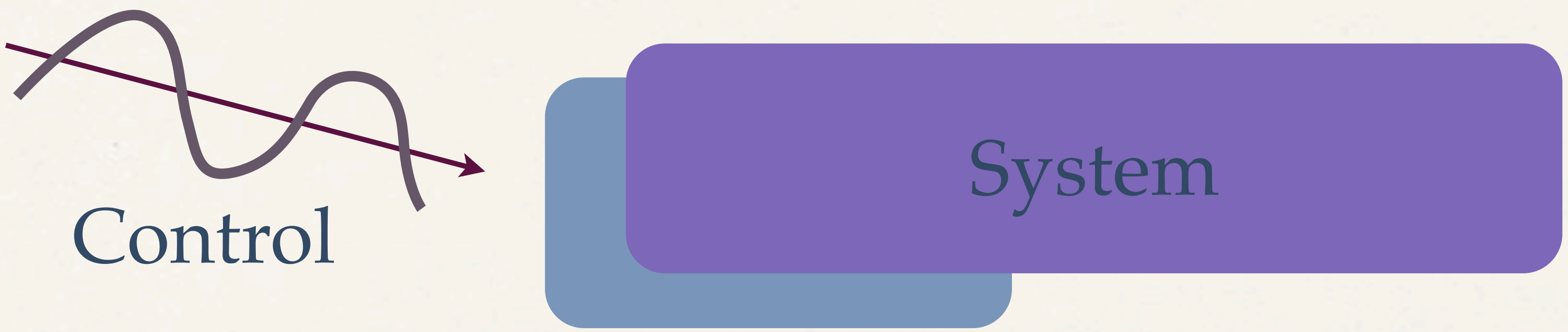


- Simple and versatile optimal control technique
- Unique optimal control integrated with tensor network methods (t-DMRG, ...)
- Works for open systems

Control

Many body quantum systems

CRAB optimization



- Simple and versatile optimal control technique — Control
- Unique optimal control integrated with tensor network methods (t-DMRG, ...) — Many body quantum systems
- Works for open systems — Decoherence

P. Doria, T. Calarco, SM PRL. (2011)
F. Caruso, et.al. PRA (2012)

T. Caneva, T. Calarco, SM
PRA (2011), NJP (2012)

Chopped RAndom Basis

T. Caneva, T. Calarco, and SM, Phys. Rev. A 2011

Chopped RAndom Basis

Functional minimization

T. Caneva, T. Calarco, and SM, Phys. Rev. A 2011

Chopped RAndom Basis

Reduced basis method

Functional
minimization

T. Caneva, T. Calarco, and SM, Phys. Rev. A 2011

Chopped RAndom Basis

Expand control field over n_f "randomized" basis functions

Reduced basis method

Functional minimization

T. Caneva, T. Calarco, and SM, Phys. Rev. A 2011

Chopped RAndom Basis

Expand control field over n_f
“randomized” basis functions

Reduced basis method

Multivariable
function minimization

Functional
minimization

T. Caneva, T. Calarco, and SM, Phys. Rev. A 2011

Chopped RAndom Basis

Expand control field over n_f
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Reduced basis method

Multivariable
function minimization

Functional
minimization

Direct Search
methods

T. Caneva, T. Calarco, and SM, Phys. Rev. A 2011

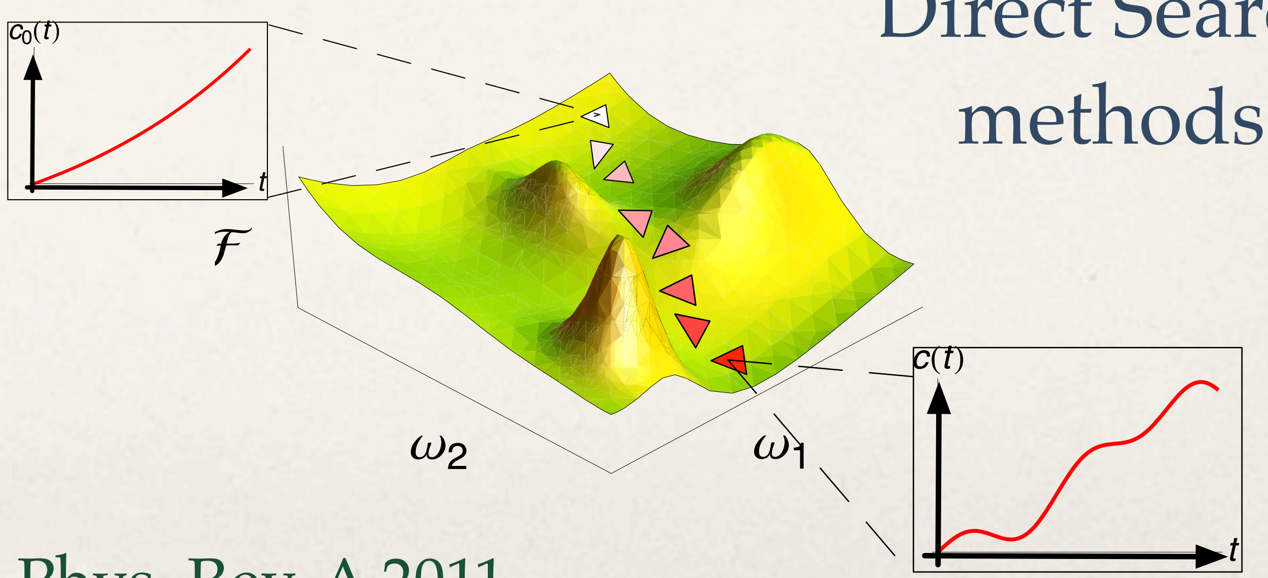
Chopped RAndom Basis

Expand control field over n_f
"randomized" basis functions

Reduced basis method

Multivariable
function minimization

Functional
minimization



Direct Search
methods

T. Caneva, T. Calarco, and SM, Phys. Rev. A 2011

Chopped RAndom Basis

Expand control field over n_f
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Reduced basis method

Multivariable
function minimization

Functional
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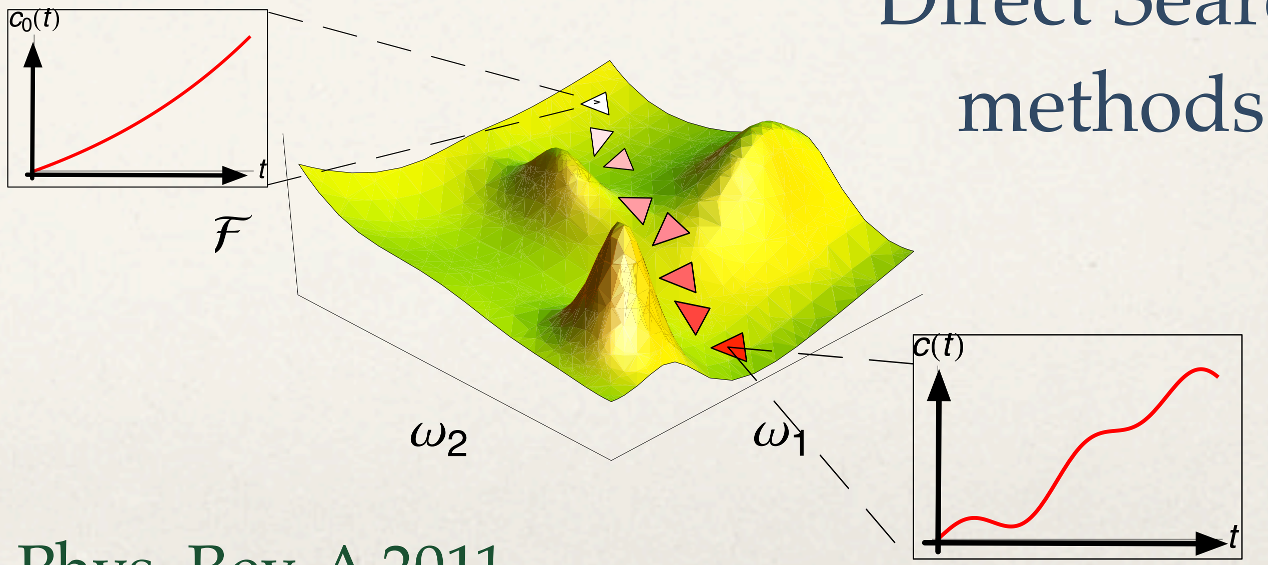
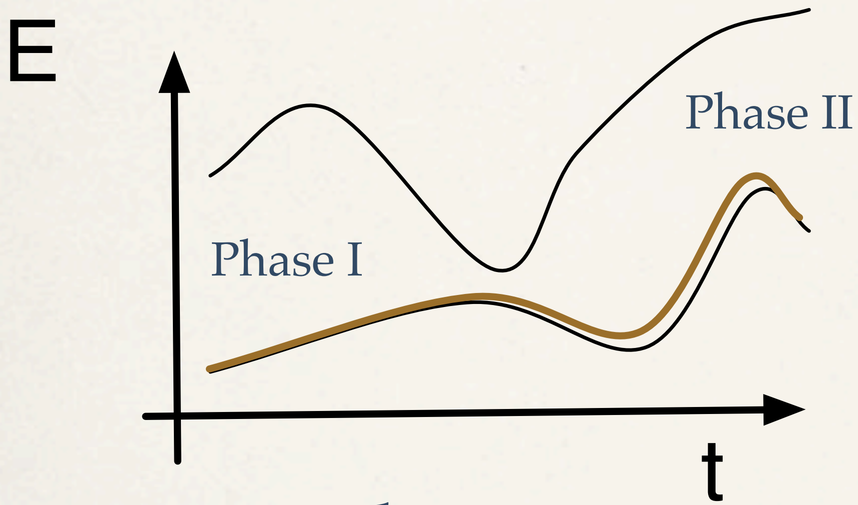
Direct Search
methods



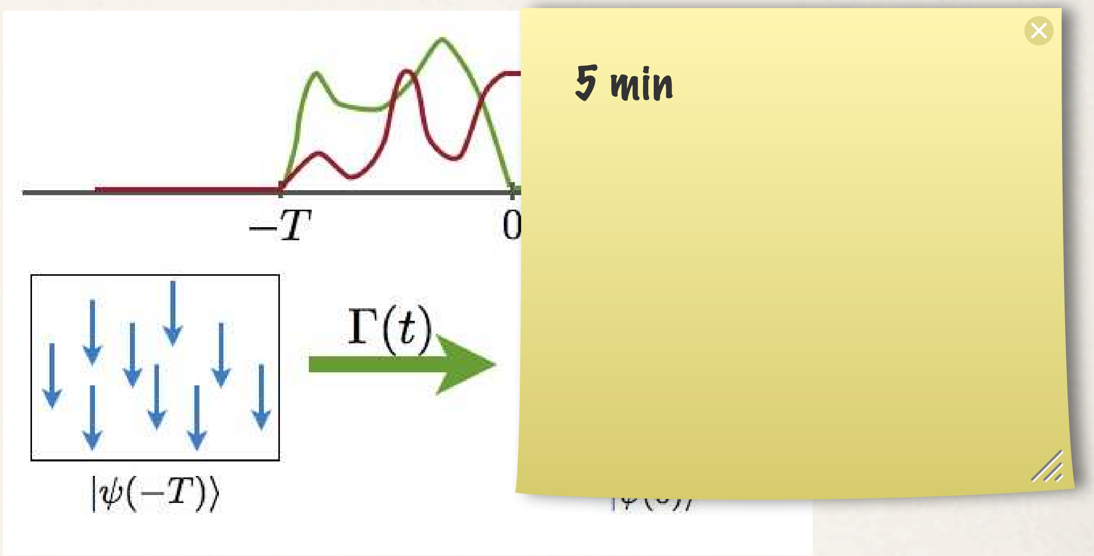
O(10) parameters!

T. Caneva, T. Calarco, and SM, Phys. Rev. A 2011

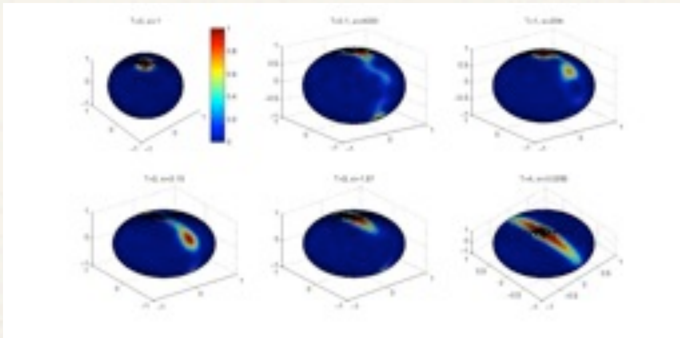
Applications

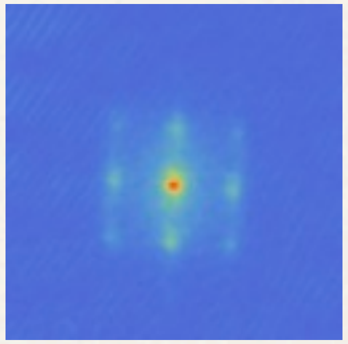


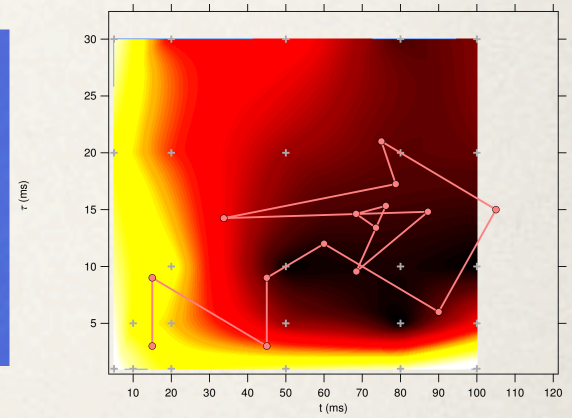
Quantum Phase Transition dynamics



Entanglement Storage units



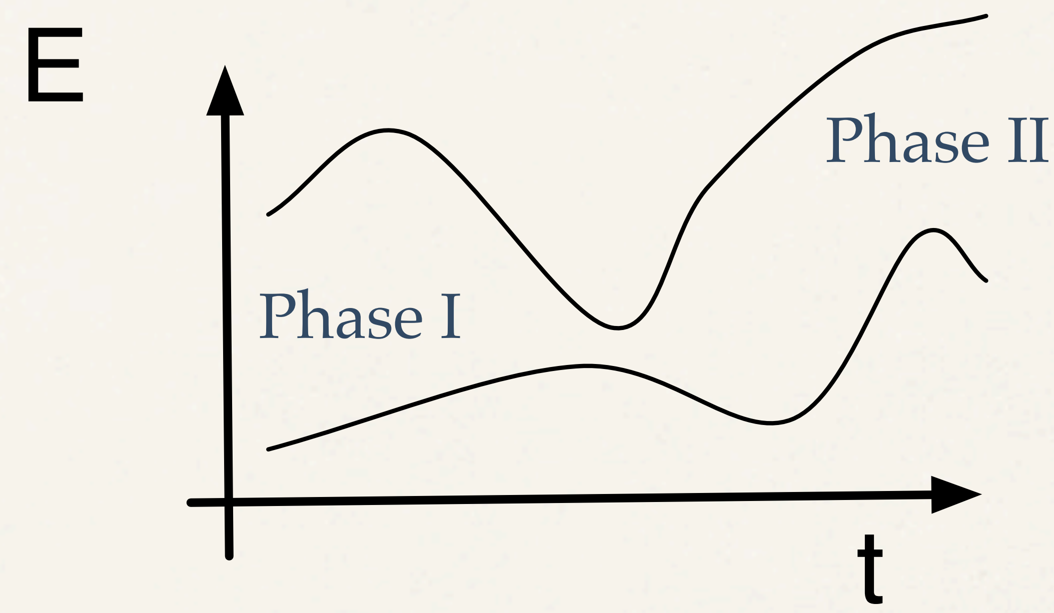




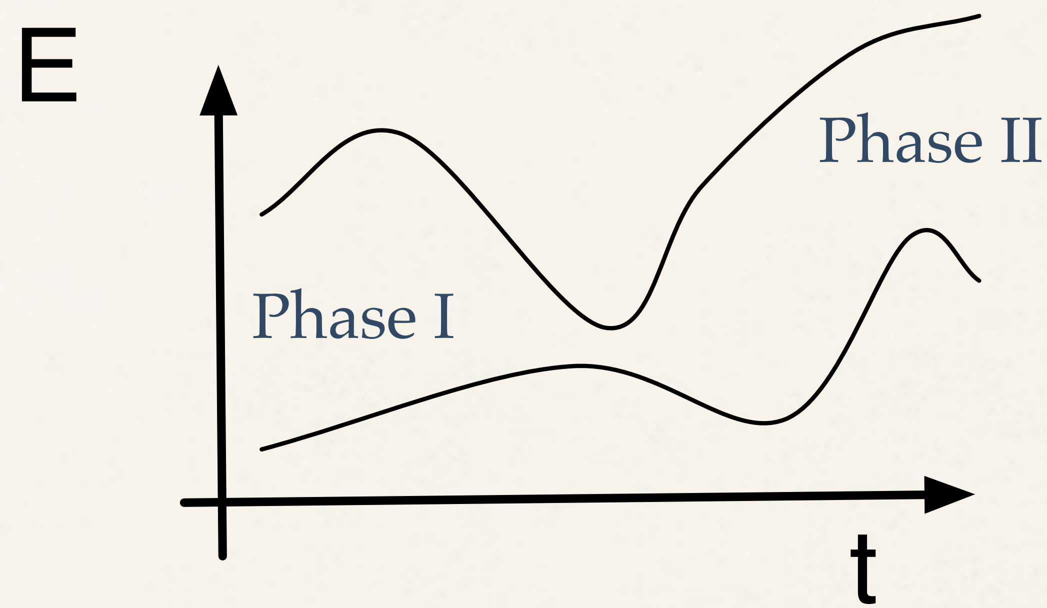
Experiment optimization

Light-harvesting dynamics

Control of QPT dynamics



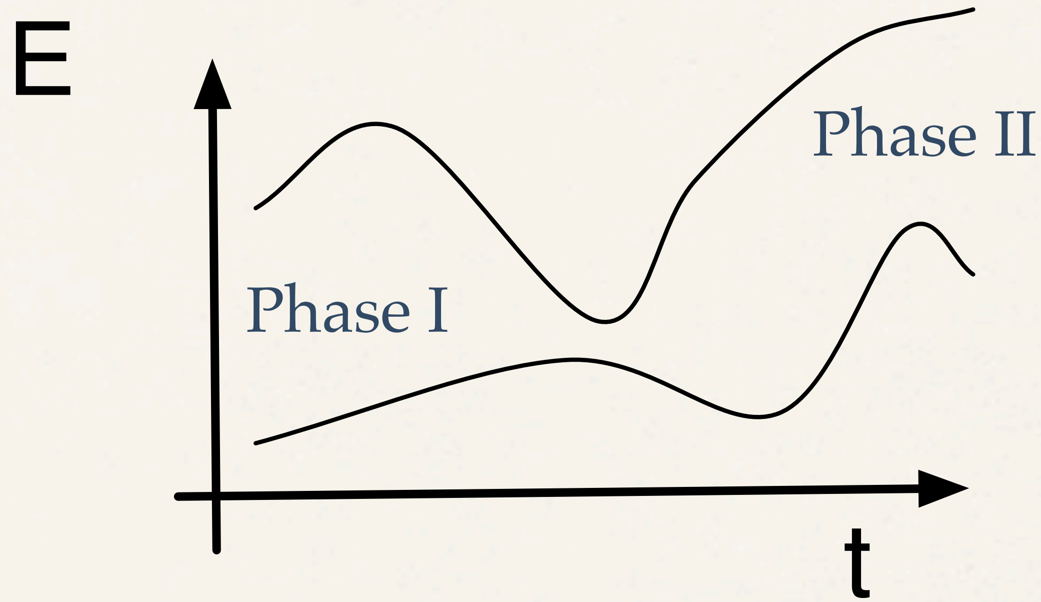
Control of QPT dynamics



Adiabatic strategy

Control of QPT dynamics

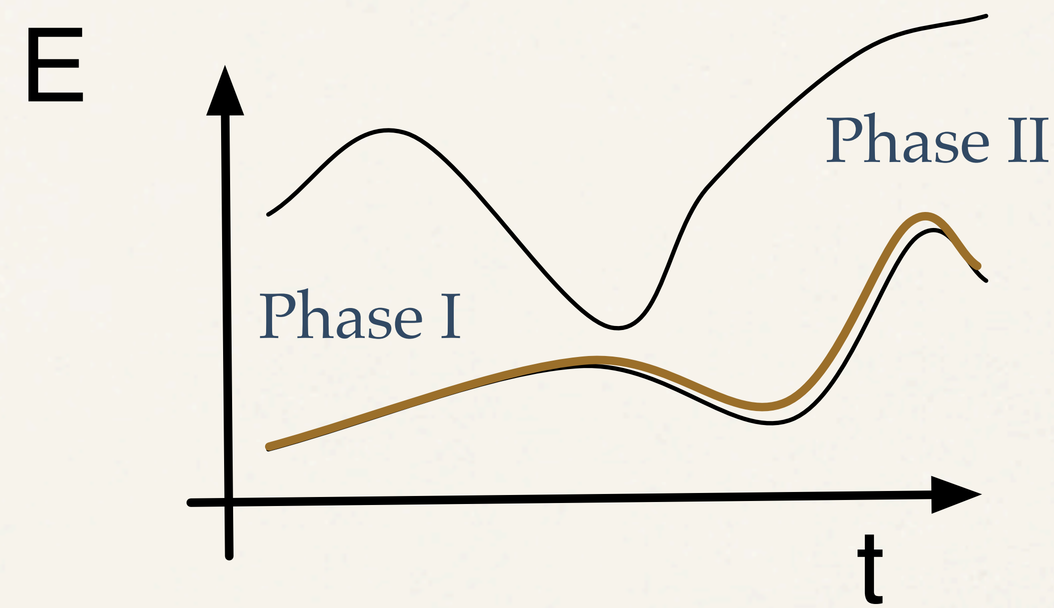
Slow



Adiabatic strategy

Control of QPT dynamics

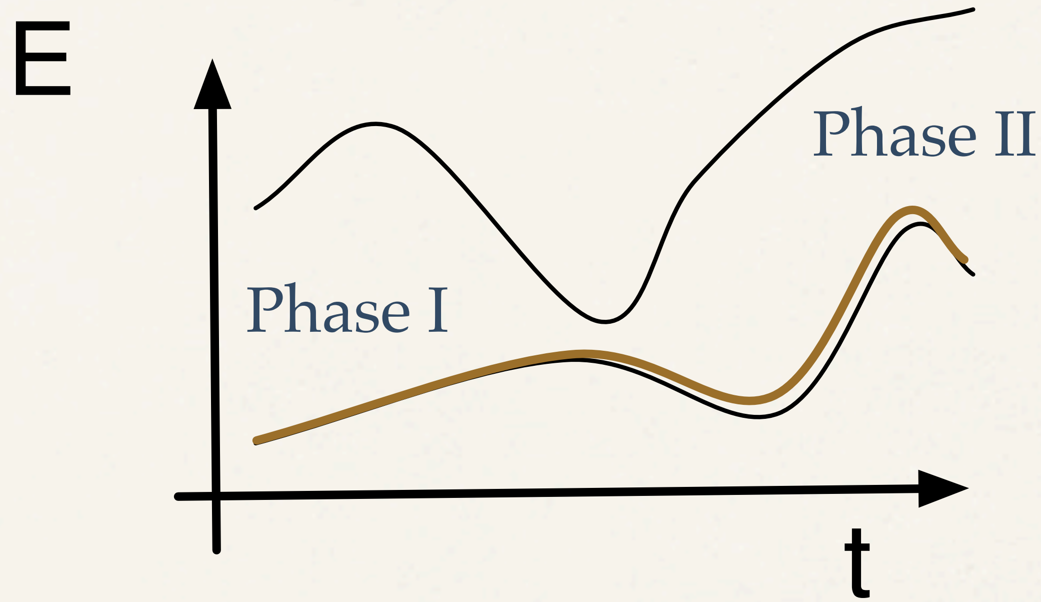
Slow

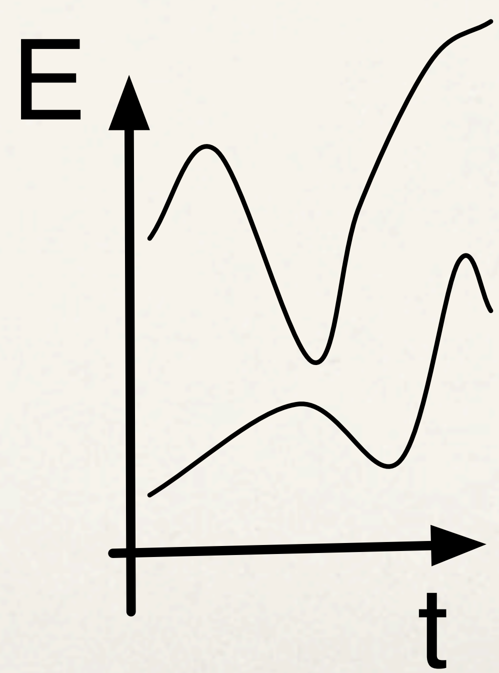


Adiabatic strategy

Control of QPT dynamics

Slow

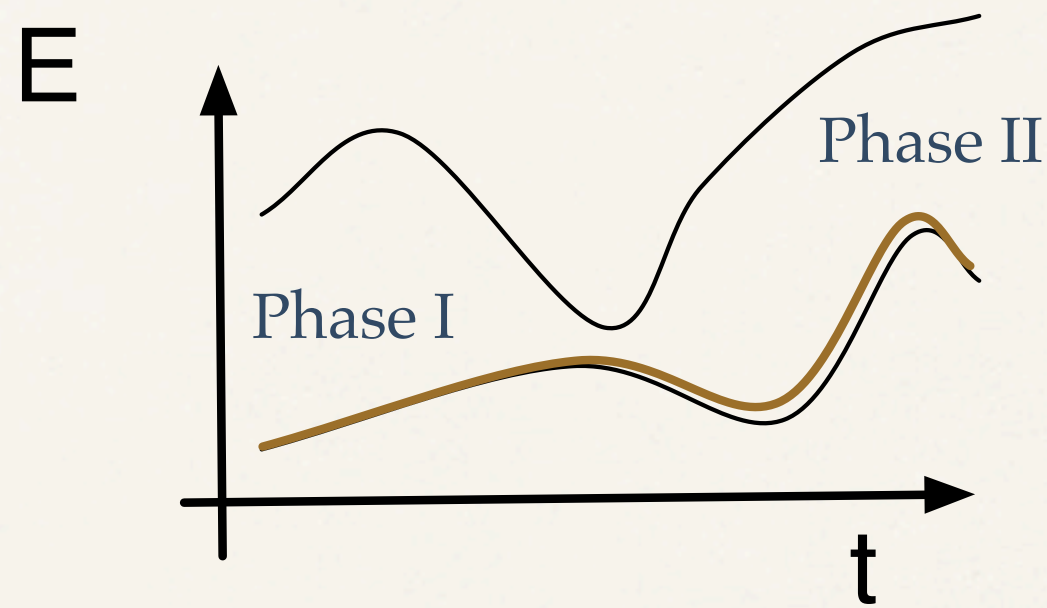




Adiabatic strategy

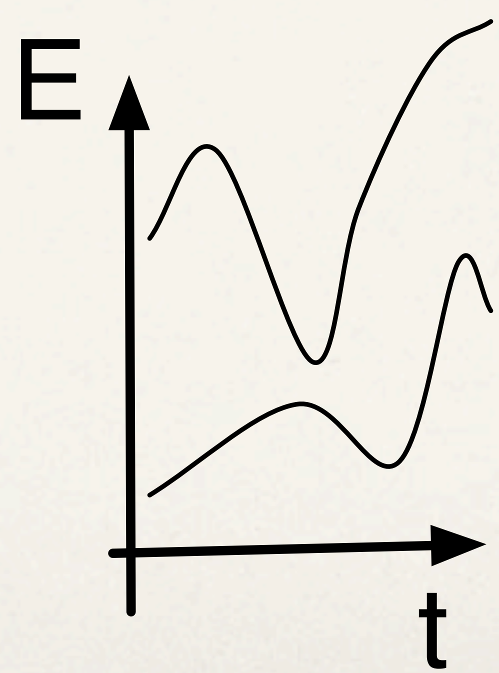
Control of QPT dynamics

Slow



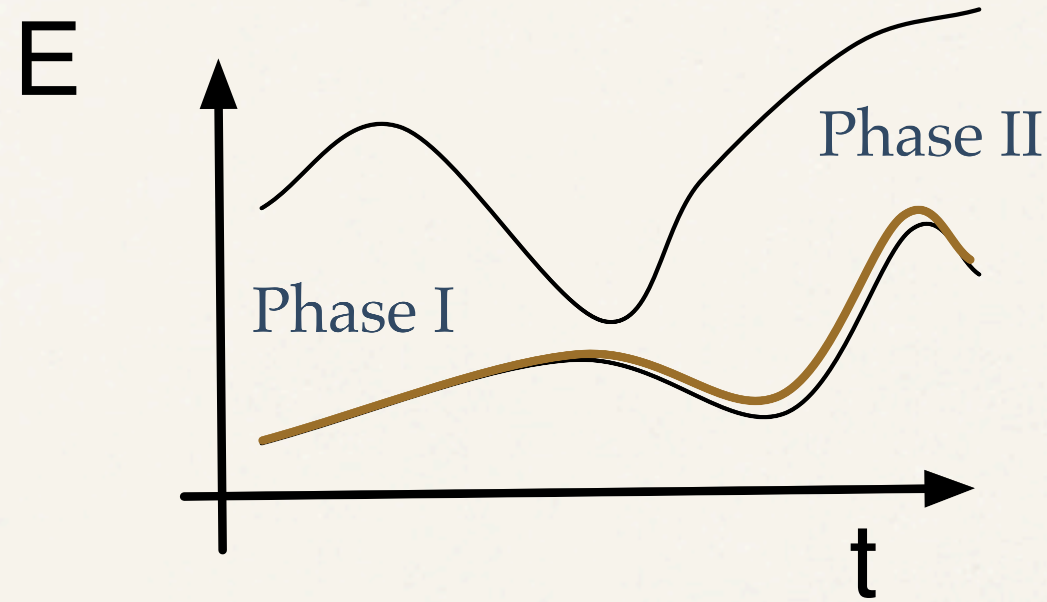
Adiabatic strategy

Fast



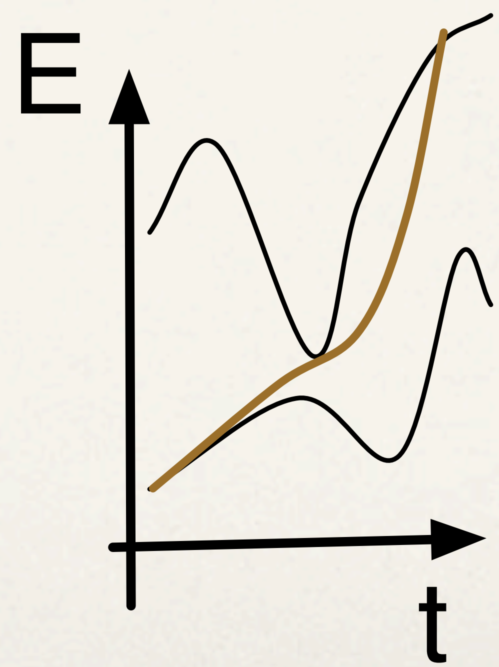
Control of QPT dynamics

Slow



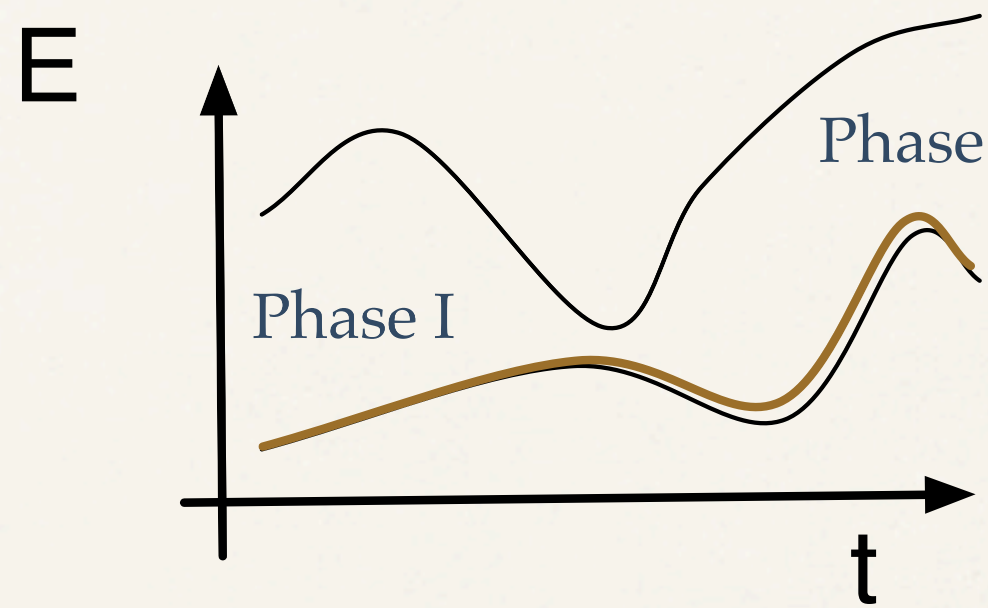
Adiabatic strategy

Fast



Control of QPT dynamics

Slow



Adiabatic strategy

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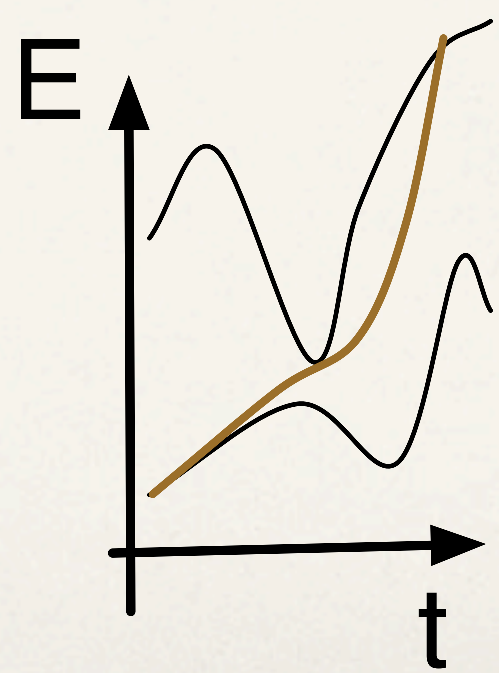
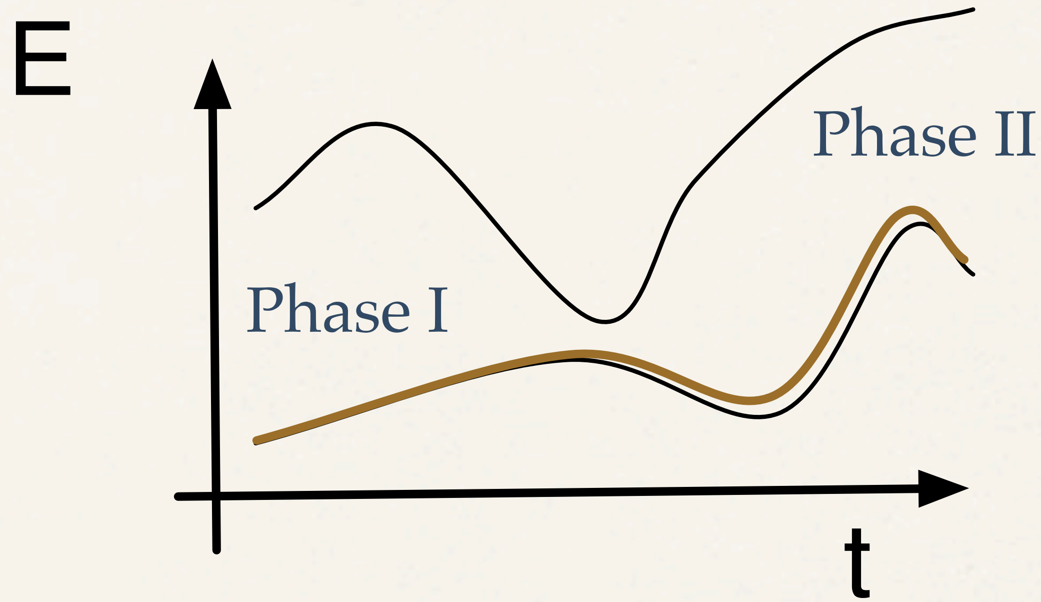


Optimal control

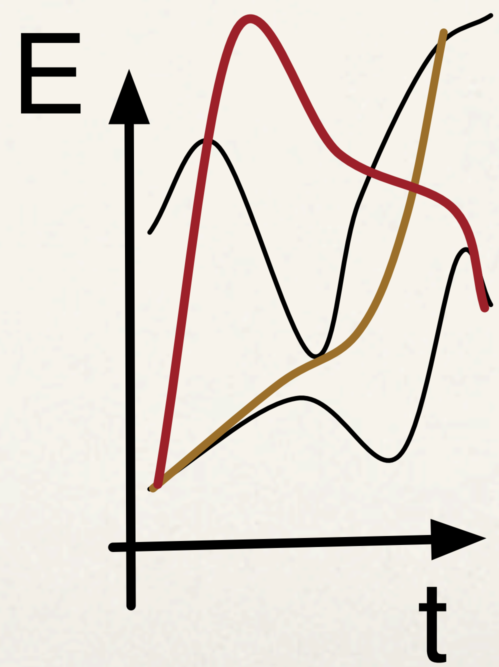
Control of QPT dynamics

Slow



Adiabatic strategy

Fast



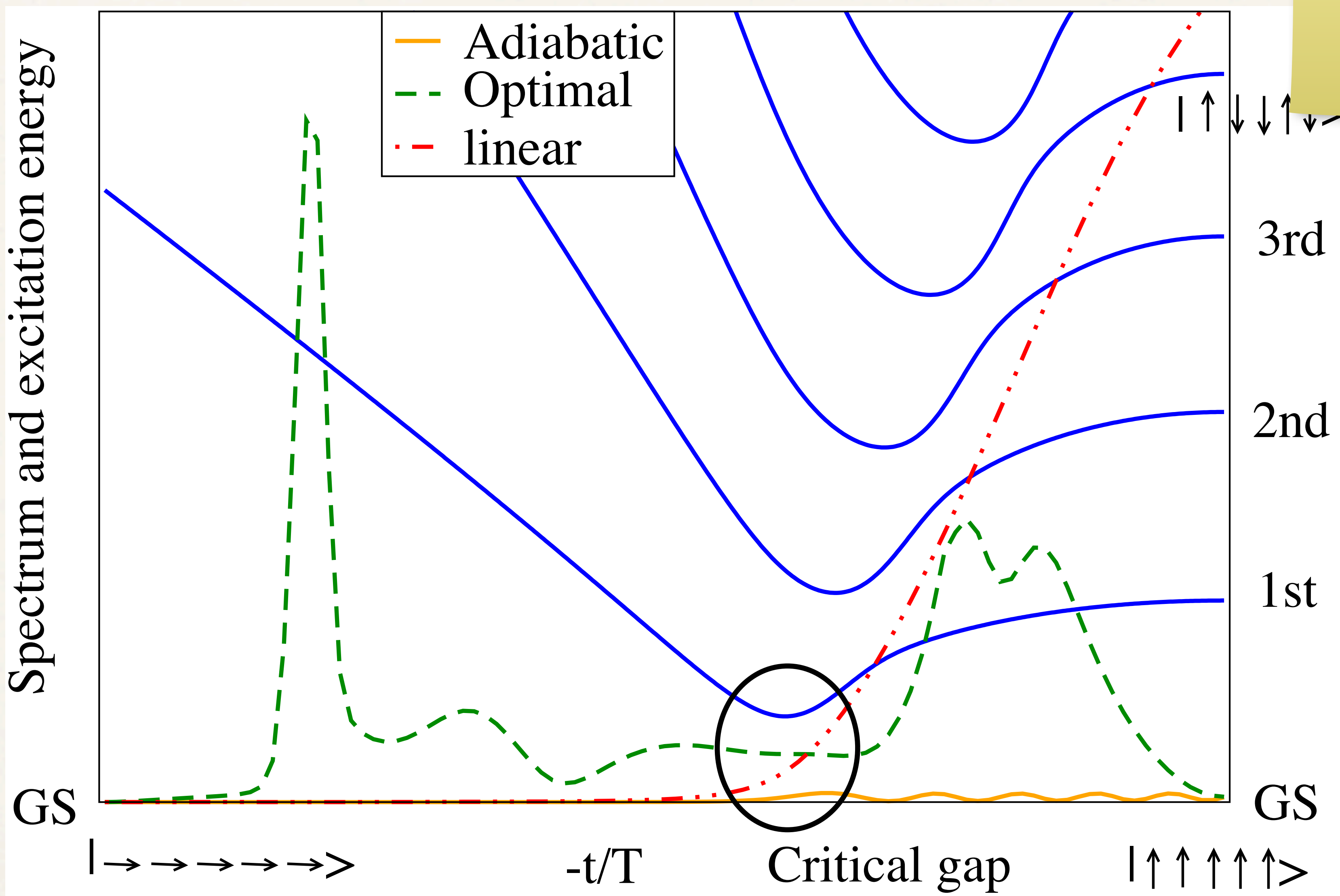
Optimal control

Optimal QPT crossing

atoms in cavities!

Dicke model + adiabatic elimination

LMG model



$$H = -\frac{1}{N}\sum_{i<j}(\sigma_i^x\sigma_j^x + \gamma\sigma_i^y\sigma_j^y) - \Gamma(t)\sum_i^N \sigma_i^z$$

Mott-Insulator Superfluid QPT

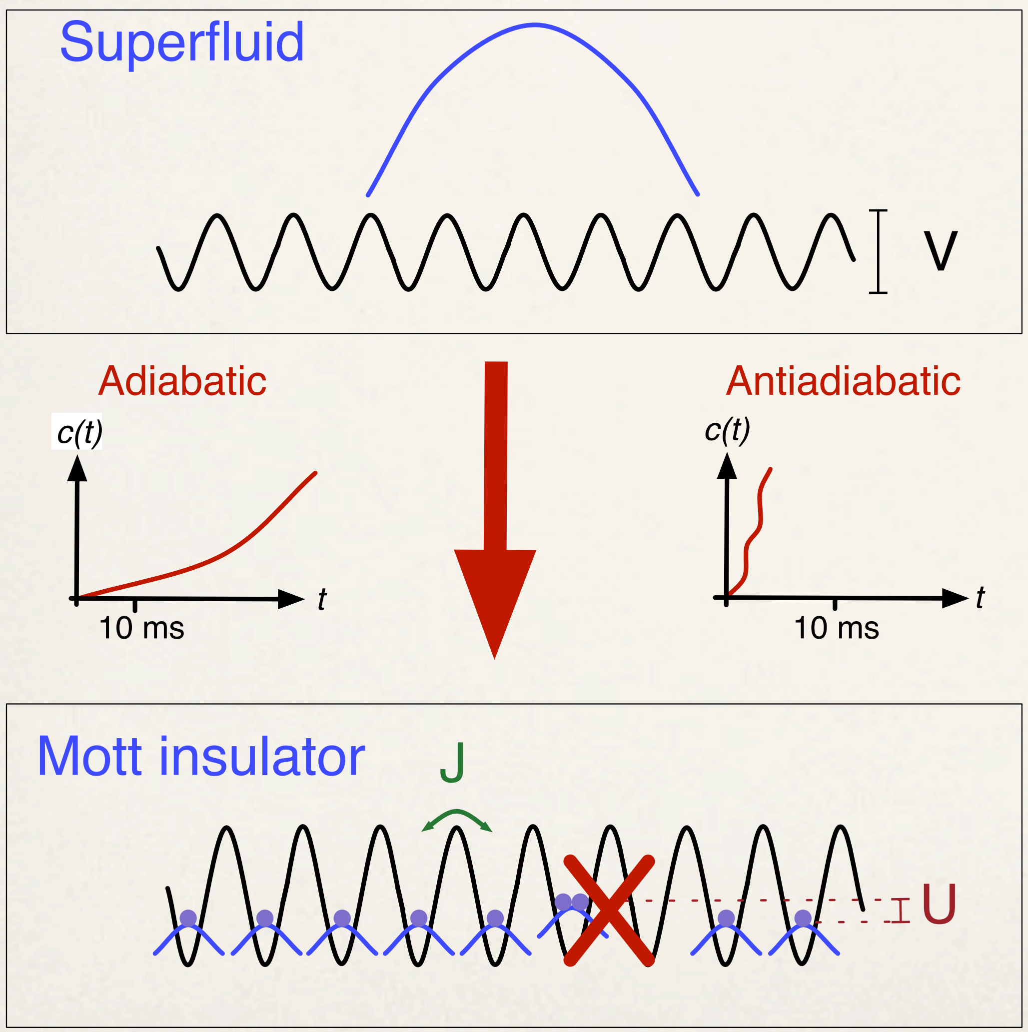
Mott-Insulator Superfluid QPT

$$\mathrm{H} = \sum_j [-J(b_j^\dagger b_{j+1} + \mathrm{h.c.}) + \Omega(j - \frac{N}{2})^2 n_j + \frac{U}{2}(n_j^2 - n_j)]$$

J Hopping
U Onsite energy
Ω Trapping

Mott-Insulator Superfluid QPT

$$\mathrm{H} = \sum_j [-J(b_j^\dagger b_{j+1} + \mathrm{h.c.}) + \Omega(j - \frac{N}{2})^2 n_j + \frac{U}{2}(n_j^2 - n_j)]$$



J/U >> 0.1

J Hopping

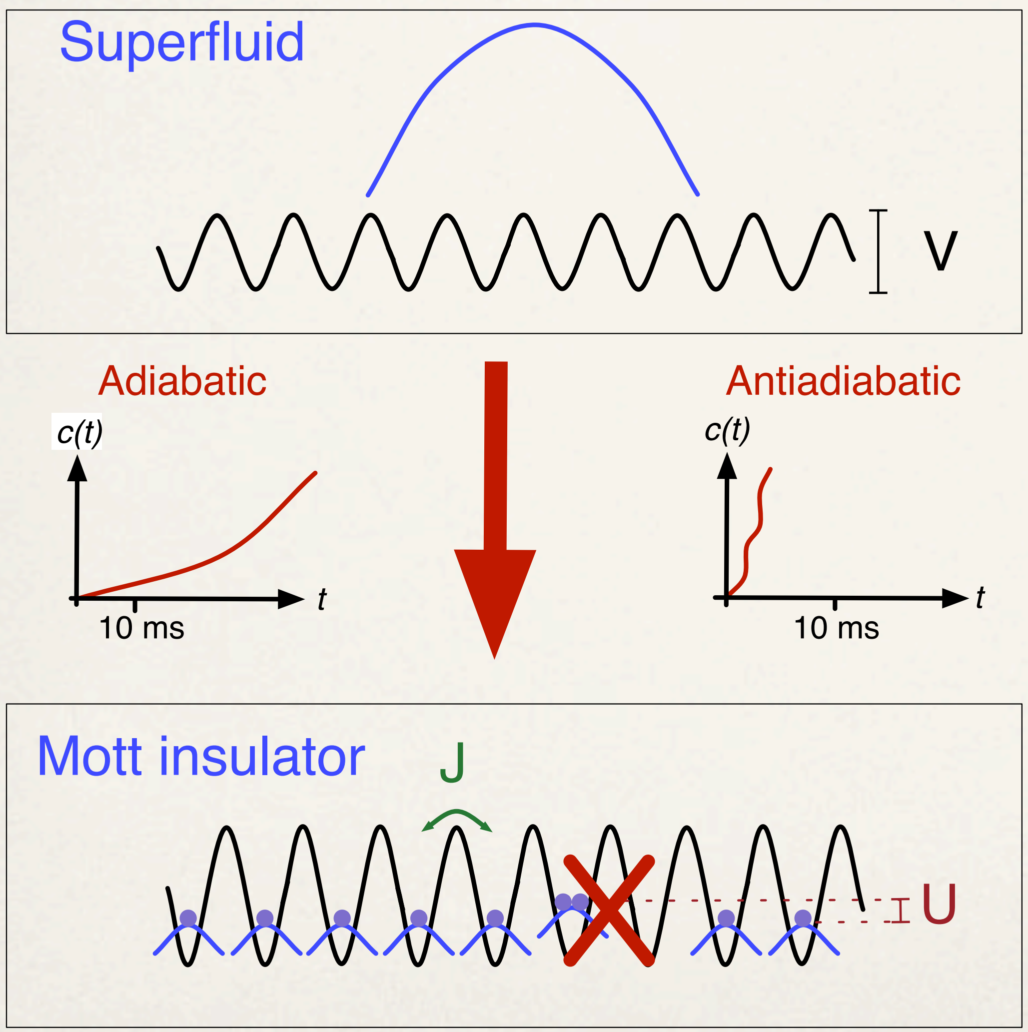
U Onsite energy

Ω Trapping

J/U << 0.1

Mott-Insulator Superfluid QPT

$$\mathrm{H} = \sum_j \left[-J(b_j^\dagger b_{j+1} + \mathrm{h.c.}) + \Omega\left(j - \frac{N}{2}\right)^2 n_j + \frac{U}{2}(n_j^2 - n_j) \right]$$



J/U >> 0.1

J Hopping

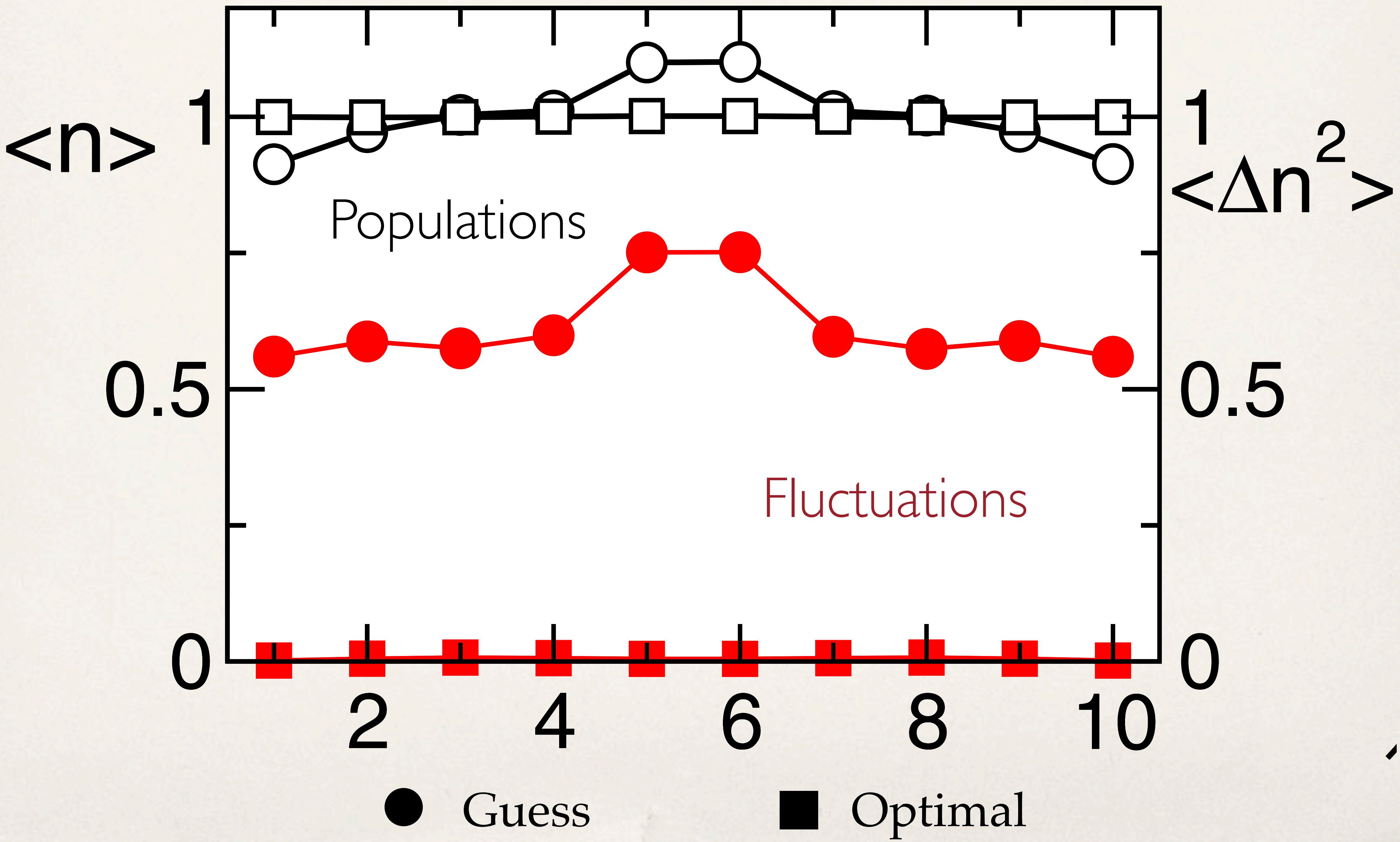
U Onsite energy

Ω Trapping

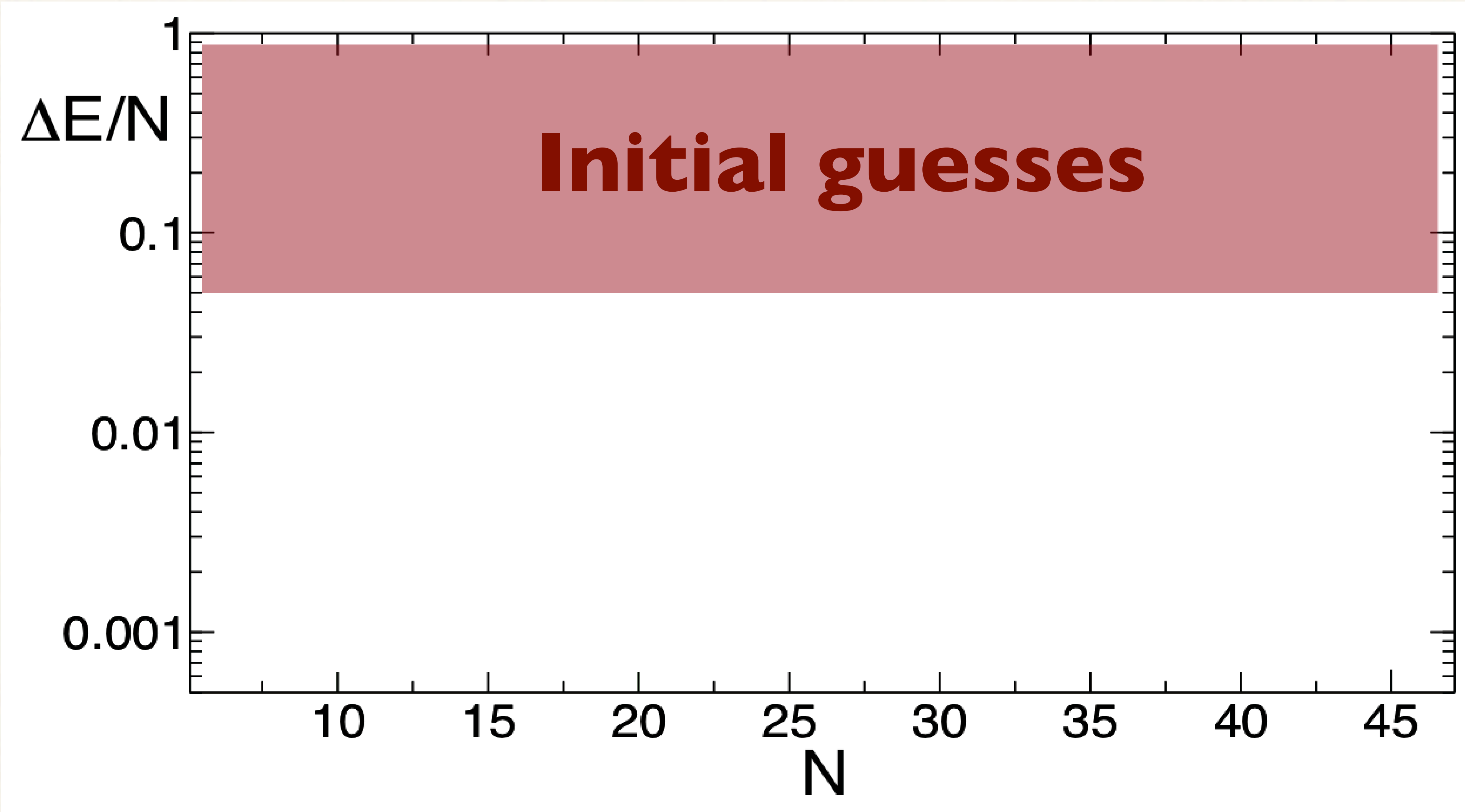
J/U << 0.1

M. Greiner, O. Mandel, T. Esslinger, T.W. Hansch and I. Bloch, Nature 415, 39 (2002).

CRAB Optimized dynamics

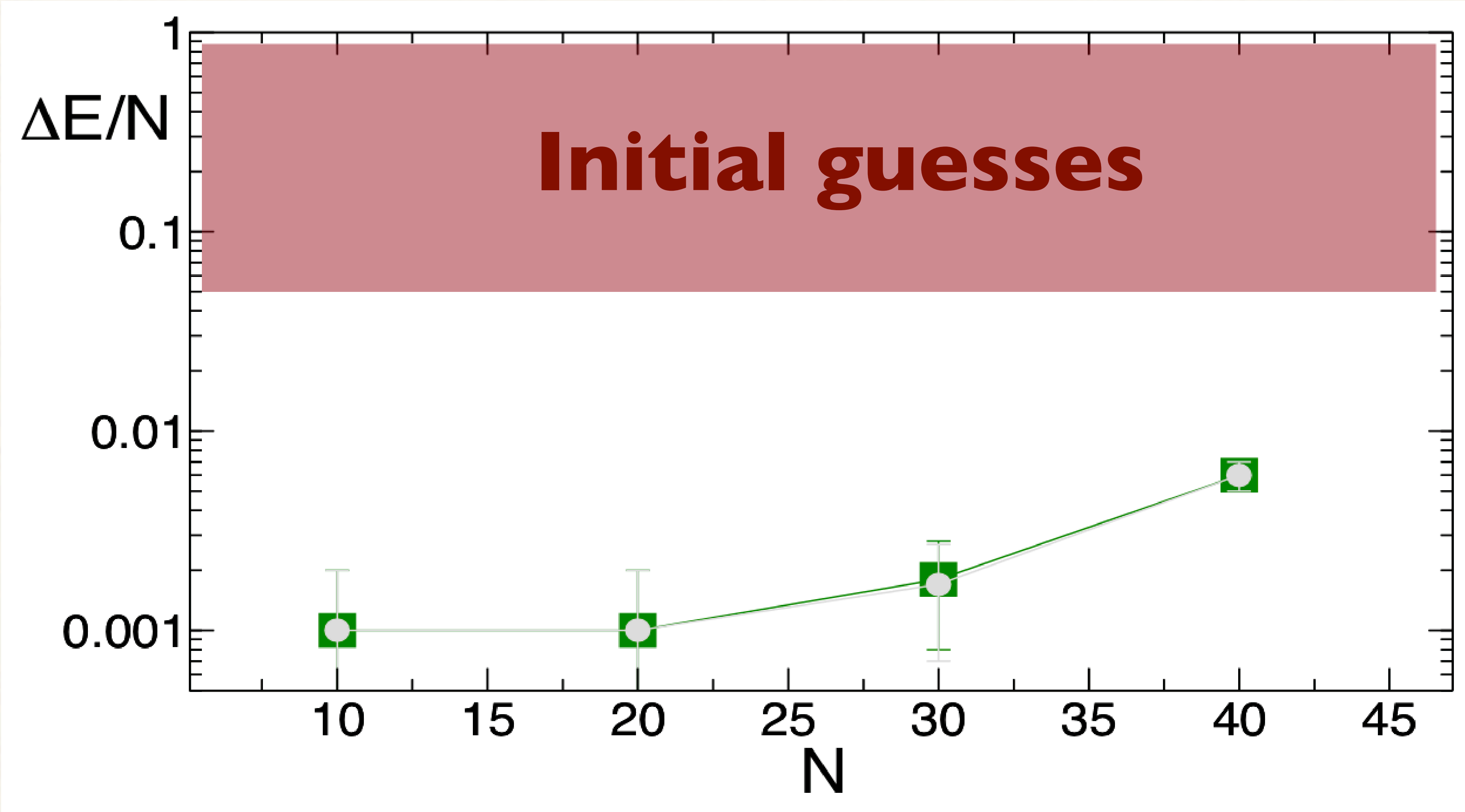


Residual density of defects



T= 3ms

Residual density of defects



T= 3ms

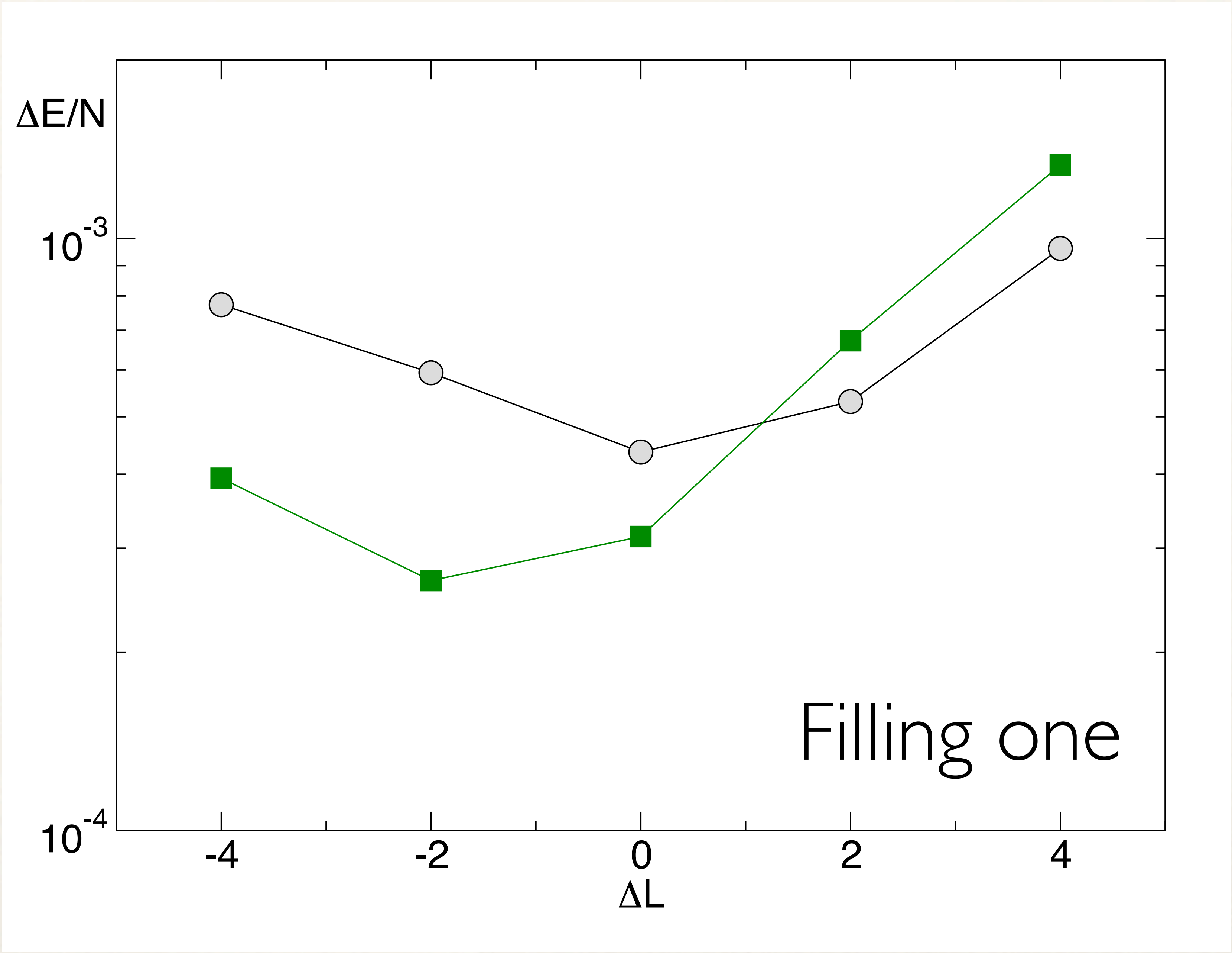
Homogeneous system

Trapping potential
T. Esslinger group
PRL (2004)

P. Doria, T. Calarco, SM Phys. Rev. Lett. 106, 190501 (2011)

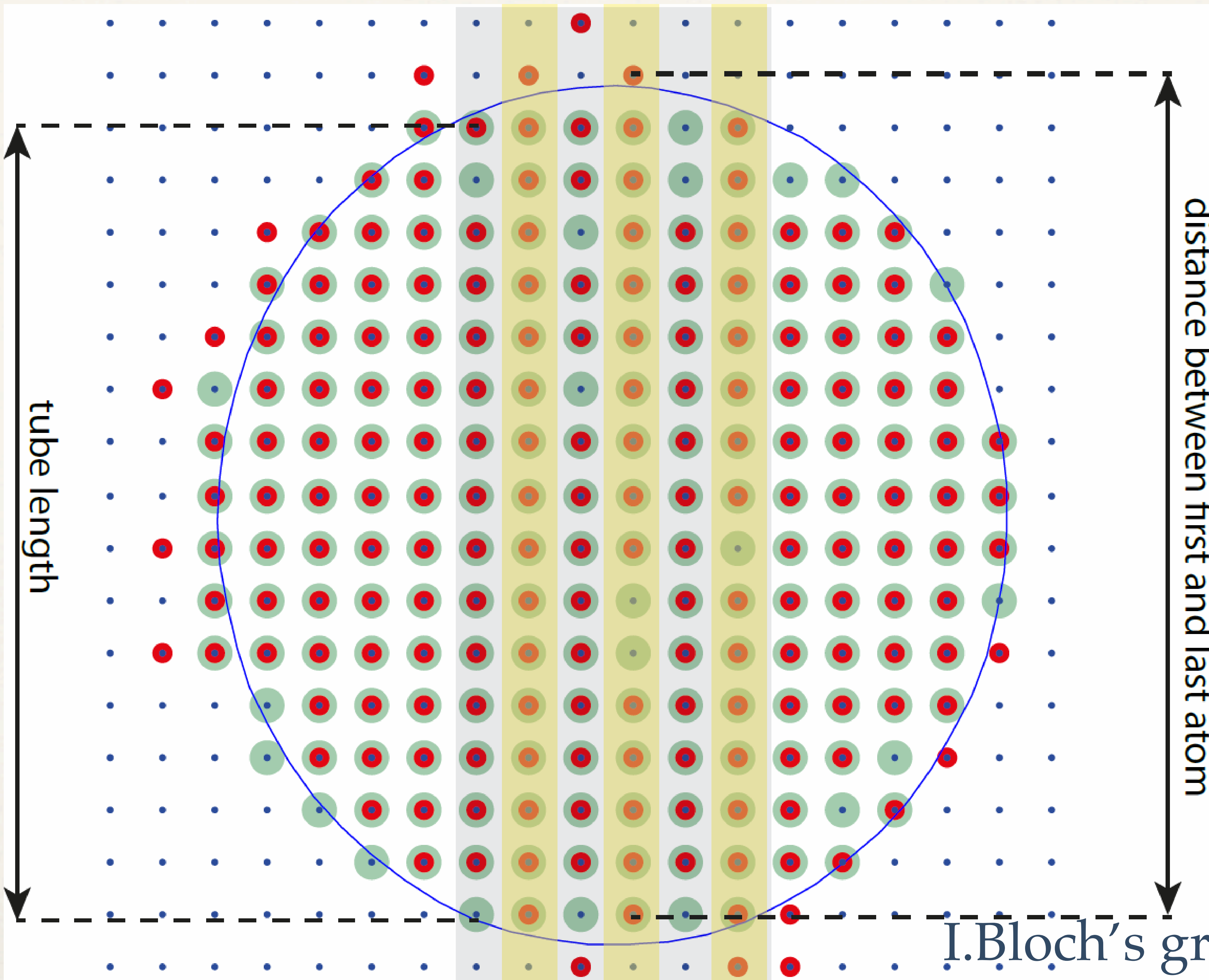
Atom number fluctuations

Optimal pulse
for L=20



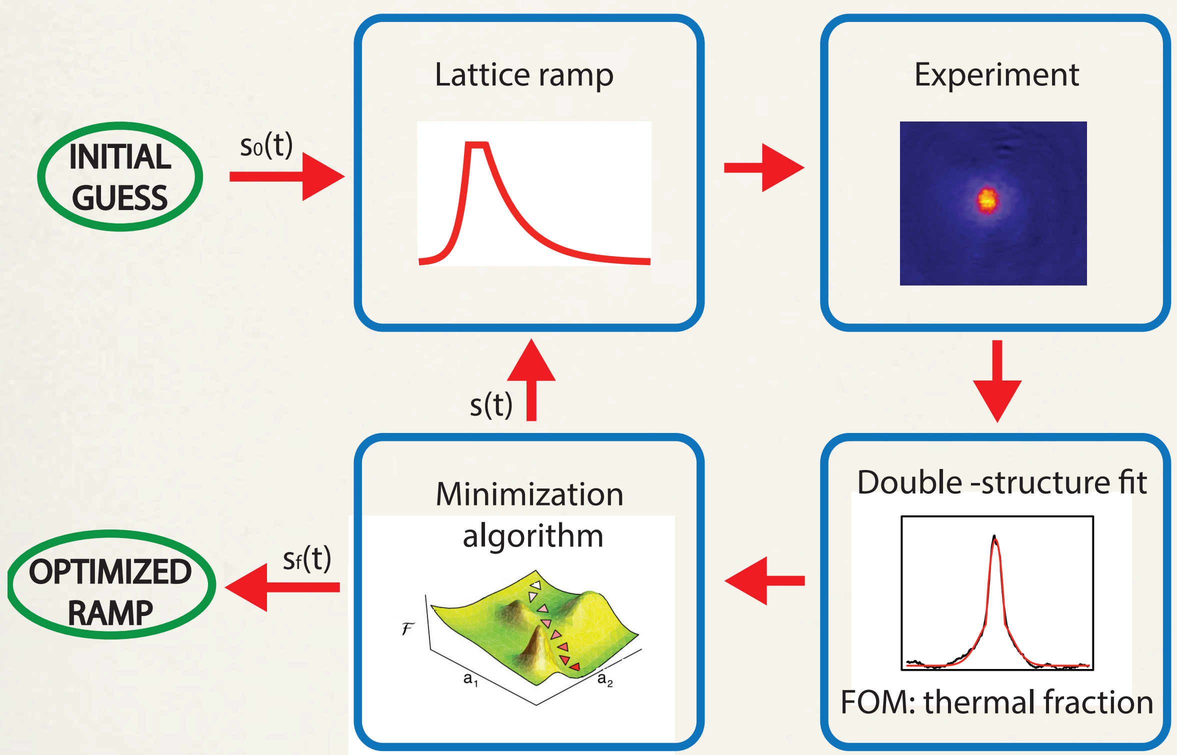
Open loop optimization

Open loop optimization



I.Bloch's group

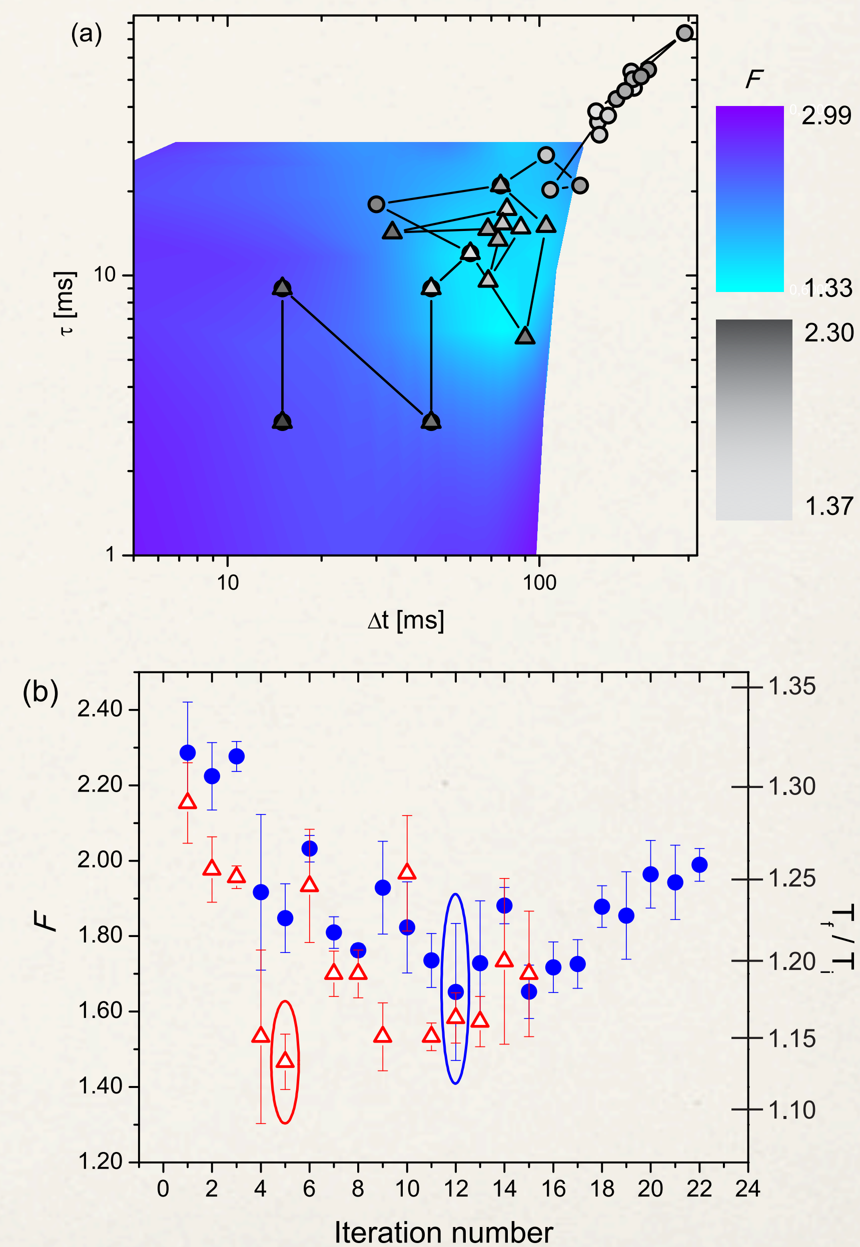
Closed loop optimization



3D-1D crossover and QPT

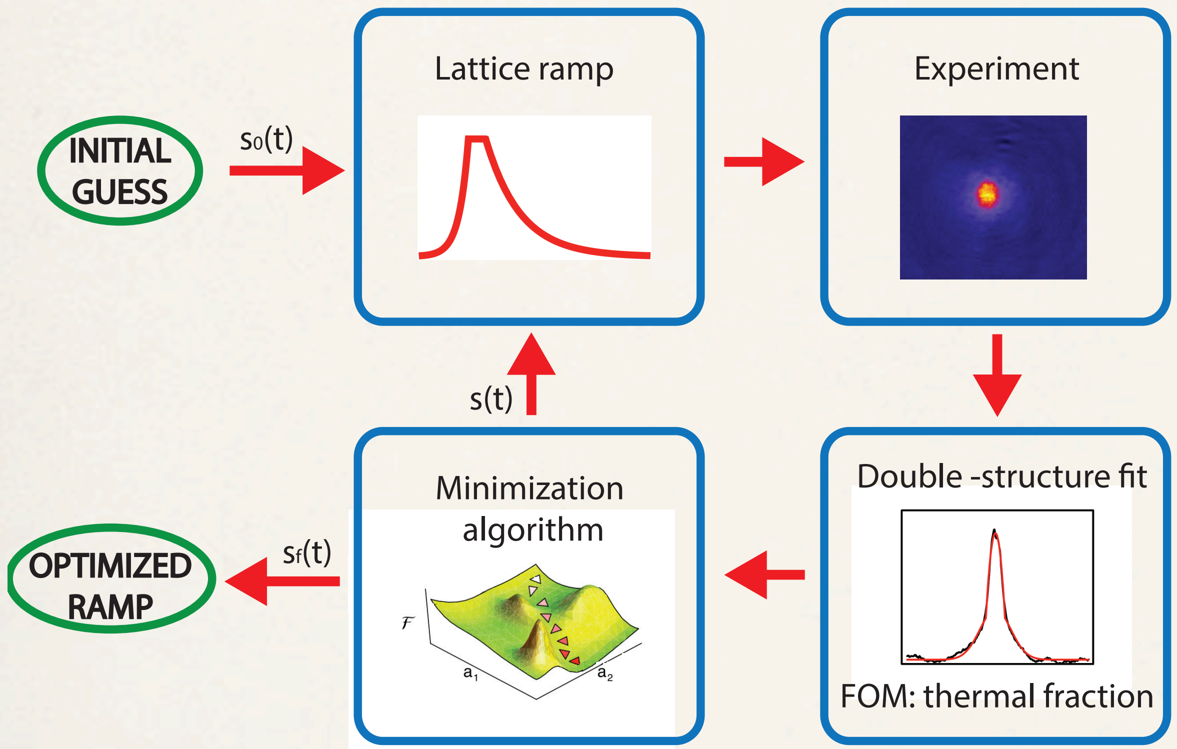
$$T_{opt} \sim T_{ad}/3$$

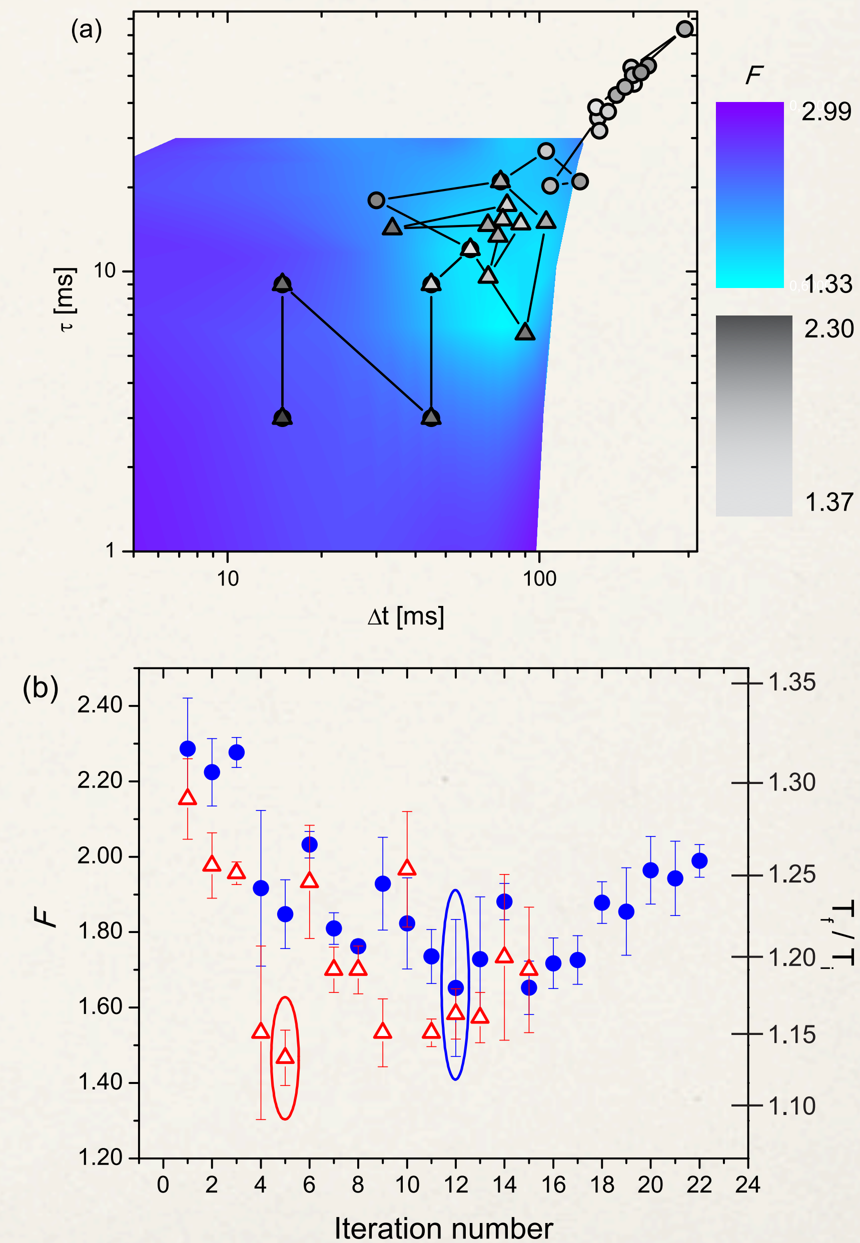
$$FOM_{opt} \sim 0.9\ FOM_{ad}$$



Run	Δt_{opt}	τ_{opt}	F_{uncorr}	Best F_{opt}
1	154 ms	35 ms	2.30 ± 0.03	1.73 ± 0.02
2	45 ms	9 ms	2.30 ± 0.03	1.40 ± 0.06

Closed loop optimization





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S. Rosi, et. al. in preparation

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Paradigm shift

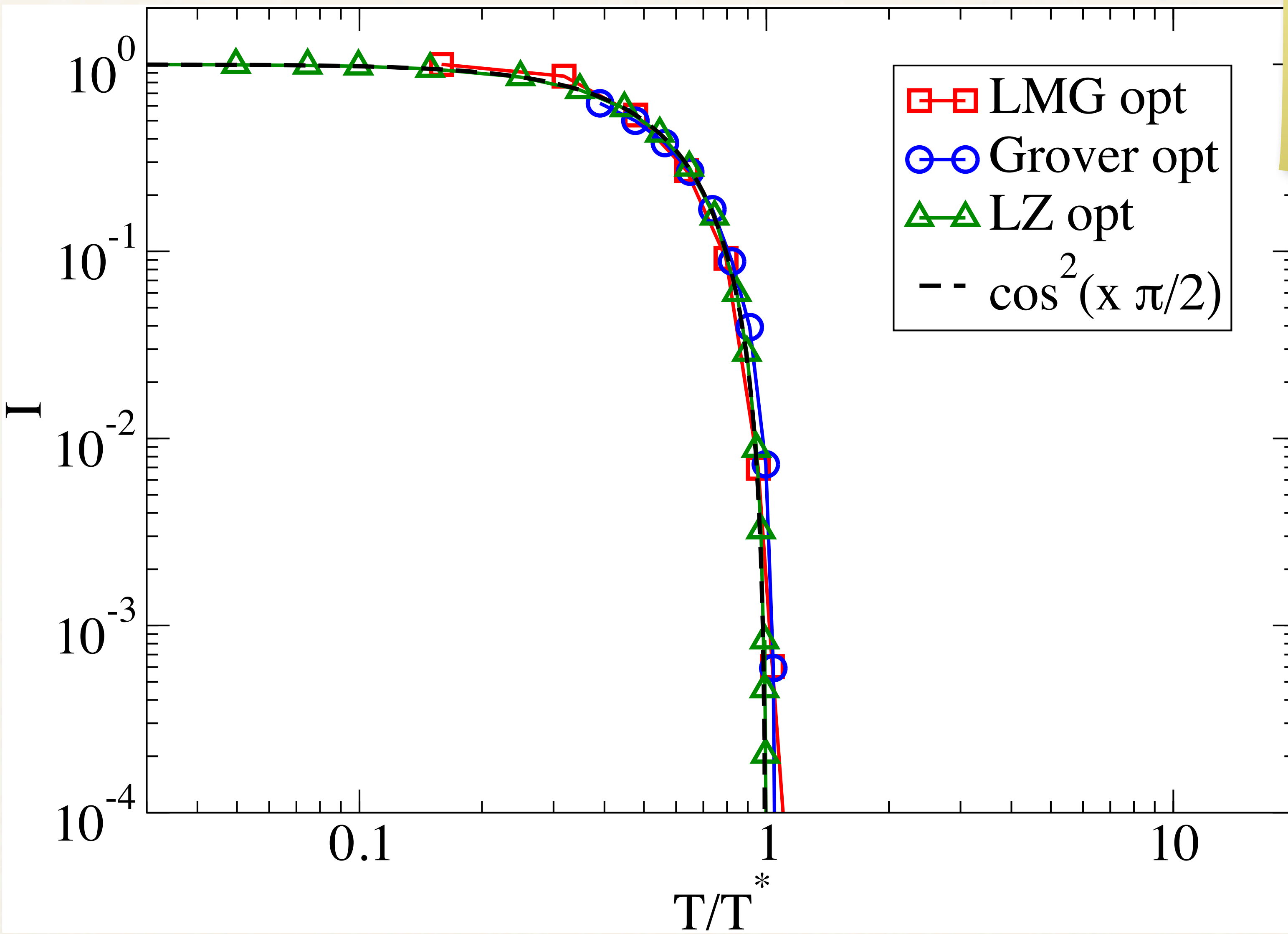
Paradigm shift

How do we control MBQS?

Paradigm shift

Under which conditions
can we control MBQS?

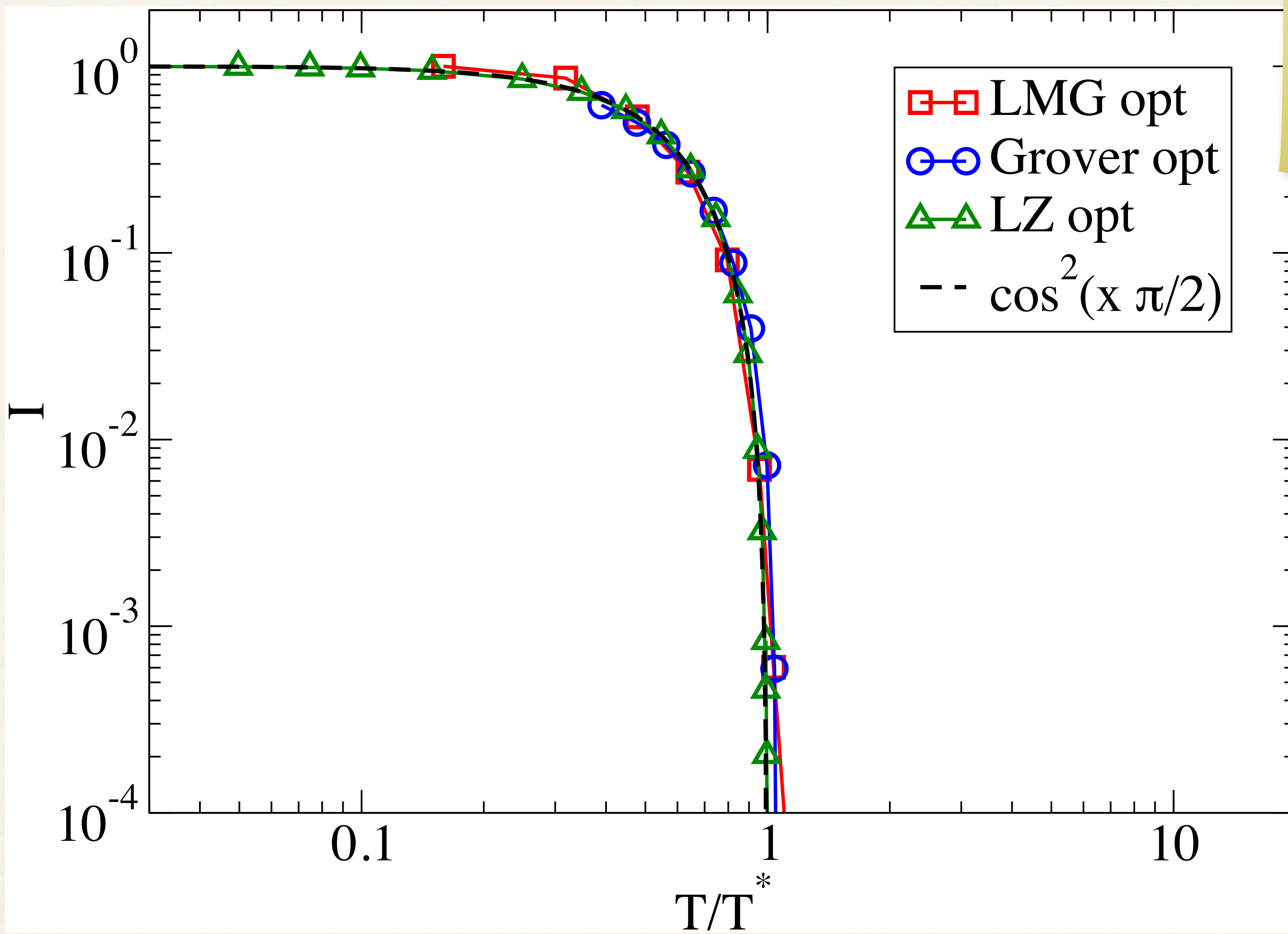
Quantum speed limit



pfeiffer 1993
battacharrya 1983
margolus and levitin 1998
alberto carlini

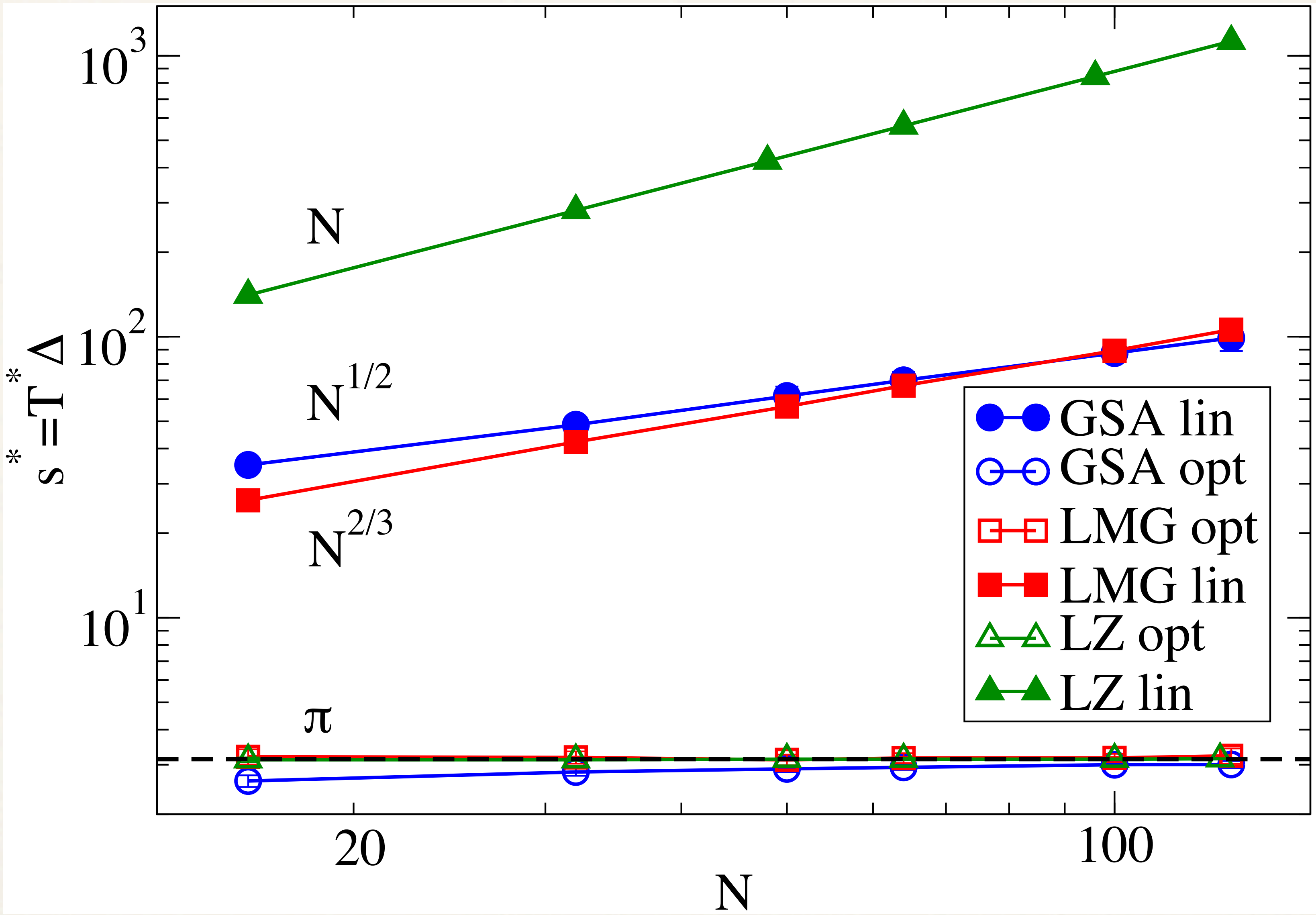
Quantum speed limit

pfeiffer 1993
battacharrya 1983
margolus and levitin 1998
alberto carlini



see also T. Caneva, M. Murphy, T. Calarco, R. Fazio, SM, V. Giovannetti, and G. E. Santoro, Phys. Rev. Lett. 103, 240501 (2009).

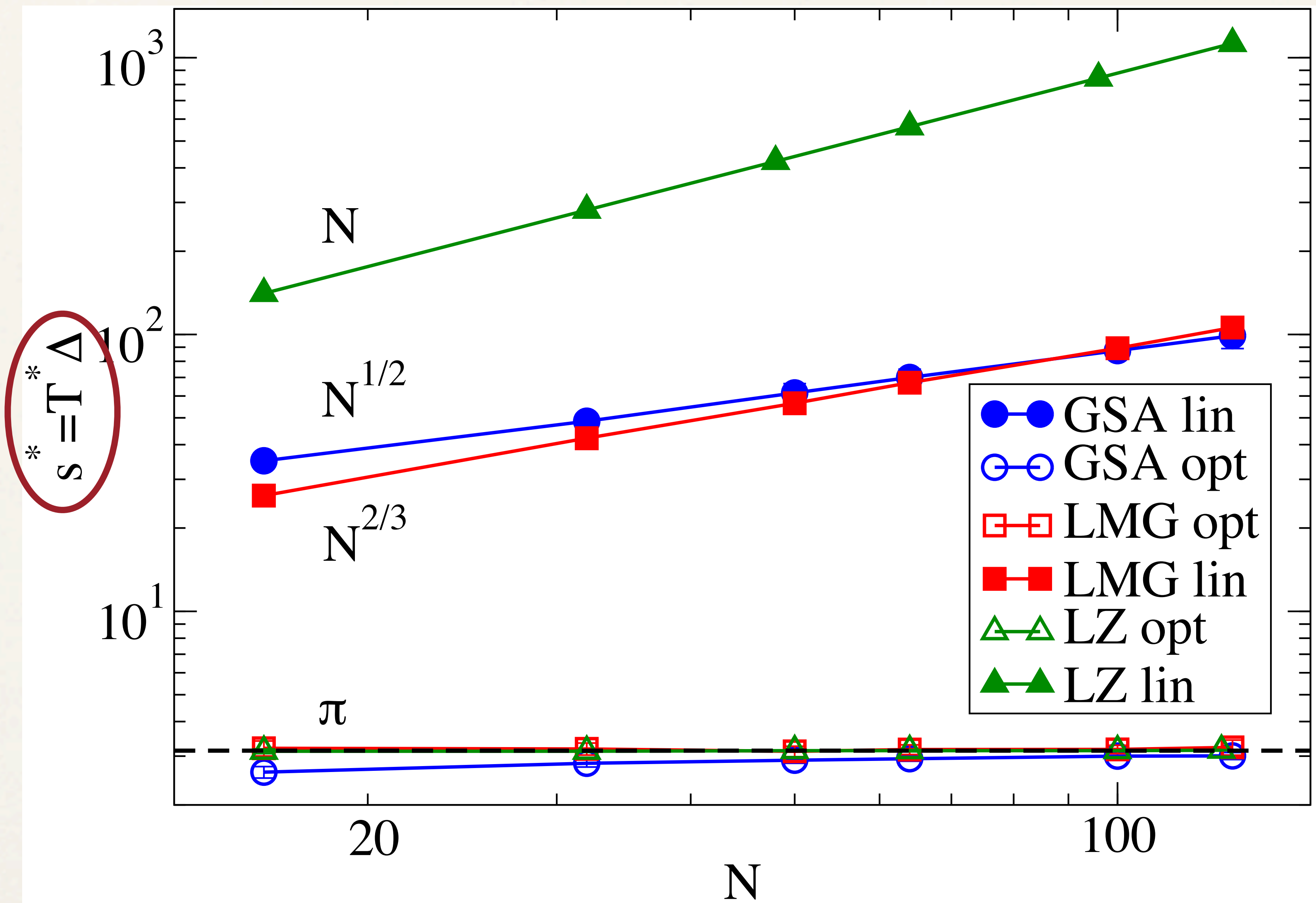
Optimal action



Optimal scaling

Optimal action

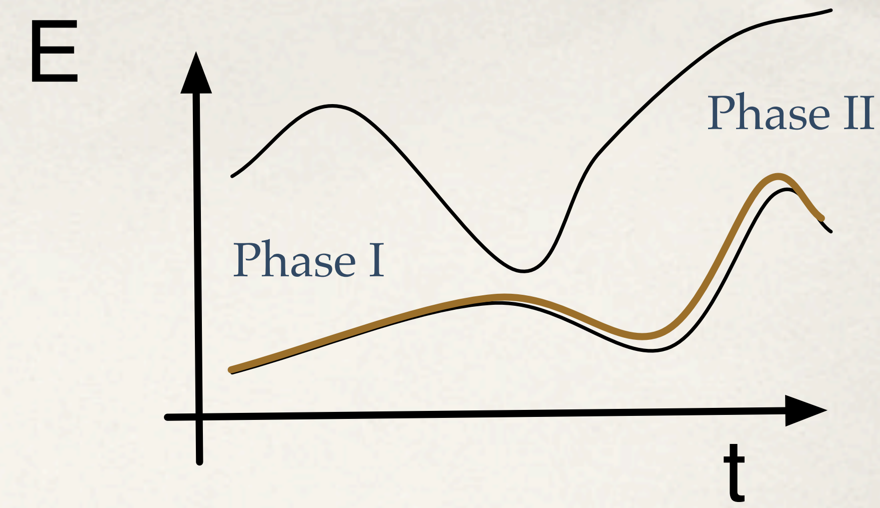
Optimal action QSL



Optimal scaling

T. Caneva, T. Calarco, R. Fazio, G. E. Santoro, and SM Phys. Rev. A 84, 012312 (2011)

QPT crossing scenario



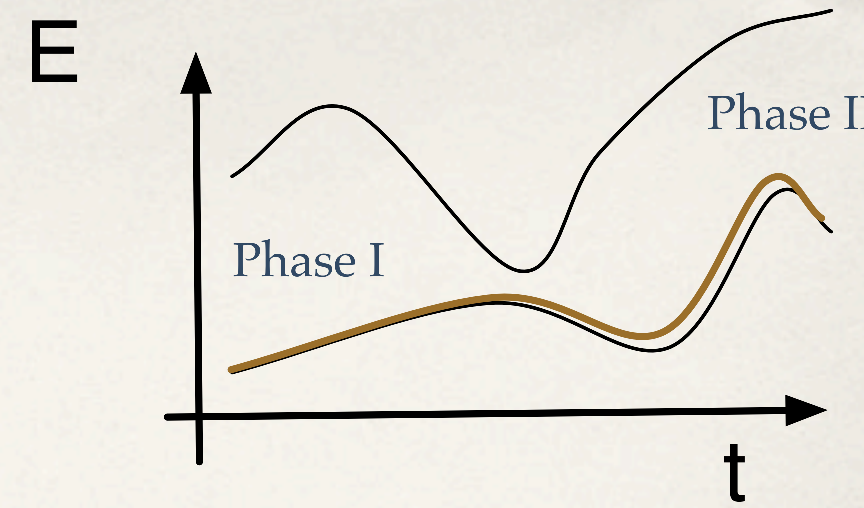
$T \ll \Delta^{-1}$ $\quad$ $T \sim \Delta^{-1}$ $\quad$ $T \gg \Delta^{-1}$

$I \sim O(1)$ $\quad$ $I(s \gg 1) \to 0$

Linear s $\quad$ π $\quad$ ∞

Kibble Zurek $\quad$ QSL $\quad$ Adiabatic

QPT crossing scenario



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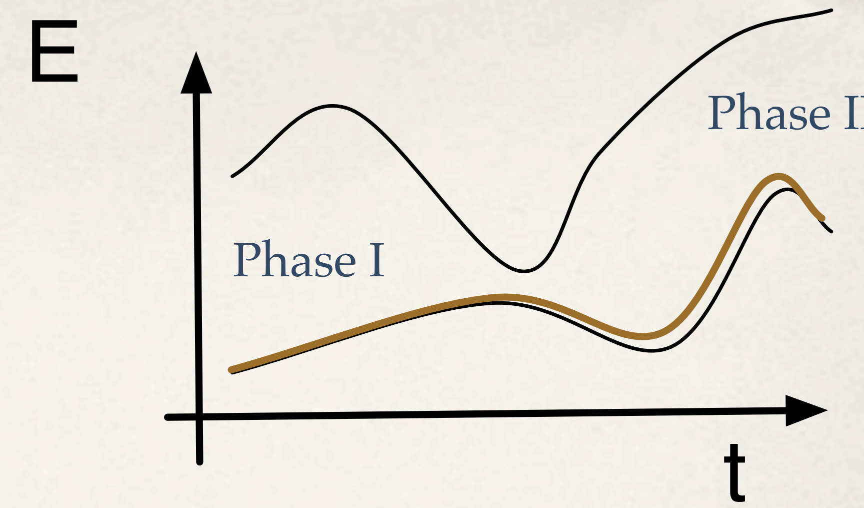
Linear $\quad s$ ———— π ————→ ∞

Optimal

Kibble Zurek $\qquad$ QSL $\qquad$ Adiabatic

QPT crossing scenario



$T \ll \Delta^{-1}$ $\qquad$ $T \sim \Delta^{-1}$ $\qquad$ $T \gg \Delta^{-1}$

$I \sim O(1)$ $\qquad$ $I(s \gg 1) \rightarrow 0$

Linear $\quad s$ ——— π ———→ ∞

$I = \cos^2(s/2)$ $\qquad$ $I = 0$

Optimal $\quad s$ ——— π ———→ ∞

Kibble Zurek $\qquad$ QSL $\qquad$ Adiabatic

T. Caneva, T. Calarco, R. Fazio, G. E. Santoro, and SM Phys. Rev. A 84, 012312 (2011)

New questions

time 15 min

New questions

How do we control MBQS?

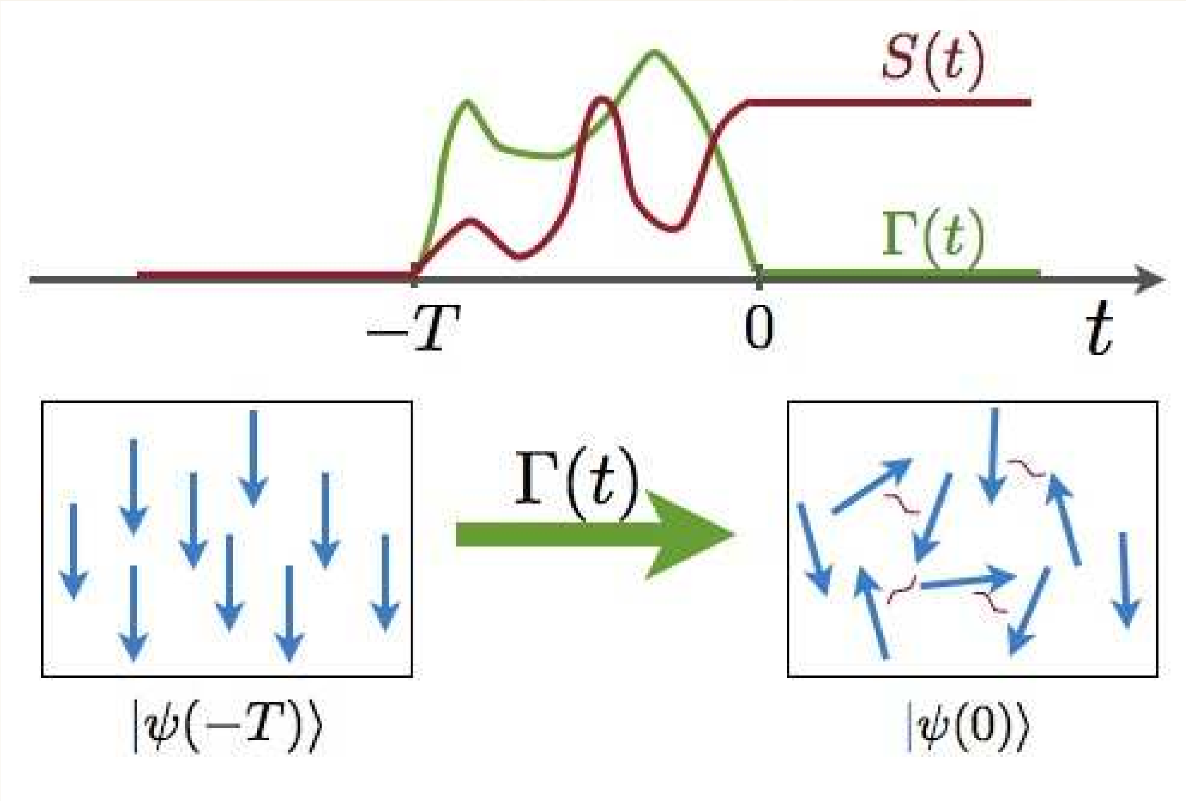
time 15 min

New questions

time 15 min

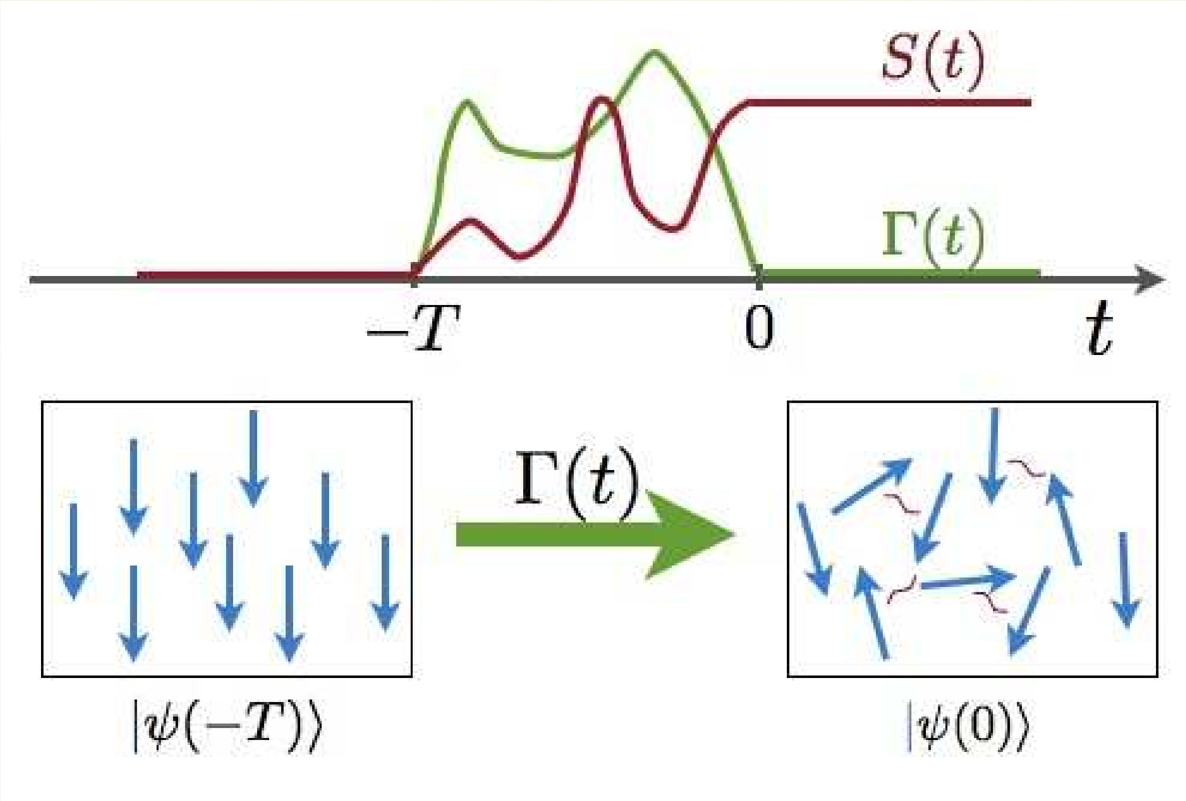
Is there something "new"
we can learn/achieve/gain
exploiting the control MBQS?

Entanglement Storage Units



inset:
T VS noise intensity

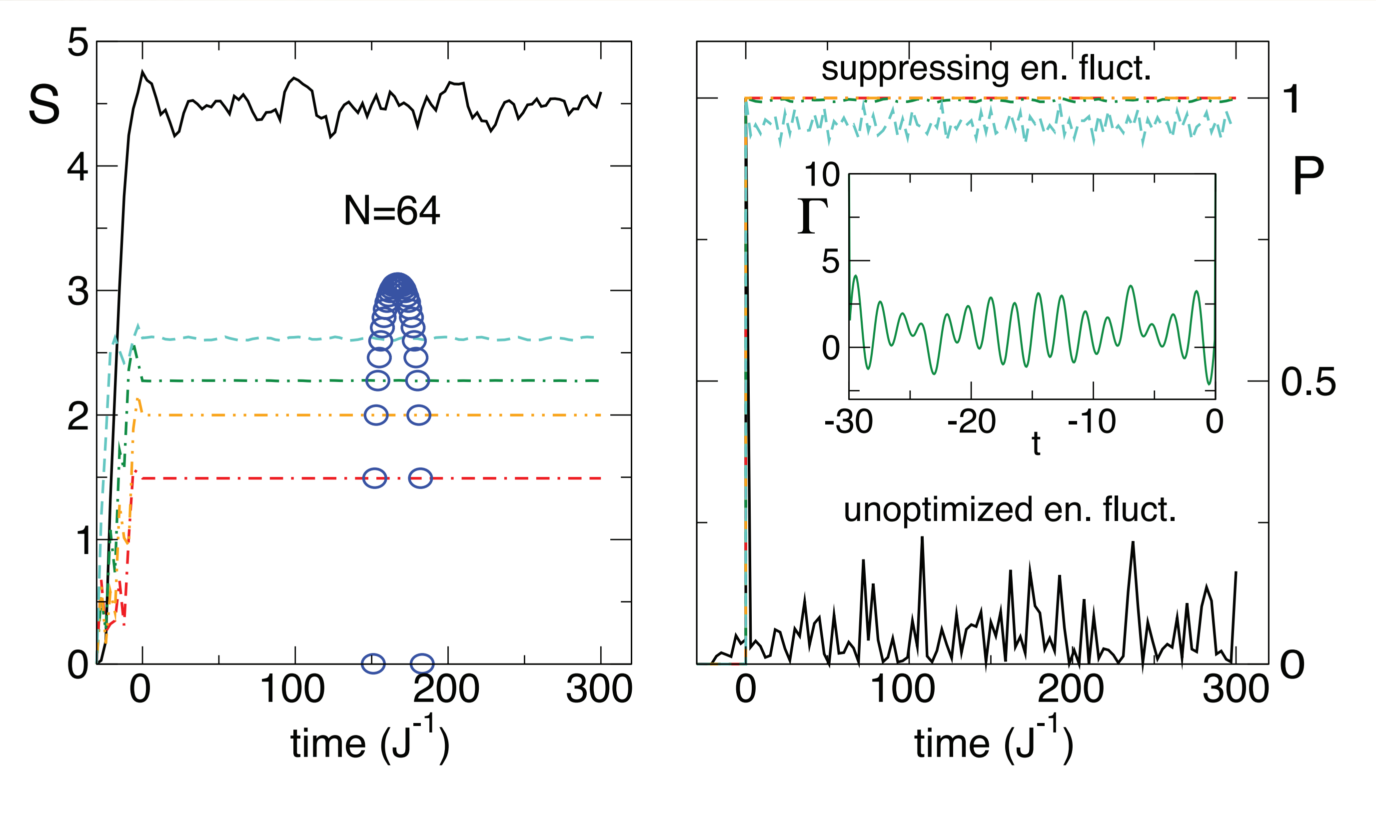
Entanglement Storage Units



inset:
T VS noise intensity

T. Caneva, T. Calarco, SM, New J. Phys. 14 093041 (2012)

Entanglement Storage Units

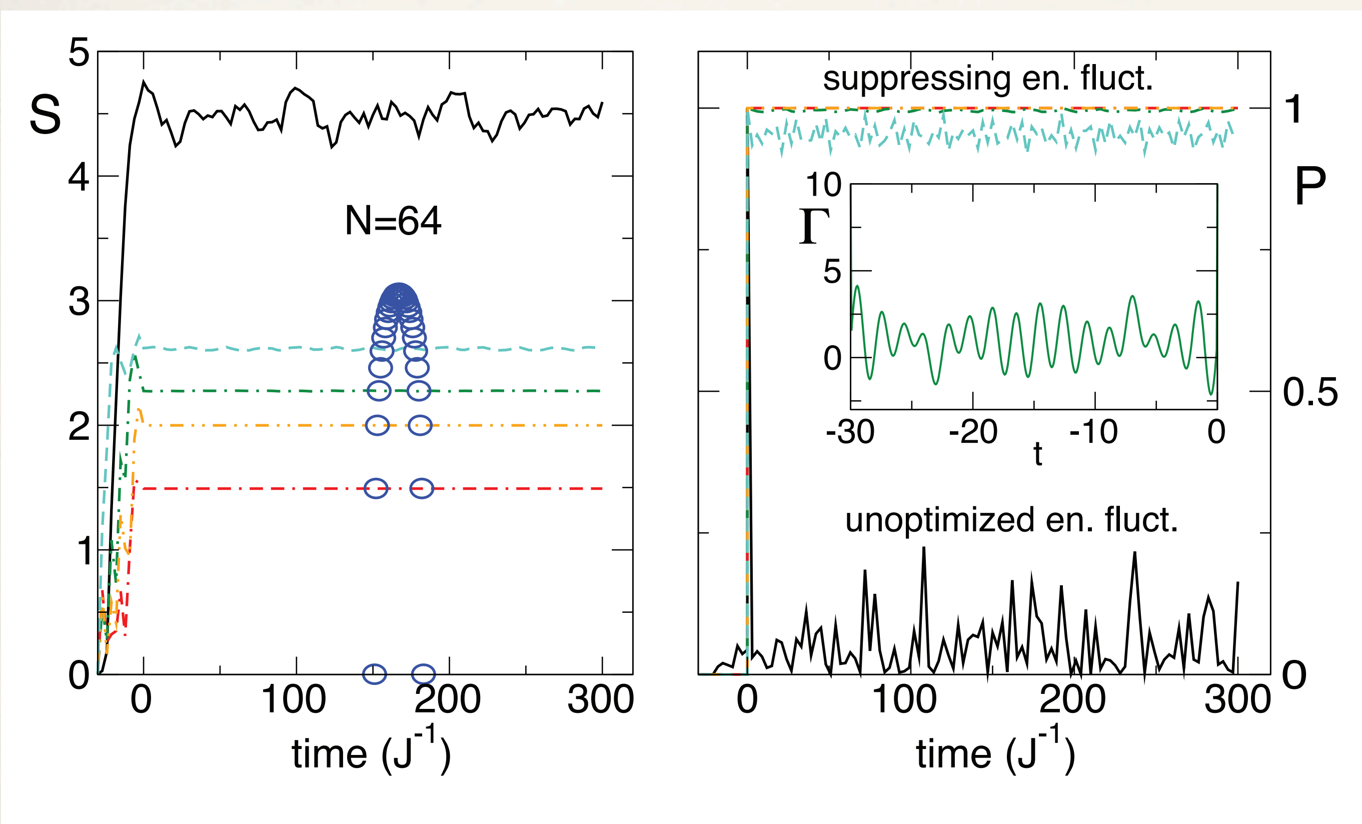


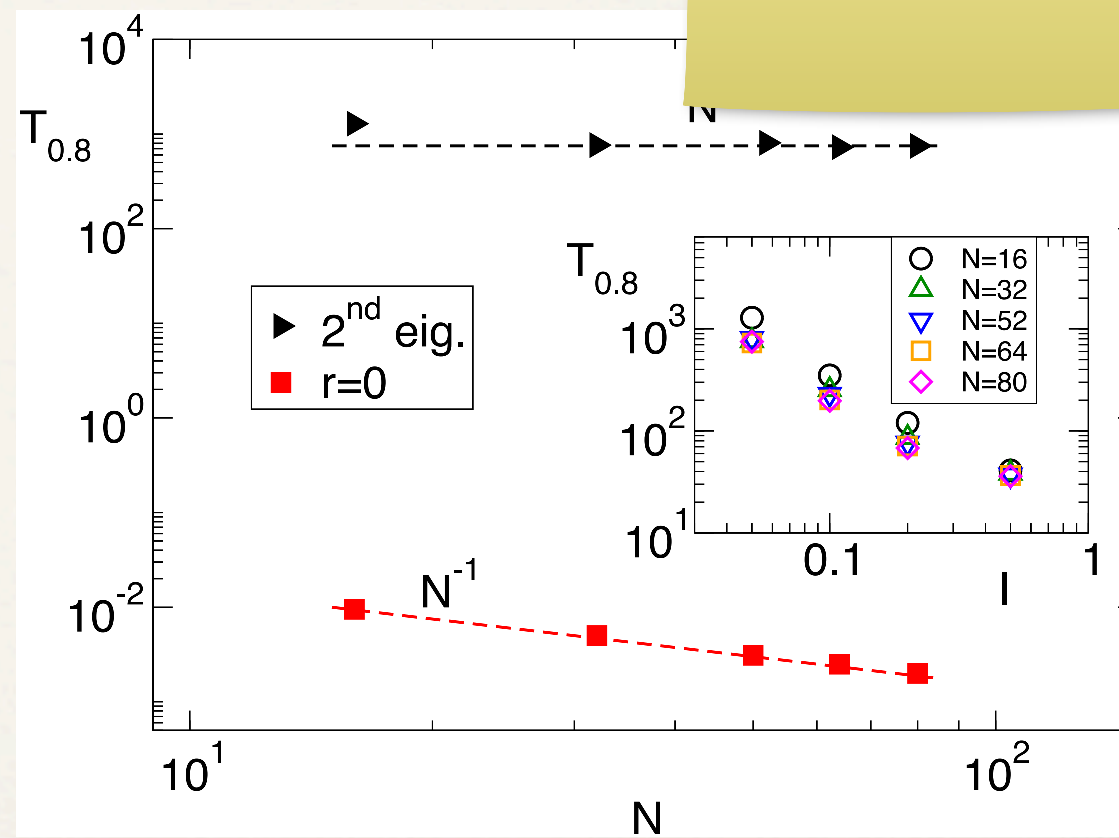
inset:
T VS noise intensity

T. Caneva, T. Calarco, SM, New J. Phys. 14 093041 (2012)

Entanglement Storage Units

inset:
T VS noise intensity

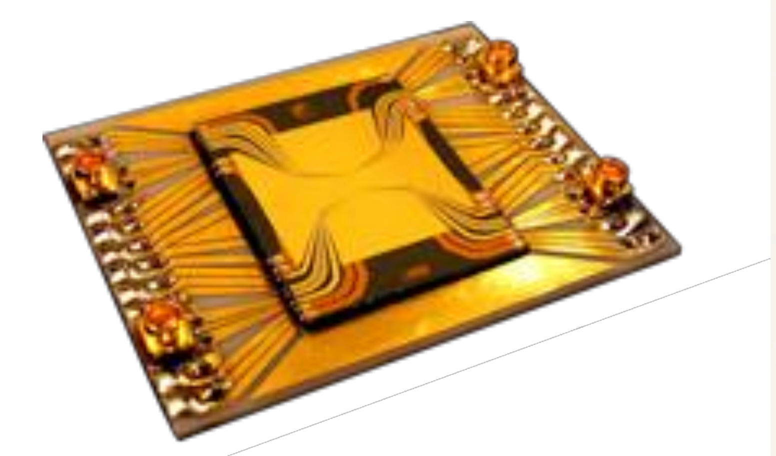




T. Caneva, T. Calarco, SM, New J. Phys. 14 093041 (2012)

Atom chip experiments at QSL

see also R. Büker et. al. Nat. Phys. 2011



Picture: P. Treutlein (Basel, CH)

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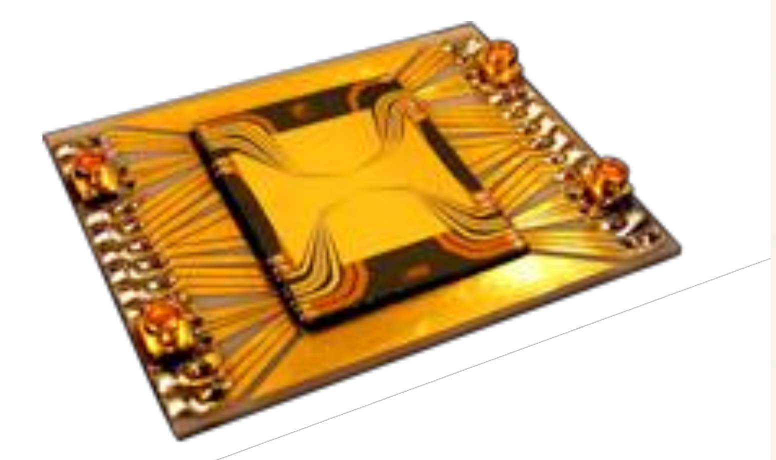

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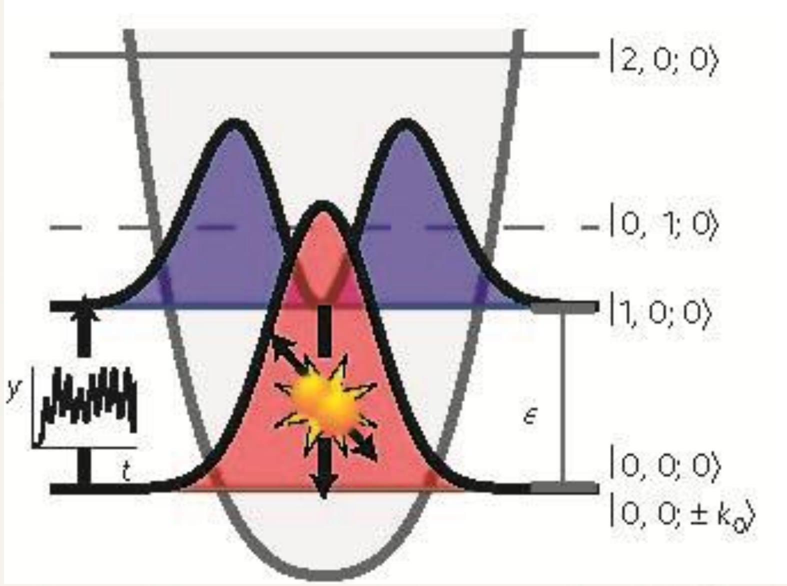

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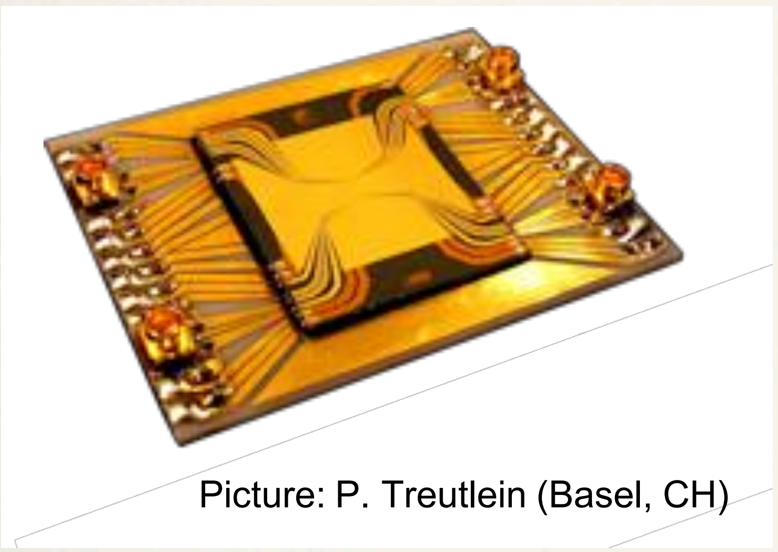
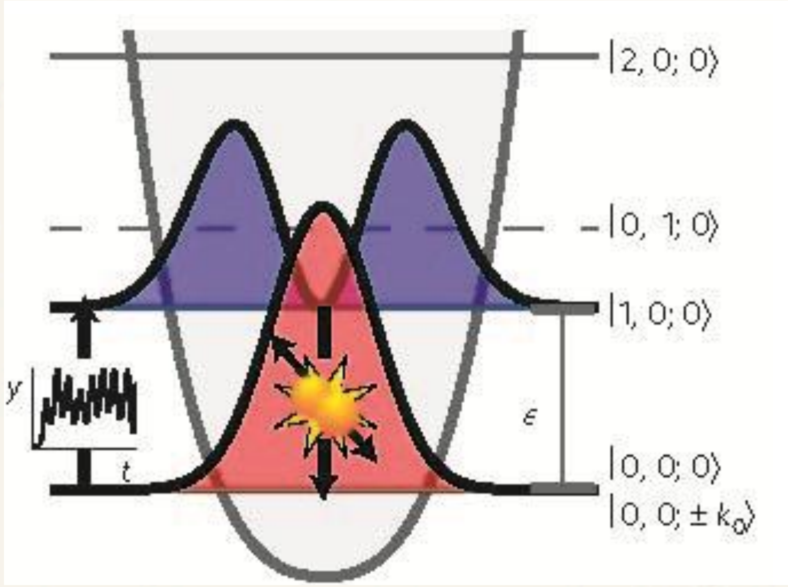
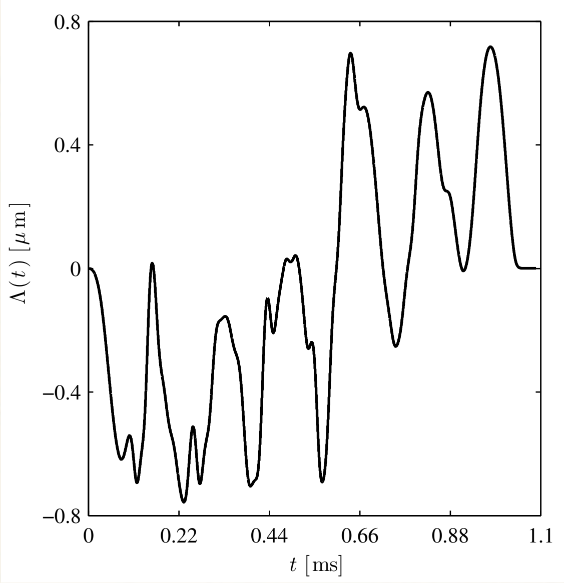


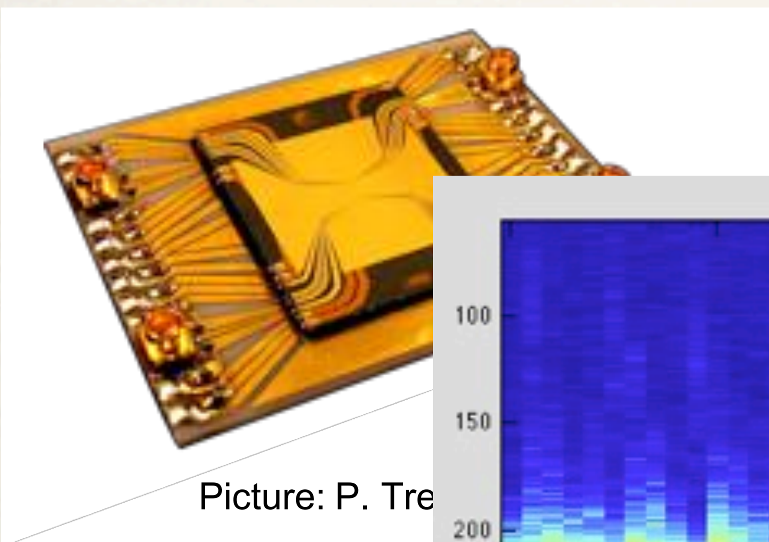


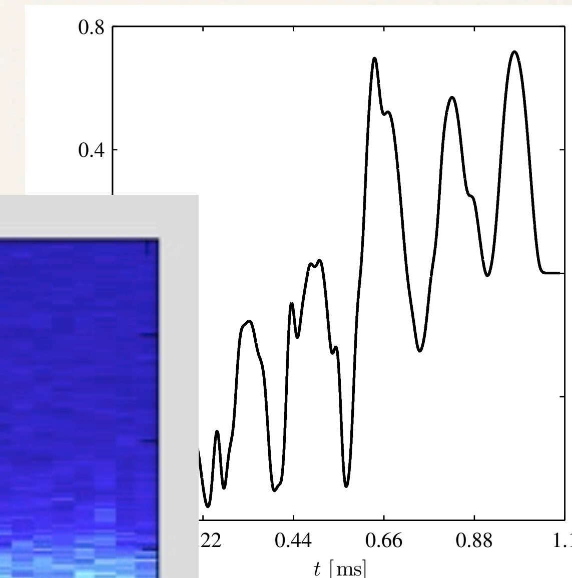


97%

Atom chip experiments at QSL

see also R. Büker et. al. Nat. Phys. 2011





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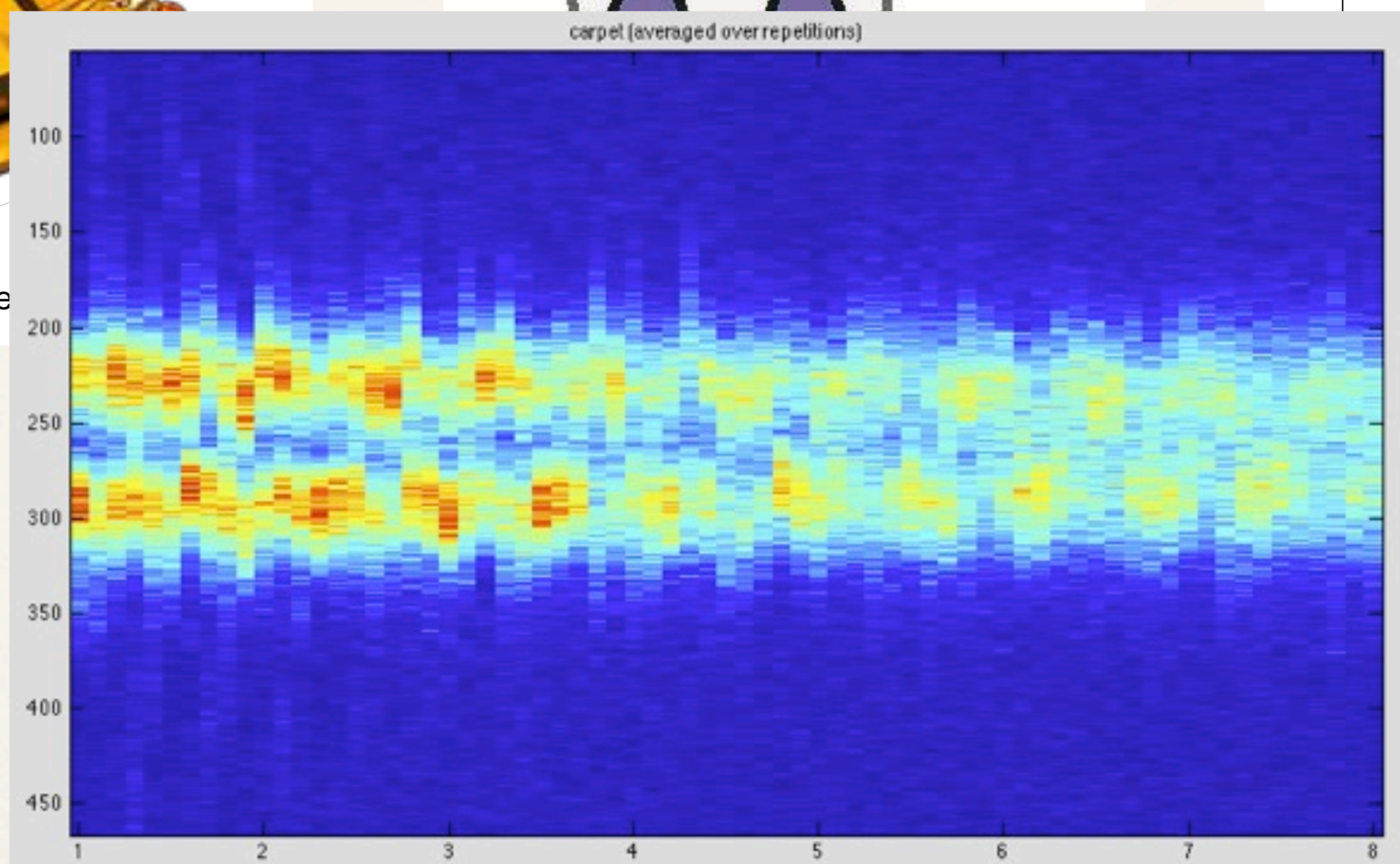

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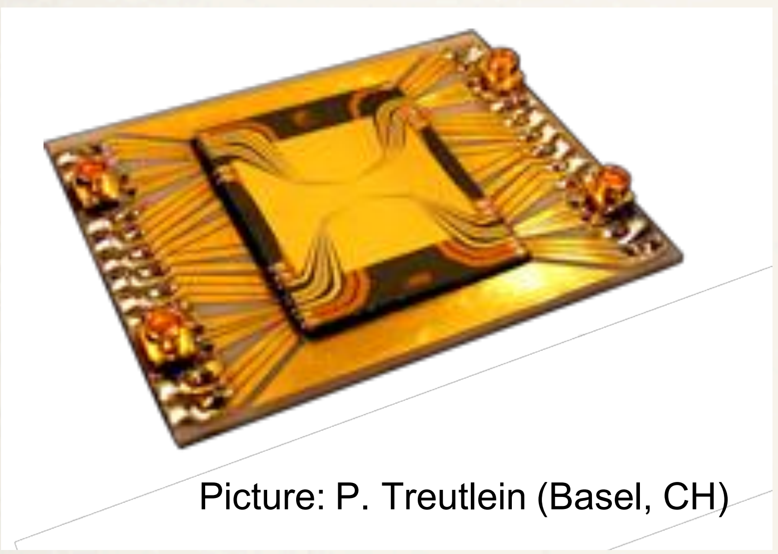

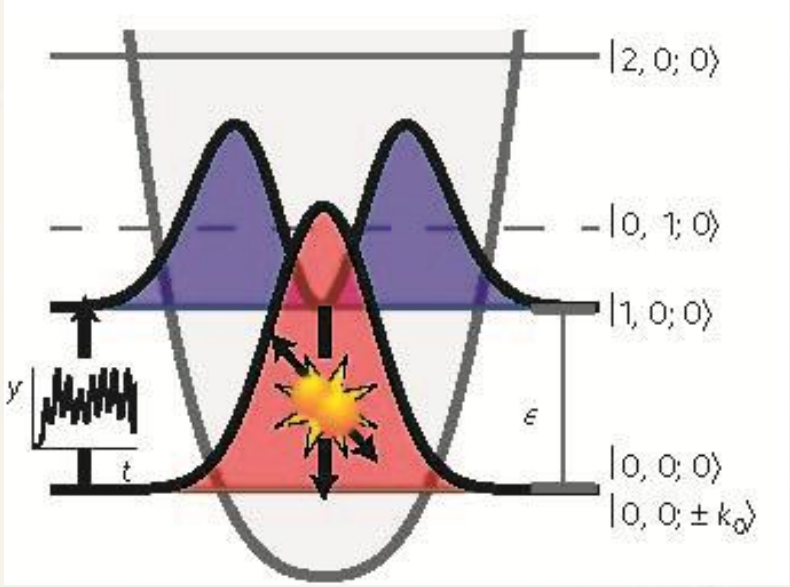

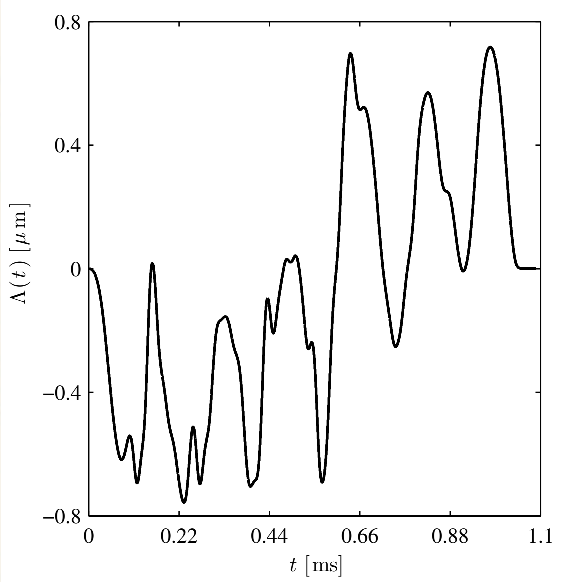

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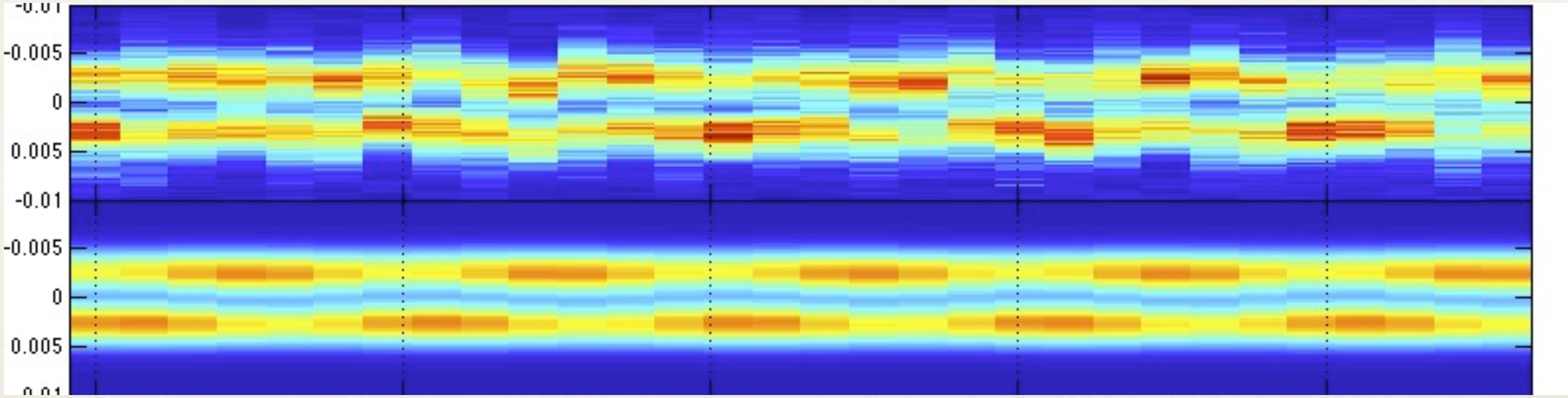

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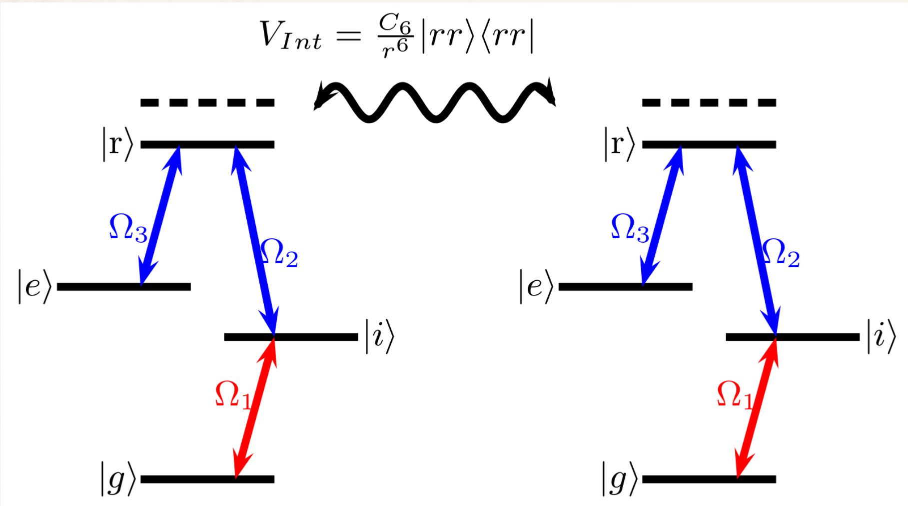
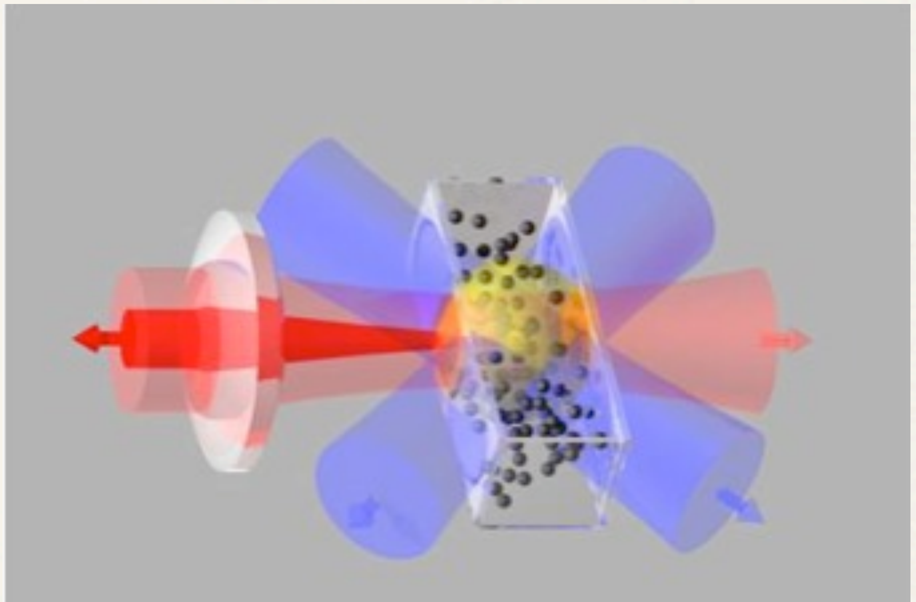
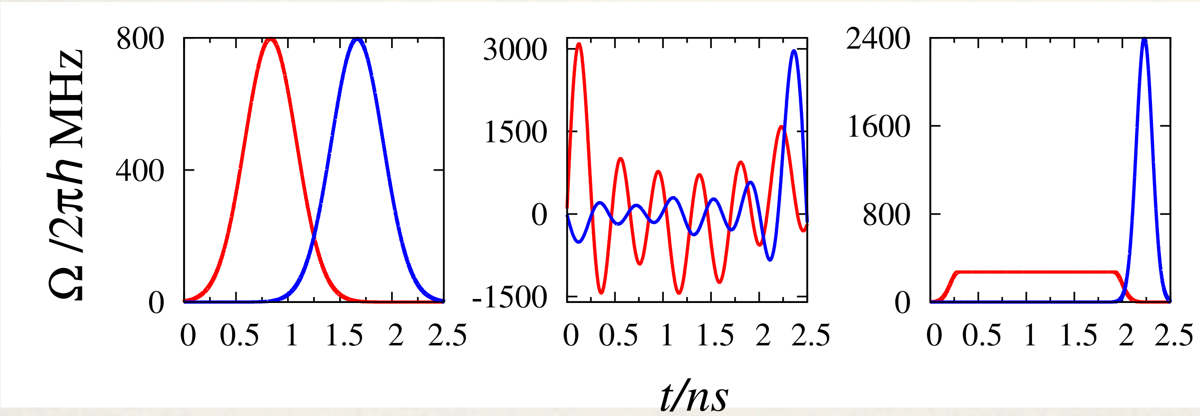
Single Photon Source at 30

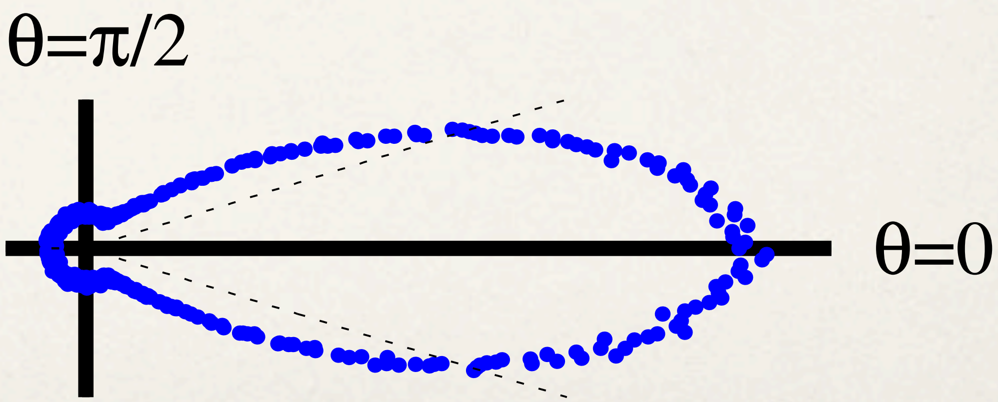
20 min

Rydberg atoms




Picture: 5th institute of Physics, Stuttgart





Single Photon Source at 30

20 min

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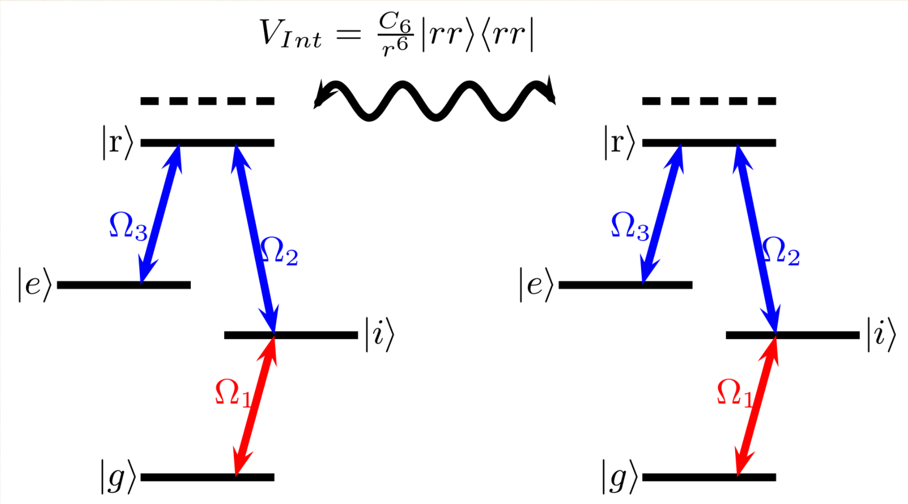

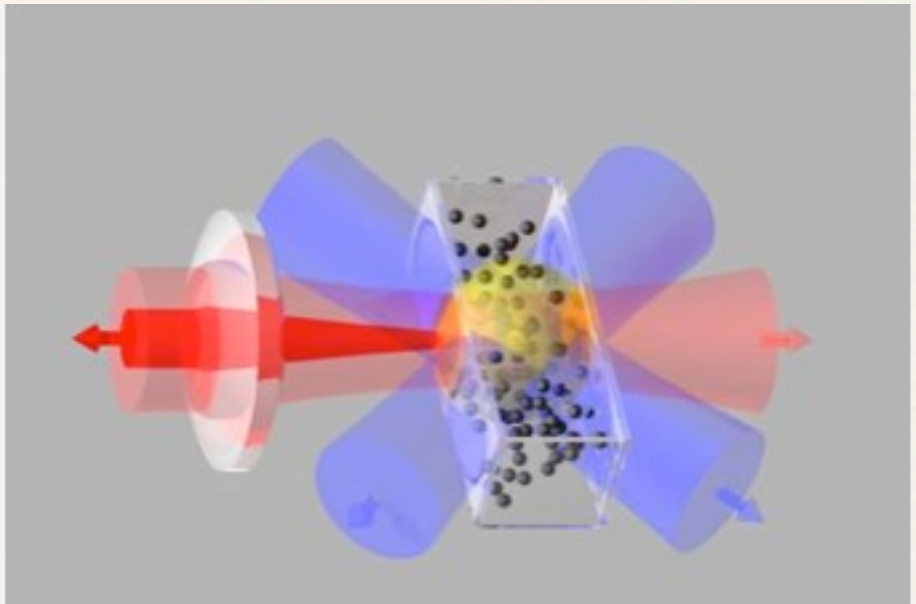

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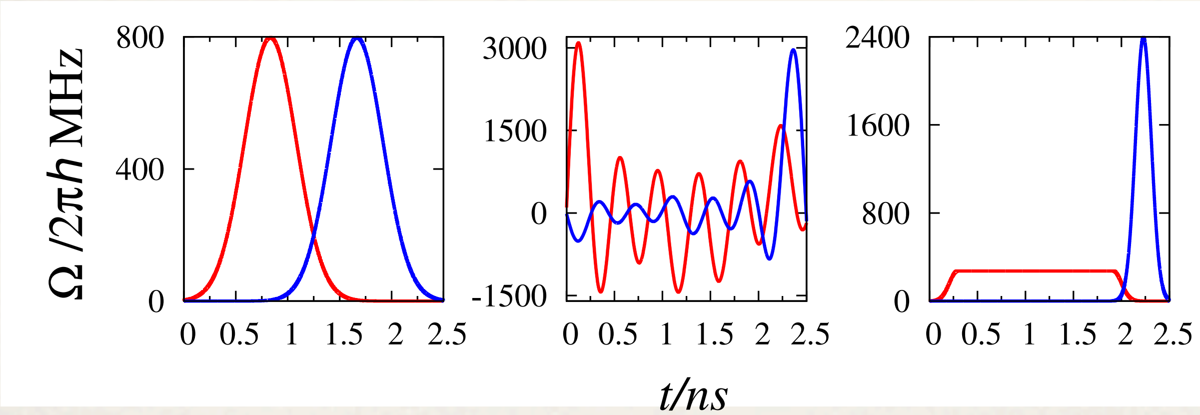

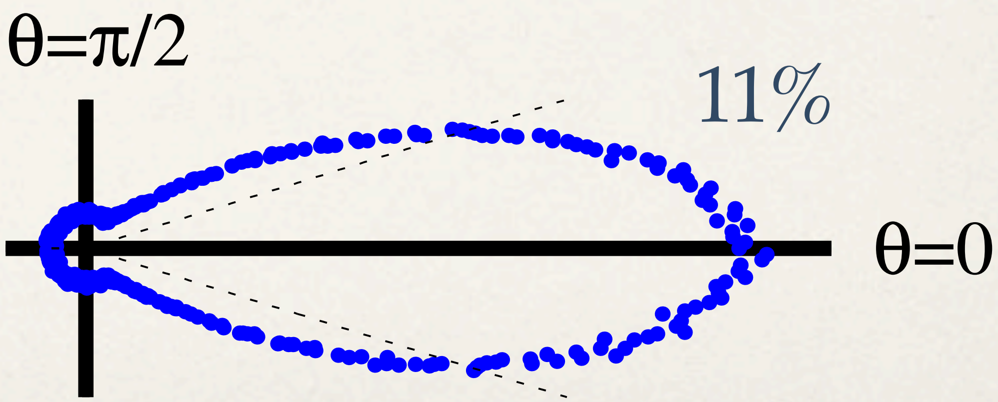

M. Mueller, et.al. arxive: 1212.2811

Dudin & Kuzmich, Science 2012

Optimal control limits

Optimal control limits

- What are the physical limits of control of MBQS?

Optimal control limits

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Controllability, Reachability, Quantum Speed Limit, ...

Control complexity

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Control complexity

- What are the physical limits of control of MBQS?

  Controllability, Reachability, Quantum Speed Limit, ...

- Are there any algorithmic/informational limits?

- How to characterize the complexity of the optimization task?

T. Caneva, A. Silva, R. Fazio, T. Calarco, S. Montangero, Arxive: 1301.6015

Assumptions

- Closed systems
- Many-body
- State to state transformation
- $$H(t) = \underbrace{H_0}_{\text{Drift}} + \underbrace{\sum \lambda_j(t) H_j}_{\text{Controls}}$$

Drift   Controls

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Drift Controls

$$|\psi_0\rangle$$

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$$|\psi_0\rangle \longrightarrow |\psi_G\rangle$$

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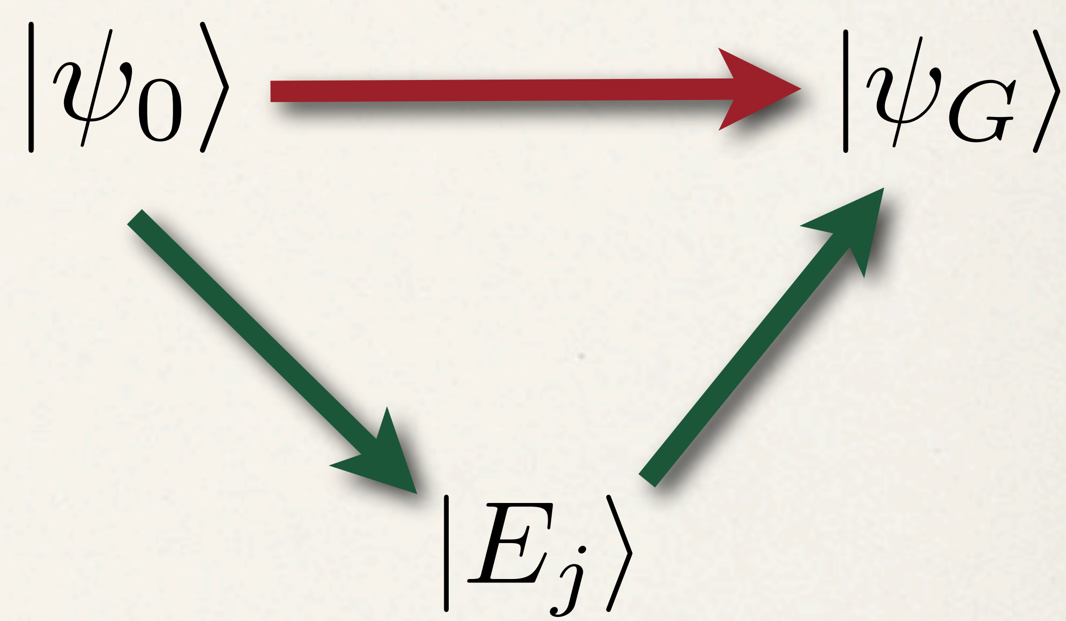

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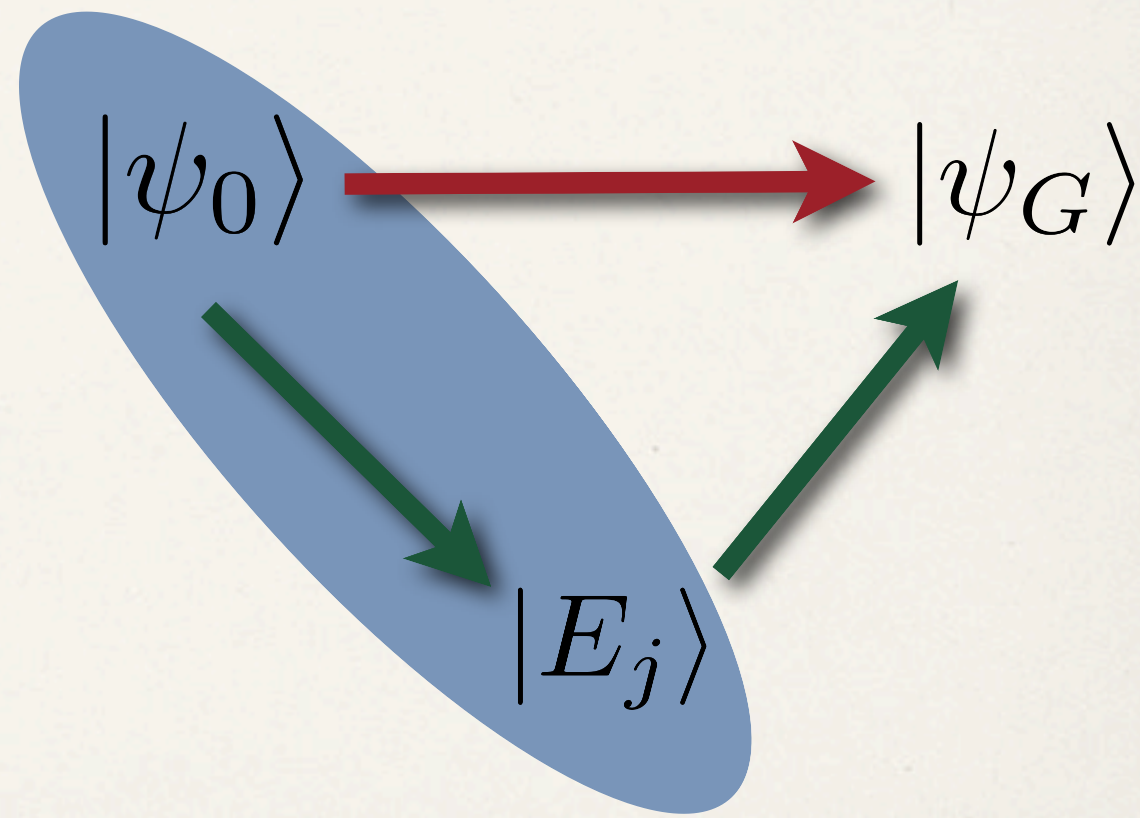
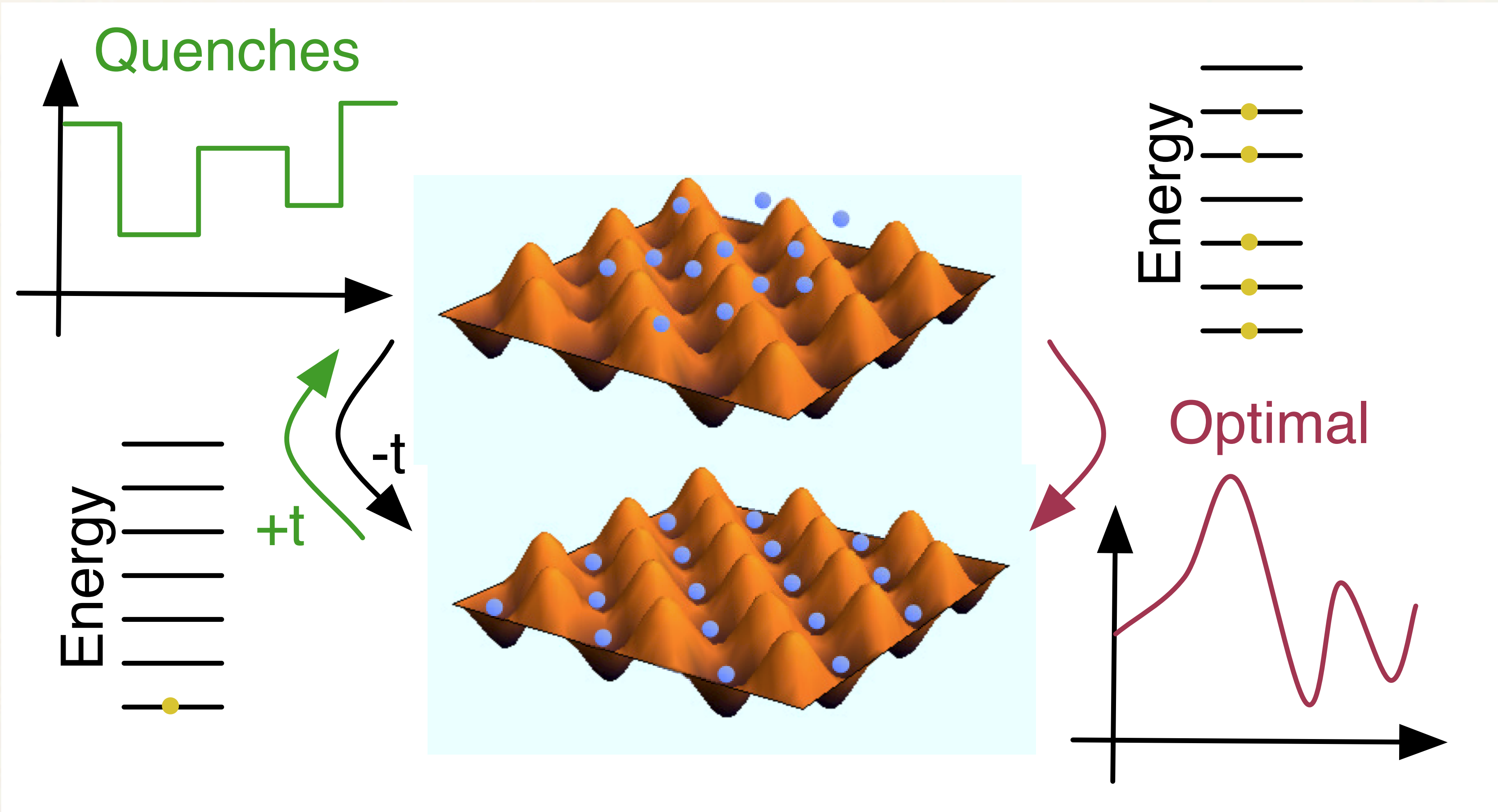


Reversibility



Is it possible? Is it difficult?

Diagonal Entropy

we are interested in quantifying the state complexity

$$S_d = \sum \rho_{nn} \log \rho_{nn}$$

$$\rho = \sum \rho_{nm} |E_n(t)\rangle\langle E_m(t)|$$

$$H(t) = \sum E_n(t) |E_n(t)\rangle\langle E_n(t)|$$

✓ Introduces a preferre sis

✓ At equilibrium for d equal to VN entropy (positive, additive, 0 for T=0)

✓ Constant for stationary (diagonal) states

✓ Constant for adiabatic processes

✓ Only increases from stationary states in closed systems $S_d(T) \geq S_d(0)$

✓ Obeys fundamental Thermodynamical equation:

$$\Delta E = T\Delta S + \sum_j \left.\frac{\partial E}{\partial \lambda_j}\right|_S \Delta\lambda_j$$

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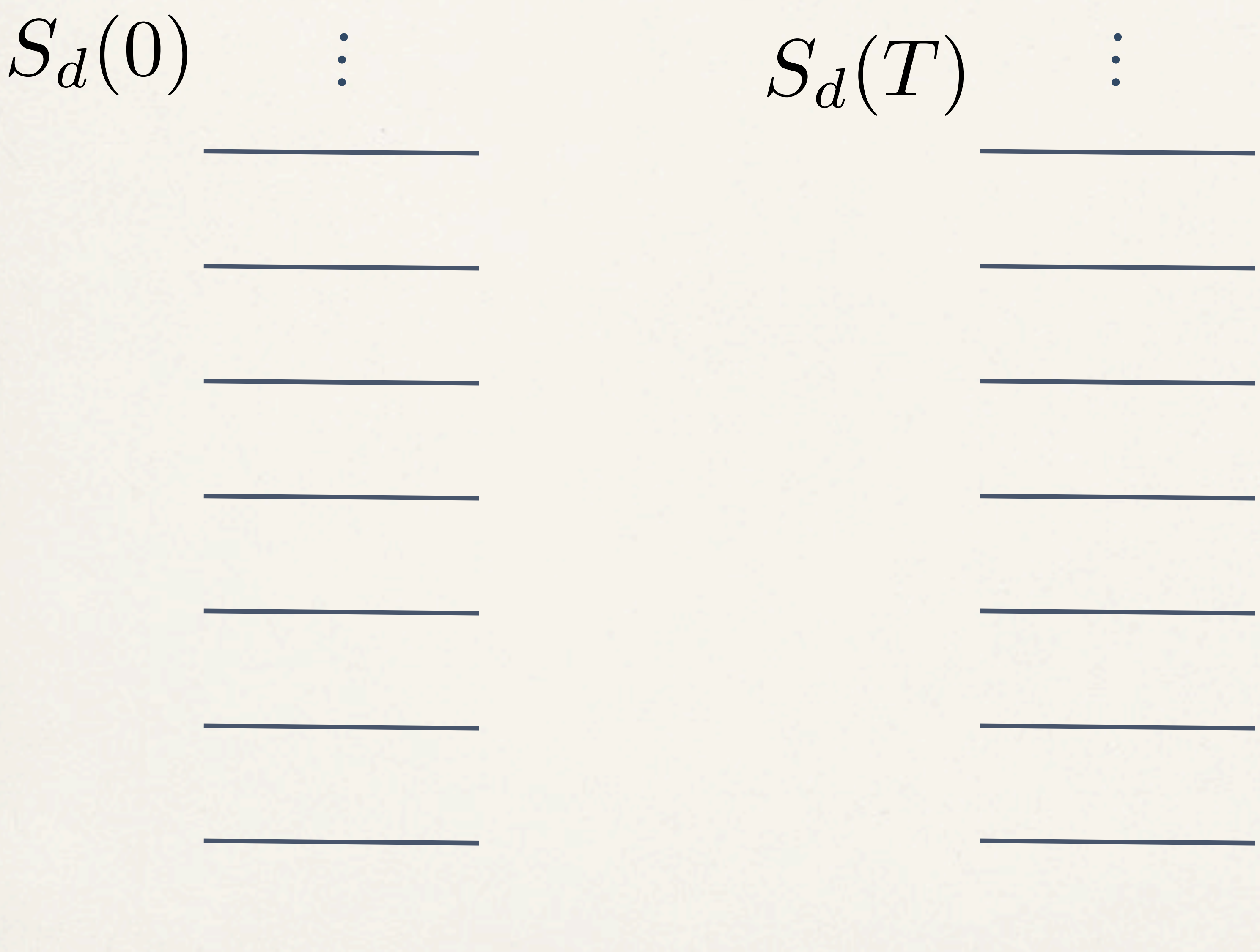
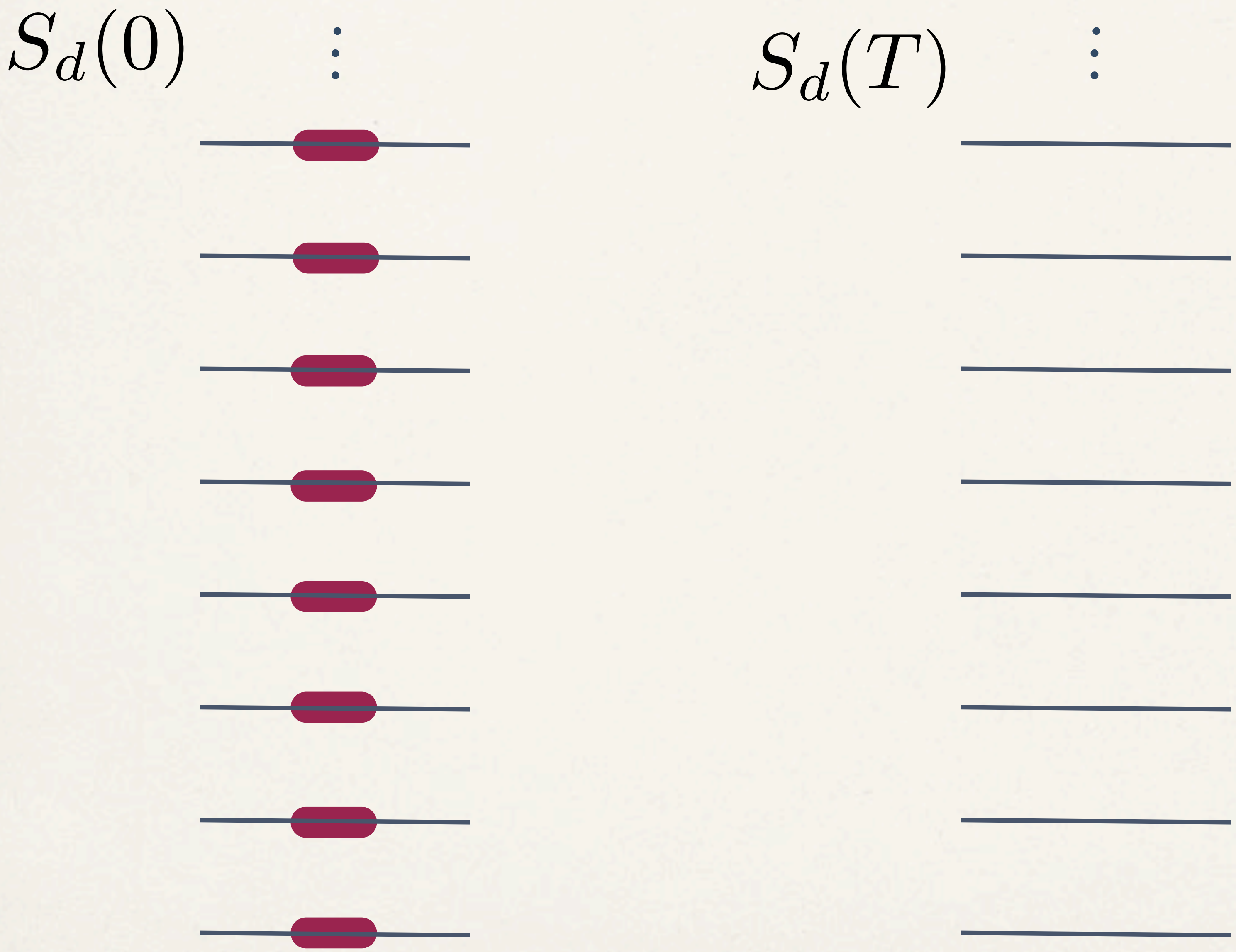
A. Polkovnikov, Annals of Physics (2011).

Diagonal entropy reduction



Control field?

Diagonal entropy reduction



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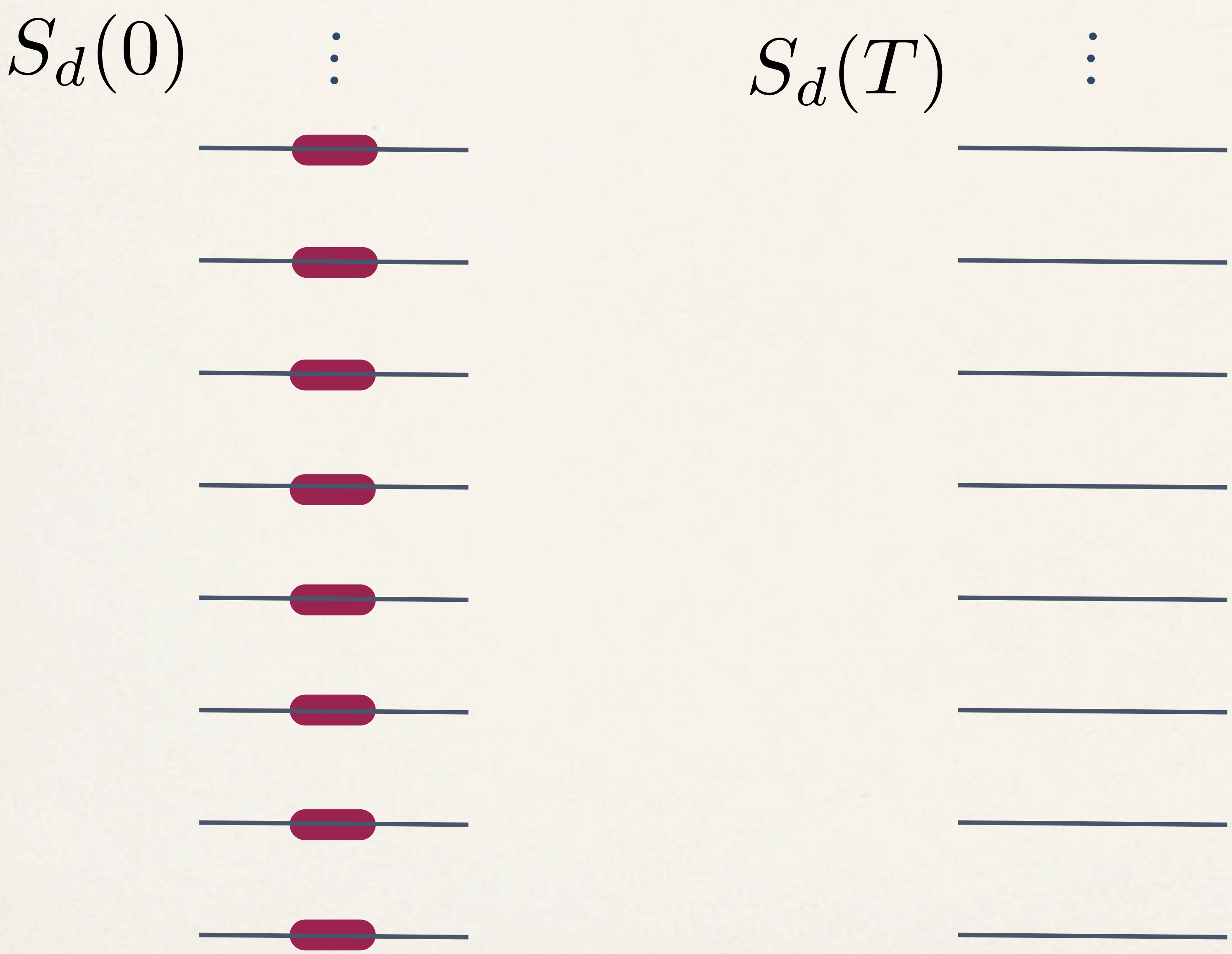
Diagonal entropy reduction



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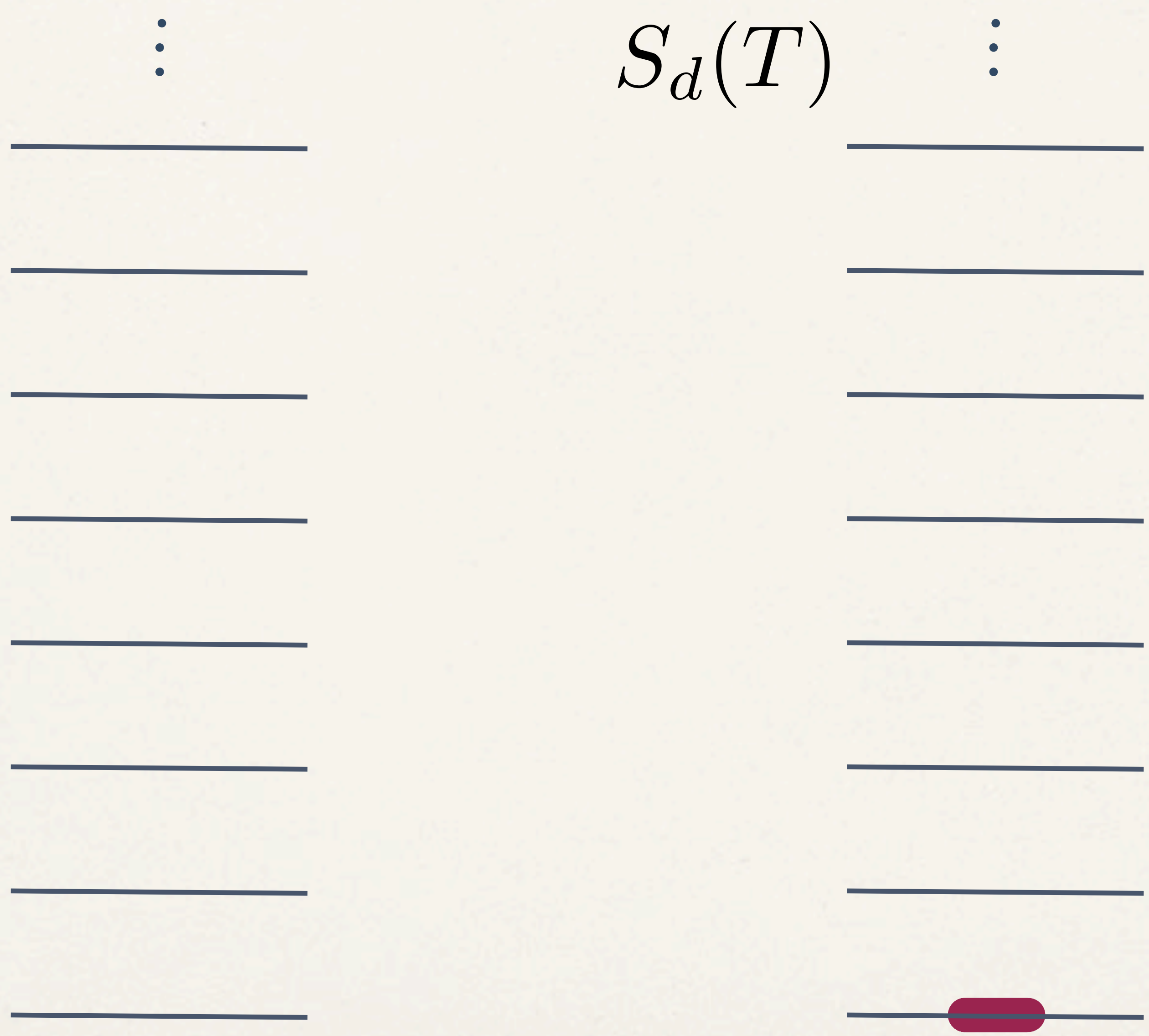
Diagonal entropy reduction

$S_d(0)$ $S_d(T)$



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Reversed quantum dynamics

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Multiple random (time & strength) quenches

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Initial ground state

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CRAB optimization

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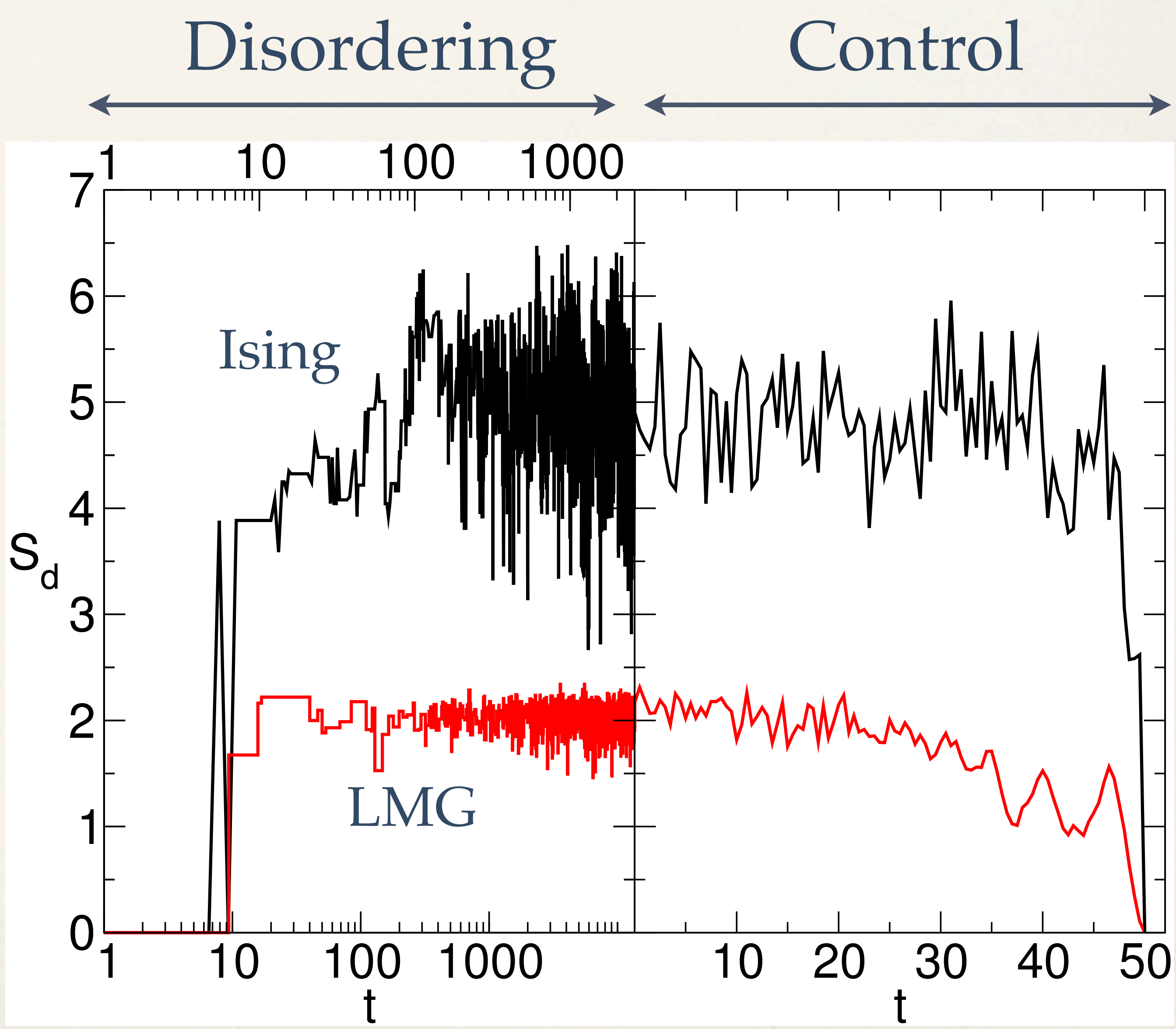

Robustness

Is all that robust against noise?

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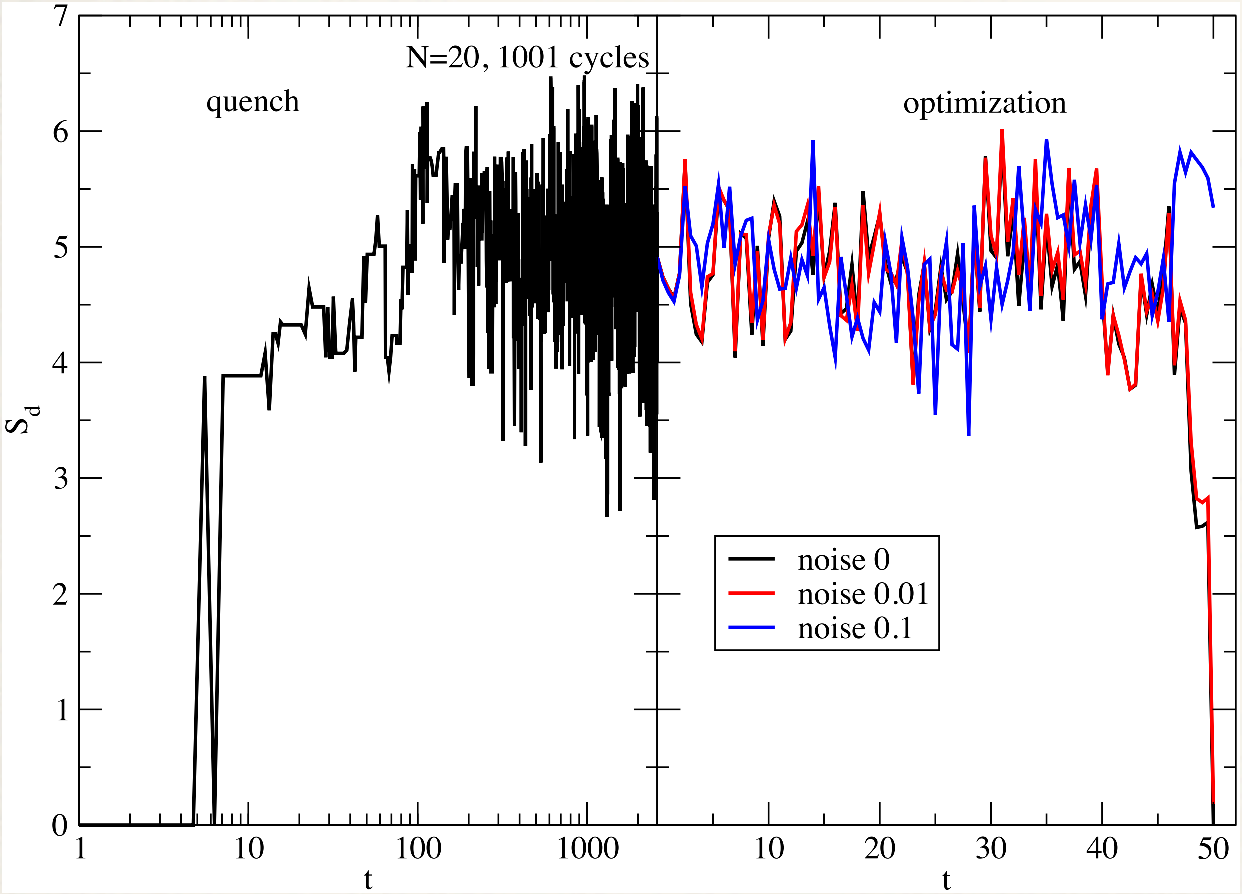

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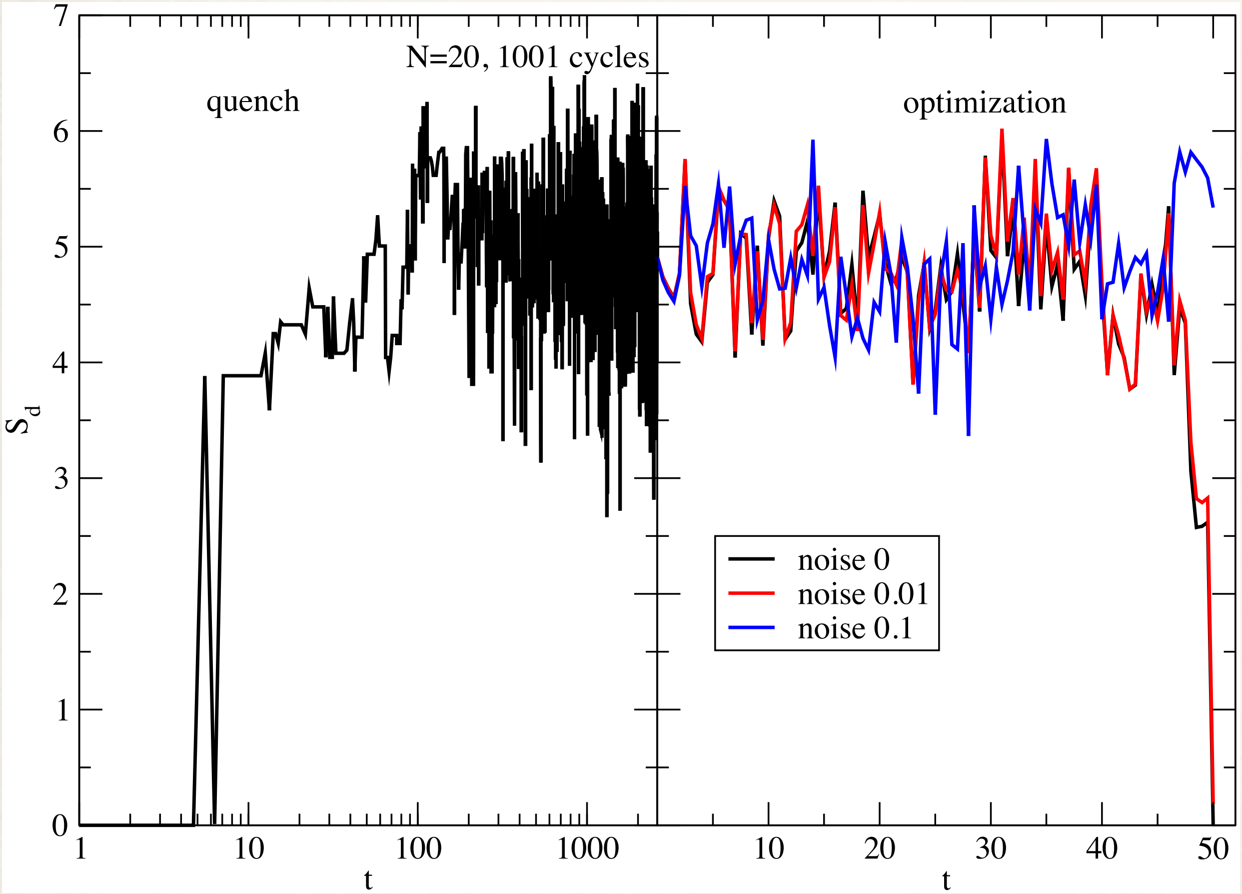
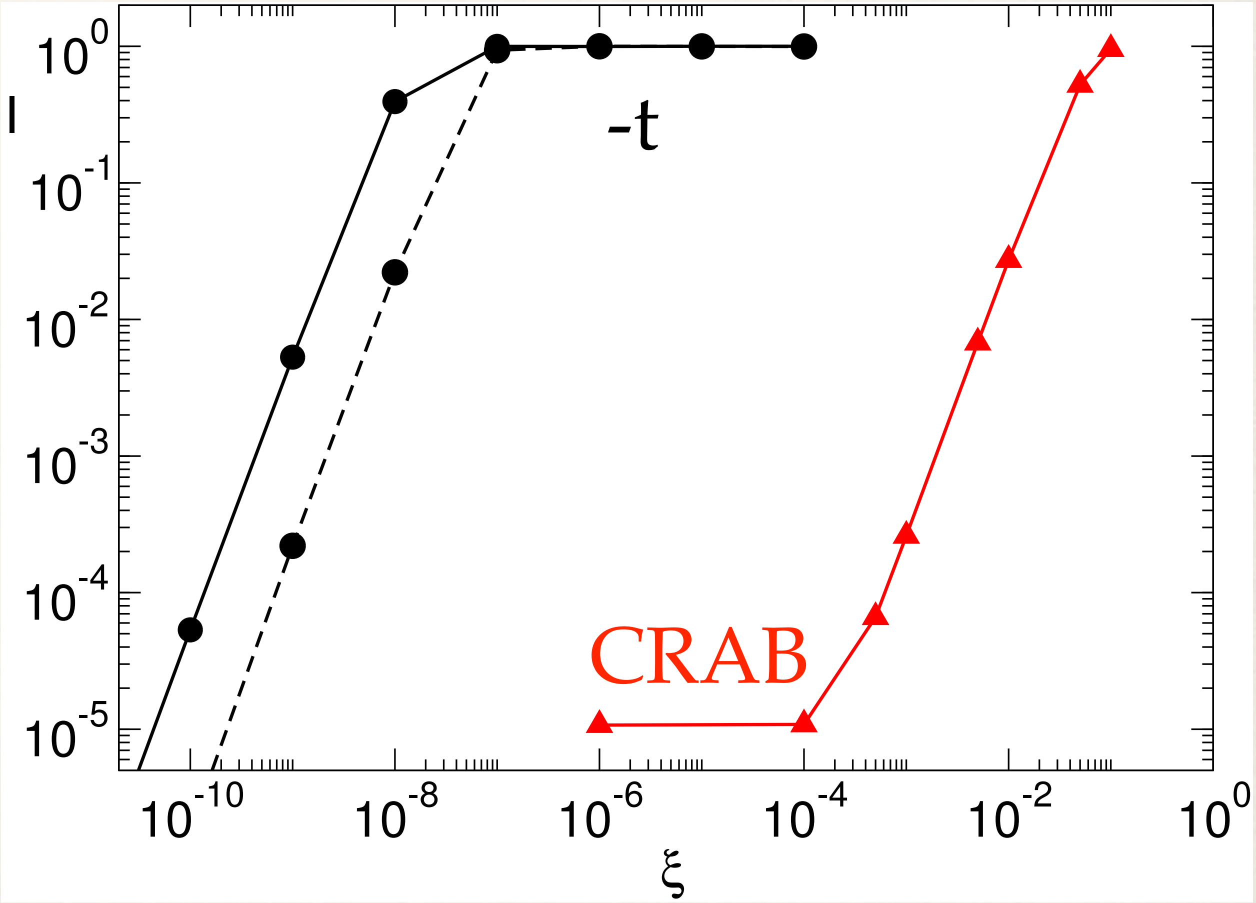
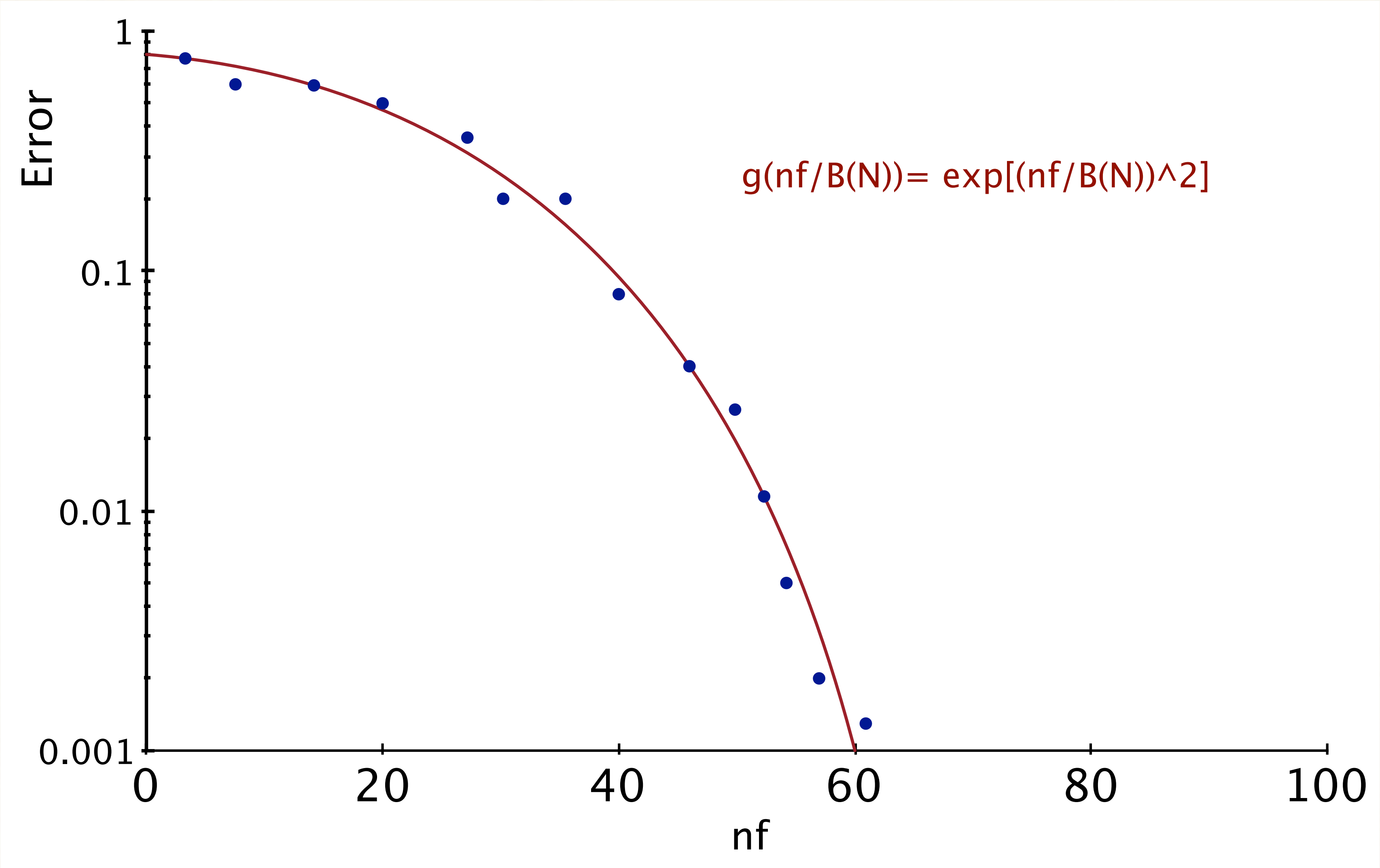




Complexity



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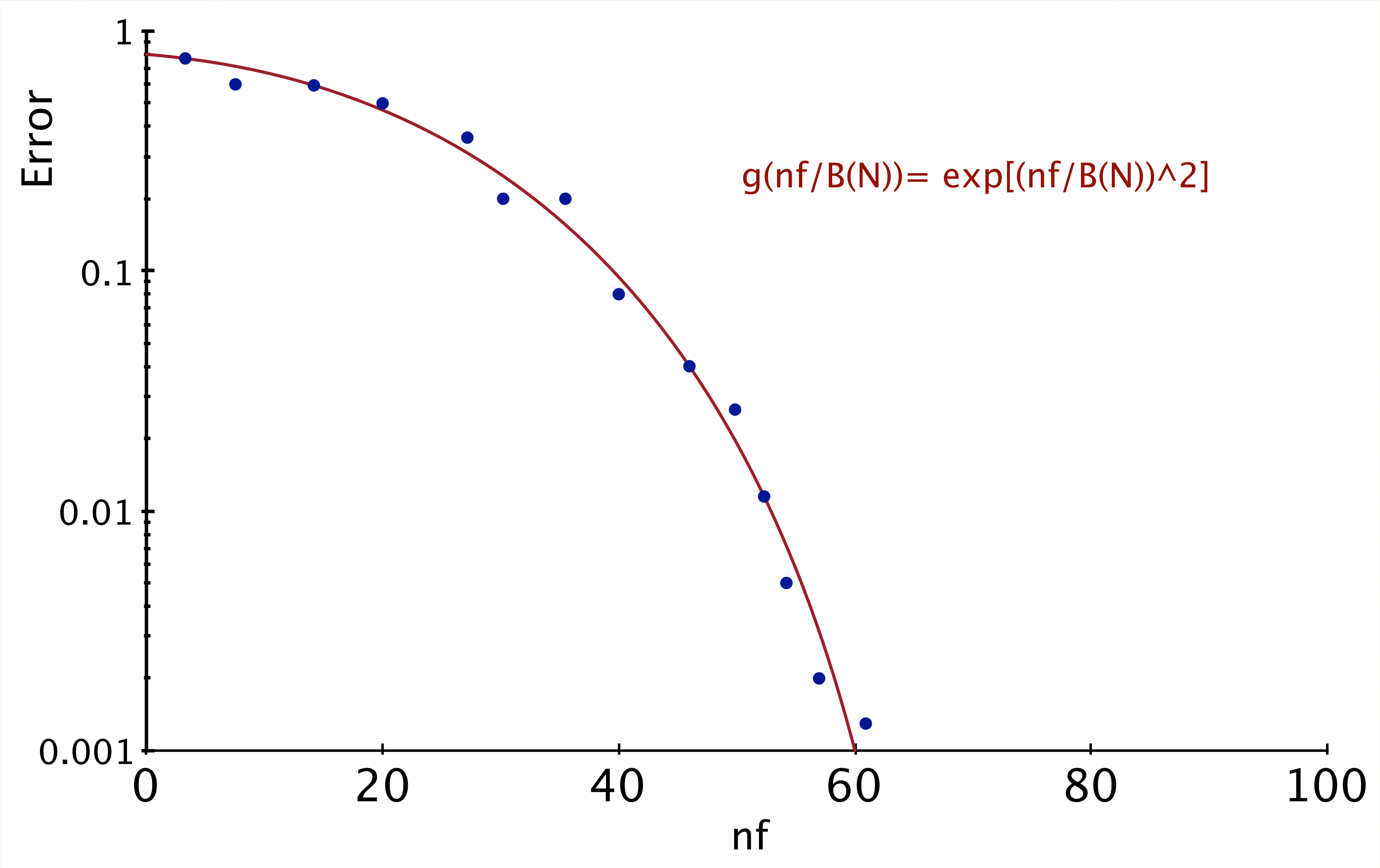
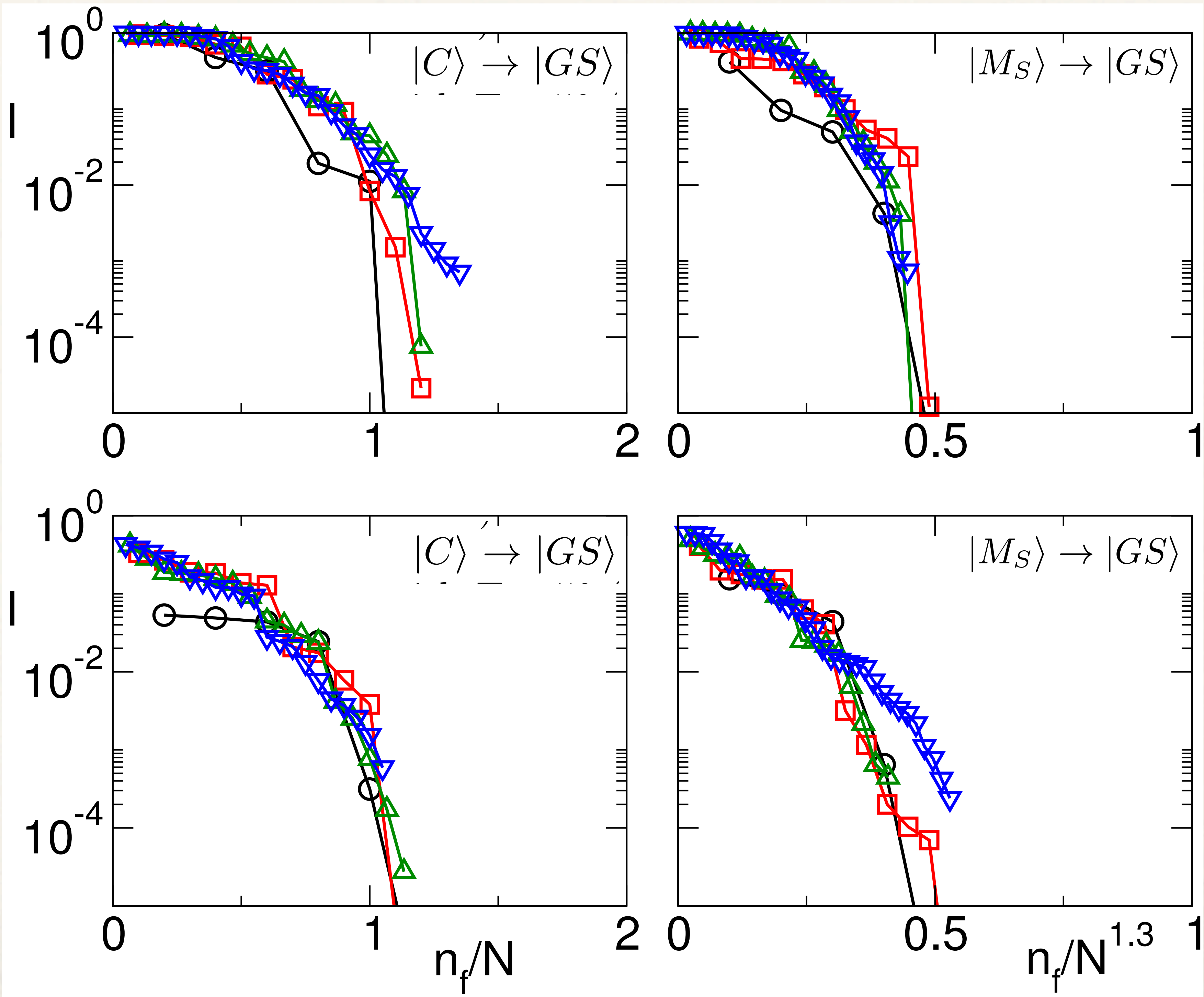


Scaling of the number of parameters with the system size B(N)

Reversibility and Information

Ising

LMG



N=10,...,40

Conjecture

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$$H = -\sum_{i,j} J_{ij}\sigma_i^x\sigma_j^x - \Gamma(t)\sum_i^N \sigma_i^z - J_x\sum_i^N \sigma_i^x$$

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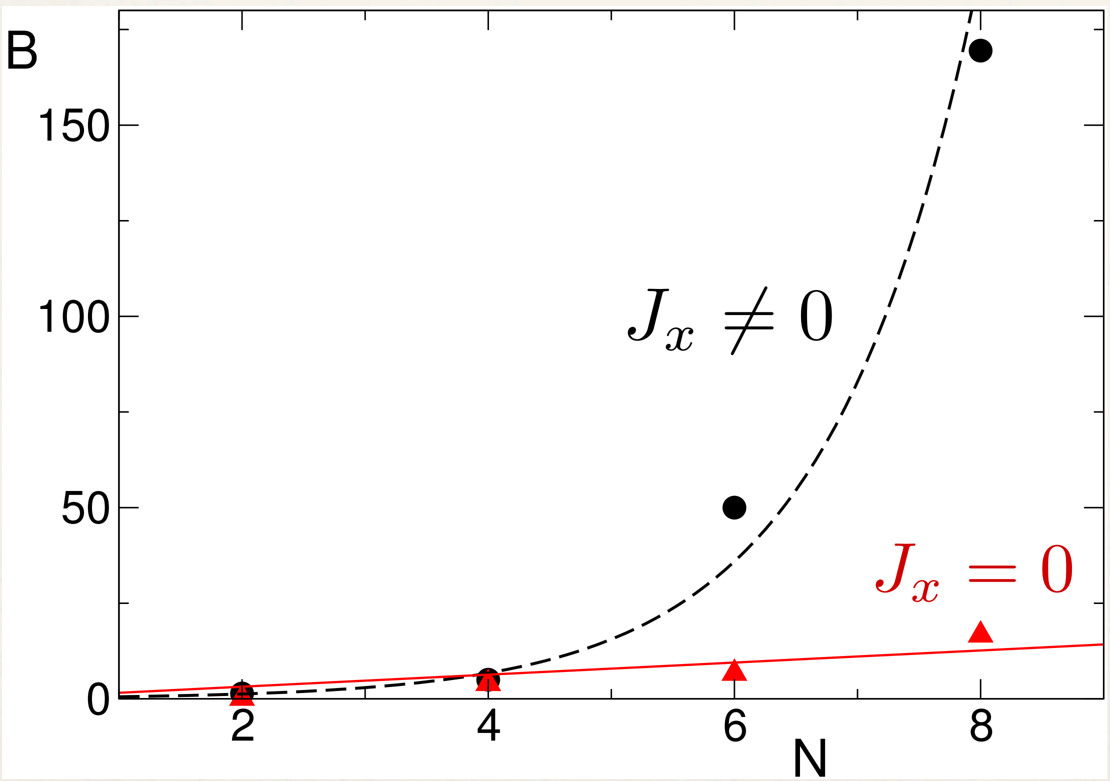

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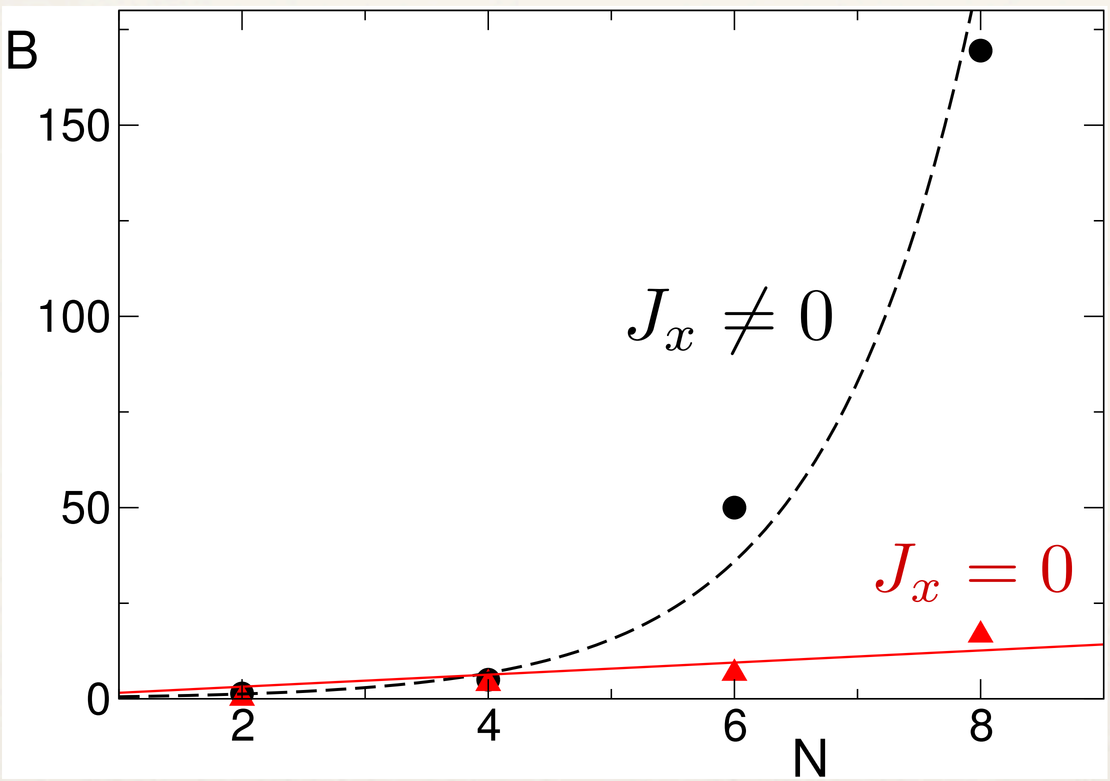
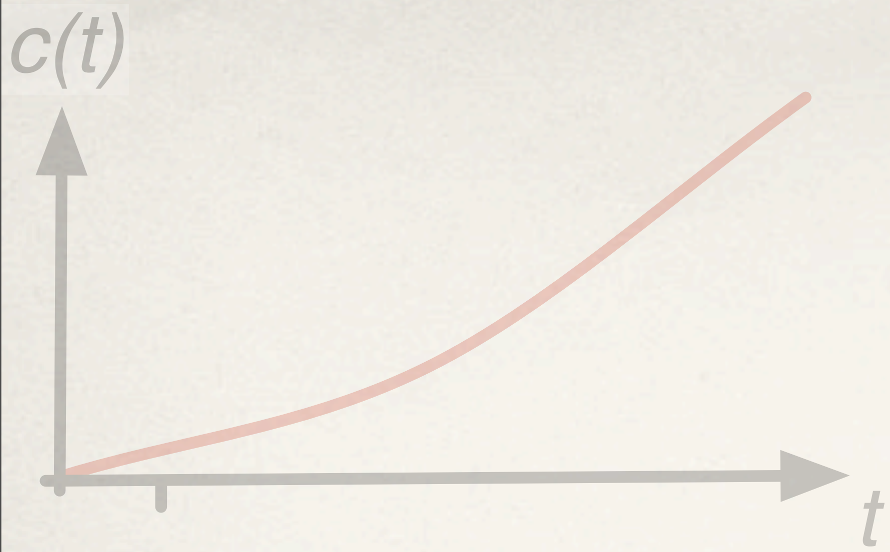
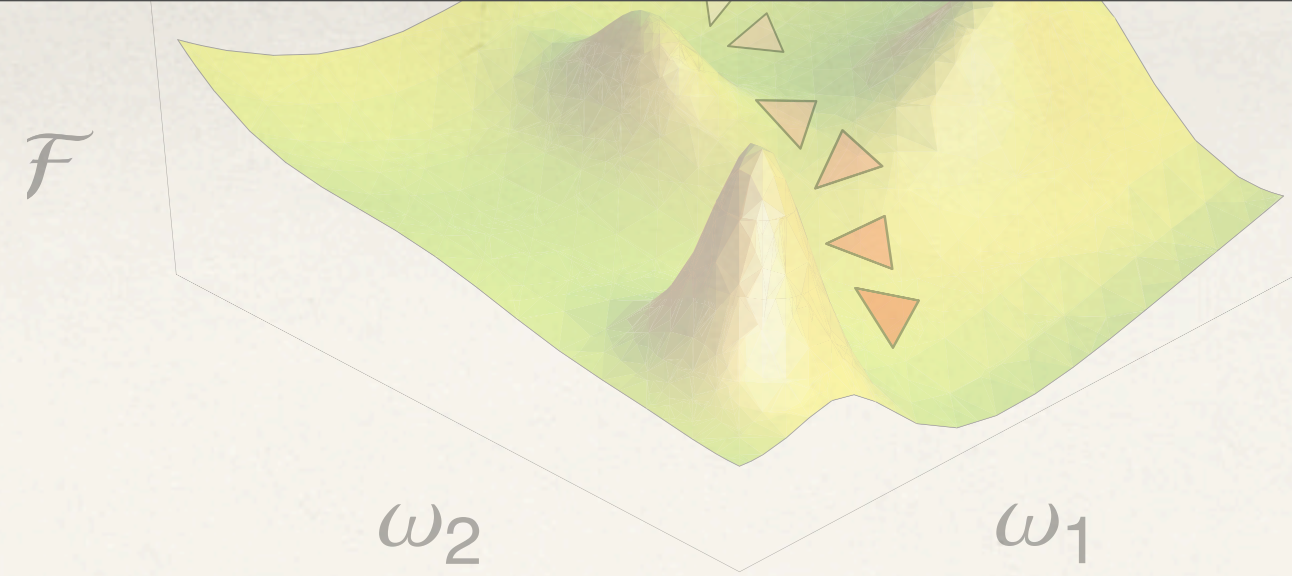


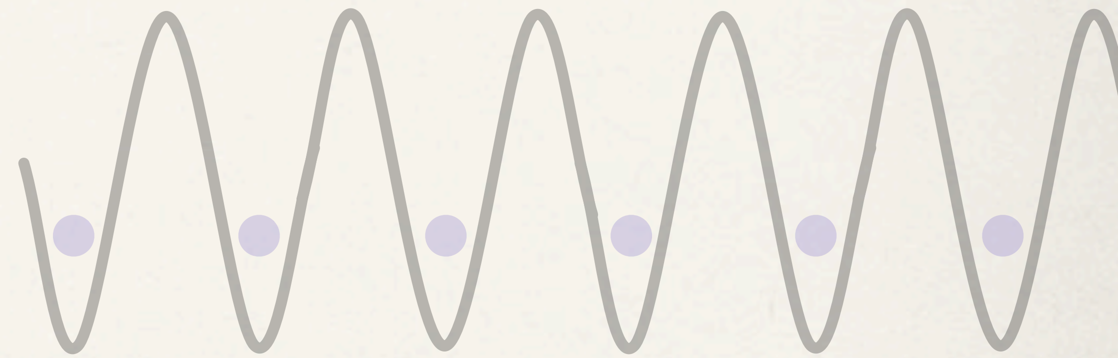
$$B(N) \propto D_m(N)$$

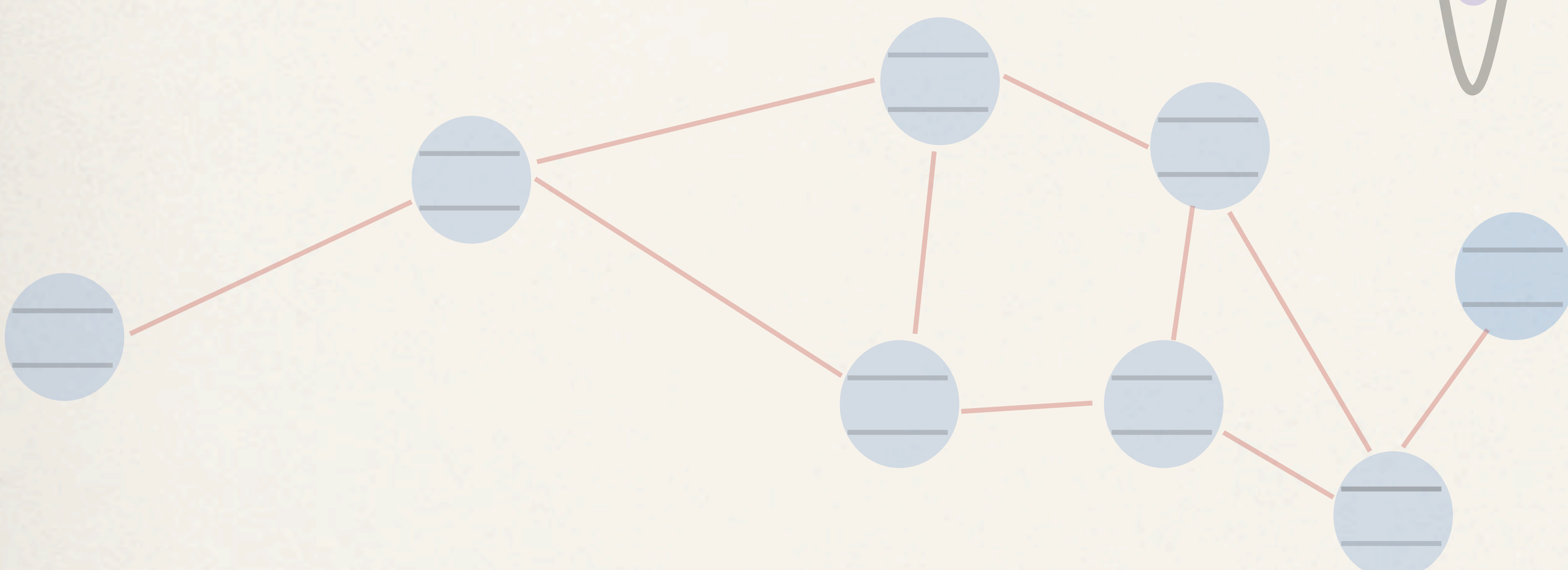
Conclusions

- CRAB optimization can be applied successfully to MBQS dynamics opening new perspectives.
- Using optimal control it is possible investigate qualitatively new phenomena
- Optimal trajectories are robust with respect to noise and perturbations.
- Complexity of control task can be characterized by the degrees of freedom of the optimal driving field.
- Non-integrable MBQS are exponentially complex to be optimized


c(t)
t


$\mathcal{F}$
ω_2
ω_1





Thank you for your attention!

In collaboration with:
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Thorsten Schumm
Sandrine van Frank
Wolfgang Rohringer

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SFB / TRR21 Co.Co.Mat.
DFG Deutsche Forschungsgemeinschaft

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