Dynamic melting and condensation of topological dislocation modes

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Objectives

1. Does the signature of topological dislocation modes survive in translationally inert insulators, reached from a translationally active topological insulator (TATI) via a real time ramp?
2. Can topological dislocation modes be dynamically generated via a ramp, taking the system into a TATI phase from translationally inert insulators?

Introduction

- **Bulk dislocation lattice defects**: To identify translationally active topological insulators (TATIs), featuring band inversion at a finite momentum ($K_{inv}$).
- Characterization and $K \cdot b$ rule: Burgers vector ($b$), $\Phi_{dis} = K_{inv} \cdot b$
- $\Phi_{dis} = 0$ in the $\Gamma$ phase, as $K_{inv} = 0$.
- **M-phase**: $b$ and $K_{inv}$ are such that $\Phi_{dis} = \pi$ (modulo $2\pi$).

M-phase dislocation mode and ramping scheme

$$H = \hbar \sum_{\alpha, \tau} \sin(k_\alpha c_\alpha) c_\alpha + \hbar \sum_{\alpha, \tau} \cos(k_\alpha a_\alpha) - m_z c_\tau.$$  

$\tau$: orbital degrees of freedom

Results: Limit (II) $\rightarrow$ Ramping into TATI

- In quantum materials, midgap dislocation modes can only be occupied upon filling all the negative energy bulk states.
- **Particle-hole symmetry**: a half-filled system displays a uniform average electronic density equal to one always.
- **Mixed density matrix**:
  $$\rho(0) = \frac{1}{N+1} \sum_{i=1}^{N+1} |\Psi_i\rangle \langle \Psi_i|.$$  

Results: Limit (II) density matrix with pure state $\Psi$

Results: Limit (II) $\rightarrow$ Figure

Mathematical toolbox: von Neumann equation

- Time evolution of density matrix:
  $$\frac{d\rho(t)}{dt} = -i\frac{\hbar}{\tau}[H(t), \rho(t)].$$  

- **Time ramp profile**:
  $$m(t) = m_0 + (m_f - m_0) \left[1 - \exp(-\alpha t)\right].$$  

- ramp speed is given by $\alpha$. Time is measured in units of $\alpha^{-1}$.
- **Probability of finding the dislocation mode**:
  $$P(t) = \langle \Psi| \rho(t) |\Psi\rangle.$$  

- **Site resolved LDOS**:
  $$D_i(t) = \sum_{\alpha, \tau} \langle i, \alpha, \tau | \rho(t) | i, \alpha \rangle.$$  

- $i$ ($\alpha$) is the site (orbital) index, and $| i, \alpha \rangle$ is the single particle state vector at site $i$ with orbital $\alpha$.

Results: Limit (I) $\rightarrow$ Ramping out of TATI

- Melting: Ramping out of the TATI phase: dislocation modes survive for sometime and show periodic revive irrespective of the nature of the final phase.
- Site resolved LDOS displays periodic peaks and deeps at the dislocation core.
- **Condensation**: Dynamic condensation of dislocation modes is most prominent when the initial state is a normal insulator, residing close to the M phase, with the noninverted band minima near the M point.
- Slower ramp speed $\rightarrow$ better probability of dynamic generation of dislocation modes $\rightarrow$ **Adiabatic principle**.

Conclusions

References

One-dimensional topologically protected modes in topological insulators with lattice dislocations.

Topological defects and gapless modes in insulators and superconductors.

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