Fermi Surface Anomaly and Symmetric Mass Generation

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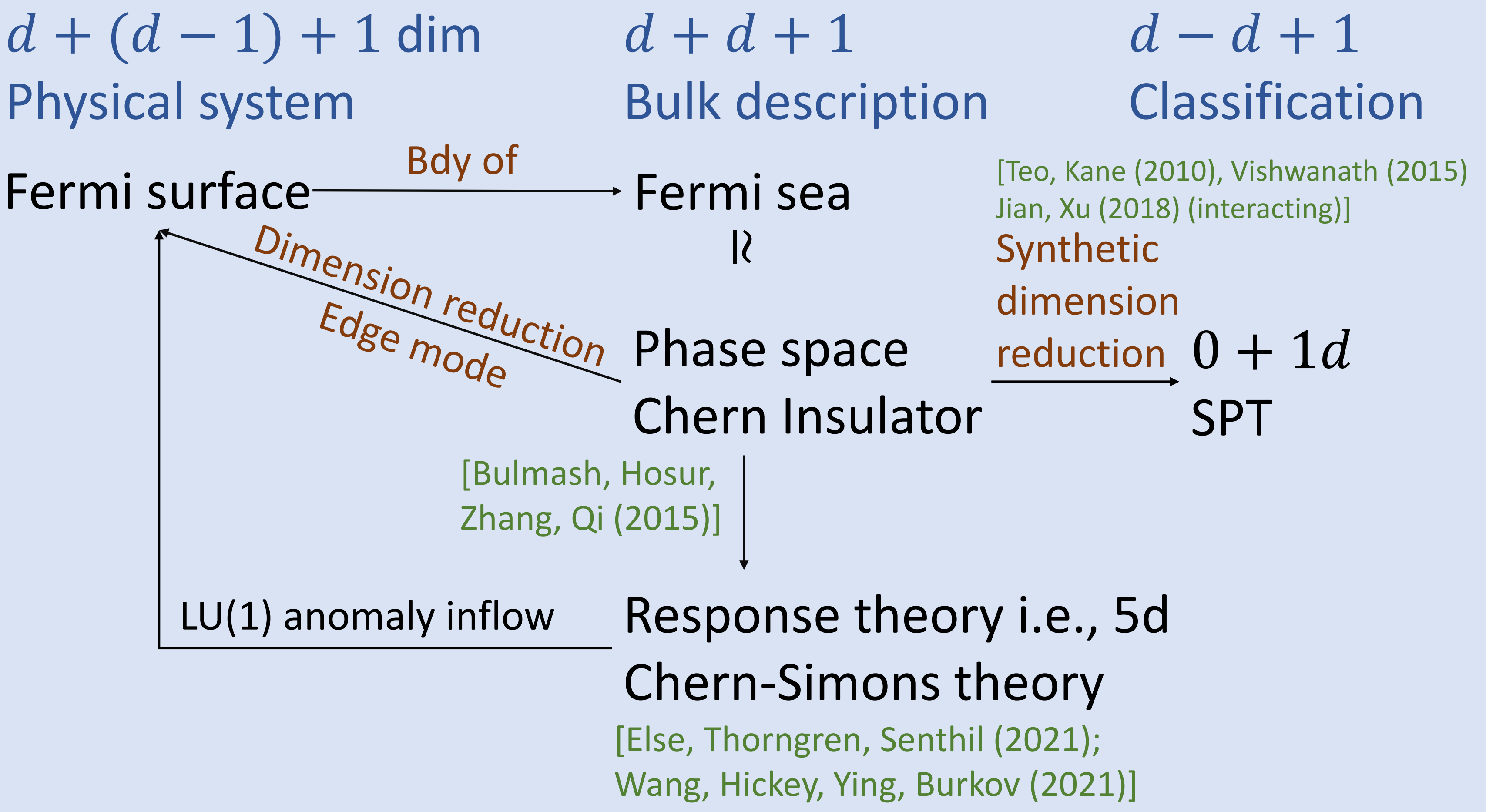
Outline



$d+(d-1)+1$ dim
Physical system

$d+d+1$
Bulk description

$d-d+1$
Classification

Fermi surface —Bdy of→ Fermi sea
≀
Phase space Chern Insulator

Dimension reduction
Edge mode

[Teo, Kane (2010), Vishwanath (2015)
Jian, Xu (2018) (interacting)]

Synthetic dimension reduction → $0+1d$ SPT

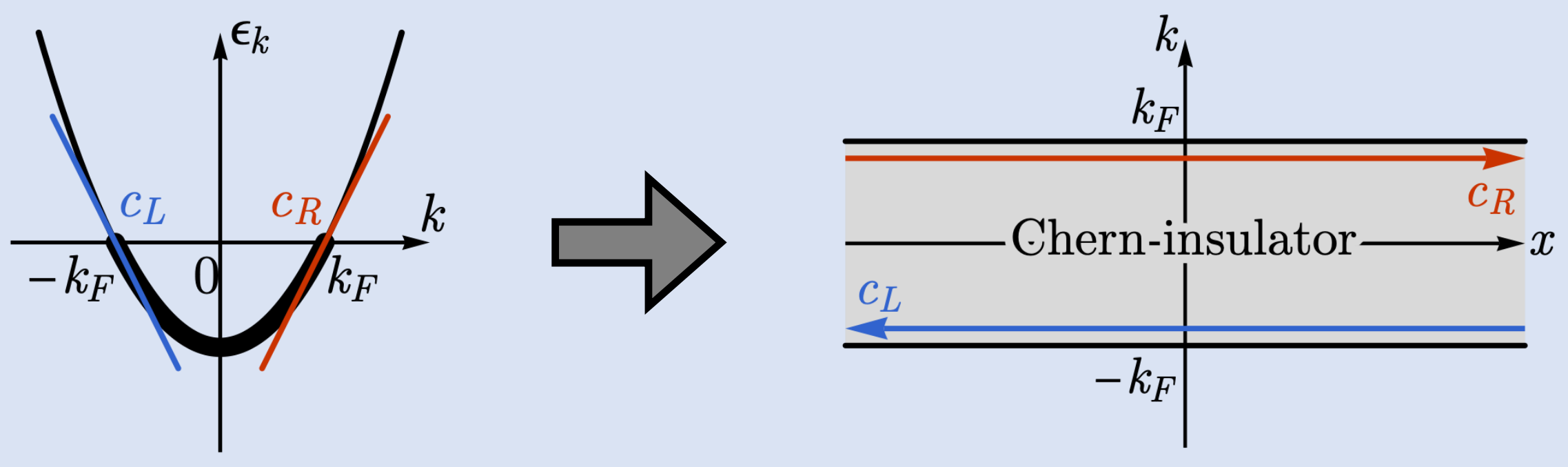
[Bulmash, Hosur, Zhang, Qi (2015)]

Response theory i.e., 5d Chern-Simons theory

LU(1) anomaly inflow

[Else, Thorngren, Senthil (2021);
Wang, Hickey, Ying, Burkov (2021)]

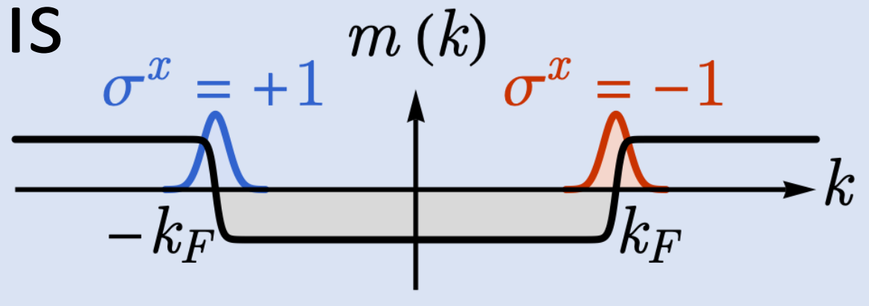
Bulk of Fermi surface ≃ Phase space Chern insulator



Given the phase space Chern insulator,

$$H=\int dx\,dk\,\psi^\dagger(i\partial_x\sigma^x+i\partial_k\sigma^y+m(x,k)\sigma^z)\psi$$

And the mass profile is



There are localized modes when mass changes sign.

$$H_L=\int dx\,\psi^\dagger(+i\partial_x)\psi\,,\qquad H_R=\int dx\,\psi^\dagger(-i\partial_x)\psi$$

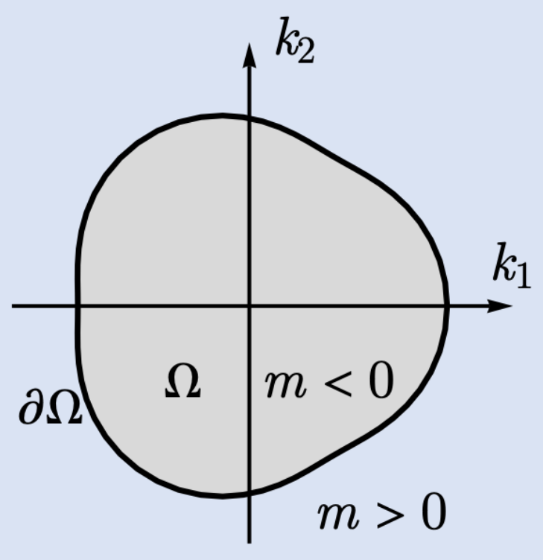
Higher dimemsnional phase space Chern insulator

In higher dimension,

$$H_{blk}=\int d^d\boldsymbol{x}\,d^d\boldsymbol{k}\,\psi^\dagger\big(i\partial_{\boldsymbol{x}}\cdot\boldsymbol{\Gamma}^x+i\partial_{\boldsymbol{k}}\boldsymbol{\Gamma}^k+m(\boldsymbol{k})\Gamma^0\big)\psi$$

Γs are from complex Clifford algebra $C\ell_{2d+1}$.
The mass profile is

$$m(\boldsymbol{k})=\begin{cases}\leq 0, \boldsymbol{k}\in\Omega\\ >0, \boldsymbol{k}\notin\Omega\end{cases}$$



To restore the commutator $[x,k]=i$, we use,

$$x=i\partial_k\qquad\Leftrightarrow\qquad k=-i\partial_x$$

The boundary theory is

$$H_{bdy}=\int_{\partial\Omega}d\boldsymbol{k}_F\int d(\boldsymbol{n}\cdot\boldsymbol{x})\,\psi^\dagger_{\boldsymbol{k}_F}i(\boldsymbol{n}\cdot\partial_{\boldsymbol{x}})\psi_{\boldsymbol{k}_F}$$

Bulk U(1) symmetry → boundary emergent loop group symmetry

$$L_{\partial\Omega}U(1):\psi_{\boldsymbol{k}_F}\to e^{i\phi(\boldsymbol{k}_F)}\psi_{\boldsymbol{k}_F},\qquad\forall\boldsymbol{k}_F\in\partial\Omega$$

Response theory

Like the ordinary Chern insulator, the theory can minimally couple to the background gauge field, tracking the global symmetry,

$$H=\int dx\,dk\,\psi^\dagger(iD_x\sigma^x+iD_k\sigma^y+m(k)\sigma^z)\psi$$

where $D_i=i\partial_i-A$. After integrating out the fermion,

$$S=\frac{1}{4\pi}\int_{(t,x,k)}\frac{1-\operatorname{sgn}m(k)}{2}A\wedge dA$$

The Chern-Simons term exists only in the Chern insulator.

The gauge connection $A=A_0dt+A_xdx+A_kdk$ has a uniform background curvature through the (x,k) plane to ensure the commutator,

$$F=dA=dx\wedge dk$$

And we can reproduce the Luttinger theorem,

$$\nu=\frac{\delta S}{\delta A_0}=\int_{-k_F}^{k_F}\frac{F_{xk}}{2\pi}=\frac{\mathrm{vol}\,\Omega}{2\pi}$$

For codimension-1 Fermi surfaces

Fermi surface d	Fermi sea d	Phase spacetime	Bulk CS theory
0	1	1+1+1=3	$A\wedge dA$
1	2	2+2+1=5	$A\wedge dA\wedge dA$
2	3	3+3+1=7	$A\wedge dA\wedge dA\wedge dA$

with background flux through every (x_i,k_i)-plane.

Classification – 0+1d SPT

Classification of $(d+d+1)D$ phase space Chern insulator = classification of $(0+1)D$ topological insulator.

- Start with the bulk phase space Chern insulator,

$$H_{blk}=\int d^d\boldsymbol{x}\,d^d\boldsymbol{k}\,\psi^\dagger\big(i\partial_{\boldsymbol{x}}\cdot\boldsymbol{\Gamma}^x+i\partial_{\boldsymbol{k}}\boldsymbol{\Gamma}^k+m\,\Gamma^0\big)\psi$$

- Using the operator relation, $i\partial_k=x$,

$$H_{blk}=\int d^d\boldsymbol{x}\,\psi^\dagger\big(i\partial_{\boldsymbol{x}}\cdot\boldsymbol{\Gamma}^x+\boldsymbol{x}\cdot\boldsymbol{\Gamma}^k+m\,\Gamma^0\big)\psi$$

- After projecting out every pair of x_i,k_i, $H_{blk}=m\,\psi^\dagger\psi$.
- The bulk state is trivial if the gapless fermion modes at $m=0$ can be symmetrically gapped by interaction.
- For $\mathbb{Z}_4$-symmetric fermions, trivialization can be achieved by $g(\psi_1\psi_2\psi_3\psi_4+h.c.)$ at multiplicity 4 ⇒ $\mathbb{Z}_4$ classified!

Conclusion

Classification of Fermi surface anomaly

Codimension-p Fermi surface anomaly of symmetry group G is classified by G symmetric interacting fermionic SPT states in p-dimensional spacetime.

Fermi surface symmetric mass generation can happen when the Fermi surface anomaly vanishes.

$$\nu=\sum_\alpha k_\alpha\frac{\mathrm{vol}\,\Omega_\alpha}{(2\pi)^d}=0\mod 1$$

- Potential applications to the pseudo-gap physics
- Spectral signatures: Green's function zeros on the Fermi surface
- Functional renormalization group of FS SMG models
- Matching Fermi surface with topological order

Collaborators and References

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DCL, Meng Zeng, Juven Wang, and Yi-Zhuang You. "Fermi Surface Symmetric Mass Generation." *arXiv:2210.16304.*

DCL, Juven Wang, and Yi-Zhuang You. "Definition and Classification of Fermi Surface Anomalies." *arXiv:2302.12731.*

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