Nonlinear Bosonization of Fermi Surfaces

Luca V. Delacretaz [1], Yi-Hsien Du [1], Umang Mehta [1], Dam Thanh Son[1]
[1]1 Kadanoff Center for Theoretical Physics, University of Chicago

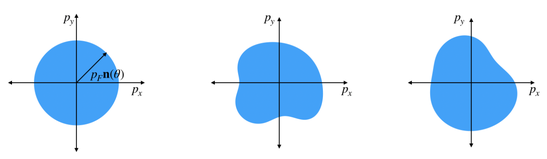
Abstract

We develop a new method for bosonizing the Fermi surface based on the formalism of the coadjoint orbits. This allows one to parametrize the Fermi surface by a bosonic field that depends on the spacetime coordinates and on the position on the Fermi surface. The Wess-Zumino-Witten term in the effective action, governing the adiabatic phase acquired when the Fermi surface changes its shape, is completely fixed. As an effective field theory the action also involves a Hamiltonian which contains, beside the kinetic energy and the Landau interaction, terms with arbitrary number of derivatives and fields. We show that the resulting local effective field theory captures both linear and nonlinear effects in Landau's Fermi liquid theory. The approach can be extended to incorporate spin degrees of freedom and the charge-2 fields corresponding to the BCS order parameter.

Landaus Fermi liquid theory

The system we study: (very) gapless system

- Gapless phases with a Fermi surface: Fermi and non-Fermi liquids.
- A closed Fermi surface in momentum space with arbitrary volume and shape characterizes the state of a Fermi liquid.



- Phase-space distribution of the quasiparticles $f(t, \mathbf{x}, \mathbf{p})$
- Liouvilles Equation of free Fermions:

$$\frac{\partial f}{\partial t} + \mathbf{v_p} \cdot \frac{\partial f}{\partial \mathbf{x}} + \left(\mathbf{E} + \mathbf{v_p} \times \mathbf{B}\right) \cdot \frac{\partial f}{\partial \mathbf{p}} = 0, \quad \mathbf{v_p} = \frac{\partial \epsilon_\mathbf{p}}{\partial \mathbf{p}} \tag{1}$$

- If initial condition of a Fermi surface (f=0 outside, f=1 inside), this would be preserved under time evolution (volume-preserving).
- Landau's Fermi liquid theory is NOT a field theory. How to formulate a field theory description? Coadjoint orbit method.

Coadjoint orbits

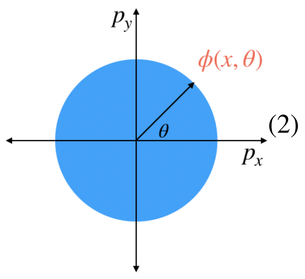
- At zero temperature, a Fermi surface is a droplet of phase space.
- Given Hamiltonian evolution for a single particle, the kinetic equation describes the time evolution of any initial droplet configuration.
- As reference state: ground state $f_0(\mathbf{p}) = \Theta(p_F - |\mathbf{p}|)$.
- Fermi liquids can be characterized as the coadjoint orbit of the group of canonical transformations.
- Coadjoint orbits: states with sharp Fermi surface with the same total number of particles (volume-preserving)
- The system's evolution takes place on the coadjoint orbit.
- Goal: we aim to parametrize the coadjoint orbit and write down the effective action $S[\phi]$.

Action for the shape of the Fermi surface

- From the ground state f_0 , not every $f(x, p)$ can be obtained

$$\partial_t f = \{H, f\} \to f = e^{Ht}\, f_0\, e^{-Ht} \tag{2}$$

allowed $f \to$ orbit f_0 of under $\mathcal{G}$



- Parametrize the degree of freedom as

$$f = U\, f_0\, U^{-1} \tag{3}$$

where parametrization group element $U = e^{-\phi} \in \mathcal{G}$ with element of algebra $\phi = \phi(x, p)$.

- Ambiguity: $U \sim UV$, V makes f_0 invariant.
- Gauge fixing : the degree of freedom $\phi = \phi(x, \theta)$ is the scalar field lives on the Fermi surface.

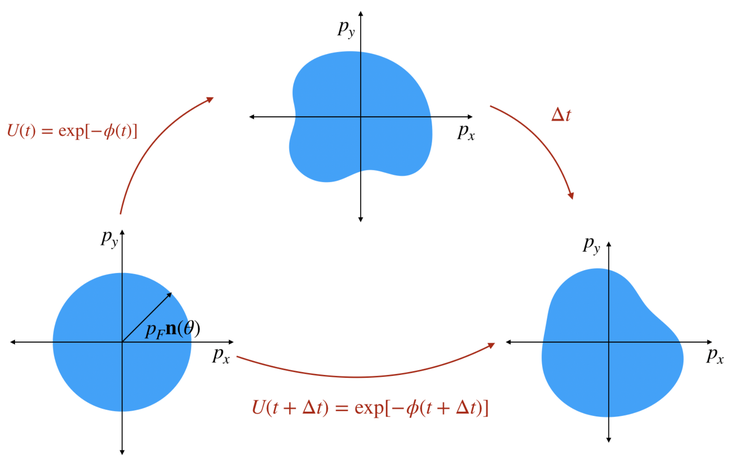


Figure 1: Fermi surface states from canonical transformation.

The action

$$S = \int dt\, Tr(f_0 U^{-1} \partial_t U) - \int dt \mathcal{H}[f] \tag{4}$$

- The first term is the Wess-Zumino-Witten term (Kirillov-Kostant-Souriau form).
- The Hamiltonian for the free fermions:

$$\mathcal{H}[f] = \int dx dp\, f(x, p, t)(\epsilon(p) + V(x)) + \cdots \tag{5}$$

- One can write down other terms based on effective field theory...

Gaussian action

$$S = -\frac{p_F^{d-1}}{2} \int \frac{dt d^d\mathbf{x} d^{d-1}\theta}{(2\pi)^d}\, \mathbf{n}_\theta \cdot \nabla\phi \left(\dot{\phi} + \epsilon'(p_F)\mathbf{n}_\theta \cdot \nabla\phi\right) \tag{6}$$

- Chiral boson on the Fermi surface.
- The result corresponds to the multi-dimensional bosonization action.

The density:

$$\rho(t, \mathbf{x}) = \frac{p_F^{d-1}}{(2\pi)^d} \int d^{d-1}\theta\, \mathbf{n}_\theta \cdot \nabla\phi(t, \mathbf{x}, \theta) \tag{7}$$

Two-point density correlator:

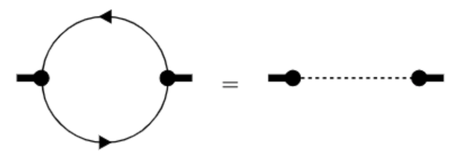


Figure 2: The density two-point function, which involves a loop in the fermion description, is captured by a tree level diagram in the boson description.

$$\langle\phi\phi'\rangle(\omega, \mathbf{q}) = i\frac{(2\pi)^d}{p_F^{d-1}} \frac{\delta^{d-1}(\theta - \theta')}{\mathbf{n}_\theta \cdot \mathbf{q}(\omega - v_F \mathbf{n}_\theta \cdot \mathbf{q})} \tag{8}$$

The density two-point function is (d=2)

$$\langle\rho\rho\rangle(\omega, q) = i\frac{p_F}{2\pi}\frac{1}{v_F}\left(1 - \frac{|\omega|}{\sqrt{\omega^2 - v_F^2 q^2}}\right) \tag{9}$$

Nonlinear Response

Nonlinear action:

$$\begin{aligned} S &= S_{\mathrm{WZW}} + S_H\,, \\ S_{\mathrm{WZW}} &= -p_F^{d-1} \int_{t,\mathbf{x},\theta} \frac{1}{2}\dot{\phi}(\mathbf{n}_\theta \cdot \nabla\phi) + \frac{1}{3!}\frac{1}{p_F}(\mathbf{n}_\theta \cdot \nabla\phi)\left(\mathbf{s}_\theta^i \cdot \nabla\phi\partial_{\theta^i}\dot{\phi} - \mathbf{s}_\theta^i \cdot \nabla\dot{\phi}\partial_{\theta^i}\phi\right) + \cdots\,, \\ S_H &= -p_F^{d-1} \int_{t,\mathbf{x},\theta} \frac{1}{2}\epsilon'(\mathbf{n}_\theta \cdot \nabla\phi)^2 + \frac{1}{3!}\frac{1}{p_F}\left(\frac{d-1}{2}\epsilon' + \epsilon'' p_F\right)(\mathbf{n}_\theta \cdot \nabla\phi)^3 + \cdots\,. \end{aligned} \tag{10}$$

The density:

$$\delta\rho = \nabla \cdot \frac{p_F^{d-1}}{(2\pi)^d} \int d^{d-1}\theta\, \mathbf{n}_\theta\phi + \frac{1}{p_F}\frac{1}{2}\mathbf{s}_\theta^i \partial_{\theta^i}\phi(\mathbf{n}_\theta \cdot \nabla\phi) + \cdots \tag{11}$$

Nonlinearity comes from:

1. Geometry of the Fermi surface.
2. Nonlinear dispersion relation: $\epsilon(p) \neq p$.

Three-point function:

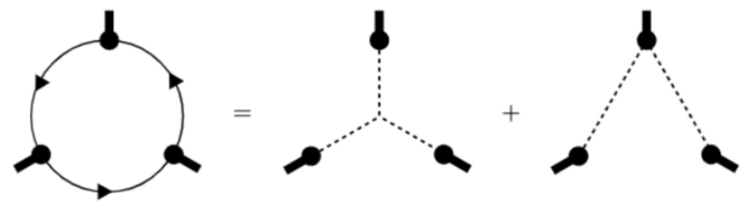


Figure 3: The density three-point function in the fermionic and bosonic descriptions.

- The fermionic side's one-loop calculation is simplified to a tree-level calculation of the bosonic side.
- Ensuring precise matching is a highly challenging task.
- Nonlinear corrections result in, e.g. density 3-pt function $\langle\rho\rho\rho\rangle$ from the kinetic theory.
- Get the action for the Luttinger liquid with two chiral bosons in (1+1) dimensions.

Beyond Fermi liquids

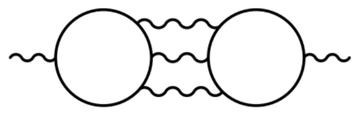
Couple the bosonized theory to a critical boson Φ:

$$\mathcal{L} = -\frac{p_F}{8\pi^2}\int_\theta \mathbf{n}(\theta) \cdot \nabla\phi(\dot{\phi} + v_F\mathbf{n}(\theta) \cdot \nabla\phi) + \int_\theta \lambda\Phi(\mathbf{n}(\theta) \cdot \nabla\phi) + \frac{1}{2}(\nabla\Phi)^2 \tag{12}$$

We get $z = 3$ at tree level:

$$\langle\Phi\Phi\rangle(\omega, q) \simeq \frac{1}{q^2 + \lambda^2 \frac{|\omega|}{\sqrt{\omega^2 - v_F^2 q^2}}} \tag{13}$$

Beyond tree-level $z \neq 3$:



Conclusions and future directions

- The nonlinear bosonized effective action for Fermi liquids can be formulated by employing the coadjoint orbit method.
- Reproduces linear and nonlinear responses.
- Studying non-Fermi liquids begans with coupling to critical bosons, gauge fields...
- Non-analytic corrections to Moyal bracket?

References

[1] L. V. Delacretaz, Y.-H. Du, U. Mehta, and D. T. Son, Nonlinear Bosonization of Fermi Surfaces: The Method of Coadjoint Orbits, Phys. Rev. Research 4, 033131 (2022), arXiv:2203.05004 [cond-mat.str-el].

[2] Work in progress.