A study on dissipative models based on Γ-matrices

Jyotsna Gidugu†, Daniel Arovas*
Department of Physics, University of California San Diego.

Abstract

We generalize the recent work of Shibata and Katsura [1], who considered an S=1/2 chain with alternating XX and YY couplings in the presence of dephasing, the dynamics of which are described by the GKLS master equation. Their model is equivalent to a non-Hermitian system which is described by the Kitaev formulation [2] in terms of a single Majorana species hopping in the presence of a Z_2 gauge field. Our generalization involves Dirac gamma matrix spin operators on a square lattice, and maps onto a non-Hermitian square lattice bilayer. In both cases, the infinite temperature state is a nonequilibrium steady state, but the various decay channels occur for nontrivial density matrices. We study the Liouvillian spectrum. We observe a phase transition in the first decay modes (similar to that in ref. [1]) in the 2d model.

We then present another dissipative model that can be solved using Gamma matrices in which we again see a phase transition in the first decay modes.

Model-I

$$\Gamma^1=\sigma^x\otimes\mathbb{I},\ \Gamma^2=\sigma^y\otimes\mathbb{I},\ \Gamma^3=\sigma^z\otimes\sigma^x$$

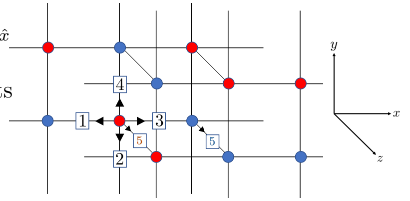
$$\Gamma^4=\sigma^z\otimes\sigma^y,\ \Gamma^5=\sigma^z\otimes\sigma^z$$

$$H=J_1\sum_{\vec R}\Gamma^1_{\vec R}\Gamma^1_{\vec R-\hat x}+J_2\sum_{\vec R}\Gamma^2_{\vec R}\Gamma^2_{\vec R-\hat y}+J_3\sum_{\vec R}\Gamma^3_{\vec R}\Gamma^3_{\vec R+\hat x}+J_4\sum_{\vec R}\Gamma^4_{\vec R}\Gamma^4_{\vec R+\hat y}$$

$$L_{\vec R}=\sqrt{\gamma}\Gamma^5_{\vec R},\ L_{\vec R+\hat x}=\sqrt{\gamma}\Gamma^5_{\vec R+\hat x}$$

$\vec R\in$ set of all red lattice points

$$i\frac{d\,|\rho\rangle}{dt}=\mathcal{M}\,|\rho\rangle$$

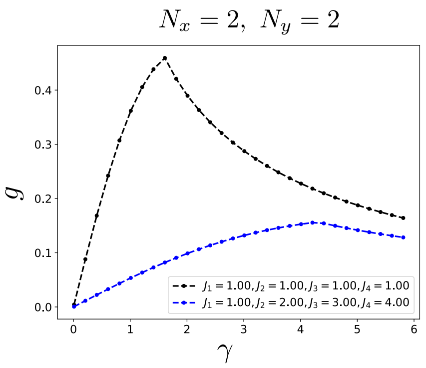


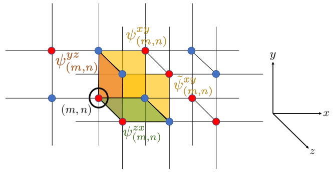
$$\mathcal{M}=H\otimes\mathbb{1}-\mathbb{1}\otimes H^{\top}+i\sum_\mu\left(L_\mu\otimes L^*_\mu-\frac{1}{2}L^\dagger_\mu L_\mu\otimes\mathbb{1}-\mathbb{1}\otimes\frac{1}{2}L^\top_\mu L^*_\mu\right)$$

$$g_{\vec F}=\min_{\substack{\vec n\\ \mathrm{Im}\Lambda_{\vec F,\vec n}\neq 0}}\left(-\mathrm{Im}\Lambda_{\vec F,\vec n}\right),\quad g=\min_{\vec F} g_{\vec F}$$

(Here, $\vec F$ refers to a flux and Wilson loop configuration. [1] [3])

Phase transition





We see a phase transition in the flux configurations corresponding to the first decay modes (for $N_x=N_y=2$). [1] [3] (Periodic boundary conditions were used.) For larger system sizes, we propose a Monte Carlo approach involving the gauge fields to arrive at the gap. (Work is underway.)

$J_1=J_2=J_3=J_4=1,\ \gamma<\gamma_c$ (16 modes)

$\psi^{xy}_{(1,1)}$	$\psi^{xy}_{(2,1)}$	$\psi^{xy}_{(1,2)}$	$\psi^{xy}_{(2,2)}$	$\psi^{yz}_{(1,1)}$	$\psi^{yz}_{(2,1)}$	$\psi^{yz}_{(1,2)}$	$\psi^{yz}_{(2,2)}$	$\psi^{zx}_{(1,1)}$	$\psi^{zx}_{(2,1)}$	$\psi^{zx}_{(1,2)}$	$\psi^{zx}_{(2,2)}$	W_x	W_y	$\tilde W_x$	$\tilde W_y$
-1	1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	-1	-1	-1

$\gamma>\gamma_c$ (80 modes)

$\psi^{xy}_{(1,1)}$	$\psi^{xy}_{(2,1)}$	$\psi^{xy}_{(1,2)}$	$\psi^{xy}_{(2,2)}$	$\psi^{yz}_{(1,1)}$	$\psi^{yz}_{(2,1)}$	$\psi^{yz}_{(1,2)}$	$\psi^{yz}_{(2,2)}$	$\psi^{zx}_{(1,1)}$	$\psi^{zx}_{(2,1)}$	$\psi^{zx}_{(1,2)}$	$\psi^{zx}_{(2,2)}$	W_x	W_y	$\tilde W_x$	$\tilde W_y$
-1	1	-1	1	-1	-1	-1	-1	1	1	-1	1	1	-1	1	-1

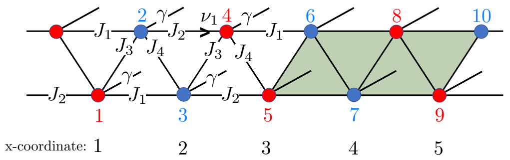
Model -II

$$H=\sum_{n=1}^{N_c}(J_1\Gamma^1_{4n-3}\Gamma^1_{4n-1}+J_1\Gamma^1_{4n}\Gamma^1_{4(n+1)-2}+J_2\Gamma^1_{4n-2}\Gamma^1_{4n}+J_2\Gamma^2_{4n-1}\Gamma^2_{4(n+1)-3}$$

$$+J_3\Gamma^3_{4n-2}\Gamma^3_{4n-3}+J_3\Gamma^3_{4n}\Gamma^3_{4n-1}+J_4\Gamma^4_{4n-2}\Gamma^4_{4n-1}+J_4\Gamma^4_{4n}\Gamma^4_{4(n+1)-4})$$

$$=\sum_{m,\mathrm{even}}\left(J_2\Gamma^1_{2m-1}\Gamma^1_{2(m+1)-1}+J_1\Gamma^2_{2m}\Gamma^2_{2(m+1)}+J_3\Gamma^3_{2m-1}\Gamma^3_{2m}+J_4\Gamma^4_{2m}\Gamma^4_{2(m+1)-1}\right)$$

$$+\sum_{m,\mathrm{odd}}\left(J_1\Gamma^1_{2m-1}\Gamma^1_{2(m+1)-1}+J_2\Gamma^2_{2m}\Gamma^2_{2(m+1)}+J_3\Gamma^3_{2m-1}\Gamma^3_{2m}+J_4\Gamma^4_{2m}\Gamma^4_{2(m+1)-1}\right)$$



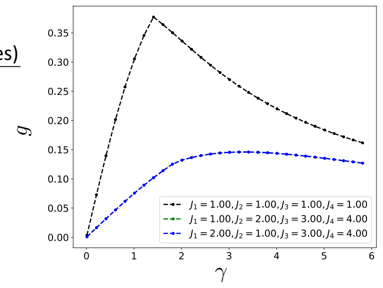
$$L_{2m-1}=\sqrt{\gamma}\Gamma^5_{2m-1},$$

$$L_{2m}=\sqrt{\gamma}\Gamma^5_{2m}$$

First decay modes: Ref. [1]

$$J_1=J_2=J_3=J_4=1$$

$\gamma<\gamma_c$ (4 modes)	$\gamma>\gamma_c$ (16 modes)
$\phi^{\triangle}=$ [-1, -1]	$\phi^{\triangle}=$ [-1, -1]
$\phi^{\nabla}=$ [-1, -1]	$\phi^{\nabla}=$ [1, 1]
$\phi^{L}=$ [-1, -1]	$\phi^{L}=$ [-1, 1]
$\phi^{R}=$ [1, 1]	$\phi^{R}=$ [1, 1]
$\tilde\phi^{\triangle}=$ [1, 1]	$\tilde\phi^{\triangle}=$ [1, -1]
$\tilde\phi^{\nabla}=$ [1, 1]	$\tilde\phi^{\nabla}=$ [1, -1]
$W_z=$-1	$W_z=$-1
$\tilde W_z=$ -1	$\tilde W_z=$ 1



Summary

- We simplified the above two models based on Γ-matrices using fluxes and Wilson loops. [3]
- We observed a phase transition (in the first decay modes) at the cusps in the g vs γ plots. [1]

References

[1] Dissipative spin chain as a non-Hermitian Kitaev ladder, N. Shibata and H. Katsura, Phys. Rev. B 99, 174303 (2019).
[2] Anyons in an exactly solved model and beyond, A. Kitaev, Annals of Physics 321, 2 (2006), ISSN 0003-4916
[3] Γ-matrix generalization of the Kitaev model, C. Wu, D. Arovas, and H.-H. Hung, Phys. Rev. B 79, 134427 (2009).

Acknowledgements and contact details

UC San Diego PHYSICS
†email: jgidugu@ucsd.edu
*email: darovas@ucsd.edu