Local Topological Markers in Odd Spatial Dimensions and Their Application to Amorphous Topological Matter

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Local topological markers, topological invariants evaluated by local expectation values, are valuable for characterizing topological phases in materials lacking translation invariance. The Chern marker—the Chern number expressed in terms of the Fourier transformed Chern character—is an easily applicable local marker in even dimensions, but there are no analogous expressions for odd dimensions. We provide general analytic expressions for local markers for free-fermion topological states in odd dimensions protected by local symmetries: a *Chiral marker*, a local $\mathbb{Z}$ marker which in case of translation invariance is equivalent to the chiral winding number, and a *Chern-Simons marker*, a local $\mathbb{Z}_2$ marker characterizing all nonchiral phases in odd dimensions. We achieve this by introducing a one-parameter family P_ϑ of single-particle density matrices interpolating between a trivial state and the state of interest. By interpreting the parameter ϑ as an additional dimension, we calculate the Chern marker for the family P_ϑ. We demonstrate the practical use of these markers by characterizing the topological phases of two amorphous Hamiltonians in three dimensions: a topological superconductor ($\mathbb{Z}$ classification) and a topological insulator ($\mathbb{Z}_2$ classification).