Local topological markers in odd spatial dimensions and their application to amorphous matter

Julia D. Hannukainen, Miguel F. Martínez, Jens H. Bardarson, and Thomas Klein Kvorning
Department of Physics, KTH Royal Institute of Technology, 106 91, Stockholm, Sweden
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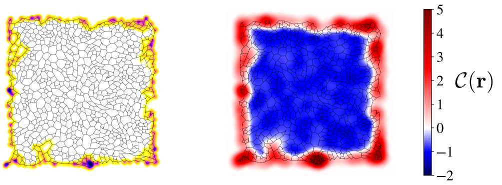
Local topological markers are important for categorising topological phases of amorphous matter which lack long range order

The topological phases of the Amorphous Chern insulator can be characterised using the local Chern marker, the Fourier transform of the Chern character [1].

$$\mathcal{C}(\mathbf{r}) = 2\pi\ \mathrm{Im}\ \langle \mathbf{r}|[\hat{Q}\hat{x},\hat{P}\hat{y}]|\mathbf{r}\rangle, \qquad \mathcal{C} = \frac{1}{A_{\mathrm{sys}}}\sum_{\mathbf{r}} \mathcal{C}(\mathbf{r}) \tag{1}$$

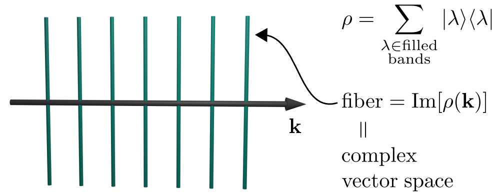


Amorphous Chern insulator: Density of states and the local Chern marker [2].

We **provide** a general expression for **local topological markers** for **odd-dimensional** free fermion states, analogous to the local Chern marker, previously missing from the literature.

The topological classification of Slater determinant states in terms of their single particle density matrix, ρ, is equivalent to a classification of vector bundles [3]

translation invariance $\Longrightarrow \rho(\mathbf{k}) =$ diagonal smooth in $\mathbf{k}$



The local Chern marker as a bundle invariant in D even dimensions:

$$\mathcal{C}(\mathbf{r}) = \sum_{\alpha} \frac{\varepsilon^{i_1,\ldots,i_D}[\rho X_{i_1}\rho X_{i_2}\cdots X_{i_D}\rho]_{(\mathbf{r},\alpha),(\mathbf{r},\alpha)}}{(D/2)!/(2\pi i)^{D/2}} \tag{2}$$

The **odd** dimensional **Chern-Simons bundle invariant** cannot be expressed as a function of ρ $\Longrightarrow$ **No local marker.**

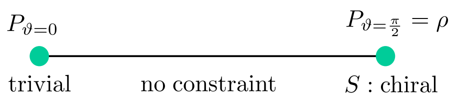
The local chiral marker, a $\mathbb{Z}$-invariant characterising 3 out of 5 AZ-classes in odd dimensions, is constructed using Eq. (2) by adding an extra parameter dimension ϑ:

$$\nu(\mathbf{r}) = \sum_{\alpha}\int_0^{\pi/2} d\vartheta \frac{\varepsilon^{i_0,\ldots,i_D}[P_\vartheta X_{i_0} P_\vartheta \cdots X_{i_D} P_\vartheta]_{(\mathbf{r},\alpha),(\mathbf{r},\alpha)}}{[(D+1)/2]!/[2i(2\pi i)^{(D-1)/2}]} \tag{3}$$

where D is odd and $X_0 = i\partial_\vartheta$.

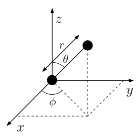
$$P_\vartheta = \frac{1}{2}[1-\sin(\vartheta)(1-2\rho)-\cos(\vartheta)S] \tag{4}$$

is a family of single-particle density matrices interpolating between a trivial state and the chiral state ρ, obeying the chiral constraint: $\{\rho, S\} = S$.



The topological phases of the topological superconductor in three-dimensions (Class DIII) are characterised by the local chiral marker

$$\mathcal{H}_{ij} = -\delta_{ij}M\tau_z - 2t_{ij}\tau_z +$$
$$t_{ij}\{[\sigma_z\cos\theta + e^{-i\phi}(\sigma_x + i\sigma_y)\sin\theta\ ][i\tau_x + t\tau_y] + h.c.\}. \tag{5}$$

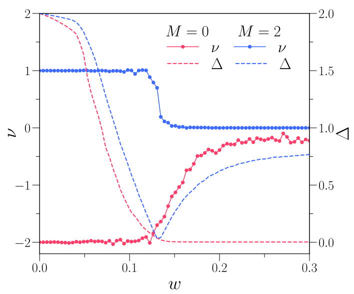


$$t_{ij} = \frac{1}{4}e^{1-|\mathbf{r}_i-\mathbf{r}_j|/a}$$

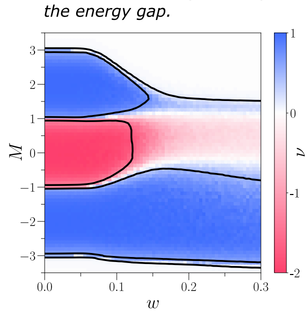
Each lattice site is drawn from a Gaussian distribution with standard-deviation w, centred at the lattice site positions of a cubic lattice.

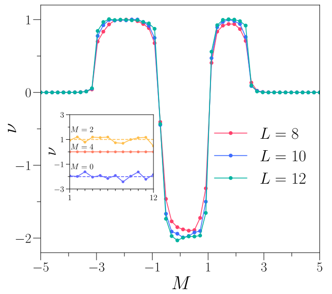
The coordination number is fixed at six nearest neighbours.

The chiral marker (left) and the energy gap (right) as a function of the width.



The phase diagram in parameter space (M,w). The line is the closing/opening of the energy gap.





The chiral marker as a function of the mass-parameter for w=0.1, for a linear lattice size L.

The marker value fluctuates over sites and is averaged over 25 sites and four lattice realisations.

The local Chern-Simons marker ($\mathbb{Z}_2$-invariant) characterises the topological phases in odd dimensions without a chiral constraint:

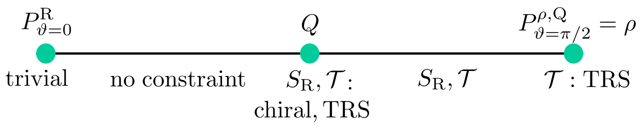
$$\nu_{\mathrm{cs}} = \nu \mod 2 \tag{6}$$

The path is the sum: $P_\vartheta^{\mathrm{R}} + P_\vartheta^{\rho,\mathrm{Q}}$ where $P_\vartheta^{\rho,\mathrm{Q}}$ yields a zero contribution to ν_{cs}.

$$P_\vartheta^{\mathrm{R}} = \frac{1}{2}[1-\sin(\vartheta)(1-2Q)-\cos(\vartheta)S_{\mathrm{R}}] \tag{7}$$

$$P_\vartheta^{\rho,\mathrm{Q}} = \frac{1}{2}[1-\sin(\vartheta)(1-2\rho)-\cos(\vartheta)(1-2Q)] \tag{8}$$

Q is invariant under both the chiral symmetry and the symmetry of ρ.



The topological phase of the three-dimensional topological insulator (class AII), which obeys a time-reversal constraint, is characterised by the local Chern-Simons marker

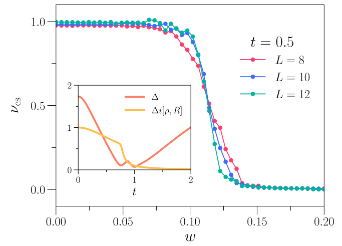
$$Q = \frac{1}{2}\left(1 + i|[\rho, S_{\mathrm{R}}]|^{-1}[\rho, S_{\mathrm{R}}]\right) \qquad \{Q, S_{\mathrm{R}}\} = S_{\mathrm{R}}$$
$$S_{\mathrm{R}} = 1 - 2R$$

Choose R such that: $\mathcal{T}Q\mathcal{T}^{-1} = Q$
$i[\rho, S_{\mathrm{R}}]$ is gapped $\Longrightarrow Q$ local

The Chern-Simons marker as a function of width, w, for a linear system size L, for t=0 in Eq. (5).

$$S_{\mathrm{R}} = -\tau_y$$

The inset is the energy gap and the gap of $i[\rho, R]$ as a function of the parameter t.



References:
[1] R. Bianco and R. Resta Phys. Rev. B 84, 241106(R) (2011)
[2] Q. Marsal, D. Varjas, and A.G. Grushin Proc. Natl. Acad. Sci. U.S.A. 117, 30260 (2020)
[3] A. Kitaev AIP Conf. Proc. 1134, 22 (2009)

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