Many-Body Real Space Invariants for Interacting 2D Topology

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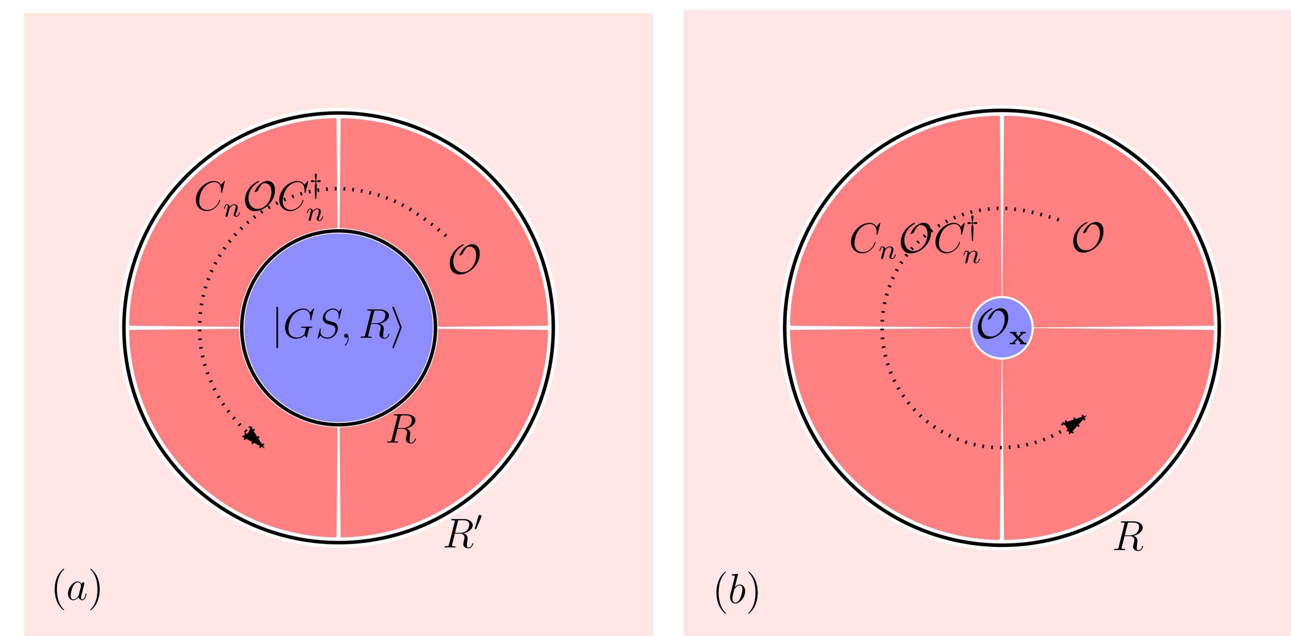
1. Many-Body Real Space Invariants

- Topological indices are global in momentum space → local in real space, act as "Noether charges"
- RSIs as quantum numbers: define symmetry operators on OBC's s.t. spectrum independent of OBC's
- Space group operators give local angular momentum and local charge as RSIs (n even, spin-less):

$$e^{i\frac{\pi}{n}\hat{N}}C_n\,|GS\rangle = e^{i\frac{\pi}{n}\Delta_1}\,|GS\rangle\,,\ C_n^2\,|GS\rangle = e^{i\frac{2\pi}{n/2}\Delta_2}\,|GS\rangle\,,\quad \Delta_1,\Delta_2 \in \mathbb{Z}_{2n}\times\mathbb{Z}_{n/2}$$

- Proof makes use of a "Many-body atomic limit" of arbitrarily strong interactions, but onsite-local:

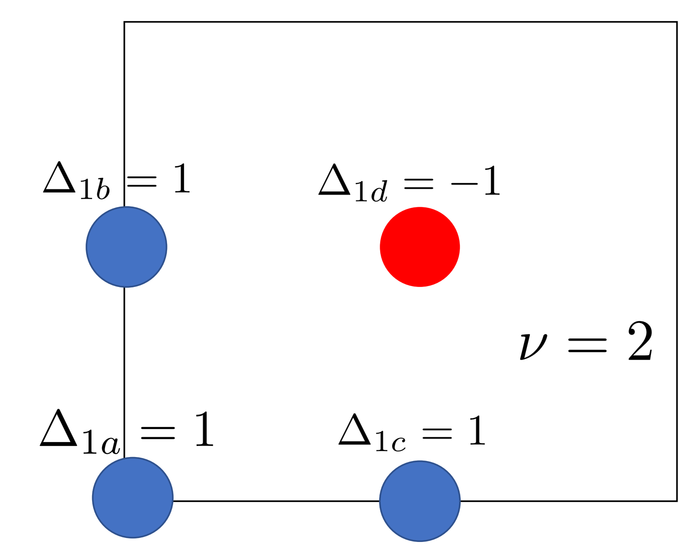
$$|GS,R'\rangle = \prod_{i=0}^{n-1}\mathcal{O}_i\,|GS,R\rangle\,,\quad \mathcal{O}_i = \prod_{\mathbf{R}+\mathbf{r}\in\mathcal{D}} C_n^i\mathcal{O}_{\mathbf{R},\mathbf{r}}C_n^{\dagger i}$$



2. Many-Body Fragile Topology

- Definition: Many-body Fragile Topology iff adiabatically disconnected from many-body atomic state at fixed particle number, but connected at higher particle number.
- Interplay between U(1) charge and space group G. Topological indices are obstructions in the form of inequalities.
- RSIs form an abelian group. In a fragile phase, there exists n such that $\Delta = \Delta_{\nu+n} - \Delta_n$ defined by coupling trivial states.
- Intuition: nonzero RSIs (implying local angular momentum or charge) require local operators to carry them. Formally,

$$\nu \geq \sum_{\mathbf{x}=1a,1b,1c,1d} N_{\mathbf{x}} \geq \sum_{\mathbf{x}=1a,1b,1c,1d} |\Delta_{\mathbf{x}}|$$ holds in all atomic limit states. Violating this inequality presents an obstruction.

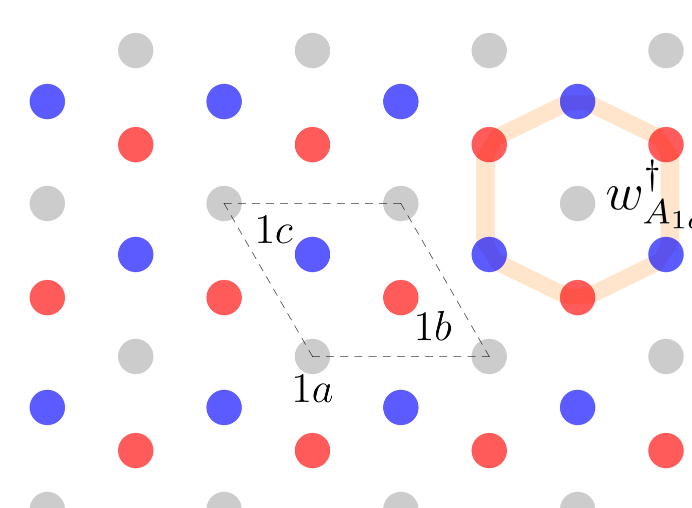


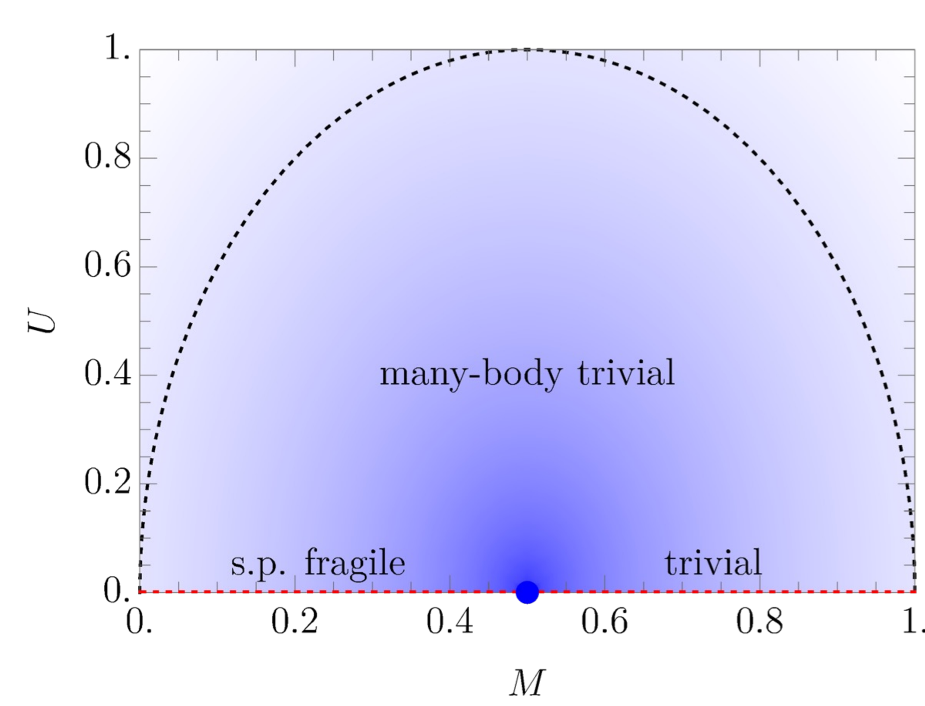
3. Weak Coupling Limit, Reduction of Single-Particle Topology, and Model Examples

- Understand RSIs at weak-coupling by acting on product states

$$e^{i\frac{\pi}{2}\hat{N}}C_2\,|GS\rangle = e^{i\frac{\pi}{2}(m(A)+m(B))+i\pi m(B)}\,|GS\rangle\,,\qquad \Delta = m(A)-m(B)\quad \mathrm{mod}\ 4$$

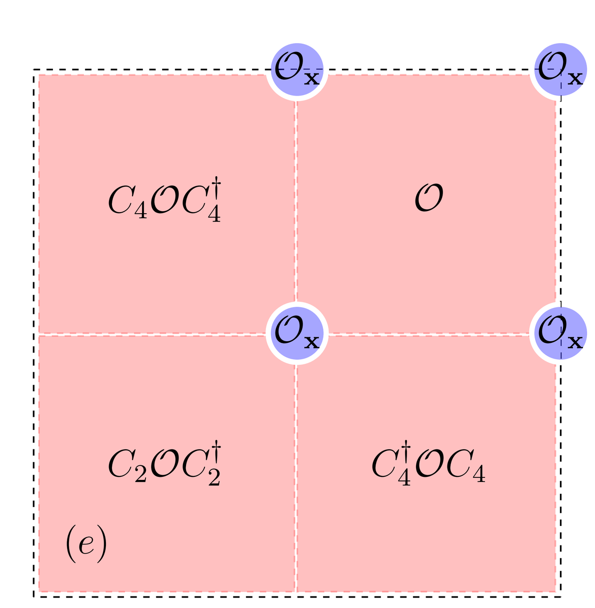
- Interpretation from single-particle RSI $\Delta = \delta \mod 4$.
- Momentum space computation at weak-coupling and reduction of single-particle affine-monoid structure.
- Some single-particle fragile phases are many-body fragile. Some are not. Example:





4. Many-Body Stable Topology and Interaction-Enabled Topological Phases

- Proof of many-body RSIs in atomic and fragile states relied on a gap for open boundary conditions. No such gap in stable topological phases!
- Proposal: define <u>global</u> many-body RSIs as the quantum numbers of the RSI operators on PBCs (gapped groundstate).
- We prove (1) all global RSIs (defined above) vanish in atomic and fragile states, and (2) Chern insulators have non-zero global RSIs
- Classification suggests strongly correlated topological states that do not exist at weak coupling. Ex: Chern insulator with nonzero momentum.



4. Topological Response Theory: RSIs as Wen-Zee Coefficients

- Topological insulators are gapped states with no low-energy electronic excitations. Their response to external probes is described by TQFTs.
- Recent work identified Wen-Zee terms that describe the response to geometric/curvature perturbations $d\omega$ (defects in spatial symmetries).
- Lagrangian is $\mathcal{L} = \frac{C}{4\pi}A\,dA + \frac{s}{2\pi}A\,d\omega + \frac{\ell}{4\pi}\omega\,d\omega$. We propose $s = \Delta_1 - 2\Delta_2,\ \ell = \frac{1}{2}\Delta_1$. Local response = local invariant.