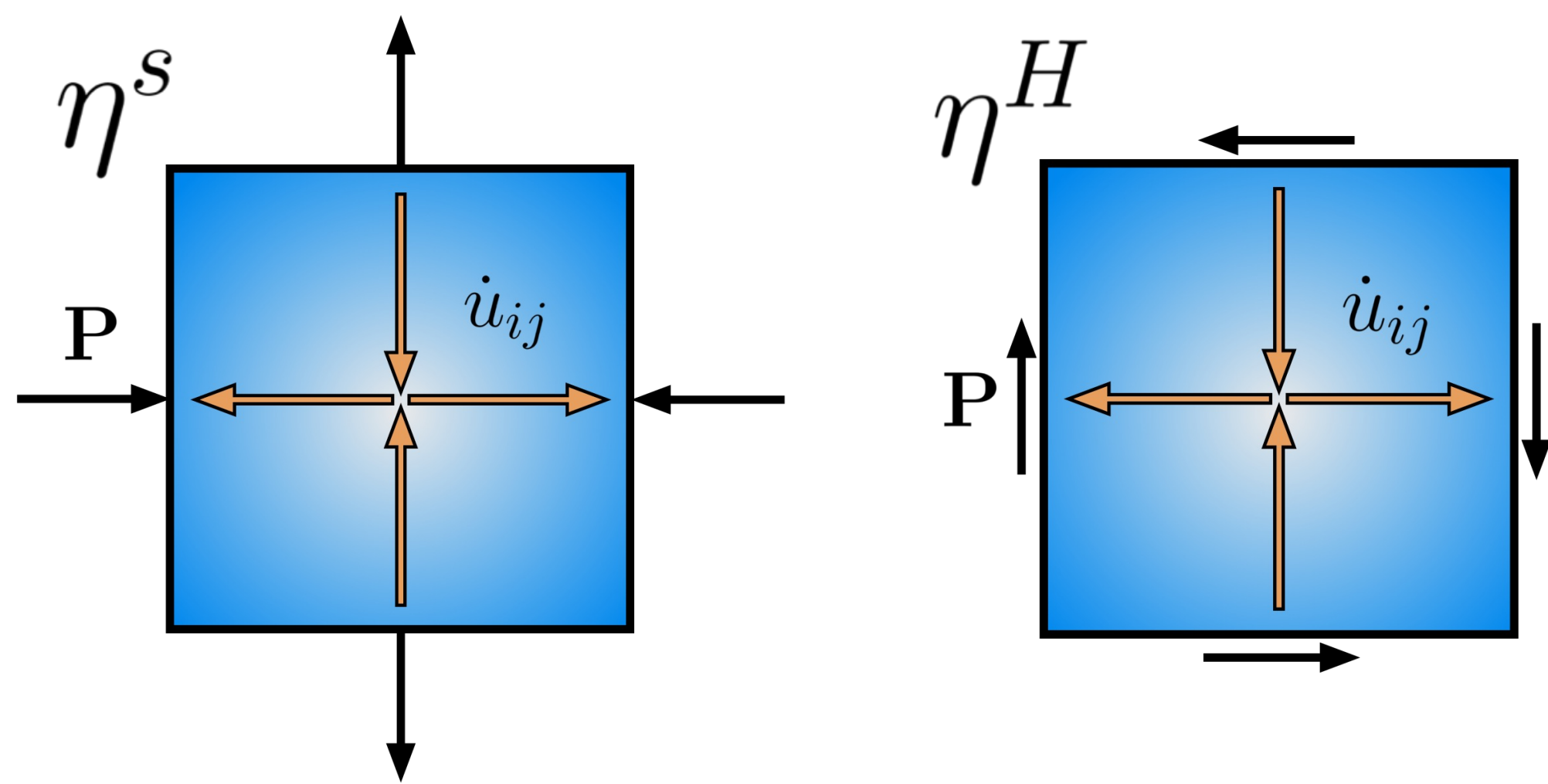


Summary

- Hall viscosity is a dissipationless viscosity arising in fluids with broken time reversal. Despite its importance in a range of contexts – including the fractional quantum Hall effect – **observing Hall viscosity in quantum systems has proven to be a big challenge**
- We propose explicit protocols for measuring Hall viscosity in **realistic** rotating BEC systems where the bosons condense into a single LLL orbital
- These protocols exploit the effect of Hall viscosity on superfluid vortices. For example, **Hall viscosity leads to precession of vortex dipoles**

Why Hall viscosity?

- The non-dissipative part of the viscosity tensor. It is allowed in two dimensional systems which break time reversal; in isotropic systems it is the only such term. In incompressible quantum systems it is given by half the average angular momentum per unit area. It is thus proportional to the average "orbital spin" per particle.



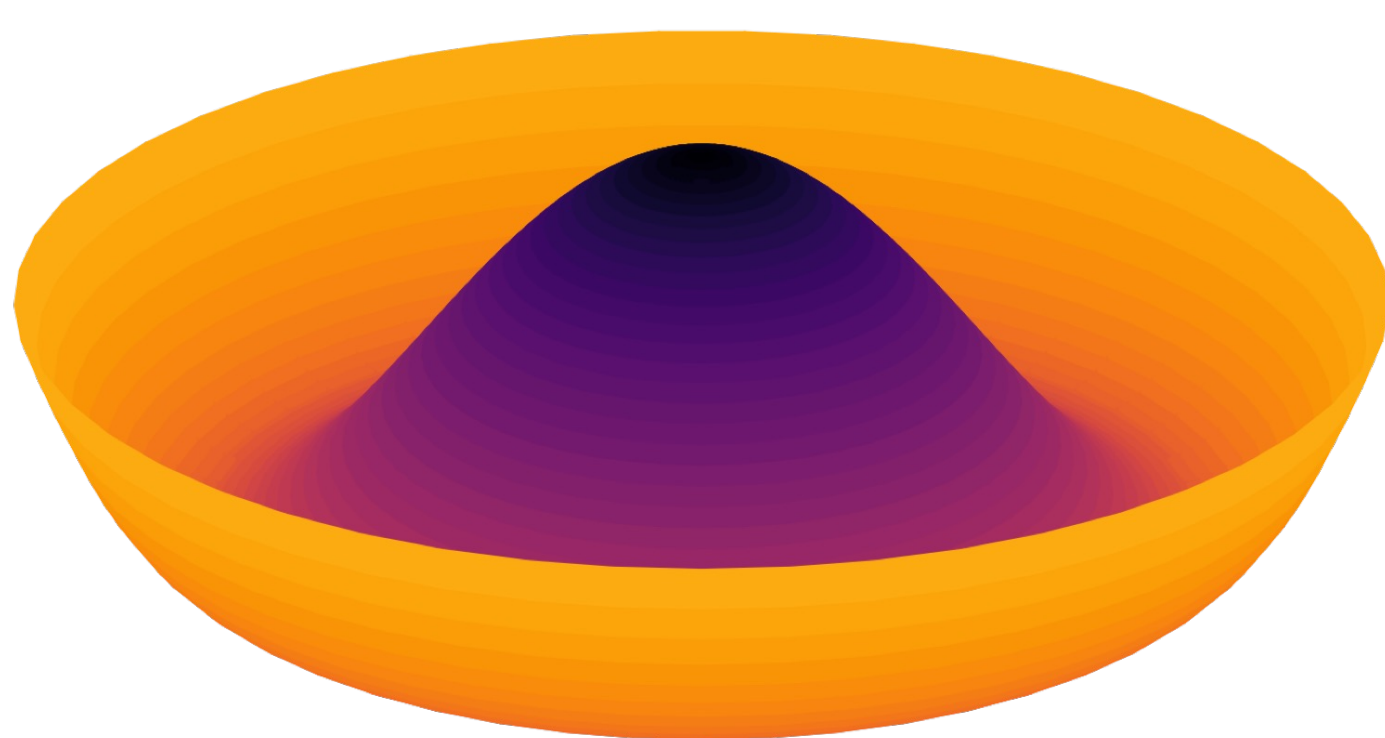
Cartoon of forces exerted by shear and Hall viscosity

Experimental system

- Rotating Bose Einstein condensates (BECs) of neutral bosons experience the Coriolis force. This is equivalent to a magnetic field proportional to the rotation frequency
- Application of a confining potential can then squeeze the BEC into a specific state with an average angular momentum

$$\Psi(r_1, \dots, r_{N_b}) = \prod_{\alpha=1}^{N_b} \psi_{\text{LLL}}(r_\alpha)$$

- We propose a confining potential with a minimum at a nonzero radius. The BEC will then be a condensate of a symmetric gauge wavefunction with a controllable angular momentum per particle



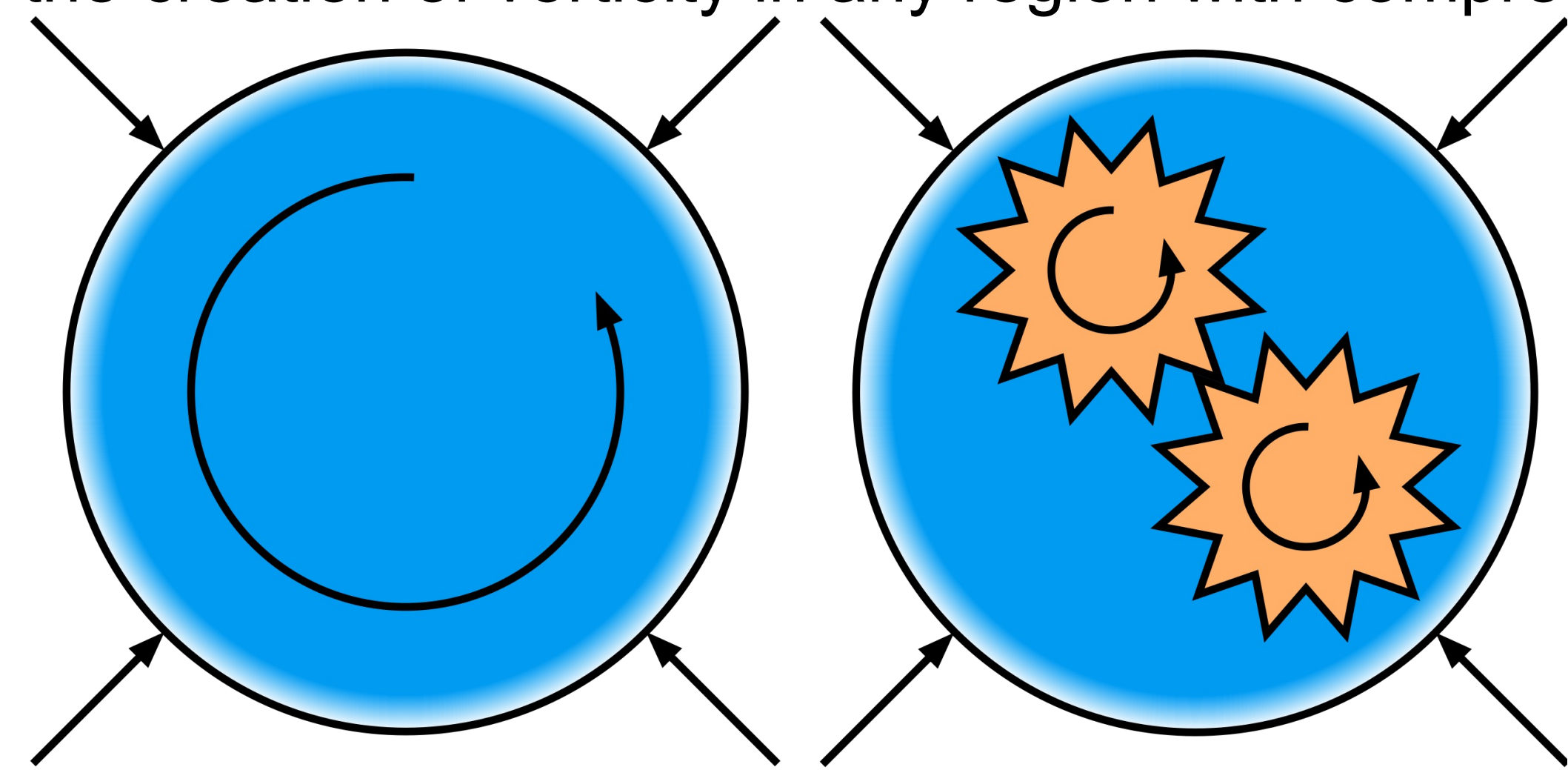
- Current experiments use an anisotropic potential to squeeze the bosons into a strip; they will also possess an angular momentum but will require regularization

Hydrodynamic phenomenology

- In a hydrodynamic system, e.g. the LLL superfluid, Hall viscosity can be rephrased in terms of its effect on the vorticity of the fluid, Ω :

$$\frac{\partial \Omega}{\partial t} + \partial_i(\Omega v^i) = \nu_s \nabla^2 \Omega + \nu_H \nabla^2(\partial_i v^i) - \omega_c \partial_i v^i$$

- Shear viscosity leads to vorticity diffusion, while Hall viscosity will lead to the creation of vorticity in any region with compression

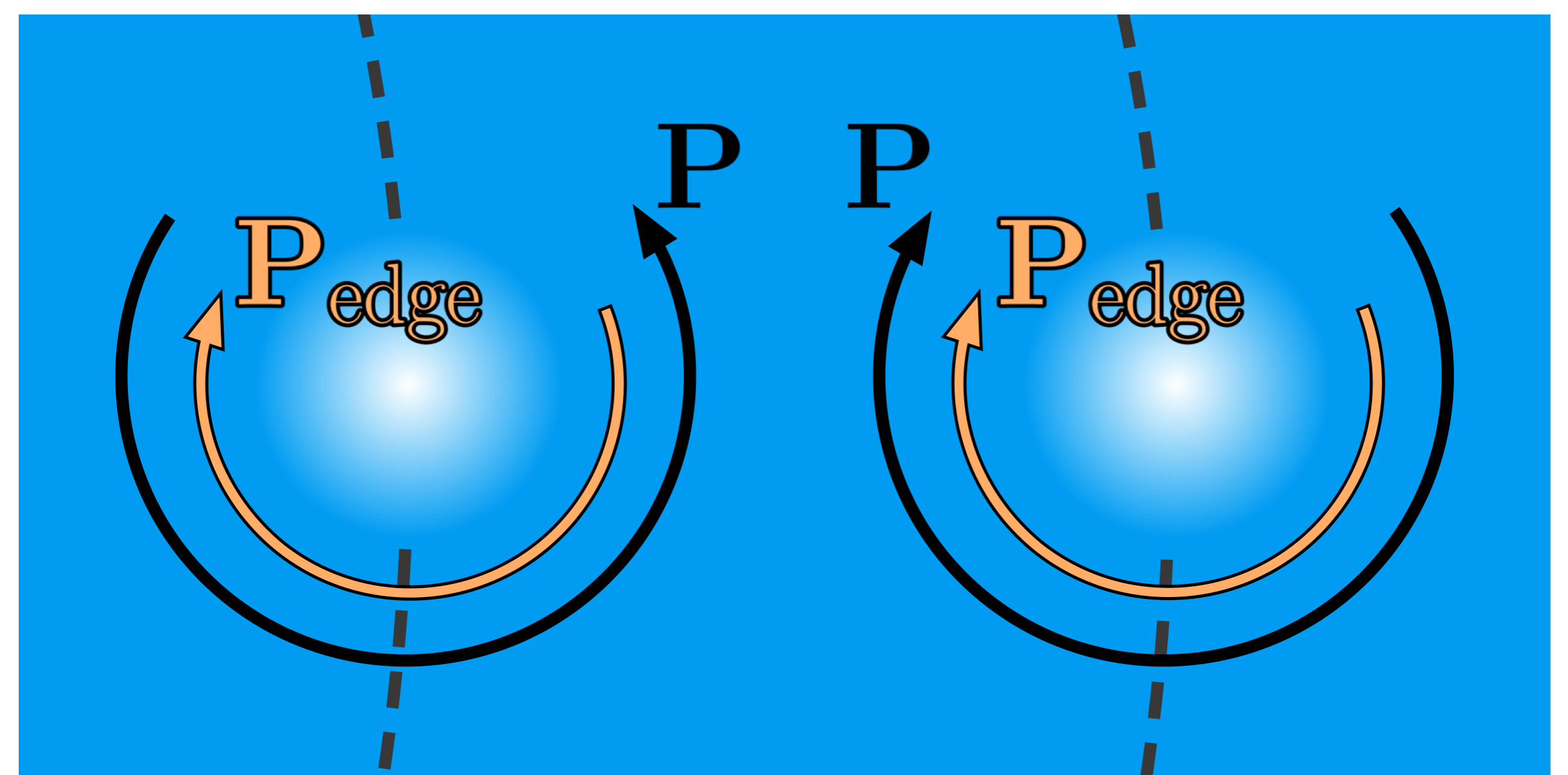


- E.g. if the microscopic particles are spinning gears; compression will cause them to interlock and turn their internal rotation into vorticity
- The equations of motion can be rearranged to express the *total* vorticity as:

$$\Omega_{\text{total}} = \nabla \times \frac{P}{\rho} = \Omega_{\text{phase}} + \omega_c - \nu_H \nabla^2 \log \rho$$

where P is the momentum current and ρ is the mass density

- This will enhance (suppress) the flow around positive (negative) vortices. In particular, this will cause a vortex dipole to rotate



Microscopic foundations

- The full theory, including coupling to geometry, can be derived via the coherent states approach
- The Hall viscosity can then be understood to result from restriction to a subspace of the full Hilbert space with a given angular momentum, l , per particle
- The particles are charged under $SO(2)$ rotations and require a coupling to the gauge field of rotations, i.e. the spin connection
- In a curved geometry this will give rise to the Wen Zee term, which specifies the parallel transport of particles with spin

$$S_{BH}[n, \theta, A, \omega, g] = \int dt d^2x \sqrt{g} \left[\hbar n \left(-\mathcal{D}_t \theta + \frac{\hbar}{2m} \mathcal{D}^i \theta g_{ij} \mathcal{D}^j \theta \right) + \frac{\ell}{2} n B + \frac{U}{2} n^2 + \frac{\hbar^2}{2m} (\partial^i \sqrt{n}) g_{ij} (\partial^j \sqrt{n}) \right],$$

$$\mathcal{D}^\mu \theta = \partial^\mu \theta - A^\mu - \ell \omega^\mu$$