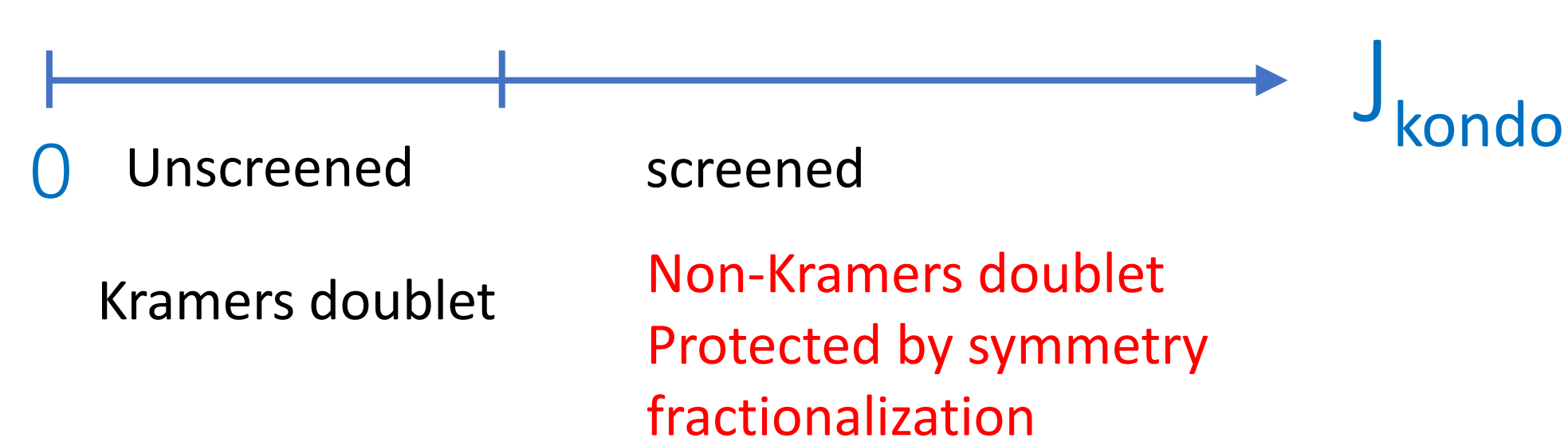


### Introduction

- Quantum spin liquids (QSLs) have exotic properties that go beyond the Landau scheme of symmetry breaking, and they are of great interest due to the presence of anyons that obey fractional statistics.
- Experimental detection of GAPPED QSLs is challenging due to the lack of charge and gapless modes.
- We propose using the Kondo effect to detect the symmetry fractionalization in gapped  $Z_2$  QSLs.
- the Kondo screening phase will feature a non-Kramers doublet localized at the impurity site, protected by fractionalized crystalline symmetries in the  $Z_2$  QSL.

### Theory

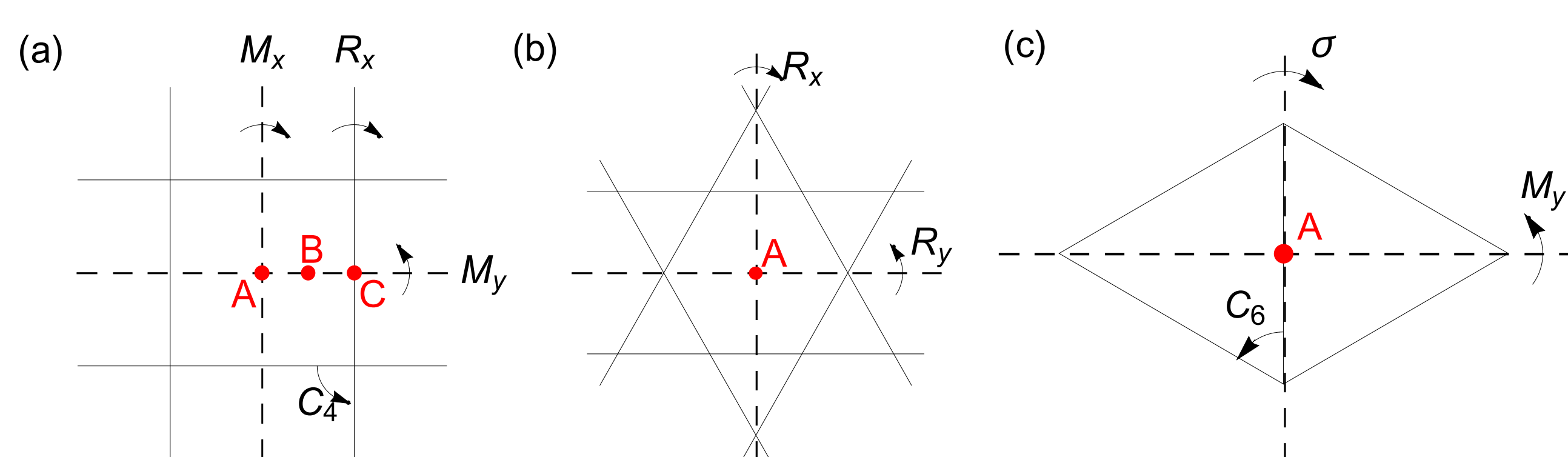
- Kondo screening



- Symmetry fractionalization

- A, C**:  $M_x M_y M_x^{-1} M_y^{-1} = -1$
- B**:  $(M_x \mathcal{T})^2 = 1$
- Does not have trivial representation and has local degeneracy

### Results



- High-symmetry impurity sites** that can be used to detect the symmetry fractionalization of spinons in a gapped symmetric  $Z_2$  spin liquid.

(a)	Algebraic identity	$\omega \in \mathcal{H}^2(G, \mathcal{A})$	$\omega^e$	$\omega^\epsilon$	Impurity site
	$(R_x)^2$	$\omega_{R_x, R_x}$	$(-1)^{p_4}$	$\eta_\sigma$	-
	$(M_y)^2$	$\omega_{M_y, M_y}$	$(-1)^{p_3+p_4}$	$\eta_\sigma \eta_{xpx}$	-
	$(C_4 R_x)^2$	$\omega_{C_4 R_x, C_4 R_x}$	$(-1)^{p_4+p_7}$	$\eta_\sigma \eta_{\sigma C_4}$	-
	$M_x M_y M_x^{-1} M_y^{-1}$	$\frac{\omega_{M_x, M_y}}{\omega_{M_y, M_x}}$	$(-1)^{p_1}$	$\eta_{xy}$	A
	$(M_y \mathcal{T})^2$	$\omega_{M_y \mathcal{T}, M_y \mathcal{T}}$	$(-1)^{p_3+p_8+1}$	$-\eta_t \eta_{xpx}$	B
	$R_x M_y R_x^{-1} M_y^{-1}$	$\frac{\omega_{R_x, M_y}}{\omega_{M_y, R_x}}$	$(-1)^{p_2}$	$\eta_{xpy}$	C

(b)	Algebraic identity	$\omega \in \mathcal{H}^2(G, \mathcal{A})$	$\omega^e$	$\omega^\epsilon$	Impurity site
	$(R_x)^2$	$\omega_{R_x, R_x}$	$(-1)^{p_2+p_3}$	$\eta_\sigma$	-
	$(R_y)^2$	$\omega_{R_y, R_y}$	$(-1)^{p_2}$	$\eta_\sigma \eta_{\sigma C_6}$	-
	$R_x R_y R_x^{-1} R_y^{-1}$	$\frac{\omega_{R_x, R_y}}{\omega_{R_y, R_x}}$	$(-1)^{p_1}$	$\eta_{12}$	A

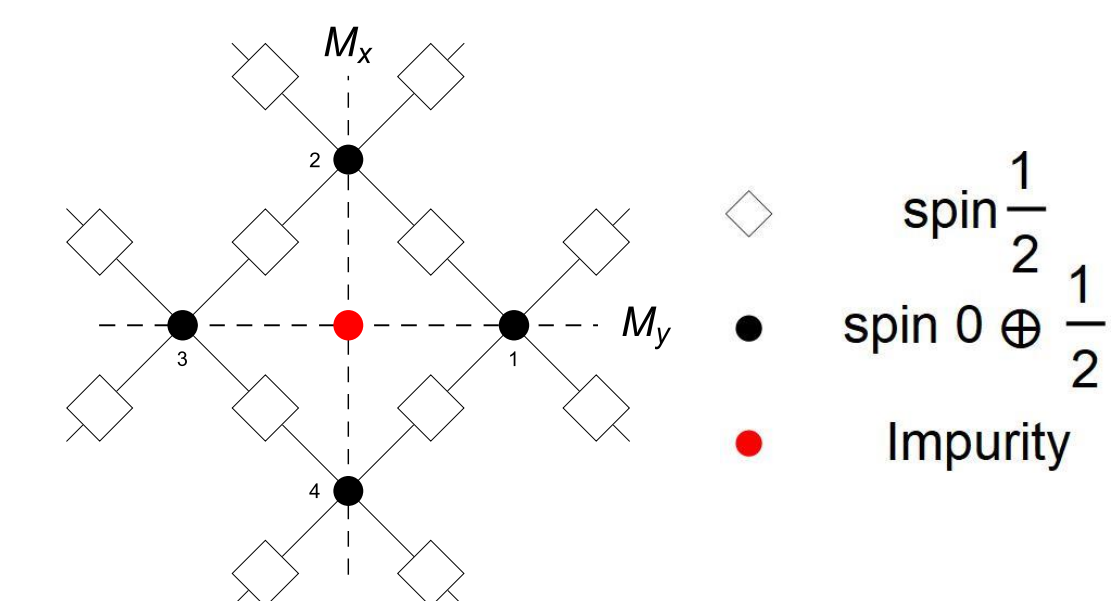
(c)	Algebraic identity	$\omega \in \mathcal{H}^2(G, \mathcal{A})$	$\omega^e$	$\omega^\epsilon$	Impurity site
	$\sigma^2$	$\omega_{\sigma, \sigma}$	$(-1)^{p_2}$	$\eta_\sigma$	-
	$(M_y)^2$	$\omega_{M_y, M_y}$	$(-1)^{p_2+p_3}$	$\eta_\sigma C_6$	-
	$\sigma M_y \sigma^{-1} M_y^{-1}$	$\frac{\omega_{\sigma, M_y}}{\omega_{M_y, \sigma}}$	$(-1)^{p_1}$	$\eta_{12}$	A

- In the presence of  $SO(3)$  spin rotational symmetry, the classification of symmetric  $Z_2$  spin liquids on these lattices are summarized in TABLE above.
- A part of the symmetry fractionalization data can be detected by presence/absence of non-Kramers doublets for Kondo-screened magnetic impurities located at different high-symmetry sites.

### Method 1: Exactly solvable model

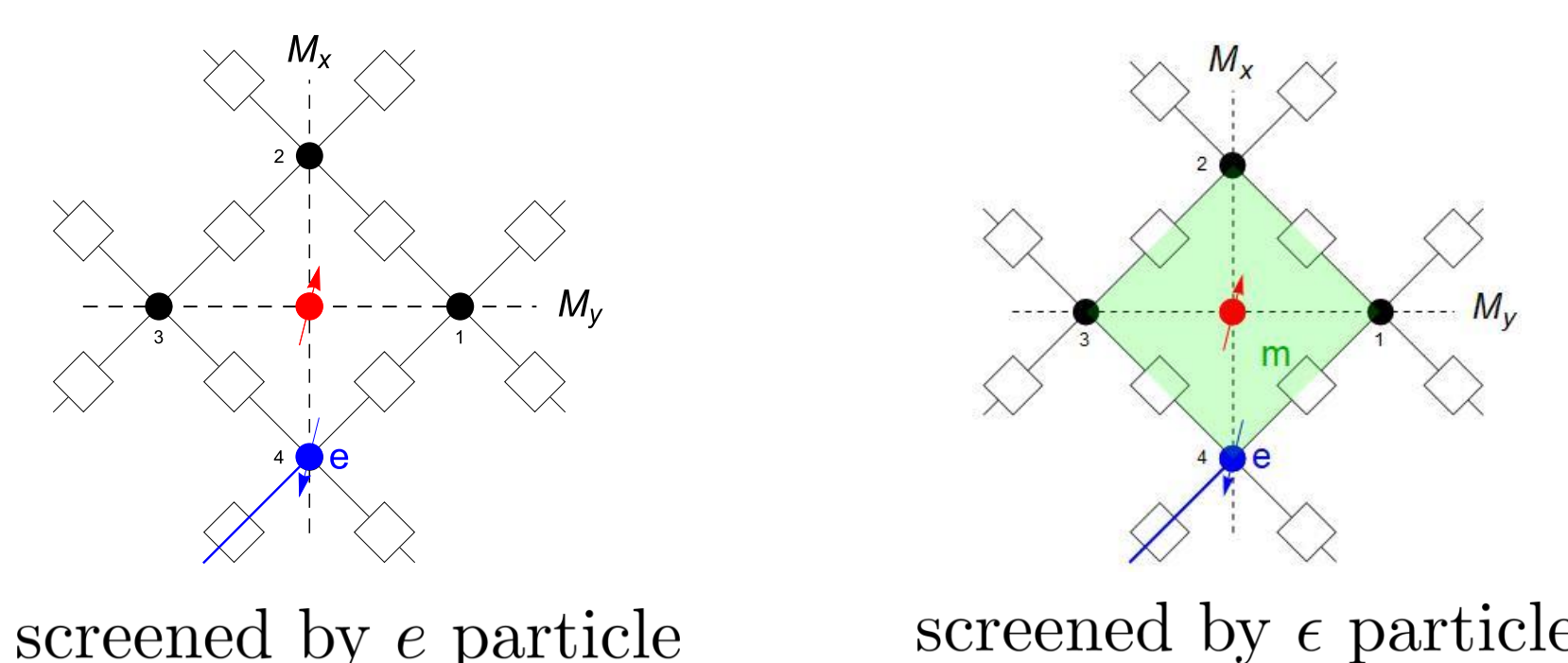
- Toric code model + spin-1/2 + impurity

$$\hat{H} = - \sum_s A_s - \sum_p B_p - \sum_s \Delta \left( \frac{1}{2} (A_s + 1) P_s (S=0) + \frac{1}{2} (1 - A_s) P_s (S=1/2) \right) + J \sum_{\langle i, imp \rangle} \vec{S}_i \cdot \vec{S}_{imp} + \dots$$



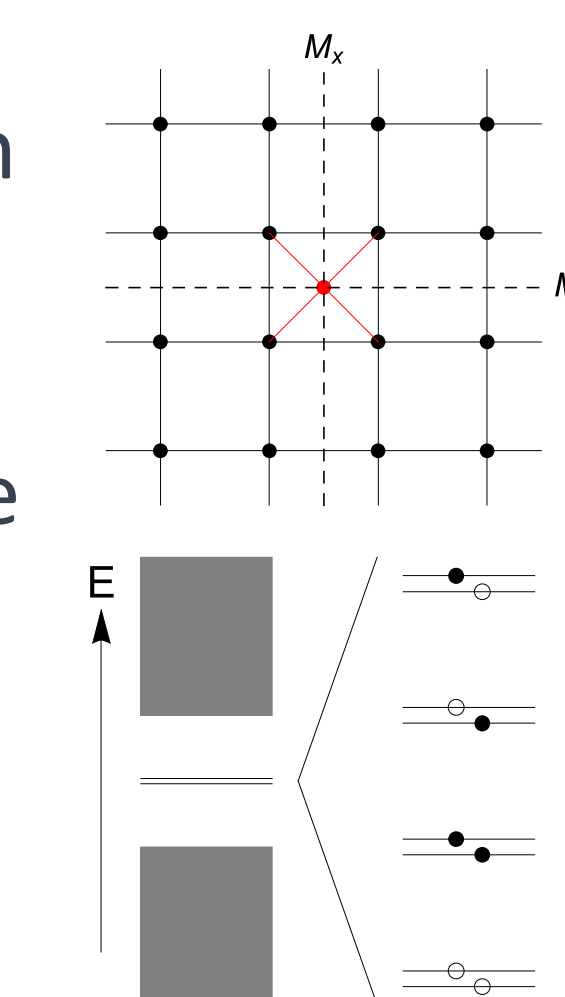
- The  $SU(2)$  symmetry is implemented in the Hilbert space of  $\text{spin } 0 \oplus 1/2$  on each vertex.
- The model shares the same ground state as toric code where all vertex spins are in the spin-0 state.
- The low energy excitations are spin-0  $m$  particles, and  $e, \epsilon$  particles each carry spin-1/2.

- Ground state is unique if impurity is screened by  $e$  ( $M_x M_y M_x^{-1} M_y^{-1} = 1$ )
- Ground state is degenerate if screened by  $\epsilon$  ( $M_x M_y M_x^{-1} M_y^{-1} = -1$ )

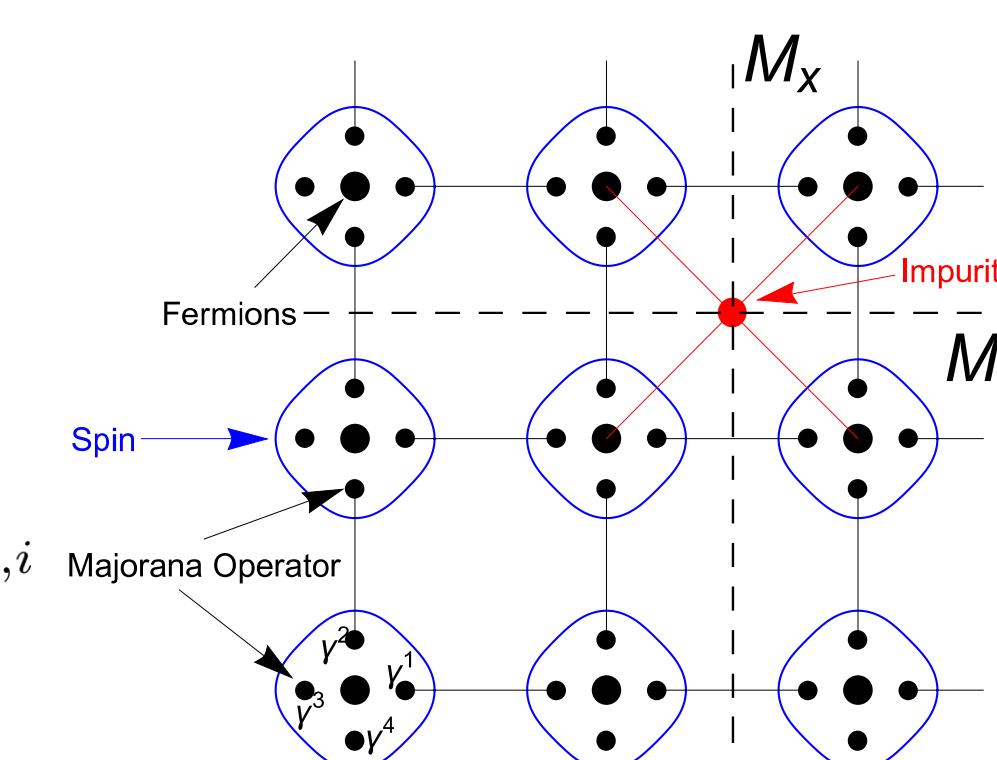


### Method 2: large-N mean field theory

- The parton construction of quantum spin liquids provides another aspect to look at the Kondo effect in  $Z_2$  spin liquids.
- In the case of a nontrivial fractionalization class with  $M_x M_y M_x^{-1} M_y^{-1} = (-1)^{N_F}$  each energy level in the spectrum of the parton BdG Hamiltonian must be at least 2-fold degenerate
- particle-hole symmetric spectrum must have 2 zero modes
- the both filled and the both empty states are physical degenerate ground state

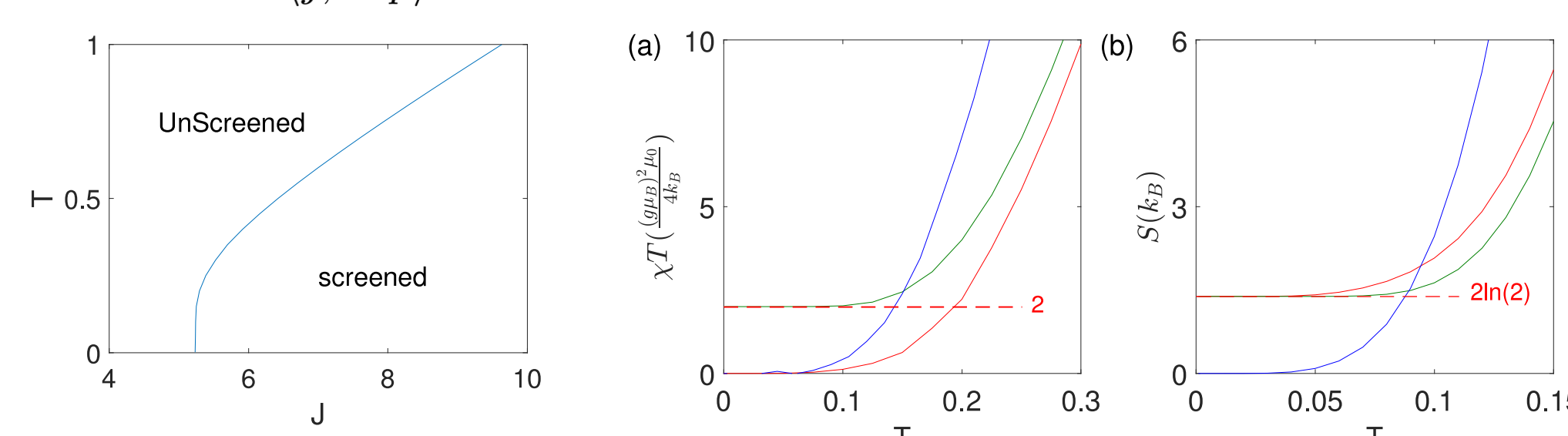


$$H_{bulk} = t \sum_{\langle i, j \rangle} \hat{u}_{i, j} (i c_{i\uparrow}^\dagger c_{j\uparrow}^a + i c_{i\downarrow}^\dagger c_{j\downarrow}^a + h.c.) + t' \sum_{j=i \pm 2\hat{x} + 2\hat{y}} \hat{u}_{i, j} (c_{i\uparrow}^\dagger c_{j\uparrow}^a + c_{i\downarrow}^\dagger c_{j\downarrow}^a + h.c.) + \Delta \sum_i c_{i\uparrow}^\dagger c_{i\downarrow}^a + g \sum_i \hat{u}_{i, i+\hat{x}} \hat{u}_{i+\hat{x}, i+\hat{x}+\hat{y}} \hat{u}_{i+\hat{x}+\hat{y}, i+\hat{y}} \hat{u}_{i+\hat{y}, i}$$



- $a$  labels  $N$  flavors. Large  $N$  limit is solvable.
- $u_{i, j} = i \gamma_i^{\alpha(i, j)} \gamma_j^{\beta(i, j)}$  is a conserved quantity and gives us nontrivial PSG
- Interaction between impurity and bulk is in form of  $sp(N)$  spin

$$H_{imp} = \sum_{\langle j, imp \rangle} \frac{J}{N} \mathbf{S}_j^{ab} \cdot \mathbf{S}_{imp}^{ba} + \frac{J'}{N^3} (\mathbf{S}_j^{ab} \cdot \mathbf{S}_{imp}^{ba})^2$$



- thermodynamic quantities from the mean-field calculation to illustrate the experimental implications of the anomalous Kondo effect.