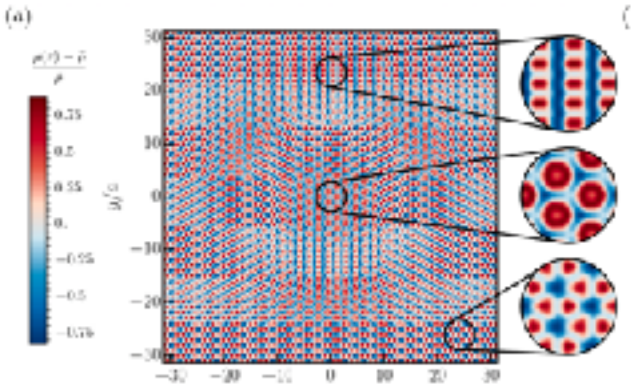
From Abelian anyons in moiré matter to non-Abelions in synthetic quantum systems



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Acknowledgement

Patrick Ledwith

Dan Parker

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Eslam Khalaf

"Vortexability"
Ledwith, AV, Parker (arXiv:2209.15023)
Ledwith, Tarnopolsky, Khalaf, AV (PRR 2020)

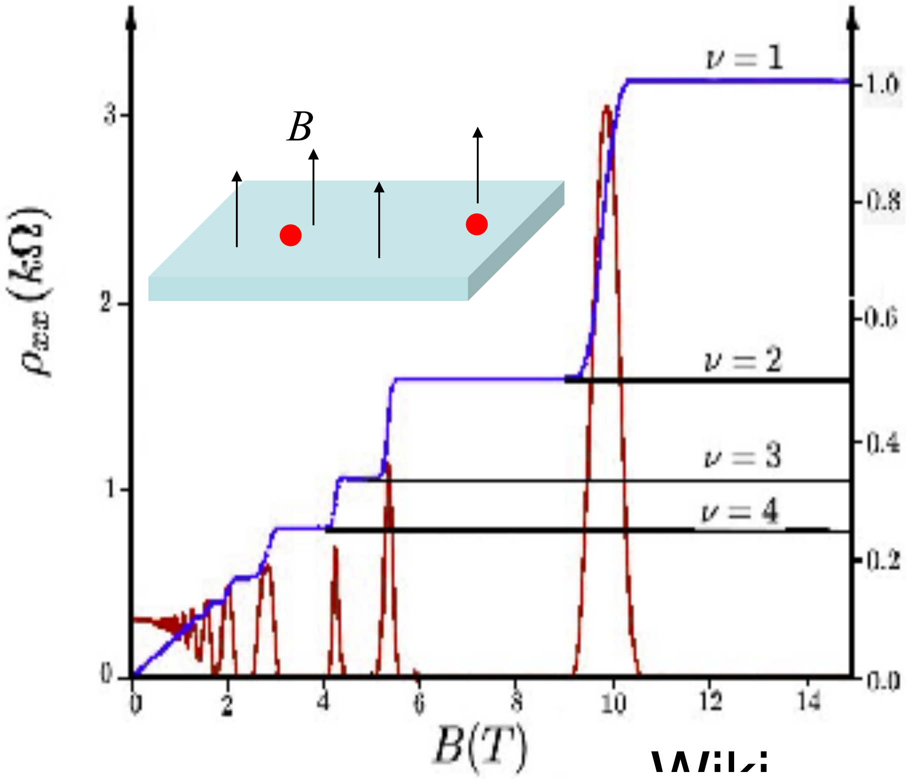
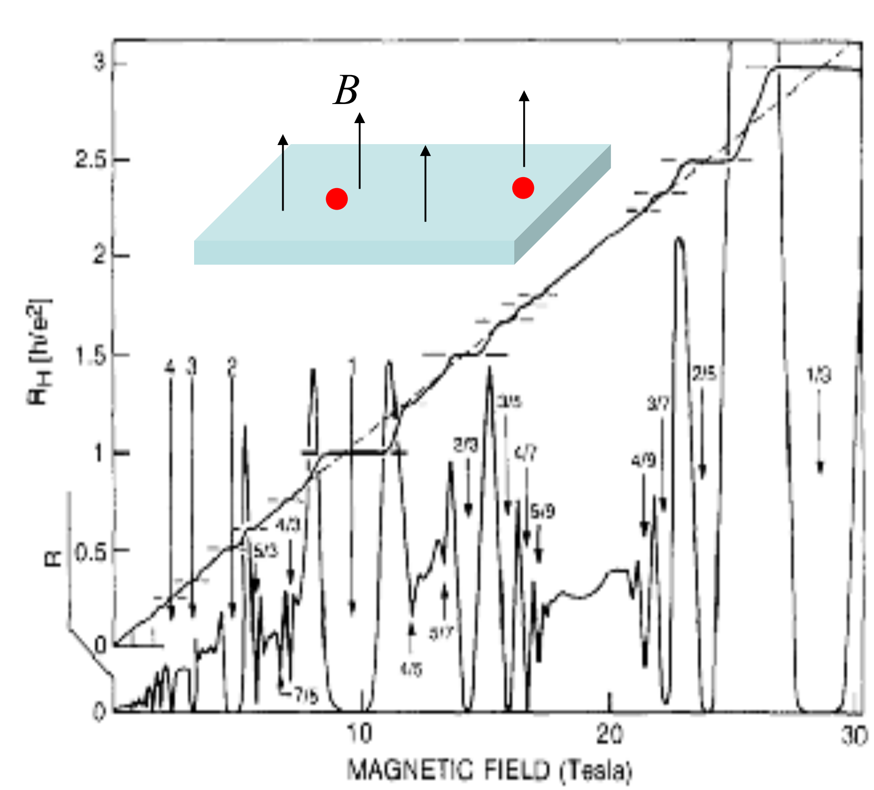
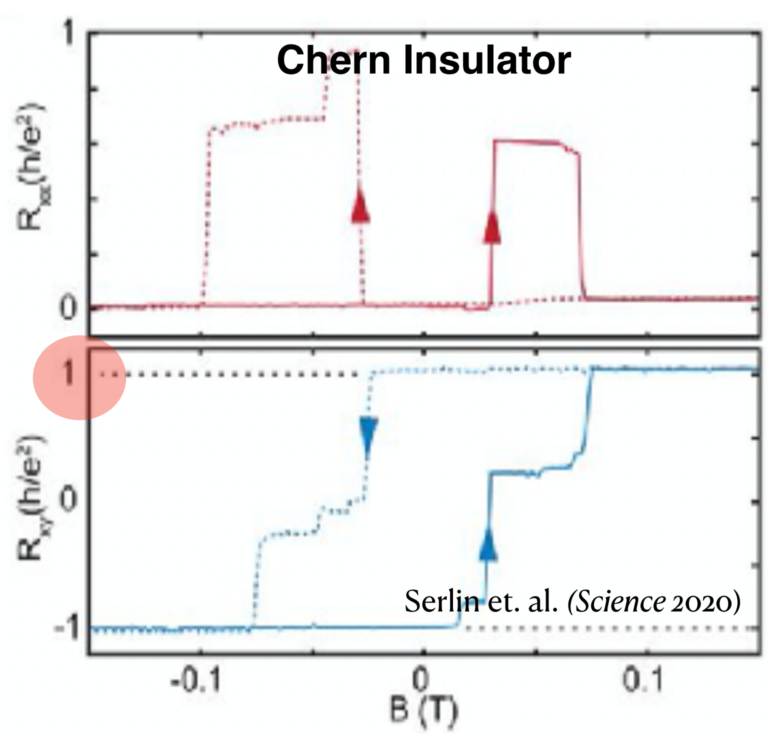
"C > 1"
Ledwith, AV, Khalaf (PRL 2022)
Dong, Ledwith, Khalaf, Lee, AV (arXiv:2210.13477)

+Theory: G. Tarnopolsky, J. Y. Lee

DMRG Collaboration: Tomo Soejima, Johannes Hauschild, Mike Zaletel

Experimental Collaboration: Yacoby and Jarillo-Herrero Groups

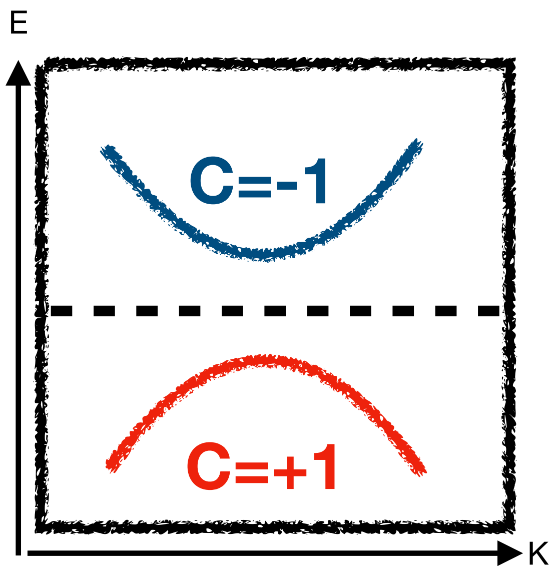
Fractional Chern Insulators in Moire' Materials

	Integer Charge	Fractional Charge
B-Field Landau level		
Band Topology/ Geometry		

Magic angle graphene (+ aligned hBN)

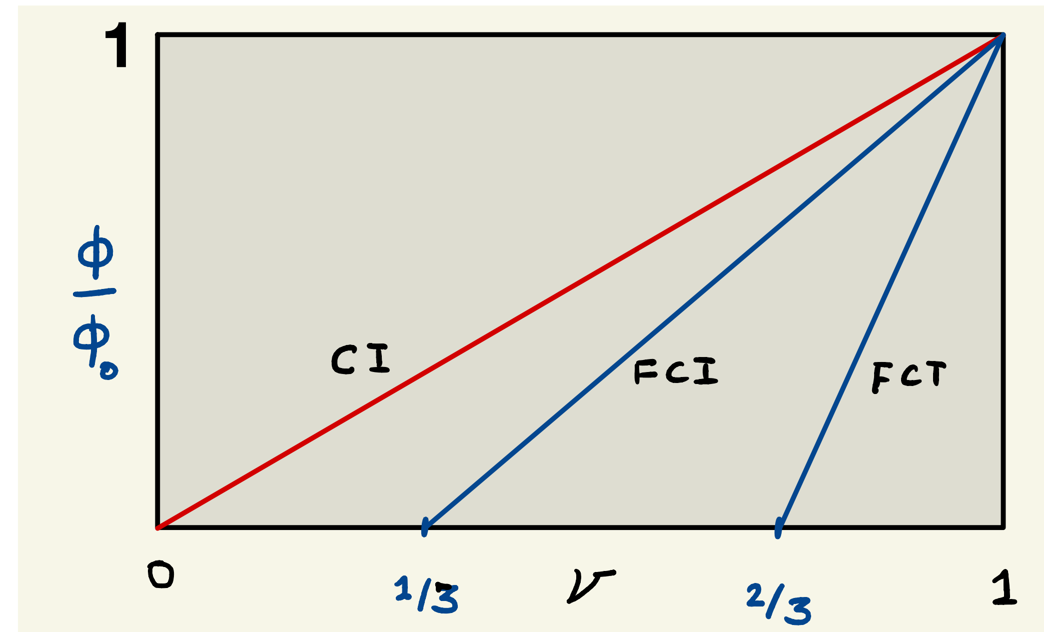
Sheng et al, \`11. Neupert et al, \`11. Tang et al, '11. Regnault & Bernevig \`11. X. Qi \`11.

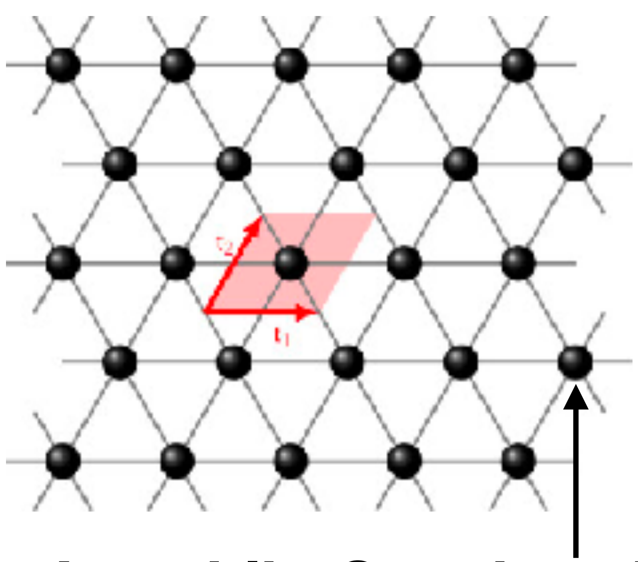
FCIs- Symmetry Enriched Topological Order



Streda Formula
Incompressible State

$$\nu = t\frac{\phi}{\phi_0} + s$$





Laughlin Quasiparticle attached to each lattice site

$$(t,s) = (\frac{2}{3},\frac{1}{3})\text{ and }(\frac{1}{3},\frac{2}{3}) \implies \textbf{FCI}$$

$$(t,s) = (\frac{1}{3},0) \implies \textbf{FQHE}$$

Translation symmetry *enriched* Topological Order

Chern Band Geometry and FCIs

Chern insulators - band topology dictates the state, But

Realizing FCIs by *partially* filling Chern band. *Wavefunctions important!*

Find theoretically well motivated starting points + extend with numerics

Historical Approach: *mimic* Landau levels by requiring:

Lowest Landau Level
(FQHE)

- Zero Dispersion
- Uniform Berry Curvature

Crucial Ingredient?

N =4 Landau Level
(Stripe/Bubble Phase*)

- Zero Dispersion
- Uniform Berry Curvature

*Coulomb or short ranged interactions

Roy (`14), Parameswaran, Roy, Sondhi (`13),

Chern Band Geometry and FCIs

Trace Condition (LLL)

$$|\mathrm{Tr}[g(k)]| = |\Omega(k)|$$

Quantum Metric Berry Curvature

Historically: Uniform Berry curvature + Trace condition

=> $g=g_{LLL}$ AND $\Omega = \Omega_{LLL}$

Mimics the LLL -> GMP Algebra.

BUT hard to achieve

Here: Isolate Trace condition.

Easier to realize.

FCIs + physics beyond LLL

Physical meaning of Trace condition?

Vortexable Bands

Roy (`14), Parameswaran, Roy, Sondhi (`13), Claassen et al (`15), Jackson et al. (`15), Varjas et al. (`21). Mera & Ozawa (`21).Ledwith, Tarnopolsky, Khalaf, AV `20.

Chern Band Geometry and FCIs

Here: Isolate Trace condition.
Easier to realize.
FCIs + physics beyond LLL

Physical meaning of Trace condition?
Vortexable Bands

- What does it imply? Why is it important?
- Can we find bands that naturally satisfy it?
- Experimental consequence?
- Extend to `beyond' Landau level physics ?

Ledwith, Tarnopolsky, Khalaf, AV `20. Jie Wang, Cano, Millis, Liu, Yang `21. Ledwith, AV, Parker (arXiv:2209.15023)

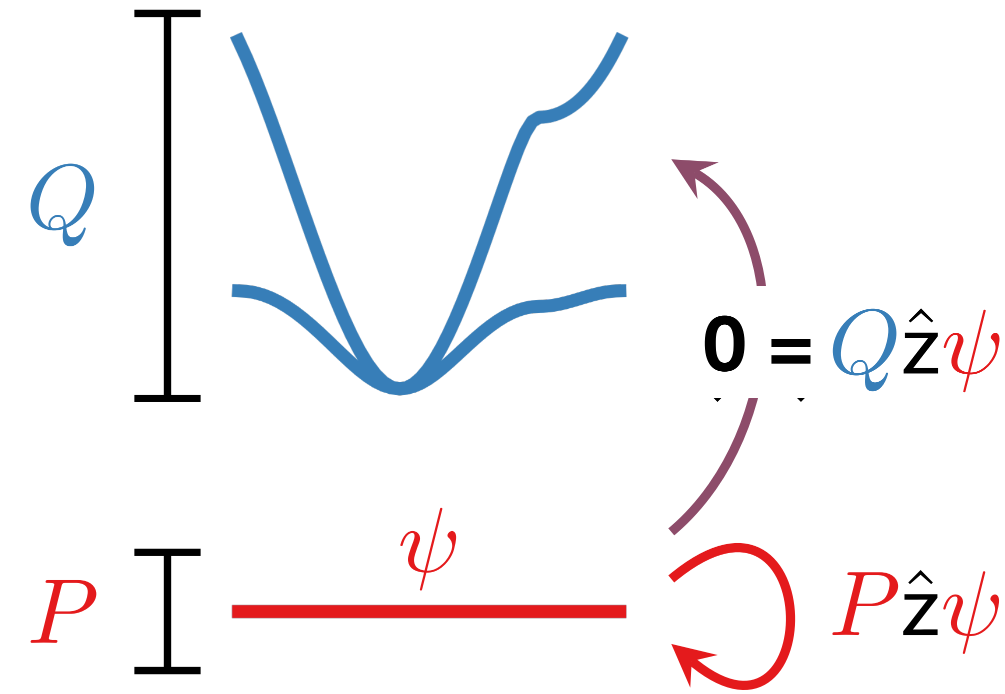
Vortexable Bands

$$\psi(x,y) = \sum a_k \psi_k(x,y)$$

$\psi \in$ **Vortexable Chern Band**

$\Longrightarrow z\,\psi \in$ **Vortexable Chern Band**

where $\mathbf{z} = x + iy$



"Vortexable band" since $z\,|\psi\rangle$ attaches a vortex without leaving the band.

Equivalently, define the band projector:

$$\hat{P} = \sum_k |\psi_k\rangle\langle\psi_k| \quad \textbf{and } \hat{Q} = 1 - \hat{P}$$

$$\boxed{z\,|\psi\rangle = \hat{P}\,z\,|\psi\rangle} \text{ for all } |\psi\rangle = P\,|\psi\rangle \qquad \boxed{\text{or } Qz\,|\psi\rangle = 0}$$

Ledwith, AV, Parker (arXiv:2209.15023)

Vortexability & FCIs

Consequences of Vortexability:

$$Qz\psi(x,y) = 0 \implies Qz^n \psi(x,y) = 0 \implies Qf(z)\,\psi(x,y) = 0$$

Many Body Wave-function: $Q\,f(z_1, z_2 \ldots z_n)\,\psi(z_1, z_2, \ldots z_n) = 0$

Write FCI Laughlin wave function *entirely within* vortexable band.

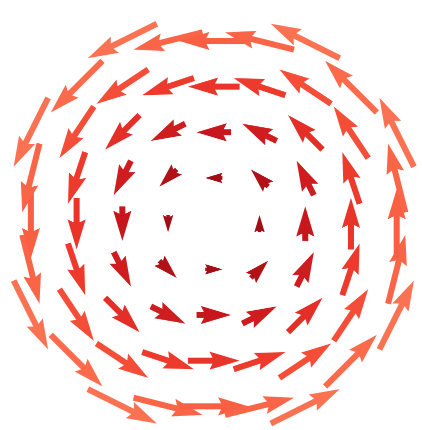
$$|\Psi^{\mathrm{FCI}}_{\nu=\frac{1}{2s+1}}\rangle = \prod_{i<j} (z_i - z_j)^{2s}\,|\,\text{Filled } \nu{=}1\rangle$$

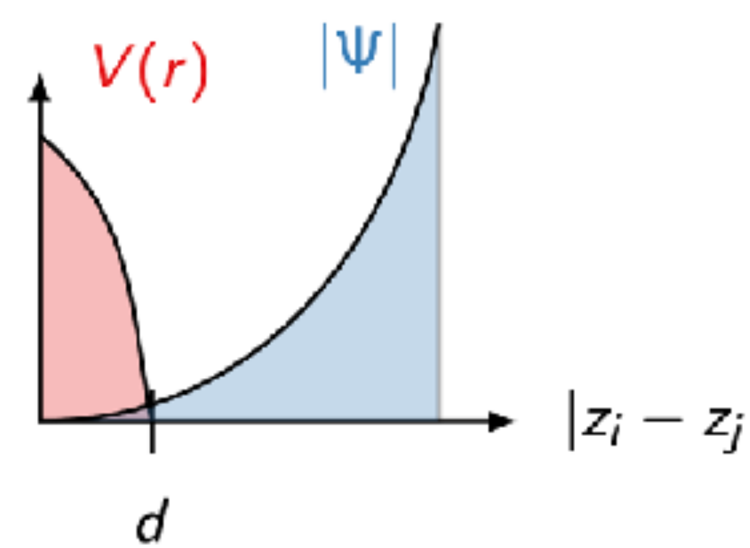
Exact ground state for sufficiently short ranged repulsive interactions:

Eg: $\nu = 1/3$;

Interaction:

$V(r_1 - r_2) = V_1 l^2 \nabla^2 \delta(r_1 - r_2)$





Cf. Trugman-Kivelson argument for Laughlin State in LLL.

Momentum Space Consequences of Vortexability

Vortexability

$$z\psi = Pz\psi$$

Trace Condition

$$\mathrm{Tr}\, g(\mathbf{k}) = \boldsymbol{\Omega}(\mathbf{k})$$

K-space Holomorphicity

$$|u_{\mathbf{k}}\rangle = |u_{k_x+ik_y}\rangle$$

- Holomorphic in `k': $|\psi_{\mathbf{k}}\rangle = e^{i\mathbf{k}\cdot\mathbf{r}}|u_{\mathbf{k}}\rangle$

$$\frac{\partial}{\partial \bar{k}}|u_{k_x+ik_y}\rangle = 0 \implies Q\,z\,\psi_k = 0$$

$$u_{k=k_x+ik_y}(x,y) = \exp\left(-\frac{i}{2}k\bar{z}\right)\frac{\sigma(z+il^2k)}{\sigma(z)}\psi_\Gamma(\mathbf{r})$$

Eg. Chiral MATBG Wfns.

Vortexability

- [x] What does it imply? Why is it important?
- [ ] Can we find bands that naturally satisfy it?
- [ ] Experimental consequence?
- [ ] Extend to `beyond' Landau level physics ?

Realizations of Vortexable Bands

PRINCIPLE:

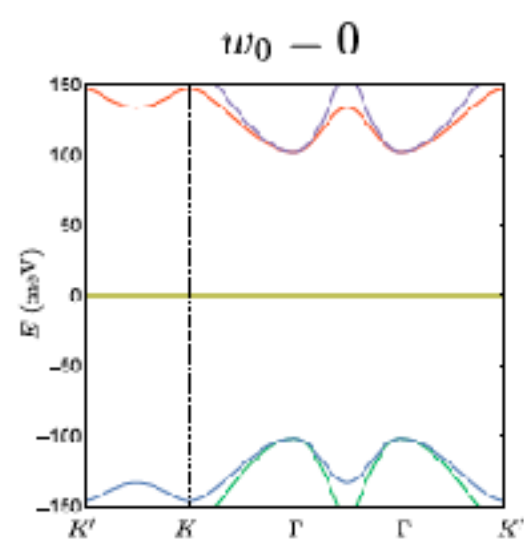
Flat band from zero mode: $D_A \psi_k = 0$ Where: $D_A = i\frac{\partial}{\partial \bar{z}} \otimes \mathbf{1} - \hat{A}$

Automatically Vortexable: $D_A \psi = 0 \implies D_A z \psi = 0$

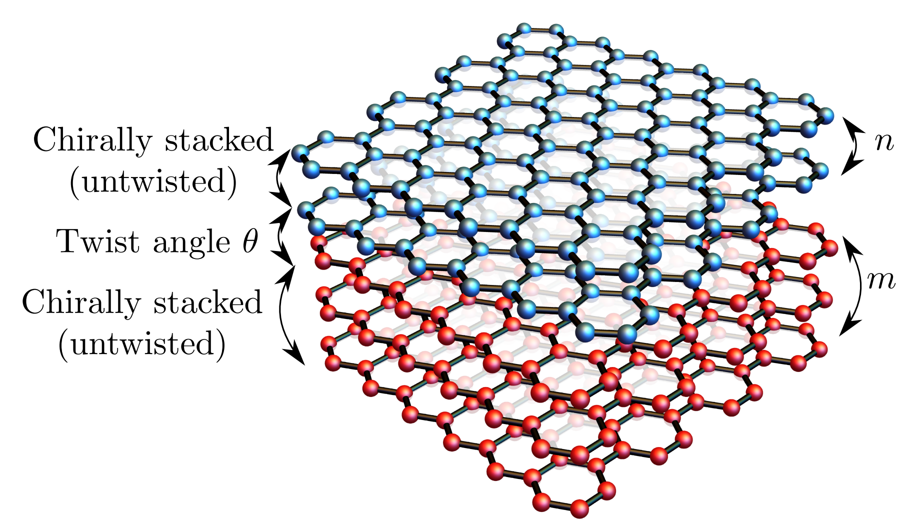
Examples: Lowest Landau level: $H = (p - eA) \cdot \sigma$

- Magic angle twisted bilayer graphene in the *chiral limit*.

$$\hat{A} = \begin{pmatrix} 0 & w_1 U_1(\boldsymbol{r}) \\ w_1 U_1(-\boldsymbol{r}) & 0 \end{pmatrix}$$



- Chirally stacked multilayer graphene (*magic angle* + *chiral limit*). Gives C>1!



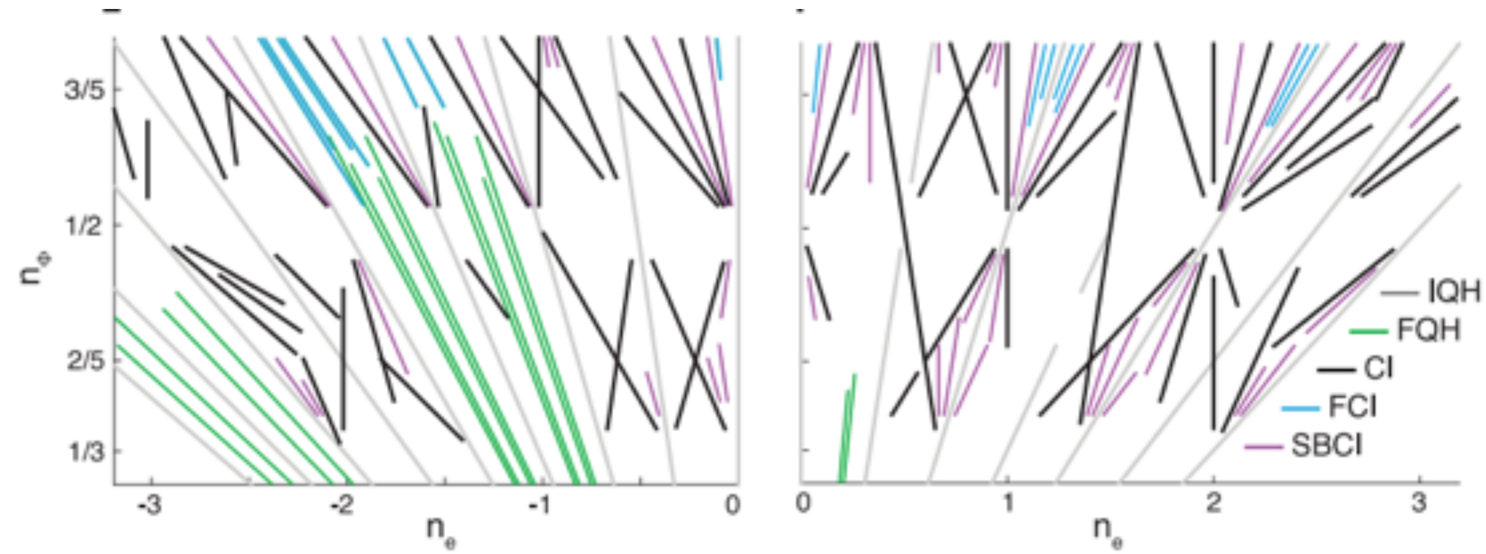
Vortexability

- ☑ What does it imply? Why is it important?
- ☑ Can we find bands that naturally satisfy it?
- Experimental consequence?
- Extend to `beyond' Landau level physics ?

FCI in Experiment

1. In Hofstadter Bands
High Field FCI (~25 Tesla)

where B-field creates Chern bands



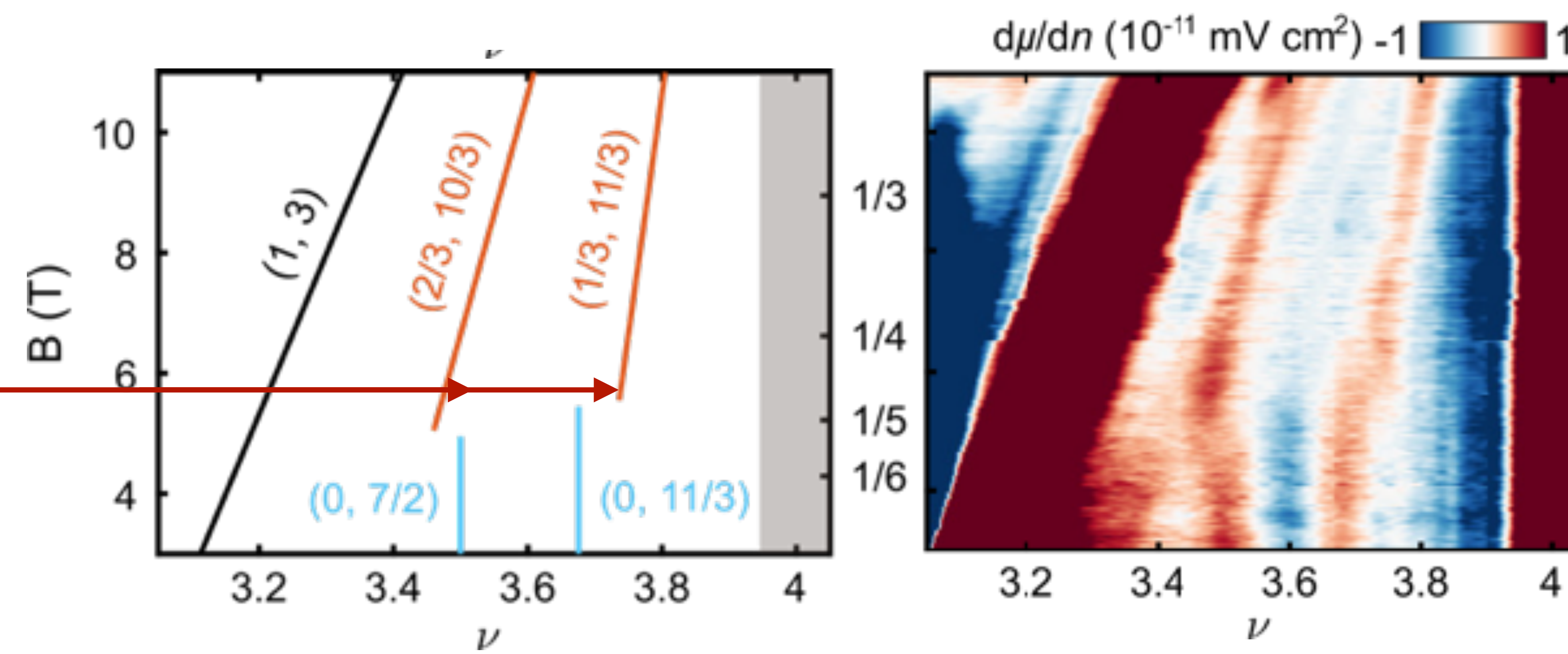
Spanton..A. Young. Science (2018)

2. Magic angle graphene + aligned hBN substrate:

Intrinsic zero-field Chern bands
{D. Goldhaber-Gordon; A. Young}
$[\nu = 3;\ C = 1]$

Compressibility measurements show for $B \gtrsim 5\,\mathrm{T}$

Fractional Chern Insulators!



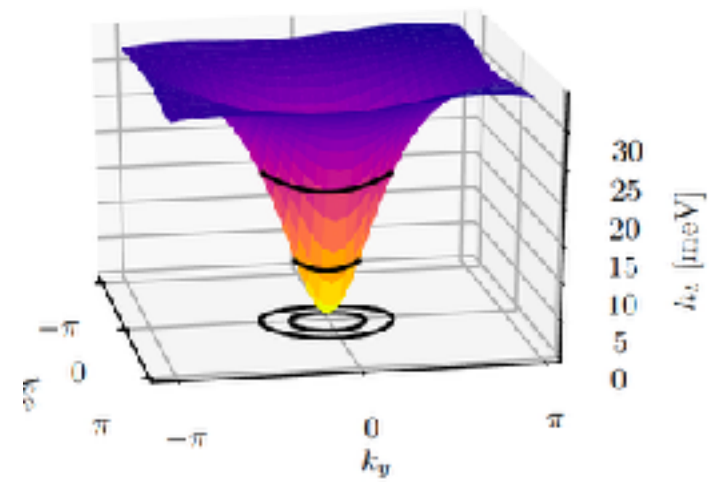
Xie,… Jarillo-Herrero, Yacoby Nature (Nature, 2021)

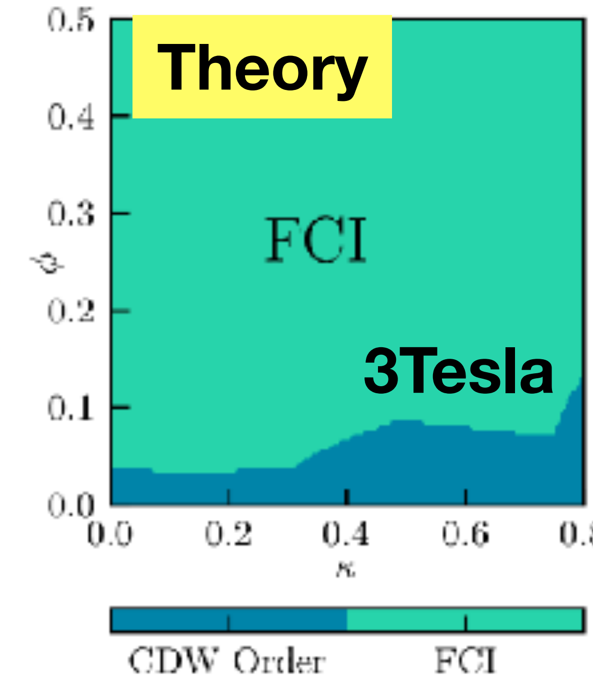
Theory/Numerics: Ledwith, Tarnopolsky, Khalaf, AV \`20. . Repellin, Senthil \`20 Abouelkomsan, Liu, Bergholtz \`20 Wilhelm, Lang, Läuchli

FCI in Magic Angle Graphene

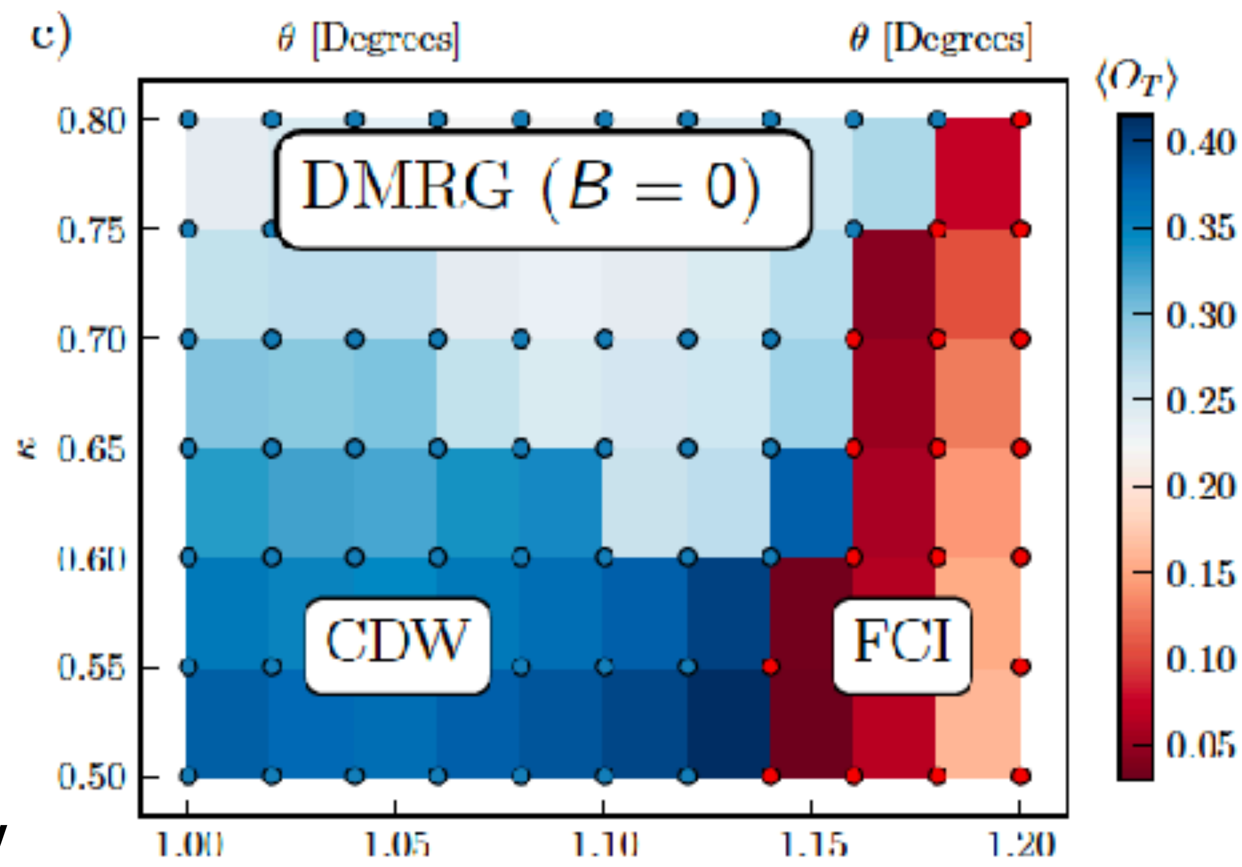
- Small magnetic fields *improves* band geometry and dramatically *reduces* bandwidth.

arXiv: 2112.13837
Parker *et al*, 2021





- Zero field FCI?
  - DMRG Numerics points to slightly higher angles.



Parker, Ledwith, Khalaf, Soejima, Hauschild, Xie, Pierce, Zaletel, Yacoby, **AV**

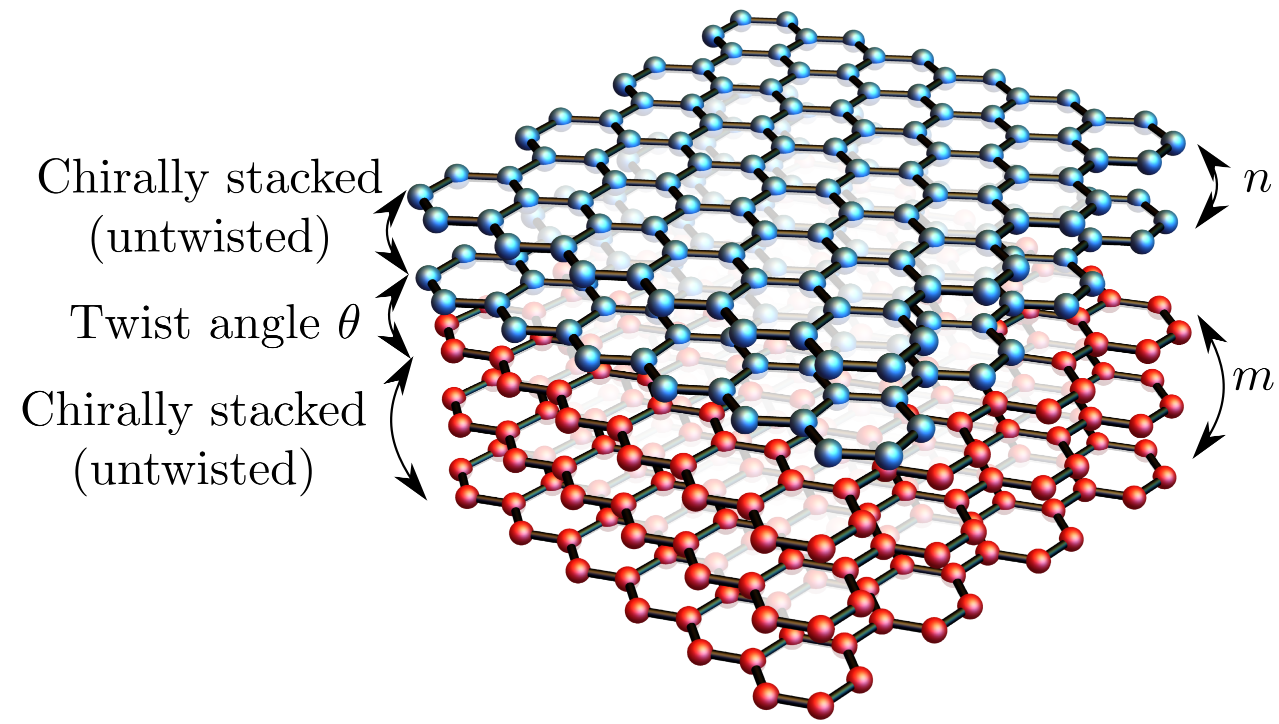
Vortexability

- ☑ What does it imply? Why is it important?
- ☑ Can we find bands that naturally satisfy it?
- ☑ Experimental consequence?
- Extend to `beyond' Landau level physics ?

Higher Chern numbers in Chiral Multilayers

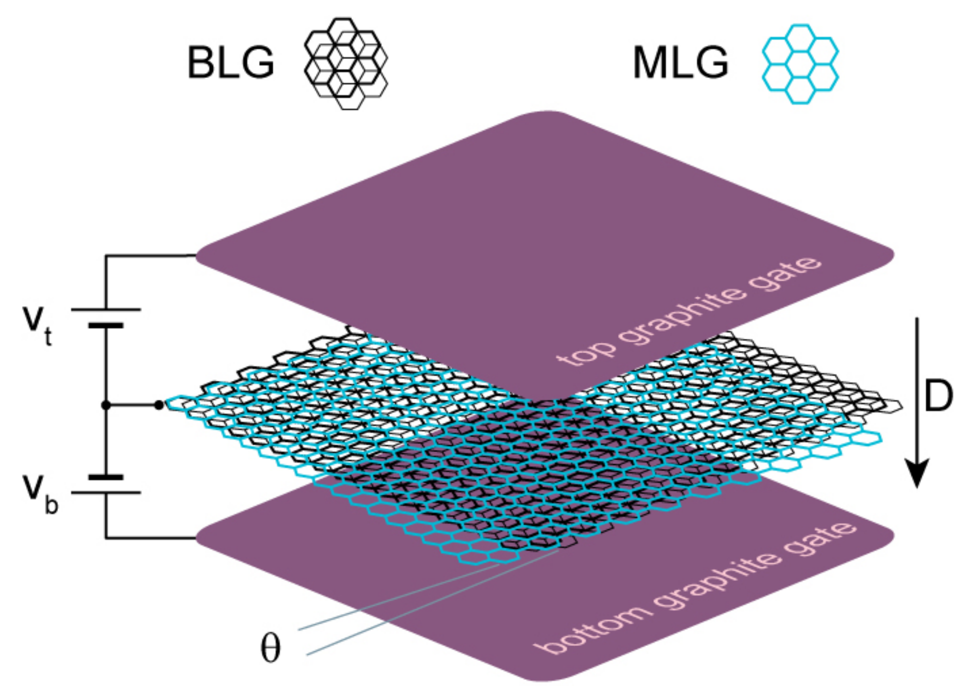
***Flat* & *vortexable* Chern bands**
Same magic angle as *chiral* TBG
Any Chern number!

(σ, σ')	Chern A	Chern B
$(+,+)$	n	$-m$
$(-,+)$	1	$-(n+m-1)$



Examples:

<u>Mono-Bi graphene:</u>
n=2; m=1
|C| = 2; 1



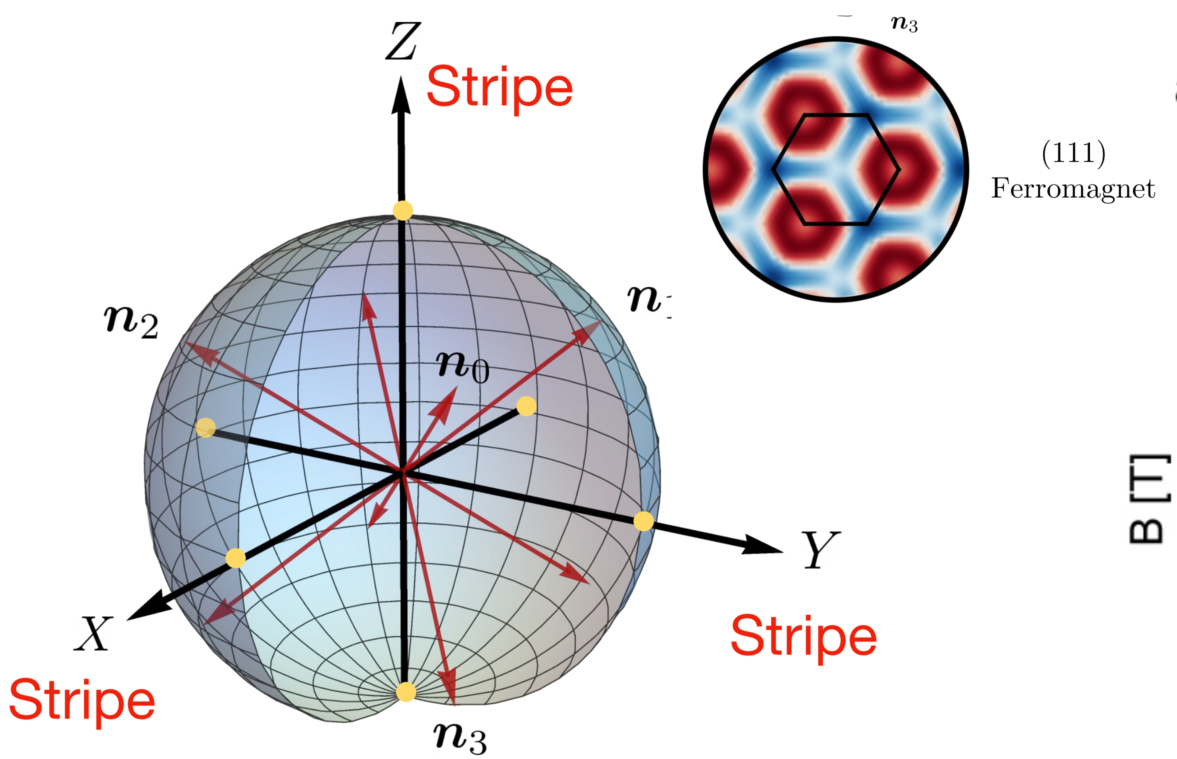
Ledwith, AV, Khalaf (PRL 2022)
Wang, Liu 2109:10325
Y. Zhang..Senthil (2018)

Experiment:
Polshyn, et al. (Nat. Phys. 2022)

Higher Chern Vortexable Bands

Theory: Chern C=2 at *Half* filling:

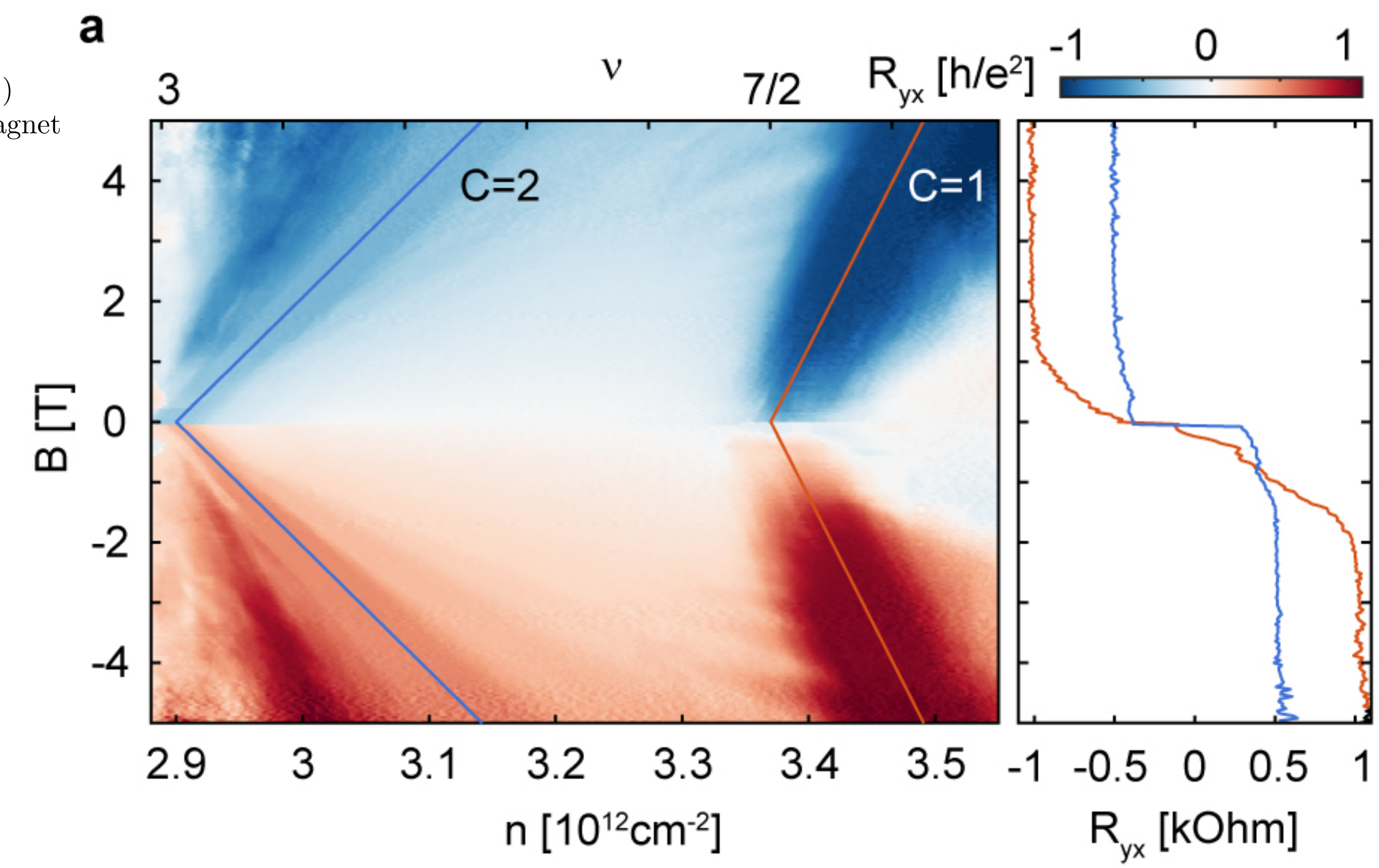
Generalized Ferromagnet (CDW) with C=1



(spinless)

Expt: Mono-Bilayer @ n=3
spin+valley polarized and C=2;

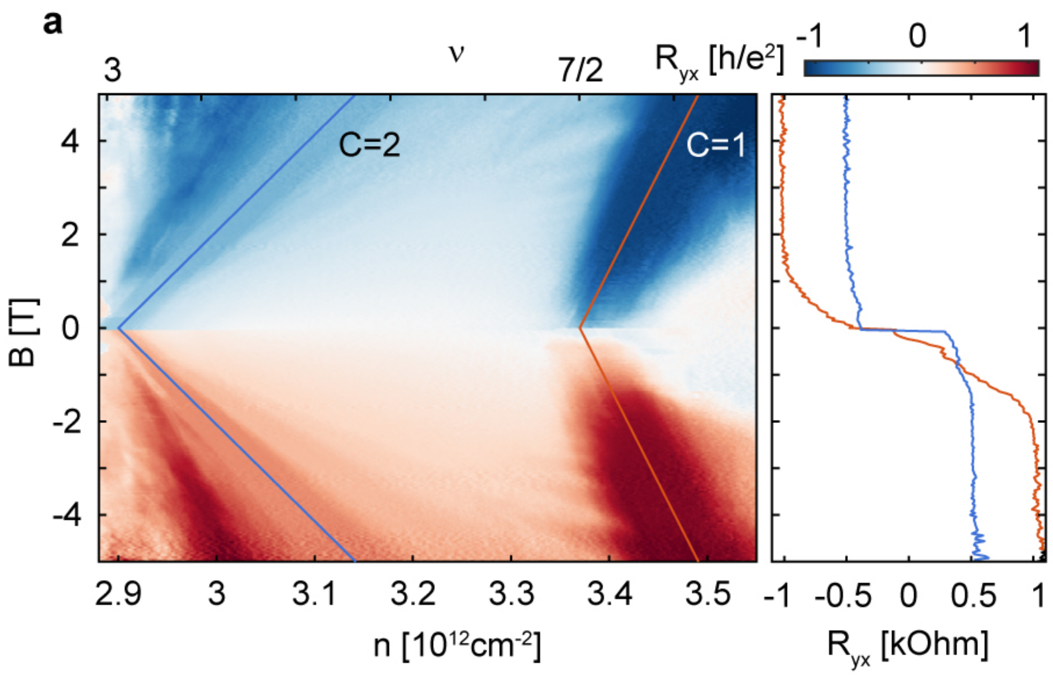
@ n= 3+1/2 Generalized Ferromagnet
(eg. CDW) with C=1



Dong, Ledwith, Khalaf, Lee, AV (2210.13477)
Wilhelm, Lang, Scheurer, Lauchli (2204.05317)
Wu, Regnault, Bernevig ’13; Kumar, Roy, Sondhi `14

Polshyn, et al. (Nat. Phys. 2022)

Higher Chern Vortexable Bands

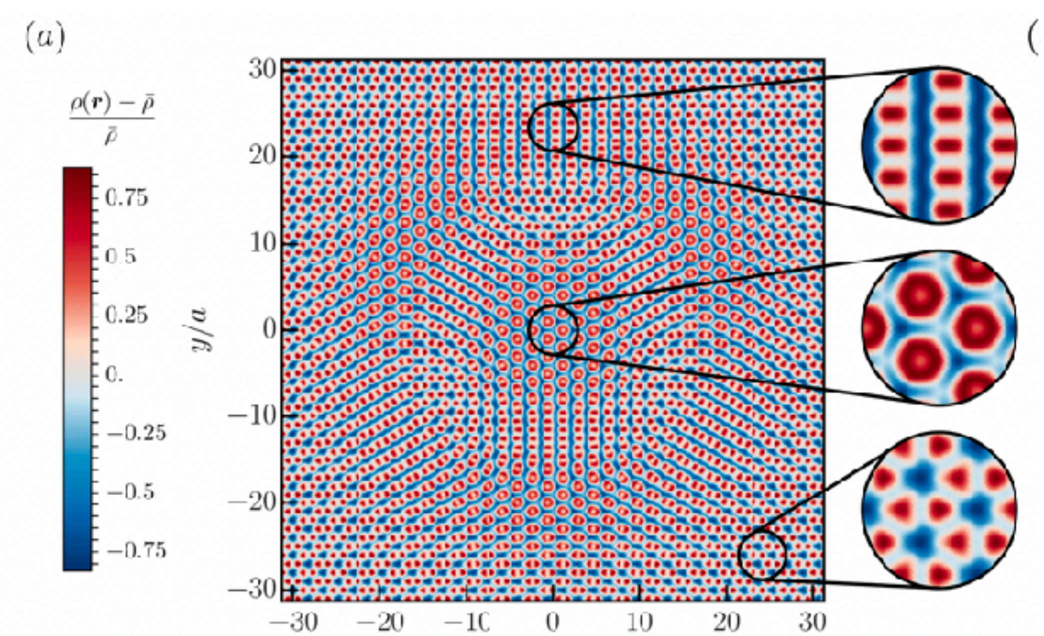


Mono-Bilayer at
n=3 (C=2); n= 3+1/2 (C=1)

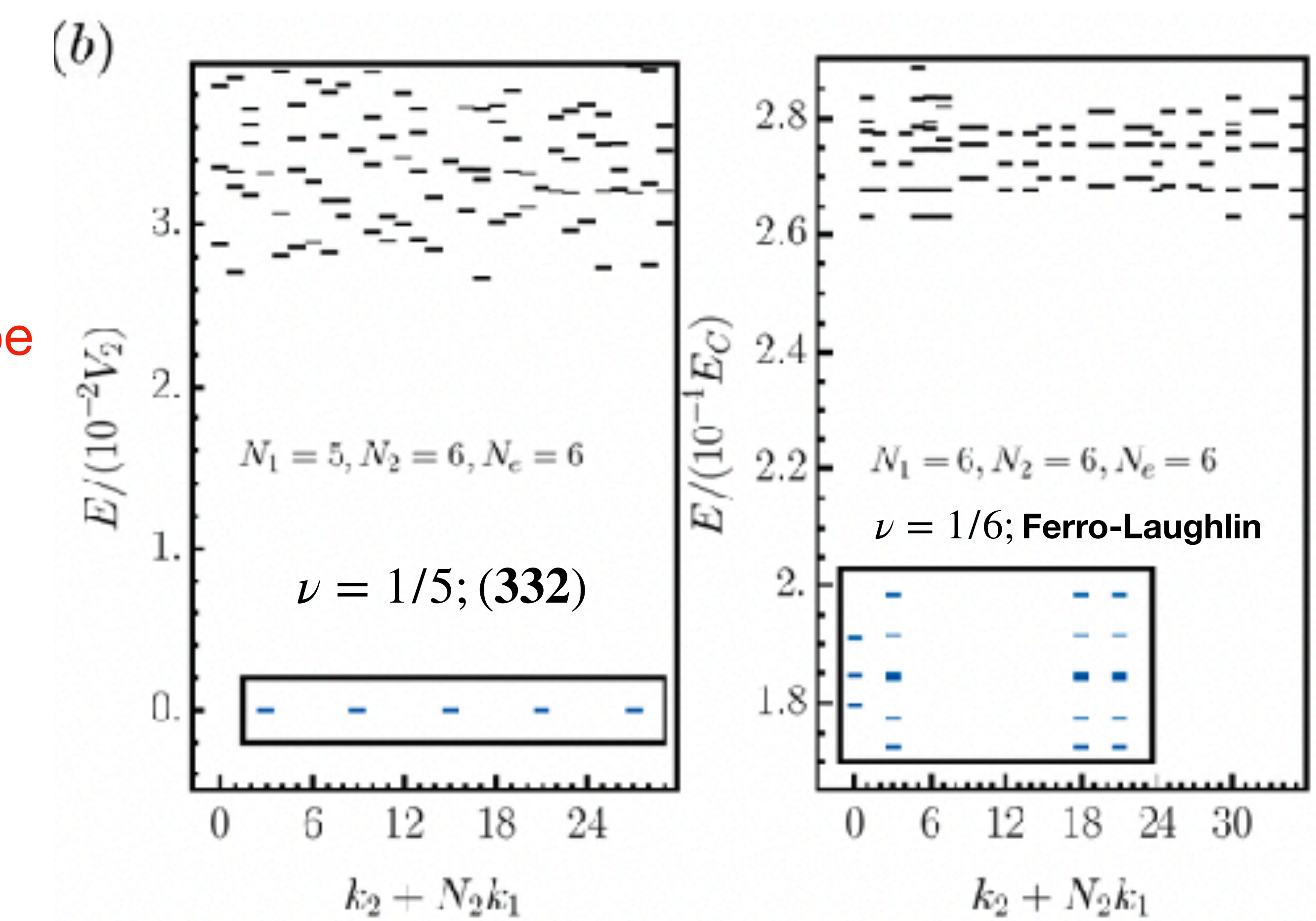
Polshyn, et al. (Nat. Phys. 2022)

FCIs from Vortex Attachment

Skyrmion of topological CDWs



Stripe



Dong, Ledwith, Khalaf, Lee, **AV**(2210.13477)

Conclusions

- Vortexability - starting point for identifying promising FCI candidates. Many physical details + strain + substrate need to be understood

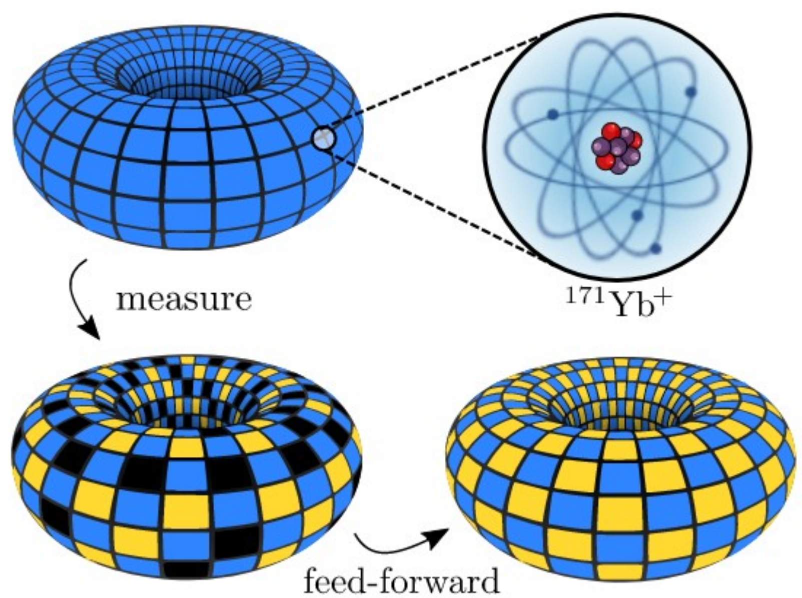
- Can be generalized, different vortex functions: $\tilde{u}_{\boldsymbol{k}} = e^{-i\boldsymbol{k}\cdot\boldsymbol{\phi}(\boldsymbol{r})}\psi_{\boldsymbol{k}}$: where $\boldsymbol{\phi}(\boldsymbol{r}+\boldsymbol{a}) = \boldsymbol{\phi}(\boldsymbol{r}) + \boldsymbol{a}$

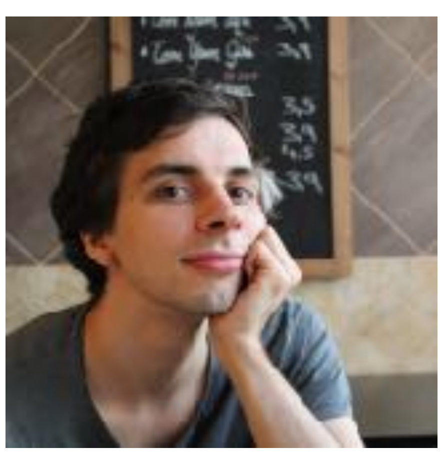
- Higher Chern - can we find states supporting `genon' non Abelian defects?

- "Nearly Vortexable" bands -periodically strained graphene (Gao-Khalaf et al, Sun et al.) & TMDs (Crepel-Fu, Reppelin, Wang, Cano et al.)

- Towards ideal bands and zero field FCI by relaxation engineering - eg. in alt-twist penta-layer. (Ledwith, Khalaf et al.)

…non-Abelions in synthetic quantum systems





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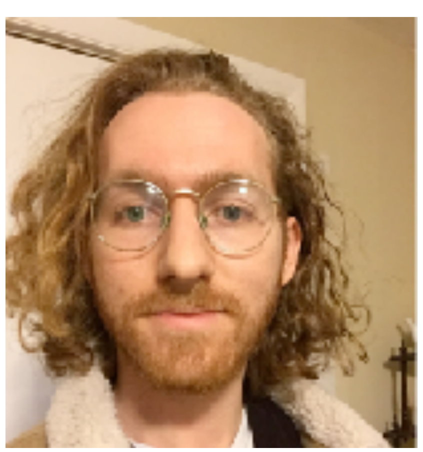

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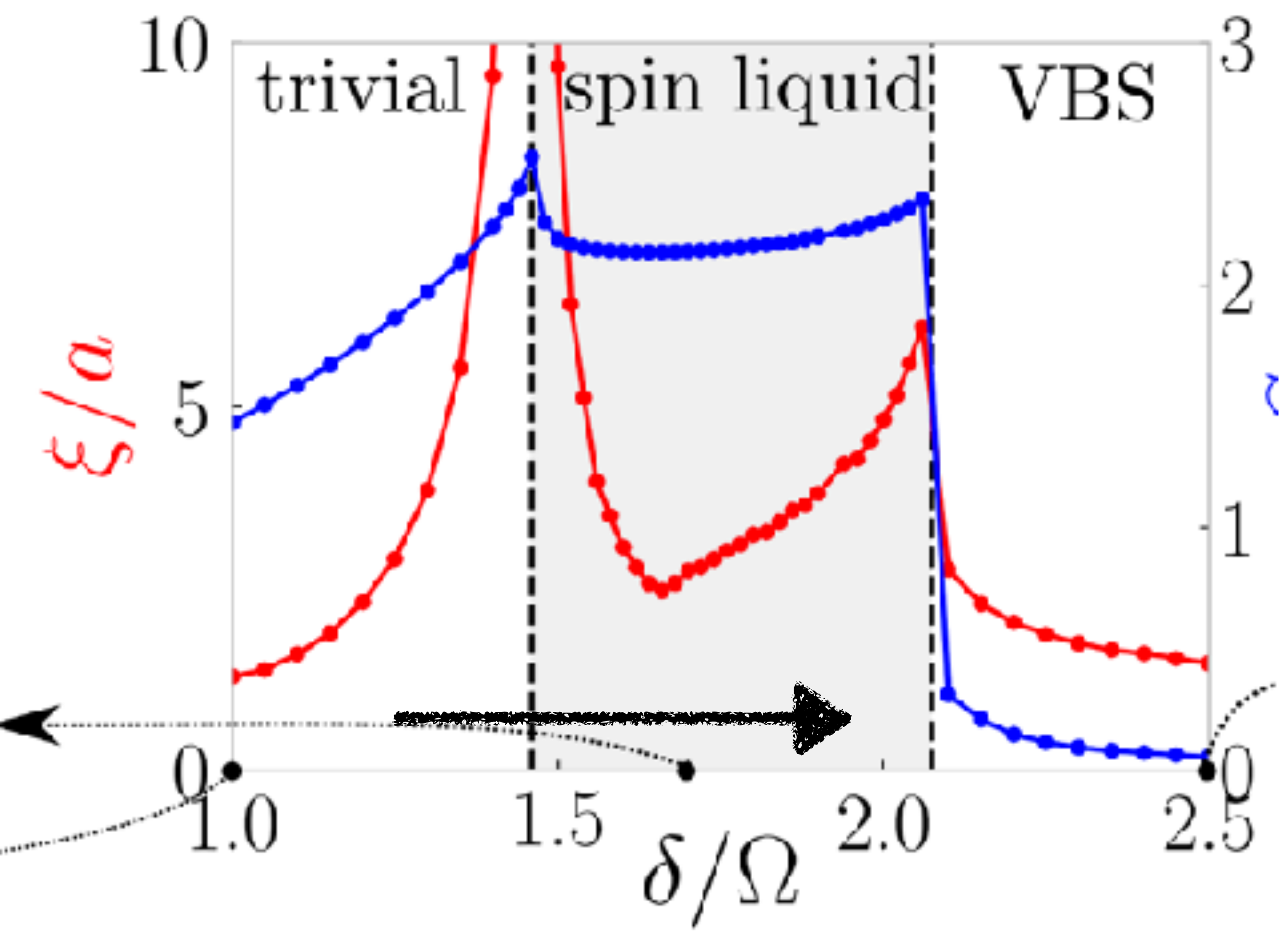
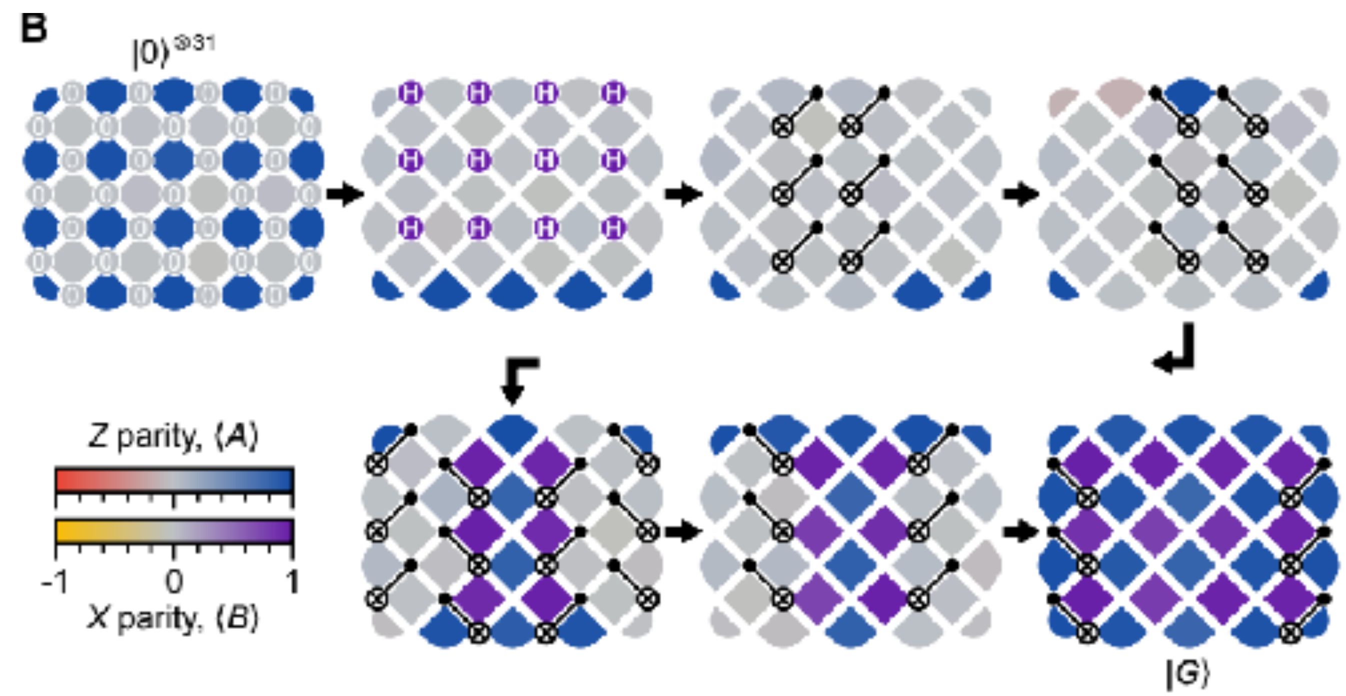
Preparation of topological phases



Verresen, Lukin, Vishwanath '20
Semeghini et. al. '21



Satzinger et.al. '21

Tune adiabatically through phase transition. Time ∝ System size

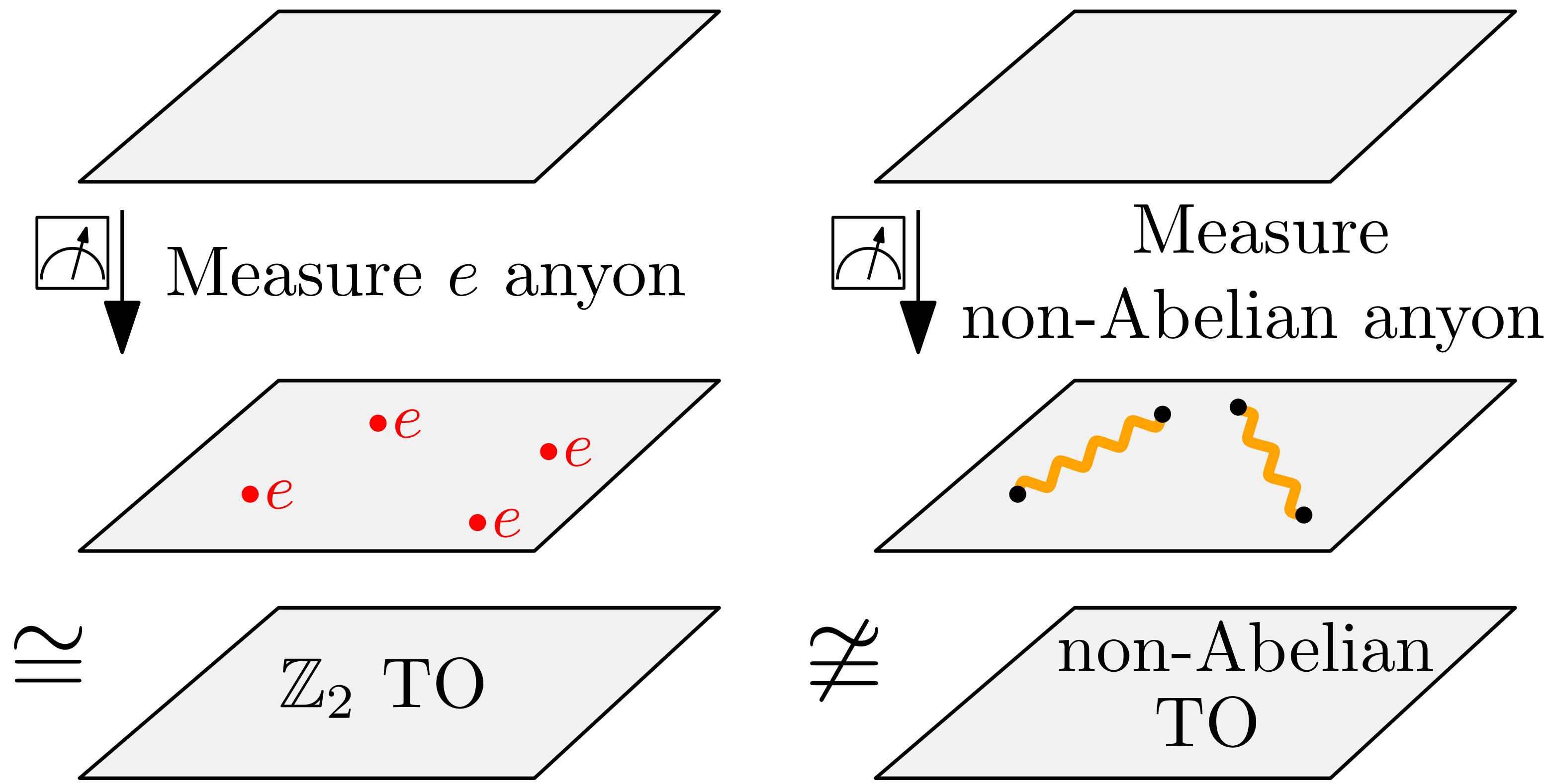
Linear depth circuits required
Bravyi, Hastings, Verstraete '06 ; Chen, Gu Wen '10

It seems that all such preparations are <u>slow</u> (not scalable)?

Efficient Preparation of Non-Abelian states With MEASUREMENT

Non-Abelian anyons allow for **universal** quantum computation *(Kitaev '01, Mochon '03)*

Were believed to be **inaccessible** via measurement!



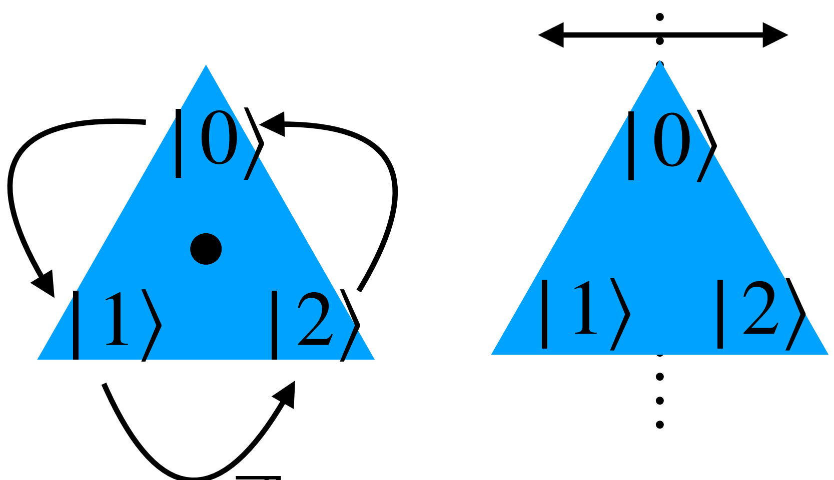
Raussendorf-Bravyi

cannot be paired up in finite time
(Shi '17)

Solution: sequentially gauge abelian symmetries: pair up abelian anyons in each step

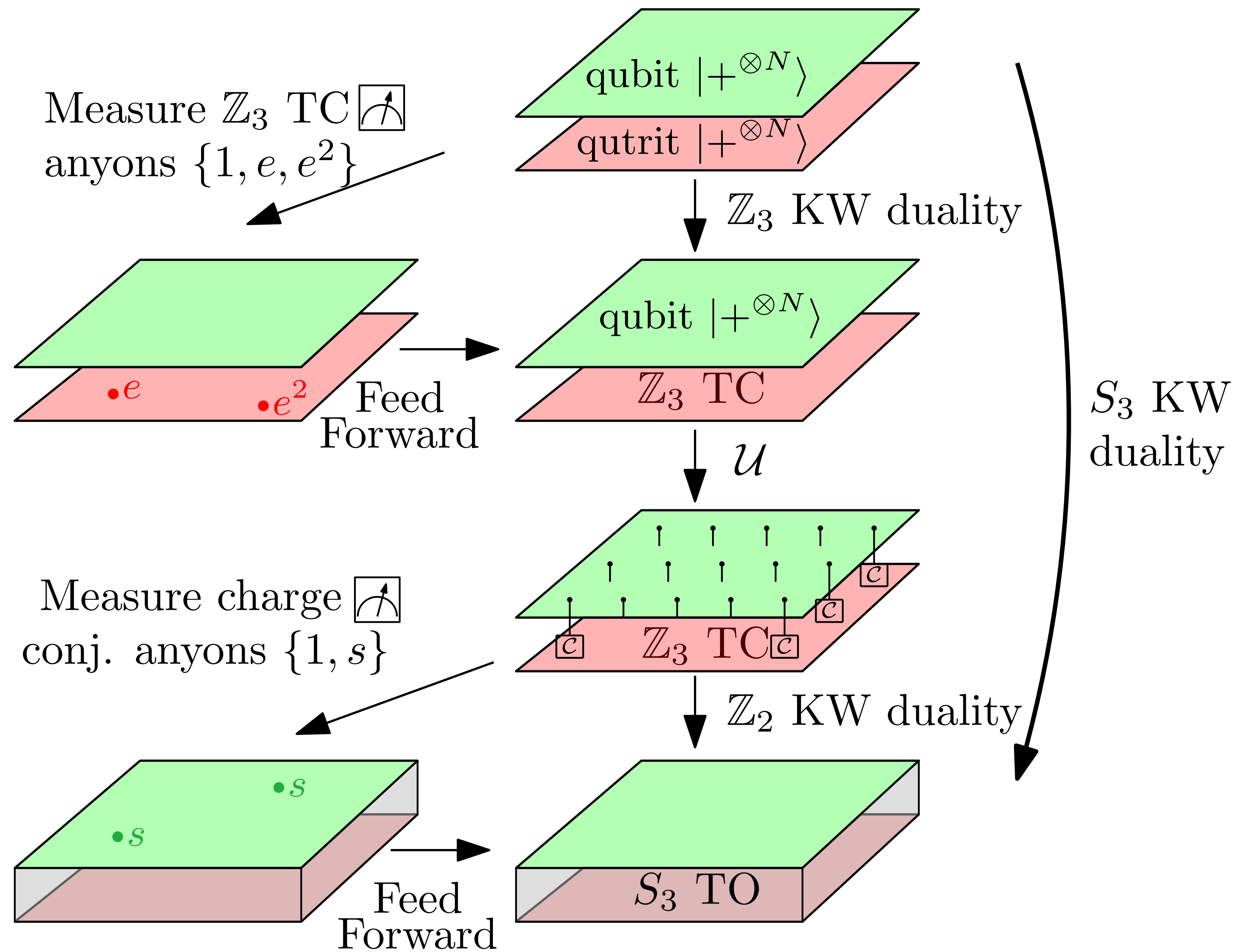
Preparing S_3 topological order

$$\mathbb{Z}_3 \rtimes \mathbb{Z}_2$$



$\mathbb{Z}_2$ acts as “charge conjugation” $\mathscr{C}$ on qutrit $\mathbb{Z}_3$

$$\mathscr{C} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$



Gos and No Gos

Can prepare the (twisted) quantum double any **solvable group**

Commutator subgroup — Commutator subgroup

$$\mathbb{Z}_1 \lhd N_1 \lhd N_2 \lhd \cdots \lhd N_{l_G-1} \lhd N_{l_G} = G$$

Gauge N_1 — Gauge N_2/N_1 — Gauge G/N_{l_G-1}

NT, Vishwanath, Verresen (see Bravyi, Kim, Kliesch, Koenig for a related protocol)

Non-solvable polynomials

Évariste Galois

Theorem (Abel-Ruffini):
A polynomial of degree ≥ 5 has no solution in terms of radicals

Non-solvable groups

❌

Conjecture (with physicist's "proof"): Quantum doubles for the symmetric group S_n for $n \geq 5$ cannot be prepared with a finite depth of circuits and measurements