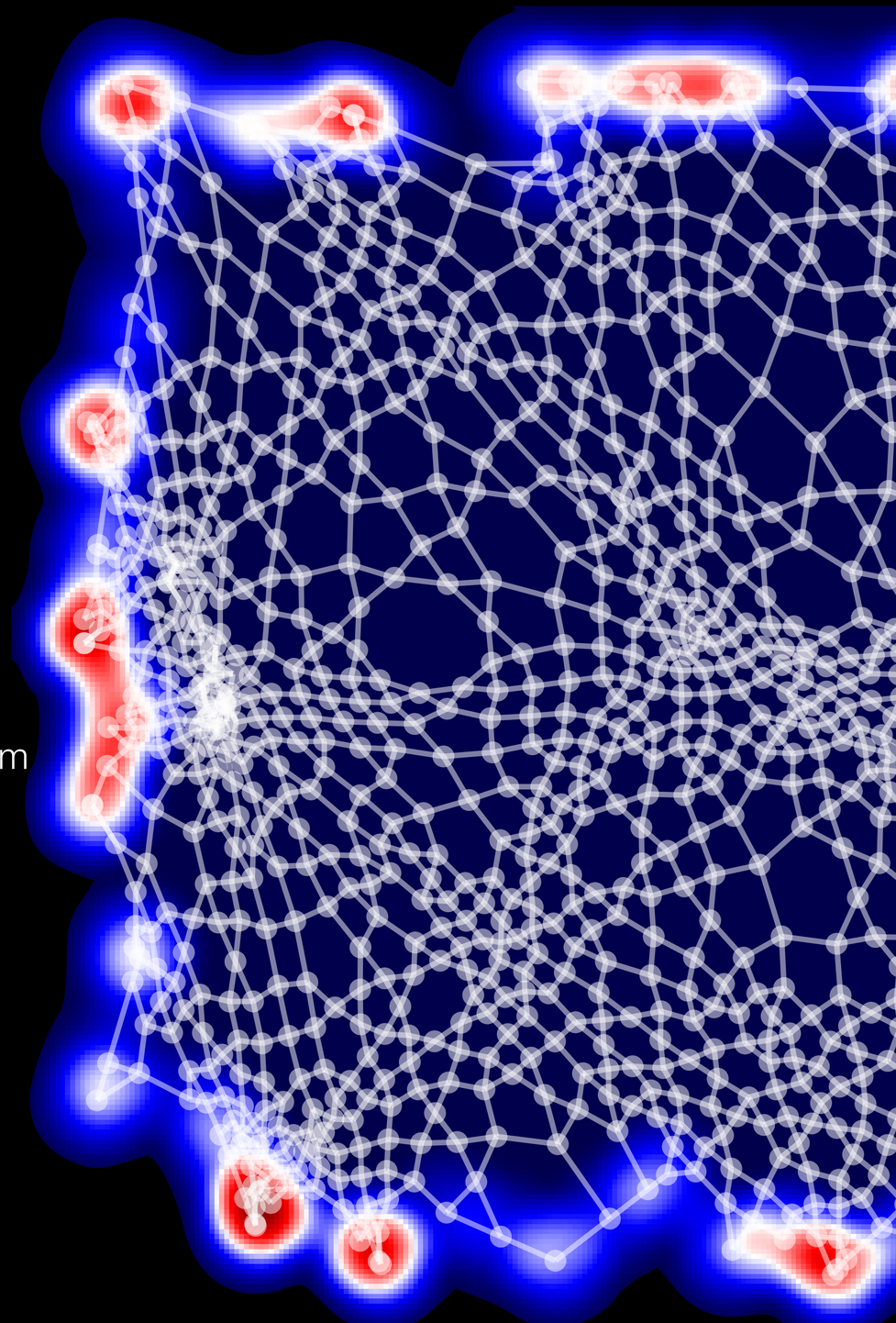
Amorphous topological solids

from flat bands to chiral spin liquids

Adolfo G. Grushin, Néel Institute, CNRS

KITP— March 30th, 2023

A Quantum Universe in a crystal: Symmetry and Topology across the Correlation Spectrum



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Amorphous topological solids

from flat bands to chiral spin liquids

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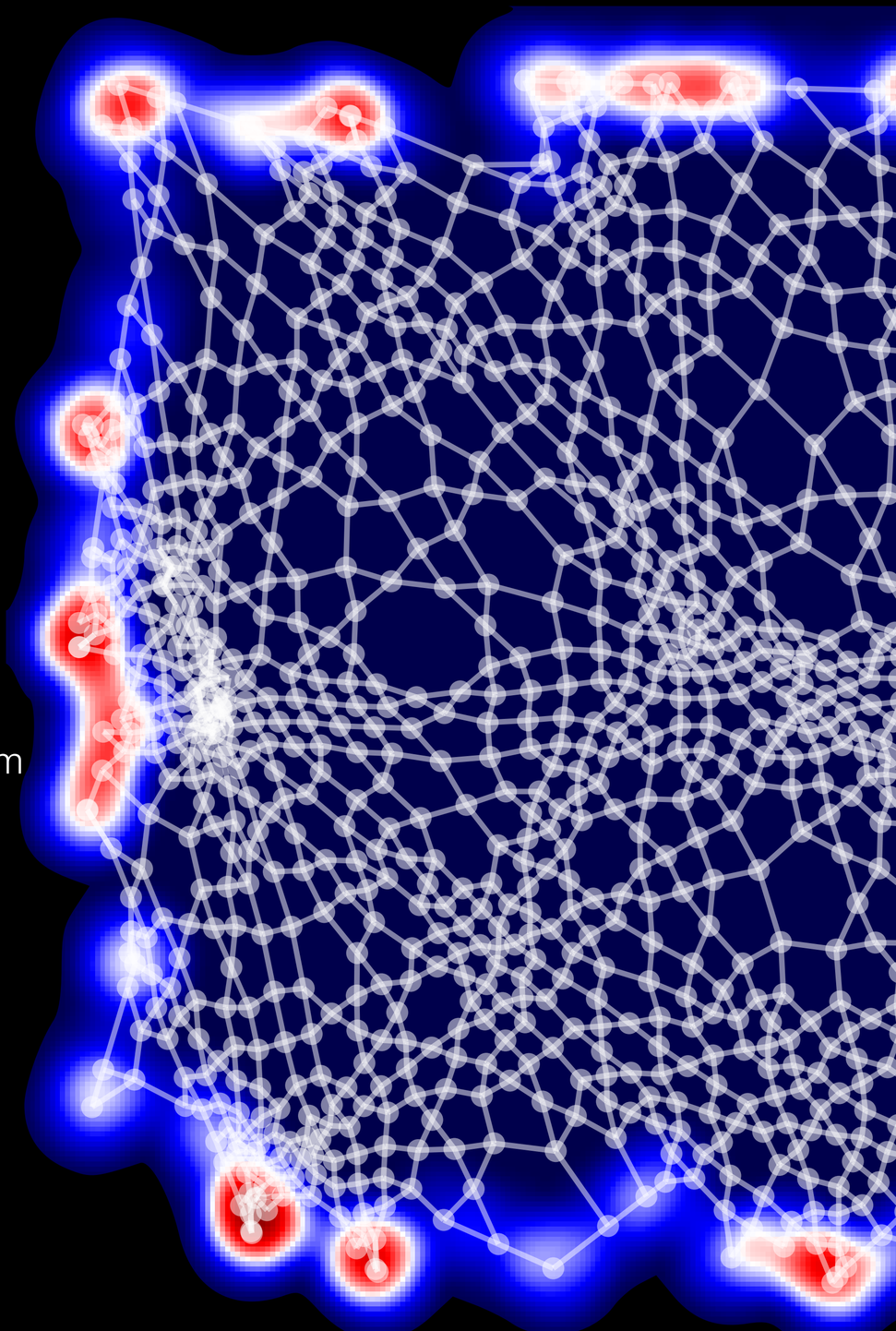
A Quantum Universe in a ~~crystal~~: Symmetry and Topology across the Correlation Spectrum

lattice



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ANR

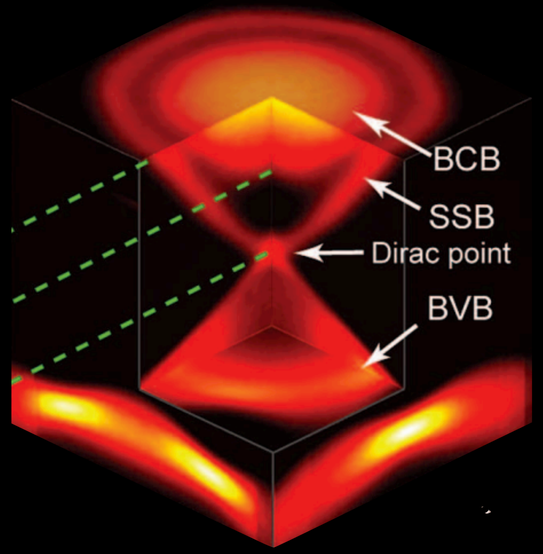
Topological solids

Topological >50%	Crystals

Vergniory et al. Nature (2019)

Zhang et al. Nature (2019)

Tang et al. Nature (2019)



Topological Materials Database

Compound Contains: Bi Se | Only these elements: Exclude | ICSD Number: eg. 123456 | Search

Show Advanced Search

H																	He
Li	Be											B	C	N	O	F	Ne
Na	Mg											Al	Si	P	S	Cl	Ar
K	Ca	Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	At	Rn
Fr	Ra	Ac	Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn	Nh	Fl	Mc	Lv	Ts	Og
			Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu	
			Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr	

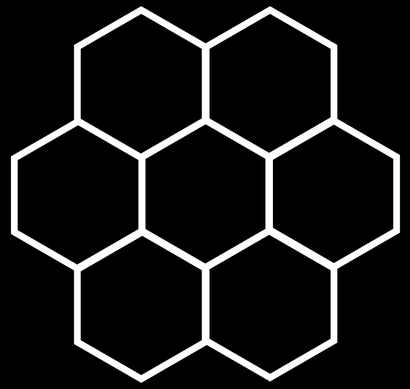
92 Entries found for **Bi, Se**, showing:

ALL (92) | TI (18) | SM (14) | Trivial (60)

Compound	Symmetry Group	Topological Indices	Crossing Type	Type
Bi1 Se1	225 (*Fm-3m*)		Point	ESFD
Bi1 Se1	12 (*C2/m*)	$Z_{2w,1}$=0, $Z_{2w,2}$=0, $Z_{2w,3}$=1, Z_4=0		SEBR
Bi1 Se1	164 (*P-3m1*)	$Z_{2w,1}$=0, $Z_{2w,2}$=0, $Z_{2w,3}$=1, Z_4=0		SEBR
Bi1 Se2	12 (*C2/m*)			LCEBR
Bi2 Se2	164 (*P-3m1*)	$Z_{2w,1}$=0, $Z_{2w,2}$=0, $Z_{2w,3}$=1, Z_4=0		SEBR
Bi2 Se3	62 (*Pnma*)			LCEBR
Bi2 Se3	166 (*R-3m*)	$Z_{2w,1}$=0, $Z_{2w,2}$=0, $Z_{2w,3}$=0, Z_4=3		SEBR

How do we find new topological insulators?

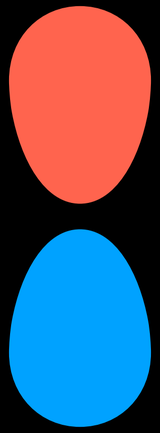
Space Group



— lattice symmetries

translations, rotations, inversions, mirrors

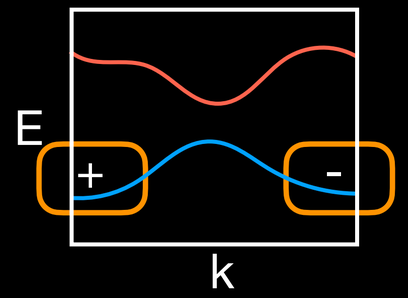
Orbitals

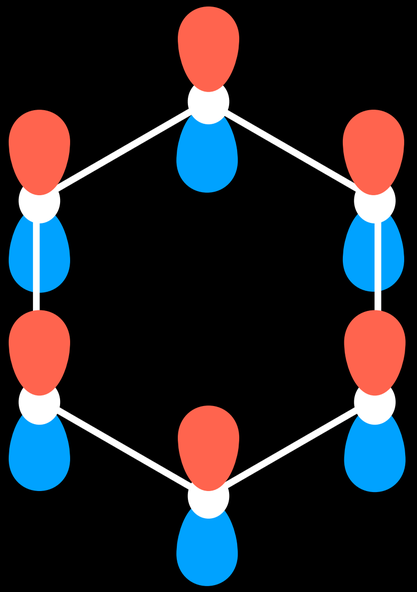


— wavefunctions

s, p, d…

Atomic positions



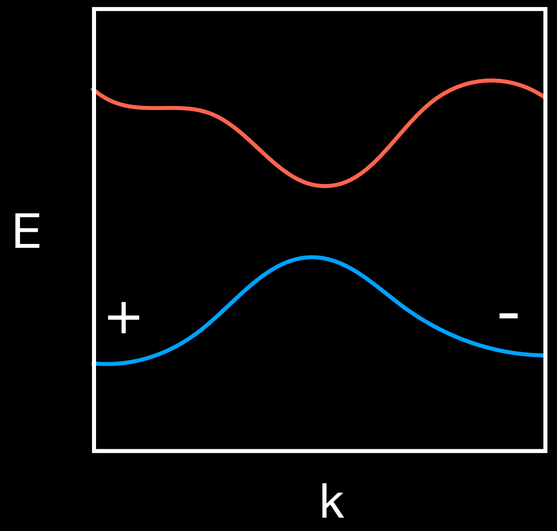


— band connectivity + symmetries labels

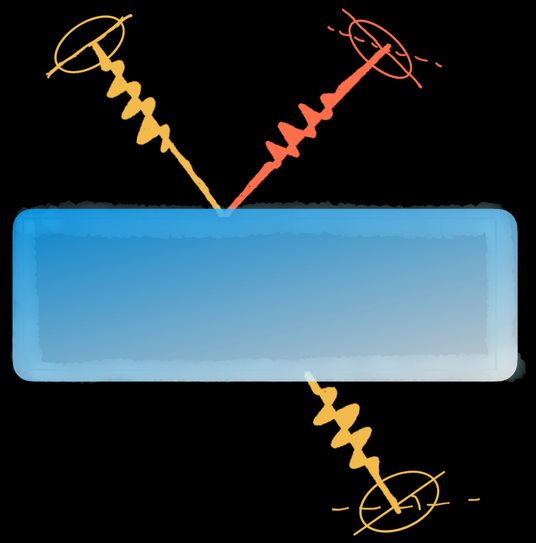
Bradlyn et al Nature (2017) Kruthoff et al. Phys. Rev. X (2017) Po et al. Nat. Comm (2017) Song et al. Nat. Comm (2018)

How do we find new topological insulators?

band connectivity + symmetries labels → Topological invariants → Quantized responses

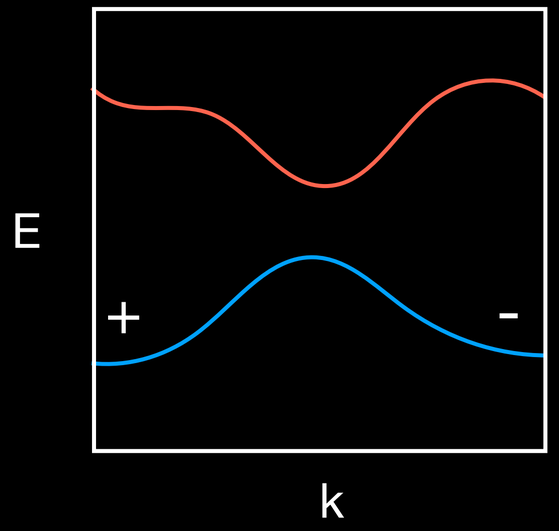


$\mathbb{Z}$



How do we find new topological insulators?

band connectivity + symmetries labels

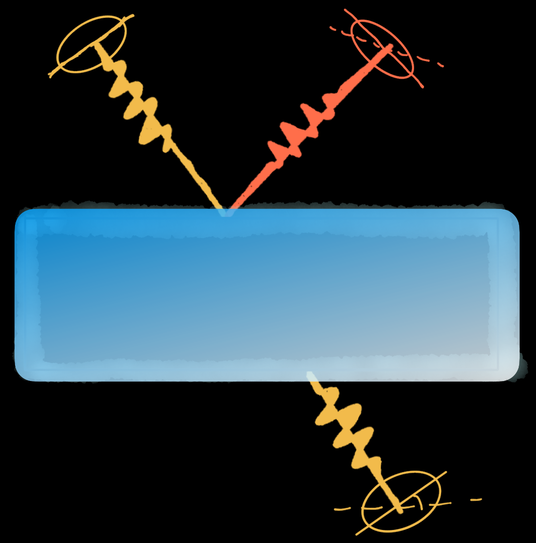


Topological invariants

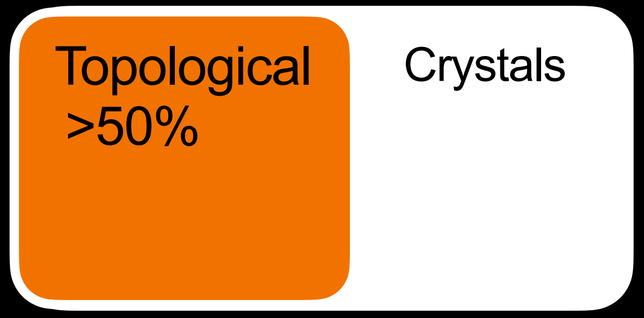
$$(-1)^{\nu} = \prod_{\text{TRIM}} \zeta_i$$

symmetry eigenvalues

Quantized responses



Topological solids



Vergniory et al. Nature (2019)

Zhang et al. Nature (2019)

Tang et al. Nature (2019)

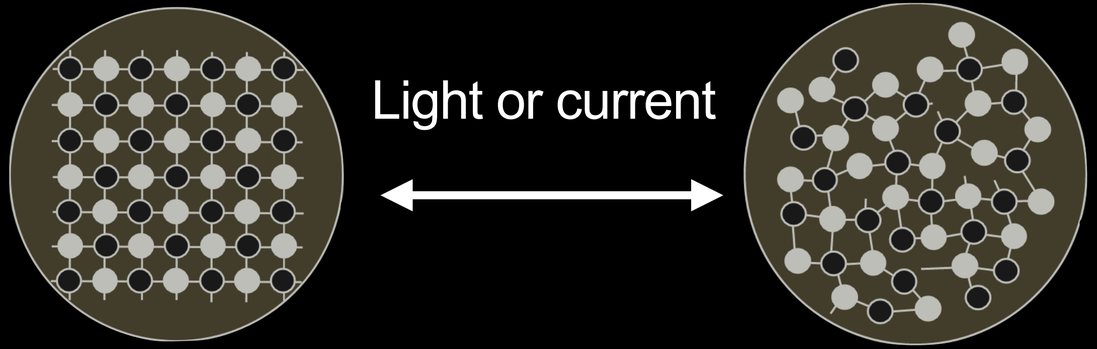
Topological solids

Amorphous > Crystal

"Nearly all materials can […] be prepared as amorphous solids"

R. Zallen, *Amorphous Solids*

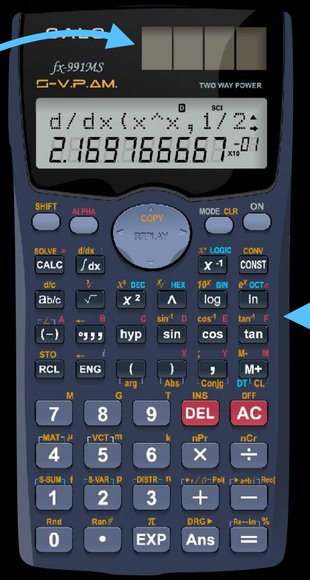
Different capabilities



Mathias, Naboro Nat Mat (2007)
Q. Marsal, D. Varjas, AGG Phys Rev B, (2023)

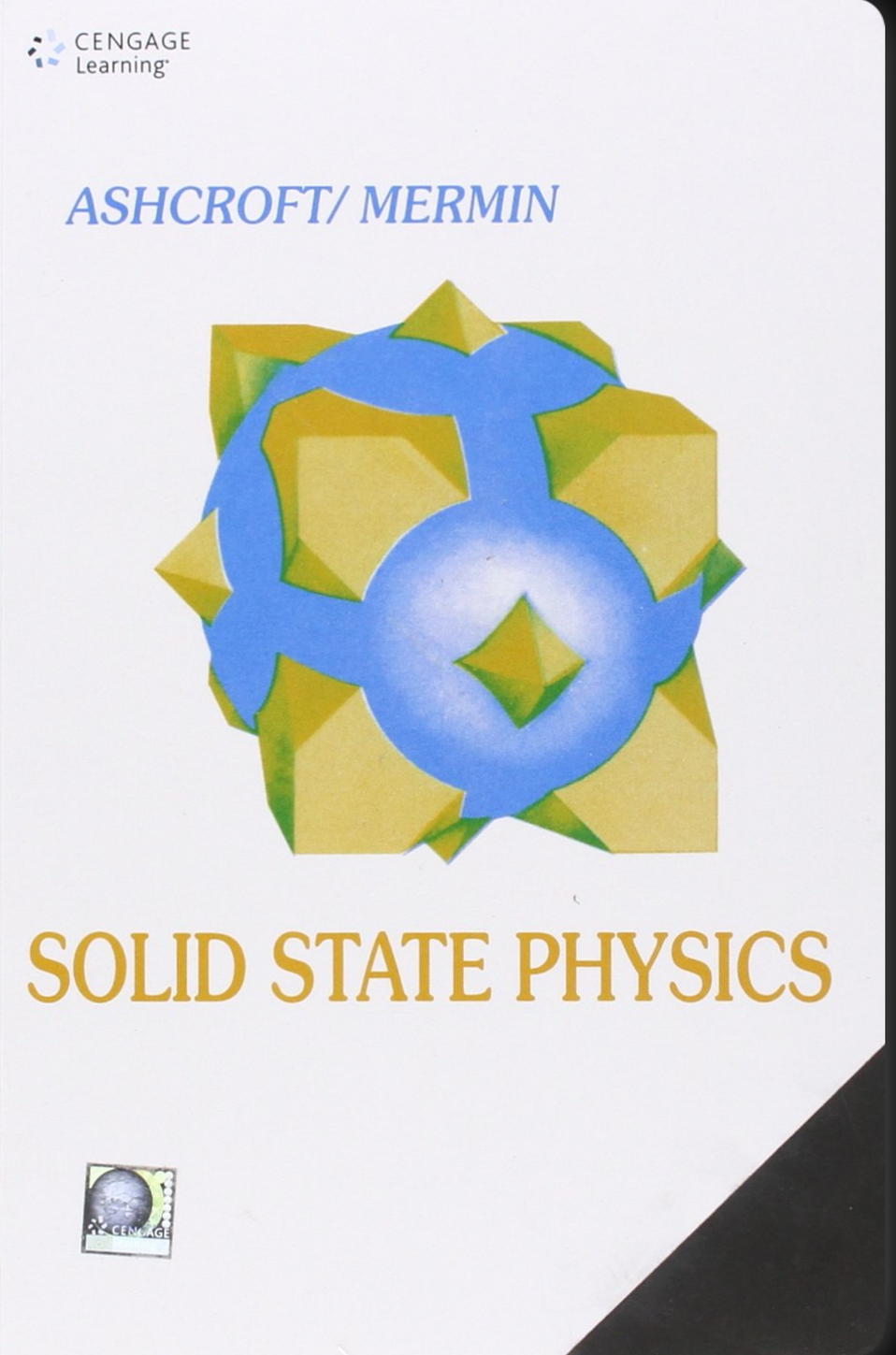
Cheap and scalable

Solar cells



Memories

Why are we missing amorphous solids?



Absent in solid-state textbooks (typically)

amorphous solids

topological phases

How do we find amorphous topological insulators?

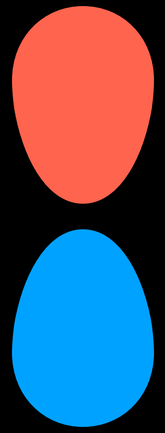
Space Group

— lattice sy tries

translations, ions, inversions, mirrors

No long-range order

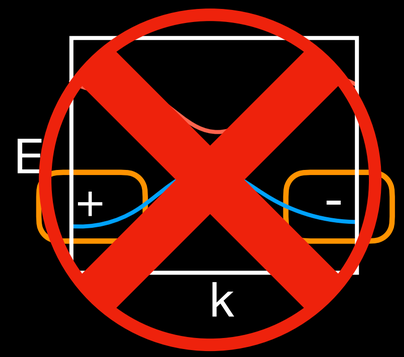
Orbitals

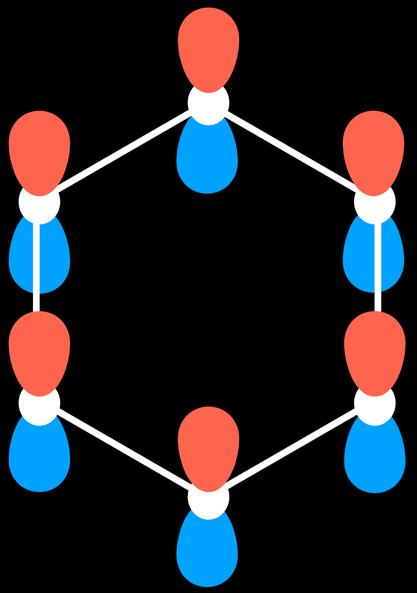


— wavefunctions

s, p, d…

Atomic positions





— band connectivity + symmetries labels

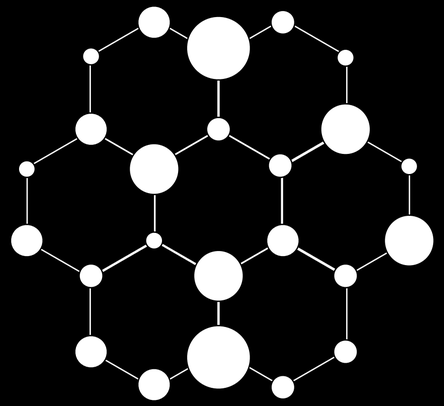
Bradlyn et al Nature (2017) Kruthoff et al. Phys. Rev. X (2017) Po et al. Nat. Comm (2017) Song et al. Nat. Comm (2018)

Topology survives disorder

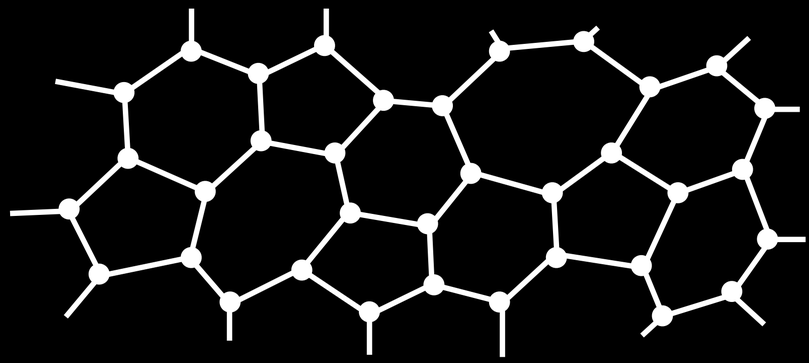
Some choices

Bond disorder

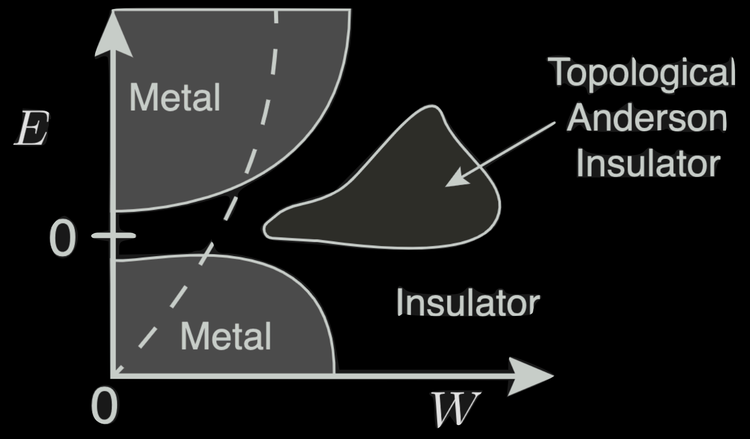
Onsite disorder



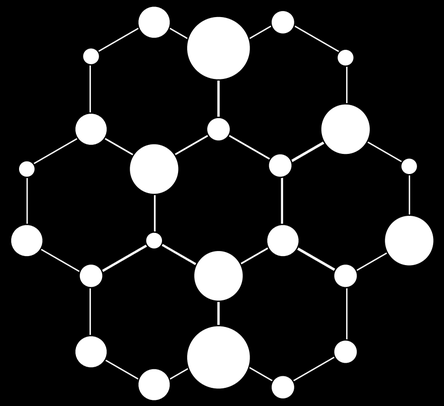
Structural disorder



Topology survives disorder



Onsite disorder

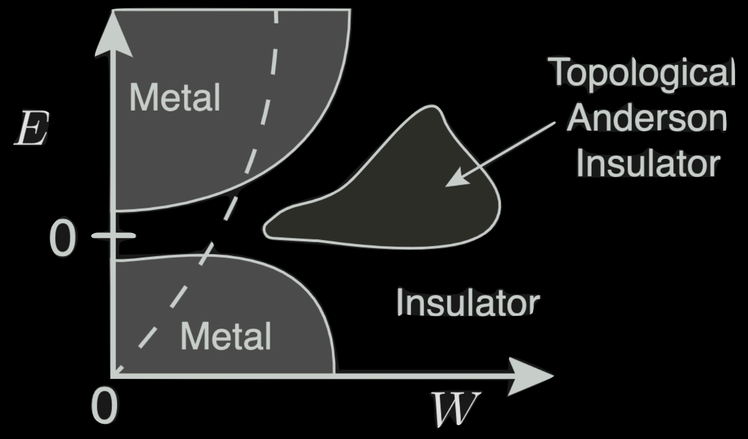


Li et al PRL (2009)
Groth et al. PRL (2009)

Topology survives disorder

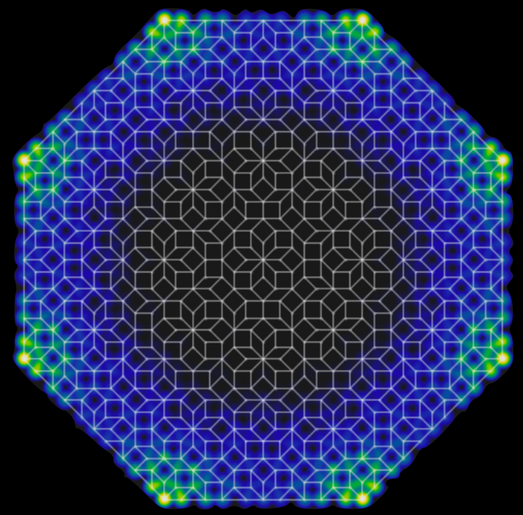
topology is not restricted by translational invariance

Disordered systems



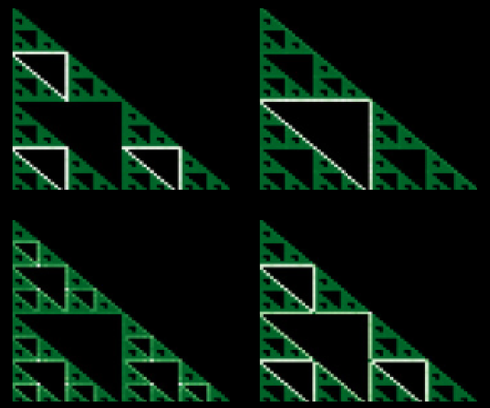
Li et al PRL (2009)
Groth et al. PRL (2009)

Quasicrystals



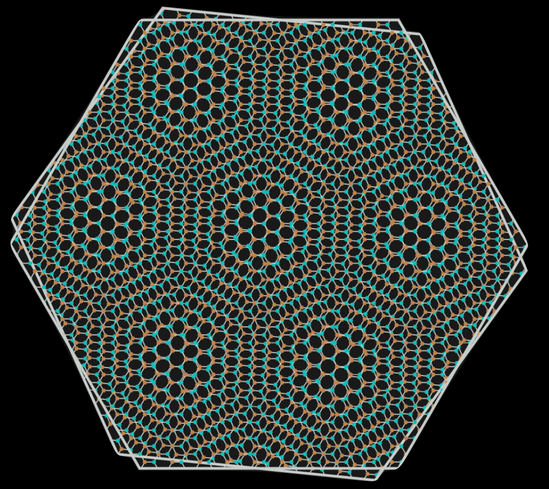
Fuchs and Vidal PRB (2016)
Varjas et al. PRL (2019)

Fractals



Brzezinska et al. PRB (2018)

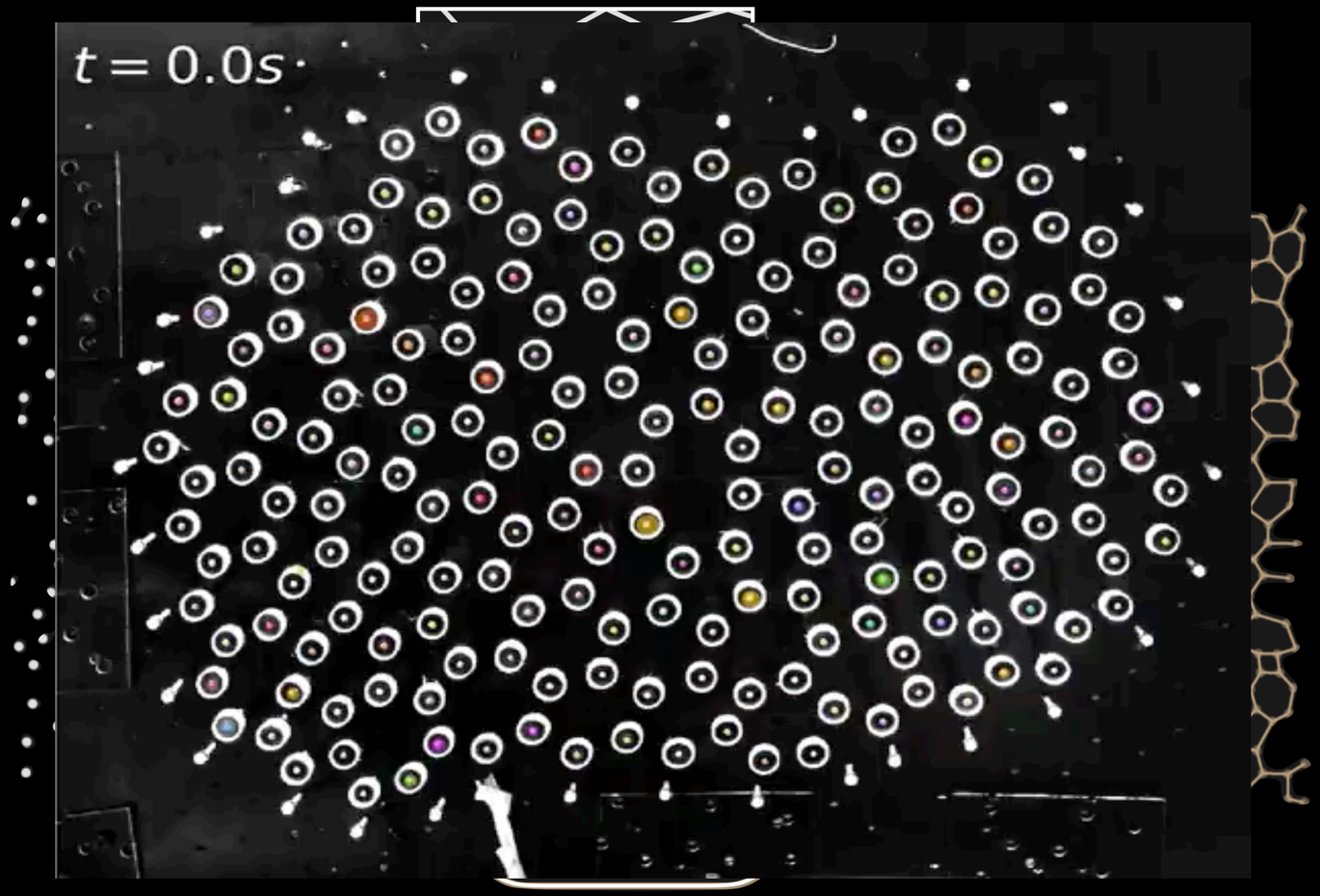
Moiré superlattices



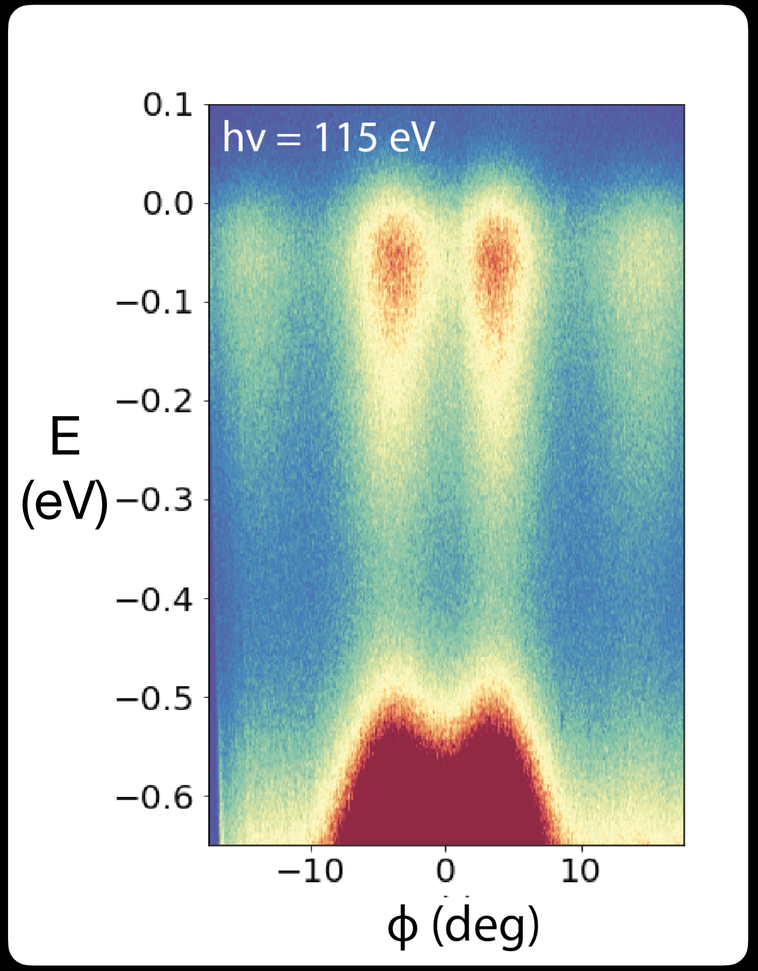
Cao et al. Nature (2018)

A recent addition: amorphous topological phases

Synthetic systems:



solid state: a-Bi_2Se_3



Theory:
Agarwala, Shenoy PRL (2017)
Xia and Fan PRB (2017)
Mansha and Shong PRB (2018)

Exp: Mitchell, et al. Nat Phys (2018)
Liu et al PRL (2020)
Zhou et al Light: Science and App. (2020)
Jia, et al. arXiv: 2211.12719
Zhang, et al. Sci. Advances (2023)

Corbae et al, AGG, Lanzara, Hellmann Nat Materials (2023)

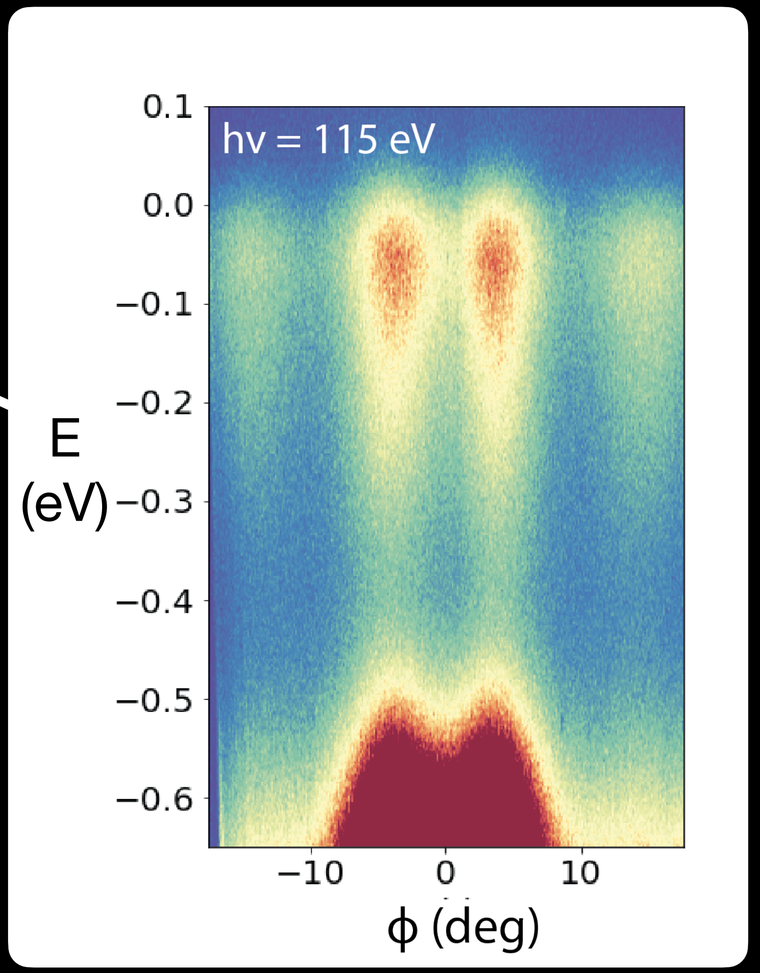
Cyocis, Marsal et al, Hellmann, AGG, Lanzara 2302.05945

Overlooked topological solids?

We just know one!

= hard to predict

solid state: a-Bi_2Se_3



Corbae et al, AGG, Lanzara, Hellmann Nat Materials (2023)

Cyocis, Marsal et al, Hellmann, AGG, Lanzara 2302.05945

How do we find topological amorphous solids?

Any different physics compared to crystals?

Amorphous graphene

Amorphous graphene

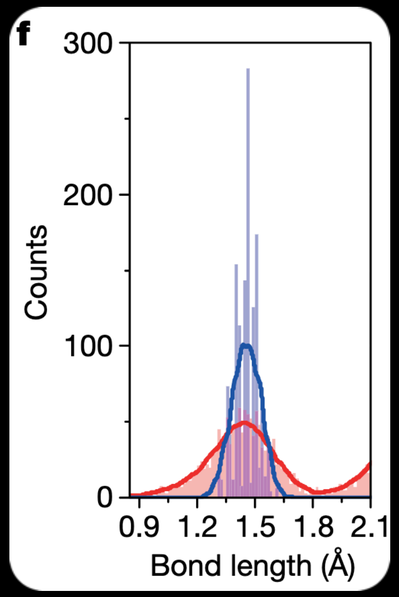
Amorphous graphene

Amorphous graphene

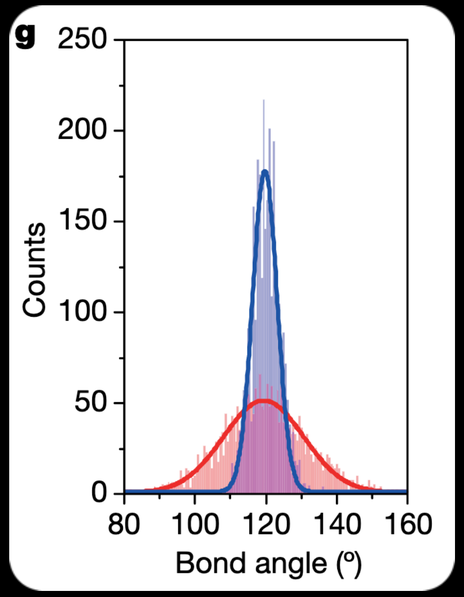
Local order

Fixed coordination

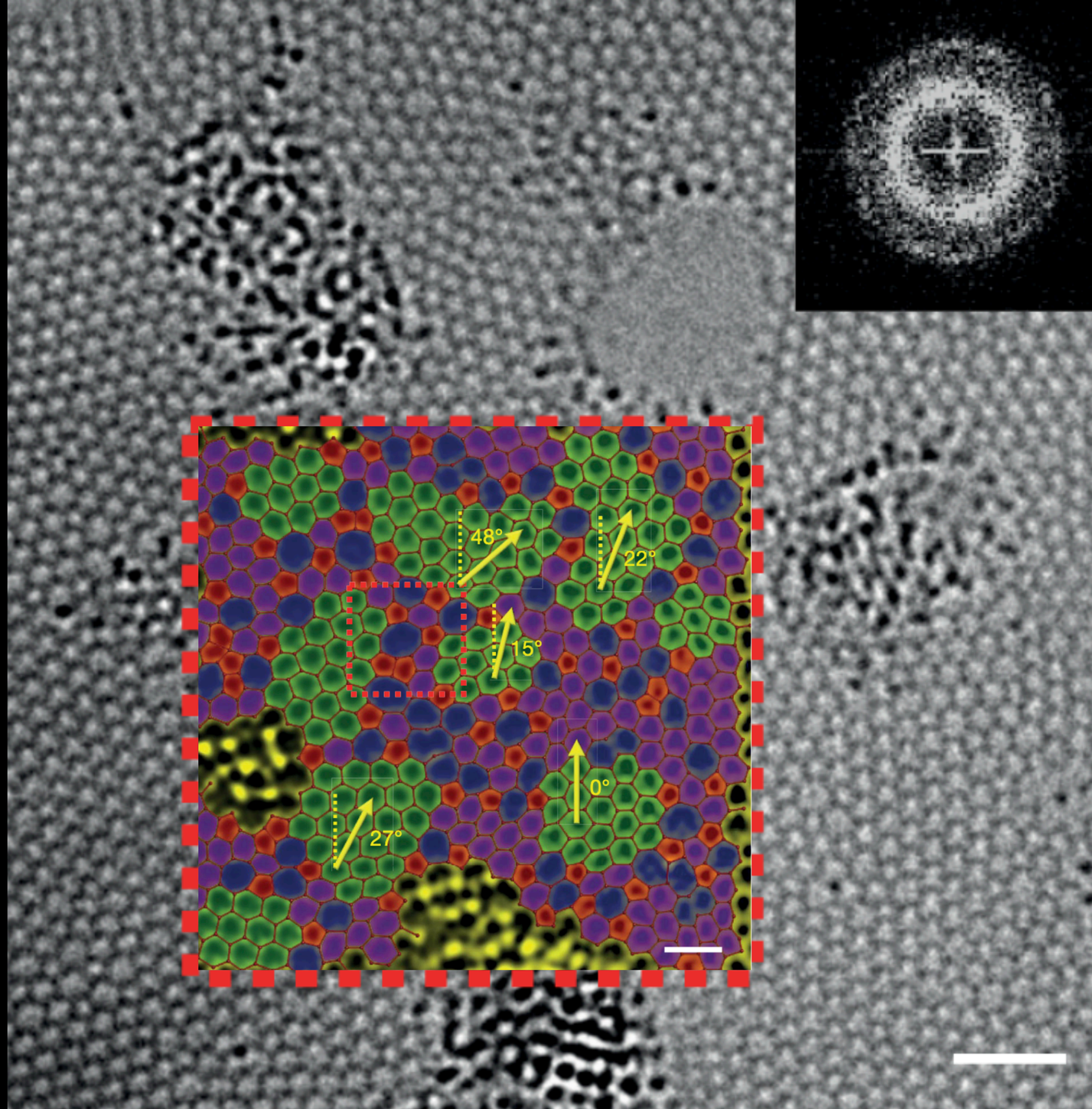
Bond lengths



Bond-angles



Crystalline and amorphous regions coexist



Toh et al. Nature (2020)

What's new?

Flat-band switch

Q. Marsal, D. Varjas, AGG Phys Rev B, (2023)

Quentin Marsal
Néel Institute

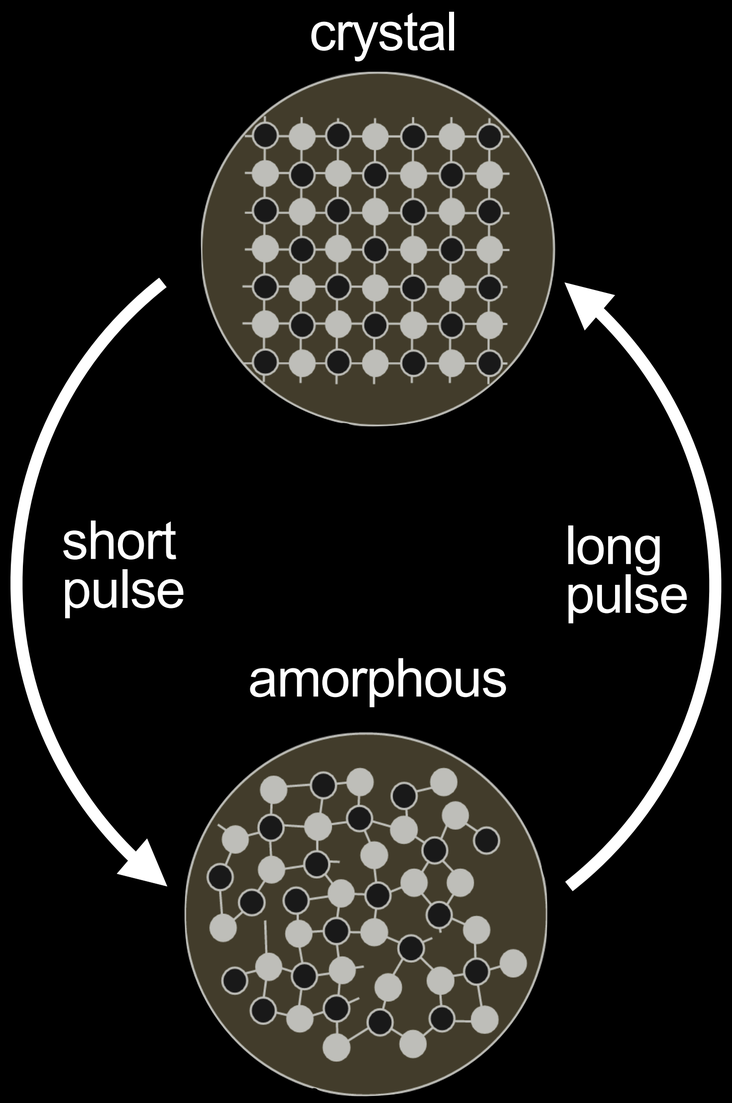
Daniel Varjas
MPI PKS

Amorphous chiral spin-liquid

AGG, C. Repellin 2210.13548

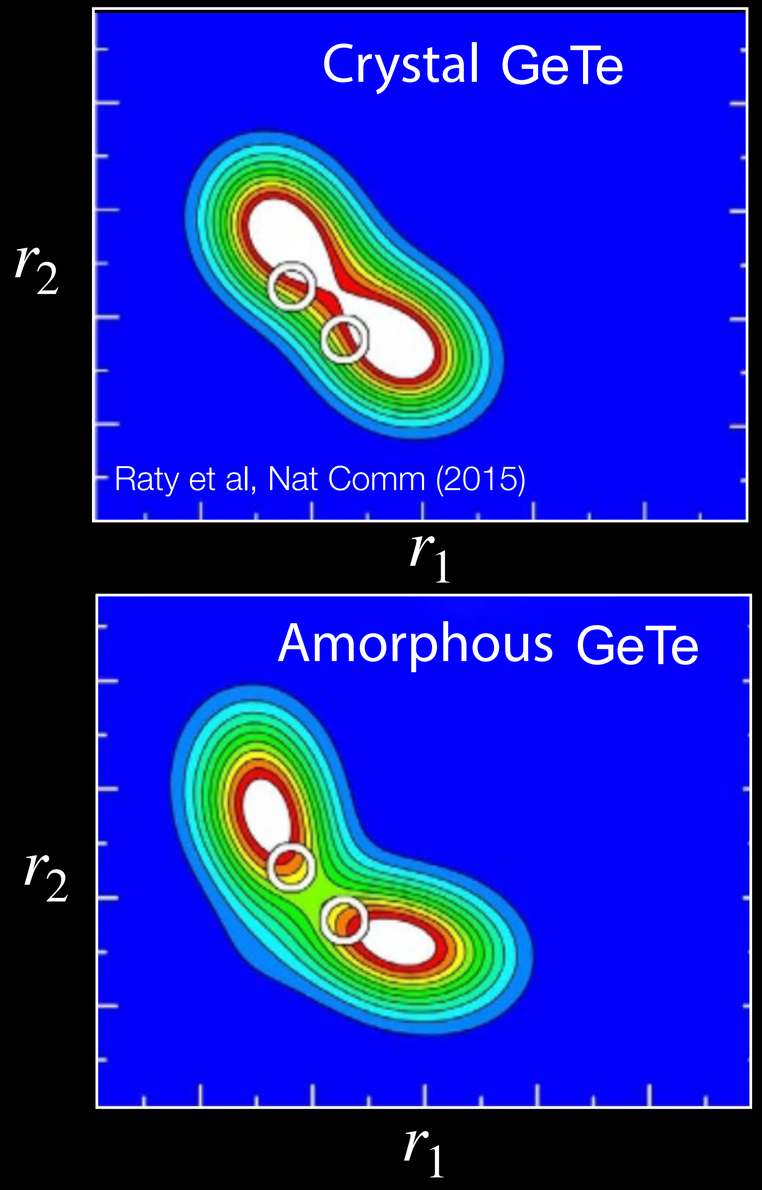
Cécile Repellin
LPMMC / Grenoble

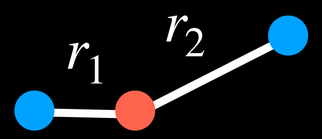
Phase-change materials

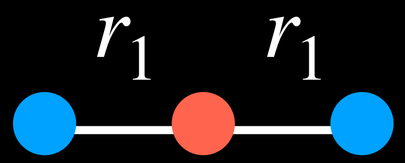


Mathias, Naboro Nat Mat (2007)

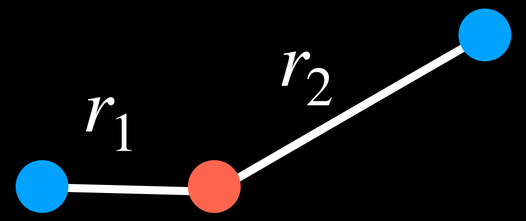
Pair distribution function







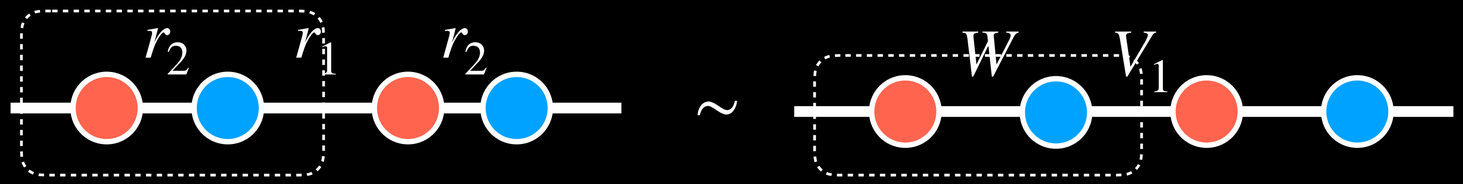
No Peierls distortion



Average Peierls distortion

Gaspard, Compte Rendus Physique (2016)

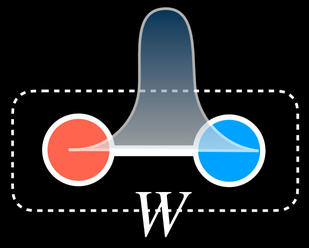
1D Peierls phase (SSH)

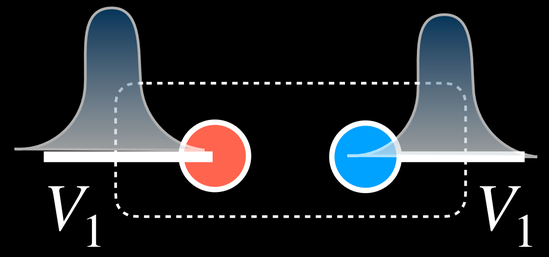


$W \gg V_1$

Two atomic limits

$W \ll V_1$





Can’t be deformed into each other while preserving gap and inversion

= obstructed atomic limits

Peierls (1955)

Su, Schrieffer and Heeger PRL (1979)

Song et al PRL (2017), Xu et al 2106.10276

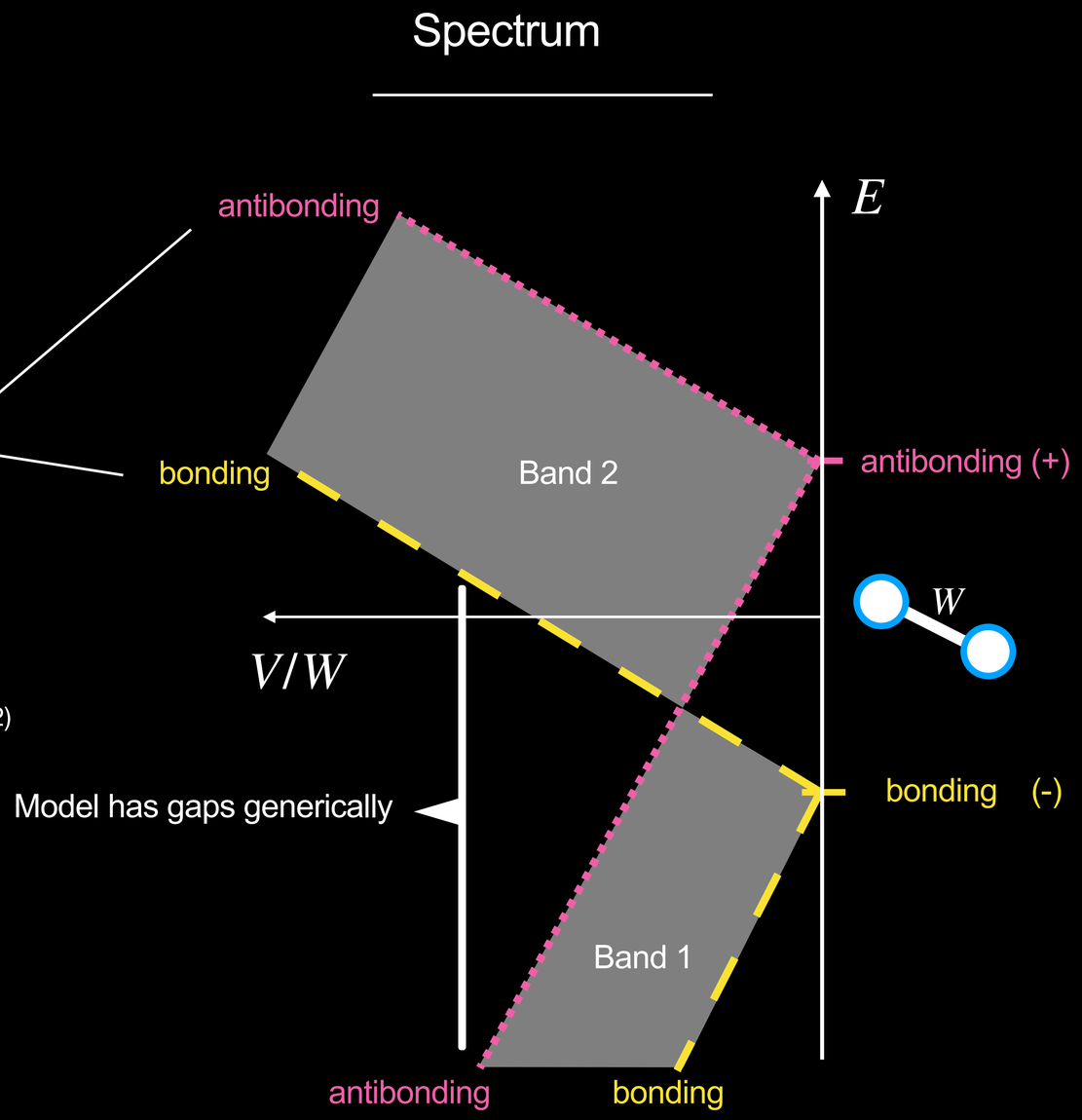
Fixed coordination = symmetry indicators

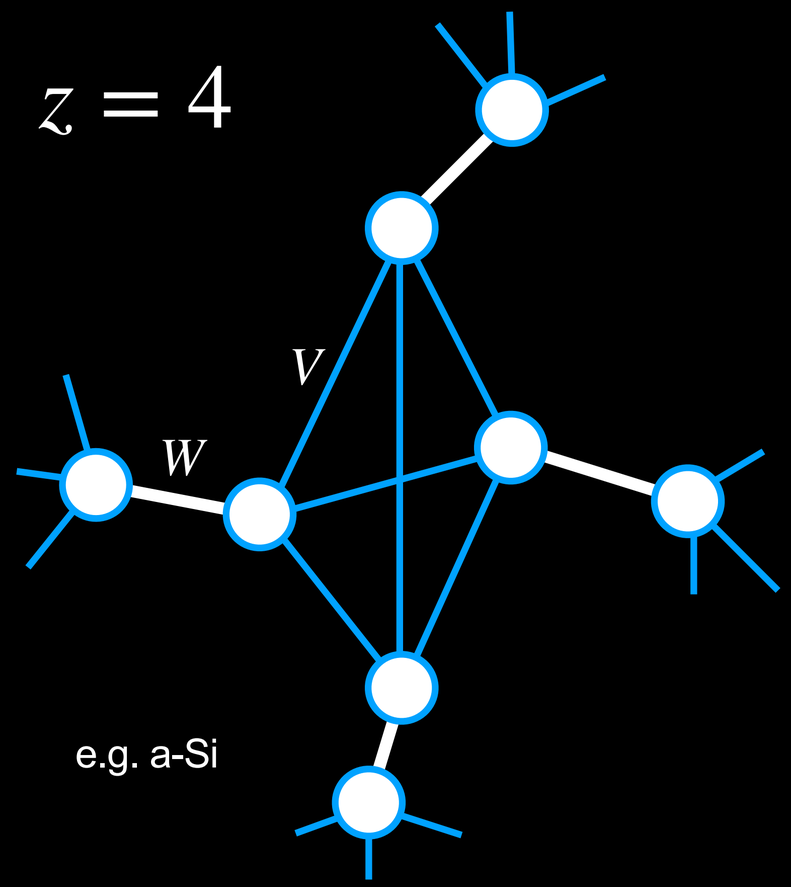
Q. Marsal, D. Varjas, AGG PNAS, (2020)

Band edges:

1. Labelled by symmetry
   Weaire, Thorpe PRB (1971)

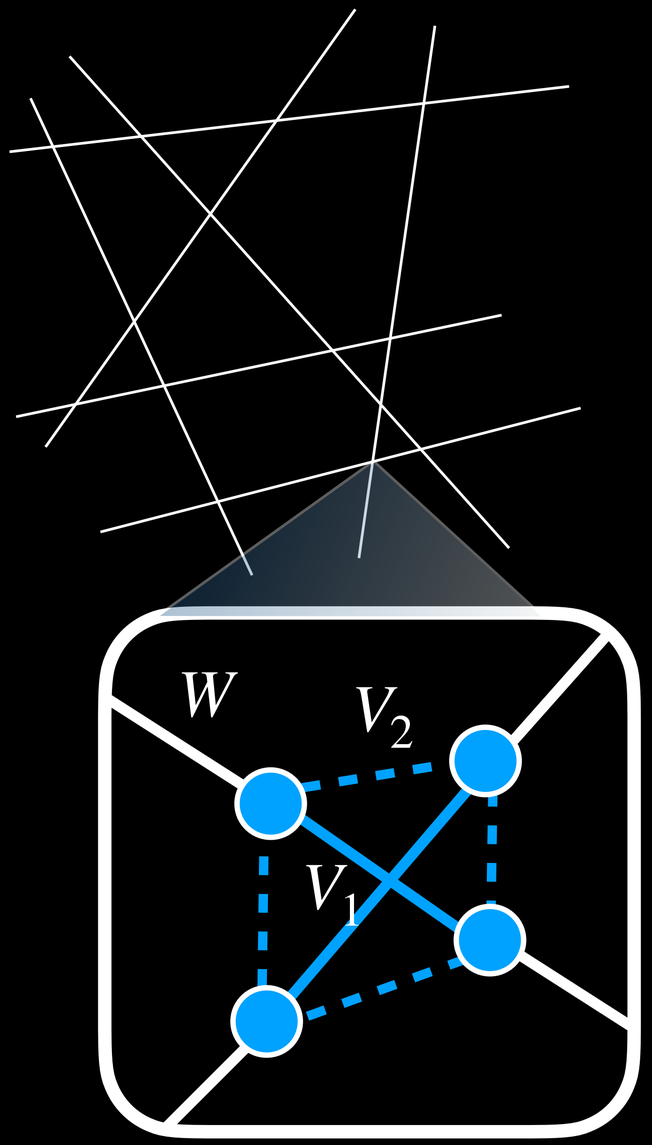
2. Obtained analytically
   Schwartz, Ehrenreich PRB (1972)





Weaire, Thorpe PRB (1971)

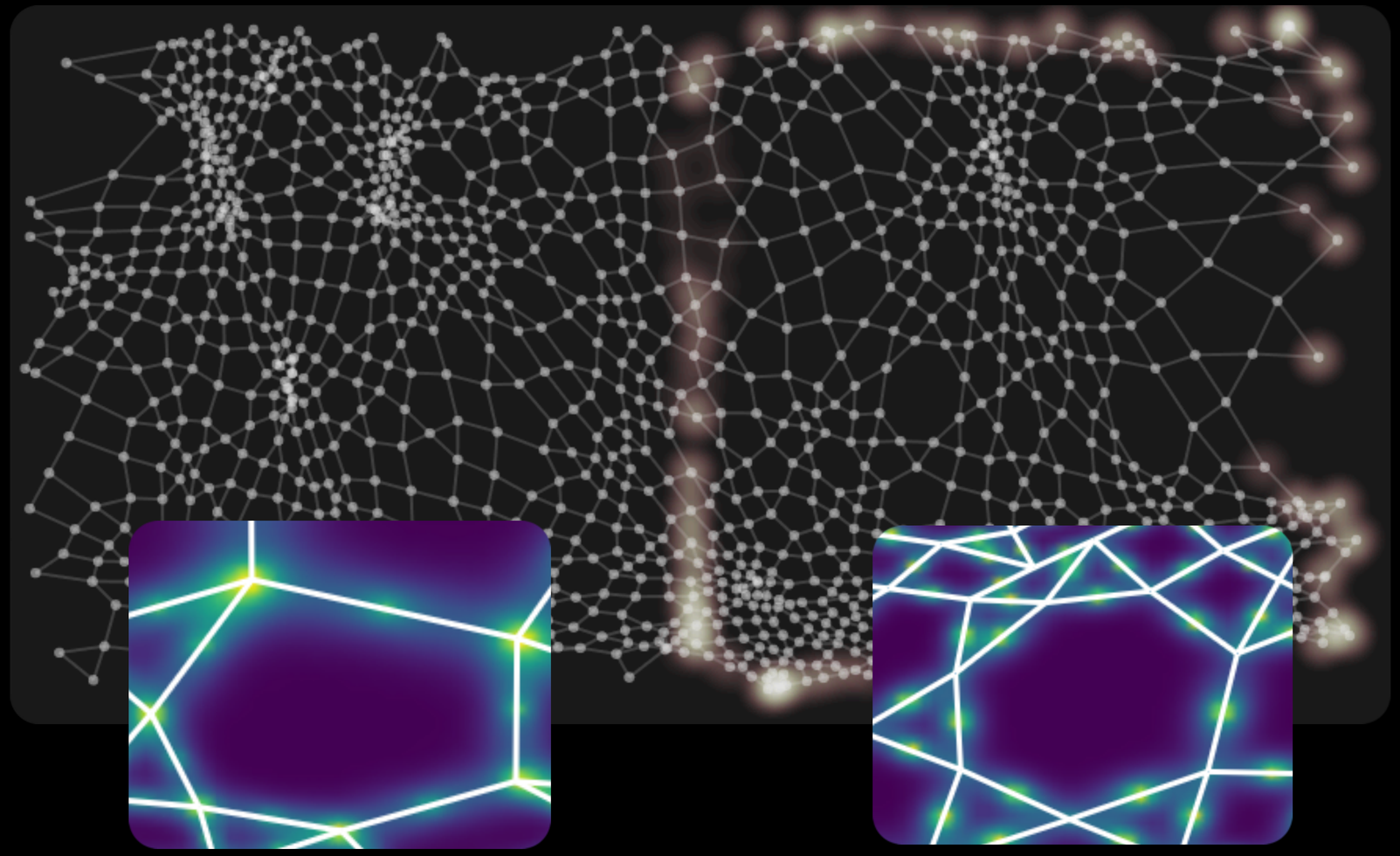
Mikado model (2D or 3D)



Two obstructed limits

$W \ll V_{1,2}$

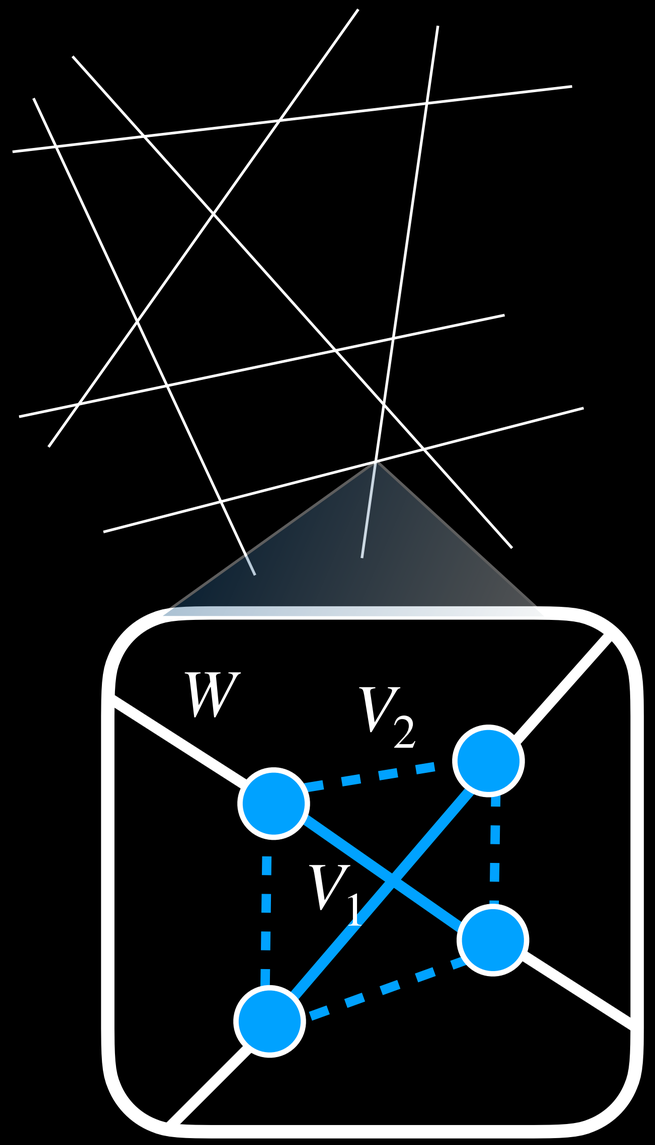
$W \gg V_{1,2}$



charge on sites

charge on bonds

Mikado model (2D or 3D)

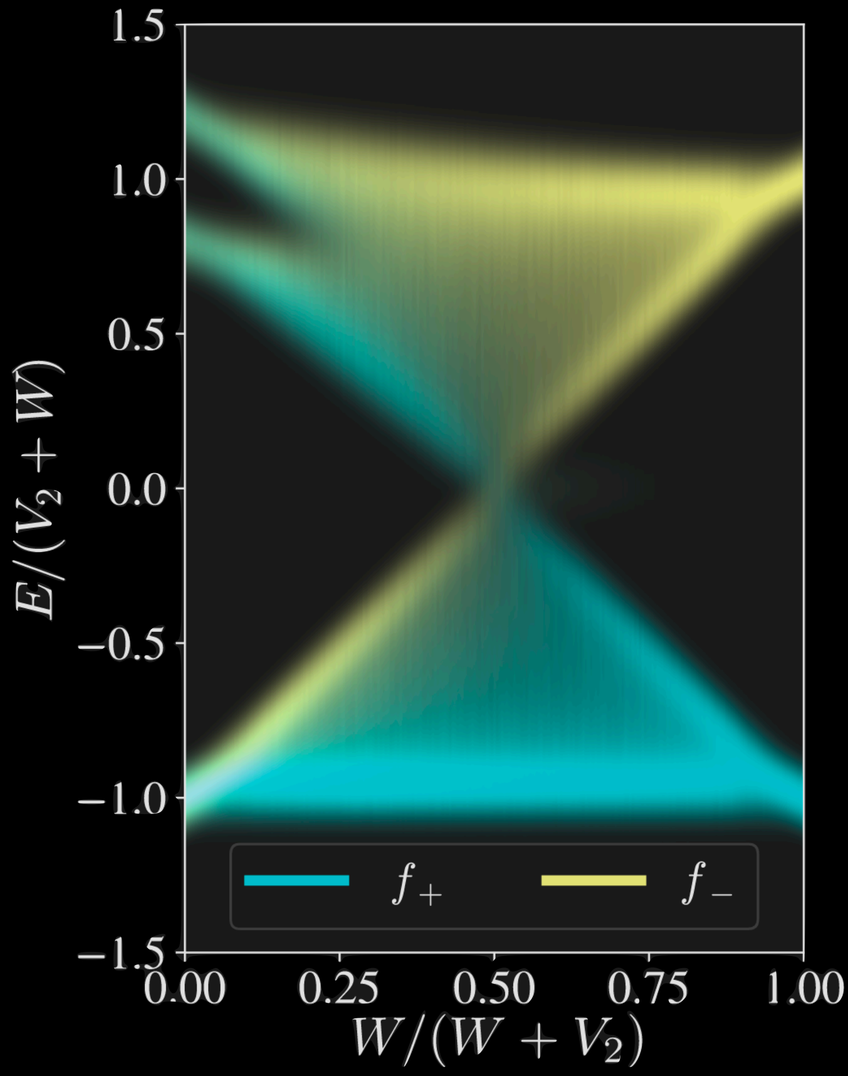


Symmetry indicated

Q. Marsal, D. Varjas, AGG PNAS, (2020)

$W \ll V_{1,2}$ $W \gg V_{1,2}$

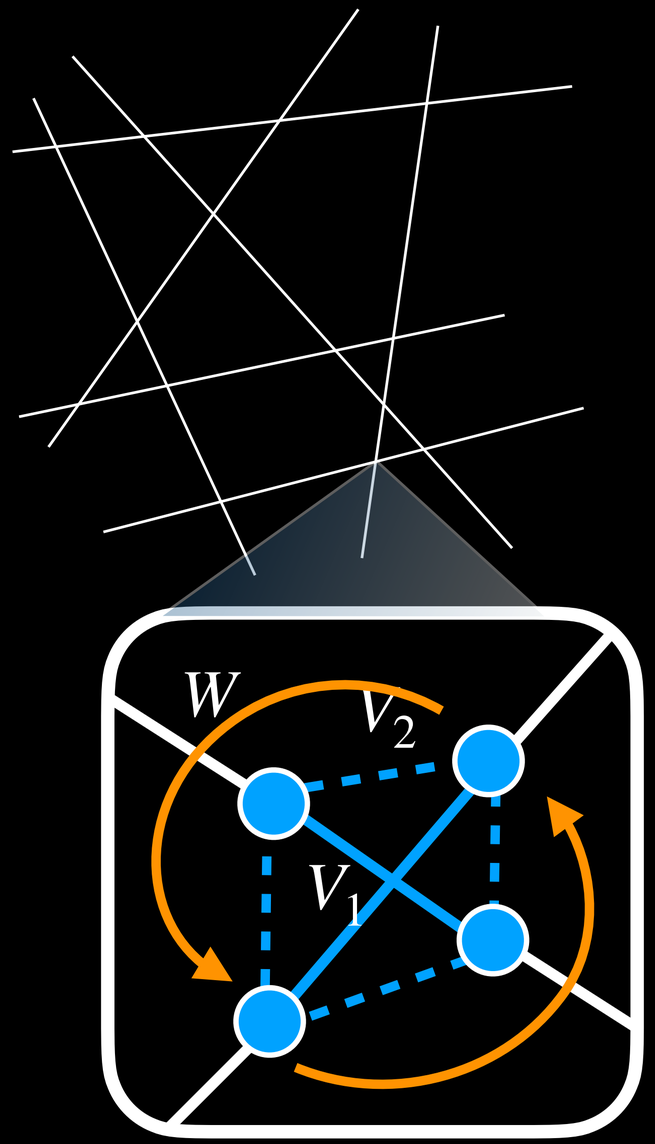
DOS



anti-bonding

bonding

Mikado model (2D or 3D)

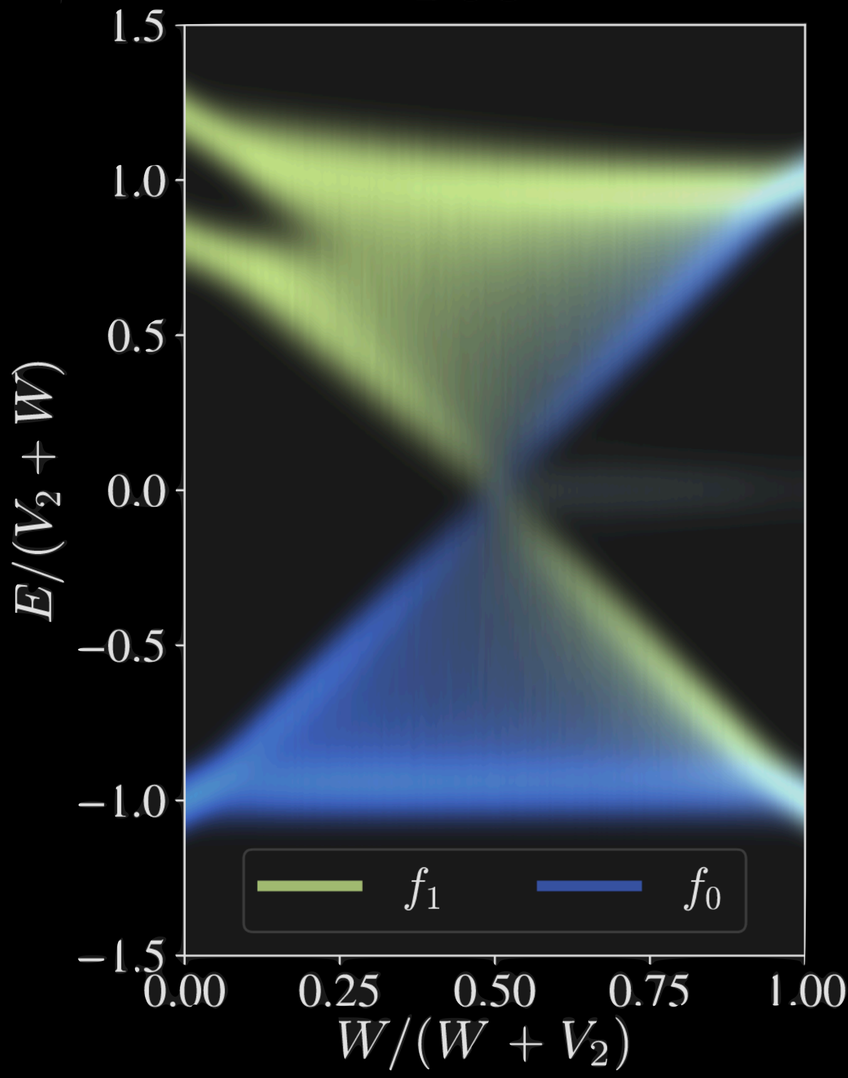


Symmetry indicated

Q. Marsal, D. Varjas, AGG PNAS, (2020)

$W \ll V_{1,2}$ $W \gg V_{1,2}$

DOS



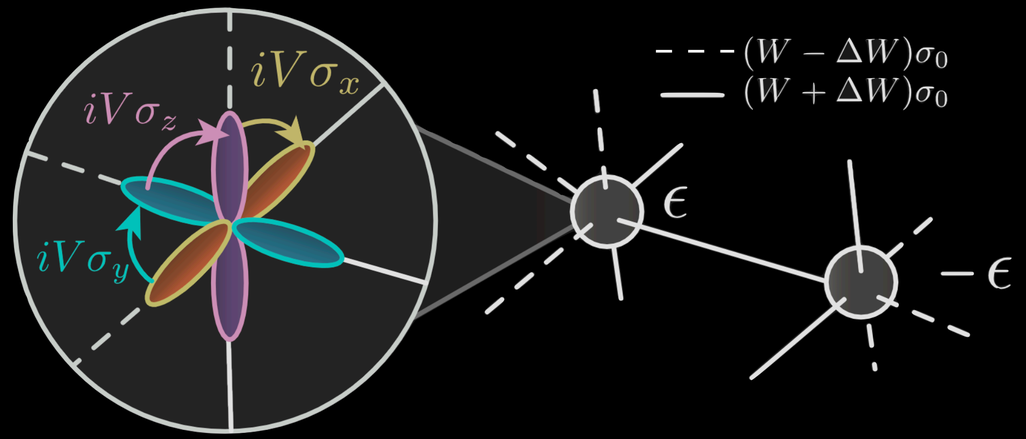
local C2 eigenvals

F_1

F_0

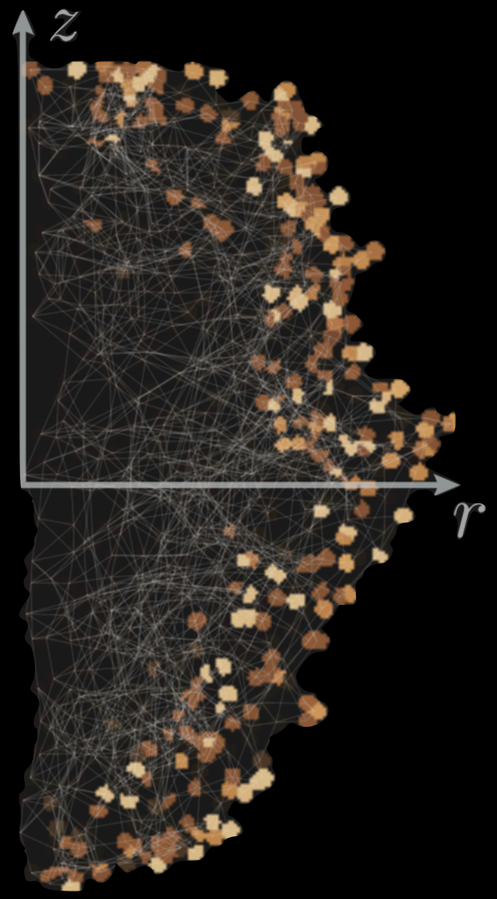
Phase change material GeTe

3-orbital tight-binding model



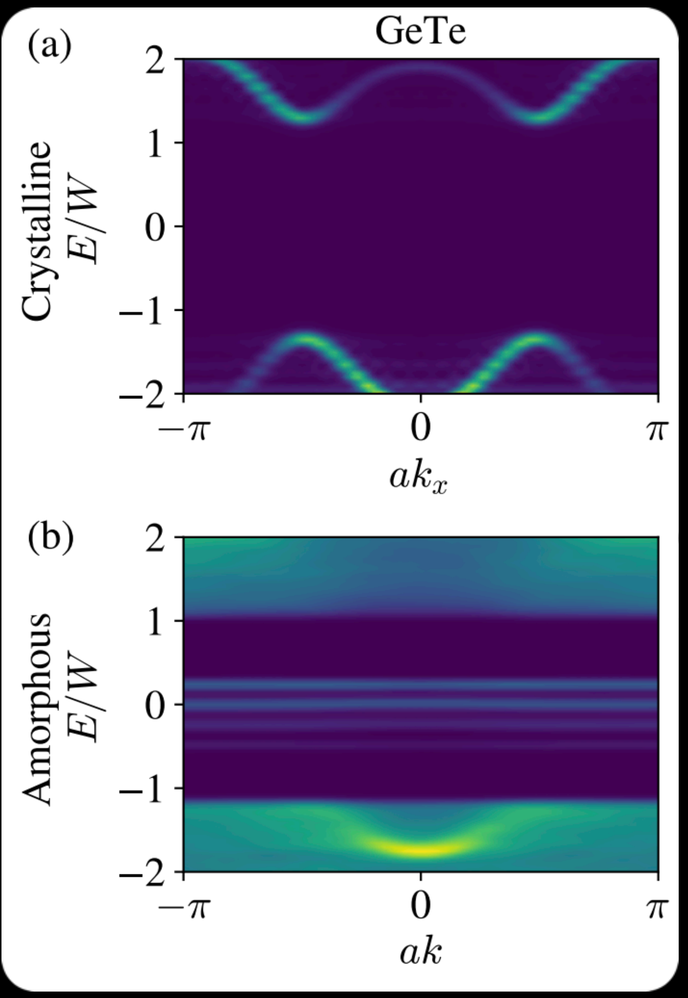
hopping + SOC + onsite energy diff

in-gap LDOS



2D layer of localized surface states

ARPES

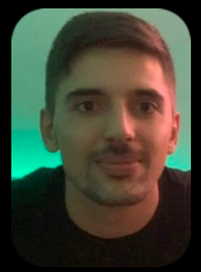


A flat band switch!

Predicting amorphous topological solids efficiently?

Structural spillage

D. Muñoz-Segovia, P. Corbae, F. Hellman, D. Varjas, S. Griffin, AGG, arXiv: 2301.02686



Daniel Muñoz-Segovia

DIPC

Paul Corbae

Berkeley

Frances Hellmann

Berkeley

Daniel Varjas

MPI Dresden

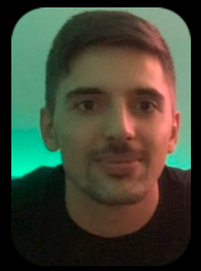
Sinead Griffin

LBNL Berkeley

Predicting amorphous topological solids efficiently?

compare crystal — known topology

with amorphous — unknown topology



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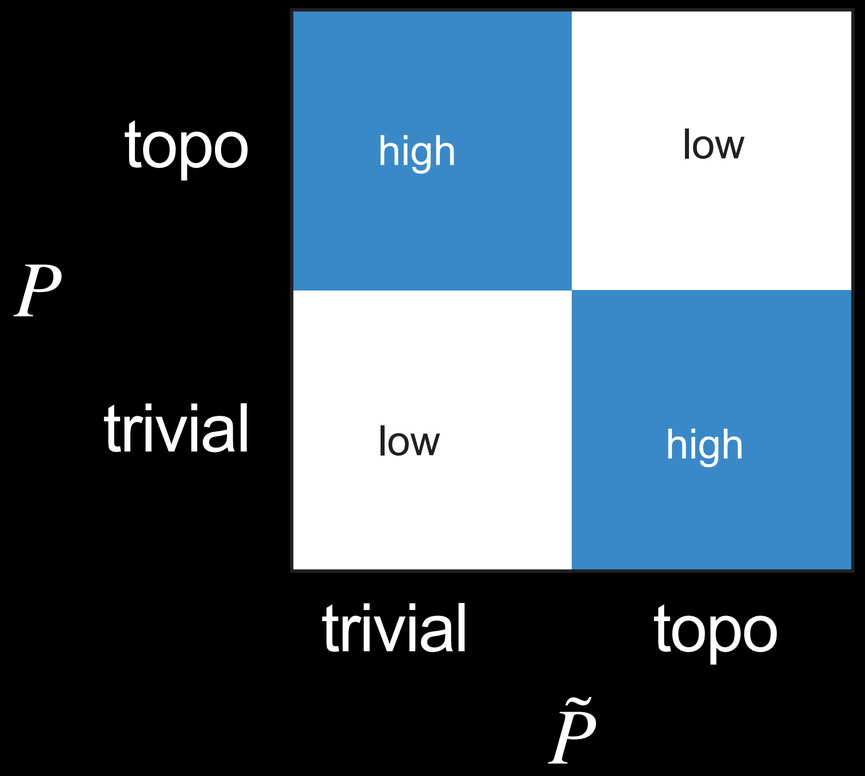
Spillage



$$P = \sum_{n \in \text{occ}} |\psi_n\rangle \langle \psi_n|$$

$$\gamma = \text{Tr}[(P - \tilde{P})^2]/2$$

Test system (unknown)

Reference system (known)

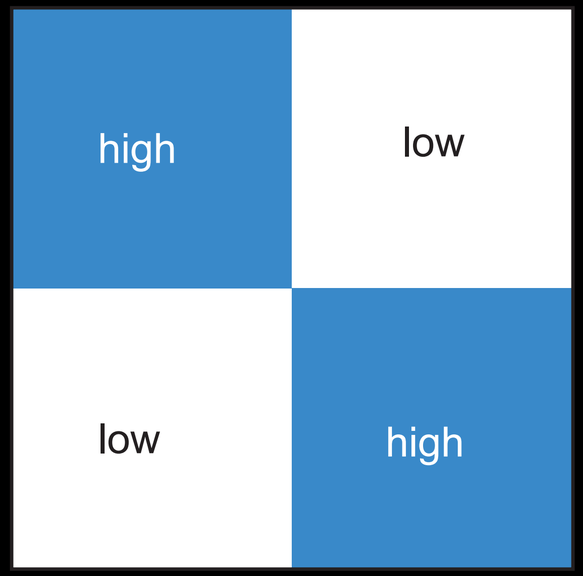
spin-orbit spillage — with spin-orbit vs without spin-orbit

Liu and Vanderbilt PRB (2014)

structural spillage — amorphous vs crystalline

D. Muñoz-Segovia, P. Corbae, D. Varjas, S. Griffin, AGG, arXiv: 2301.02686

Spillage



$$P = \sum_{n \in \text{occ}} |\psi_n\rangle \langle \psi_n|$$

$$\gamma = \text{Tr}[(P - \tilde{P})^2]/2$$

Test system (unknown)

Reference system (known)

*k and G are well defined

Momentum resolved

$$\gamma_{\text{qB}}(\boldsymbol{k}) = \frac{1}{2}\left\{ \left[\sum_{\boldsymbol{G}\alpha} P^{\alpha\alpha}_{\boldsymbol{k}+\boldsymbol{G},\boldsymbol{k}+\boldsymbol{G}} \right] + \tilde{n}_{\text{occ}}(\boldsymbol{k}) - \sum_{\boldsymbol{G}\alpha} \sum_{\boldsymbol{G}'\beta} \left[P^{\alpha\beta}_{\boldsymbol{k}+\boldsymbol{G},\boldsymbol{k}+\boldsymbol{G}'} \tilde{P}^{\beta\alpha}_{\boldsymbol{k}+\boldsymbol{G}',\boldsymbol{k}+\boldsymbol{G}} + \tilde{P}^{\alpha\beta}_{\boldsymbol{k}+\boldsymbol{G},\boldsymbol{k}+\boldsymbol{G}'} P^{\beta\alpha}_{\boldsymbol{k}+\boldsymbol{G}',\boldsymbol{k}+\boldsymbol{G}} \right] \right\}$$

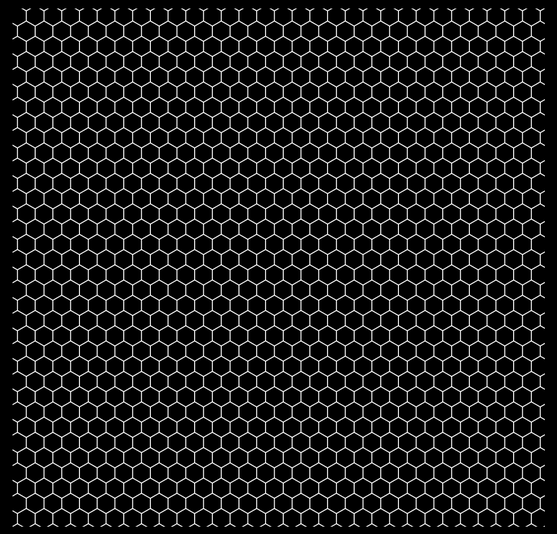
structural spillage

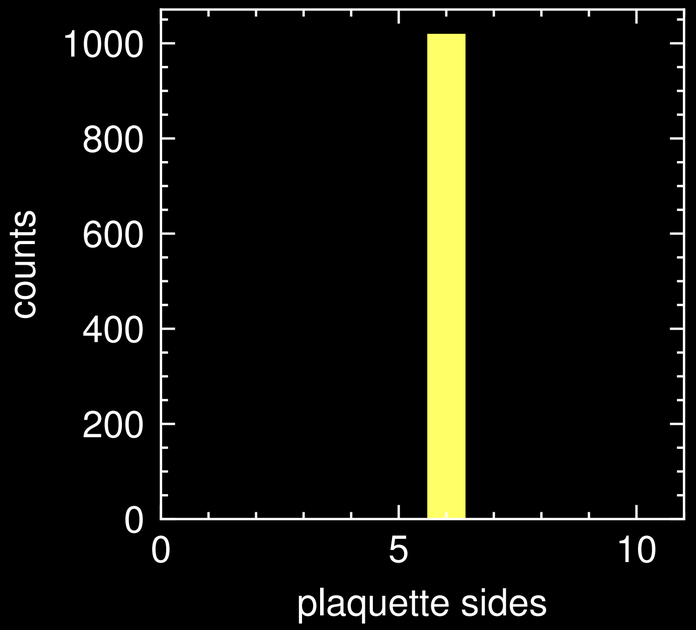
amorphous vs crystalline

D. Muñoz-Segovia, P. Corbae, D. Varjas, S. Griffin, AGG, arXiv: 2301.02686

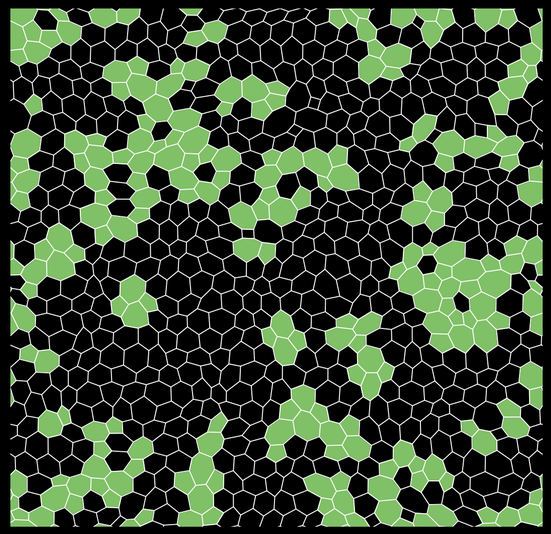
A new knob

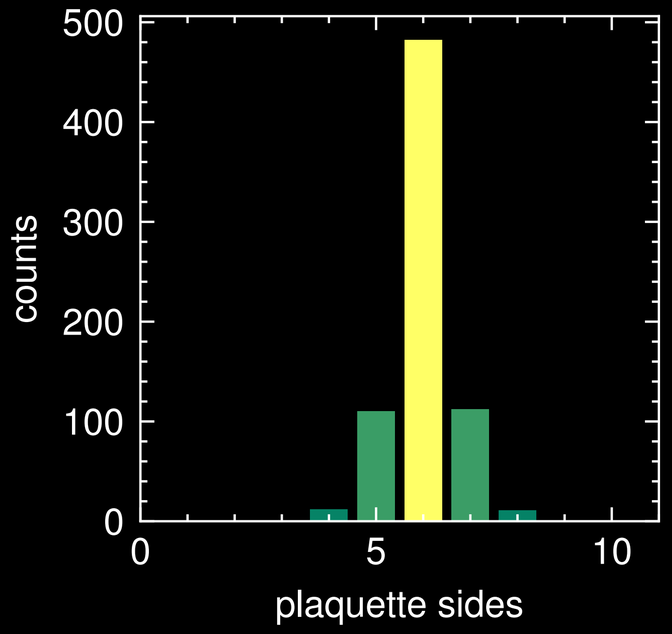
crystal



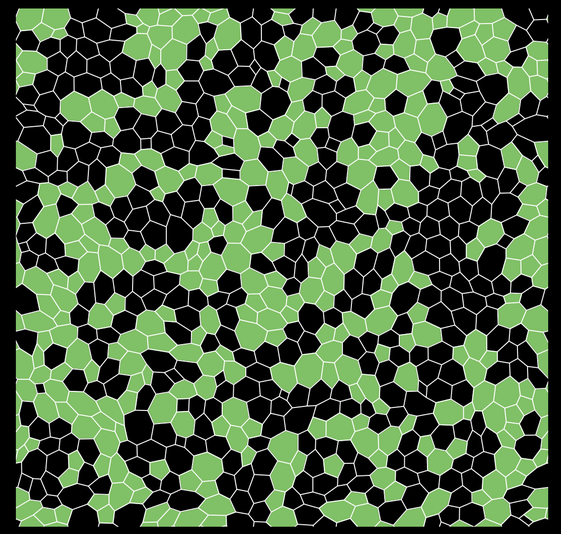


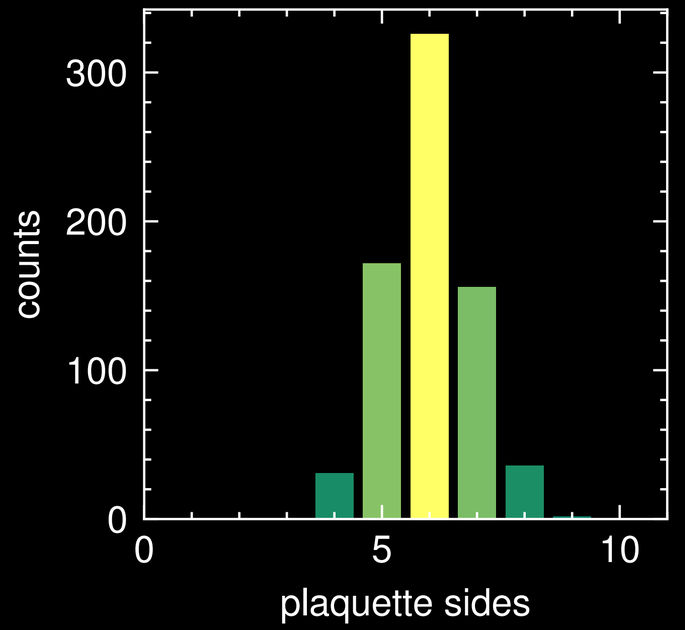
polycrystal





amorphous

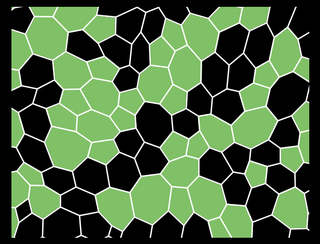


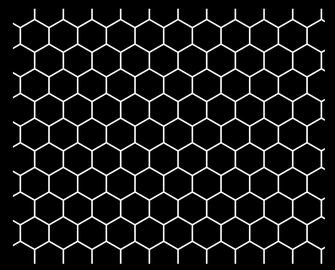


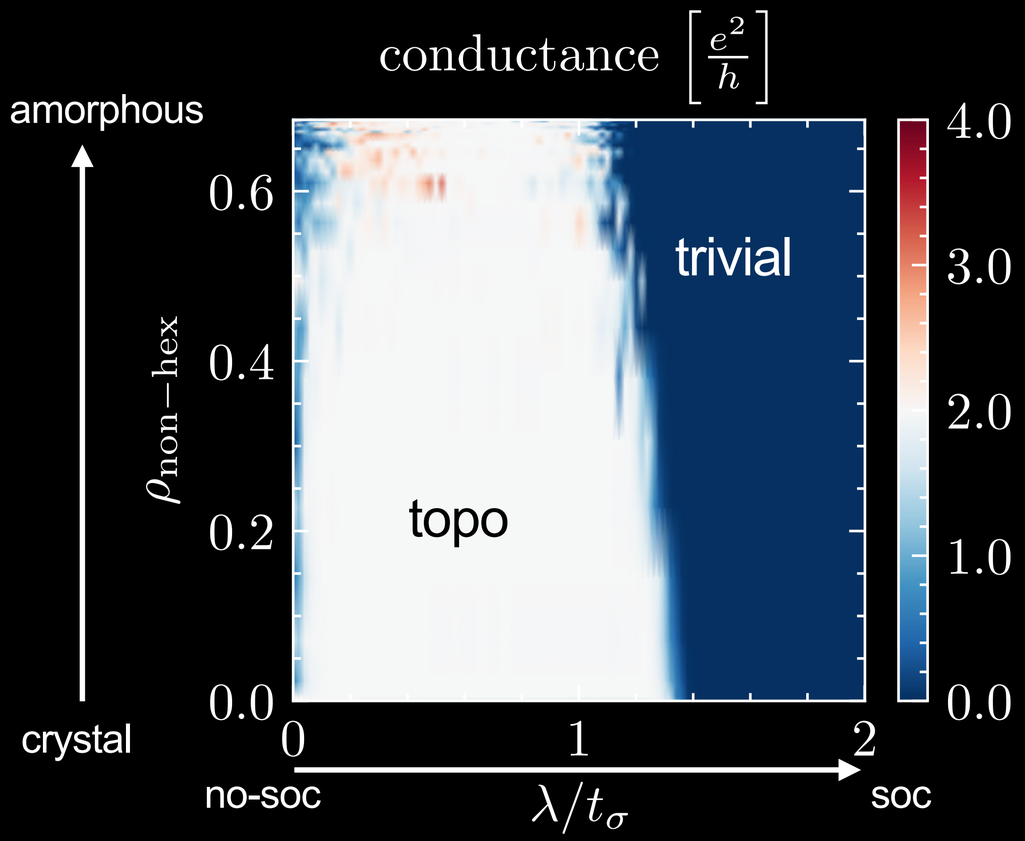
Tight-binding benchmark

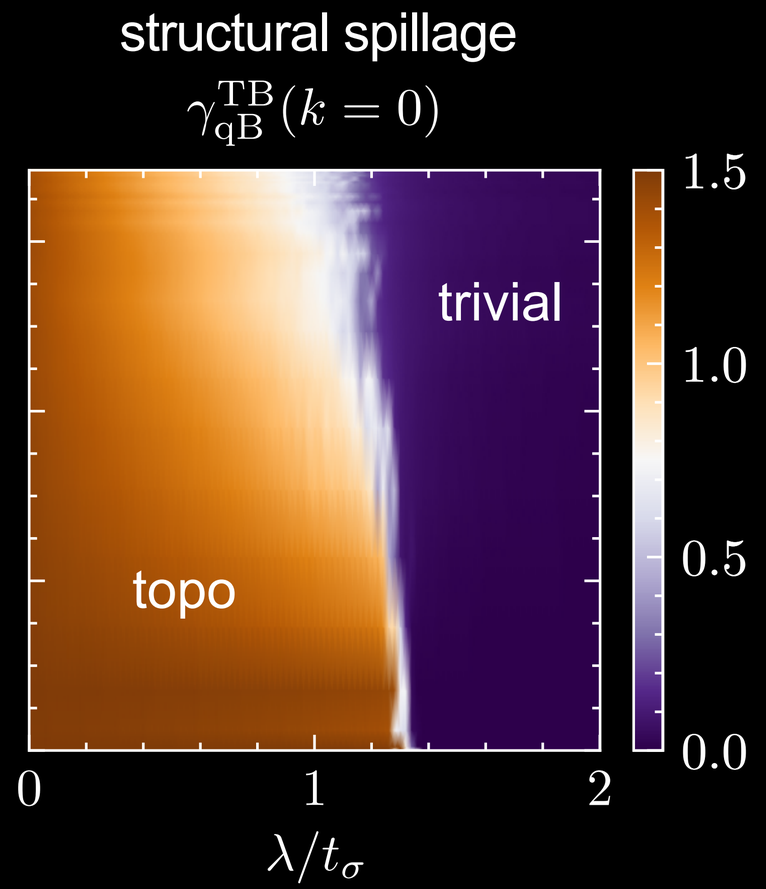
Amorphous Bismuthene

Costa, et al Nano Letters (2019)
Focasio et al 2D Materials (2021)









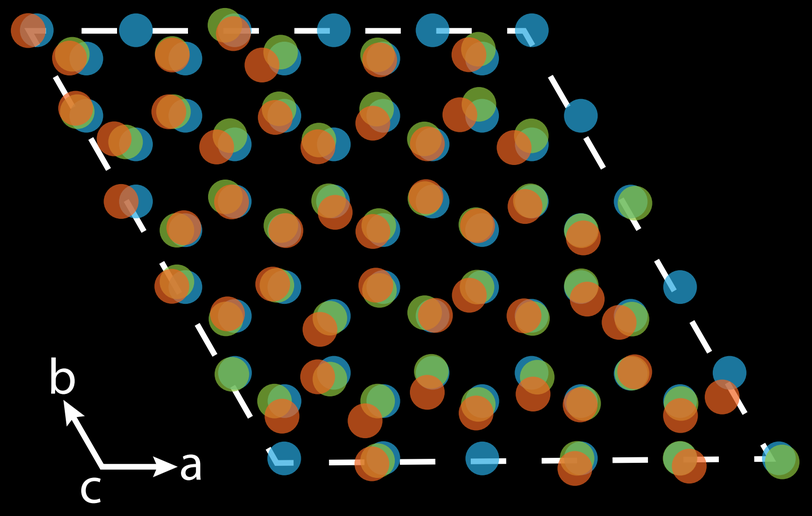
D. Muñoz-Segovia, P. Corbae, D. Varjas, S. Griffin, AGG, arXiv: 2301.02686

DFT benchmark

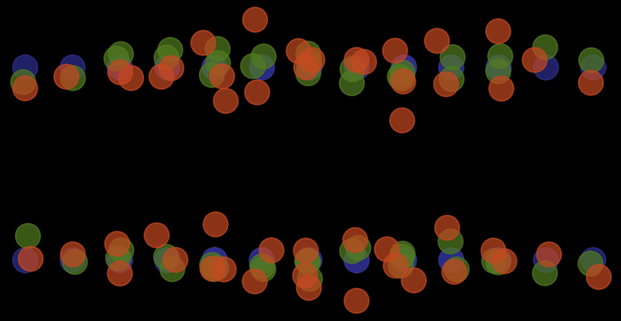
amorphous Bismuth bilayer

DFT compatible = high-throughput search!

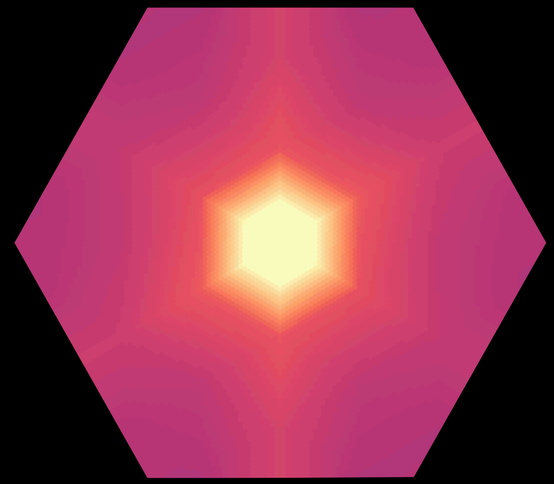
top view



side view



structural spillage



D. Muñoz-Segovia, P. Corbae, D. Varjas, S. Griffin, AGG, arXiv: 2301.02686

What's new?

Flat-band switch

Q. Marsal, D. Varjas, AGG Phys Rev B, (2020)

Quentin Marsal
Néel Institute

Daniel Varjas
MPI PKS

Amorphous chiral spin-liquid

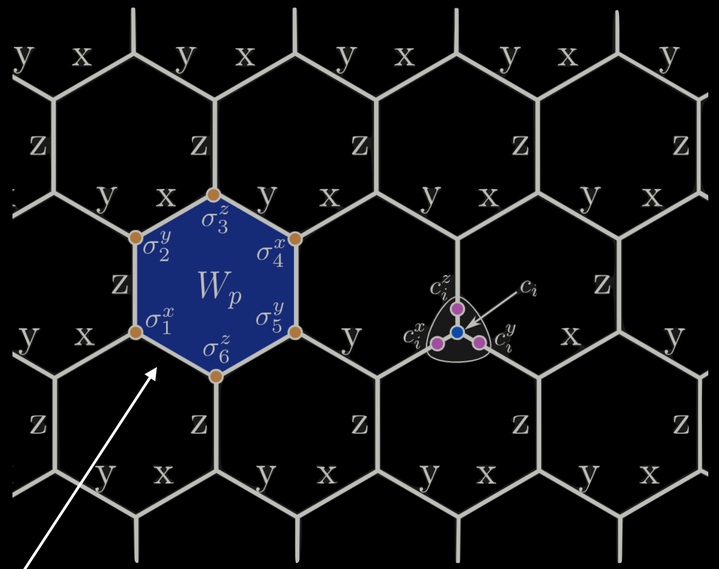
AGG, C. Repellin 2210.13548

Cécile Repellin
LPMMC / Grenoble

Kitaev models

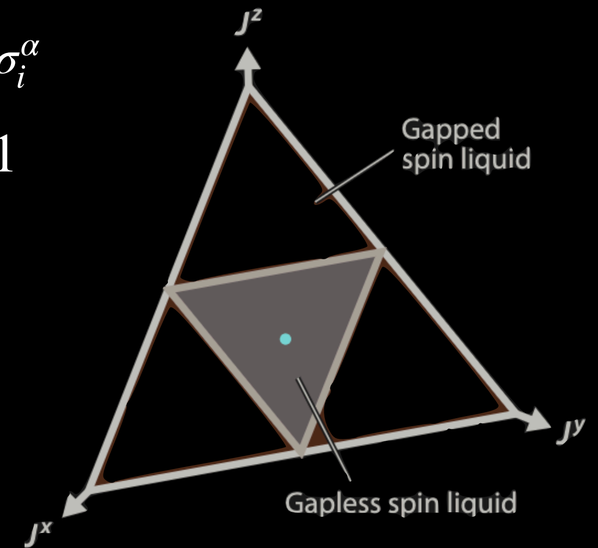
Honeycomb

Kitaev Ann. Phys. (2006)



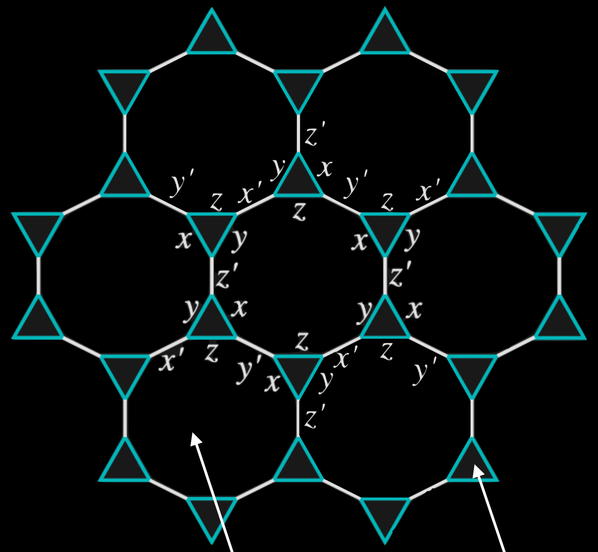
$$W_p = \prod \sigma_i^\alpha$$

$$\phi_p = \pm 1$$



Decorated Honeycomb

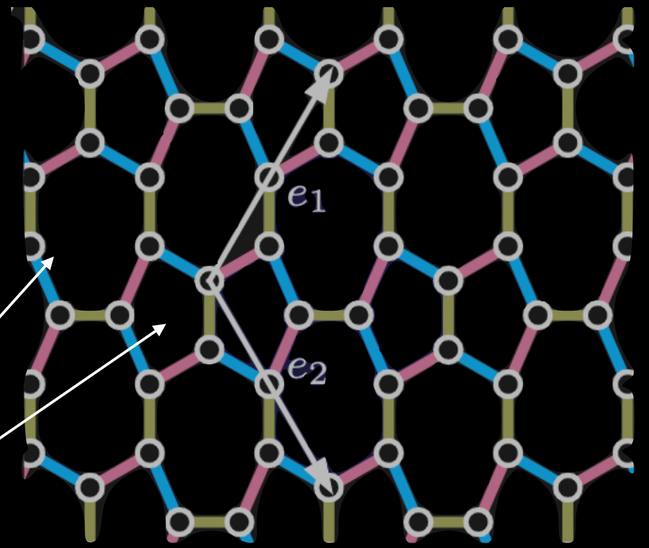
Yao and Kivelson PRL (2007)



$$\phi_p = \pm 1 \qquad \phi_p = \pm i$$

Pentaheptite lattice

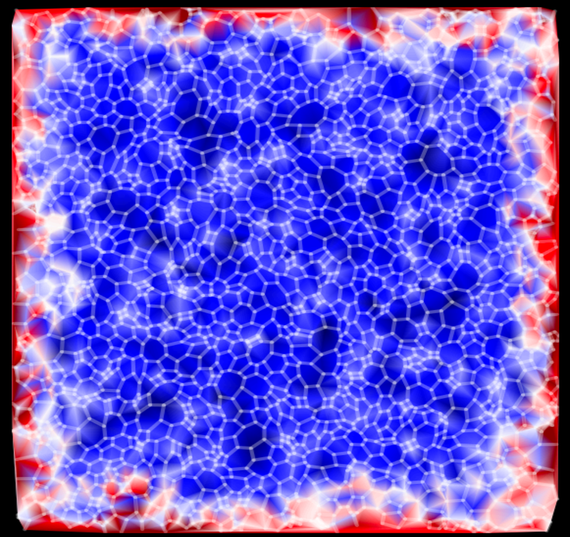
Peri et al PRB (2020)



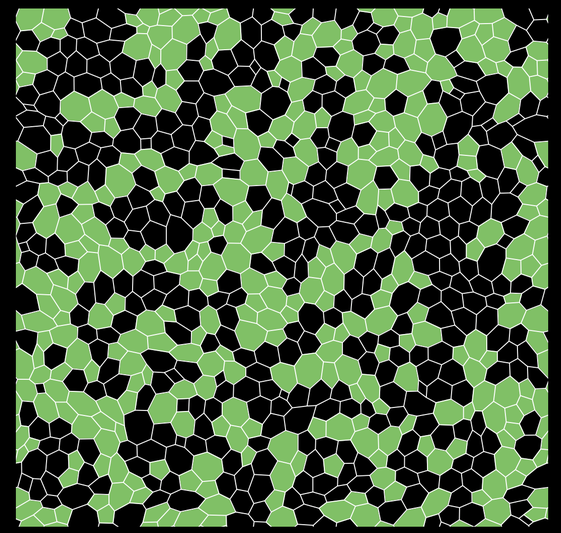
Odd plaquettes break TRS → Gapped chiral spin-liquid!

Amorphous chiral spin liquid?

Local Chern marker



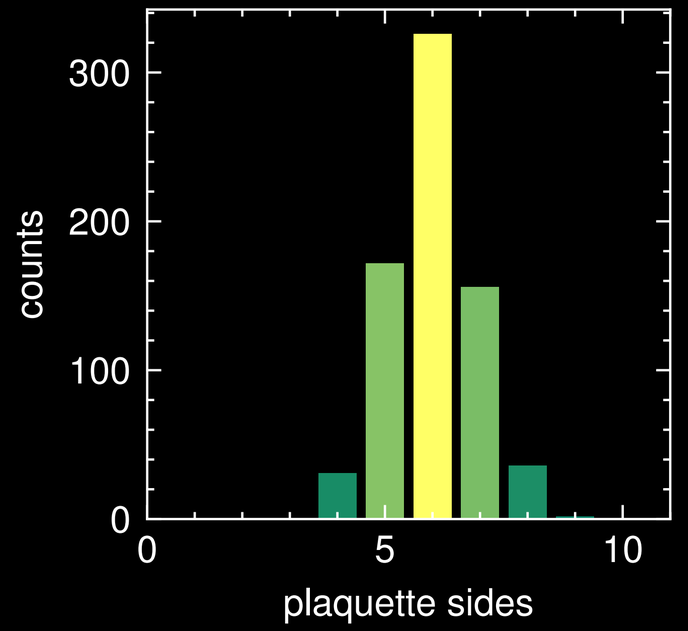
amorphous



Gapped chiral spin-liquid!

$$\phi_p = -(\pm i)^{n_{sides}}$$

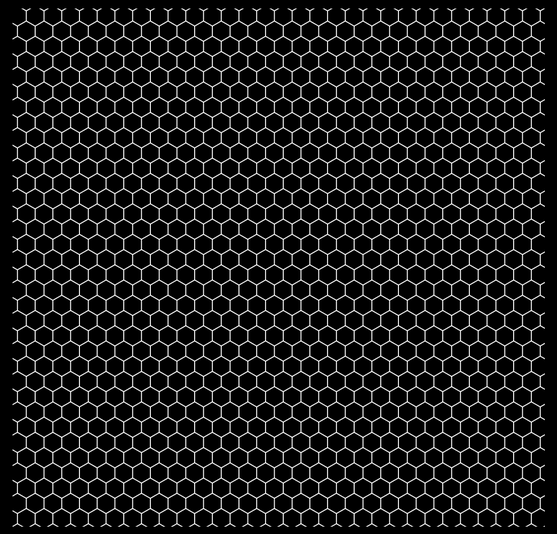
G. Casella et al 2208.08246

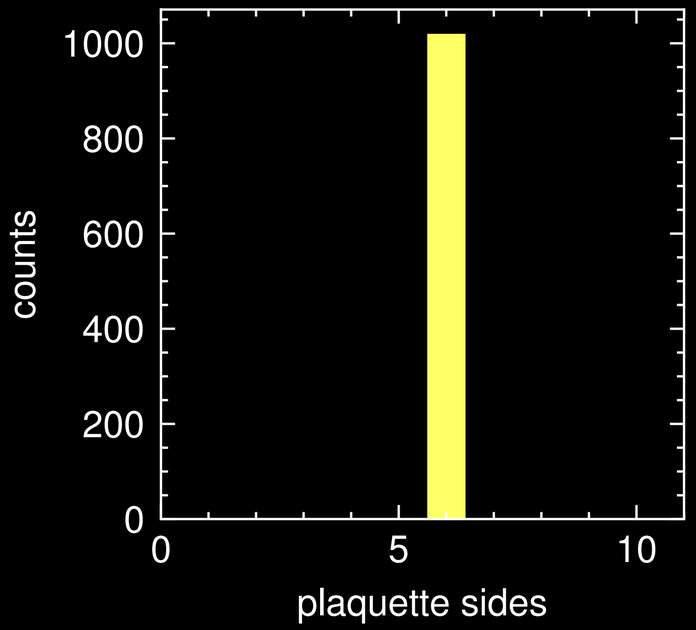


Effect of the new knob?

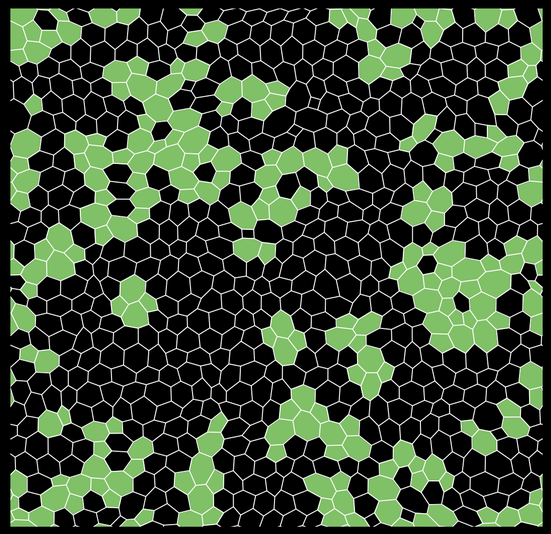
AGG, C. Repellin 2210.13548

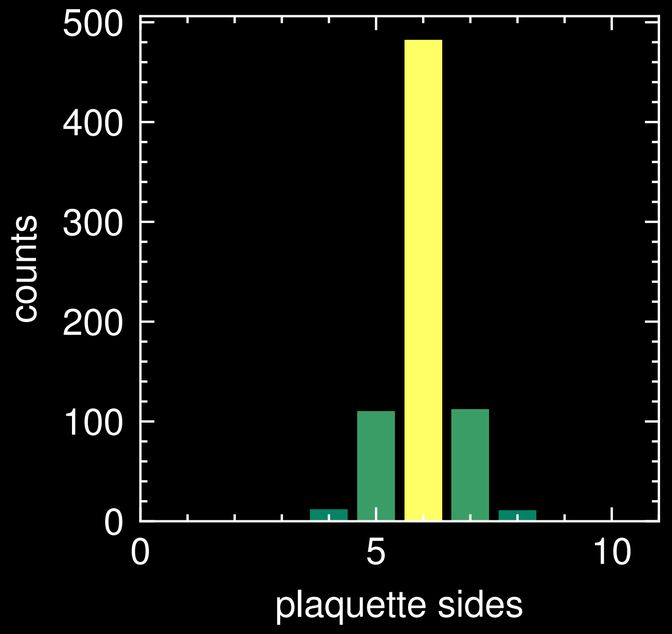
crystal



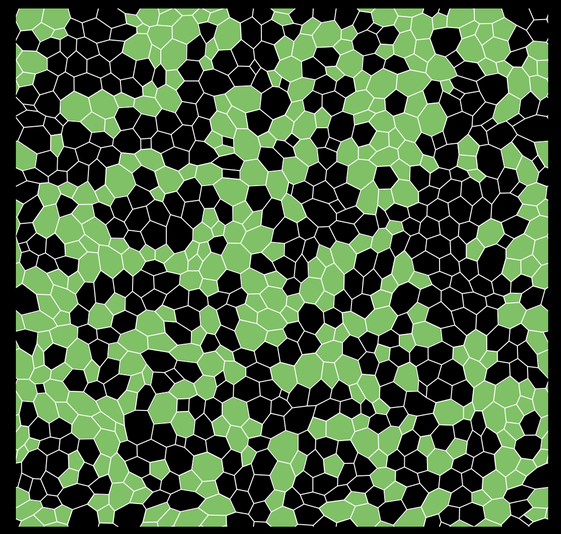


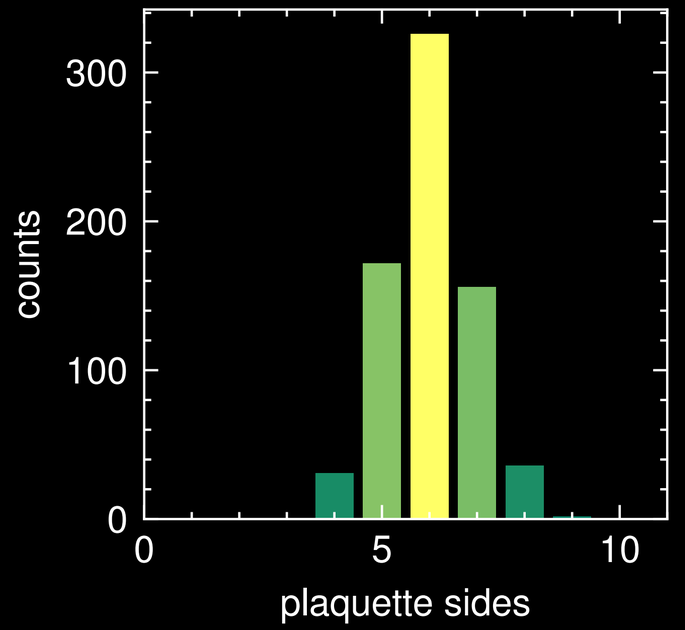
polycrystal





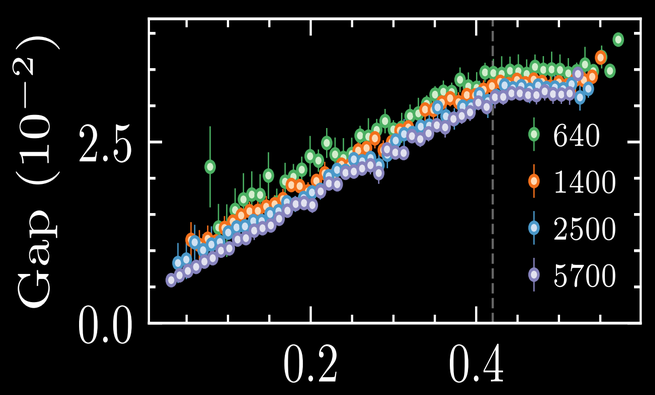
amorphous

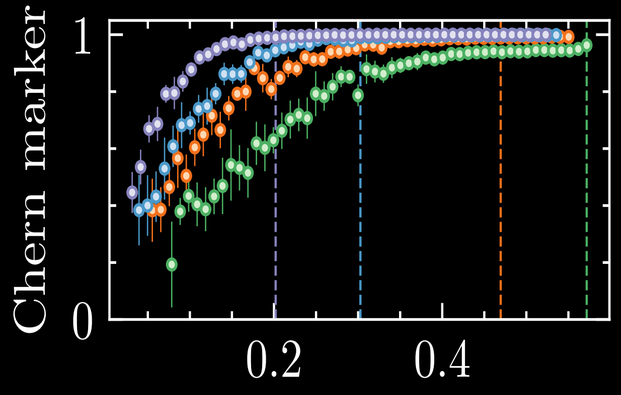




A topological gap as disorder is increased

AGG, C. Repellin 2210.13548

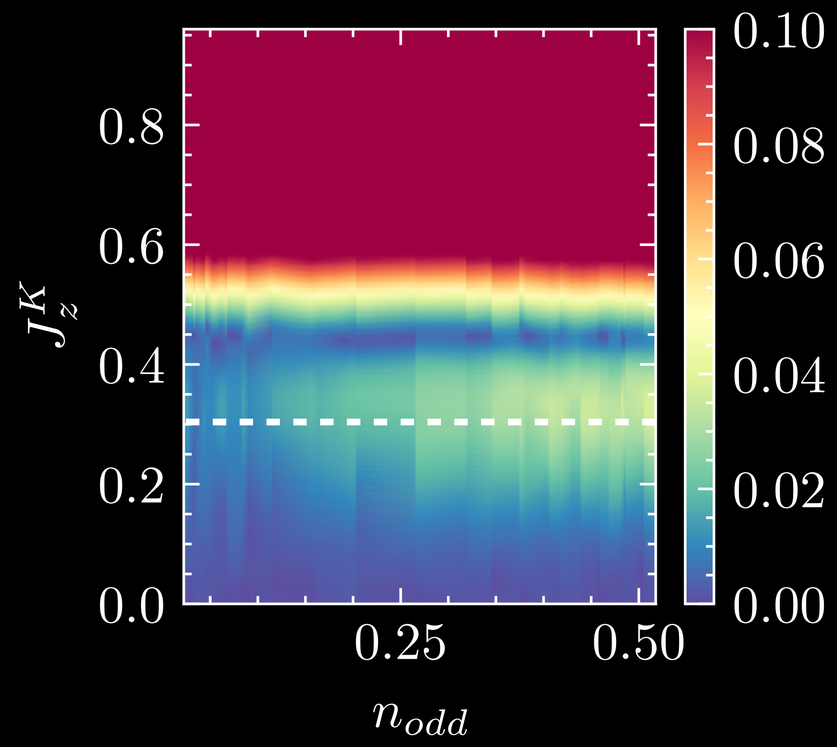




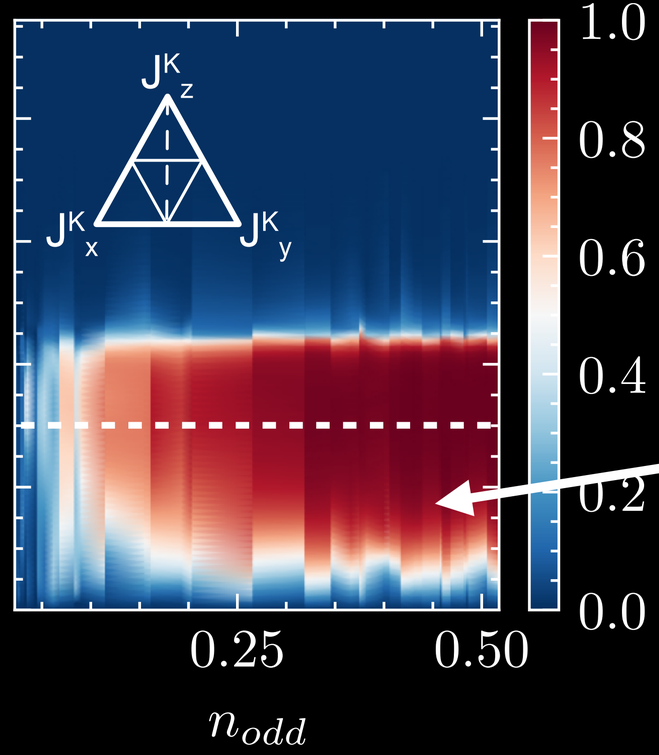
small density of odd plaquettes is enough

stable to Heisenberg interactions (confirmed by ED)

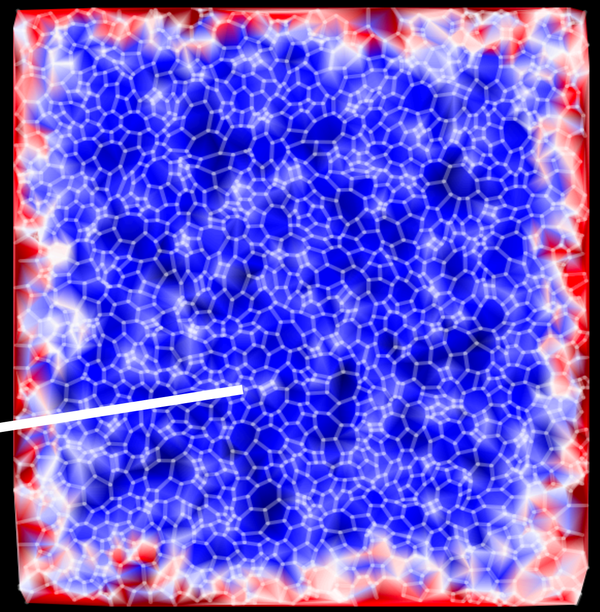
gap



Chern marker



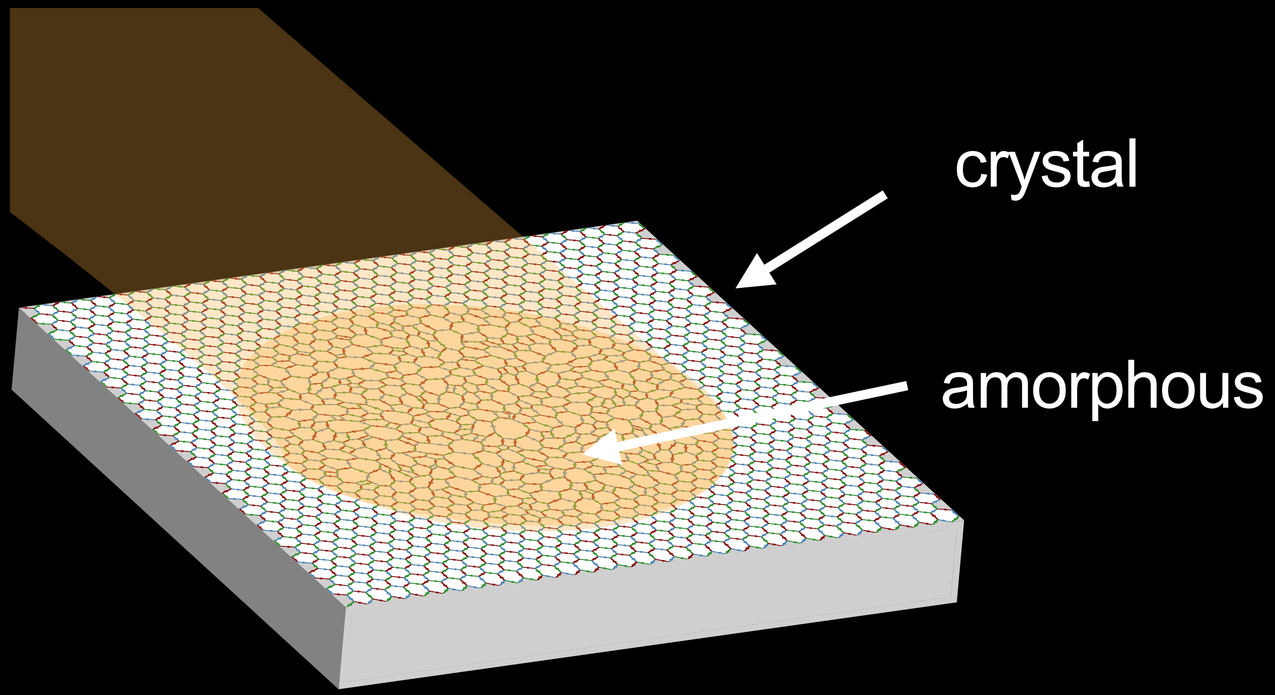
Local Chern marker



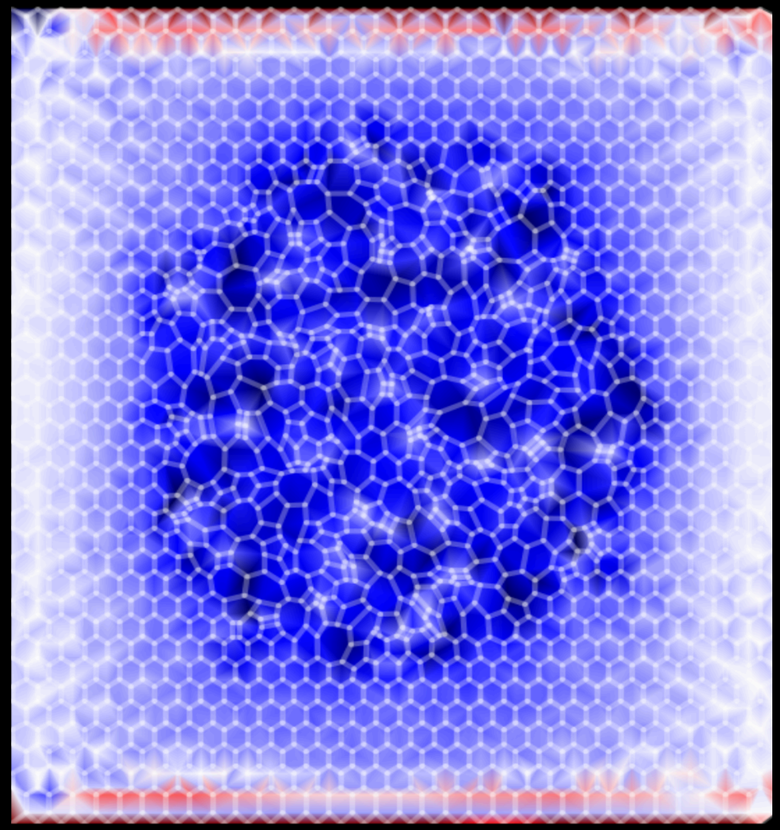
A new knob to control

AGG, C. Repellin 2210.13548

focused ion beam



Local Chern marker



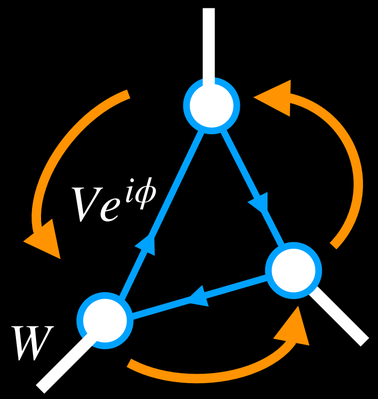
*Thanks to Julie Karel (Monash U.) for the idea

Conclusions

Finding amorphous solids efficiently

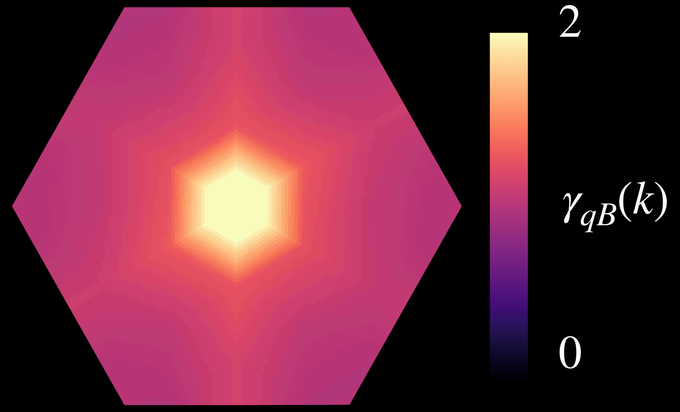
Symmetry indicators

Q. Marsal, D. Varjas, AGG PNAS, (2020)



Structural spillage

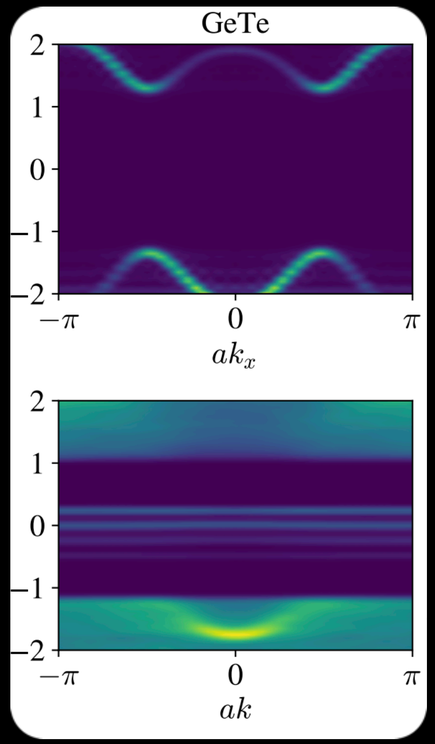
D. Muñoz-Segovia,et al arXiv: 2301.02686



New control mechanisms

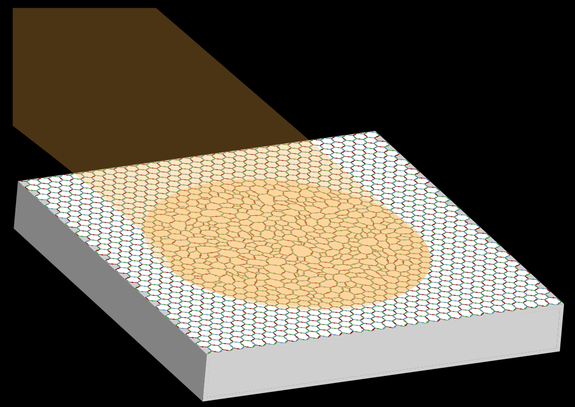
Controllable flat-bands

Q. Marsal, D. Varjas, Phys. Rev. B (2023)



Chiral spin liquids

AGG, C. Repellin 2210.13548



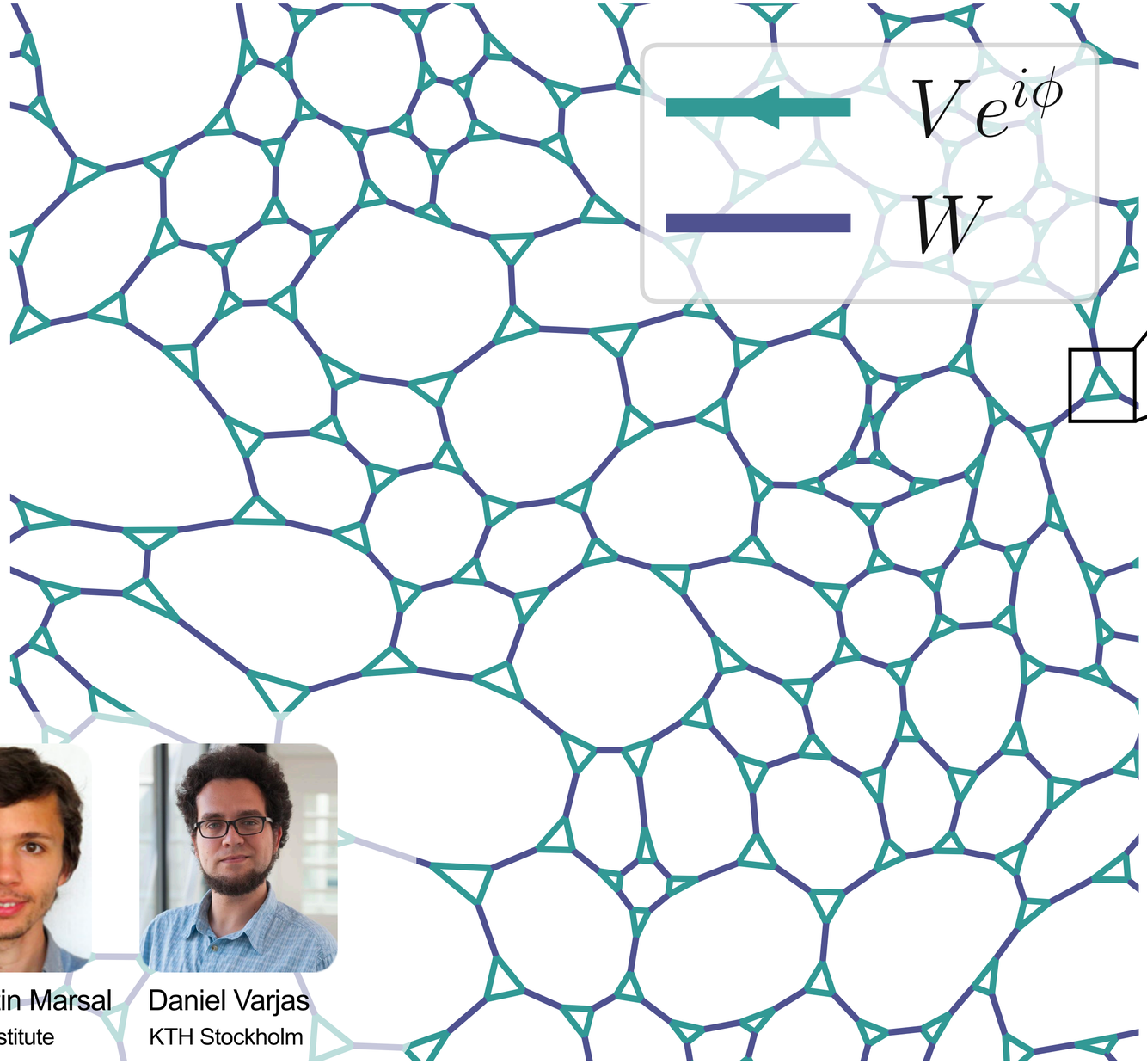
Exp: Corbae et al, AGG, Lanzara, Hellmann Nat Materials (2023)
Cyocis, Marsal et al, Hellmann, AGG, Lanzara 2302.05945

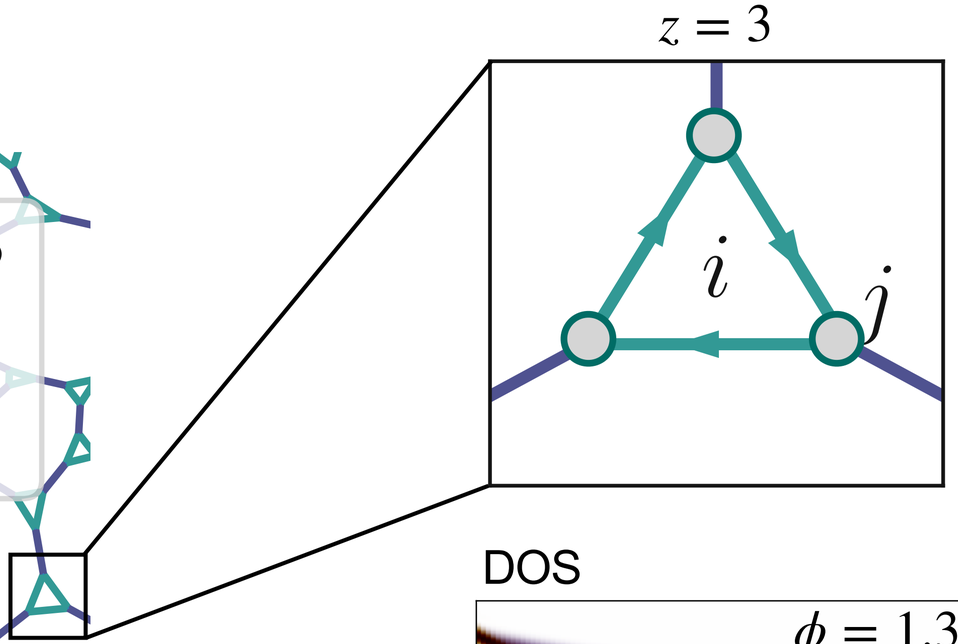
Review: P. Corbae, J. Hannukainen, Q. Marsal, D. Muñoz-Segovia, AGG
arXiv:2301.04176 (EPL in press)

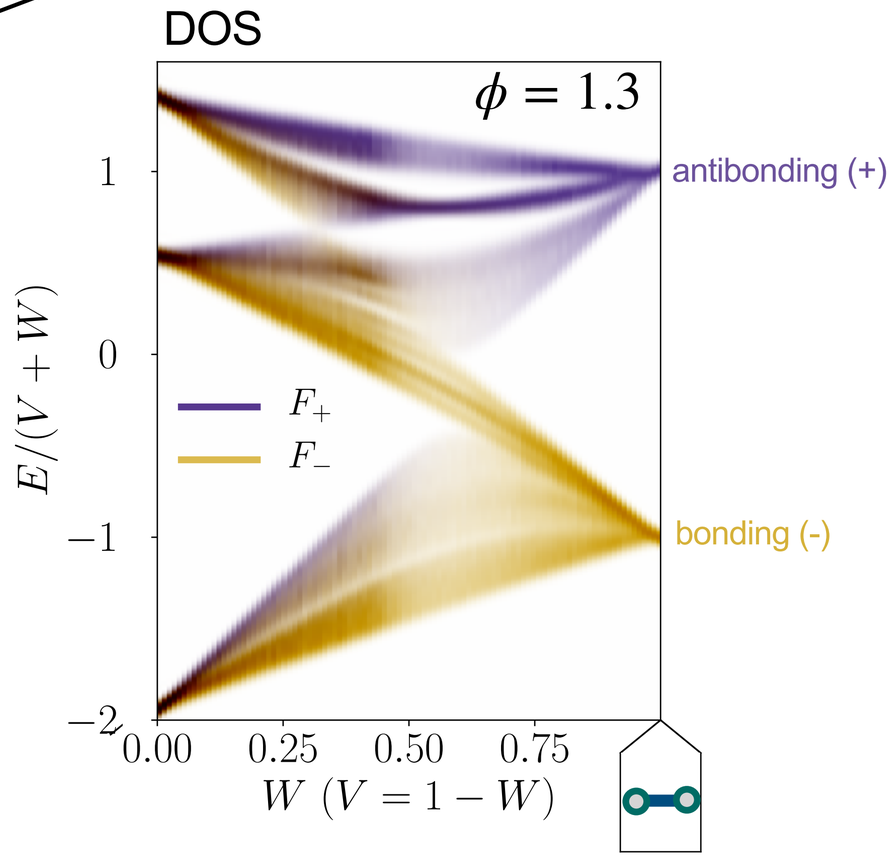
Technical details: Symmetry indicators

Topological Weaire-Thorpe model

Q. Marsal, D. Varjas, AGG PNAS, (2020)





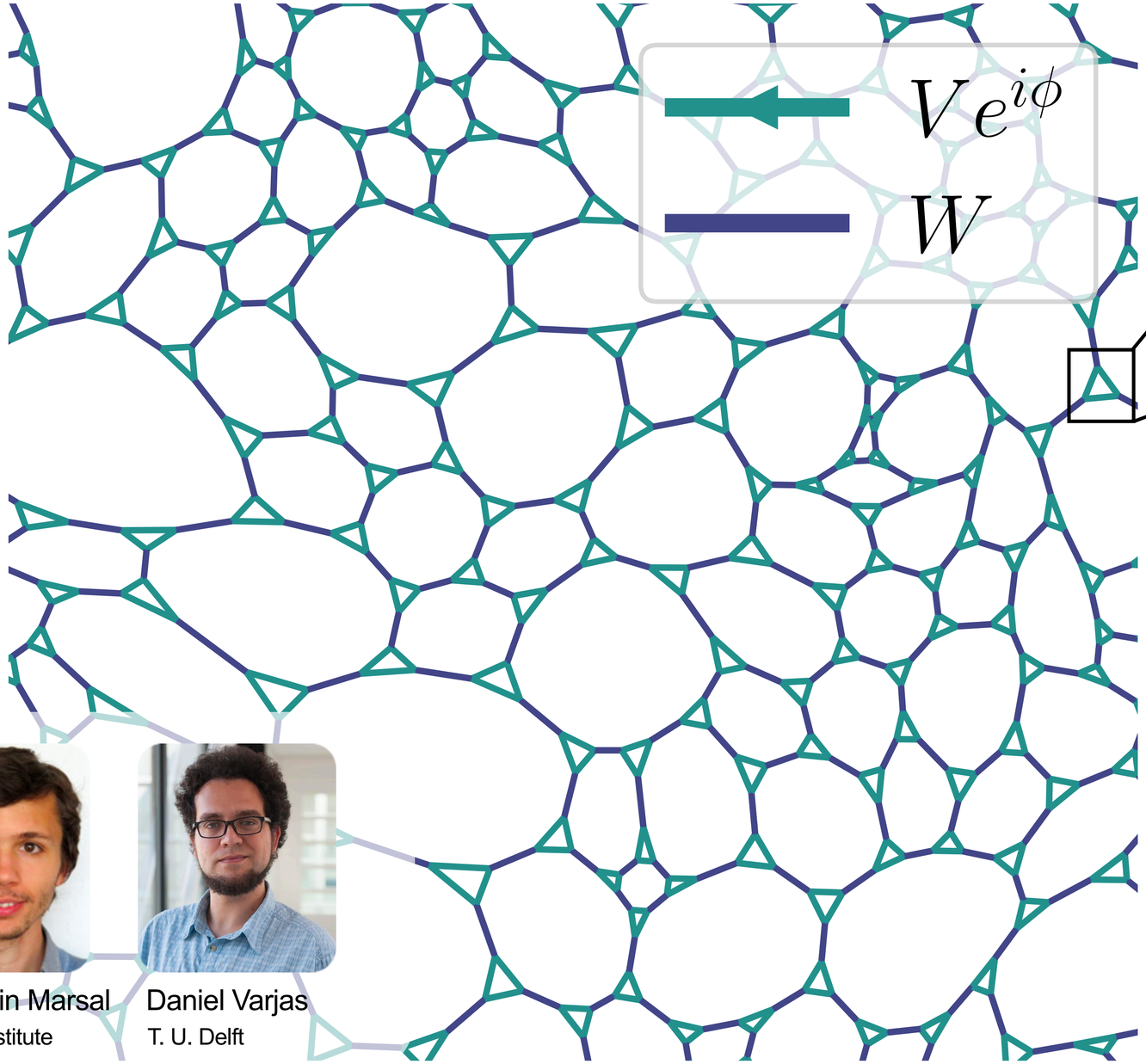


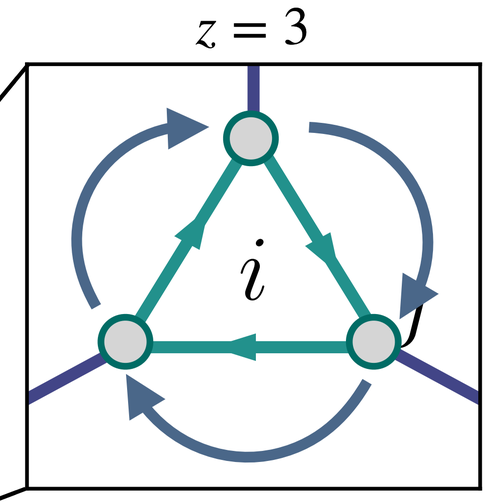
Quentin Marsal
Néel Institute

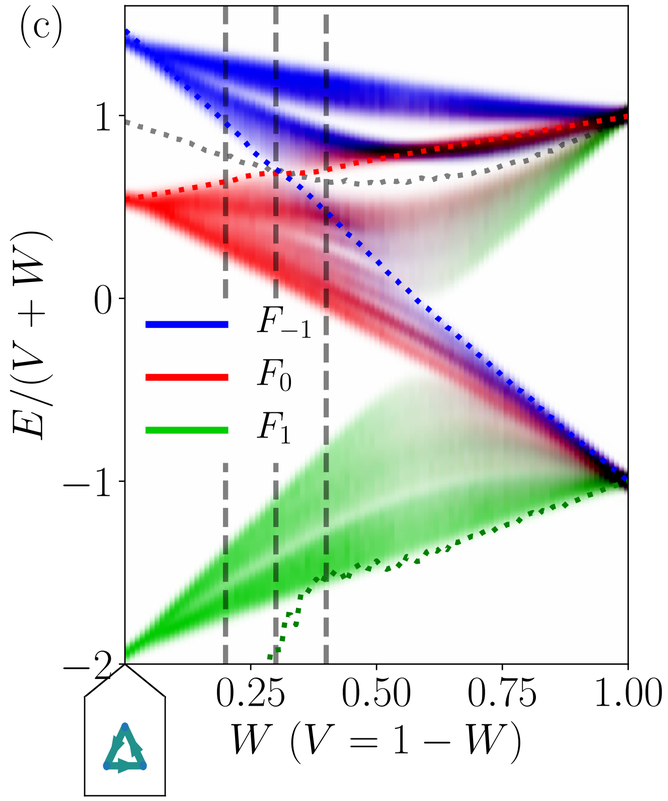
Daniel Varjas
KTH Stockholm

Topological Weaire-Thorpe model

Q. Marsal, D. Varjas, AGG PNAS, (2020)







Quentin Marsal

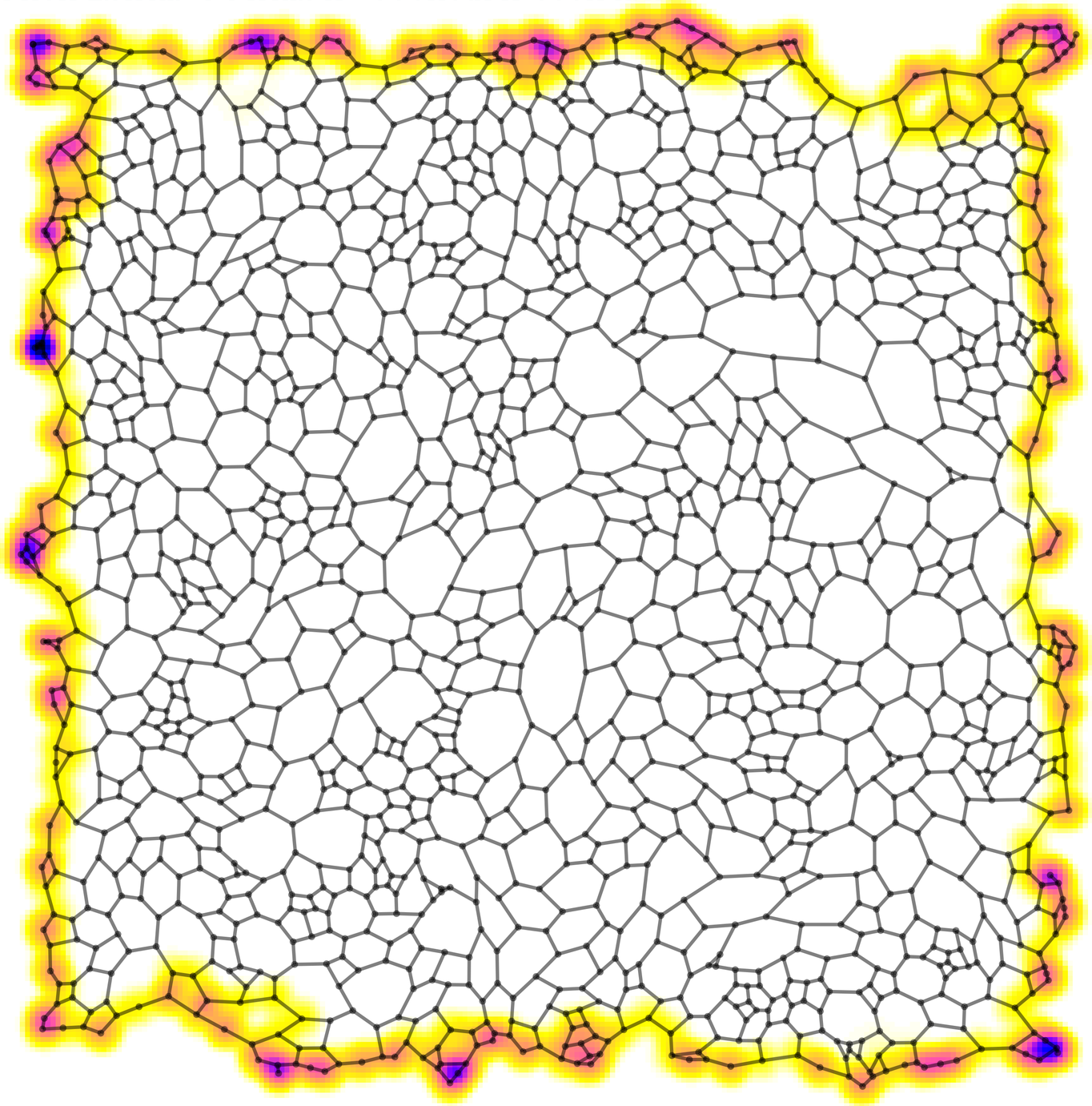
Néel Institute

Daniel Varjas

T. U. Delft

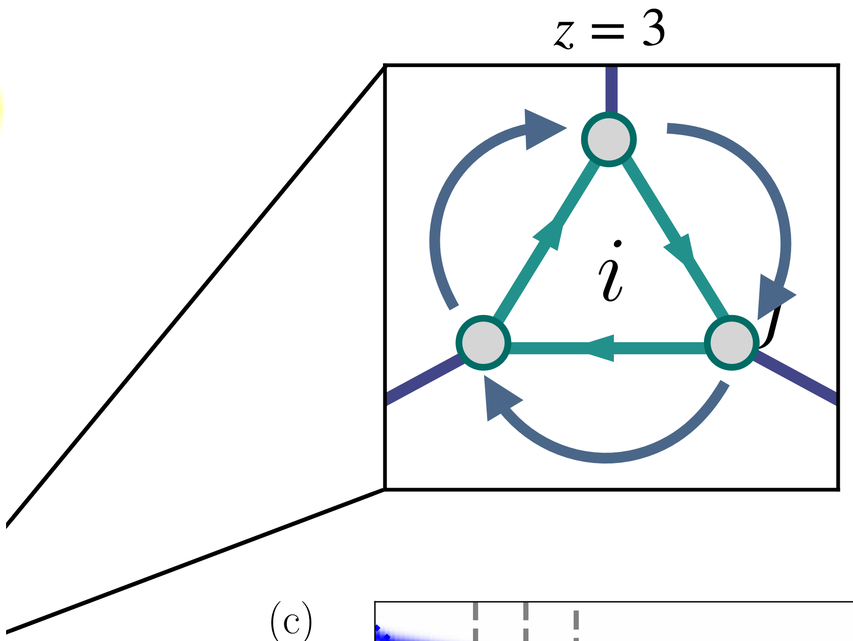
Topological Weaire-Thorpe model

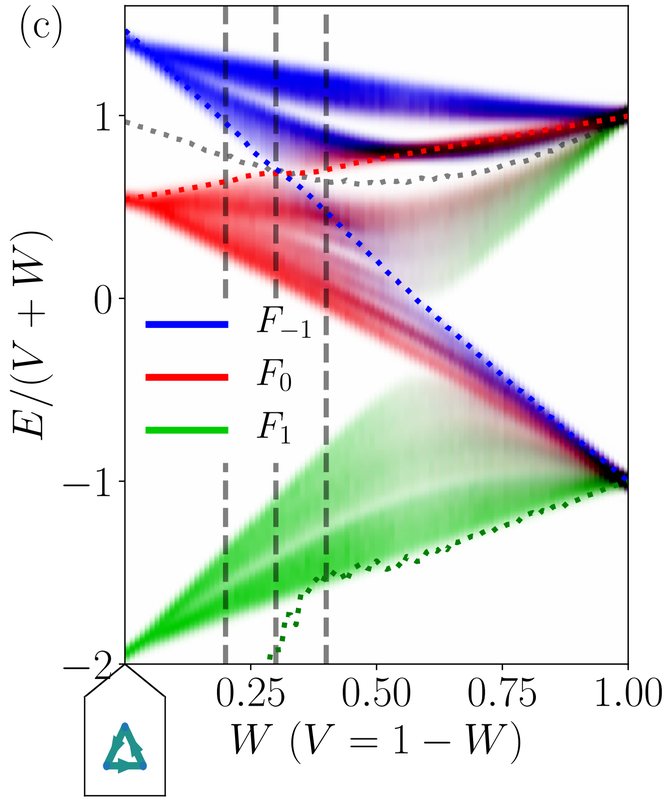
Q. M

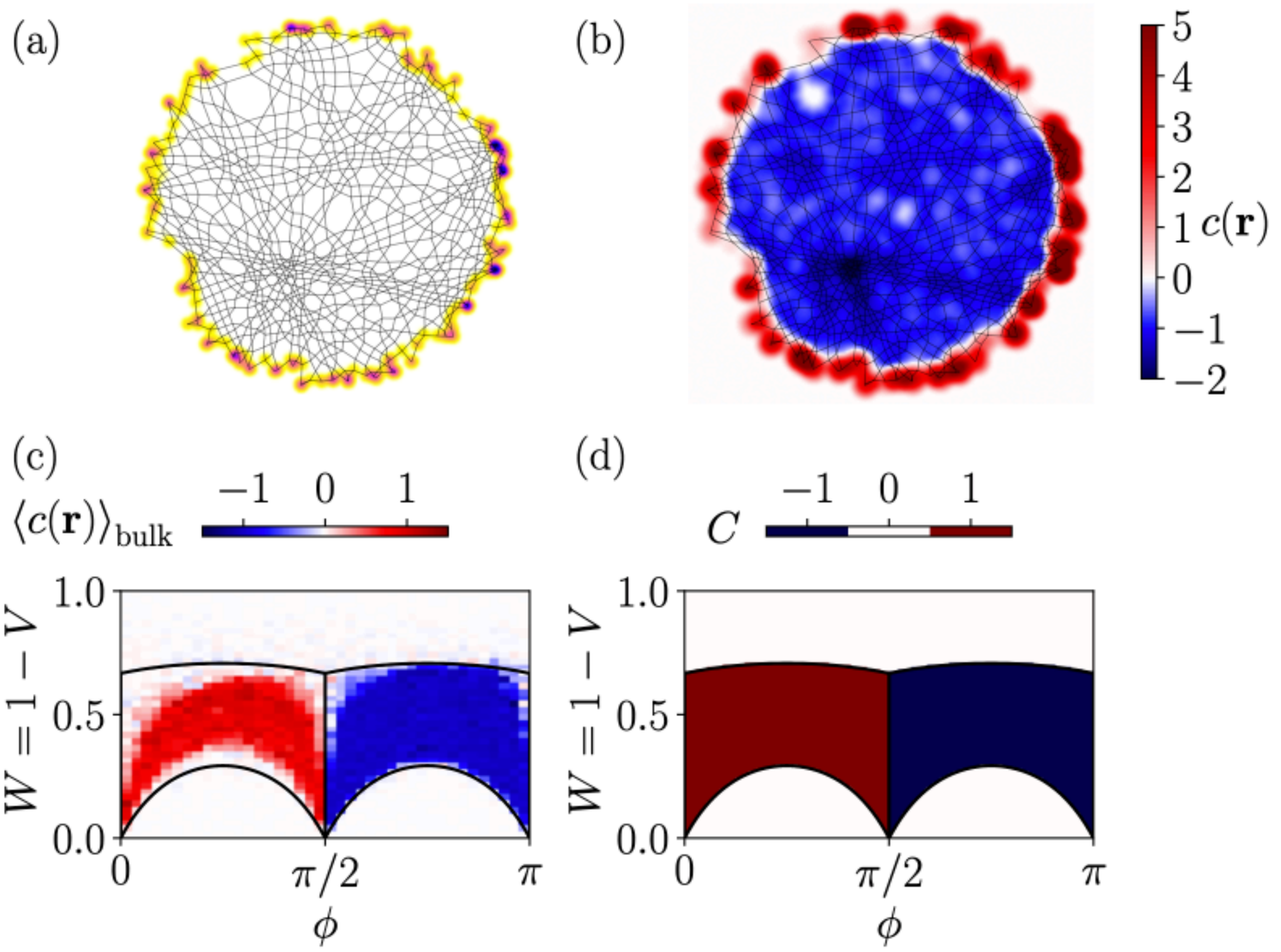


Que

Néel

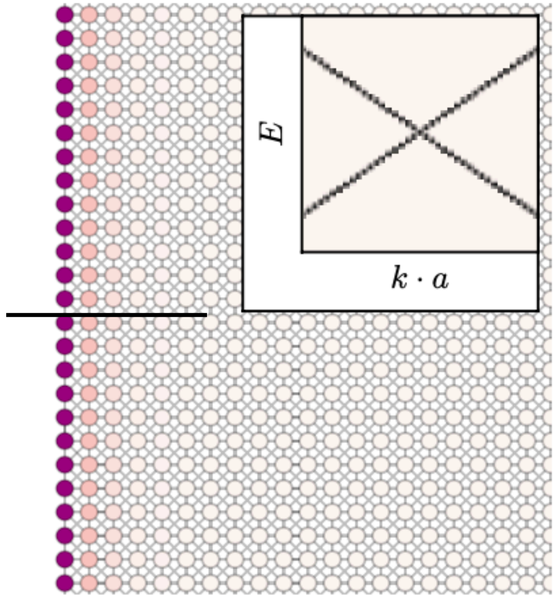





(a)
(b)
5
4
3
2
1
0
−1
−2
$c(\mathbf{r})$
(c)
$\langle c(\mathbf{r})\rangle_{\text{bulk}}$
−1 0 1
(d)
C
−1 0 1
1.0
0.5
0.0
$W = 1 - V$
0
$\pi/2$
π
ϕ

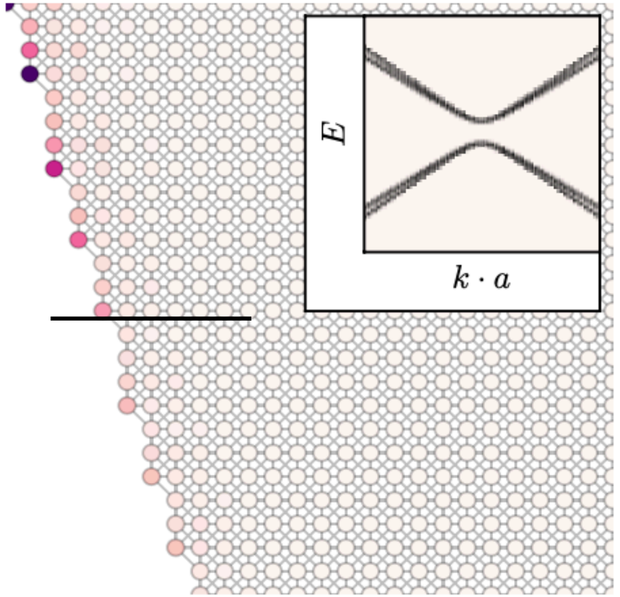
Average symmetries — enhanced robustness

Spring, Akhmerov, Varjas Sci. Post (2020)



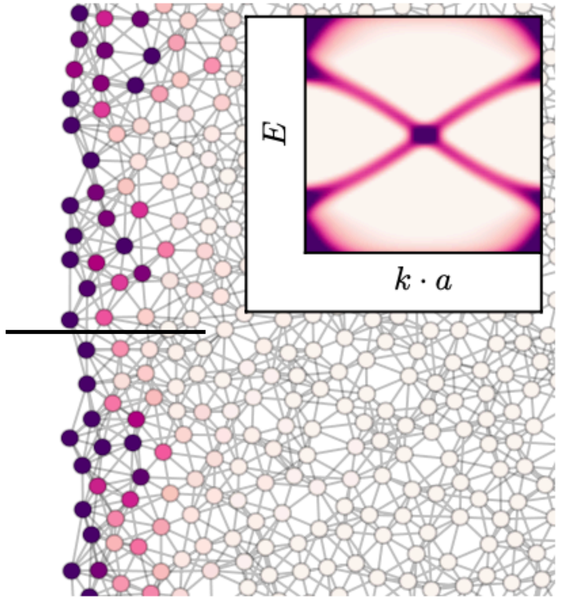
Mirror preserved

robust edge state



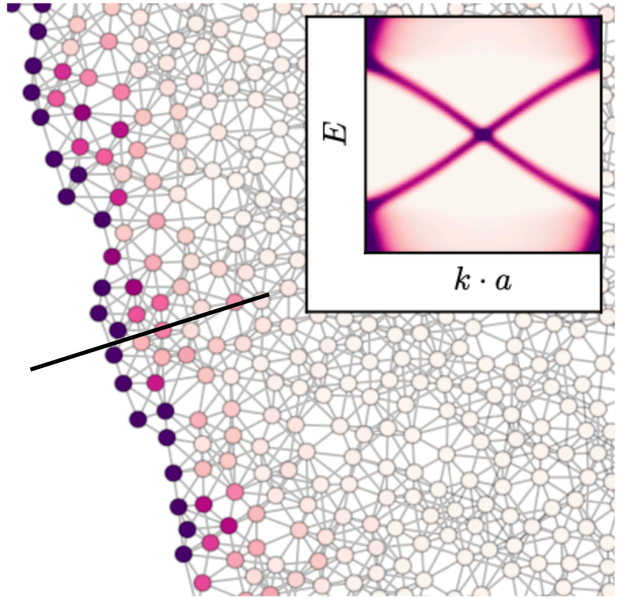
Broken mirror

no edge state



Mirror preserved
on average

robust edge state



Mirror preserved
on average

robust edge state

Technical details I

Spectral weight:

$$F_{\pm}(|\psi\rangle) = \sum_j |\langle j, \pm|\psi\rangle|^2$$

$$F_m(|\psi\rangle) = \sum_i |\langle i, m|\psi\rangle|^2$$

$$F_m(E) = \frac{1}{N}\mathrm{Tr}\left[\delta(H-E)\hat{F}_m\right]$$

Hamiltonian

$$H = \sum_{i,j\neq j'}^{z} V_{jj'}^{(i)} |i,j\rangle\langle i,j'| + \sum_{i\neq i',j}^{z} W_{ii'}^{(j)} |i,j\rangle\langle i',j|.$$

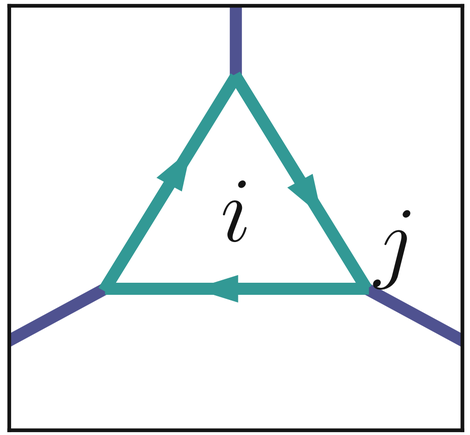
$$H_V|i,m\rangle = 2V\cos\left(\phi + m\frac{2\pi}{3}\right)|i,m\rangle \equiv \lambda_m|i,m\rangle,$$

$$R|i,m\rangle = e^{i\frac{2m\pi}{3}}|i,m\rangle \equiv \xi_m|i,m\rangle.$$

$$H_W|j,\pm\rangle = \pm W|j,\pm\rangle,$$
$$I|j,\pm\rangle = \pm|j,\pm\rangle.$$

$$H_V = V\begin{pmatrix} 0 & e^{i\phi} & e^{-i\phi} \\ e^{-i\phi} & 0 & e^{i\phi} \\ e^{i\phi} & e^{-i\phi} & 0 \end{pmatrix}$$

$$\mathcal{C}(\mathbf{r}) = 2\pi\mathrm{Im}\,\langle\mathbf{r}|\left[\hat{Q}\hat{x}, \hat{P}\hat{y}\right]|\mathbf{r}\rangle$$



Mapping to continuous media: effective Hamiltonian

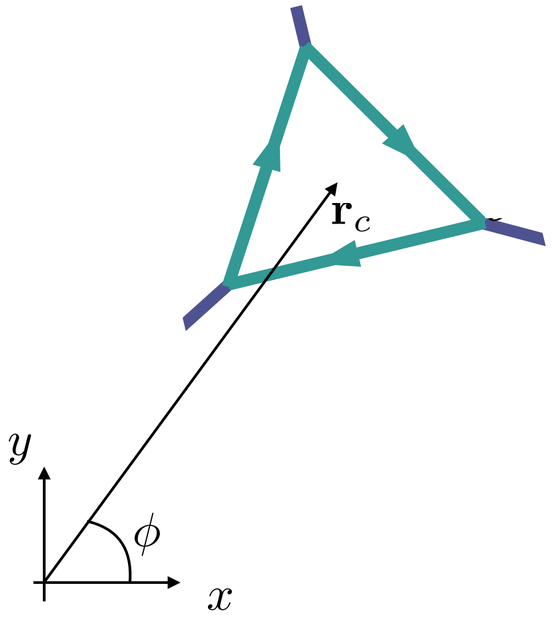
Q. Marsal, D. Varjas, AGG PNAS, (2020)

$$G_l(\mathbf{k}) = \langle \mathbf{k}l \,|\, H \,|\, \mathbf{k}l \rangle$$

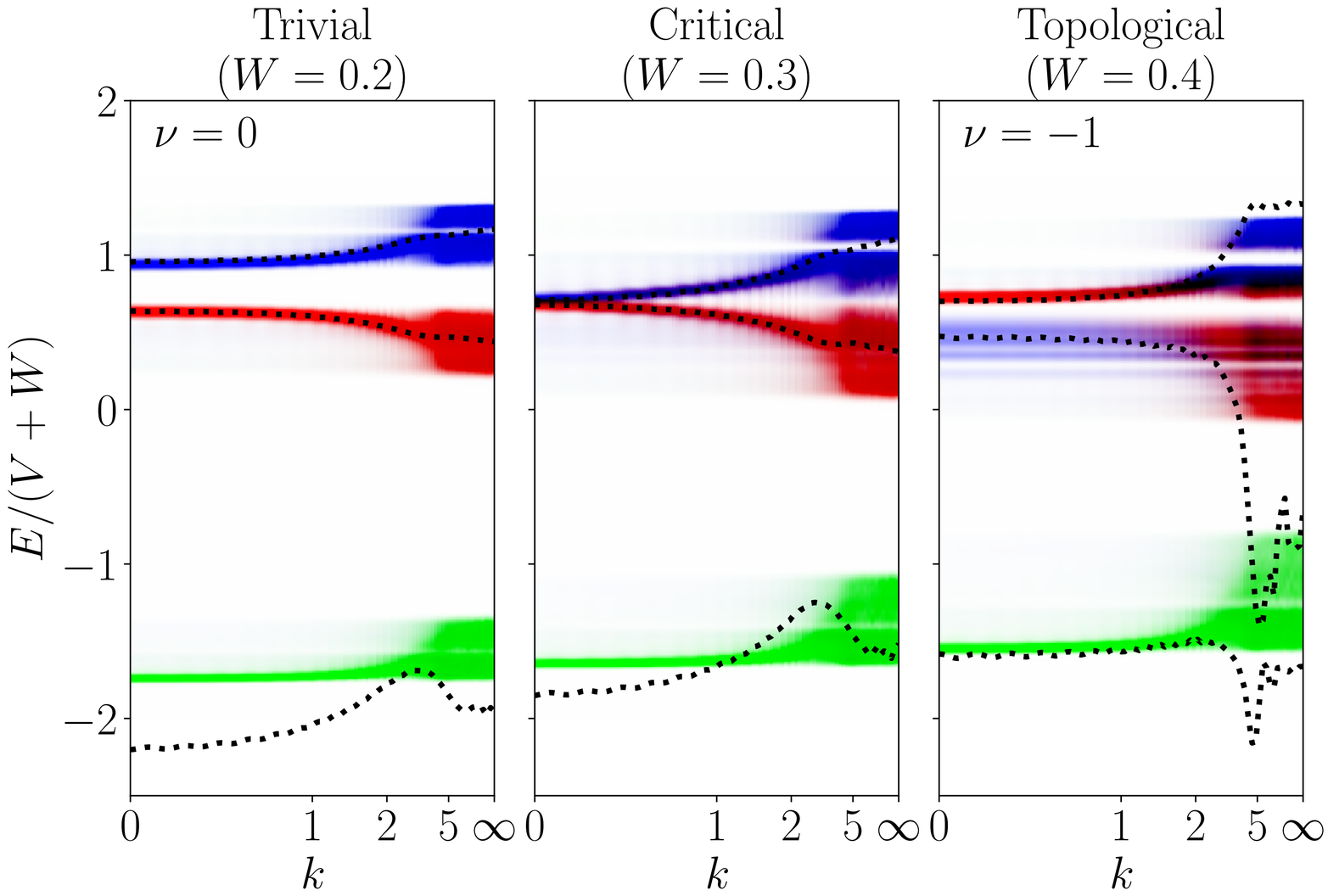
Varjas et al. PRL (2020)

$$\langle \mathbf{r}|\mathbf{k}, l\rangle = \frac{1}{\sqrt{N}} \exp(i\mathbf{k}\mathbf{r}_c) \exp(i\phi_{\mathbf{r}} l),$$

$$H_{\text{eff}}(\mathbf{k}) = G_l^{-1}(\mathbf{k}) - E_F$$



$$\nu = \sum_n l_n(|\mathbf{k}| = 0) - l_n(|\mathbf{k}| = \infty)$$



Q. Marsal, D. Varjas, AGG PNAS (2020)

Technical details: Structural spillage

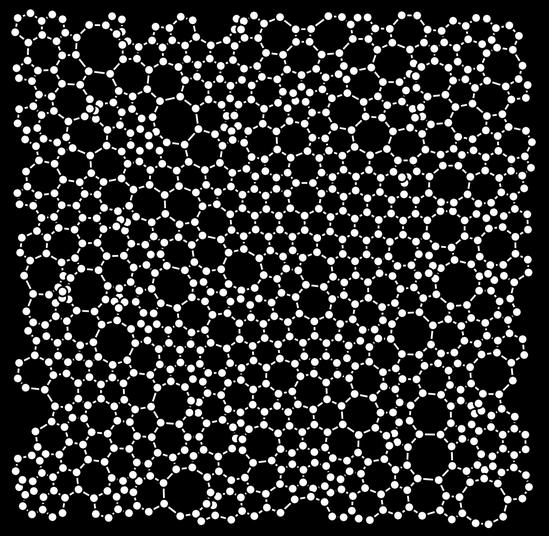
Structural spillage TB

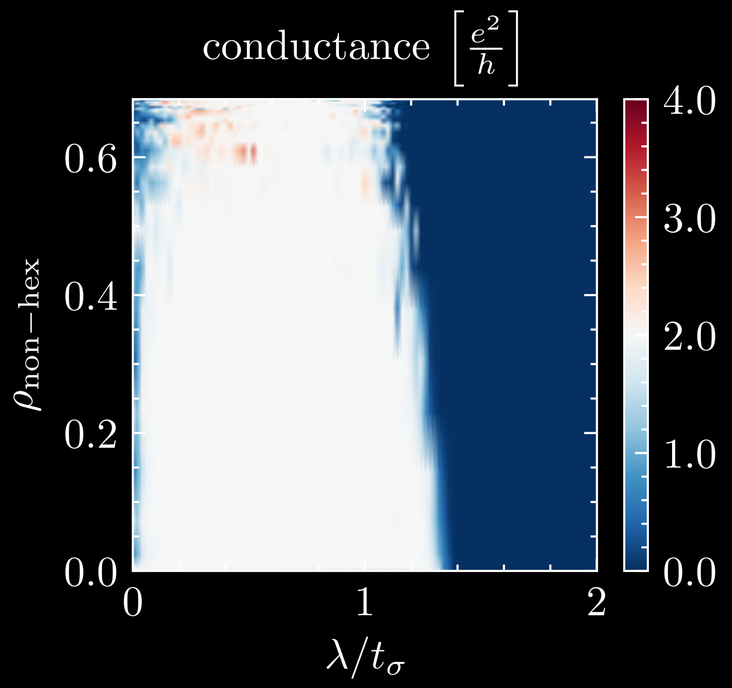
problem = plane waves not well defined in all space

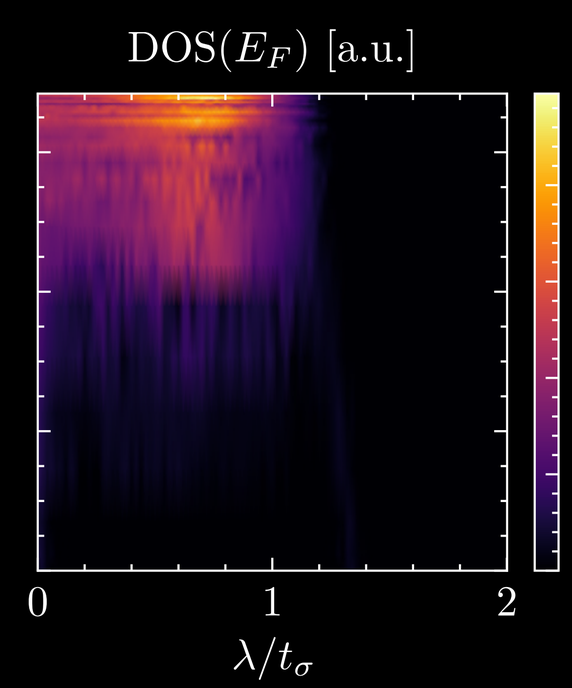
problem = different BZs

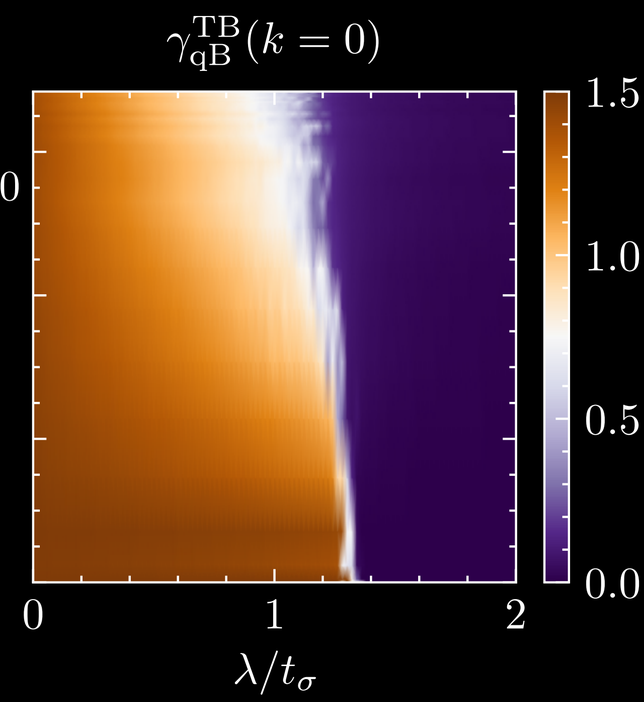
$$\gamma_{\mathrm{qB}}^{\mathrm{TB}}(\boldsymbol{k}) = \frac{1}{2}\frac{N_{\mathrm{s/c}}}{N_{\mathrm{BZs}}}\sum_{\boldsymbol{G}\in\mathrm{BZs}} \mathrm{tr}\left[\left(P_{\boldsymbol{k}+\boldsymbol{G}} - \tilde{P}_{\boldsymbol{k}+\boldsymbol{G}}\right)^2\right]$$

Tight-binding bismuthene

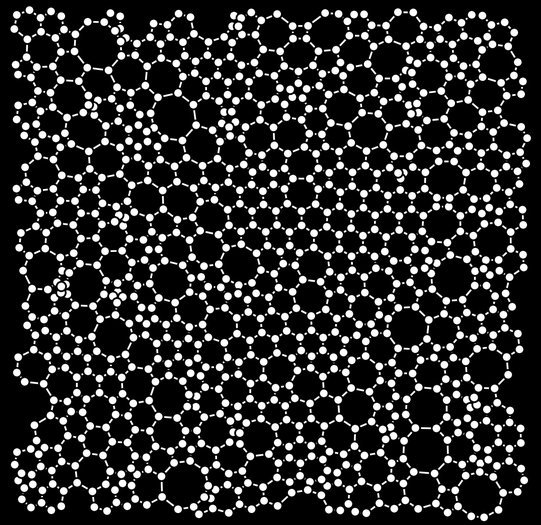


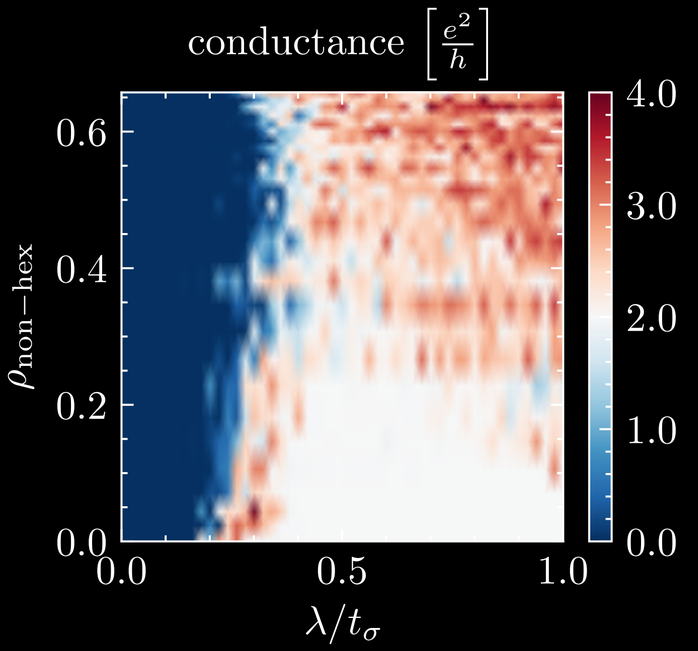


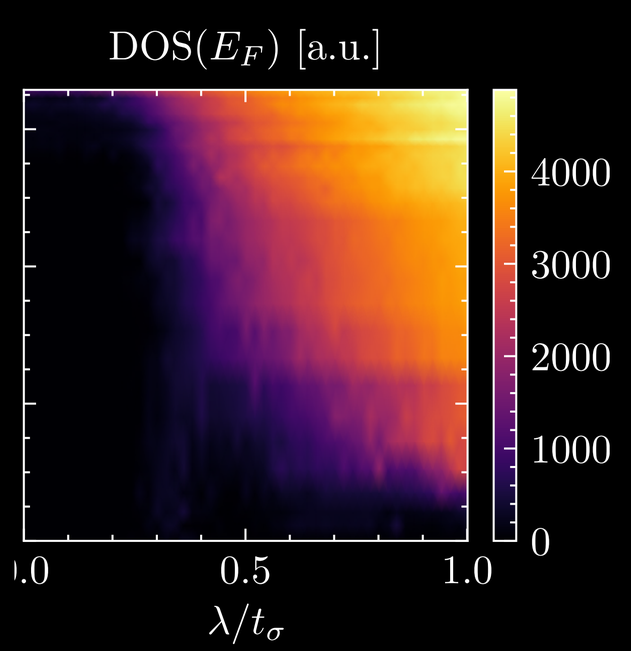


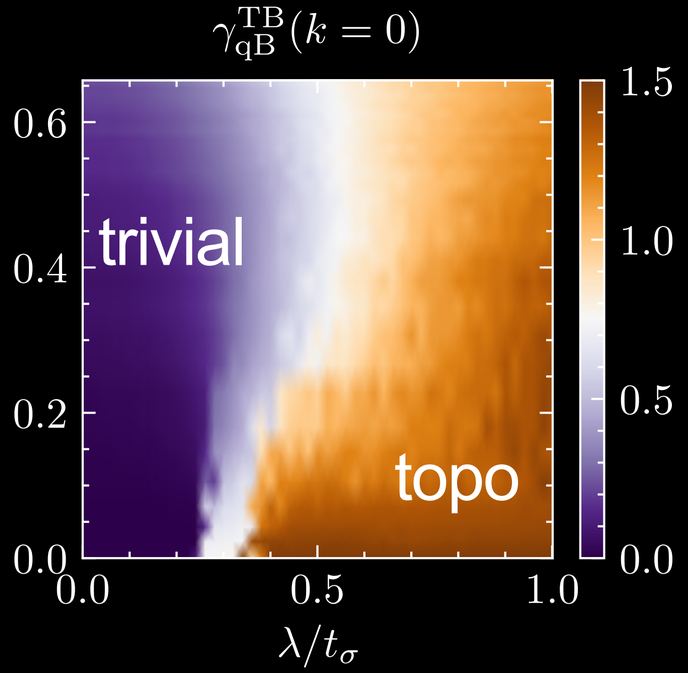


Tight-binding bismuth bilayer

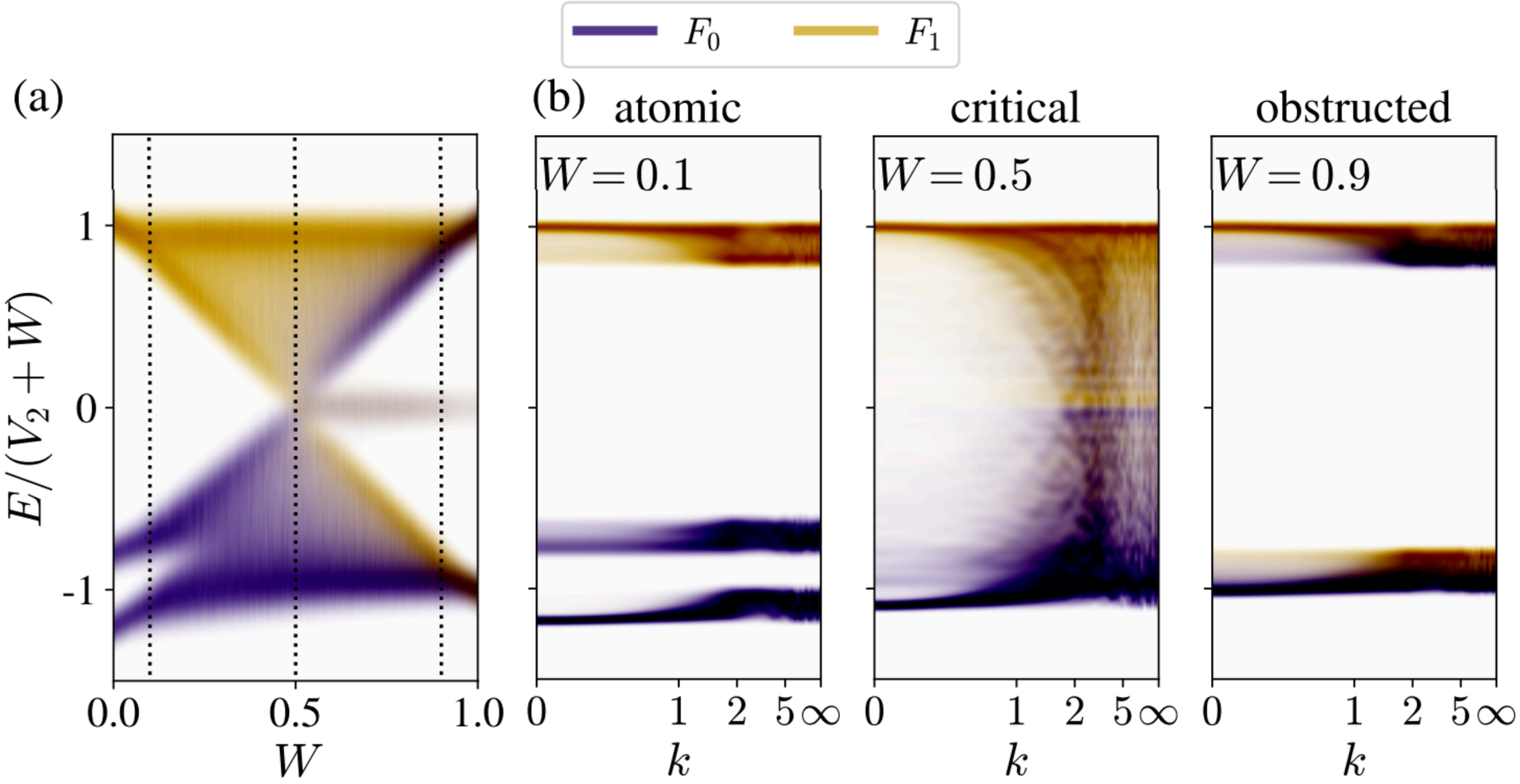


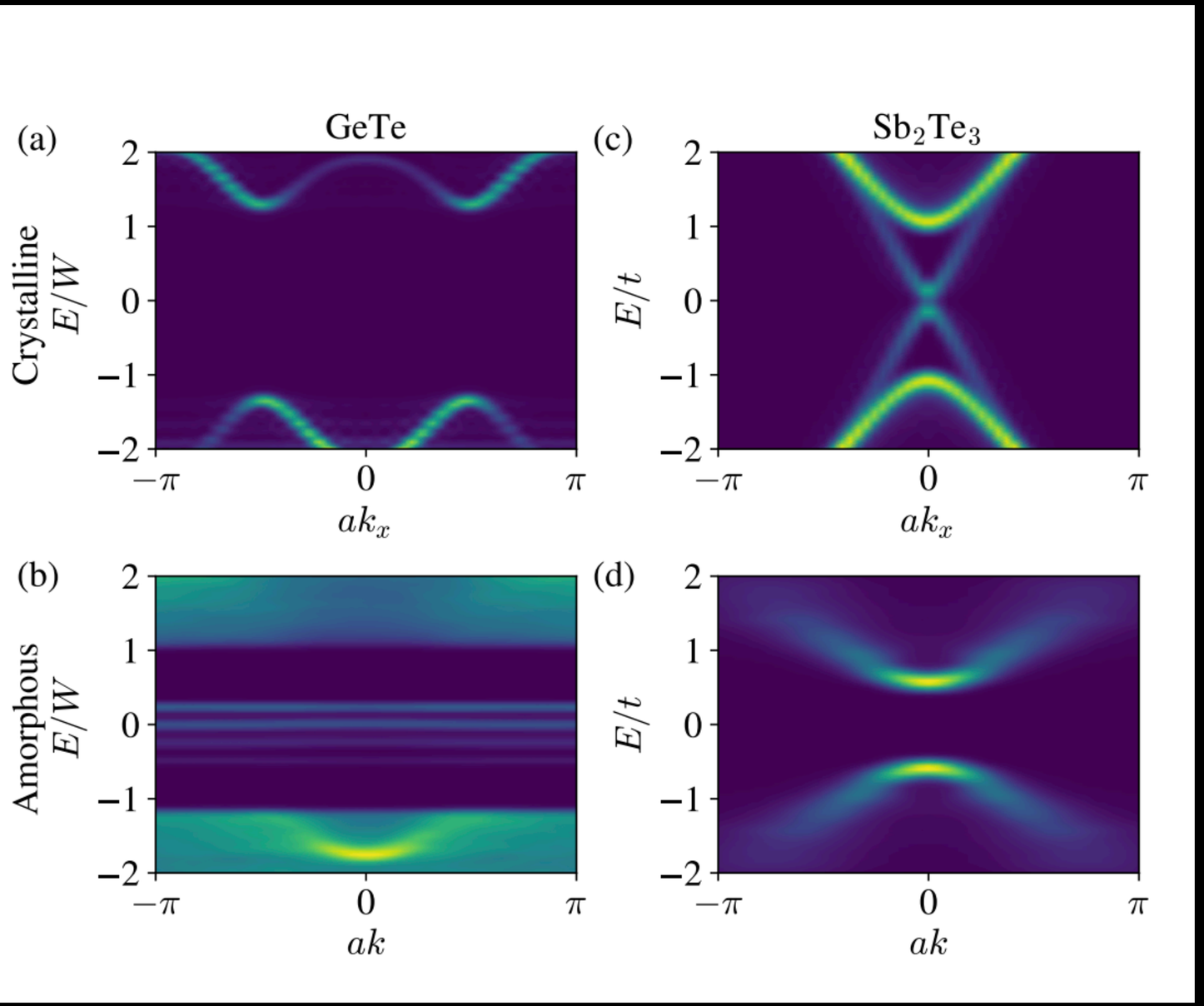






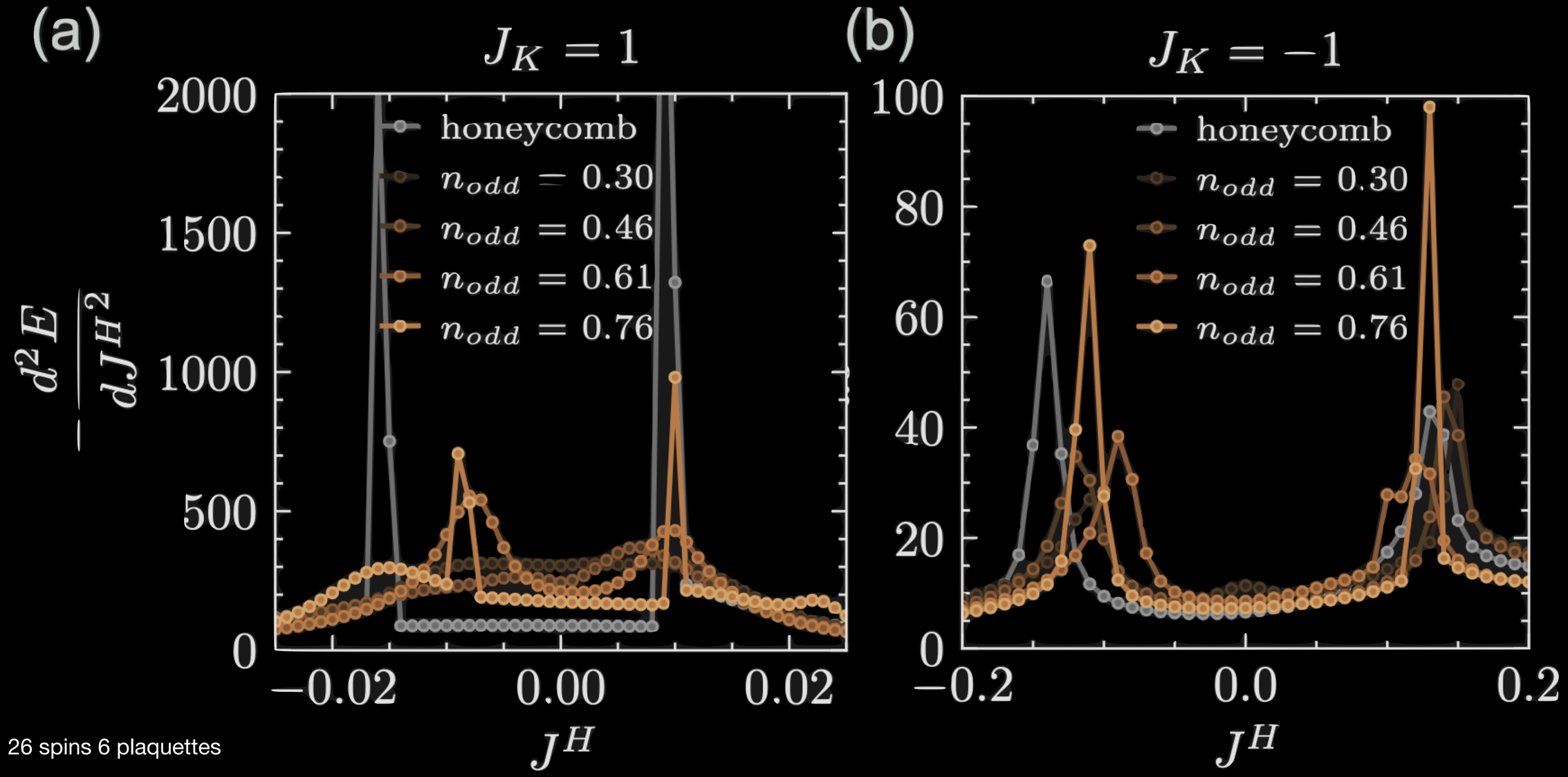
Technical details: Obstructed / phase change


F_0
F_1
(a)
(b)
atomic
critical
obstructed
$W = 0.1$
$W = 0.5$
$W = 0.9$
$E/(V_2 + W)$
1
0
-1
0.0
0.5
1.0
W
0
1
2
5
∞
k


(a)
GeTe
(c)
Sb_2Te_3
Crystalline
E/W
E/t
2
1
0
−1
−2
$-\pi$
0
π
ak_x
(b)
(d)
Amorphous
E/W
E/t
ak

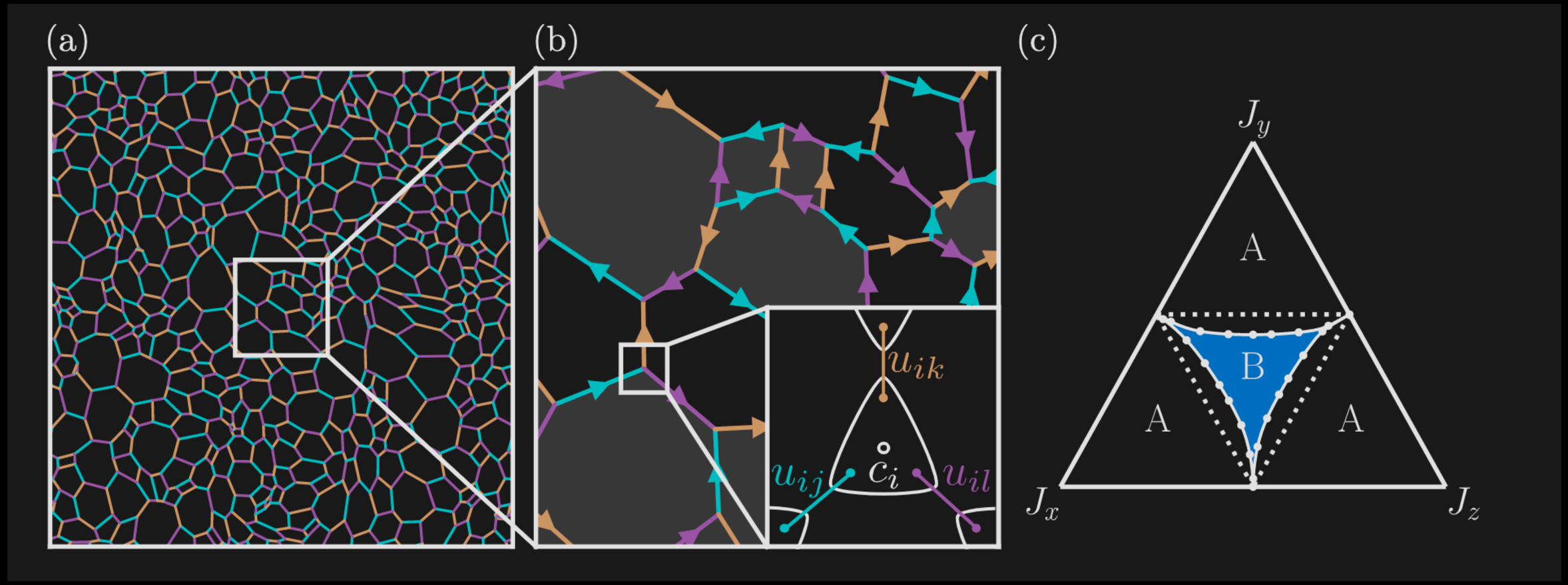
$$H = \sum_i V(\mathbf{c}_i^\dagger \wedge \mathbf{c}_i) \cdot \mathbf{S} + (-1)^i \epsilon \mathbf{c}_i^\dagger \cdot \mathbf{c}_i - \sum_{<i,j>} (W + (-1)^i \text{sgn}(\hat{d}_{ij} \cdot (\hat{e}_x + \hat{e}_y + \hat{e}_z) \Delta W)(\mathbf{c}_i^\dagger \cdot \hat{d}_{ij})(\mathbf{c}_j \cdot \hat{d}_{ji}),$$

Technical details: amorphous Kitaev



26 spins 6 plaquettes

$$H_{kh} = J^K \sum_{i,k,\alpha} \sigma_j^\alpha \sigma_k^\alpha + J^H \sum_{\langle ij \rangle} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j.$$



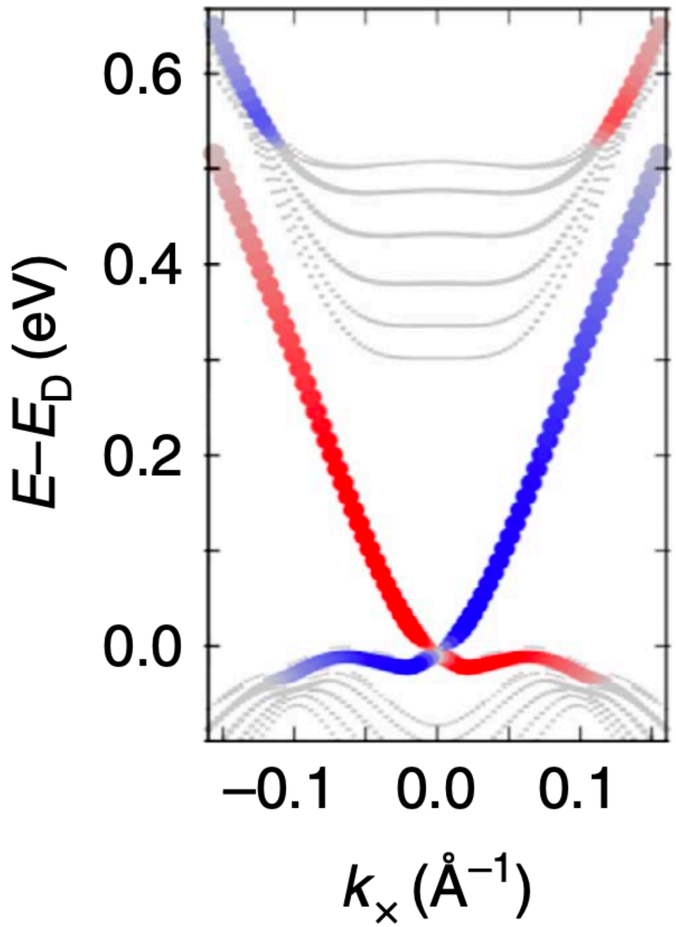
G. Casella et al 2208.08246

Technical details: amorphous Bi_2Se_3

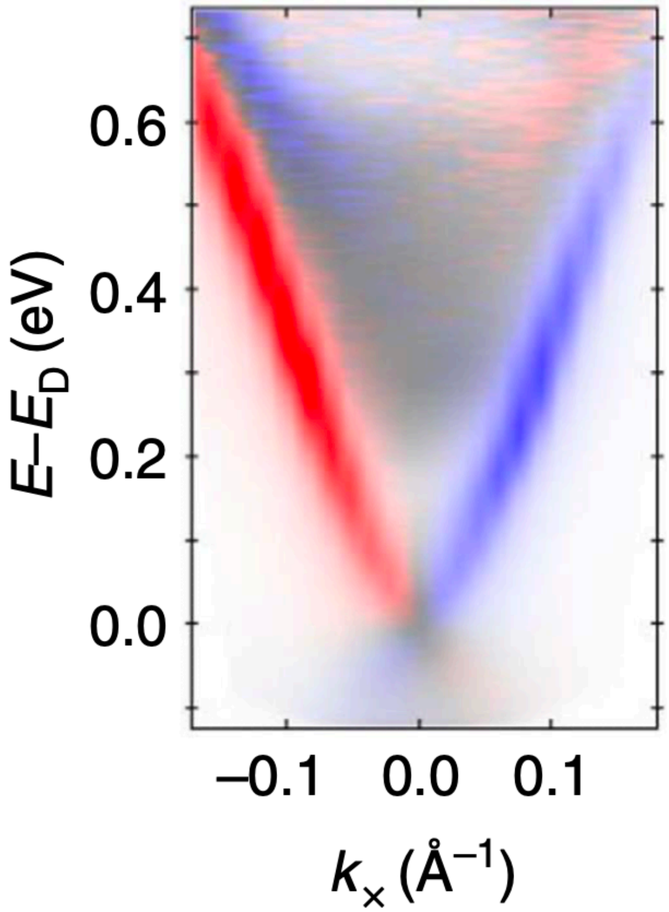
A textbook example of a 3D-topological insulator

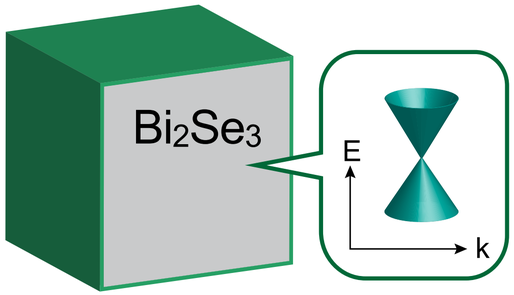
Bi_2Se_3

DFT



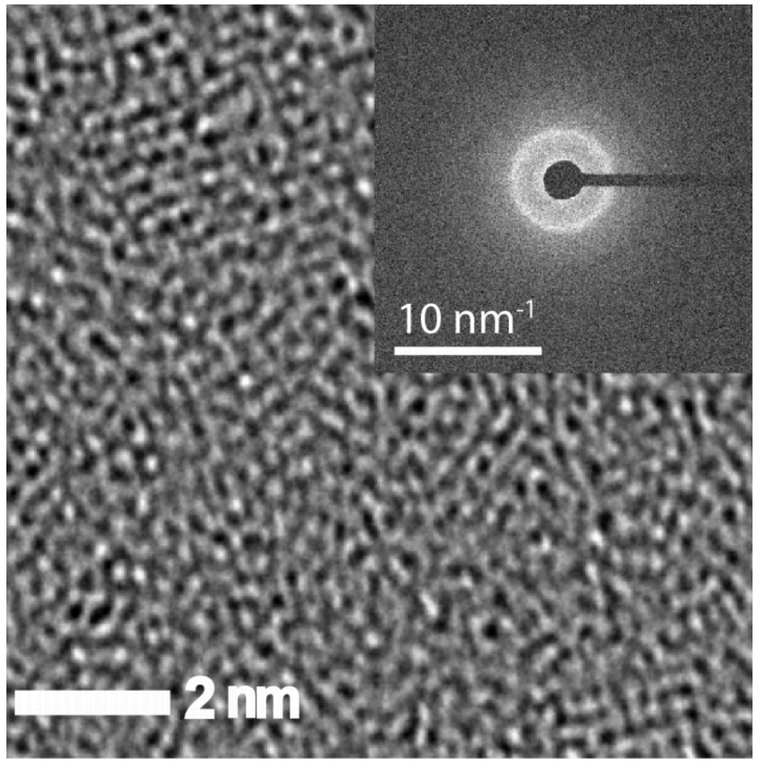
ARPES

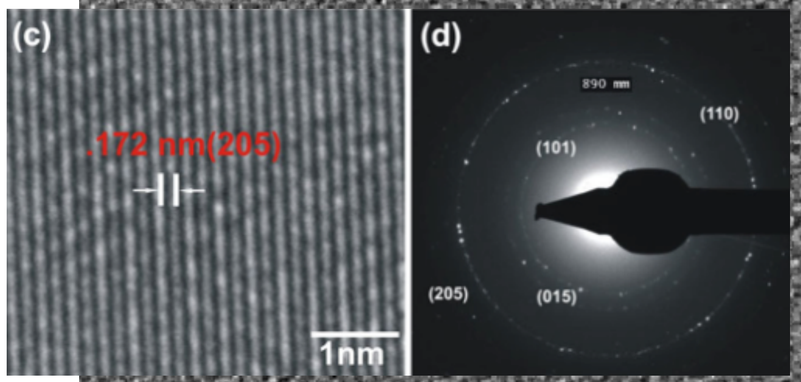




Jozwiak, C. et al. Nature Communications (2016)

Amorphous Bi_2Se_3

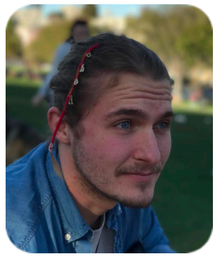




Kadel et al. Nanoscale Res. Lett. (2011)

Paul Corbae

(Hellman group, UCB)



Sam Ciocys

(Lanzara group, LBL)

[see also J. Barzola-Quiquia, et al. Journal of Applied Physics (2015)]

Amorphous Bi_2Se_3

Transport

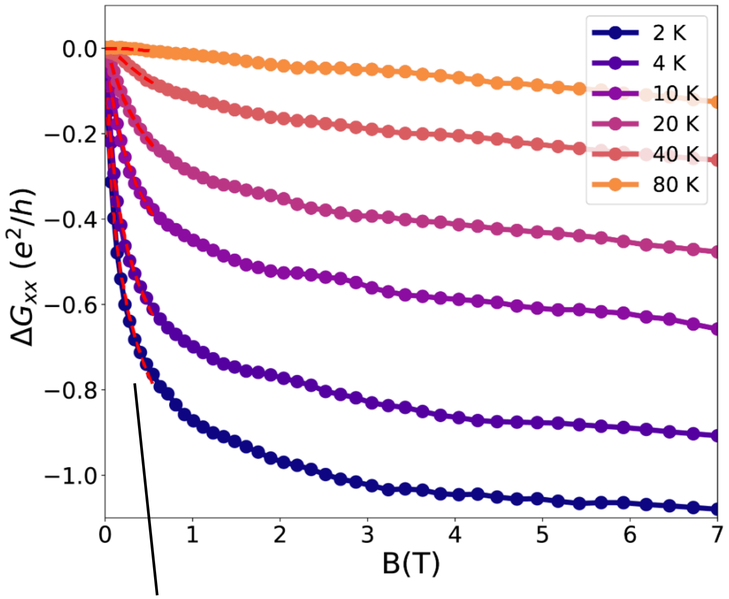
ARPES

spin resolved ARPES

P. Corbae, S. Ciocys, D. Varjas,…, AGG, A. Lanzara, F. Hellmann **arXiv:1910.13412**

Amorphous Bi_2Se_3

Transport | ARPES | spin resolved ARPES



One channel conduction large T

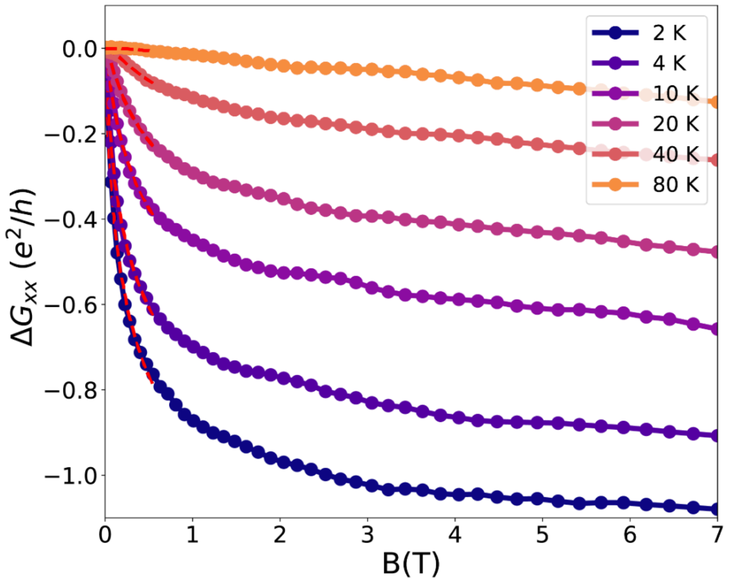
Two decoupled channel conduction low T

Weak anti-localization at low T

P. Corbae, S. Ciocys, D. Varjas,…, AGG, A. Lanzara, F. Hellmann **arXiv:1910.13412**

Amorphous Bi_2Se_3

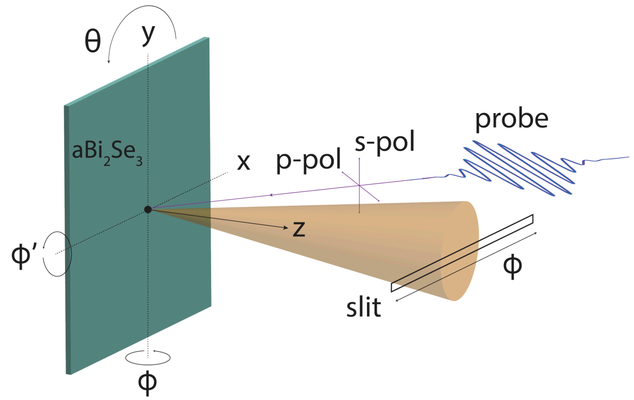
Transport

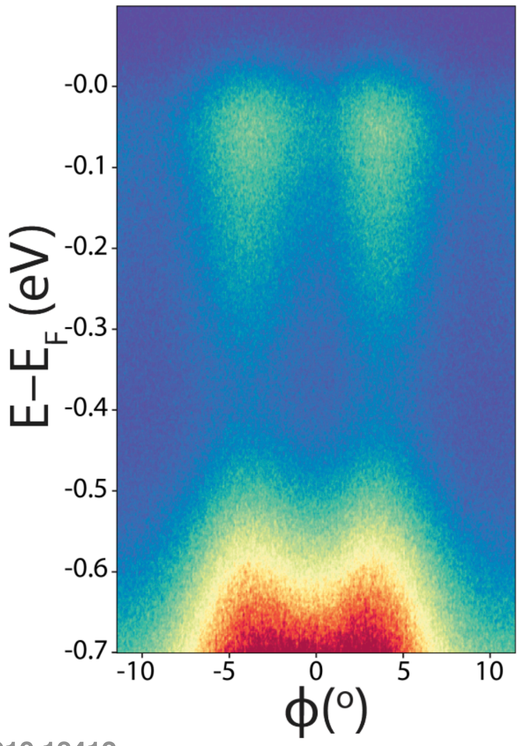


Weak anti-localization at low T

Two decoupled channel conduction

ARPES



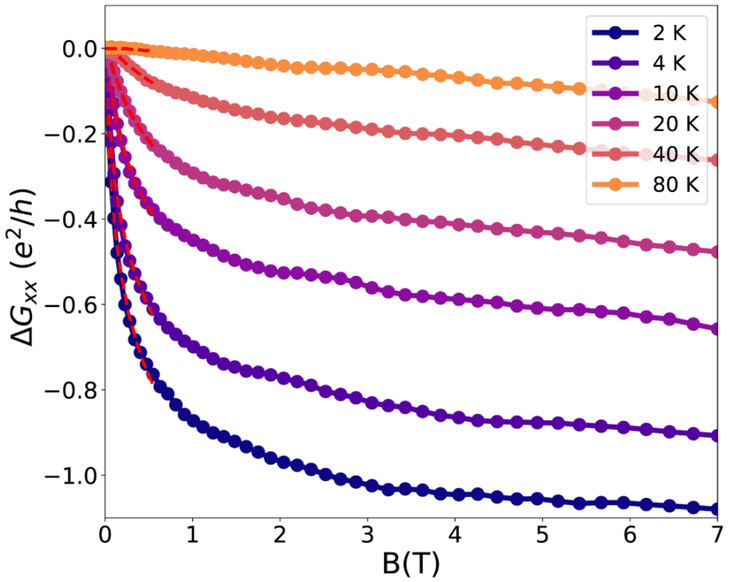


spin resolved ARPES

P. Corbae, S. Ciocys, D. Varjas,…, AGG, A. Lanzara, F. Hellmann **arXiv:1910.13412**

Amorphous Bi_2Se_3

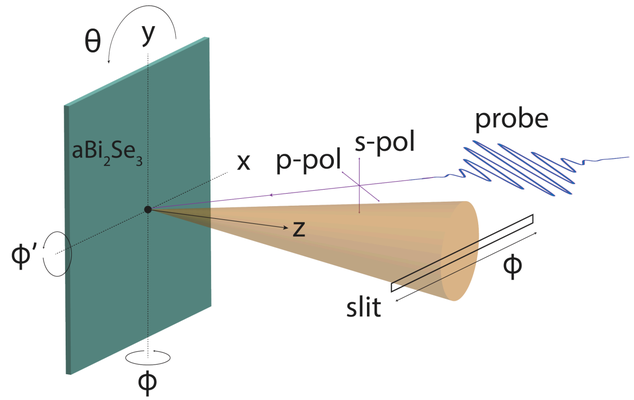
Transport

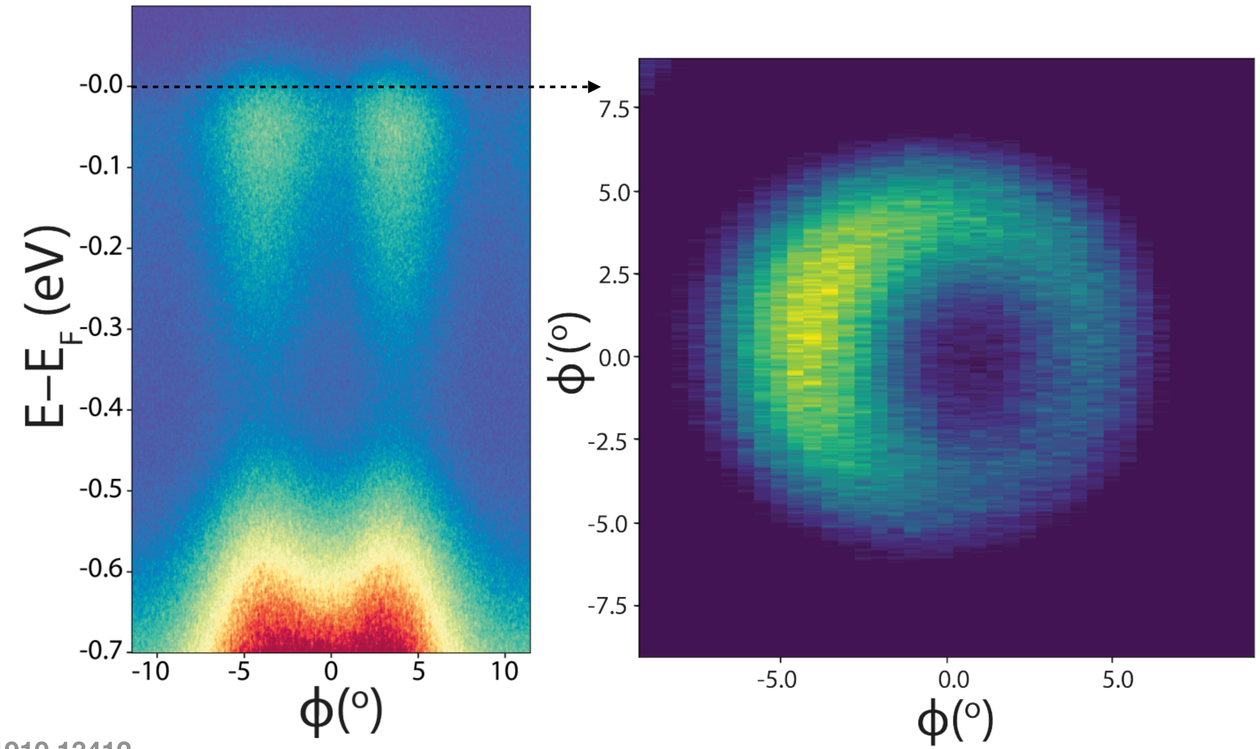


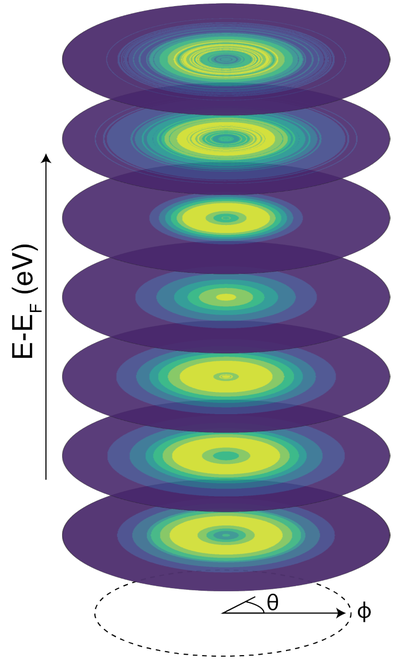
Weak anti-localization at low T

Two decoupled channel conduction

ARPES





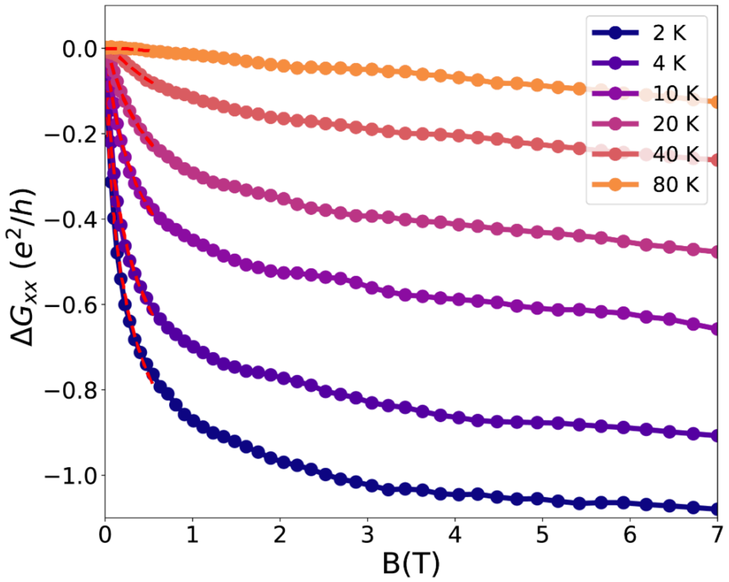


spin resolved ARPES

P. Corbae, S. Ciocys, D. Varjas,…, AGG, A. Lanzara, F. Hellmann **arXiv:1910.13412**

Amorphous Bi_2Se_3

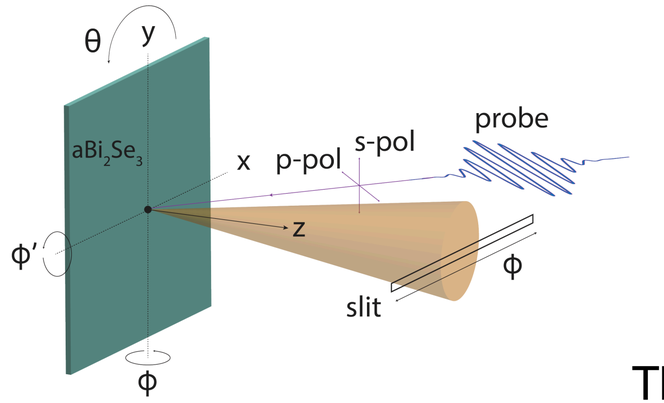
Transport

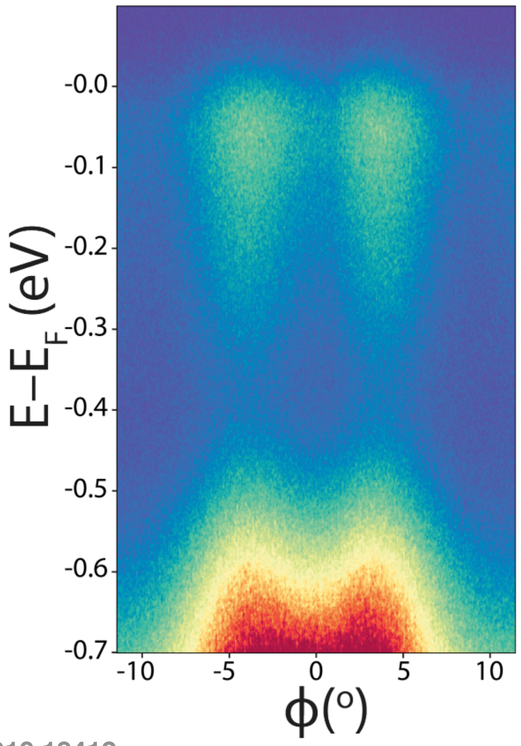


Weak anti-localization at low T

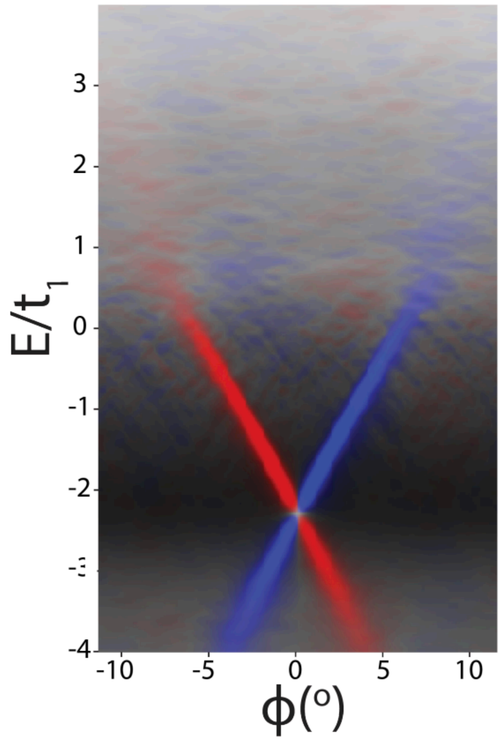
Two decoupled channel conduction

ARPES





Theory



6-coordinated amorphous BHZ

Spectral function obtained by projecting to a plane-wave basis

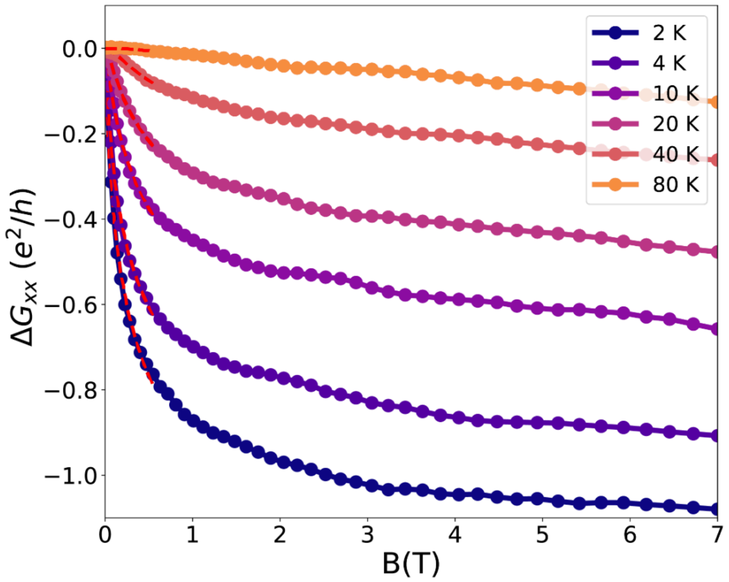
$$A(\mathbf{k}, E) = \sum_l \langle \mathbf{k}, l | \delta(H - E) | \mathbf{k}, l \rangle$$

spin resolved ARPES

P. Corbae, S. Ciocys, D. Varjas,…, AGG, A. Lanzara, F. Hellmann **arXiv:1910.13412**

Amorphous Bi_2Se_3

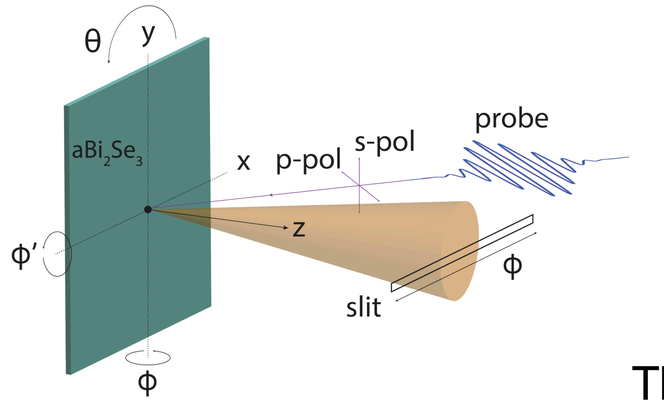
Transport

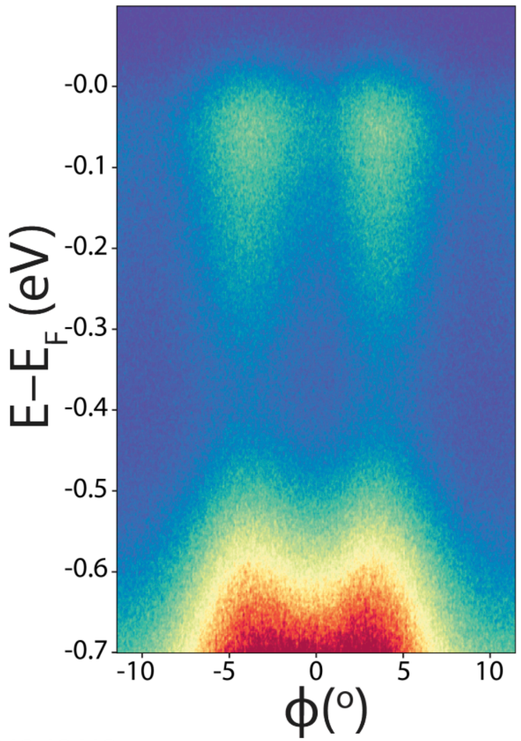


Weak anti-localization at low T

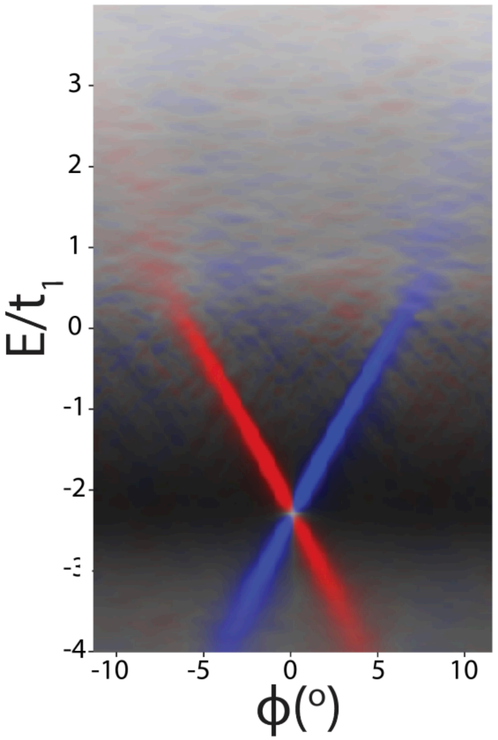
Two decoupled channel conduction

ARPES





Theory



6-coordinated amorphous BHZ

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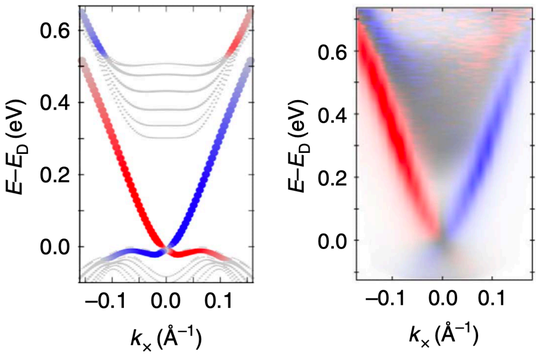
~ Crystal Bi_2Se_3



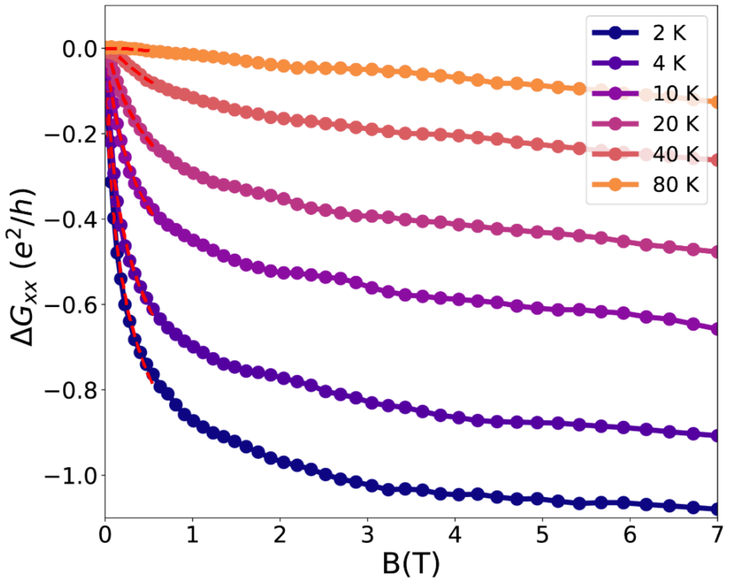
Jozwiak, C. et al. Nat. Comm (2016)

P. Corbae, S. Ciocys, D. Varjas,…, AGG, A. Lanzara, F. Hellmann **arXiv:1910.13412**

spin resolved ARPES

Amorphous Bi_2Se_3

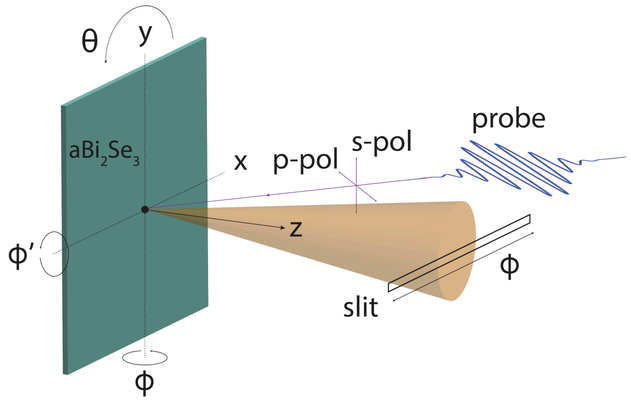
Transport

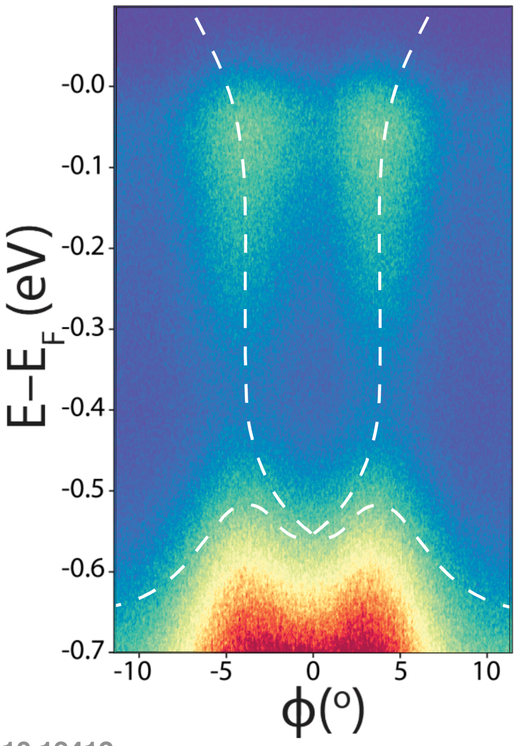


Weak anti-localization at low T

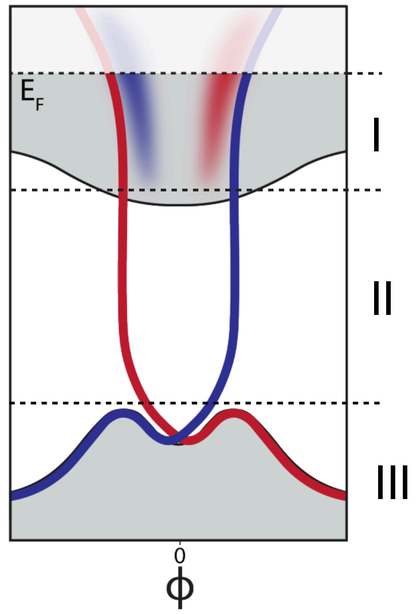
Two decoupled channel conduction

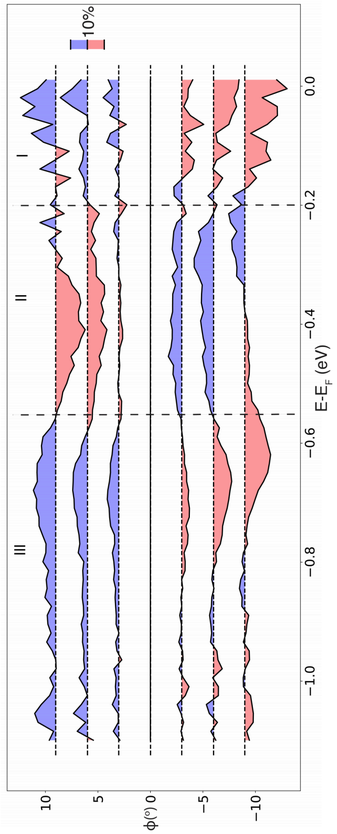
ARPES



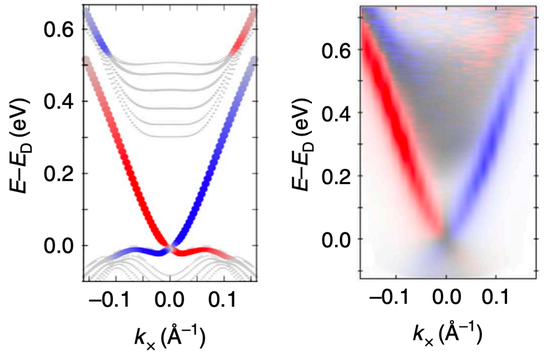


spin resolved ARPES





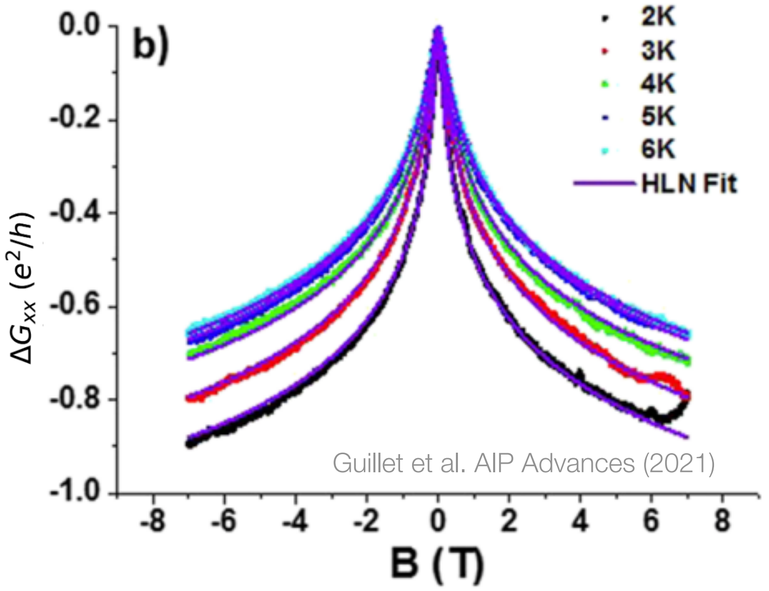
~ Crystal Bi_2Se_3

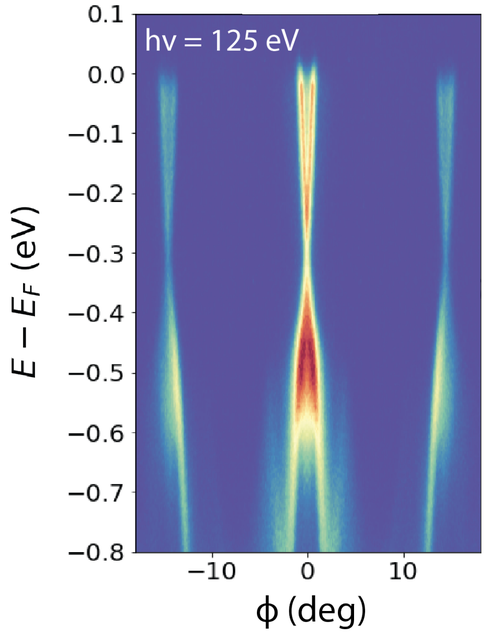


Jozwiak, C. et al. Nat. Comm (2016)

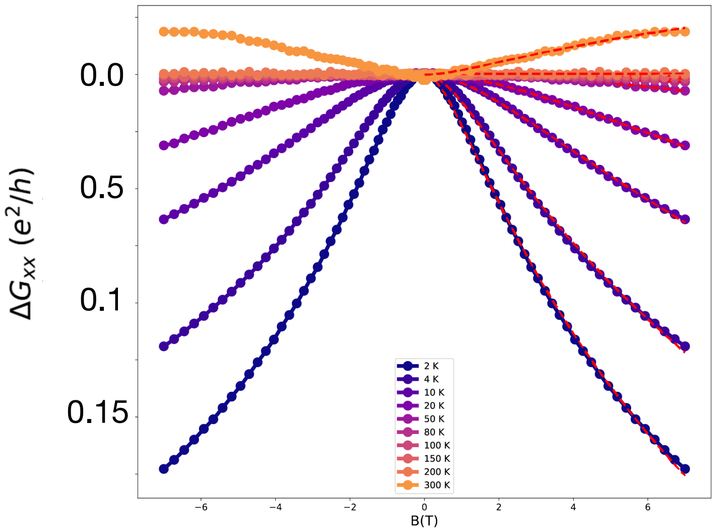
P. Corbae, S. Ciocys, D. Varjas,…, AGG, A. Lanzara, F. Hellmann **arXiv:1910.13412**

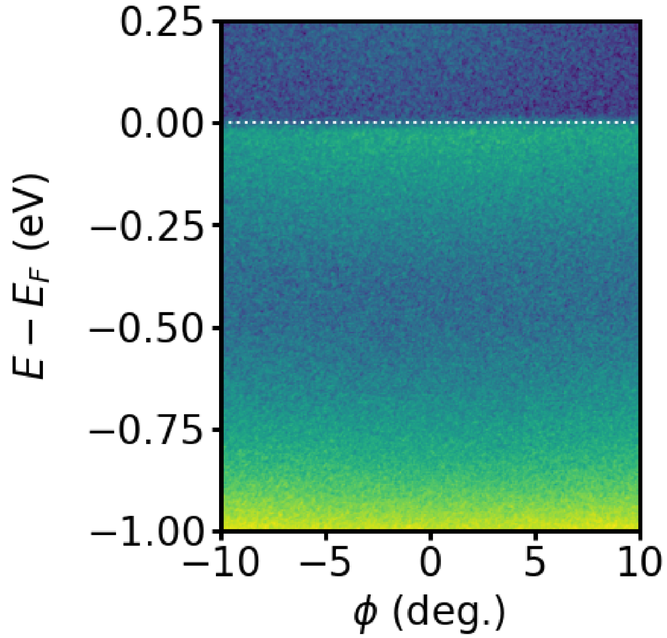
Crystalline



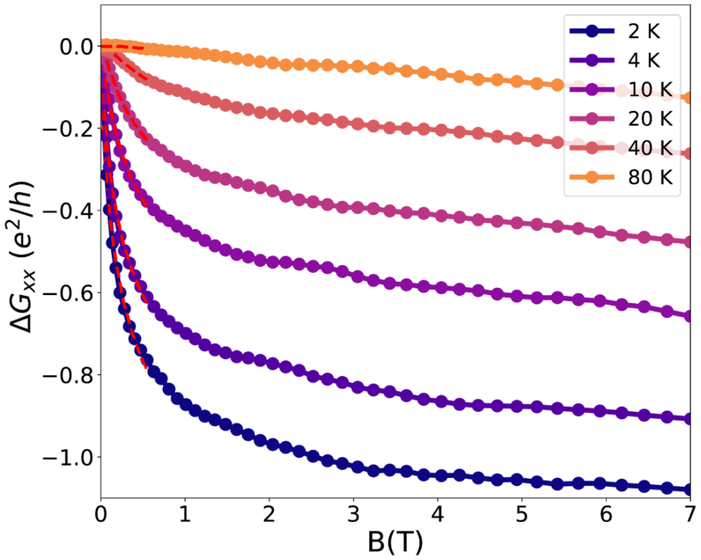


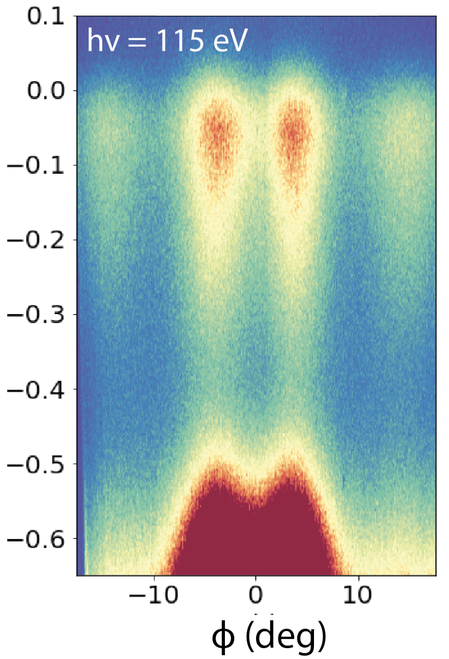
Polycrystalline





Amorphous





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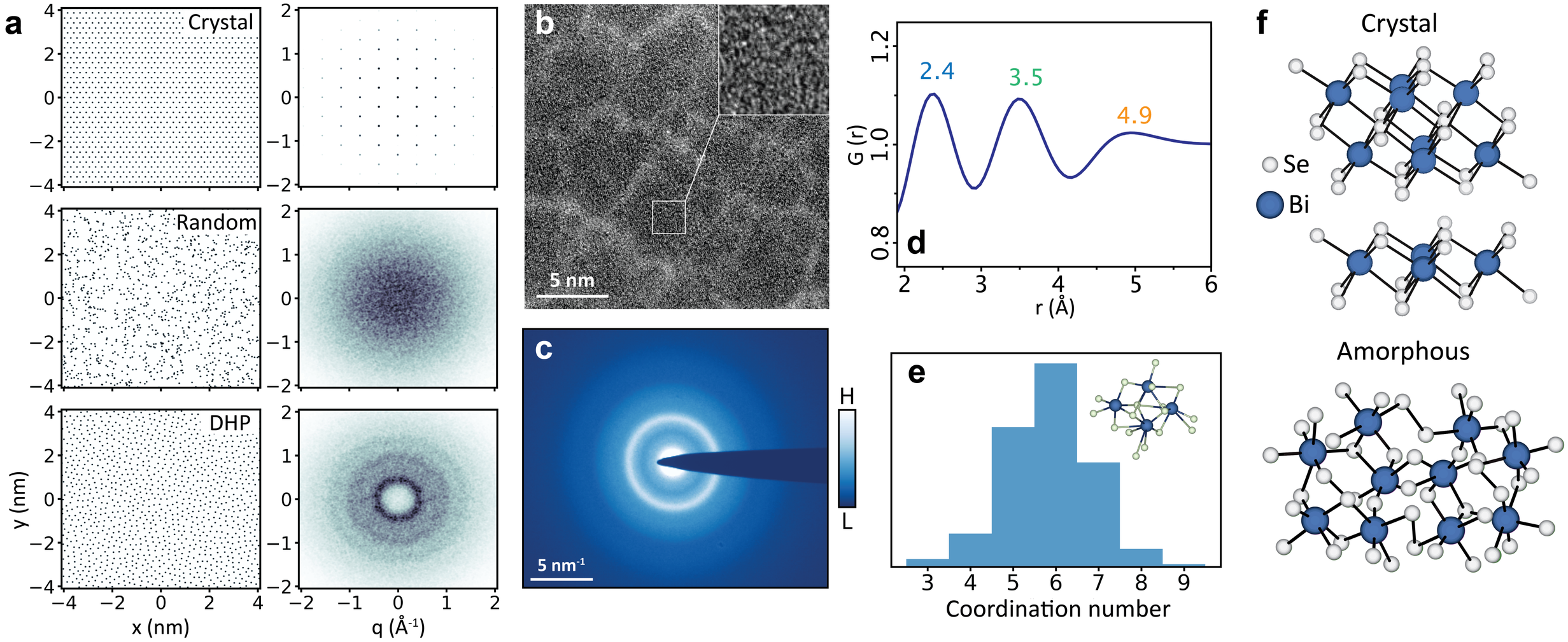
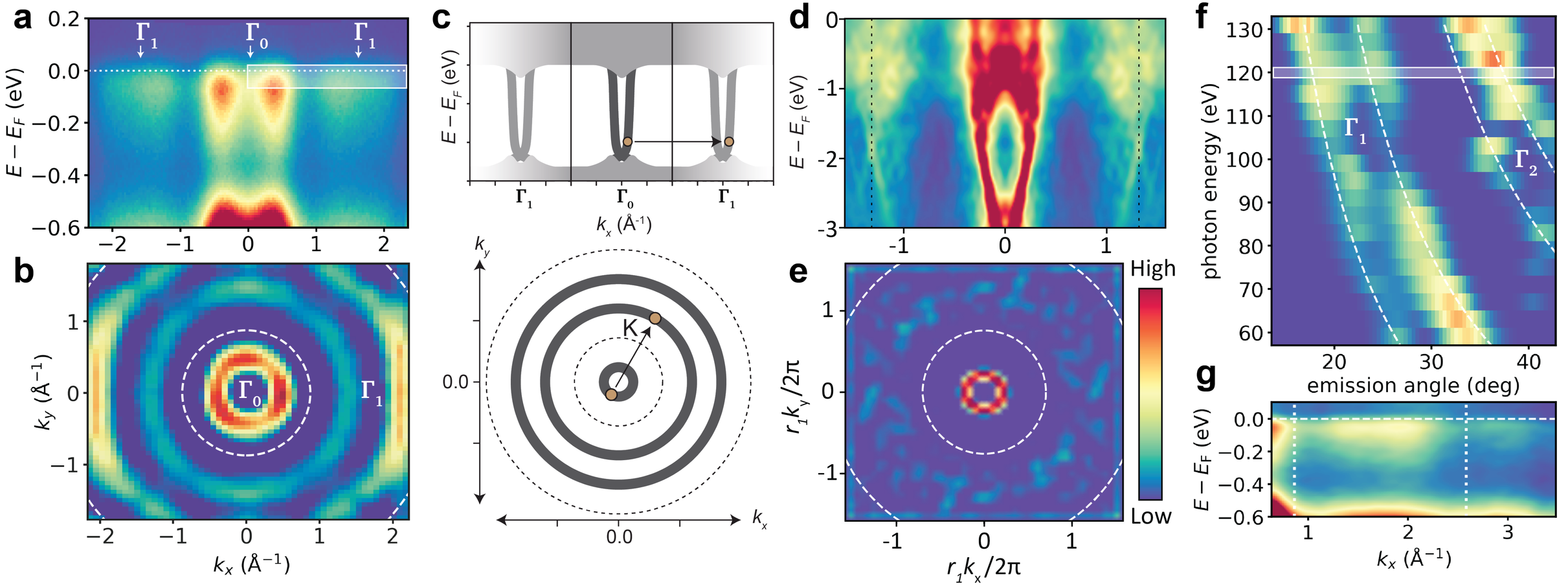
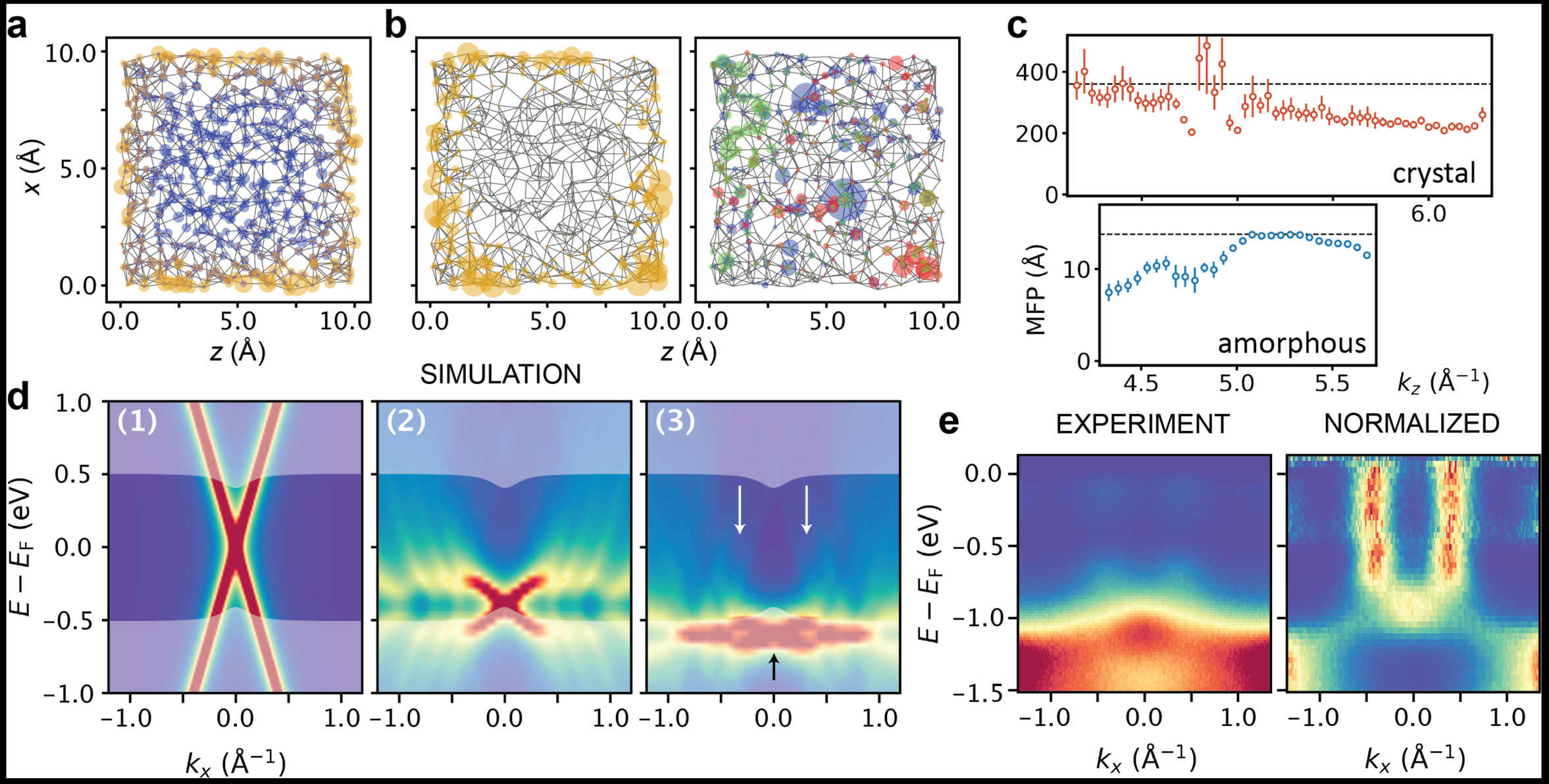
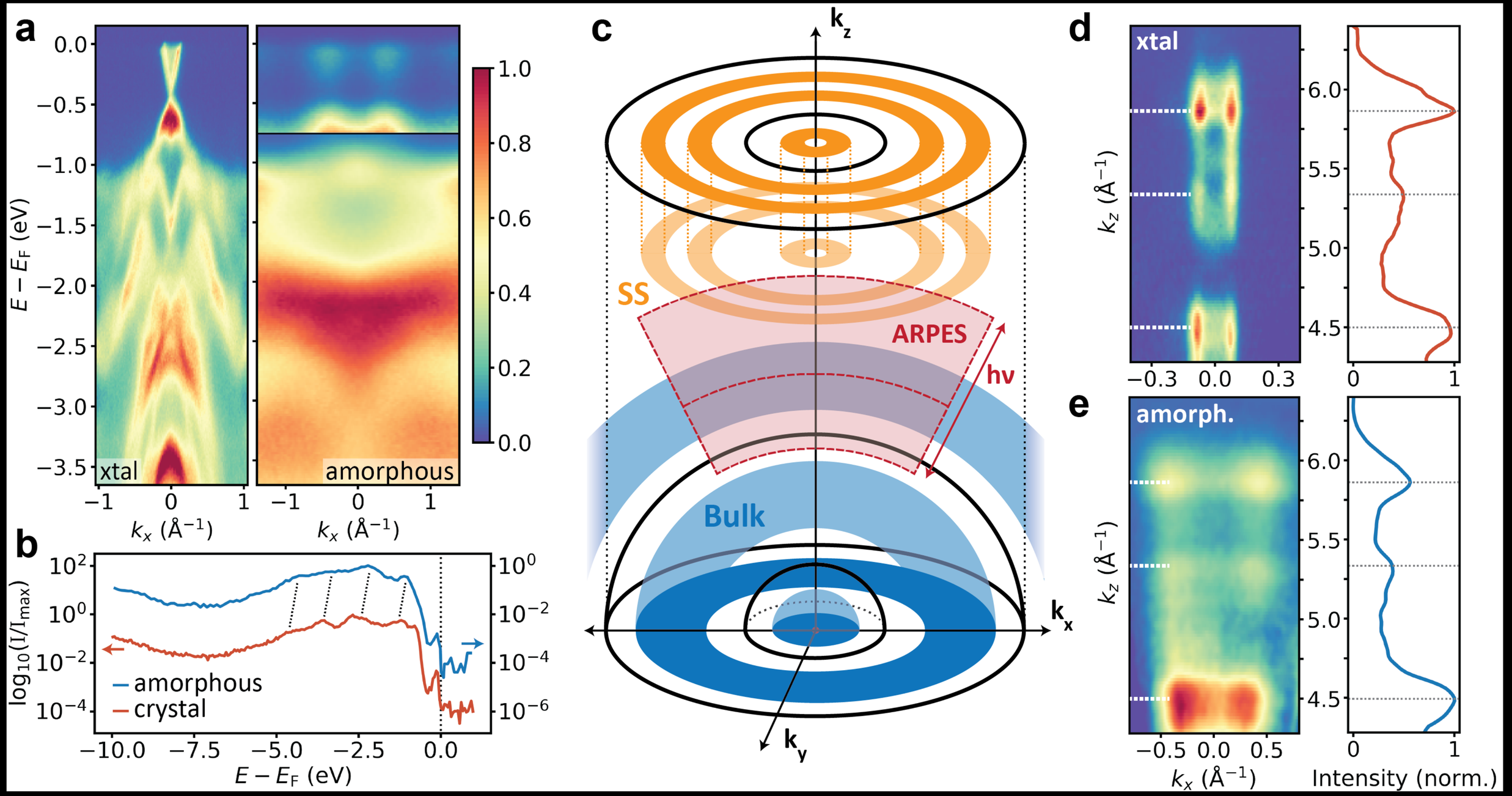

a
Crystal
Random
DHP
y (nm)
x (nm)
q (Å^{-1})
b
5 nm
c
5 nm^{-1}
H
L
d
2.4
3.5
4.9
G (r)
r (Å)
e
Coordination number
f
Crystal
Se
Bi
Amorphous


a
b
c
d
e
f
g
Γ_1
Γ_0
Γ_1
$E - E_F$ (eV)
k_x (Å^{-1})
k_y (Å^{-1})
K
k_y
k_x
0.0
High
Low
$r_1 k_x/2\pi$
$r_1 k_y/2\pi$
photon energy (eV)
emission angle (deg)
Γ_2
$E - E_F$ (eV)


a
b
c
10.0
5.0
0.0
x (Å)
0.0
5.0
10.0
z (Å)
SIMULATION
400
200
0
crystal
6.0
MFP (Å)
10
0
amorphous
4.5
5.0
5.5
k_z (Å^{-1})
d
e
EXPERIMENT
NORMALIZED
(1)
(2)
(3)
1.0
0.5
0.0
−0.5
−1.0
$E - E_F$ (eV)
−1.0
0.0
1.0
k_x (Å^{-1})
−1.5


a
b
c
d
e
xtal
amorphous
$E - E_F$ (eV)
k_x (Å^{-1})
$\log_{10}(I/I_{max})$
amorphous
crystal
$E - E_F$ (eV)
k_z
k_x
k_y
SS
ARPES
hv
Bulk
xtal
amorph.
k_z (Å^{-1})
k_x (Å^{-1})
Intensity (norm.)