Topological orbital phases beyond standard optical lattices

W. Vincent Liu
University of Pittsburgh, Pennsylvania, USA
Acknowledge Support by U.S. ARO, AFOSR, and DARPA-OLE
And NSF of China Overseas Scholar Collaboration Program (sponsor: Peking Univ/Biao Wu)
Two main thrusts in theoretical cold atom research

- Use cold atoms/optical lattices to quantum simulate important condensed matter problems: such as Mott-Superfluid in Bose-Hubbard model, high Tc superconductors, spin liquid models with ring exchange, … [extensive work by many theory groups worldwide]

- As a new type of quantum matter of no prior analogue in CM physics:
  - New many-body regimes: Fermi gases in unitarity (…, many groups …), large effective Zeeman splitting (order of $E_{\text{Fermi}}$), …
  - Quantum particles (especially bosons) in the excited “higher orbital” bands of optical lattices. Beyond the s: p, d, …
  - Quantum dynamics: great potential of studying fast dynamics in a ultracold (slow) system.

This talk
Outline

1. Introduction
   - Optical lattices. What is the p-orbital band? Why?

2. Experimental Progress

3. Quantum 120° model of “spinless” p-band fermions
   - Strong anisotropy of p orbits – new feature → direction-dependent orbit exchange

4. Z-class topological Insulator in an optical orbital ladder
   - Ladder reduced from Hamburg experimental system
   - Due to hybridization of opposite parity s and p orbitals
   - Transition to non-topological Mott insulator by interaction

5. Conclusion
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With tunneling, discrete levels become Bloch bands.

- This talk
- Compared with s: orbital degeneracy $\rightarrow$ emergent symmetry

Features of p-orbital Hubbard model: illustration with bosons in 3D cubic lattice [WVL and C. Wu, PRA (2006)]

\[ H = \sum_{r\mu} \left[ t_{||} \delta_{\mu\nu} - t_{\perp} (1 - \delta_{\mu\nu}) \right] \left( b_{\mu r + a e_{\nu}}^{\dagger} b_{\mu r} + h.c. \right) + \frac{1}{2} U \sum_{r} \left[ n_{r}^{2} - \frac{1}{3} L_{r}^{2} \right] \]

\[ \sigma \text{ bond} \]

\[ \mu, \nu = x, y, z \quad \text{or} \quad p_{x}, p_{y}, p_{z} \]

Density field operator:
\[ n_{r} = \sum_{\mu} b_{\mu r}^{\dagger} b_{\mu r} \]

Angular momentum operator:
\[ L_{\mu r} = -i \sum_{\nu\lambda} \epsilon_{\mu\nu\lambda} b_{\nu r}^{\dagger} b_{\lambda r} \]
Orbital ordering of d-electrons

[see, for example, review by Tokura and Nagaosa, science 288, 462, (2000).]

orbital: shape of the electron cloud

Orbital degeneracy in transition metal oxides:

- Charge, spin, orbital, and lattice degree of freedom entangled together
- Complex phase diagrams

\[
\begin{align*}
H &= \sum_{\langle i,j \rangle} \hat{J}_{ij}^{(\gamma)} (\vec{S}_i \cdot \vec{S}_j + \frac{1}{4}), \\
\hat{J}_{ij}^{(\gamma)} &= J (T_i^{(\gamma)} T_j^{(\gamma)} - \frac{1}{2} T_i^{(\gamma)} - \frac{1}{2} T_j^{(\gamma)} + \frac{1}{4}) \\
T_i^{(a/b)} &= \frac{1}{4} (\pm \sigma_i^z), \quad T_i^{(c)} = \frac{1}{2} \sigma_i^z
\end{align*}
\]
A new direction: p-orbital physics in optical lattices

- Orbital degeneracy ($px, py, pz$ orbitals) is considerably less understood in comparison. Implies emergent symmetry.
- Similar to spin physics but is different fundamentally.
- Strong anisotropy: Anisotropy is an interesting new feature, not a problem!
- $p$-orbitals are different than $d$-orbital in solids: Parity ODD. New possibility---$p$-orbital bosons as opposed to $d$-electrons (fermions) in solids.
- Unique to cold atom systems, “non-standard” condensed matter systems. For instance, Orbital physics of bosons has no prior analogue in CM physics?!

unique quantum phases (a main motivation of our study).
Early theoretical studies on the excited bands (pre 2010)
An incomplete list!! [red=WVL involved]

On multi-orbital
  ...

On p-orbital
- A. F. Ho, arXiv:cond-mat/0603299
- E. Zhao and WVL, Phys. Rev. Lett. (2008);
  ...
  ...

Our p-orbital work
- Nature Phys. 8, 67 (2012)
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Experiments: atoms observed in higher orbital bands

**Fermions on p-band**
- Fermions transferred into p-band by sweeping across Feshbach resonance, i.e., by strong interaction. [M. Köhl et al, PRL 94, 080403 (2005)]
- Band cross through Dirac point [ETH/Esslinger et al, Nature 483, 302 (2012)]

**Bosons on p-band**
- Pumping bosons by Raman transition [T. Mueller, I.Bloch et al., PRL 2007]
- “Unconventional (complex) multi-orbital superfluidity” on hexagonal double-well ‘super-lattice’ [P. Soltan-Panahi, …, K. Sengstock, Nature Physics (online Nov 2011)]
Pioneer Experimental observation: finite momentum BEC

In Experiment:
• selected one direction
• hence only one p sub-band,
• no orbital degeneracy

Mueller, Bloch et al [PRL 2007] and theoretical prediction
[Isacsson-Girvin, 2005; Kuklov, 2006; WVL, Wu 2006] agrees!
Experiments: atoms observed in higher orbital bands

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Boson: p- and f-band experiments – double well lattices

- Interacting chiral p+ip order [Hemmerich group, new preprint (private/unpublished)]

Hamburg/
A. Hemmerich group

Remark:
First observation of p-band BEC with C4 symmetry and hence orbital degeneracy
Experimental observation: complex p-orbital superfluids


Interpretation: Nature of orbital order

Isotropic lattice

Observed momentum distribution

complex $p_x \pm ip_y$

Theoretical understanding


Experimental finding consistent with prediction by [WVL, C. Wu, PRA 2006]
p-orbital band phase diagram

[X. Li, E. Zhao and WVL, PRA 83, 063626 (2011)]
[See also early Mott phase results by A. Isacsson and S. M. Girvin, PRA 72, 053604 (2005); and A. Collin, J. Larson, and J.-P. Martikainen, Phys. Rev. A 81, 023605 (2010)]

-Mott phases at commensurate filling
1. filling = 1
2. filling > 1

\[ \psi = p_x \pm ip_y \]

Compare Mott results
- Our theory agrees with Isacsson-Girvin MFT for filling=1
- Differs Isacsson-Girvin at higher integer fillings [who predicted \( p_x - p_y \) alternating]
- Collin-Larson-Martikainen disagrees with both Isacsson-Girvin and us

Compare: the usual s-band Mott phase is featureless.
Observation II: complex multi-orbital superfluidity

[P. Soltan-Panahi, Lühmann, Struck, Windpassinger, K. Sengstock, Nat. Phys. 8, 71 (2012)]
Other early and recent experiments of double-well superlattices

**NIST group (Porto/Phillips)**


**Bloch group**


**Stamper-Kurn group**


Superlattice (red and blue lattices overlapped) → Kagome

Does not preserve original geometric symmetry
Part 3 and 4:
- Quantum 120° orbital only model in optical lattices
- Topological insulator (Z invariant) of sp-orbital ladder

Work done (in collaboration) with:
Sankar Das Sarma, Andreas Hemmerich, Xiaopeng Li, Kai Sun, Erhai Zhao.

References:
- Background and perspective (news & views):
  * Nature Physics* 7, 101 (2011) [with M. Lewenstein]

- PRL 100, 160403 (2008)
- Nature Physics 8, 67 (2012)
- arXiv:1205.0254
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T_i^{(a/b)} & = \frac{1}{4} (-\sigma_i^z \pm \sqrt{3}\sigma_i^x), \quad T_i^{(c)} = \frac{1}{2} \sigma_i^z
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\]
Orbital quantum $120^\circ$ model

Pseudo-spin operators on frustrated lattices (triangular, honeycomb, Kagome, …)

The orbital exchange Hamiltonian is:

$$H_{120} = J_z \sum_{\mathbf{R}, j} T_j(\mathbf{R}) T_j(\mathbf{R} + \hat{e}_j)$$

lattice sites $j = 1, 2, 3$

where

$$T_1 = T_z, \quad T_2 = -\frac{1}{2} T_z - \frac{\sqrt{3}}{2} T_x$$

$$T_3 = -\frac{1}{2} T_z + \frac{\sqrt{3}}{2} T_x$$

This quantum $120^\circ$ model is closely related to the compass model and Kitaev model.
Quantum $120^\circ$ model of electron $e_g$ orbitals: van den Brink, New J. Phys. 6, 201 (2004).
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Hamburg 2D double-well lattice $\rightarrow$ sp-orbital Ladder

dimension reduction
Topological orbital ladders

Xiaopeng Li, Erhai Zhao and WVL, arXiv:1205.0254
**sp-mixing and topological nature**

**-sp-mixing**

- Hamiltonian in momentum space

\[ \mathcal{H}(k) = \begin{bmatrix} -2t_s \cos k & -2it_{sp} \sin k \\ 2it_{sp} \sin k & 2t_p \cos k \end{bmatrix} \]

\[ \mathcal{H}(k) = h_0(k) \mathbb{I} + \vec{h}(k) \cdot \vec{\sigma} \]

Berry phase is \( \pi \).
Momentum $k$

(lattice constant $a = 1$)
Edge states

- **flat band limit** (easy to show)

$$t_s = t_p = t_{sp} = t \quad E(k) = \pm 2t$$

$$H_0 \to 2t \sum_j \phi_+^\dagger(j)\phi_+(j+1) + h.c. \quad \phi_{\pm} = [a_p \pm a_s] / \sqrt{2}$$

Edge states are completely localized

- **general case**

$$H_0 = \sum_j C_j^\dagger \left[ \frac{t_p - t_s}{2} I - \frac{t_p + t_s}{2} \sigma_z - it_{sp} \sigma_y \right] C_{j+1} + h.c. \quad C_j = \begin{bmatrix} a_s(j) \\ a_p(j) \end{bmatrix}$$

Edge states decay with a width $\xi = 2 / \log(||(\sqrt{t_{sp} + t_s}) / (\sqrt{t_{sp} - t_s})||)$
Summary: topological insulator from odd parity

1. Topological insulator (index group Z class) at half filling.

2. Compare with spin-orbit coupling generated by artificial gauge field in cold atoms/Pioneer experiments:
   A. Bosons: NIST (I. Spielman et al), USTC (S.Chen, J. Pan et al) …
   B. Fermions: Shanxi U (J. Zhang et al), MIT (M. Zwierlein et al)

3. This model: No spin, but orbit. Resembles spin-orbit coupling if (s, p) space viewed pseudo-spin-1/2.

4. New result: topological phase not requiring any of previously known mechanisms: rotation, gauge field, p-wave pairing,…
Emergent Effective Spin-orbit coupling

-Hamiltonian in momentum space

\[ \mathcal{H}(k) = \begin{bmatrix} -2t_s \cos k & -2it_{sp} \sin k \\ 2it_{sp} \sin k & 2t_p \cos k \end{bmatrix} \]

\[ \mathcal{H}(k) = h_0(k)I + \vec{h}(k) \cdot \vec{\sigma} \]

\[ \sim \sigma_y \sin k_x \approx \sigma_y k_x \]
Topological phase transition – driven by rotation

-rotating individual lattice sites

\[ \delta H = \frac{\Omega^2}{\epsilon_y} C_j^\dagger \sigma_y C_j \quad \Delta_y = \frac{\Omega^2}{\epsilon_y} \]

- phase transition

\[ h_z(k) \quad h_y(k) \]

Non-trivial

\[ \Delta^c_y = 2t_{sp} \]

trivial

N. Gemelke et al, arXiv:1007.2677
Xiaopeng Li, Erhai Zhao and WVL, arXiv:1205.0254
### Domain Wall Fractional Charge

\[ H_\eta = H + \frac{\Delta v}{2} \sum_j [1 - \cos \eta(j)] C^\dagger_j \sigma_y C_j \]

\[ \eta(j = -\infty) = 0 \quad \quad \eta(j = +\infty) = \pi \]

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<th>A</th>
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<td>Non-trivial</td>
<td>Half charge</td>
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Firm Computation of Fractional Charge:
Background (auxiliary) gauge field method

• Introduce background gauge field: \((A_{\tau}, A_x)\), \(\tau = it\)

\[ D_{\tau} = \partial_{\tau} + iA_{\tau}(j, \tau), \]
\[ T_{jj+1} = e^{iA_x(j+1/2, \tau)}T_{jj+1}, \]
\[ T_{jj+1} = \begin{bmatrix} -t_s & -t_{sp} \\ ts_p & t_p \end{bmatrix}, \]
\[ T_{jj+1} = T_{jj+1}^\dagger. \]

• Effective action

\[ \tilde{S}_{\text{eff}}[A_\mu] = \int dx dt (A_x \partial_t \eta - A_t \partial_x \eta) \frac{1}{2\pi} \partial_\eta \gamma(\eta) \]

• Charge

\[ Q = \int \frac{\tilde{S}_{\text{eff}}}{\delta A_t} = - \frac{1}{2\pi} \int dx \partial_x \eta \partial_\eta \gamma(\eta) = - \int \frac{dn}{2\pi} \partial_\eta \gamma(\eta) \]

Find: \( Q = \frac{1}{2} \mod 1 \)
Effects of interaction, beyond half filling, etc.

- Topological to (non-topological) Ferro-orbital Mott insulator transition, driven by interaction [X. Li, E. Zhao, WVL, arXiv: 1205.0254]

- Away from half filling, find interesting phases: orbital density wave, pair density wave, and especially quasi-1D superconductivity with repulsive interaction, …, by RG/Bosonization [X. Li, WVL, arXiv: 1210.1859]
Topological semimetal in a fermionic optical lattice

Kai Sun, W. Vincent Liu, Andreas Hemmerich, and S. Das Sarma

System: p-orbital fermions on Double-well lattice

Ordered phase
- Breaks Time-reversal symmetry
- Topological Insulator
- New mechanism – interaction driven -- Differs from the previously known’s: artificial gauge field, rotation, spin-orbit coupling, …
Conclusion---Optical Lattice Orbital Physics

Frustrated orbital model

Orbital quantum 120° model

\[ H_{120} = J \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \left( \sigma_\mu \sigma_\nu \right) \]

\[ \mu, \nu = \sigma, \pi \]

Pseudo-spin operators on frustrated lattices (triangular, honeycomb, kagome, …)

The orbital exchange Hamiltonian is:

\[ T_\mu = \frac{1}{2} \left( c_{\mu}^\dagger c_{\nu} + c_{\nu}^\dagger c_{\mu} \right) \sigma_\mu \sigma_\nu \]

where

\[ T_1 = T_2 = -\frac{1}{2} T_1 - \sqrt{3} T_2 \]

\[ T_3 = -\frac{1}{2} T_2 + \sqrt{3} T_3 \]

This quantum 120° model is closely related to the compass model and Kitaev model.

Quantum 120° model of electron \( z \) orbitals: van den Brink, New J. Phys. 6, 201 (2004).

Interested? Perspectives in:

Topological semimetal

Topological orbital ladder

OPTICAL LATTICES

Orbital dance

Emulating condensed-matter physics with ground-state atoms trapped in optical lattices has come a long way. But excite the atoms into higher orbital states, and a whole new world of exotic states appears.

Maciej Lewenstein and W. Vincent Liu