

Probing Non-Equilibrium Dynamics Using Ultracold Quantum Gases in Optical Lattices

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\$ DARPA (OLE)

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Outline

Introduction

Single Atom Imaging

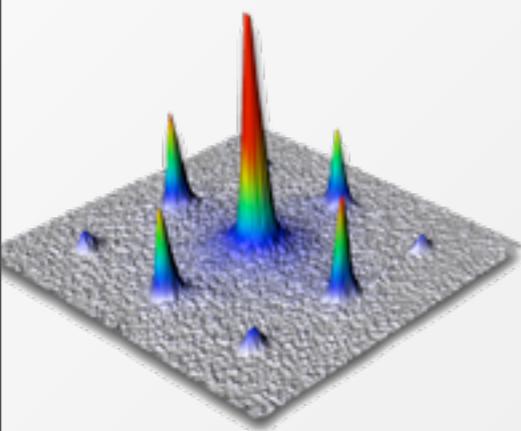
Applications

Tracking the Motion of a Single Spin Impurity

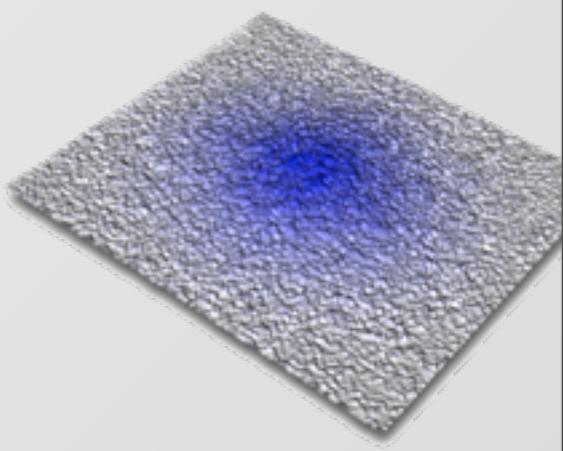
Observation of Crystalline States of Rydberg Atoms

Measuring Zak-Berry Phase of Topological Bloch Bands

$$\gamma = \frac{\text{Interaction Energy}}{\text{Kinetic Energy}} \gg 1$$

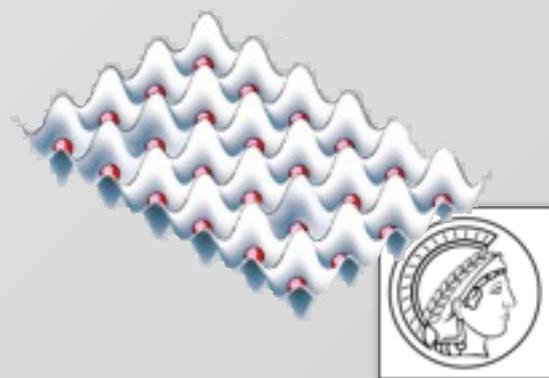
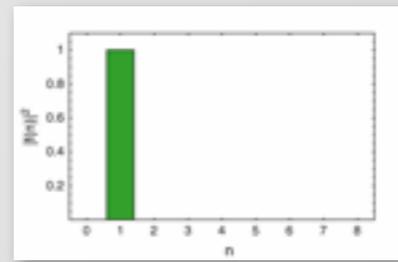
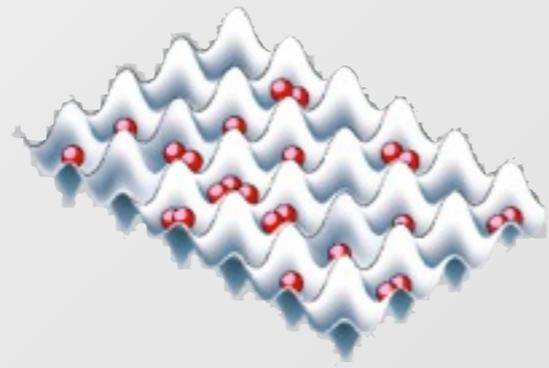
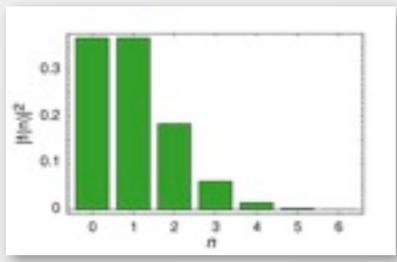


Weak Interactions



Strong Interactions

Quantum Phase Transition
See S. Sachdev & B. Keimer Phys. Today 2011

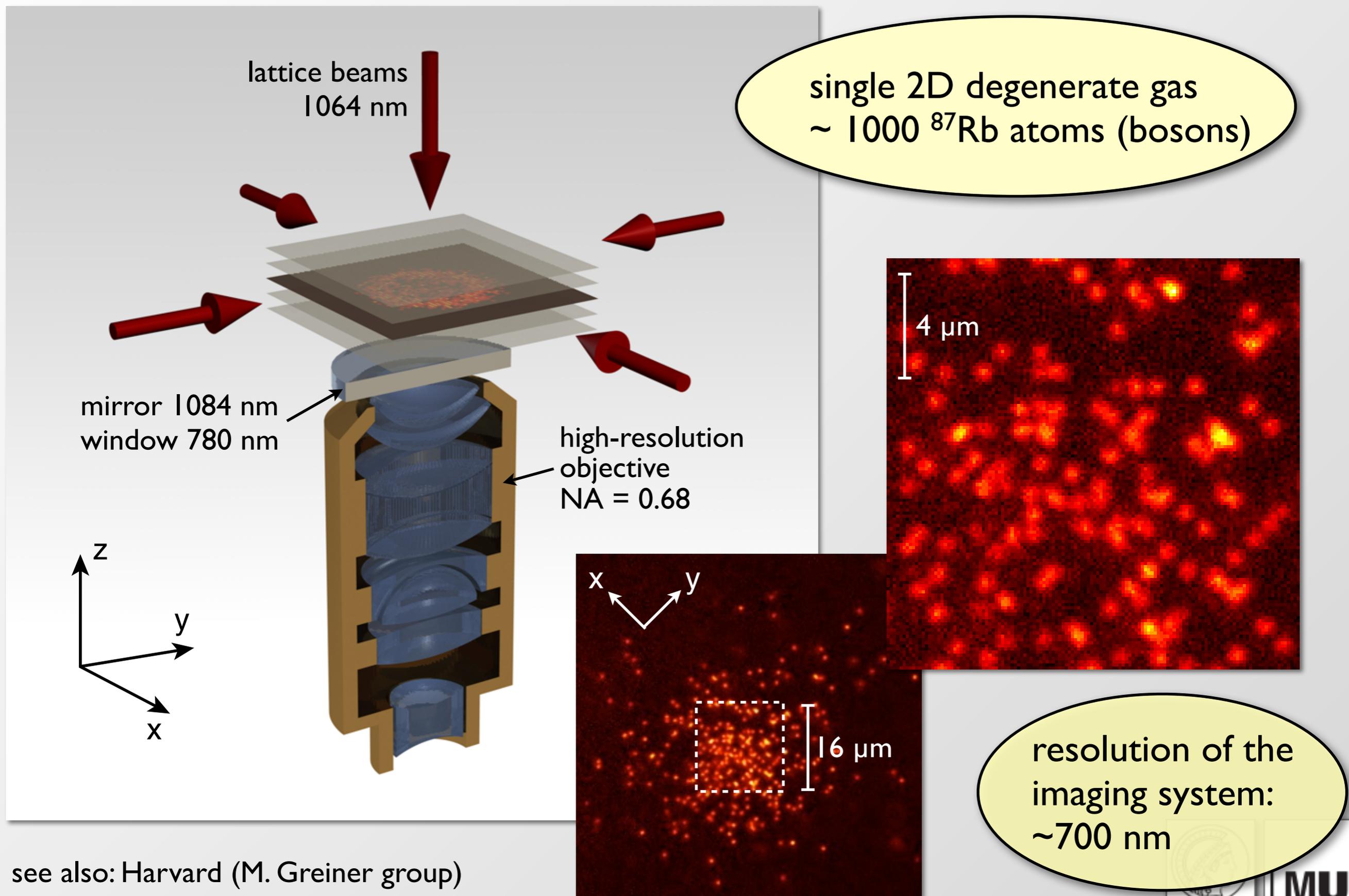


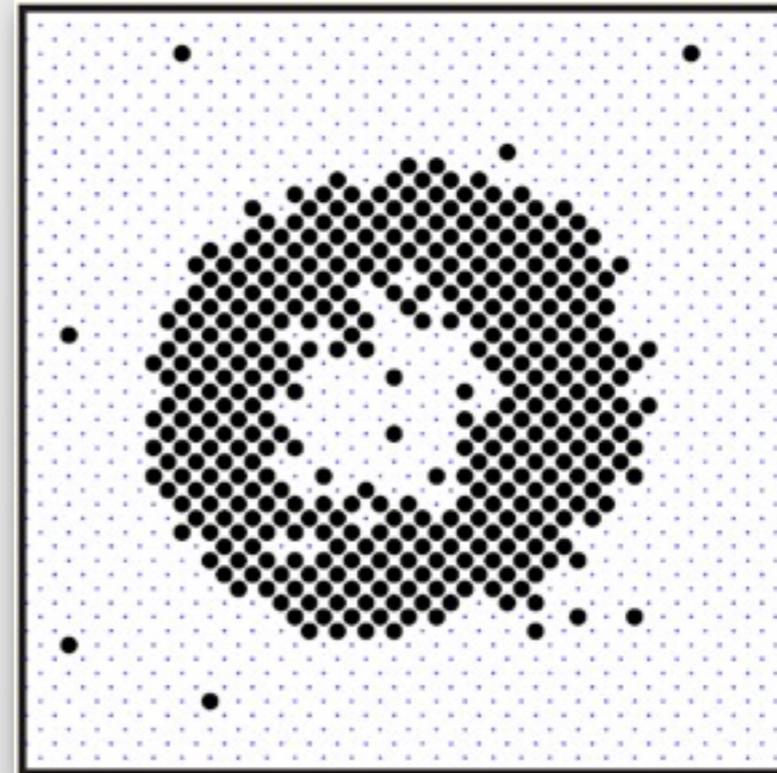
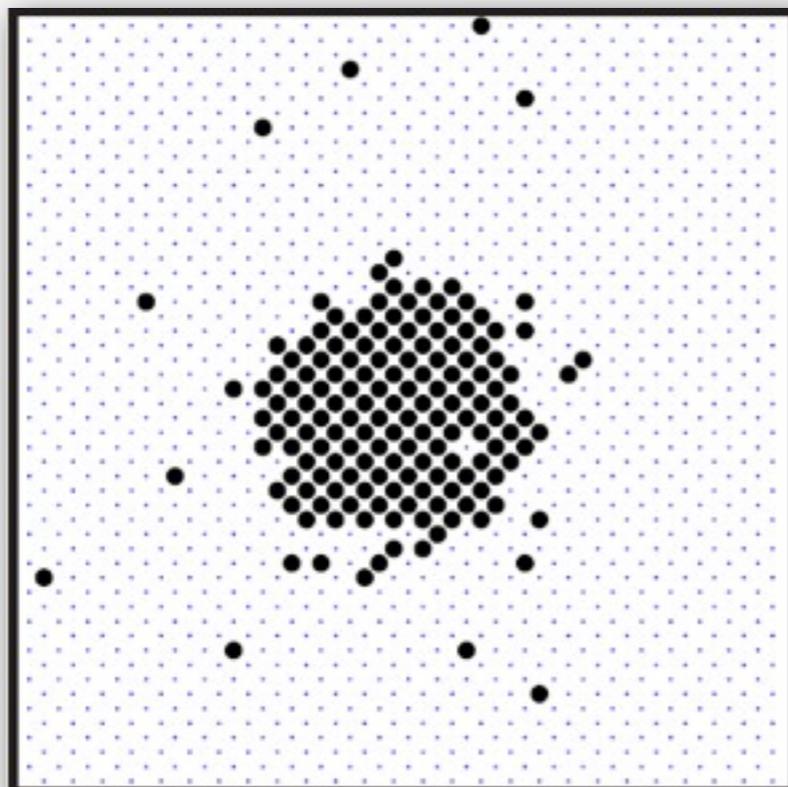
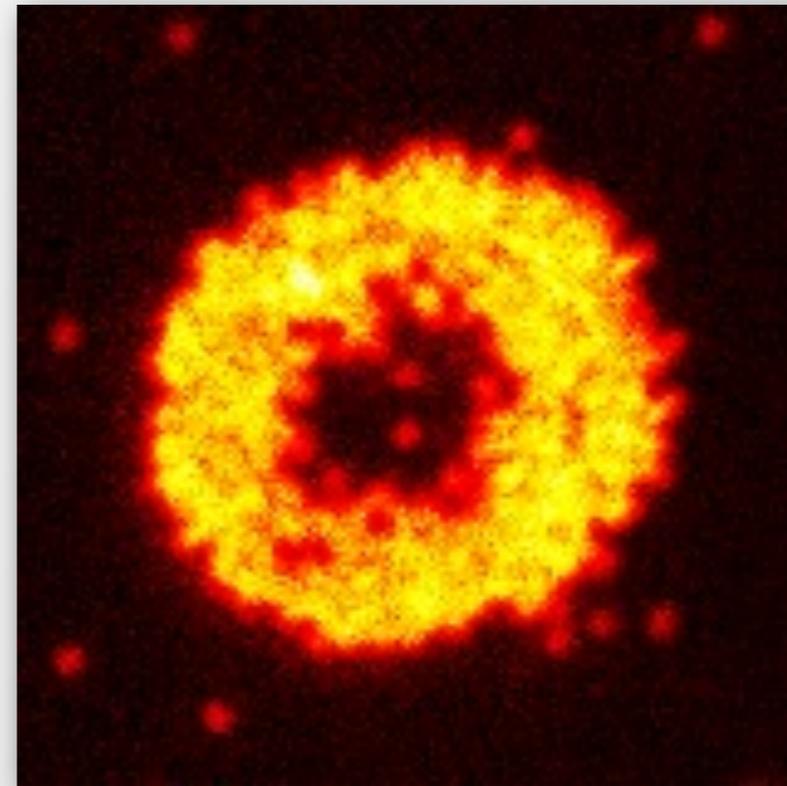
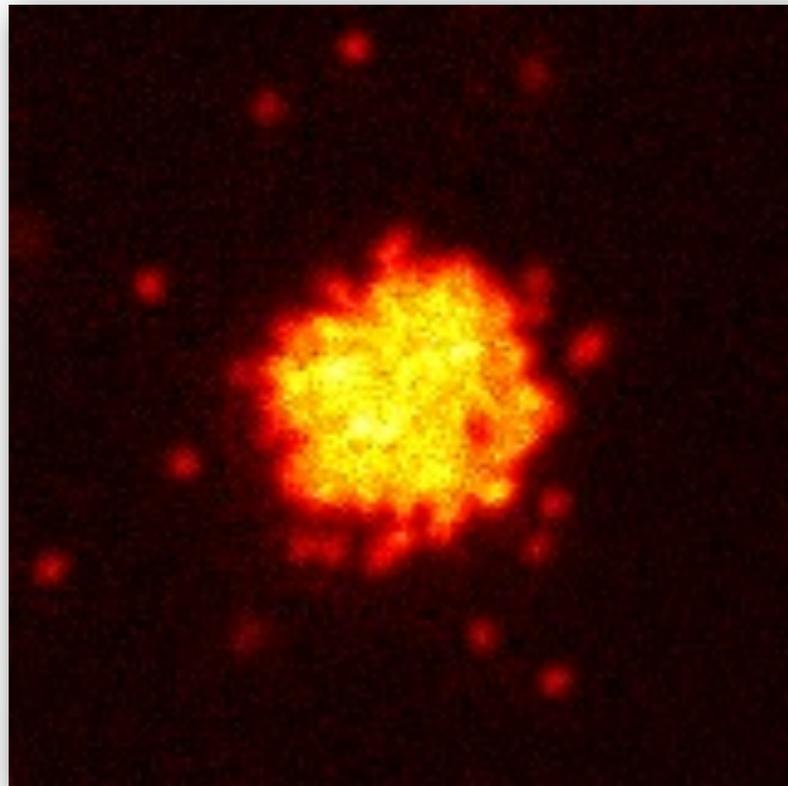
Single Atom Detection in a Lattice

J. Sherson, Ch. Weitenberg, M. Endres, M. Cheneau, T. Fukuhara, P. Schauss, J. Sherson, I.B., S. Kuhr

Sherson et al. Nature 467, 68 (2010),
see also Bakr et al. Nature (2009) & Bakr et al. Science (2010)

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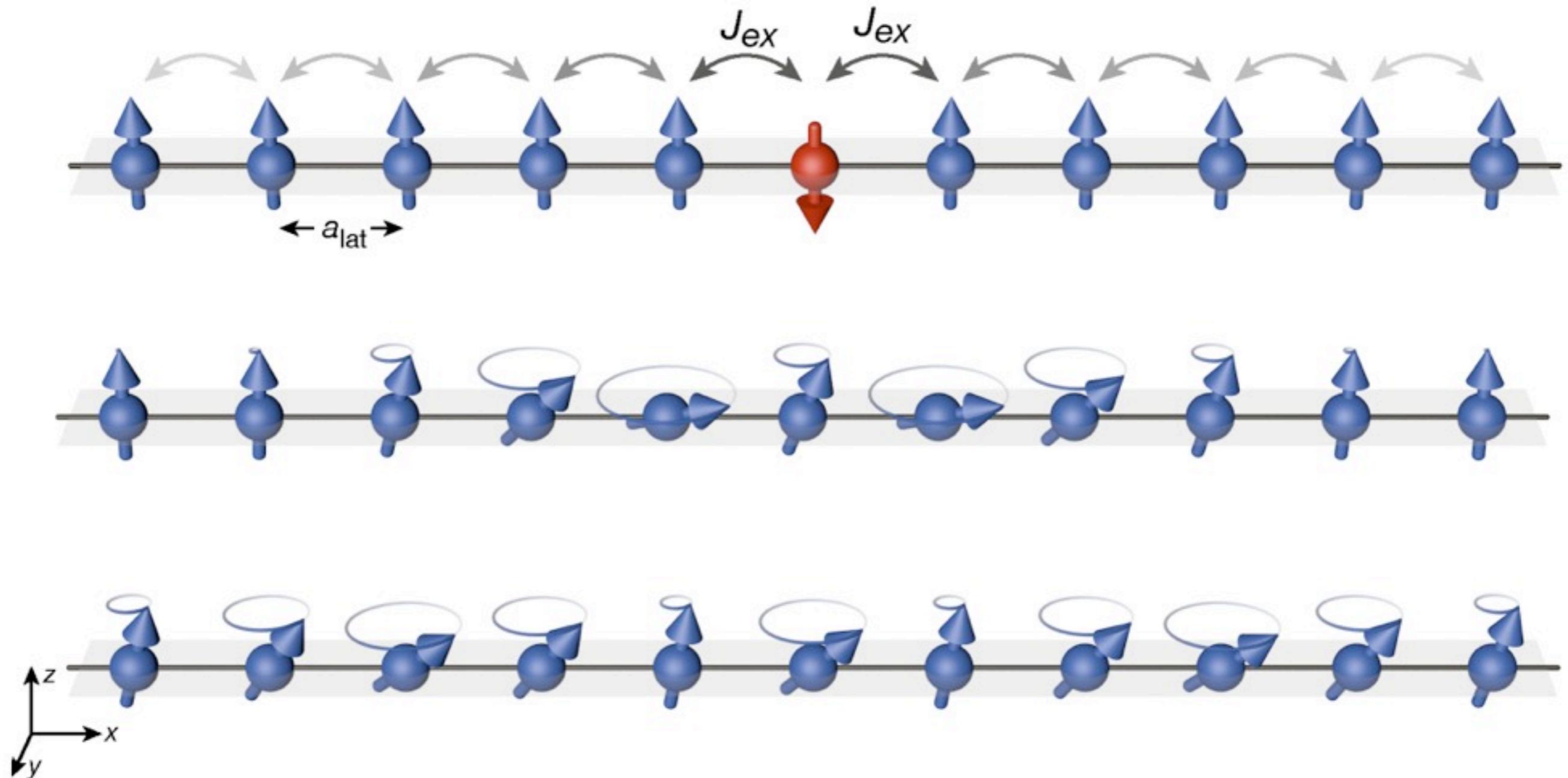


Quantum Dynamics of Mobile Single-Spin Impurity

T. Fukuhara, M. Endres, M. Cheneau P. Schauss, Ch. Gross, I. Bloch, S. Kuhr,
U. Schollwöck, A. Kantian, Th. Giamarchi



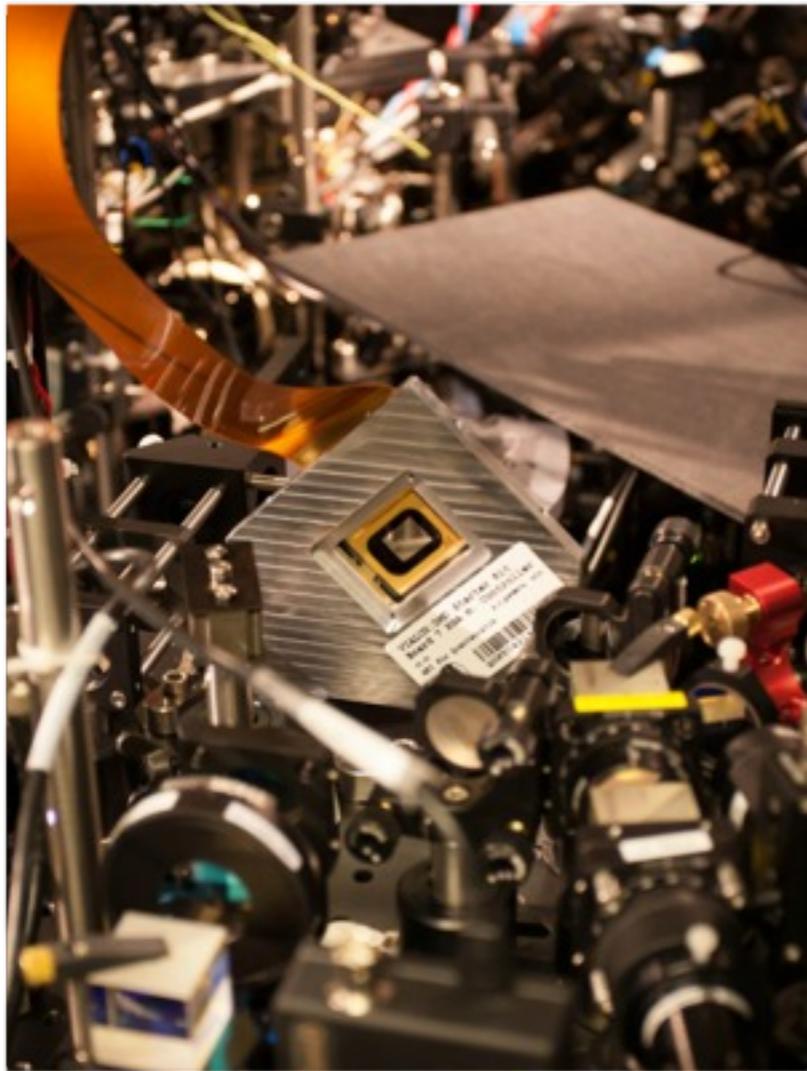
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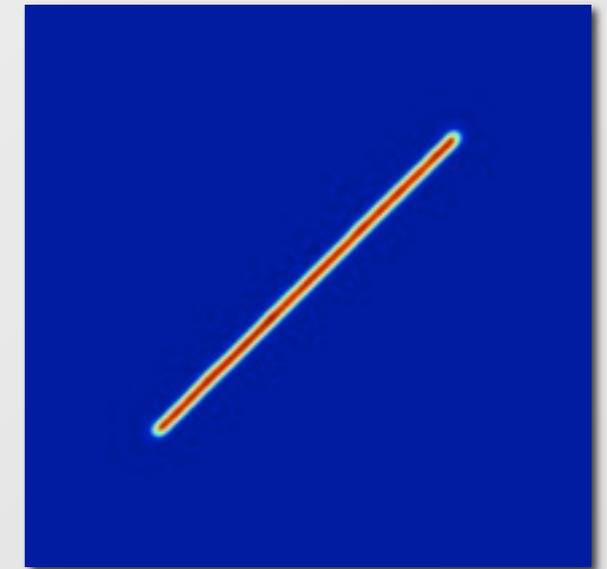
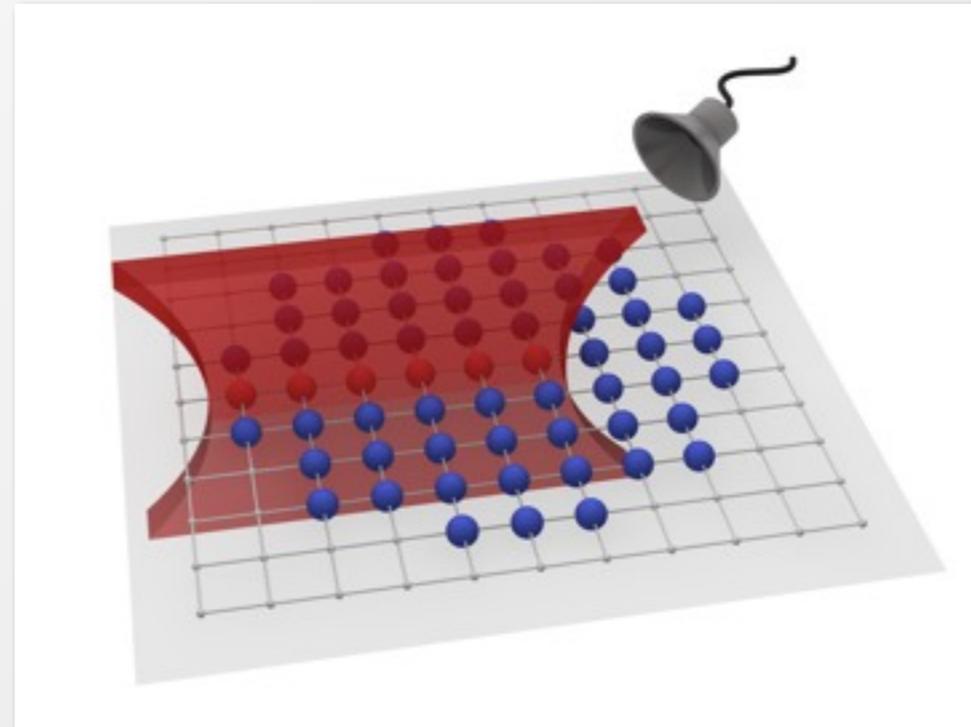
$$-J_{ex} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

Ferromagnetic Heisenberg Interaction

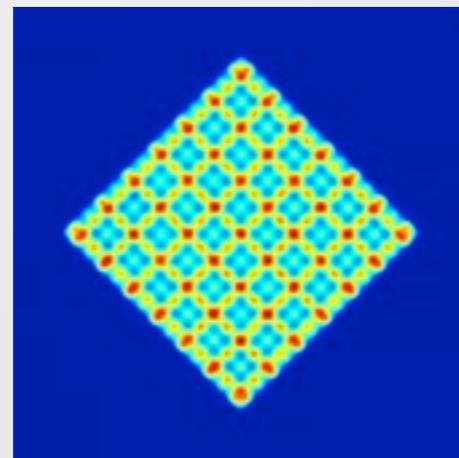




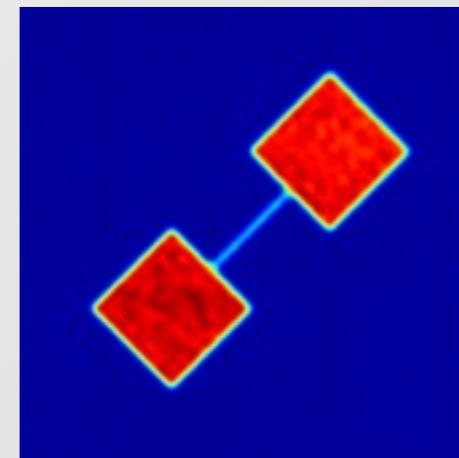
Digital Mirror Device
(DMD)



Measured Light Pattern



Exotic Lattices



Quantum Wires

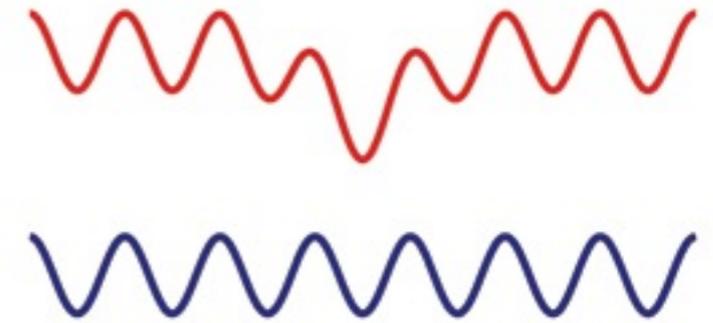
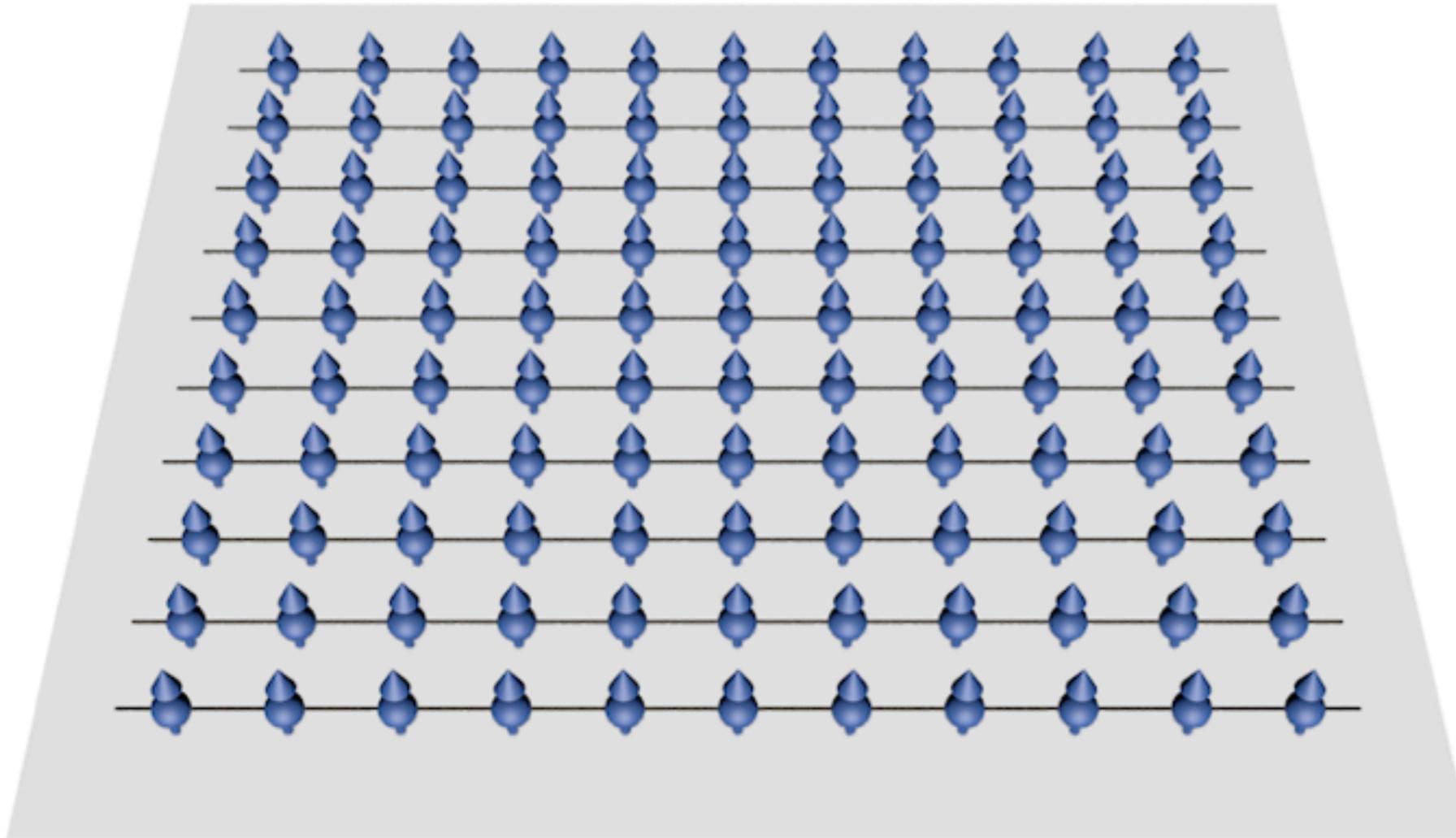


Box Potentials

Almost Arbitrary Light Patterns Possible!

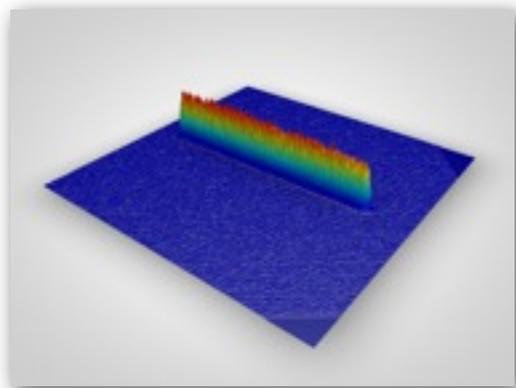
Single Spin Impurity Dynamics, Domain Walls, Quantum Wires, Novel Exotic Lattice Geometries, ...



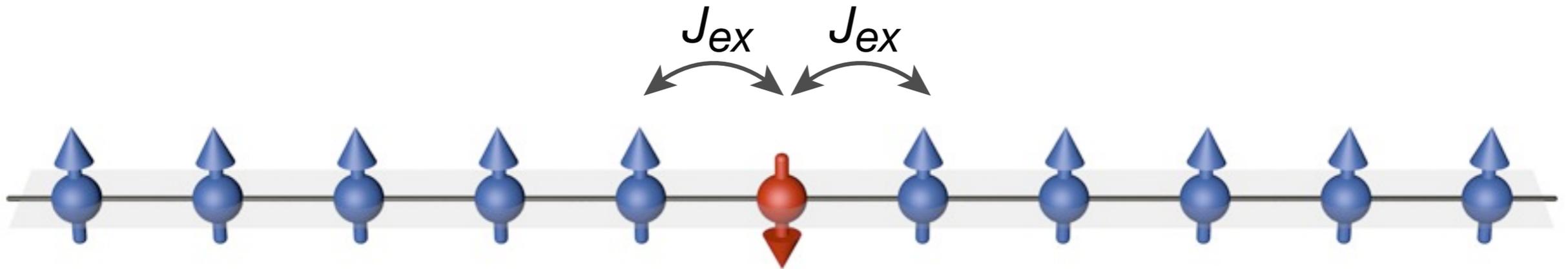


$$|2\rangle = |F=2, m_F=-2\rangle$$

$$|1\rangle = |F=1, m_F=-1\rangle$$



Line-shaped light field created with DMD SLM



Heisenberg Hamiltonian

$$H = -J_{ex} \sum \mathbf{S}_i \cdot \mathbf{S}_j = -J_{ex} \sum \left(S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z \right)$$

$$= -\frac{J_{ex}}{2} \sum \left(S_i^+ S_j^- + S_i^- S_j^+ \right) - J_{ex} \sum S_i^z S_j^z$$

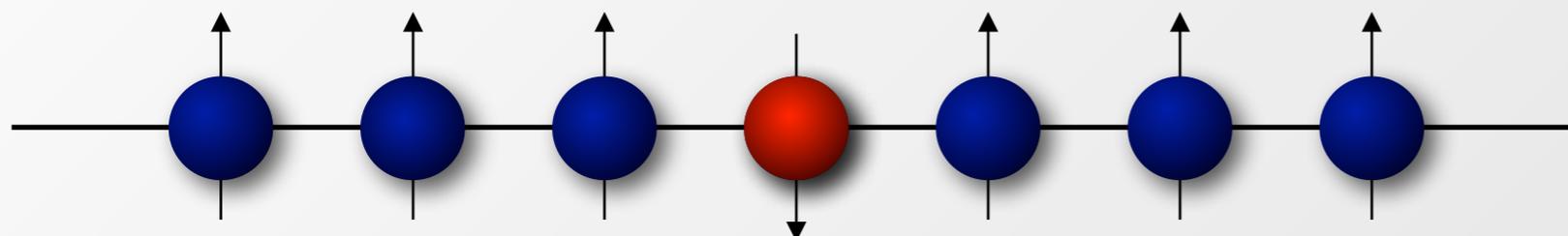
$$J_{ex} = 4 \frac{J^2}{U}$$

$$H = -J \sum \left(\hat{a}_i^\dagger \hat{a}_j + \hat{a}_i \hat{a}_j^\dagger \right) \text{ single particle tunneling}$$

Our detection is not spin-resolved.

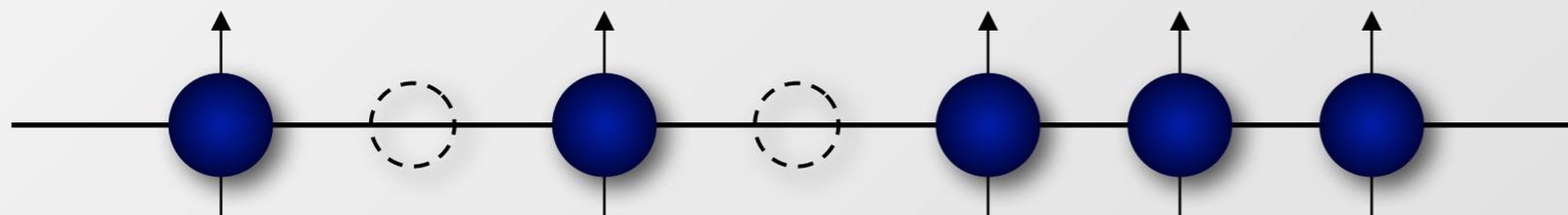
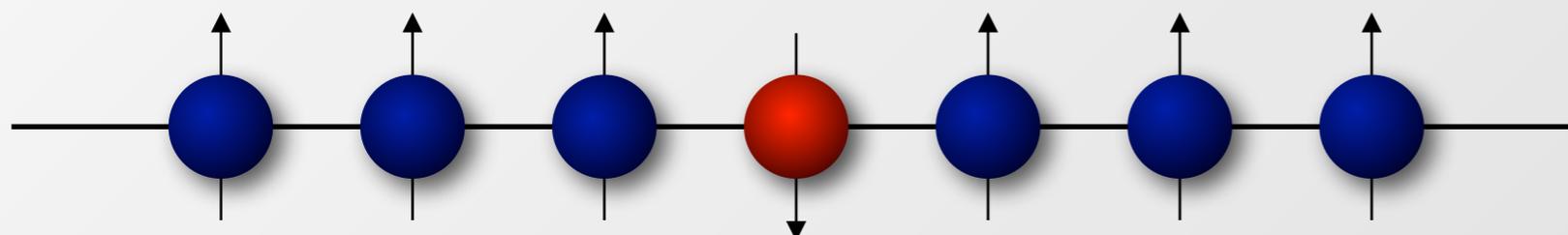
Positive image

Remove the other spin component (bath spin) before detection



Negative image

Measure spin impurities as empty sites by pushing out the impurities

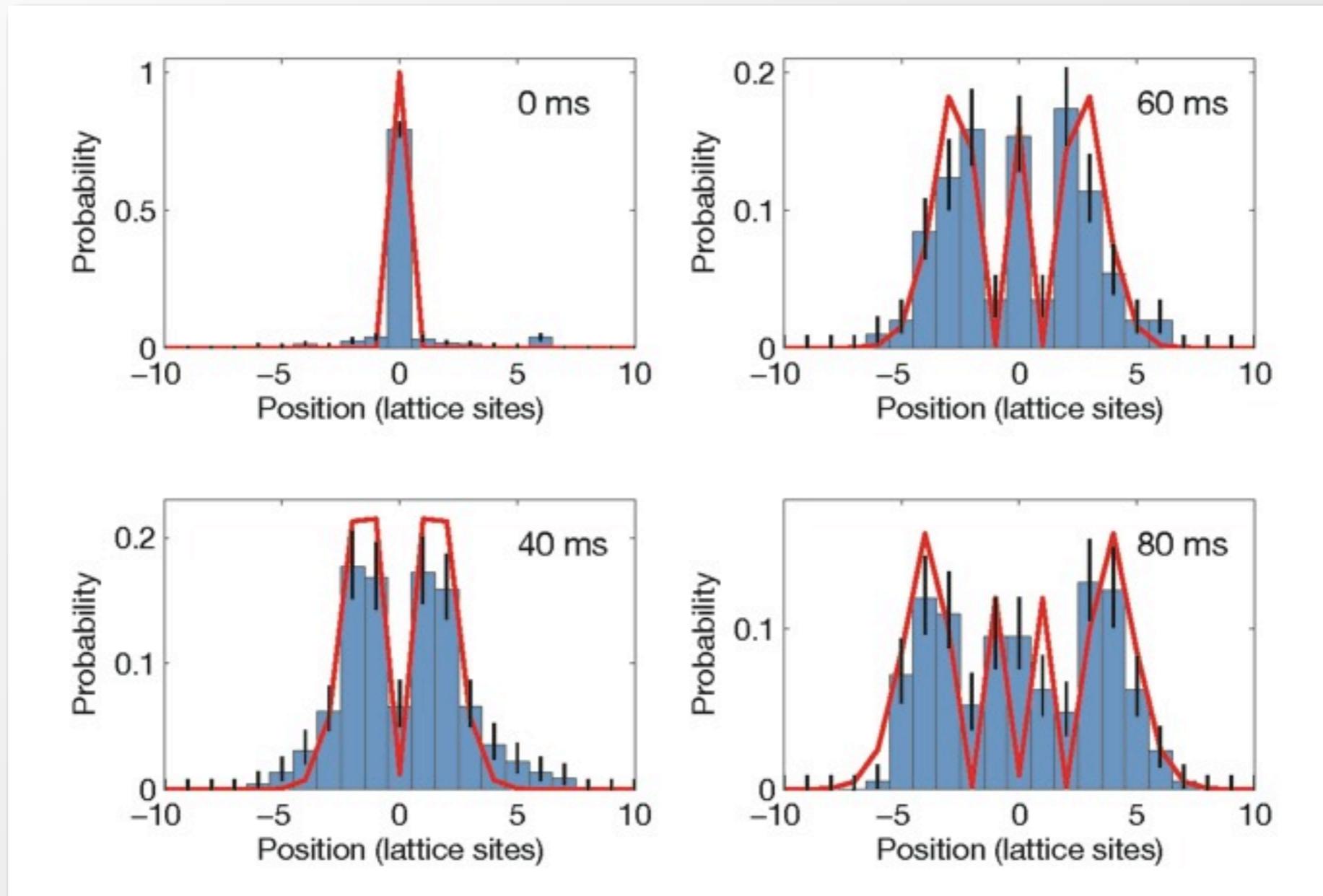


If we only use samples with one empty site ...

- the position of the impurity can be determined
- system has no extra excitation

" zero temperature"





$$V = 10 E_r$$

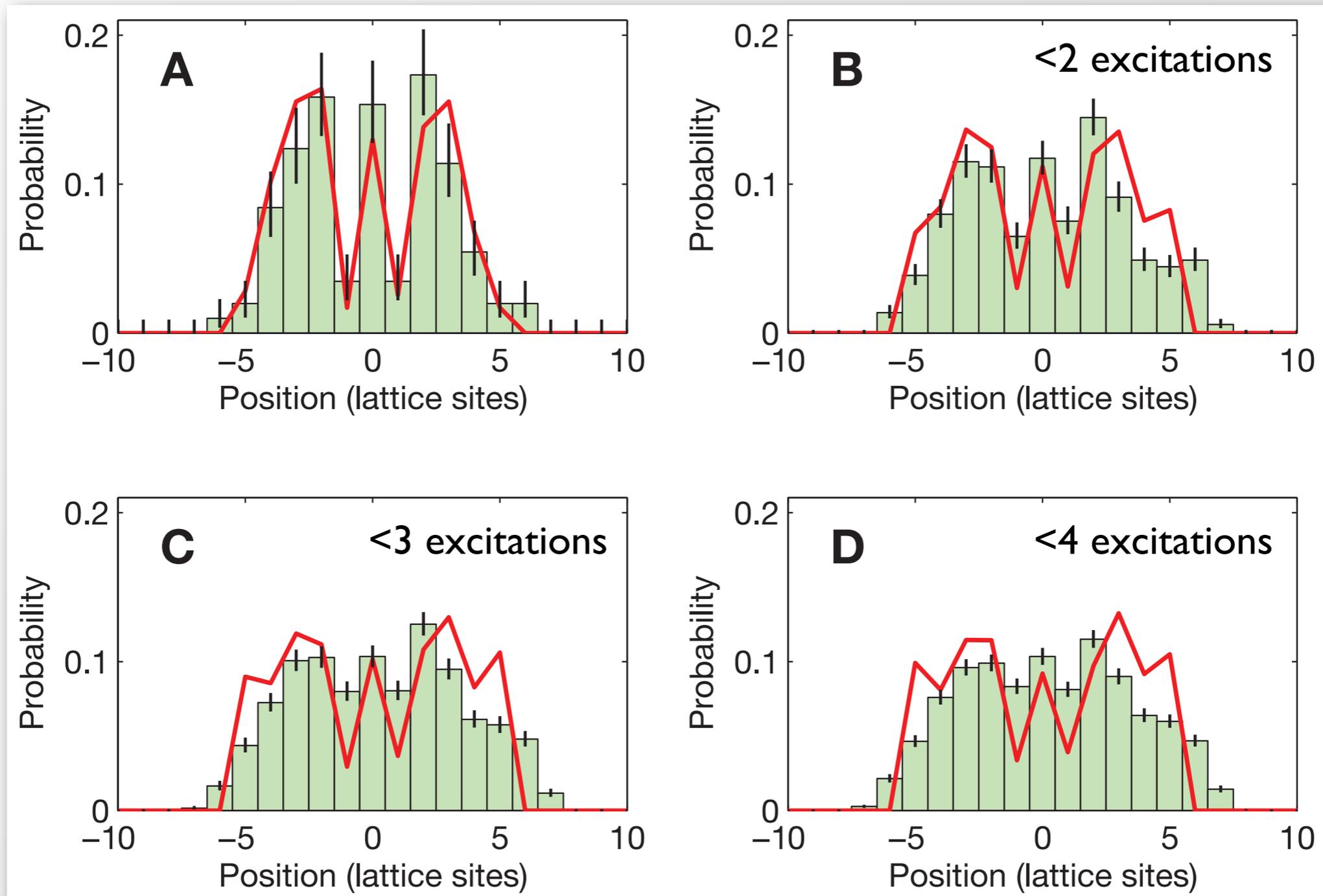
$$U/J = 19$$

$$H = -\frac{J_{ex}}{2} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+)$$

$$P_j(t) = \left[\mathcal{J}_j \left(\frac{J_{ext} t}{\hbar} \right) \right]^2$$

Bessel function of the first kind



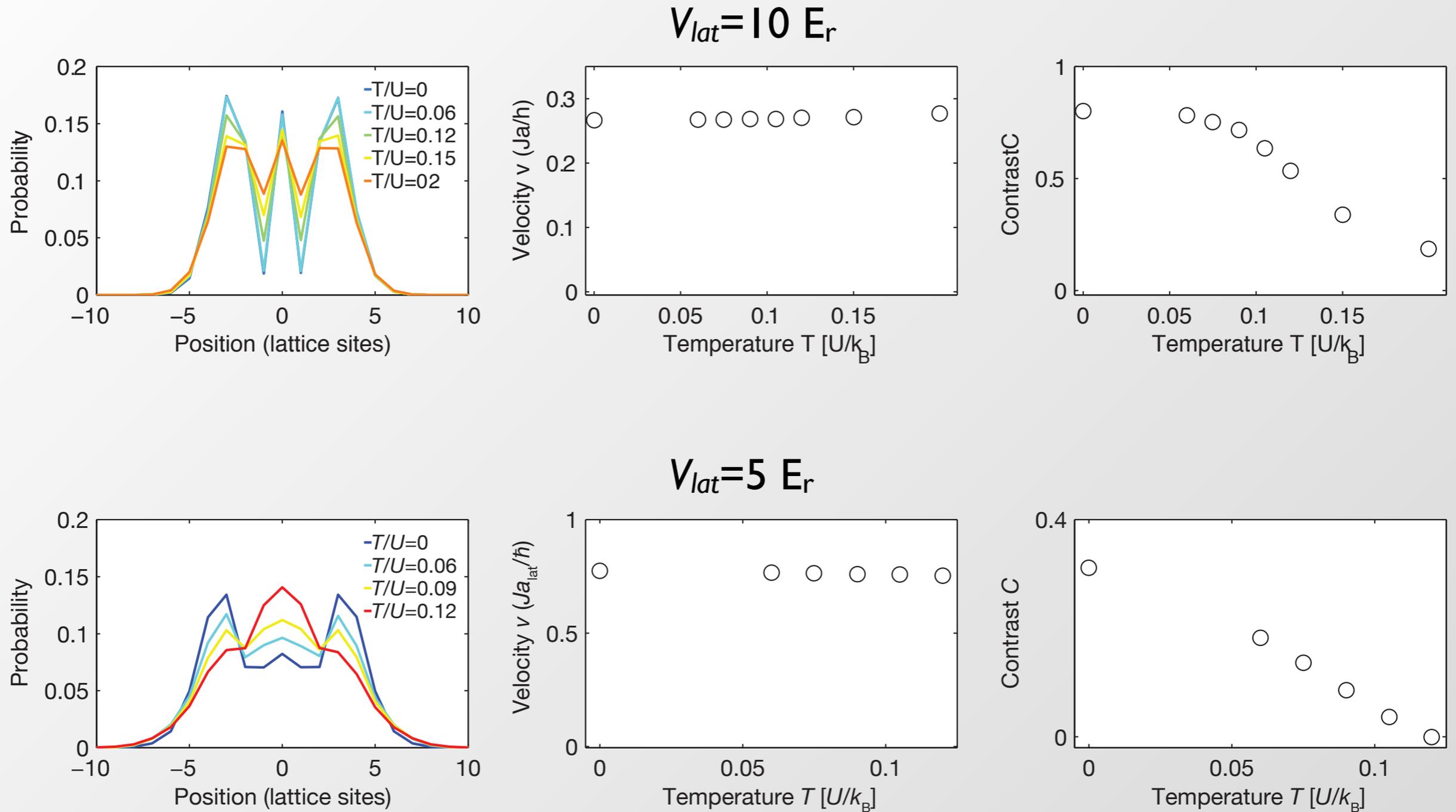


$$V = 10 E_r$$

$$U/J = 19$$

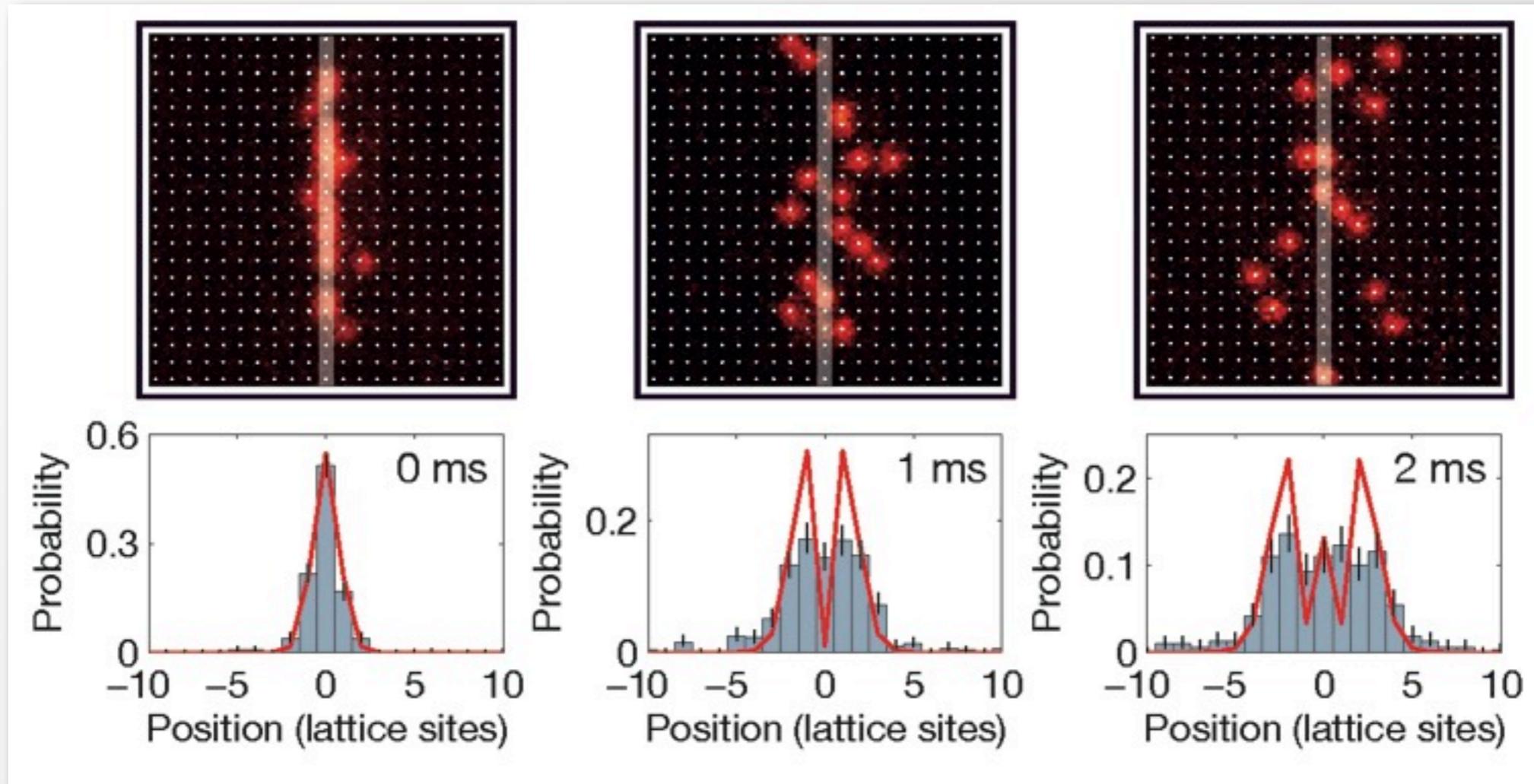
- Only visibility goes down
- Spreading speed almost independent of holes





Propagation speed almost constant, Coherence affected!

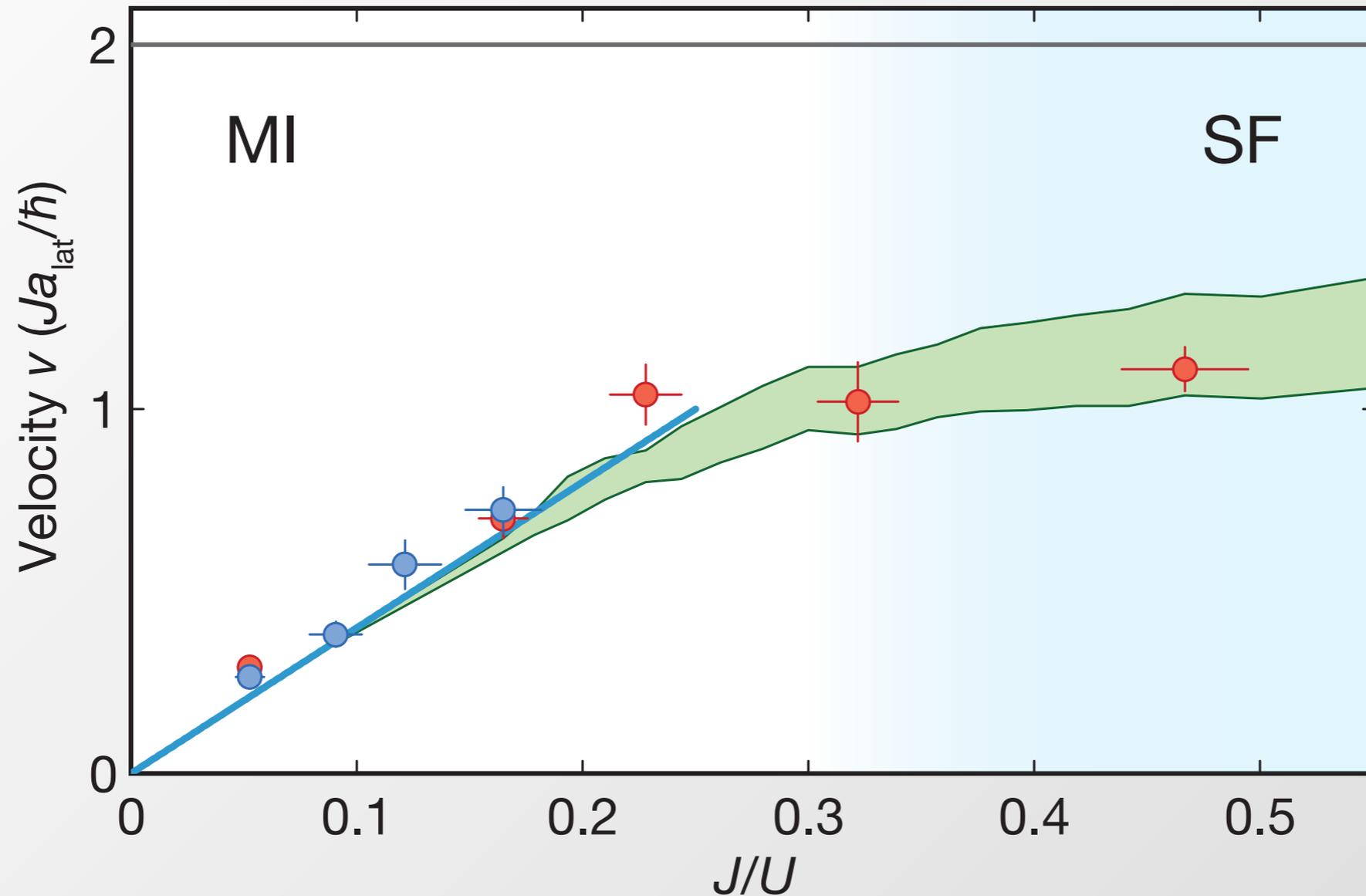




$$V = 4 E_r, J/U = 0.32$$

$$(J/U)_c \approx 0.3 \text{ for 1D}$$

Ramp-down time for pinning beam is 1 ms



Reference value:

$$v = \frac{2Ja_{\text{lat}}}{\hbar} \quad \text{Free particle tunneling}$$



Quantum Dynamics of Interacting Atoms/Spins

- Effect of Temperature/Holes on Dynamics
- Dynamics of **Magnon bound states**
- Domain Walls
- Higher Dimensions (1D, 2D, 3D)
- Entropy Transport
- Probe for **Quantum Critical Transport**
- Direct measurement of **Green's function**

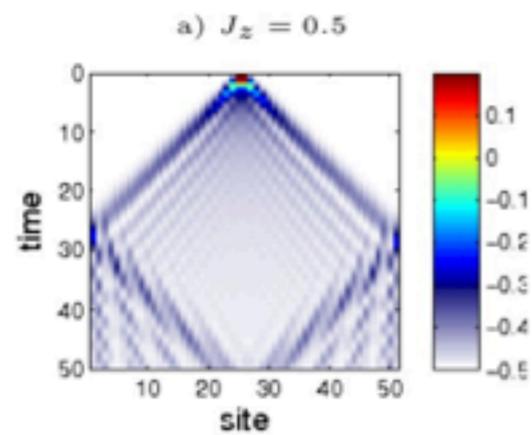
$$G(x_i, x_j, t) \propto \langle \uparrow | \hat{S}^\dagger(x_j, t) \hat{S}^-(x_i, 0) | \uparrow \rangle$$



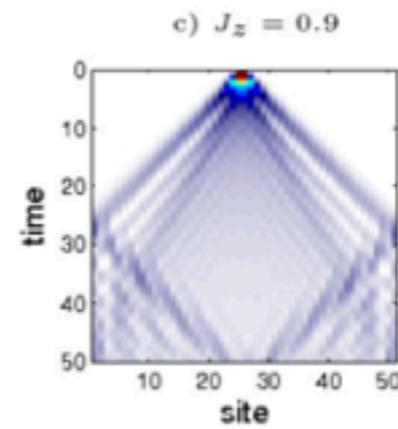
Two-spin excitation in FM



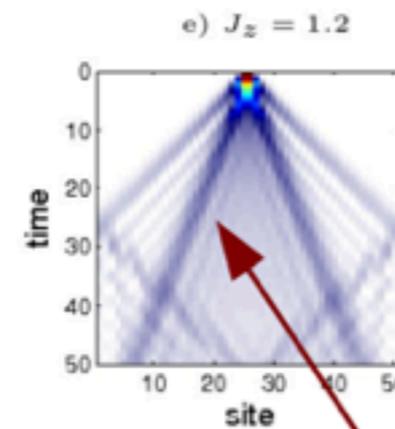
S_z



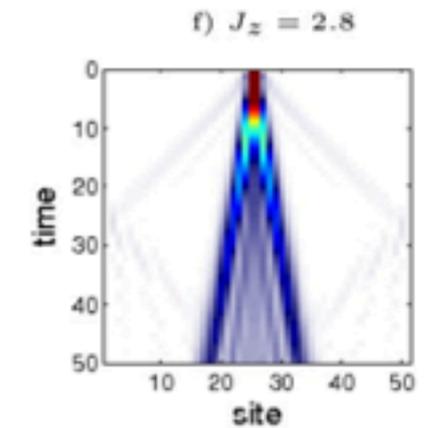
$J_z = 0.5$



$J_z = 0.9$



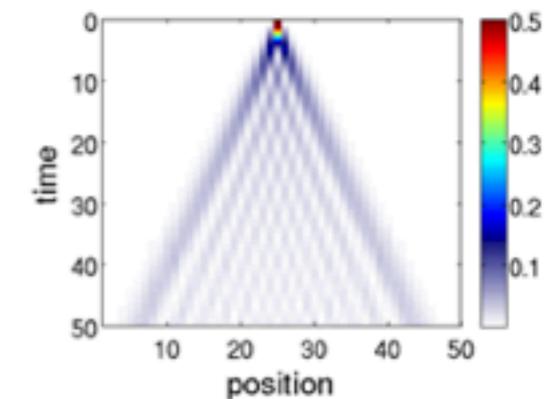
$J_z = 1.2$



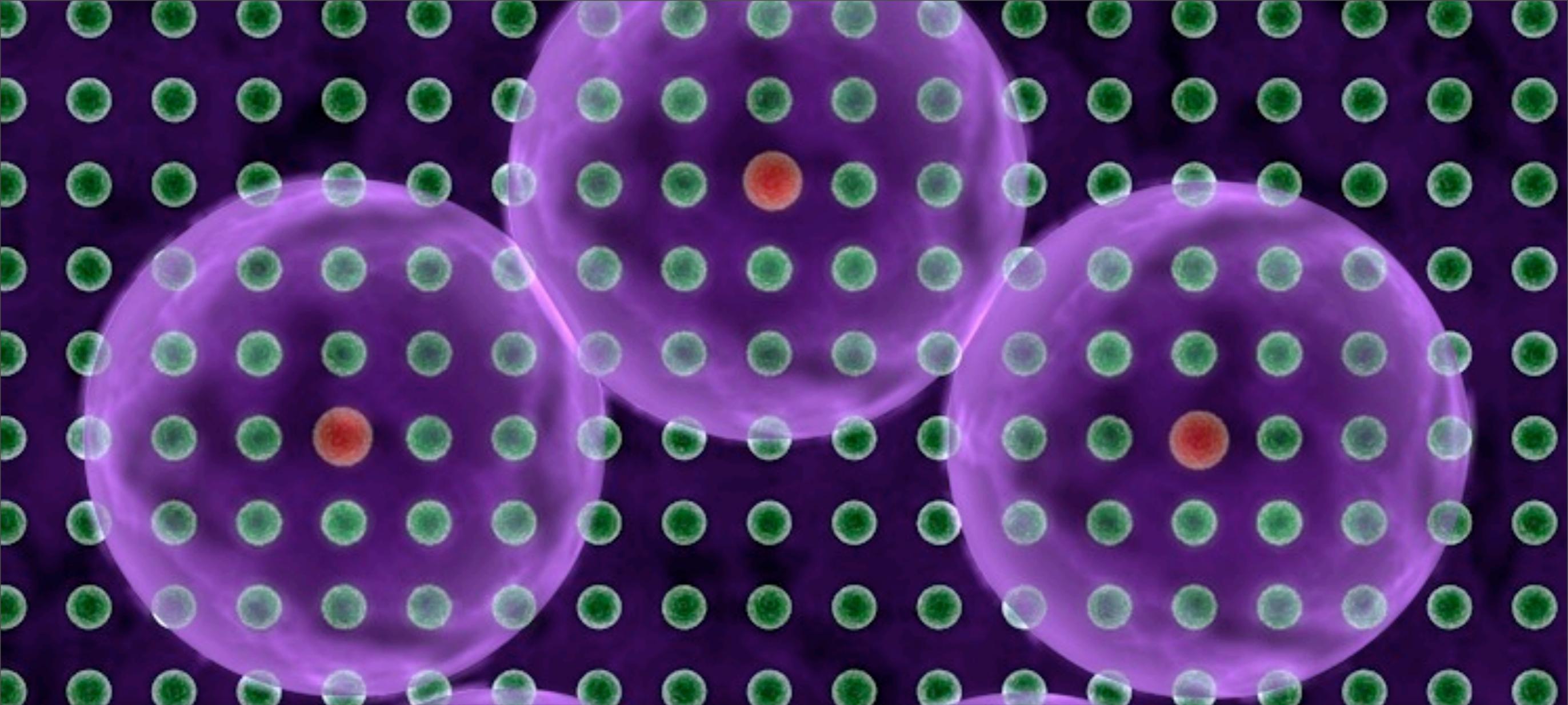
$J_z = 2.8$

- Two distinct propagation branches beyond $J_z = 0.7$
- **New lower branch is *bound state***
- **It dominates at large J_z , with decreasing velocity**
- Low entanglement entropy (see below)

$P(\uparrow\uparrow)$

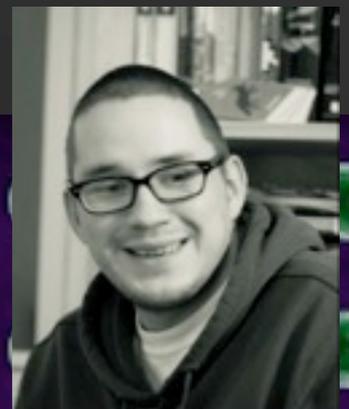


From: H.G. Evertz



Dynamical Formation of Rydberg Crystals

P. Schauss , M. Cheneau, M. Endres, T. Fukuhara, Th. Pohl, S. Kuhr & I.B.



(submitted)

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Rydberg atoms

- hydrogen-like wave function
 - quantum defect

$$E_{nlj} = - \frac{Ry}{[n - \delta_{lj}(n)]^2}$$

- Strong switchable interactions

$^{87}\text{Rb } 43S_{1/2}$

$^{87}\text{Rb } 5S_{1/2}$



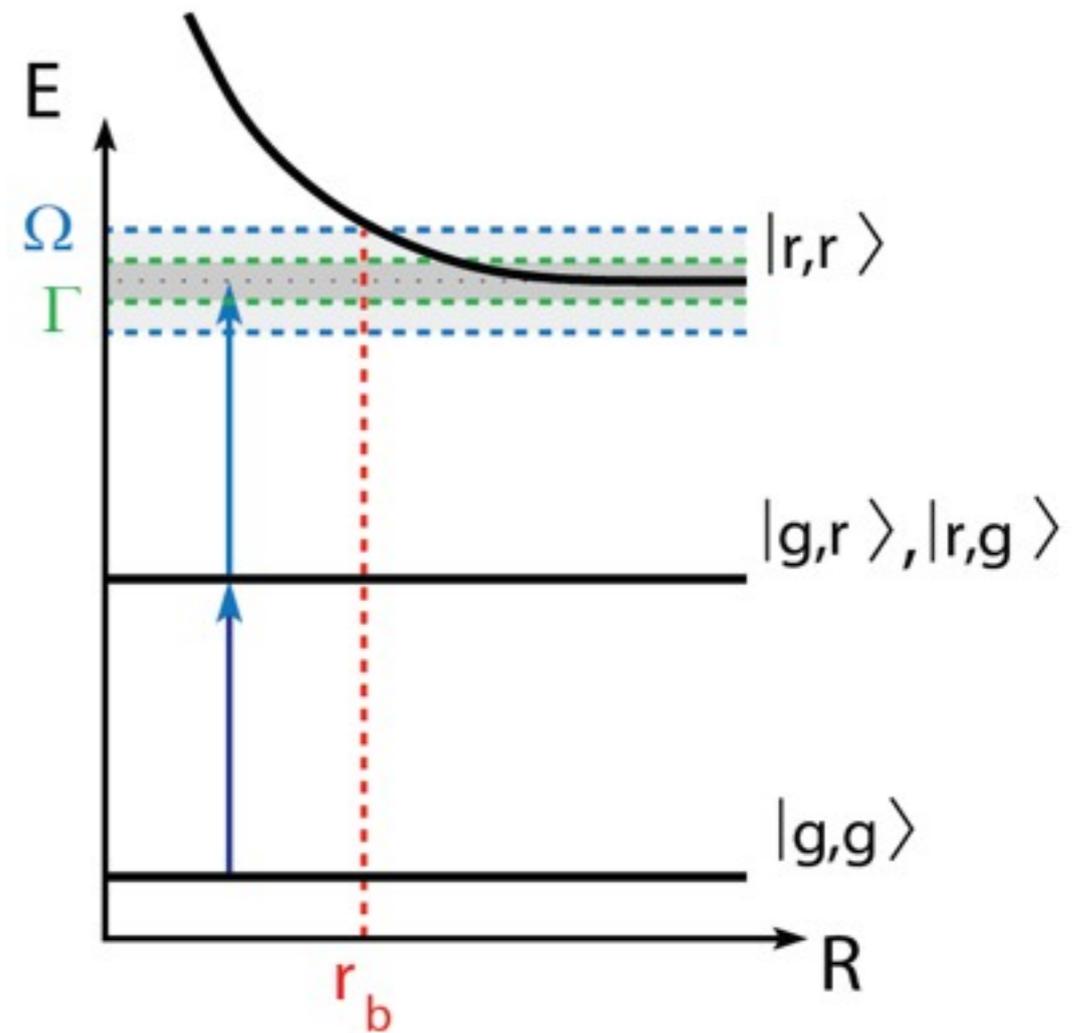
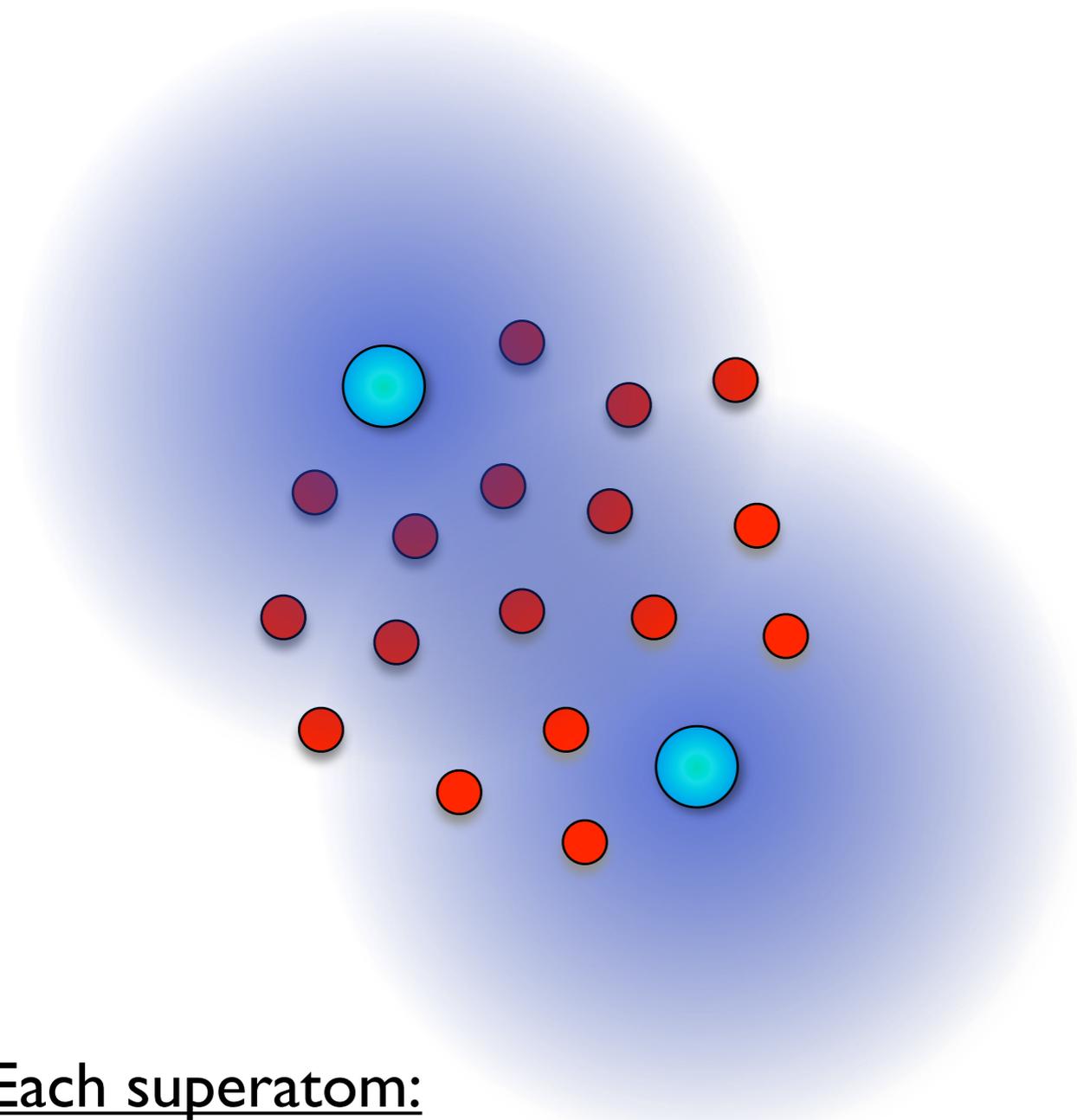
Ø 0.5nm

Ø 250 nm

Property	Scaling	$^{87}\text{Rb } 43S$
Radius	$(n^*)^2$	2400 a_0 = 127nm
Lifetime (dominated by black body radiation for large n)	$(n^*)^2$	45 μs @ 20°C
van der Waals coefficient	$(n^*)^{11}$	$C_6 = -1.7 \times 10^{19}$ a.u.
Blockade radius ($\Omega=2\pi$ 200 kHz)	$(n^*)^2$	$\sim 5 \mu\text{m}$

Saffman, Walker, & Mølmer Rev. Mod. Phys. (2010)

see work in: Madison, Palaiseau, Stuttgart, Heidelberg, Durham, Michigan....



Blockade condition

$$\mathcal{V}_{\text{vdW}} = \frac{C_6}{r^6} > \hbar \max(\Gamma, \Omega)$$

$$r_b \equiv \sqrt[6]{\frac{C_6}{\hbar\Omega}}$$

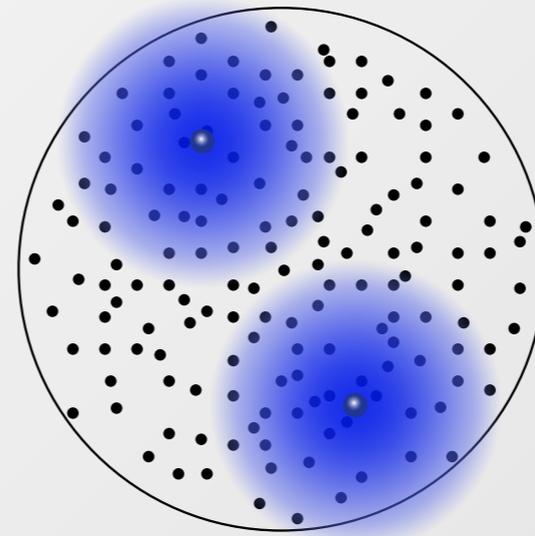
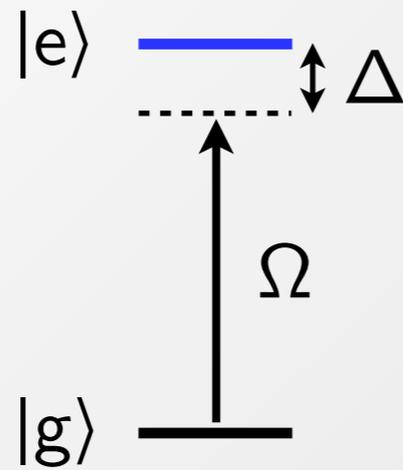
Each superatom:

$$\frac{1}{\sqrt{N}} (|r, 0, 0, 0, \dots\rangle + |0, r, 0, 0, \dots\rangle + |0, 0, 0, \dots, r\rangle)$$

M. Lukin et al. PRL (2001)



The frozen Rydberg gas - long range QM



*no mechanical motion
on the timescale of the
internal dynamics*

$$H = \frac{\hbar\Omega}{2} \sum_i \left(\sigma_{eg}^{(i)} + \sigma_{ge}^{(i)} \right) + \sum_{i \neq j} \frac{V_{ij}}{2} \sigma_{ee}^{(i)} \sigma_{ee}^{(j)} - \Delta \sum_i \sigma_{ee}^{(i)}$$

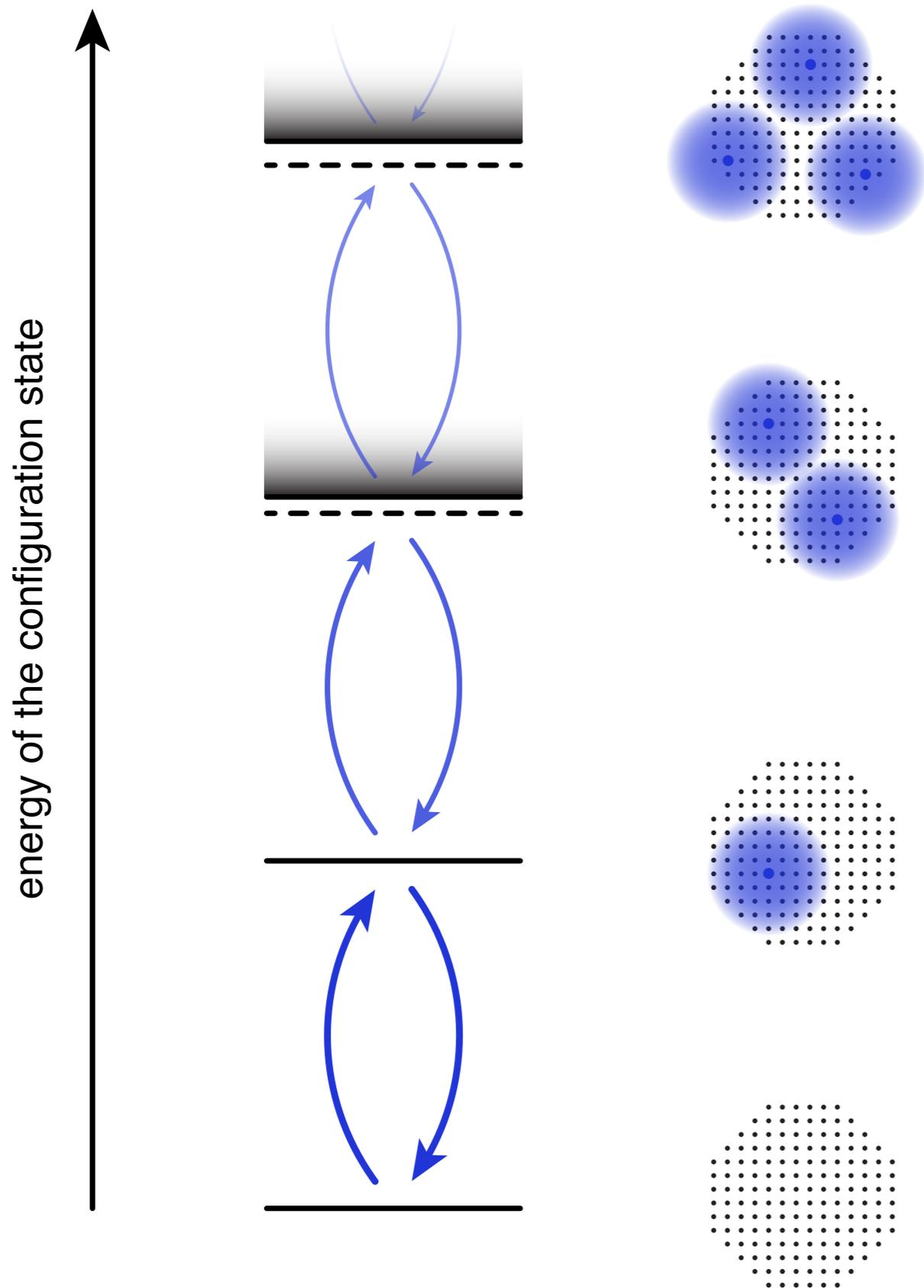
coherent coupling

interaction between
Rydberg atoms

"chemical potential"

$$V_{ij} = C_\alpha |r_i - r_j|^{-\alpha}$$

This work: $\alpha=6$, repulsive

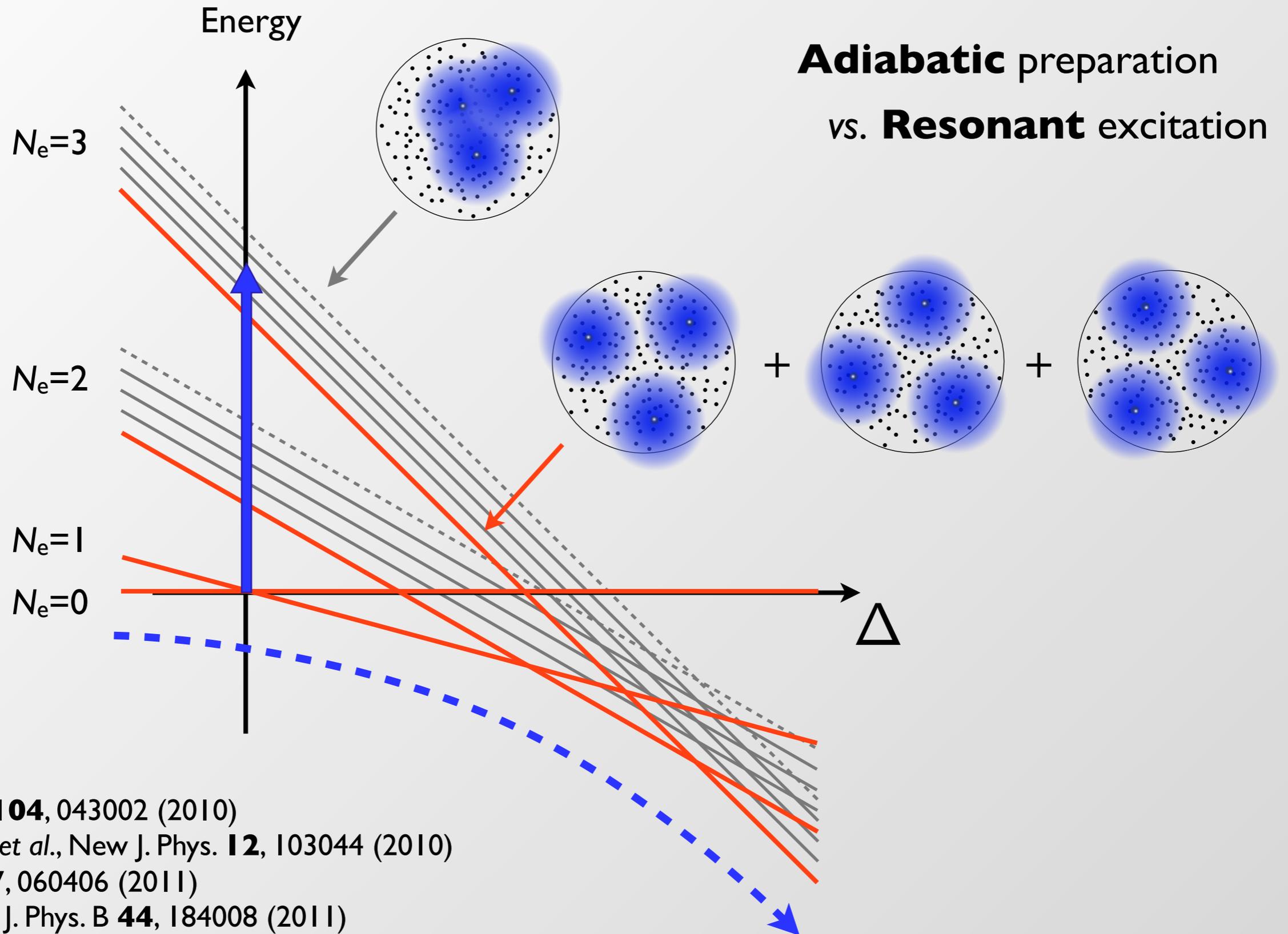


Theory see:

- H. Weimer et al., PRL 2008
- T. Pohl et al. PRL 2010
- G. Pupillo et al. PRL 2010



Dynamical crystallization of the Rydberg gas



Pohl *et al.*, PRL **104**, 043002 (2010)
Schachenmayer *et al.*, New J. Phys. **12**, 103044 (2010)
Ji *et al.*, PRL **107**, 060406 (2011)
van Bijnen *et al.*, J. Phys. B **44**, 184008 (2011)
Gärttner *et al.*, arXiv:1203.2884v2 (2012)

Dynamical Crystallization in the Dipole Blockade of Ultracold Atoms

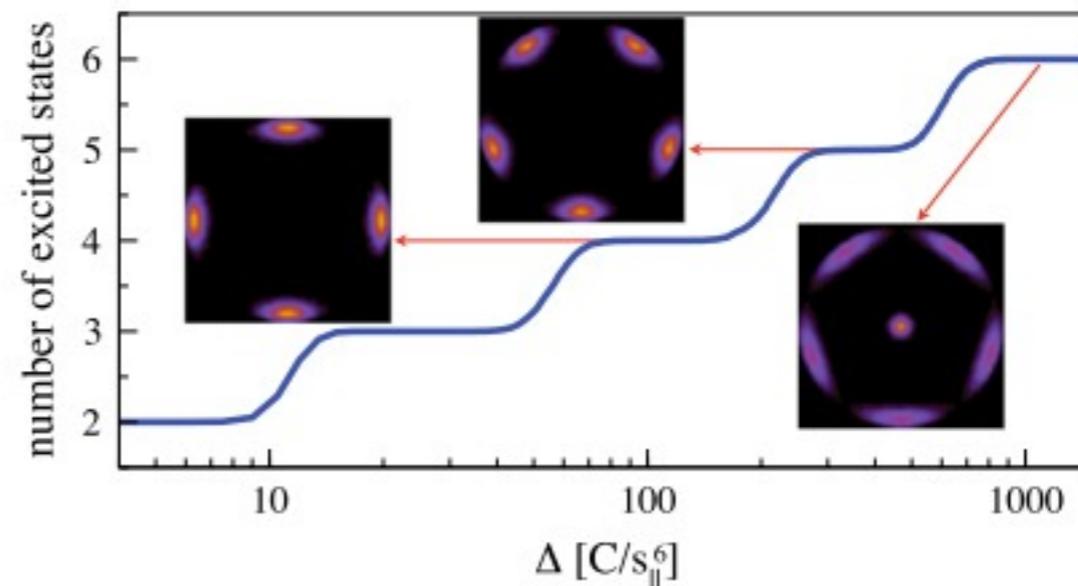
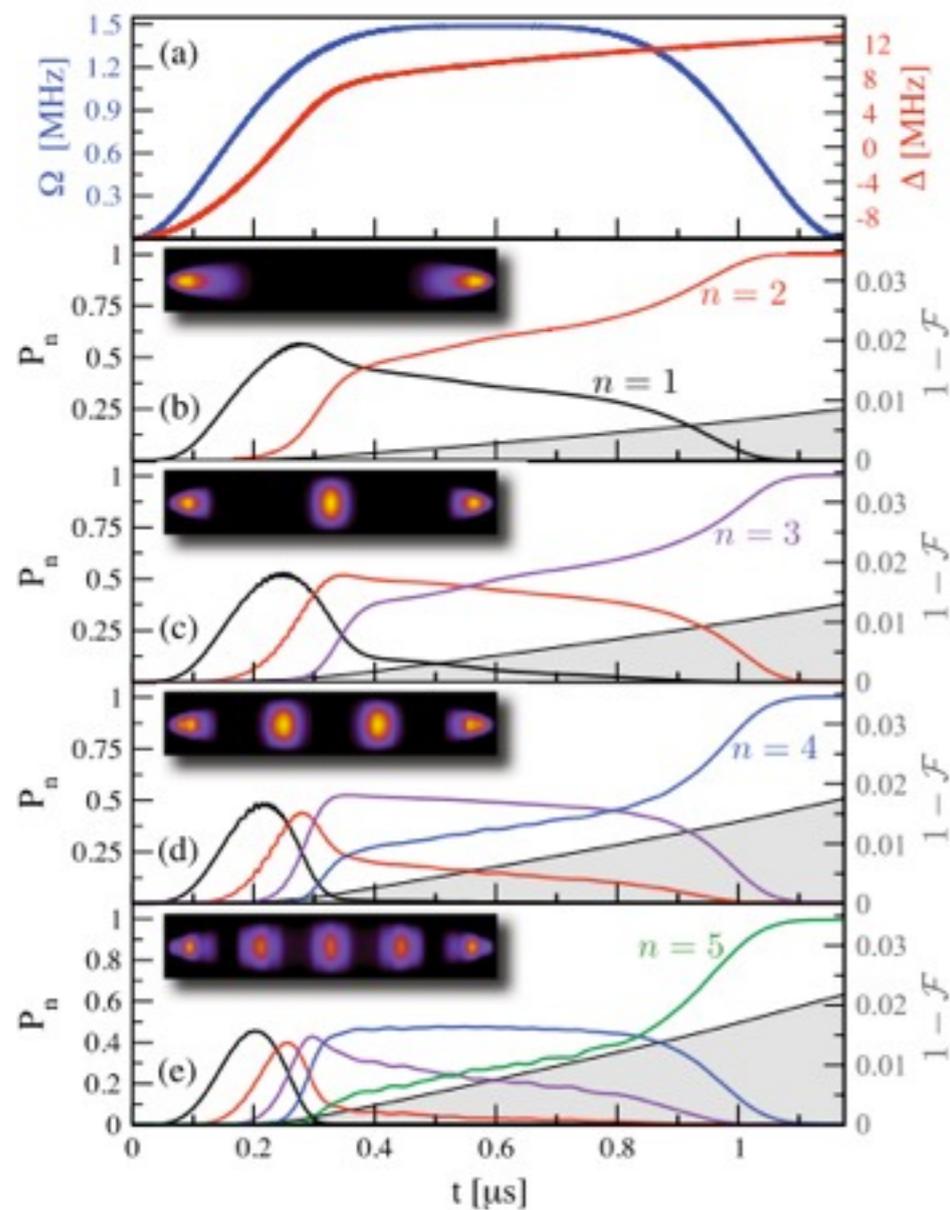
T. Pohl,^{1,2} E. Demler,^{2,3} and M. D. Lukin^{2,3}

¹Max Planck Institute for the Physics of Complex Systems, 01187 Dresden, Germany

²ITAMP-Harvard-Smithsonian Center for Astrophysics, Cambridge Massachusetts 02138, USA

³Physics Department, Harvard University, Cambridge Massachusetts 02138, USA

(Received 26 July 2009; revised manuscript received 23 October 2009; published 27 January 2010)

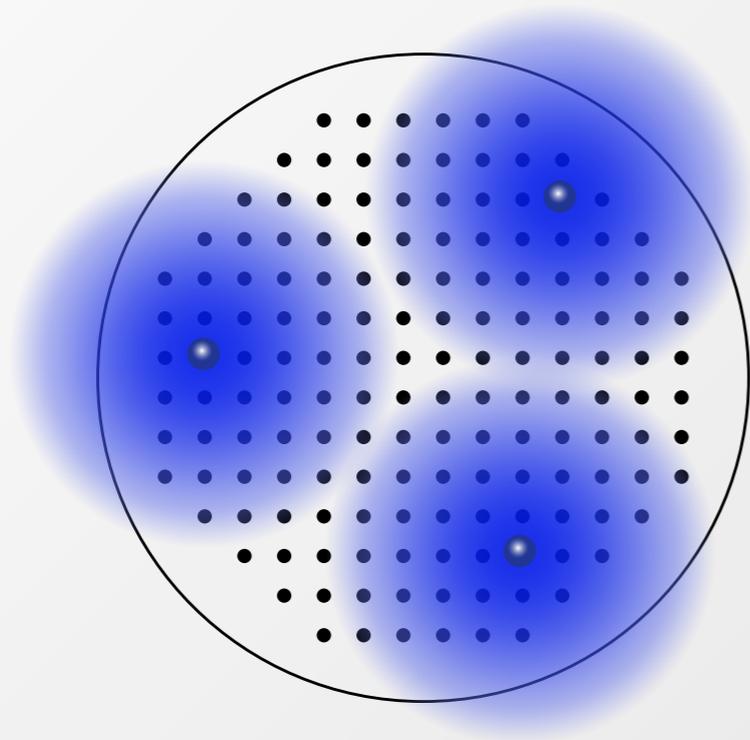


Rydberg atoms organize in crystalline excitations patterns!

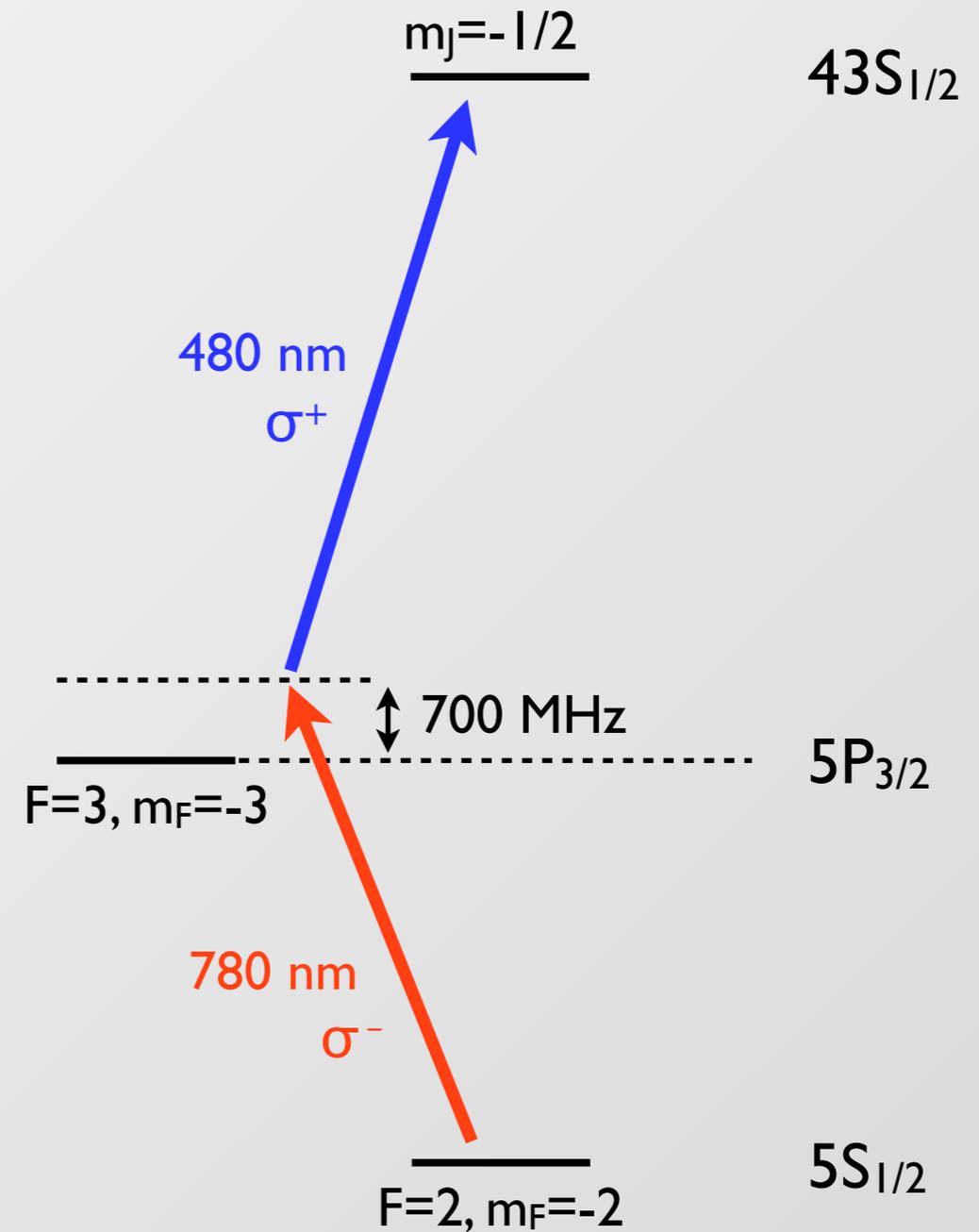
Theory see:

T. Pohl et al. PRL 2010; G. Pupillo et al. PRL 2010,
R.M.W van Bijnen et al. J. Phys. B:At. Mol. Opt. Phys. (2011)
see also: H. Weimer et al., PRL 2008

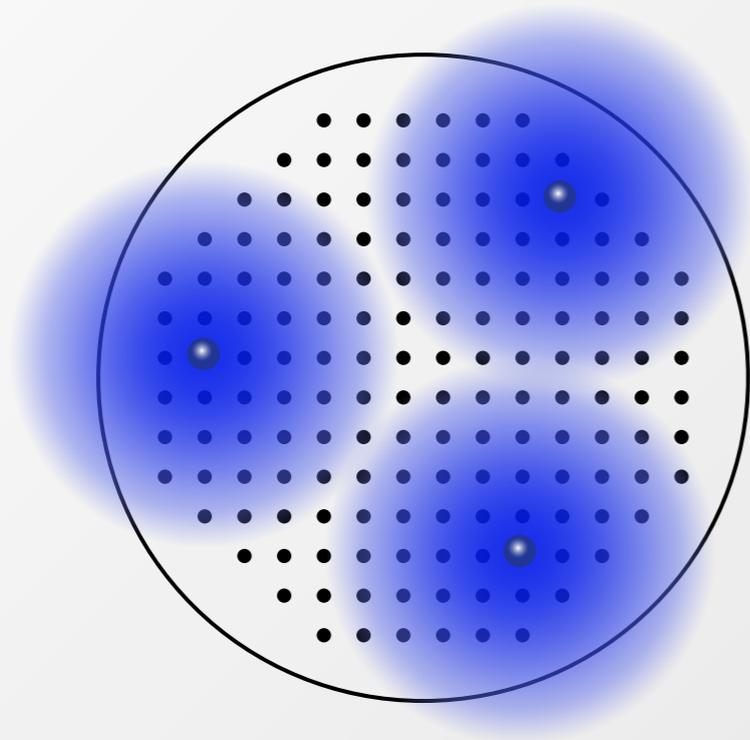
Excitation and detection of the Rydberg atoms



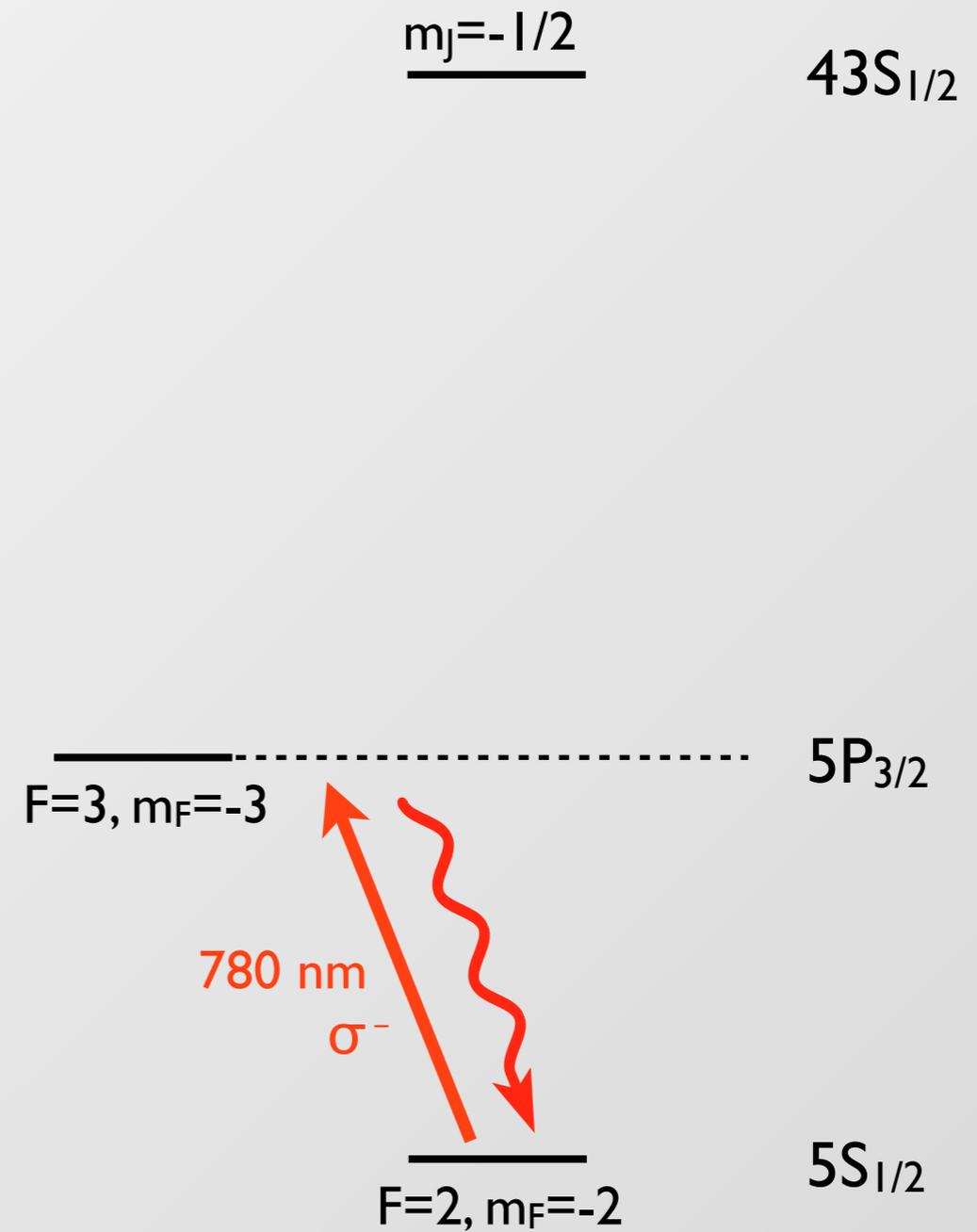
- two-photon Rabi frequency:
 $\Omega/2\pi = 170(20)$ kHz
- resonant excitation:
 $\Delta = 0$
- blockade radius:
 $R_b = 4.9(1)$ μm



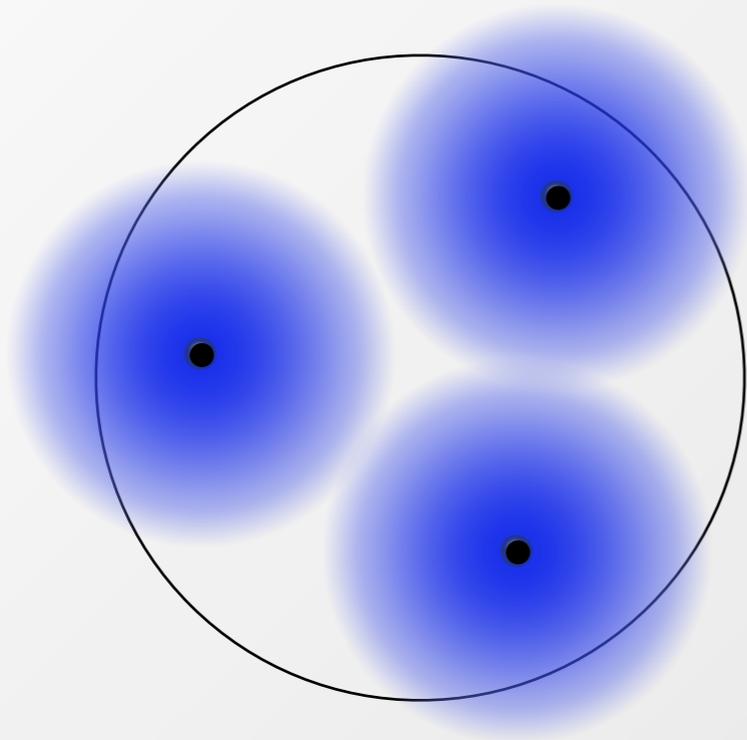
Excitation and detection of the Rydberg atoms



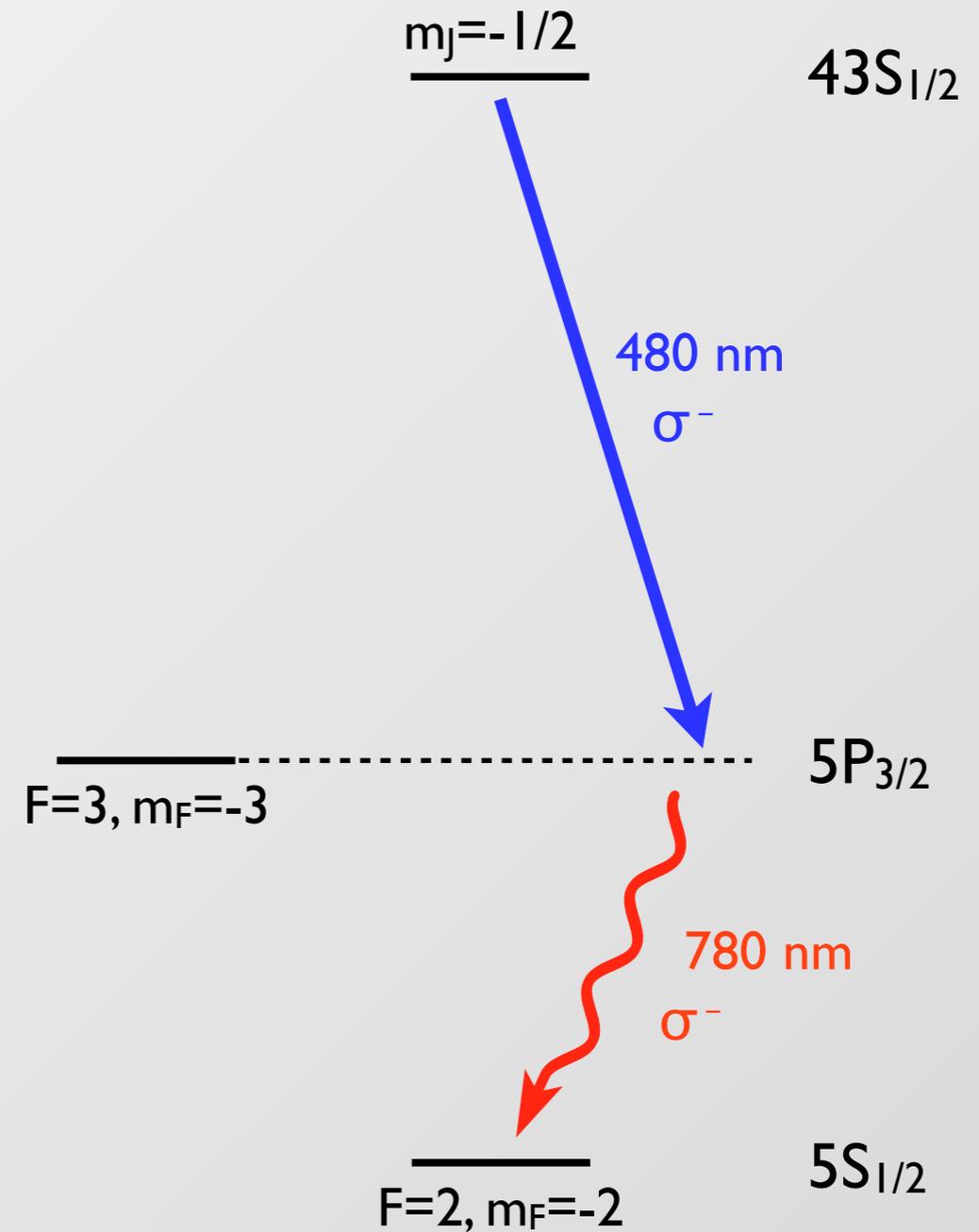
- removal pulse duration: 10 μs
- survival probability: 0.1 %



Excitation and detection of the Rydberg atoms



- deexcitation pulse duration: 2 μ s
- detection efficiency: 75(10) %
- overall resolution: \sim 500 nm



Snapshots of the excitation pattern

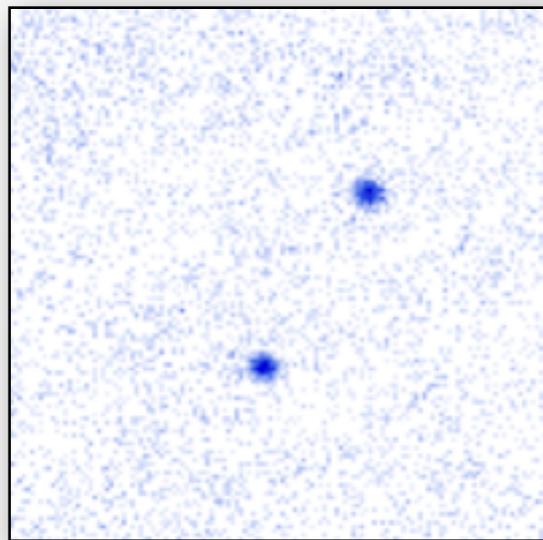
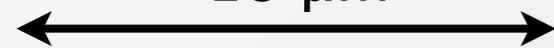
"Small" clouds:

150(30) atoms
diameter 7.2(8) μm
676 pictures

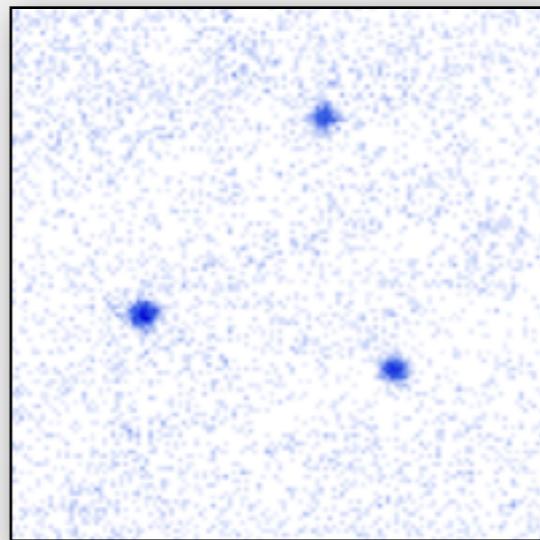
"Large" clouds:

390(30) atoms
diameter 10.8(8) μm
1654 pictures

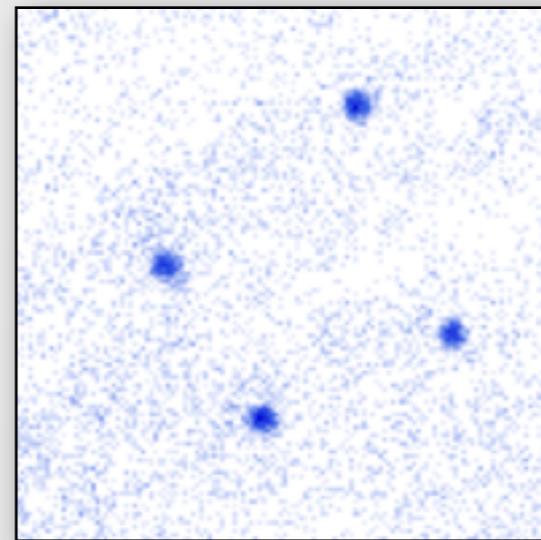
18 μm



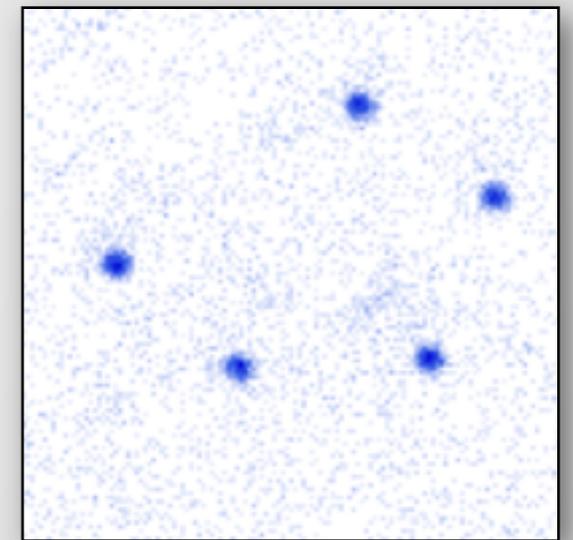
191 pictures



65 pictures

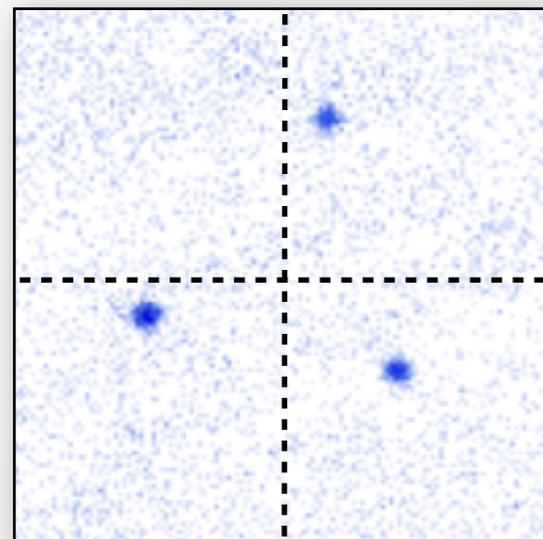


177 pictures

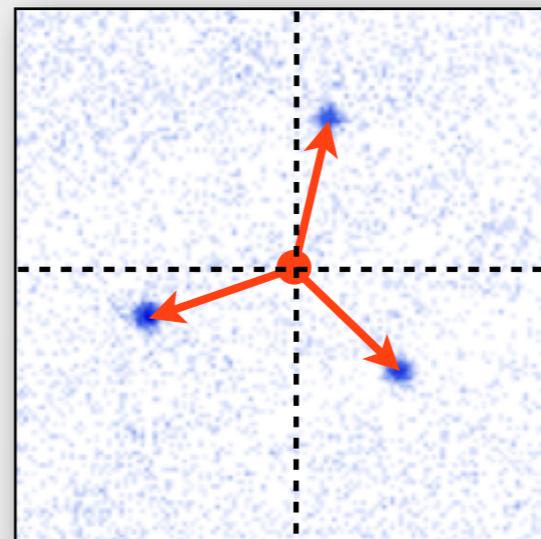
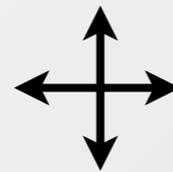


64 pictures

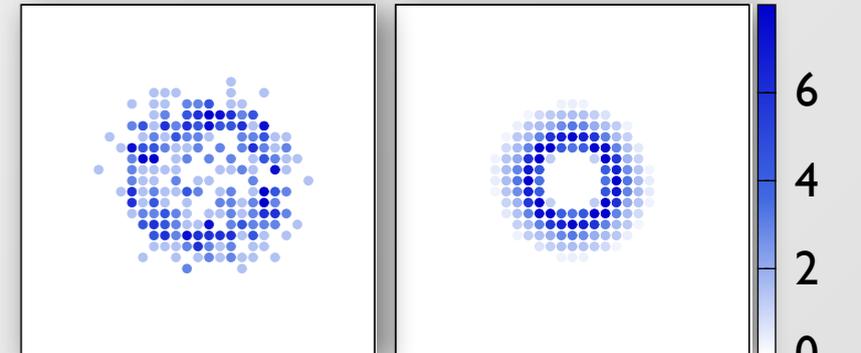
Revealing mesoscopic structures



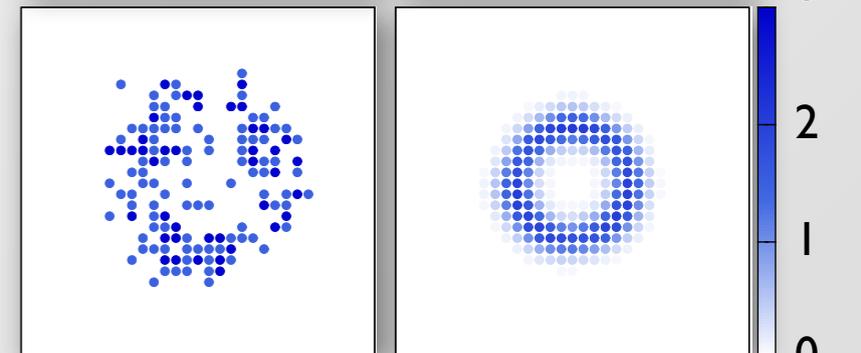
centre onto
the barycenter



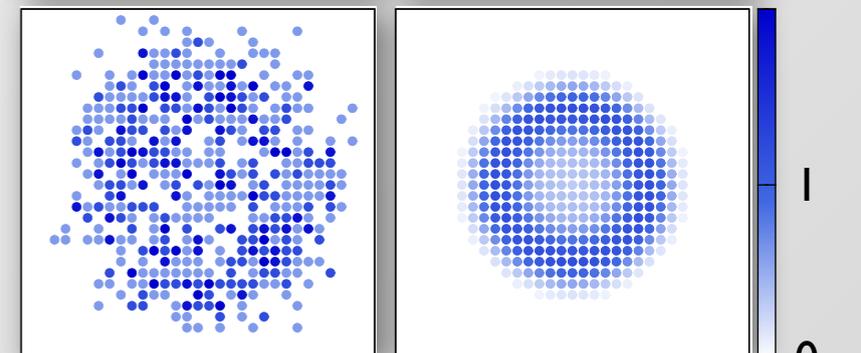
$N_e=2$



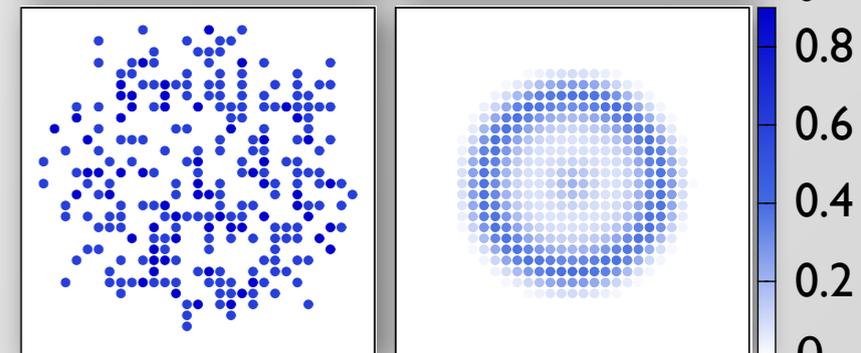
$N_e=3$



$N_e=4$



$N_e=5$



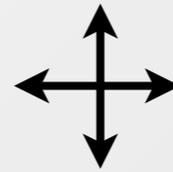
Event probability (10^{-3})

Numerical simulations:

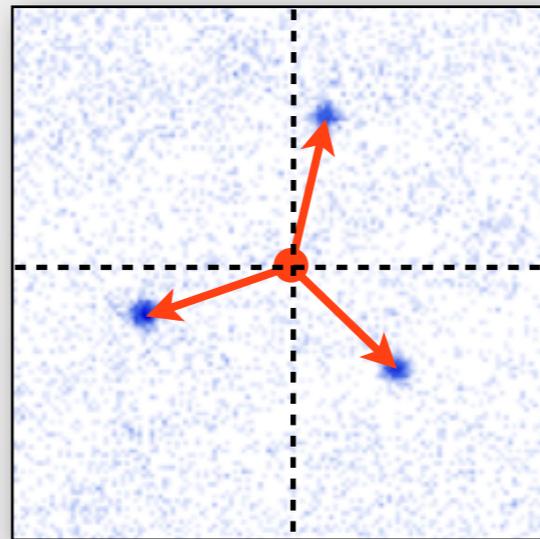
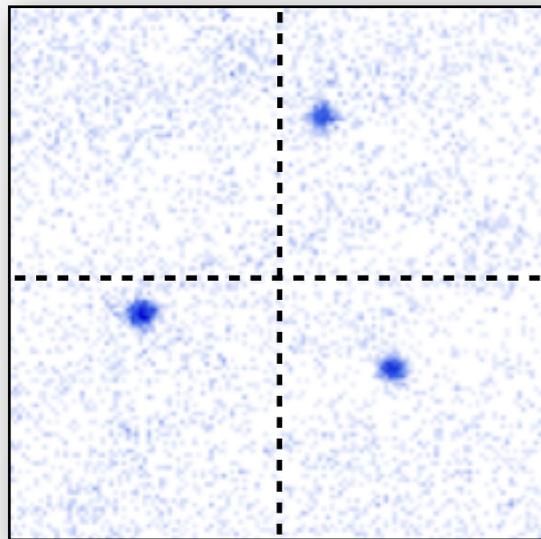
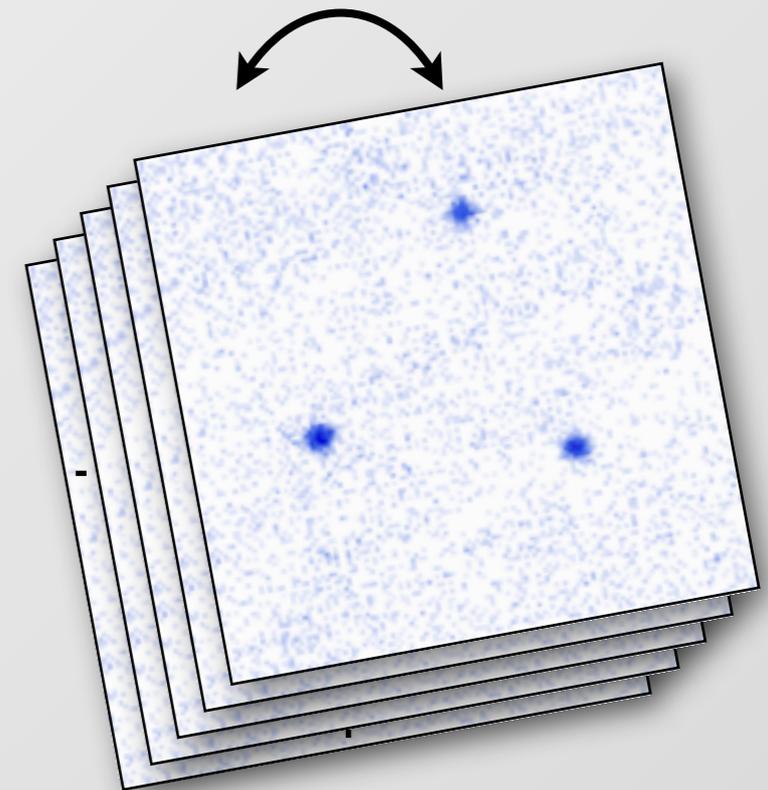
- exact dynamics (truncated Hilbert space)
- average over initial states from the grand canonical ensemble

Revealing mesoscopic structures

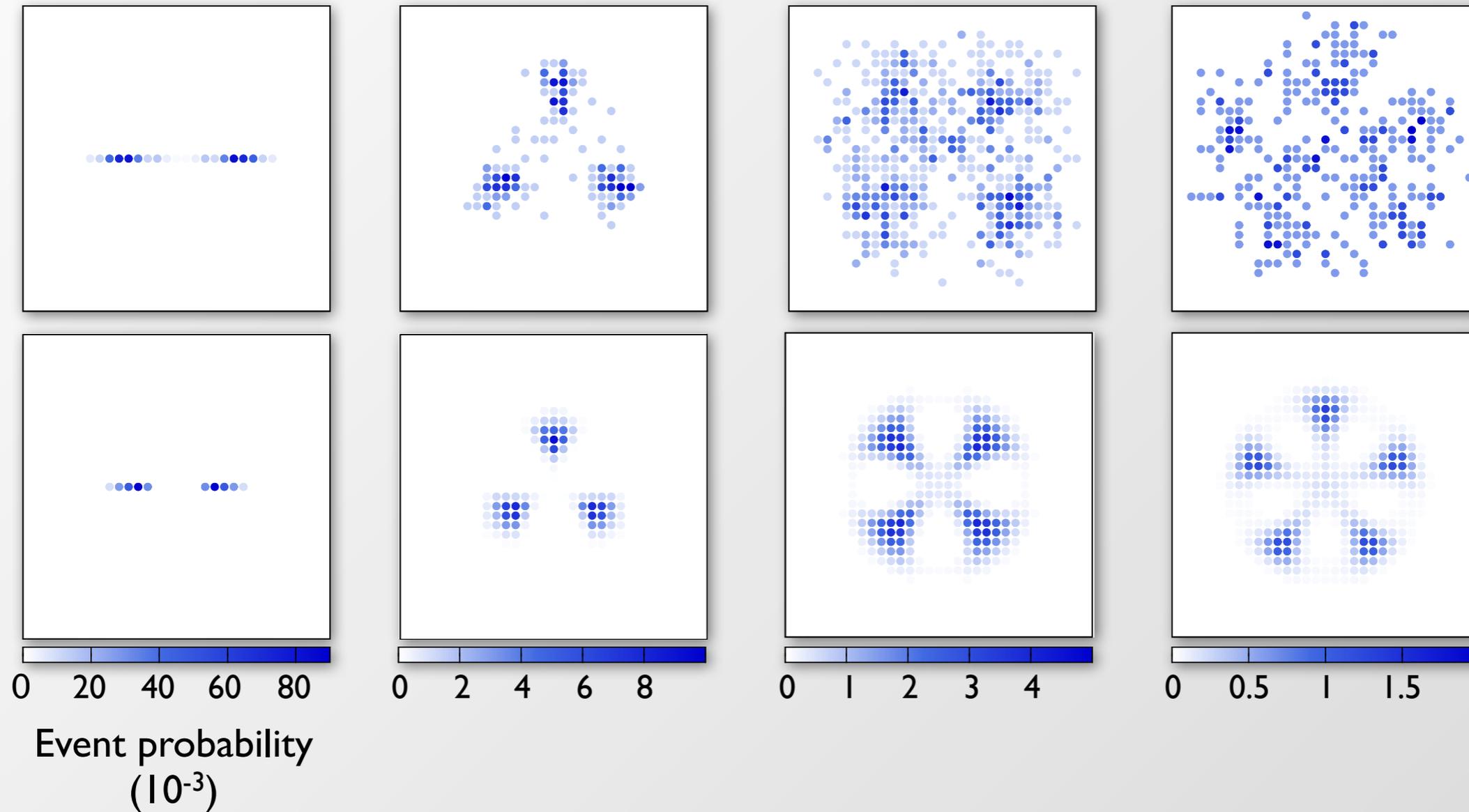
centre onto
the barycenter



align with a
reference axis

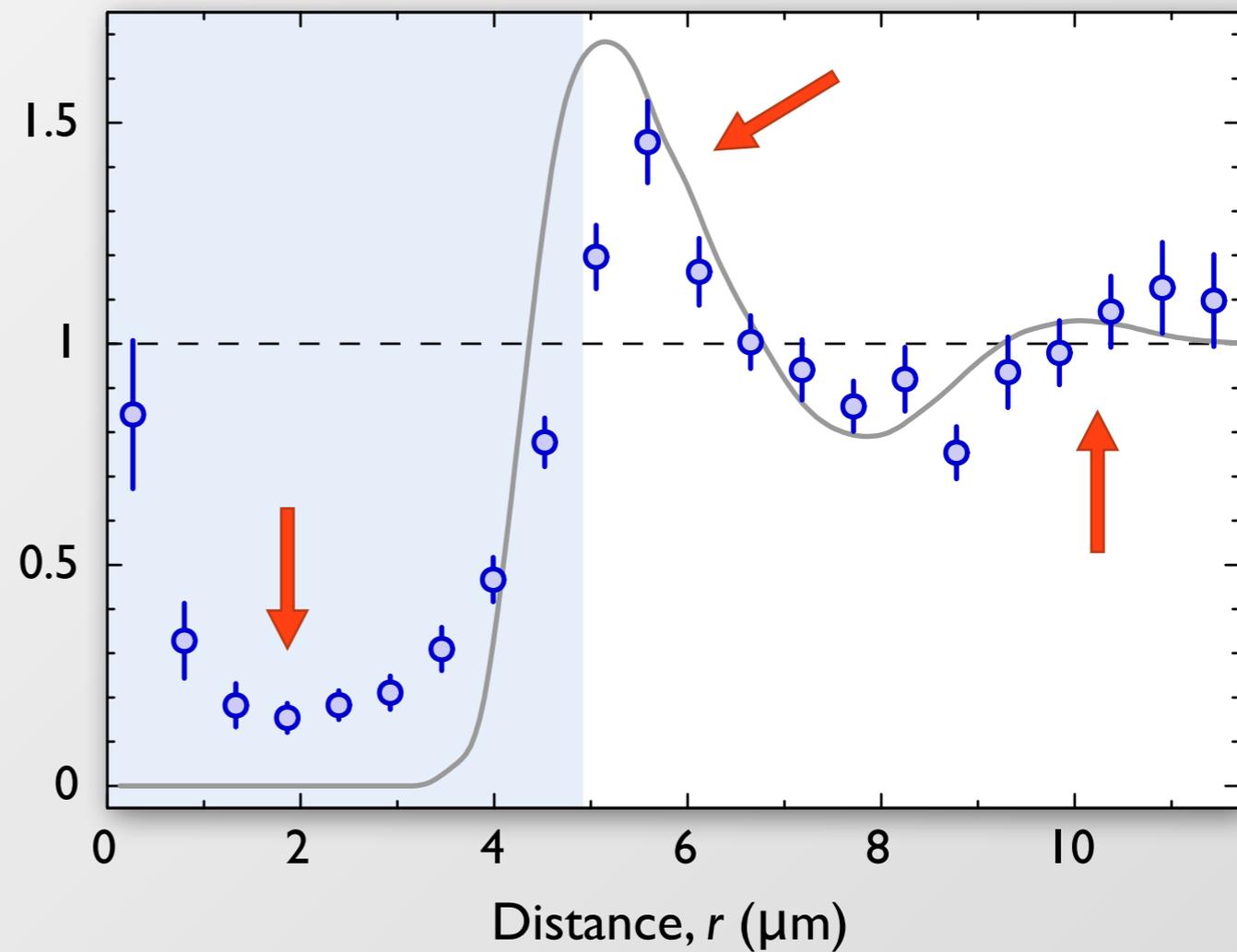
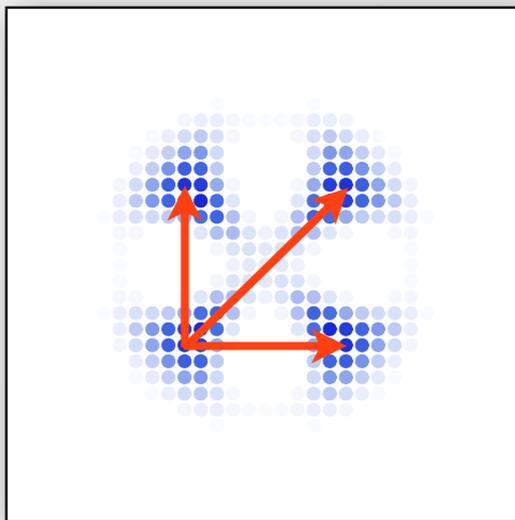


Revealing mesoscopic structures



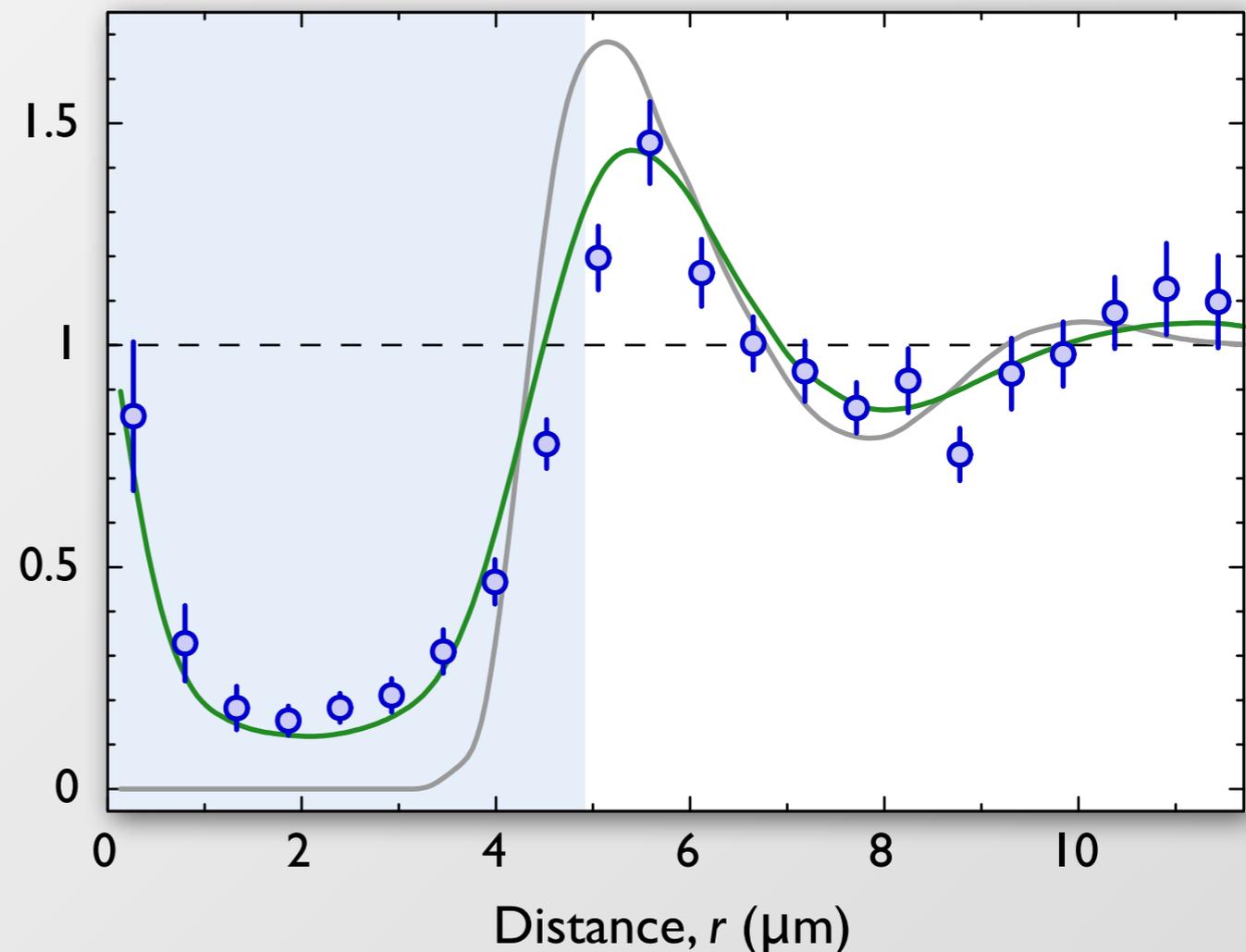
Pair correlation function:

$$g^{(2)}(r) = \frac{\sum_{i \neq j} \delta_{r, r_{ij}} \langle \sigma_{ee}^{(i)} \sigma_{ee}^{(j)} \rangle}{\sum_{i \neq j} \delta_{r, r_{ij}} \langle \sigma_{ee}^{(i)} \rangle \langle \sigma_{ee}^{(j)} \rangle}$$



Pair correlation function:

$$g^{(2)}(r) = \frac{\sum_{i \neq j} \delta_{r,r_{ij}} \langle \sigma_{ee}^{(i)} \sigma_{ee}^{(j)} \rangle}{\sum_{i \neq j} \delta_{r,r_{ij}} \langle \sigma_{ee}^{(i)} \rangle \langle \sigma_{ee}^{(j)} \rangle}$$



Deviations due to:

- hopping during the fluorescence imaging – probability $\sim 1\%$
- imperfect removal of the ground state atoms – 0.2 atoms per image
- residual motion of the Rydberg atoms before imaging – $\pm 0.5 \mu\text{m}$

Blockade radius measurement, see also: Schwarzkopf et al. PRL (2011)

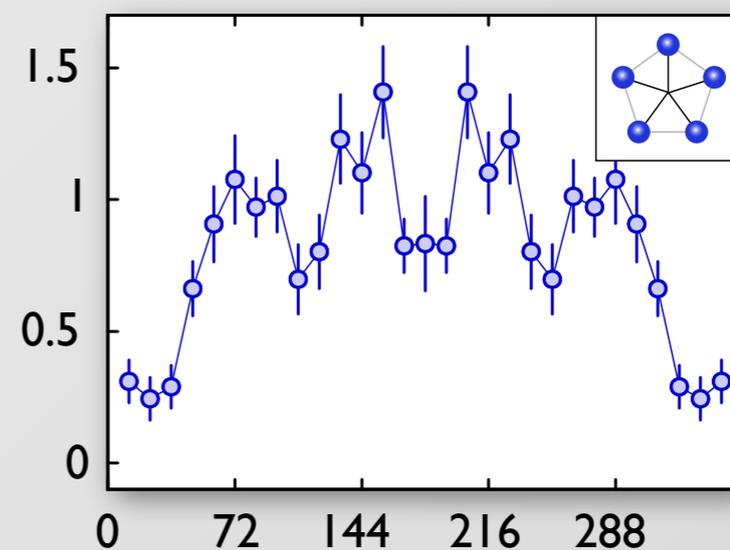
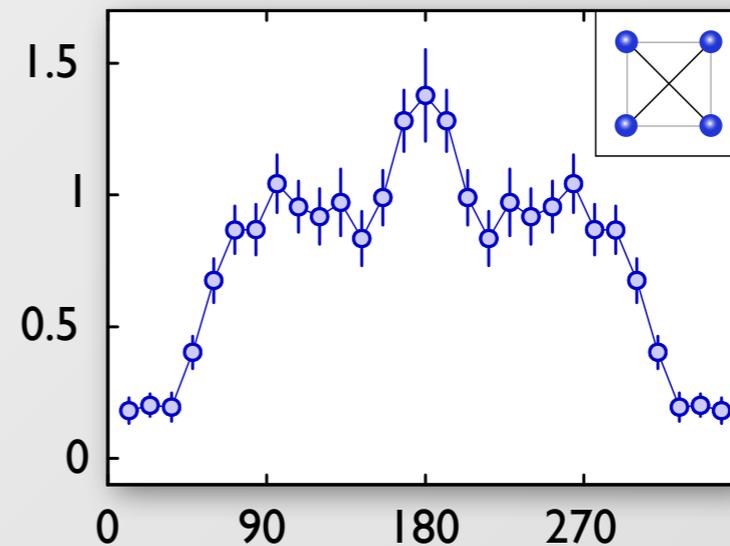
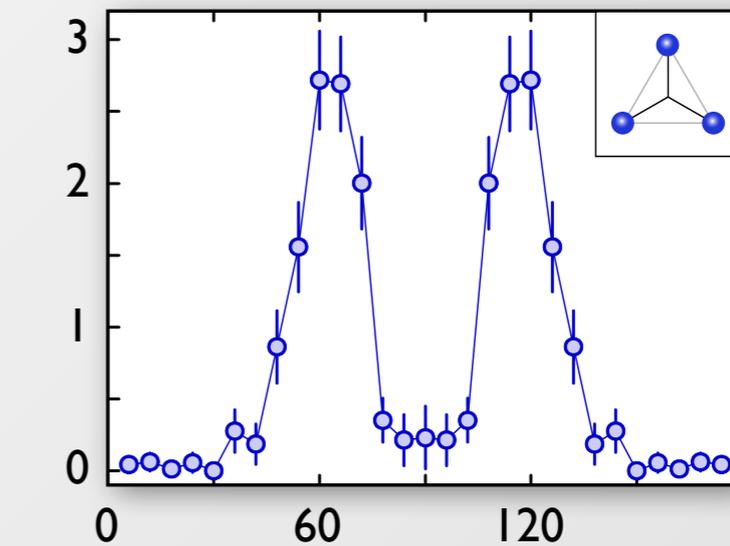
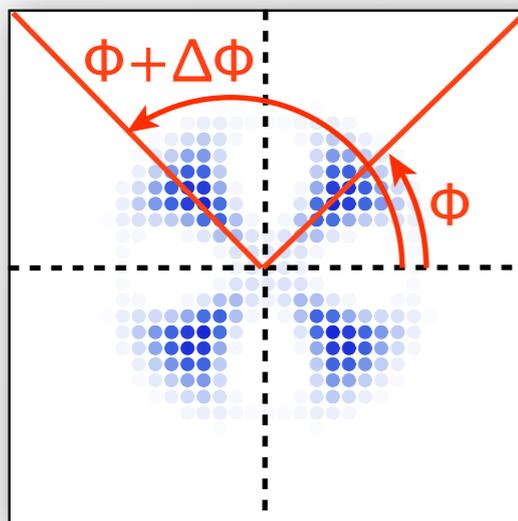
Correlation functions

effective 1D crystals along the circumference of the system

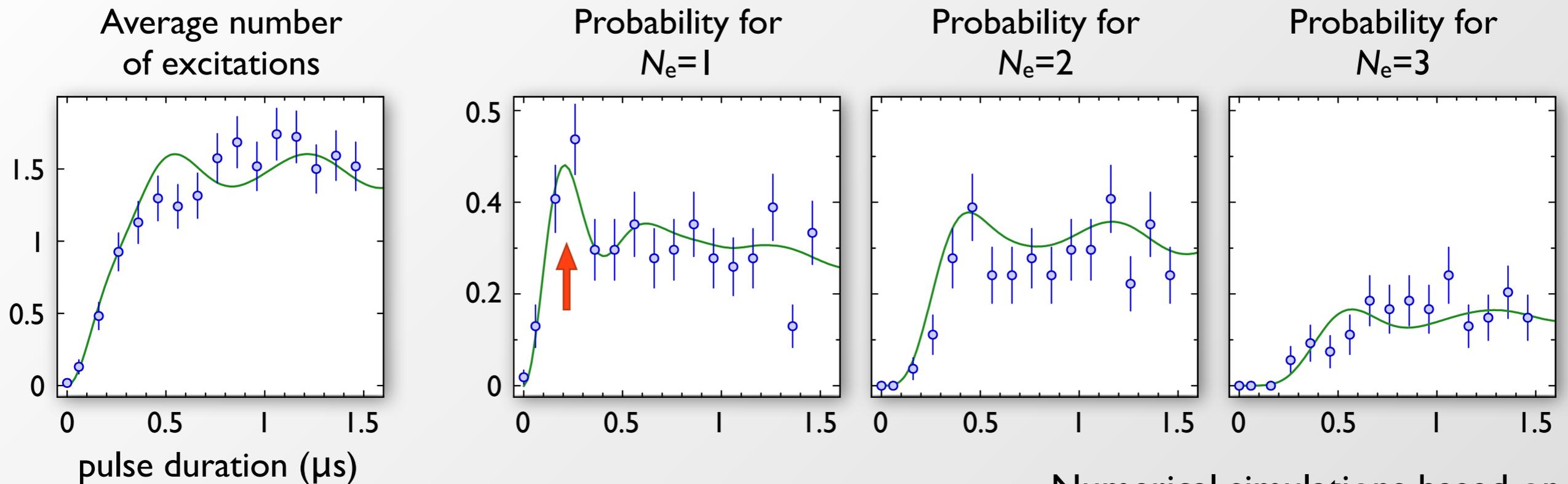
Azimuthal correlation function:

$$g_{\tilde{z}}^{(2)}(\Delta\phi) = \int \frac{d\phi}{2\pi} \frac{\langle n(\phi)n(\phi + \Delta\phi) \rangle}{\langle n(\phi) \rangle \langle n(\phi + \Delta\phi) \rangle}$$

with $n(\phi) = \sum_i \delta_{\phi, \phi_i} \sigma_{ee}^{(i)}$

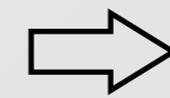


Relative angle, ΔΦ (degree)



Numerical simulations based on typical initial states observed in the experiment

- fast dynamics compared to $\pi/\Omega \approx 3 \mu\text{s}$
 \Rightarrow *collective enhancement of the coupling strength*
- different timescales for the different number of excitations
 \Rightarrow *reduced overlap between the states coupled by the optical radiation*



for $N_e = 1$:
 $\pi/(\sqrt{N_{\text{at}}}\Omega) = 220(40) \text{ ns}$

(some) evidence for the coherence of the many-body state

Smaller Blockade/Larger Cloud

- ✓ Larger Rydberg Crystals
- ✓ Larger Rydberg Atoms cp. to Lattice Spacing
- ✓ Adiabatic Sweeps to Deterministically Prepare Crystal Structures
- ✓ Show Coherence of Crystalline Superpositions! a Quantum Crystal?

T. Pohl et al, (2010), van Bijnen et al. (2011), Gärtner et al. (2012),...

Larger Blockade/Smaller Cloud

- ✓ Collectively enhanced Rabi oscillations
- ✓ Large Entangled states (e.g. EIT schemes)

M. Lukin et al. (2001), D. Moller et al. (2008), M. Müller et al. (2009), H. Weimer et al. (2009)...

Dressed Rydberg Atom Regime

- ✓ Admix controlled long range interactions

G. Pupillo et al, (2010), Henkel et al. (2010), Schachenmeyer et al. (2010), Honer et al. (2010), Cinti et al. (2010), Johnson & Rolston (2010)...



Measuring Zak-Berry Phase of Topological Bands

M. Aidelsburger, M. Atala, J. Barreiro & I.B.
D. Abanin, T. Kitagawa, E. Demler



$$\Psi(R) \rightarrow e^{i(\varphi_{\text{Berry}} + \varphi_{\text{dyn}})} \Psi(R)$$

Adiabatic evolution through closed loop

$$\varphi_{\text{Berry}} = - \oint_{\mathcal{C}} A_n(R) dR = -i \oint_{\mathcal{C}} \langle n(R) | \nabla_R | n(R) \rangle dR$$

$$\varphi_{\text{Berry}} = \int_{\mathcal{A}} \Omega_n(R) dA \quad \text{Berry Phase}$$

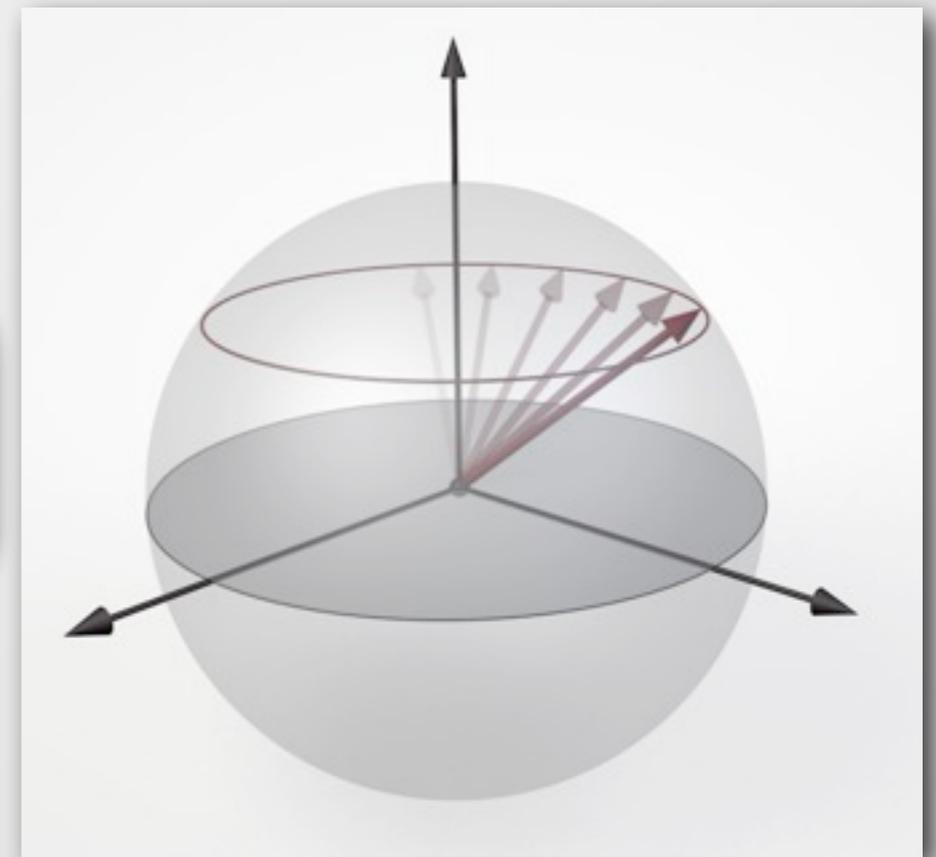
M.V. Berry, Proc. R. Soc. A (1984)

Berry connection

$$A_n(R) = i \langle n(R) | \nabla_R | n(R) \rangle$$

Berry curvature

$$\Omega_{n,\mu\nu}(R) = \frac{\partial}{\partial R^\mu} A_{n,\nu} - \frac{\partial}{\partial R^\nu} A_{n,\mu}$$

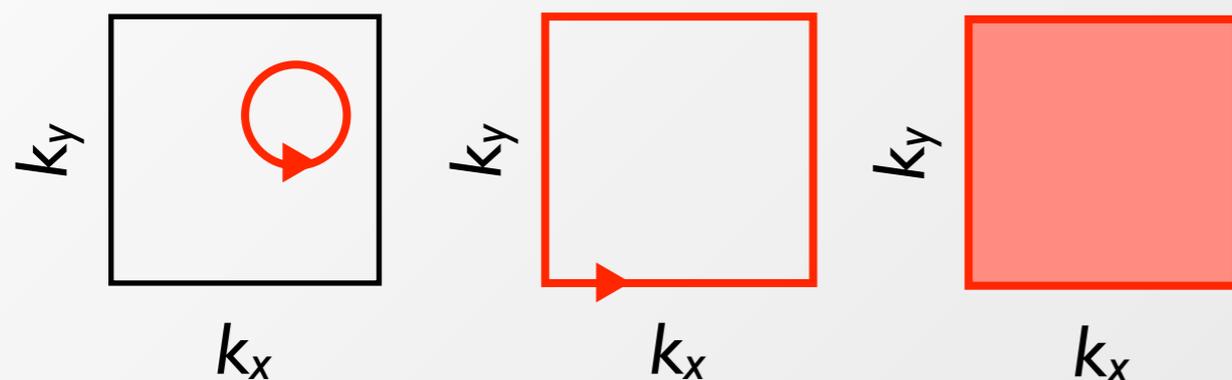


Example: Spin-1/2 particle in magnetic field



$$\Psi_k(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}} u_k(\mathbf{r}) \quad \text{Bloch wave in periodic potential}$$

Adiabatic motion in momentum space generates Berry phase!



Berry phase is fundamental to characterize topology of energy bands

$$n_{\text{Chern}} = \frac{1}{2\pi} \oint_{BZ} A_k dk = \frac{1}{2\pi} \int_{BZ} \Omega_k d^2k \quad \longleftrightarrow \quad \sigma_{xy} = n_{\text{Chern}} e^2/h$$

Chern Number (Topological Invariant)

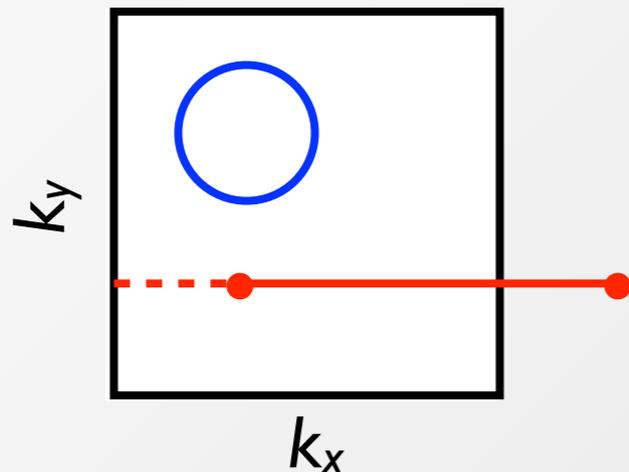
Quantized Hall Conductance

Thouless, Kohmoto, den Nijs, and Nightingale (TKNN), PRL
Kohmoto Ann. of Phys. 1985

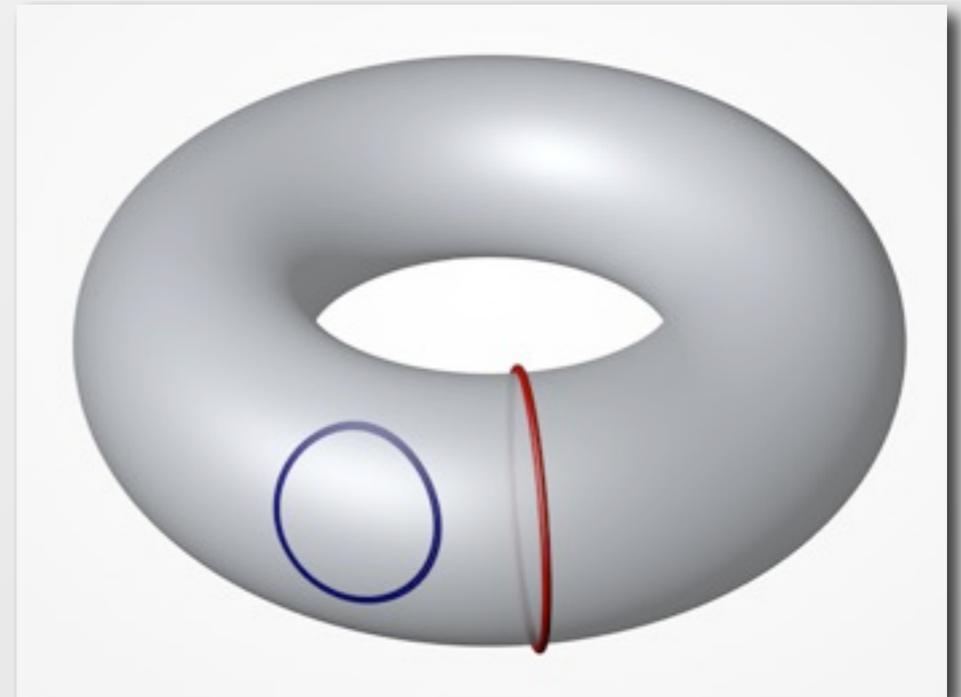
Mention Problem with going on a line is generally NOT A LOOP IN PARAMETER SPACE!

What is the extension to 1D

2D Brillouin Zone



going straight means going around!



Band structure has topology of a torus!

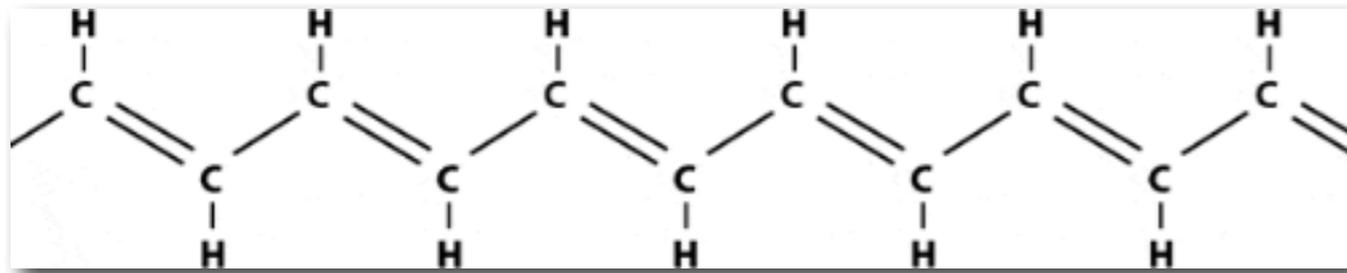
$$\varphi_{\text{Zak}} = \int_{k_0}^{k_0+G} A(k) dk = i \int_{k_0}^{k_0+G} \langle u_k | \partial_k | u_k \rangle dk$$

Zak Phase - the 1D Berry Phase

J. Zak, Phys. Rev. Lett. 62, 2747 (1989)

Non-trivial Zak phase indicates topological band.



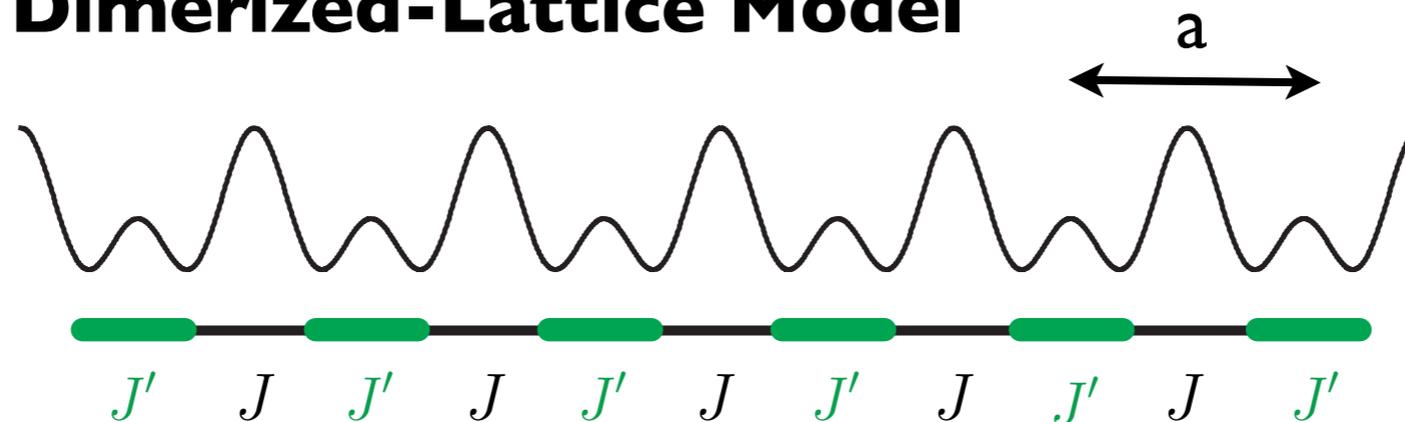


Polyacetylene

W. P. Su, J. R. Schrieffer & A. J. Heeger
Phys. Rev. Lett. 42, 1698 (1979).



Dimerized-Lattice Model



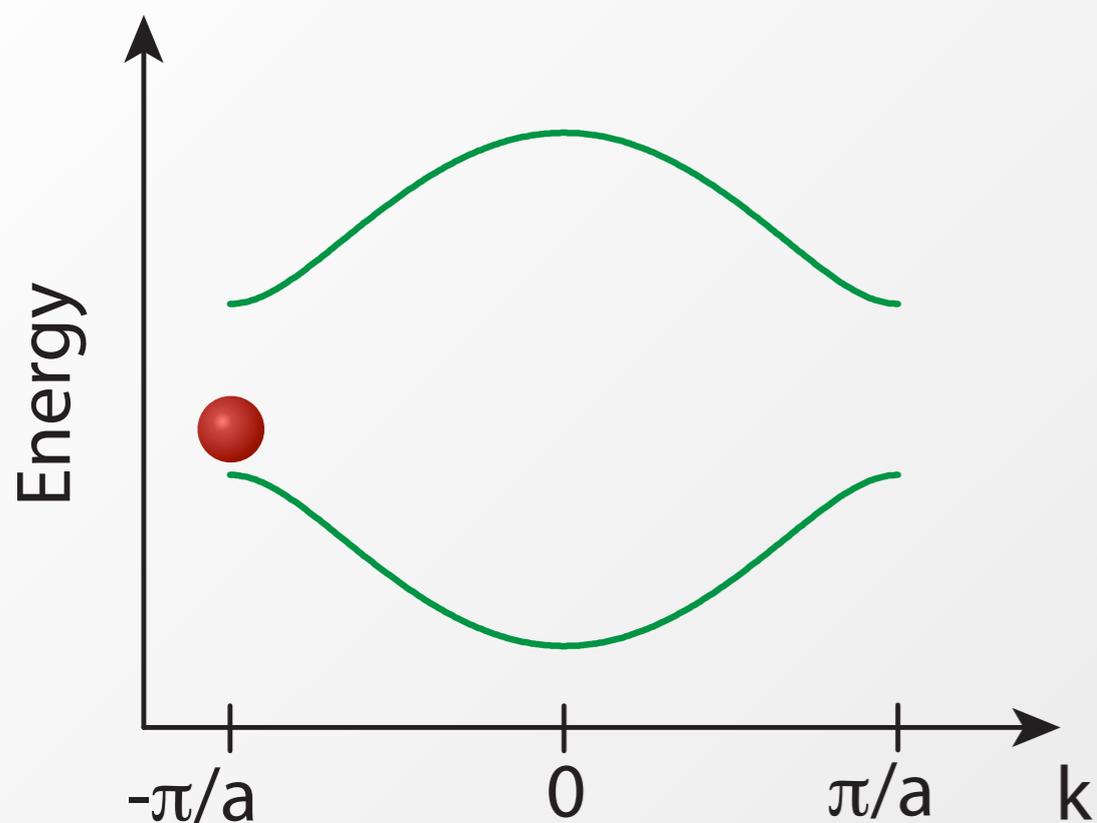
$$H_{SSH} = - \sum_n \{ J a_n^\dagger b_n + J' a_n^\dagger b_{n-1} + \text{h.c.} \}$$

- Topological Band
- Edge States (for finite system)
- Fractional Charge

R. Jackiw and C. Rebbi, Phys. Rev. D 13, 3398 (1976)

J. Goldstone and F. Wilczek, Phys. Rev. Lett. 47, 986 (1981)





...ABABABAB... Lattice Structure...

$$\psi_k(x) = \sum_n \begin{cases} \alpha_k e^{ikx_n} \\ \beta_k e^{ik(x_n+a/2)} \end{cases}$$

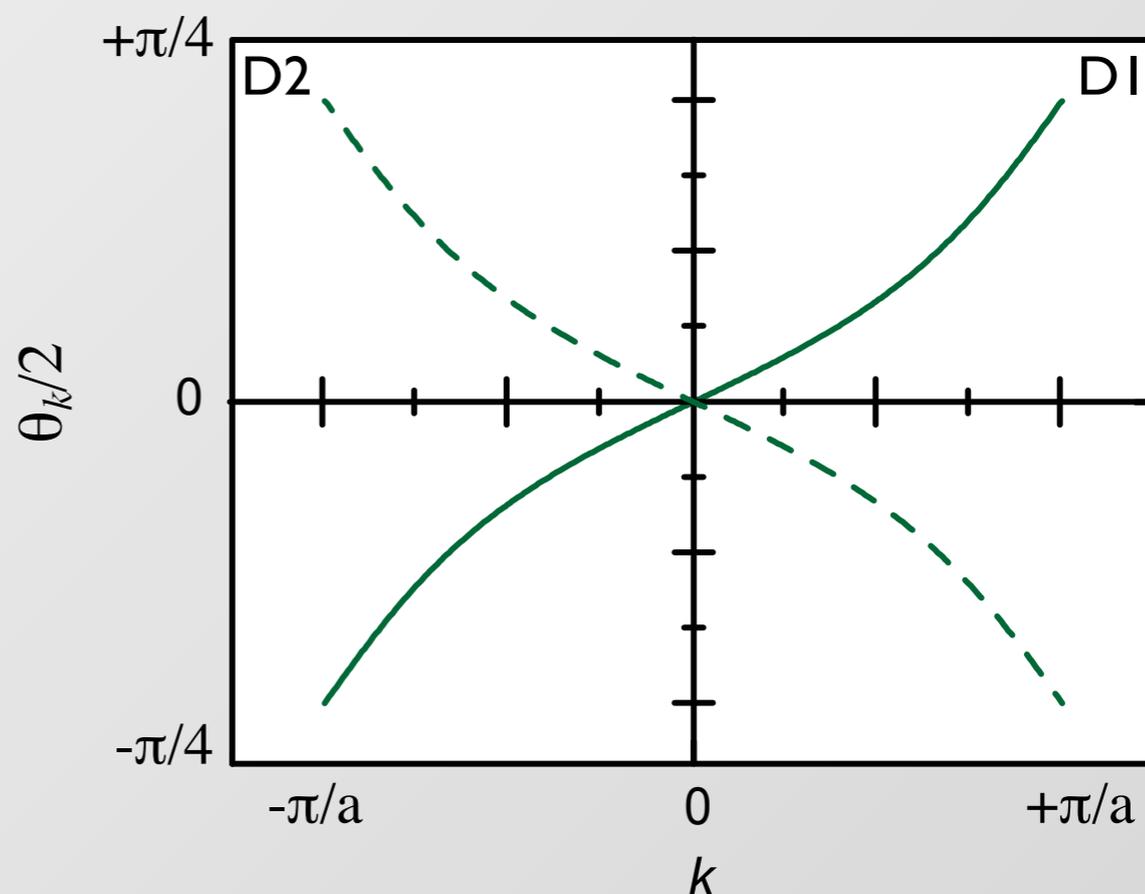
Eigenstates

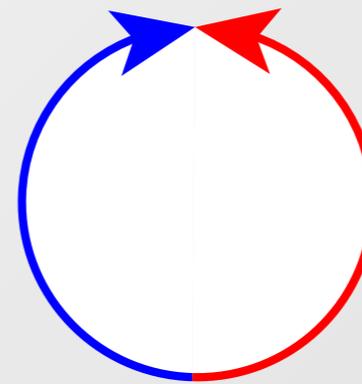
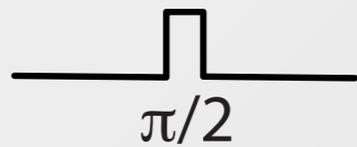
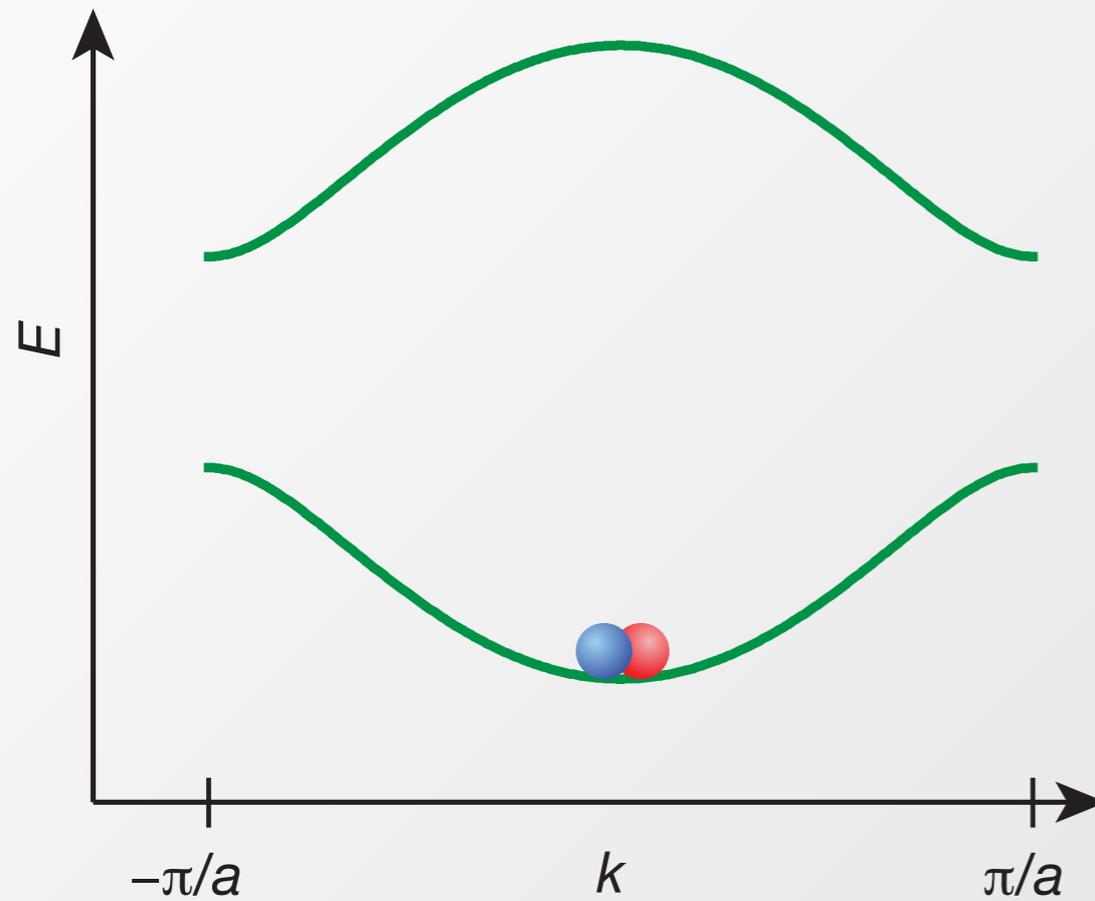
$$\begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1 \\ e^{i\theta_k} \end{pmatrix}$$

$$\varphi_{Zak} = \frac{i}{2} \int_0^G \partial_k \theta_k dk = \pm \frac{\pi}{2}$$

Zak-Berry Phase SSH Model

For the two dimerized configurations!



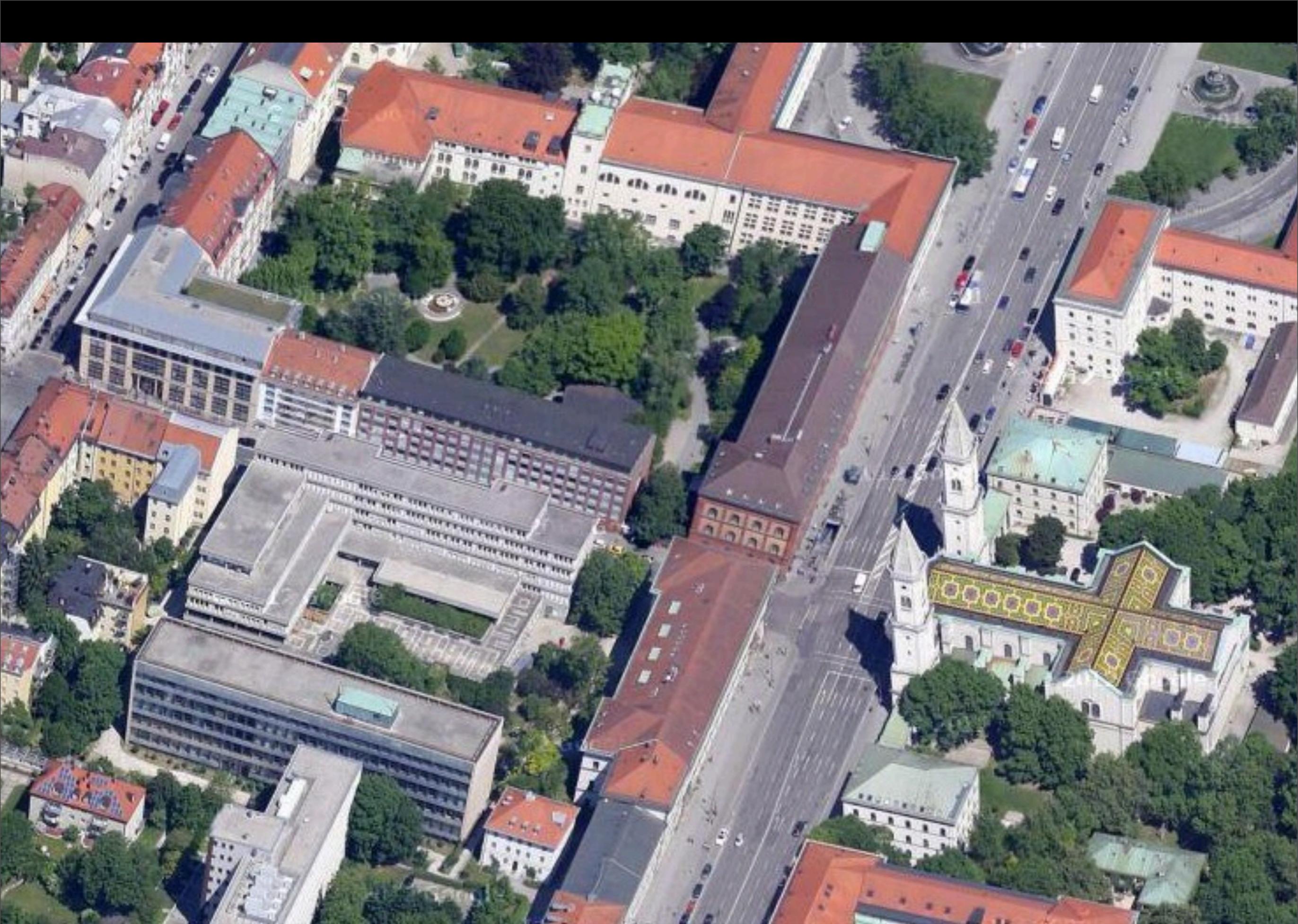


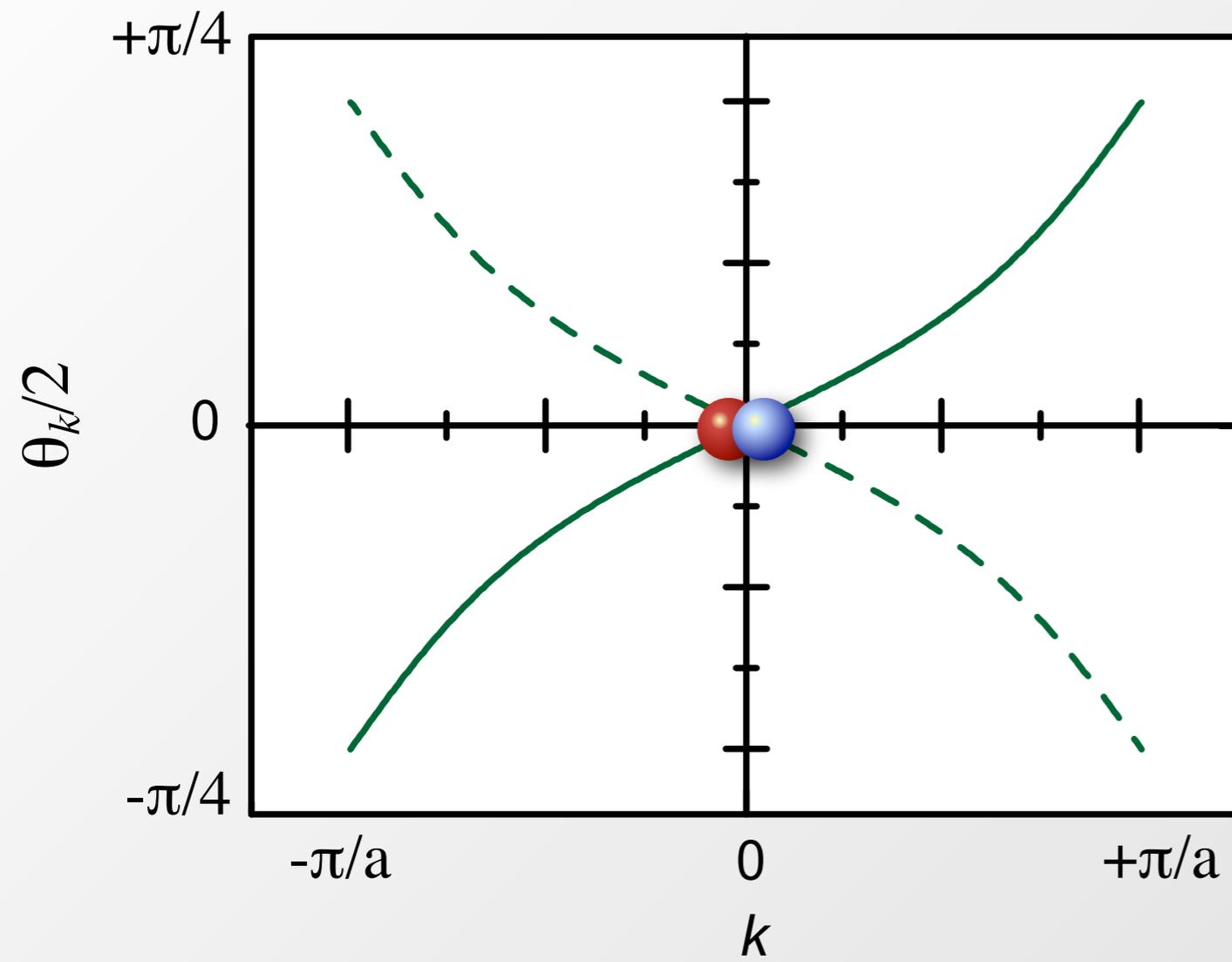
Closed Loop in
 k -Space

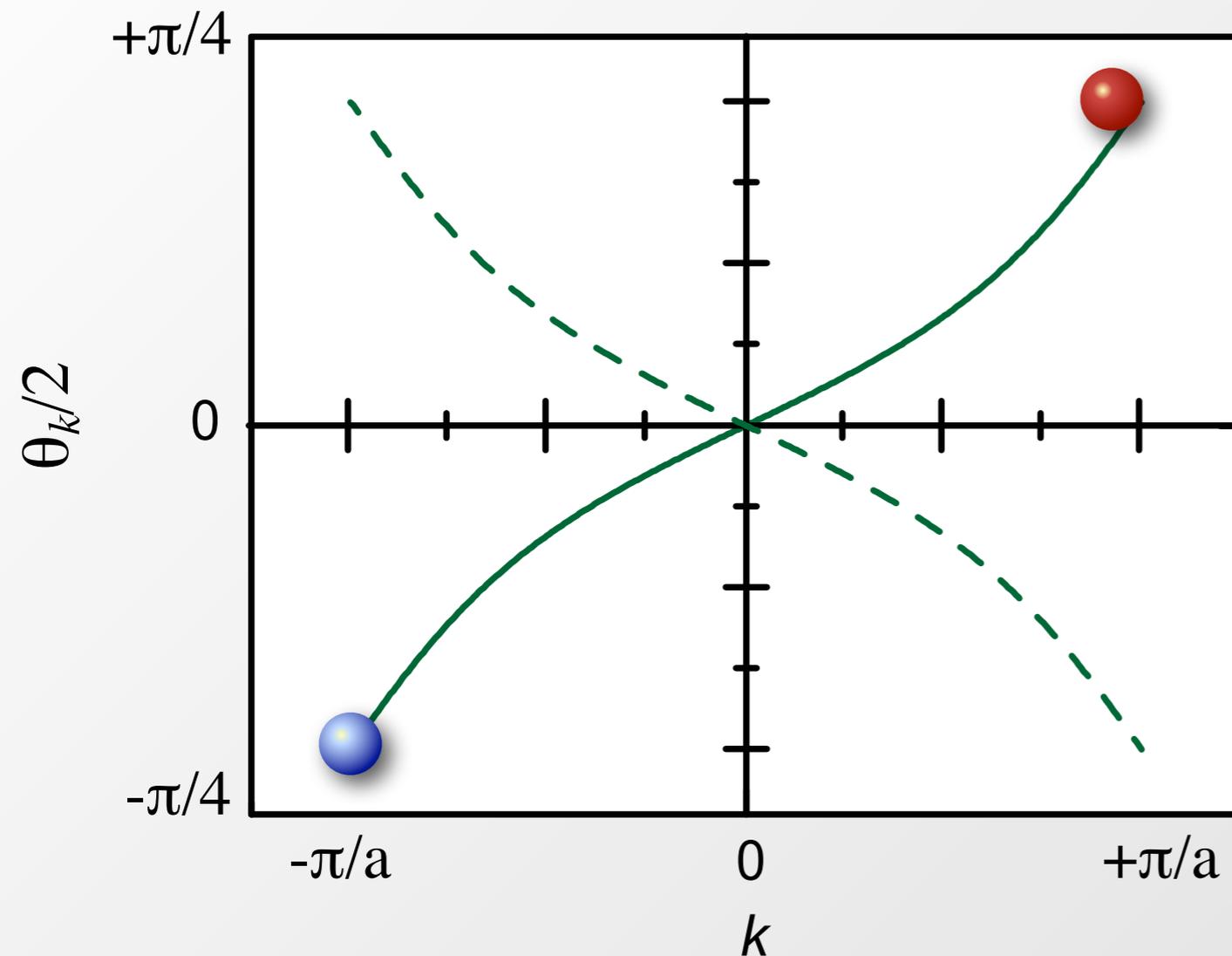
What have we measured:

$$\varphi_{\text{tot}} = \varphi_{\text{Zak}} + \cancel{\varphi_{\text{dyn}}} + \varphi_{\text{Zeeman}}$$



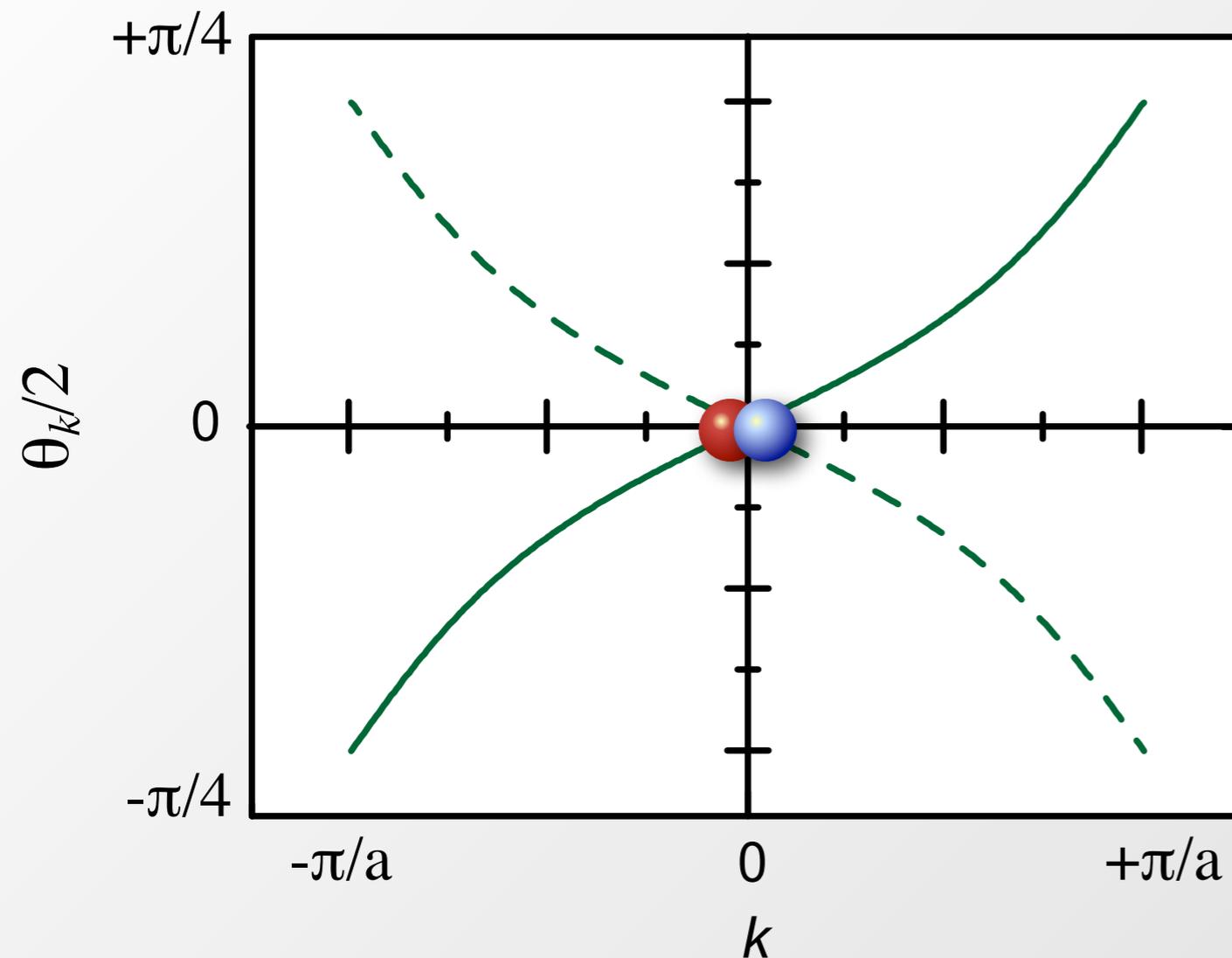


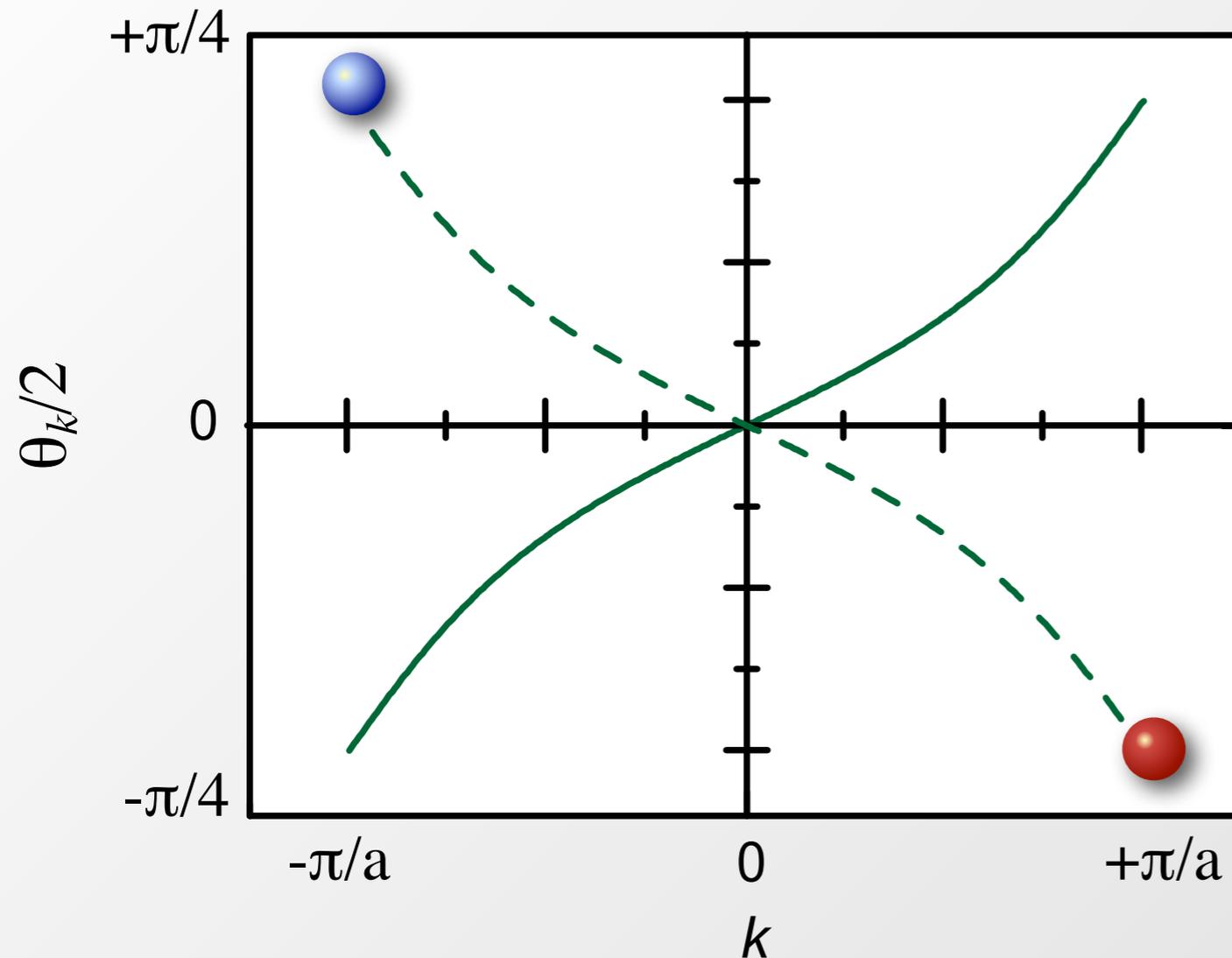




$$\varphi_{\text{tot}} = \cancel{\varphi_{\text{zak}}} + \cancel{\varphi_{\text{dyn}}} + \cancel{\varphi_{\text{Zeeman}}}$$





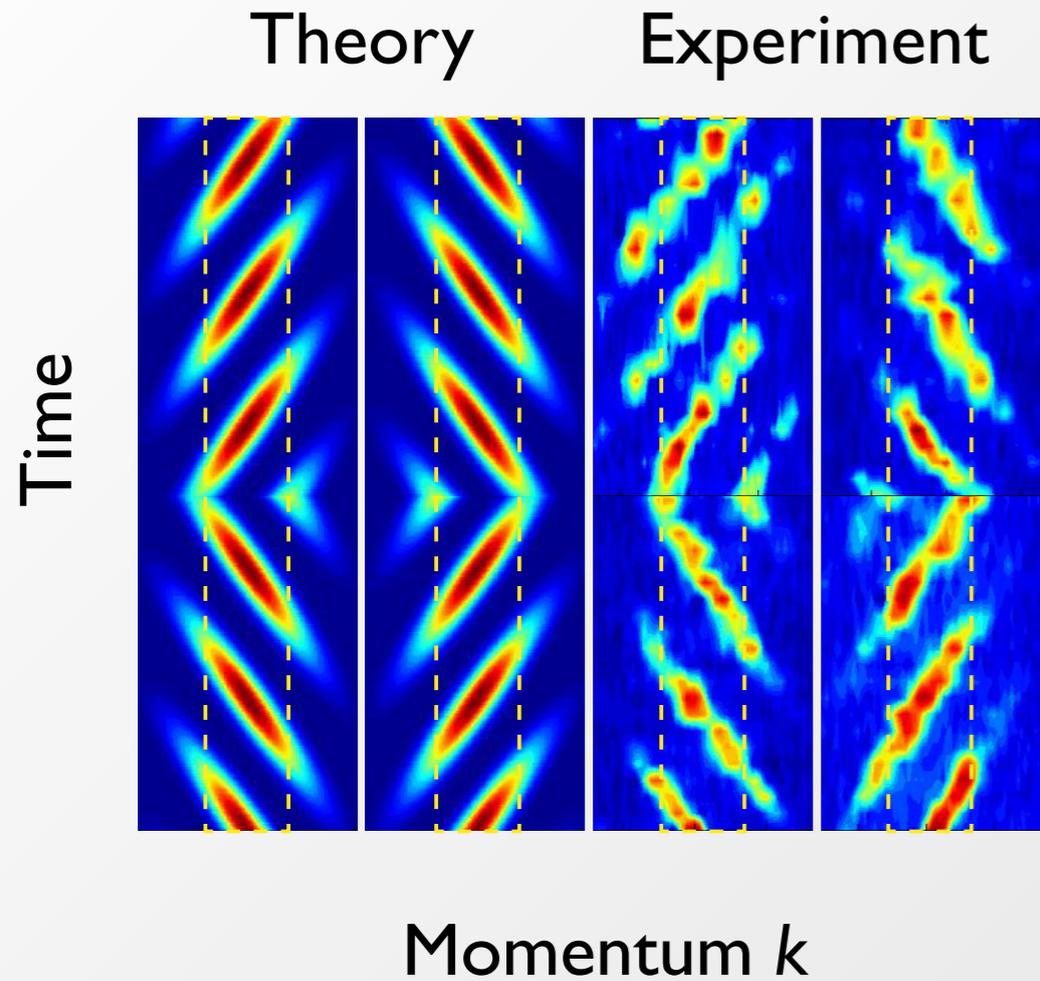


Photon Echo pi-Puls

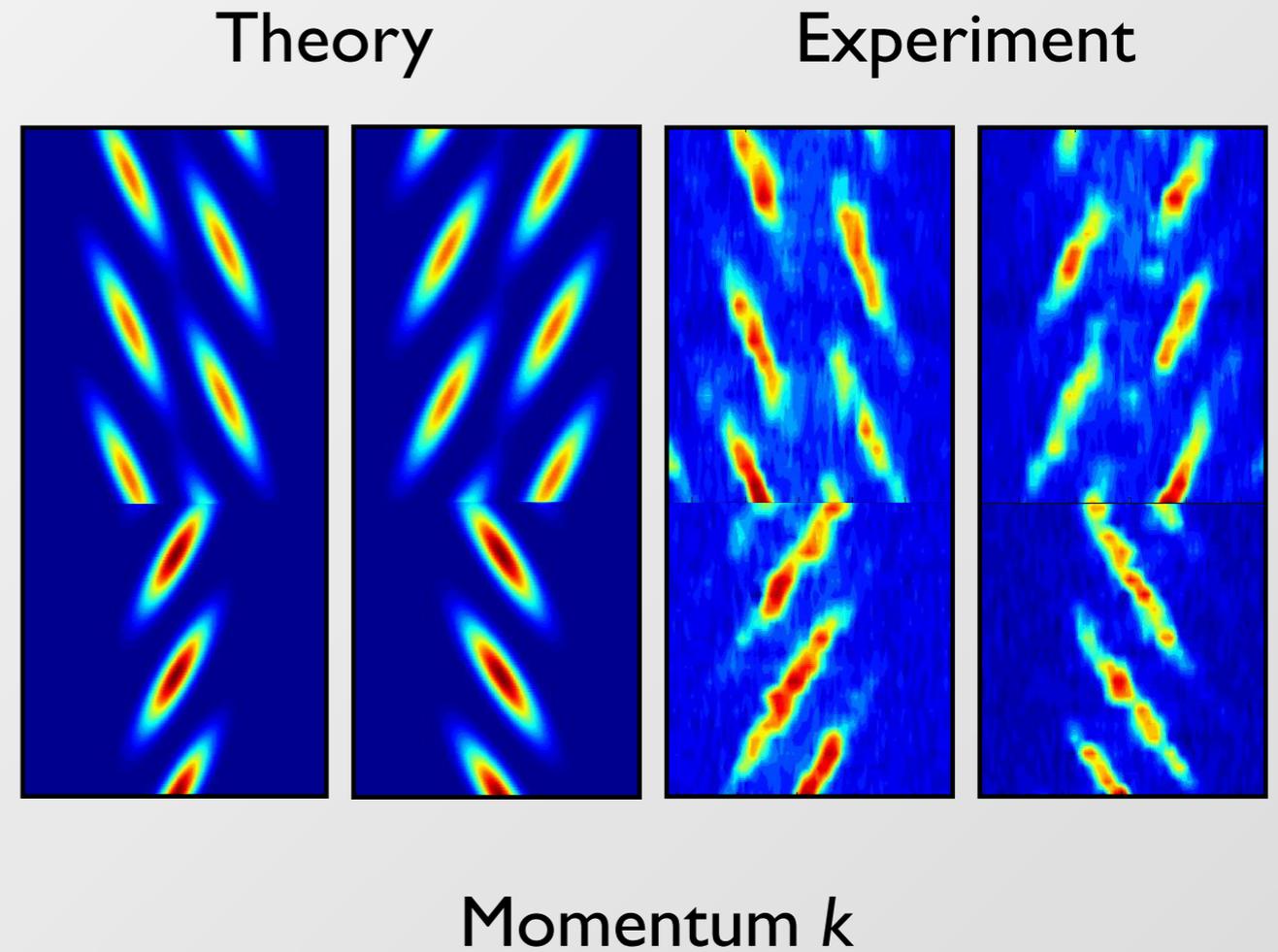
+Change of Dimerization

$$\varphi_{\text{tot}} = \left(\varphi_{\text{Zak}}^{\text{D1}} - \varphi_{\text{Zak}}^{\text{D2}} \right) + \cancel{\varphi_{\text{dyn}}} + \cancel{\varphi_{\text{Zeeman}}}$$





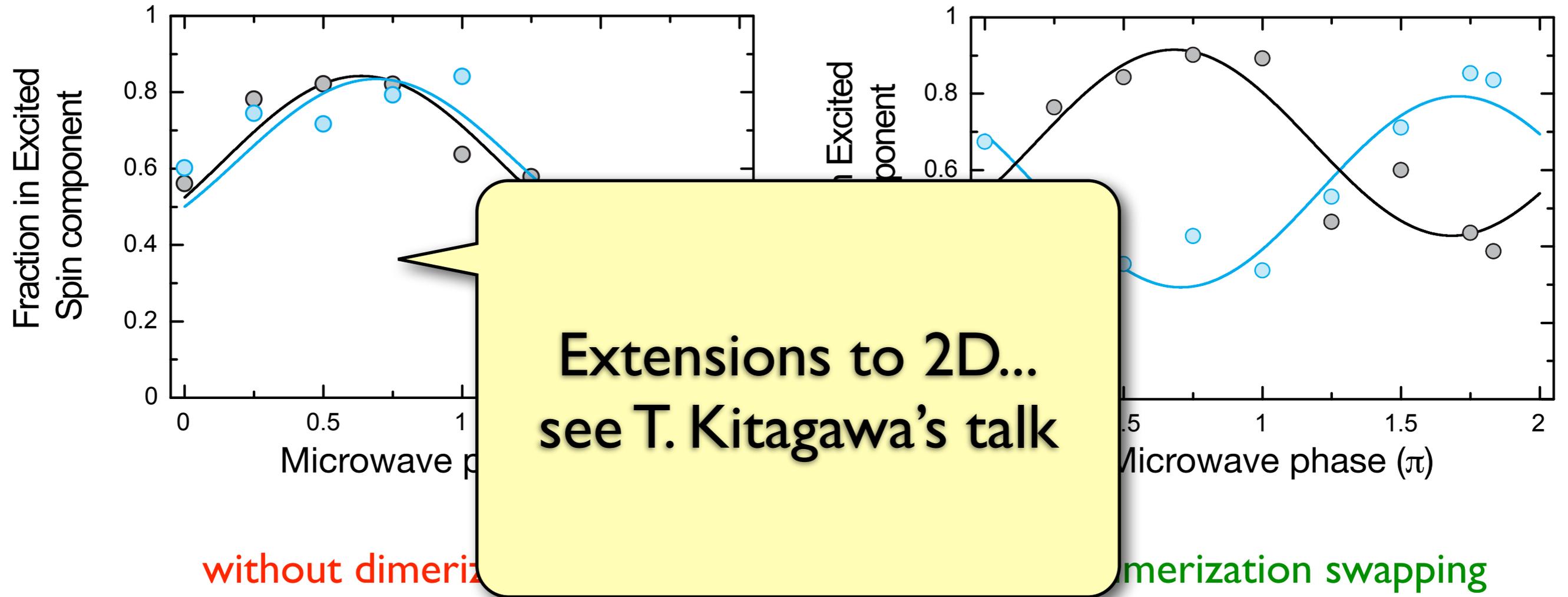
without dimerization swapping



without dimerization swapping

Bloch oscillations in gradient field!





Measured Topological Berry-Zak Phase:

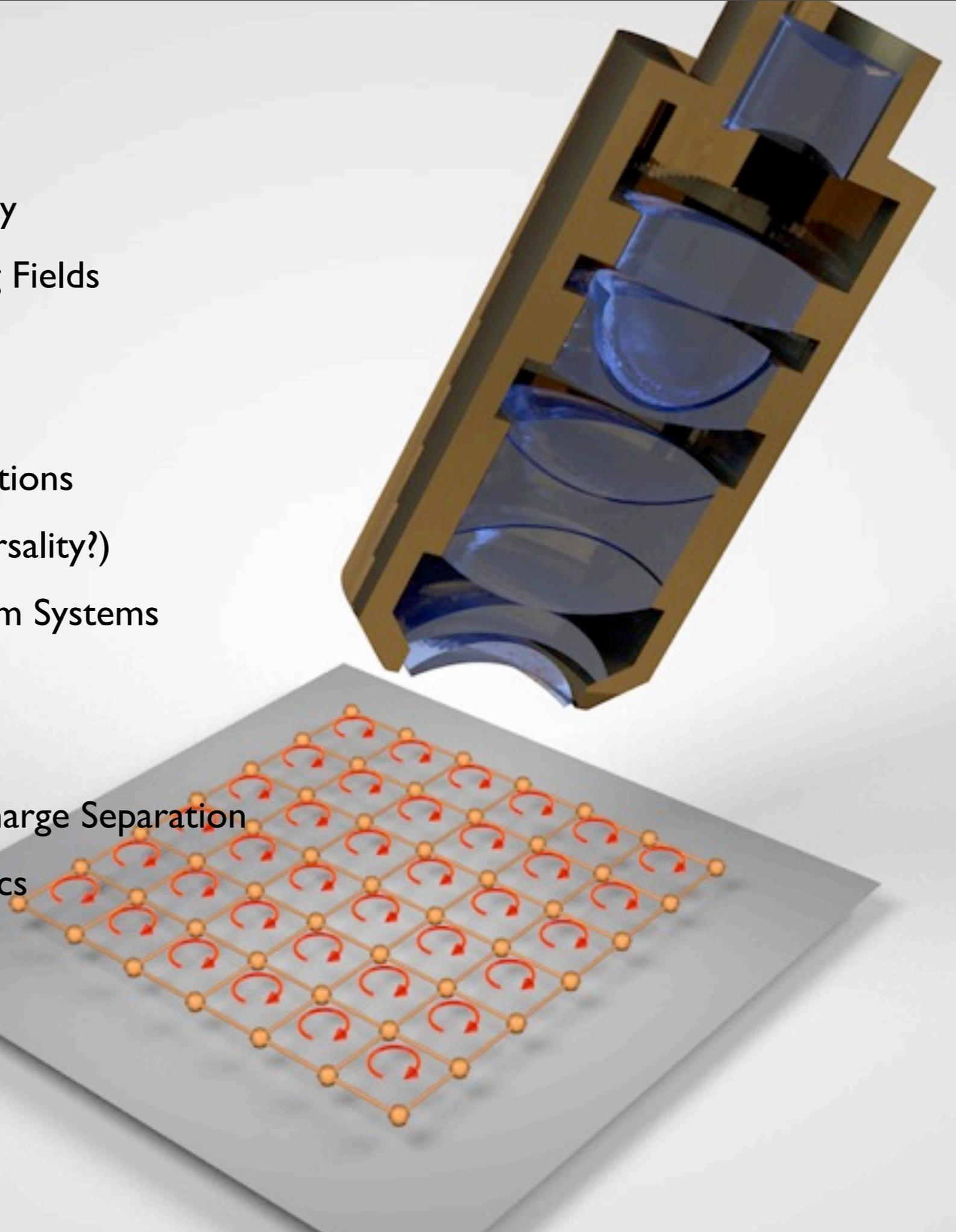
$$\Delta\varphi_{Zak} = 1.02(4)\pi$$



Outlook

- Rectified Flux, Hofstadter Butterfly
- Novel Correlated Phases in Strong Fields
- Novel Topological Insulators
- $SU(N)$ Quantum Magnets
- Controlled Quasiparticle Manipulations
- Non-Equilibrium Dynamics (Universality?)
- Thermalization in Isolated Quantum Systems
- Lieb-Robinson in 2D?
- String Order in 2D?
- Dynamical Observation of Spin-Charge Separation
- Entanglement Measures in Dynamics
- Rydberg Quantum Crystals

●
●
●



Gauge Field Team



Julio Barreiro



Christophe Salomon



Stefan Trotzky



Immanuel Bloch



Monika Aidelsburger



Sylvain Nascimbene



Yu-Ao Chen



Marcos Atala



Peter Schauß



Ahmed Omran



David Bellem



Manuel Endres



Sebastian Hild

Christof Weitenberg



Takeshi Fukuhara

Single Atom Team



Jacob Sherson



Christian Groß



Marc Cheneau



Stefan Kuhr



Immanuel Bloch

Rosa Glöckner & Ralf Labouvie

Thank you!



www.quantum-munich.de