Quantum Quenches in the Transverse Field Ising Chain

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Collaborators:
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arXiv:1205.2211, J. Stat. Mech. P07022 (2012)
arXiv:1208.1961

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I. Introduction/Definitions

1. (Global) Quantum Quench

A. Consider an **isolated** quantum system in the **thermodynamic limit**; Hamiltonian $H(h)$ (short-ranged), h e.g. bulk magnetic field

B. Prepare the system in the ground state $|\psi\rangle$ of $H(h_0)$

C. At time $t=0$ change the Hamiltonian to $H(h)$

D. (Unitary) time evolution $|\psi(t)\rangle = \exp(-iH(h)t)\,|\psi\rangle$

E. Goal: study time evolution of local (in space) observables

$\langle\psi(t)|O(x)|\psi(t)\rangle$, $\langle\psi(t)|O_1(x)O_2(y)|\psi(t)\rangle$ etc

2 main scenarios at late times after the quench

- generic systems behaves "thermally".

Deutsch '91
Srednicki '94

- integrable systems behave in a more complicated way.

Rigol, Dunjko, Yurosvki & Olshanii '07

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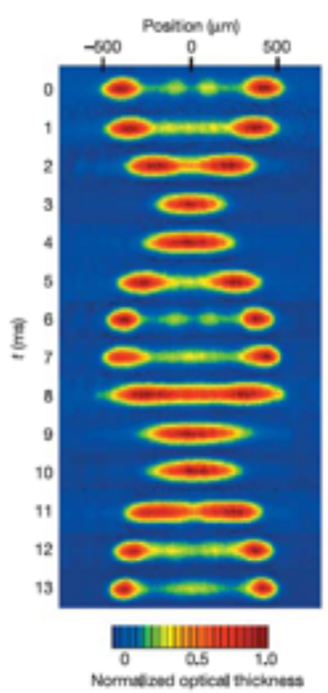
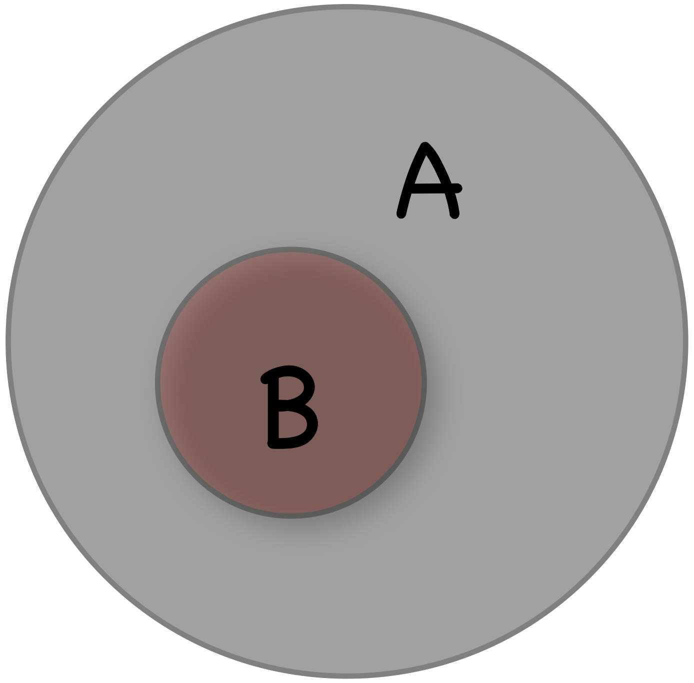


talks by J. Eisert, M. Rigol, J. Cardy, J.-S. Caux...

many (most) people in the audience

2. Reduced Density Matrix

$|\psi\rangle$ = initial (pure) state of the entire system $A \cup B$ (A infinite)



Density matrix: $\rho(t)=|\psi(t)\rangle\langle\psi(t)|$

Reduced density matrix: $\rho_B(t)=\mathrm{tr}_A\, \rho(t)$

Expectation values of local observables in B: $\langle\psi(t)|O_B(x)|\psi(t)\rangle = \mathrm{tr}_B\, [O_B(x)\ \rho_B(t)]$

3. Stationary State

If $\lim_{t\to\infty} \rho_B(t) = \rho_B(\infty)$ exists for any finite subsystem B:

→ system approaches a stationary state

4. Thermalization

Define a **Gibbs ensemble** for the entire system $A \cup B$

$$\rho_G = \exp(-\beta H(h))/Z$$

β fixed by: $\mathrm{tr}[\rho_G\, H(h)] = \langle \psi(0)|\, H(h)\, |\psi(0)\rangle$

Reduced density matrix for subsystem B:

$$\rho_{G,B} = \mathrm{tr}_A\, \rho_G$$

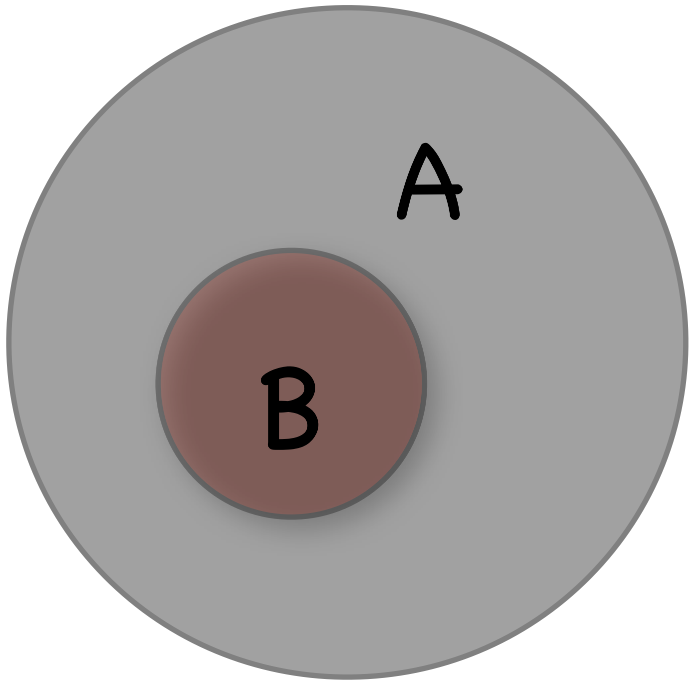
The system **thermalizes** if for any finite subsystem B

$$\rho_B(\infty) = \rho_{G,B}$$

cf **talk by J. Eisert**
Landau/Lifshitz vol 5
Goldstein et al '05
Barthel&Schollwöck '08
Cramer, Eisert et al '08 ...

4. Thermalization

A acts as a heat bath with T_{eff}



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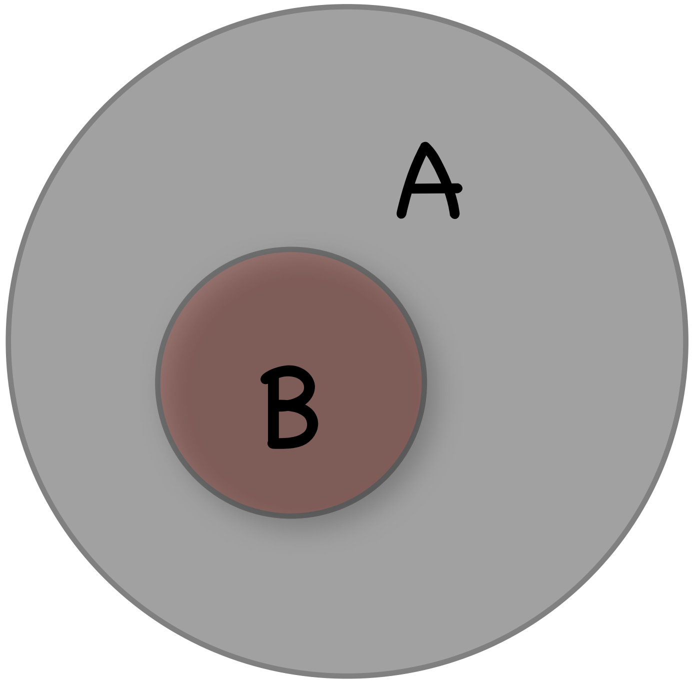


Expectation: "Generic" systems thermalize.

5. Generalized Gibbs Ensemble

Rigol, Dunjko, Yurosvki & Olshanii '07

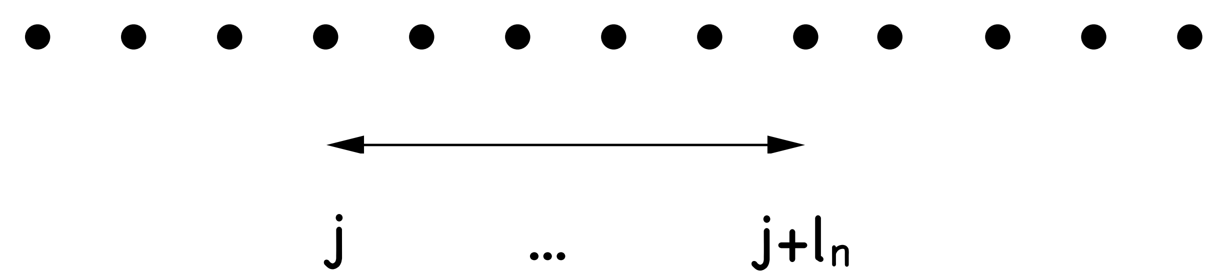
Integrable systems don't thermalize but are described by a GGE!

5. Generalized Gibbs Ensemble

Let I_m be **local** (in space) integrals of motion $[I_m, I_n]=[I_m, H(h)]=0$

in our case

$$I_n = \sum_j I_n(j, j+1, \ldots, j+\ell_n)$$



5. Generalized Gibbs Ensemble

Let I_m be **local** (in space) integrals of motion $[I_m, I_n]=[I_m, H(h)]=0$

Define GGE density matrix by: $\rho_{gG}=\exp(-\Sigma\ \lambda_m I_m)/Z_{gG}$

λ_m **fixed** by $\mathrm{tr}[\rho_{gG} I_m]= \langle\psi(0)|\ I_m\ |\psi(0)\rangle$

Reduced density matrix of B: $\rho_{gG,B}=\mathrm{tr}_A\ \rho_{gG}$

The system is described by a GGE if for any finite subsystem B

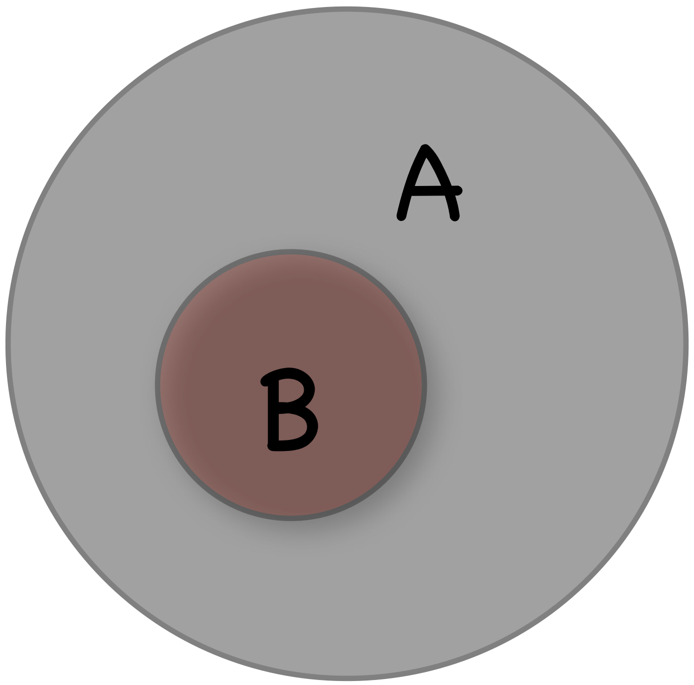
$$\rho_B(\infty)= \rho_{gG,B}$$

Barthel & Schollwöck '08

Cramer, Eisert et al '08

5. Generalized Gibbs Ensemble

A is not a standard heat bath:
∞ information about the initial
state is retained!!!



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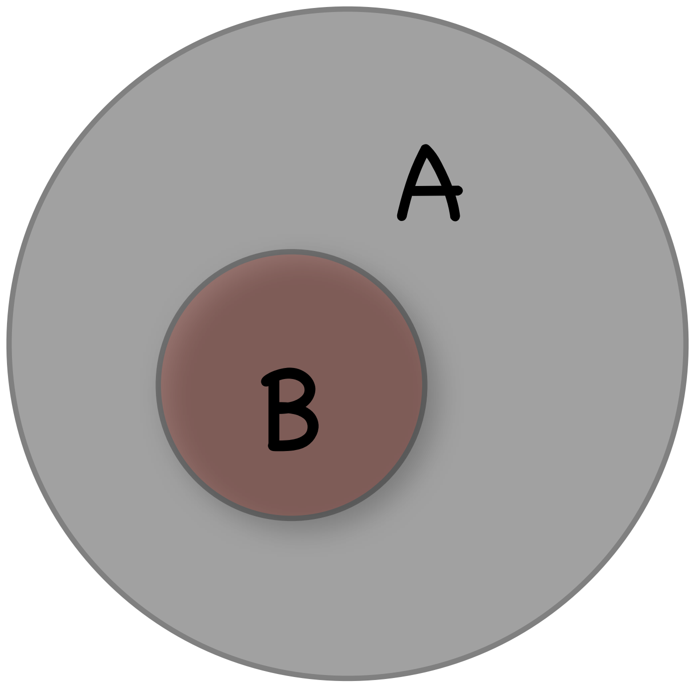


II. Goals and Questions

Derive analytic results for quench dynamics in an integrable model (should be simple, but more than just harmonic oscillators) in the thermodynamic limit.

II. Goals and Questions

Q1: What happens for $t \to \infty$?

- Is the stationary state described by a GGE?
- If it is, can we determine local observables?

Q2: How fast is the approach to the $t \to \infty$ limit?

Q3: How do local observables behave at $1 \ll t < \infty$?

$$\langle \psi(t)| O(x) O(y) |\psi(t)\rangle \;=??$$

Q4: What about dynamical response functions?

$$\langle \psi(t)| O(x,t_1) O(y,t_2) |\psi(t)\rangle \;=??$$

III. The Model: Transverse Field Ising Chain

Simplest paradigm of a T=0 **Quantum Phase Transition**

Hamiltonian:

$$H(h_0) = -J \sum_{j=1}^{L} \sigma_j^z \sigma_{j+1}^z + h_0 \sigma_j^x$$

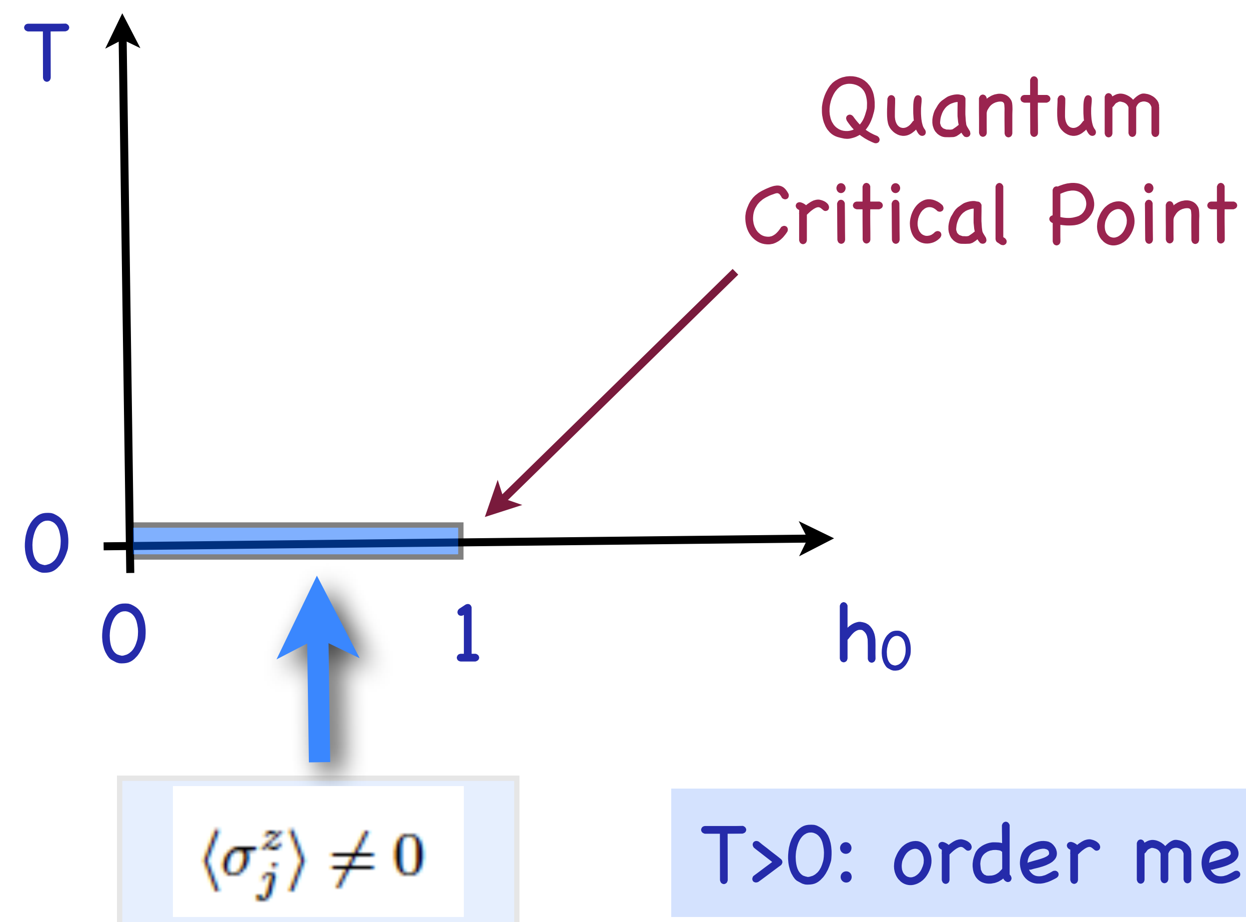
$\mathbb{Z}_2$ symmetry: rotation by π around x-axis.

$$\sigma_j^\alpha \to -\sigma_j^\alpha \ , \quad \alpha = y, z.$$

Phase Diagram:

order parameter: $\langle \sigma_j^z \rangle$

($\langle \sigma_j^x \rangle \neq 0$ always)



T>0: order melts

Jordan-Wigner transformation to spinless fermions:

$$\sigma_j^x = 1 - 2c_j^\dagger c_j \,,$$ ← **local**

$$\sigma_j^z = -\prod_{l<j}(1 - 2c_l^\dagger c_l)(c_j + c_j^\dagger).$$ ← **nonlocal**

Fourier+Bogoliubov transformations:

$$c(k) = \frac{1}{\sqrt{L}} \sum_j c_j \, e^{-ikj}.$$

$$\begin{pmatrix} c(k) \\ c^\dagger(-k) \end{pmatrix} = R_{h_0}(k) \begin{pmatrix} \alpha_k \\ \alpha_{-k}^\dagger \end{pmatrix}$$

→

$$H(h_0) = \sum_k \epsilon_{h_0}(k) \left[\alpha_k^\dagger \alpha_k - \frac{1}{2}\right]$$

$$\epsilon_{h_0}(k) = 2J\sqrt{1 + h_0^2 - 2h_0 \cos k}$$

Ground State: $\alpha_k|0\rangle = 0.$

This will be our initial state $|\Psi\rangle$

Quantum Quench $h_0 \to h$

New Hamiltonian: $$H(h) = \sum_k \epsilon_h(k) \left[\beta_k^\dagger \beta_k - \frac{1}{2}\right]$$

New vs old Bogoliubov fermions: $$\begin{pmatrix} \beta_k \\ \beta^\dagger_{-k} \end{pmatrix} = U(k) \begin{pmatrix} \alpha_k \\ \alpha^\dagger_{-k} \end{pmatrix} \qquad U(k) = R_h^\dagger(k) R_{h_0}(k)$$

linearly related → easy to calculate e.g. $\langle \Psi(t) | \sigma_j^x | \Psi(t) \rangle$

(Barouch, McCoy & Dresden '70)

σ_j^z **non-local** in fermions → hard problem.

non-local correlators may behave qualitatively different → some obervables thermal, some not??

(Rossini, Suzuki, Silva, Mussardo, Santoro '09, '10)

Note:

TFIC maps to free fermions, but **observables are spins!**

↓

TFIC in many ways close to "generic integrable models" (sine-Gordon, XXZ, Lieb-Liniger), albeit (of course) simpler.

IV. Our Work:

Developed 2 novel approaches to derive **analytic** results in the thermodynamic limit.

Q1: What happens for $t \to \infty$?

- Is the stationary state described by a GGE?.
- If it is, can we determine local observables?

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- If it is, can we determine local observables?

Showed that $\lim_{t\to\infty} \mathrm{tr}_A\, |\psi(t)\rangle\langle\psi(t)| = \mathrm{tr}_A[\rho_{gG}]$

includes quenches to critical point in scaling limit (c=1/2 CFT)

more general quenches in general CFTs → talk by J. Cardy

Q1: What happens for $t \to \infty$?

- Is the stationary state described by a GGE? Yes.
- If it is, can we determine local observables? Yes.

Calculated e.g.

$$\lim_{t\to\infty} \frac{\langle\Psi(t)|\sigma^z_{j+\ell}\sigma^z_j|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)\rangle} = C^z(\ell)e^{-\ell/\xi}\left[1+o(\ell^0)\right]$$

Have explicit expressions for correlation length and amplitude

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Have explicit expressions for correlation length and amplitude

$$\xi^{-1} = \theta_H(h-1)\theta_H(h_0-1)\ln\left[\min(h_0,h_1)\right] - \ln\left[x_+ + x_- + \theta_H((h-1)(h_0-1))\sqrt{4x_+x_-}\right]$$

$$x_\pm = \frac{[\min(h,h^{-1})\pm 1][\min(h_0,h_0^{-1})\pm 1]}{4}, \quad h_1 = \frac{1+hh_0+\sqrt{(h^2-1)(h_0^2-1)}}{h+h_0}$$

Q2: How fast is the approach to the $t \to \infty$ limit?

M. Fagotti &FHLE

Cramer et al '08
Cramer&Eisert '10
Banuls, Cirac & Hastings '11

- How close is $\rho_B(t)$ to $\rho_{gG,B}$ or to $\rho_{G,B}$?

Define distance: $d(\rho,\rho')=||\rho-\rho'||$ $\quad ||M|| \equiv \sqrt{\mathrm{Tr}(M^\dagger M)}$

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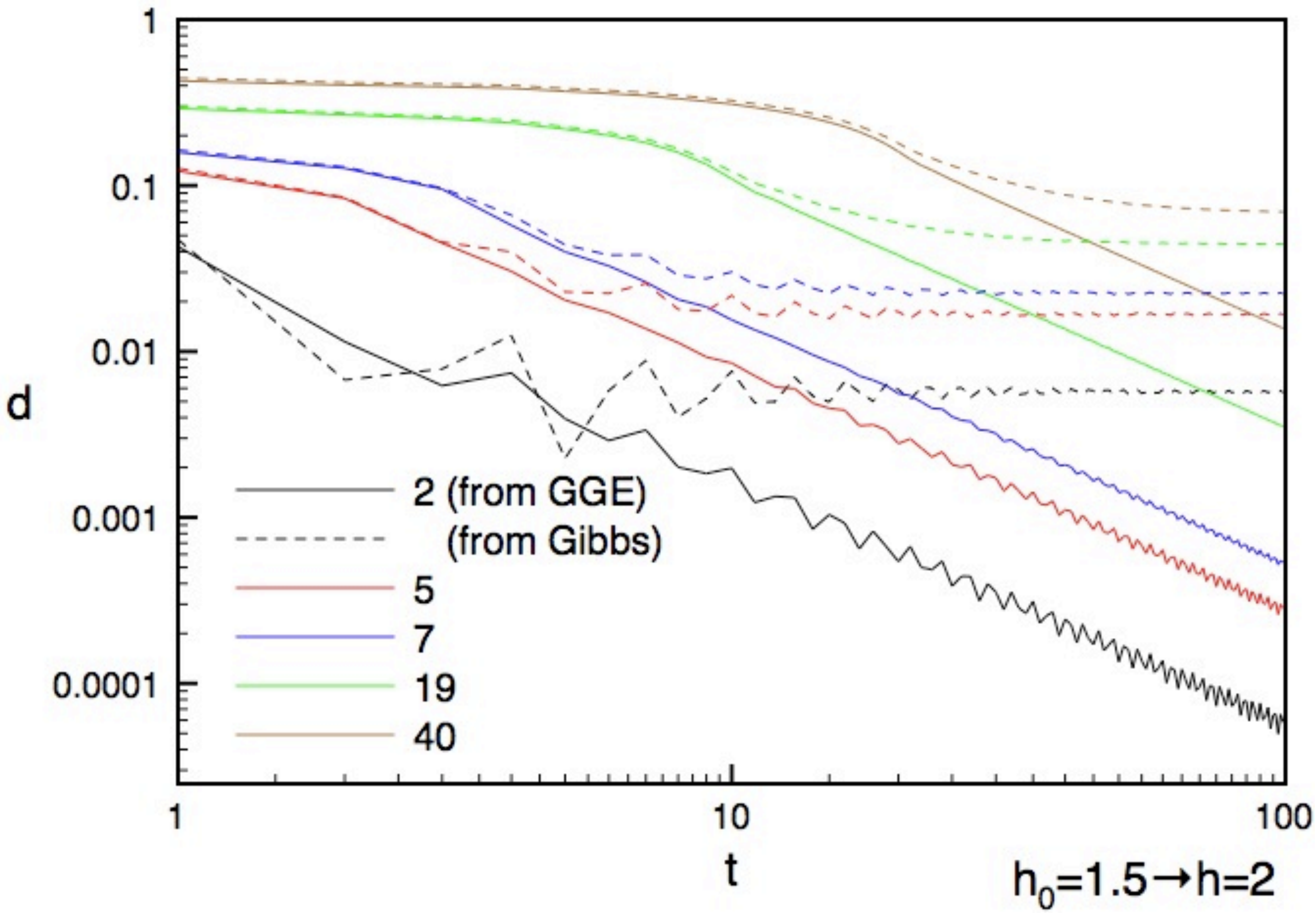


1. not thermal.

2. GGE approached as power-law in t!

3. approach slower for larger systems.

Q3: How do local observables behave at $1 \ll t < \infty$?

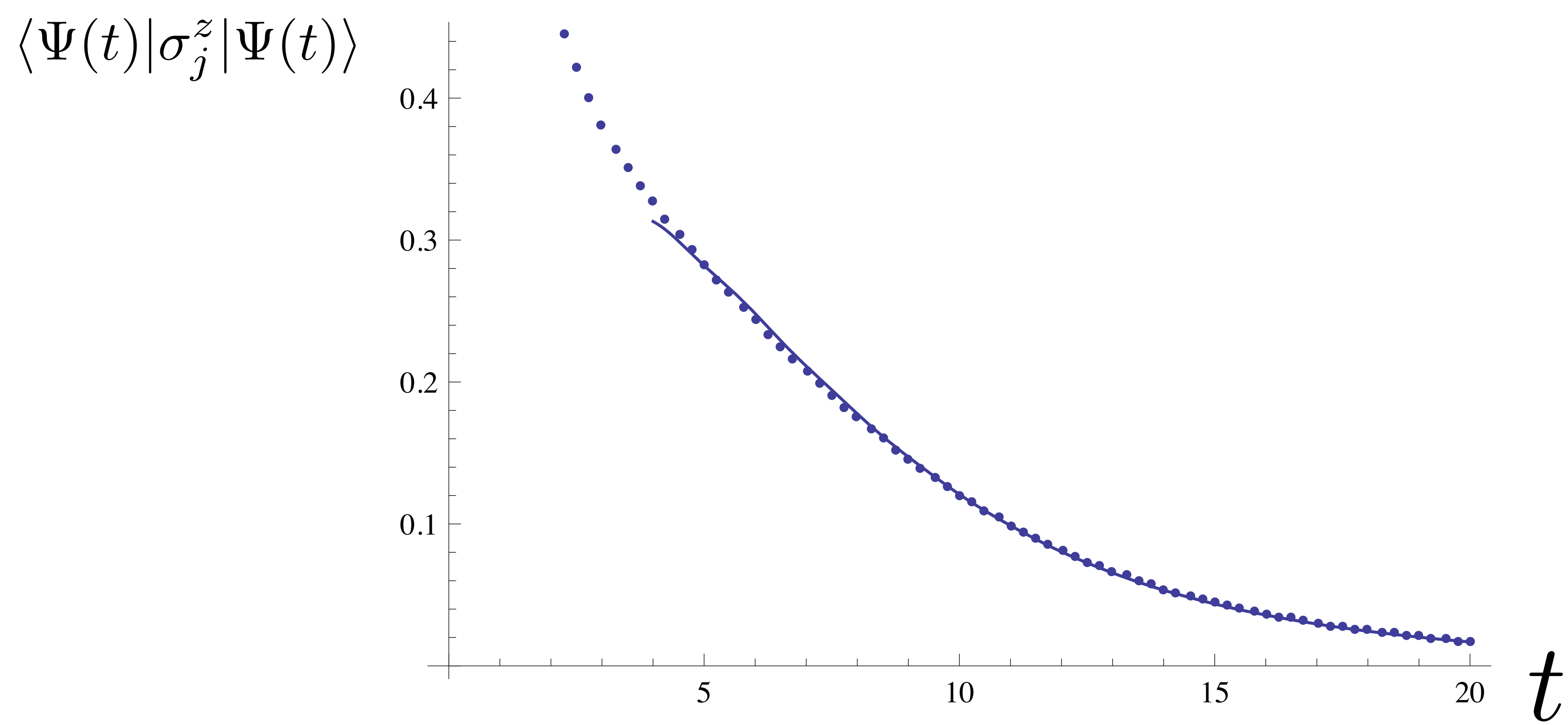
Time evolution of order parameter correlators

1. Order parameter for a quench within the ordered phase

At late times

$$\langle\Psi(t)|\sigma_j^z|\Psi(t)\rangle = \sqrt{C_{FF}}e^{-t\tau}\left[1+\mathcal{O}(t^{-1})\right]$$

Exponential decay to **zero** with exactly known decay rate **τ** and amplitude **C_{FF}.**



$h_0=0.2 \rightarrow h=0.8$

line: asymptotic result
dots: numerical data L=∞, acc. 10^{-16}

Time evolution of order parameter correlators

2. Two-point function for a quench within the ordered phase

At late times

$$\langle\Psi(t)|\sigma^z_{j+\ell}\sigma^z_j|\Psi(t)\rangle = \mathcal{C}^z_{\rm FF}\exp\left[\ell\int_0^\pi \frac{\mathrm{d}k}{\pi}\ln|\cos\Delta_k|\,\theta_H\left(2\epsilon'_h(k)t-\ell\right)\right]$$
$$\exp\left[2t\int_0^\pi \frac{\mathrm{d}k}{\pi}\epsilon'_h(k)\ln|\cos\Delta_k|\,\theta_H\left(\ell-2\epsilon'_h(k)t\right)\right]+\ldots$$

Δ_k $\mathcal{C}^z_{\rm FF}$ known simple functions

Exact in the limit t,l→∞, t/l fixed !

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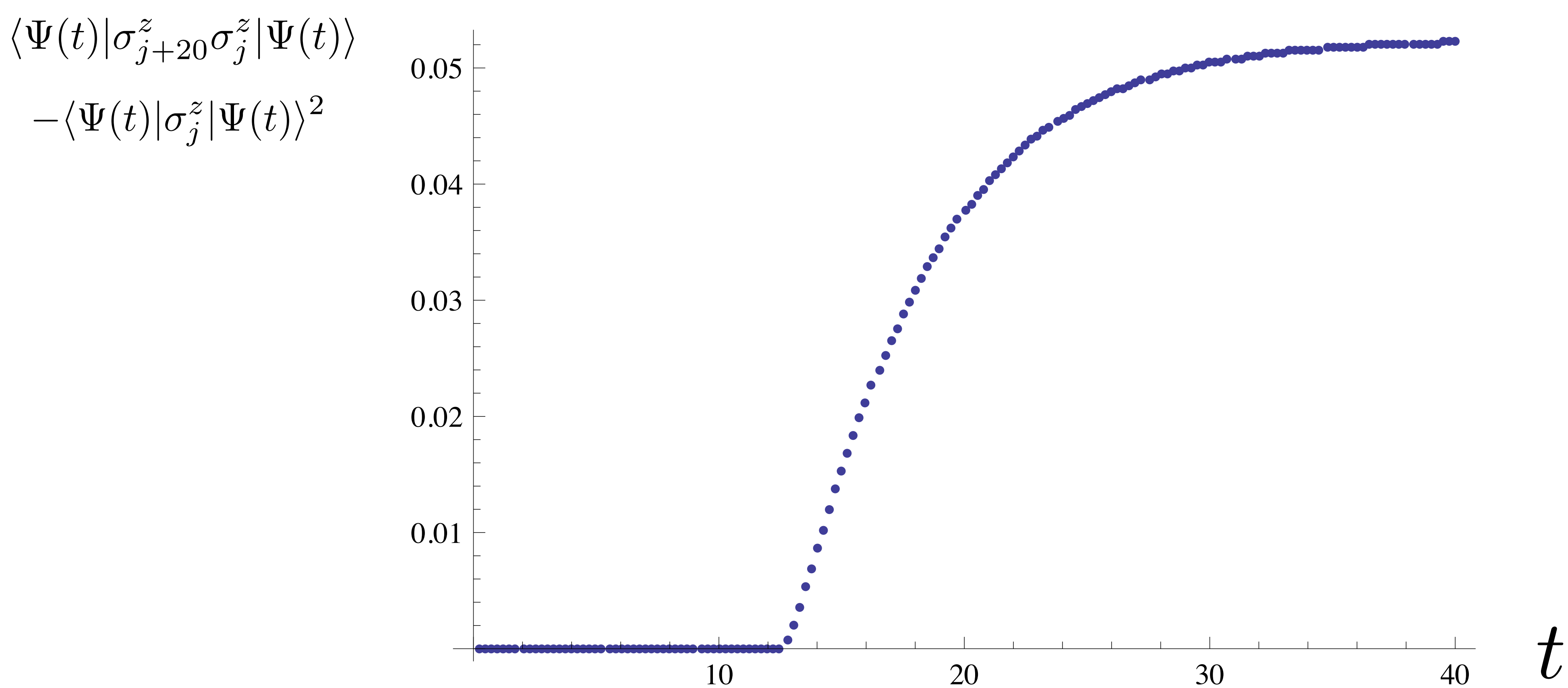

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l=20 fixed

Time evolution of order parameter correlators

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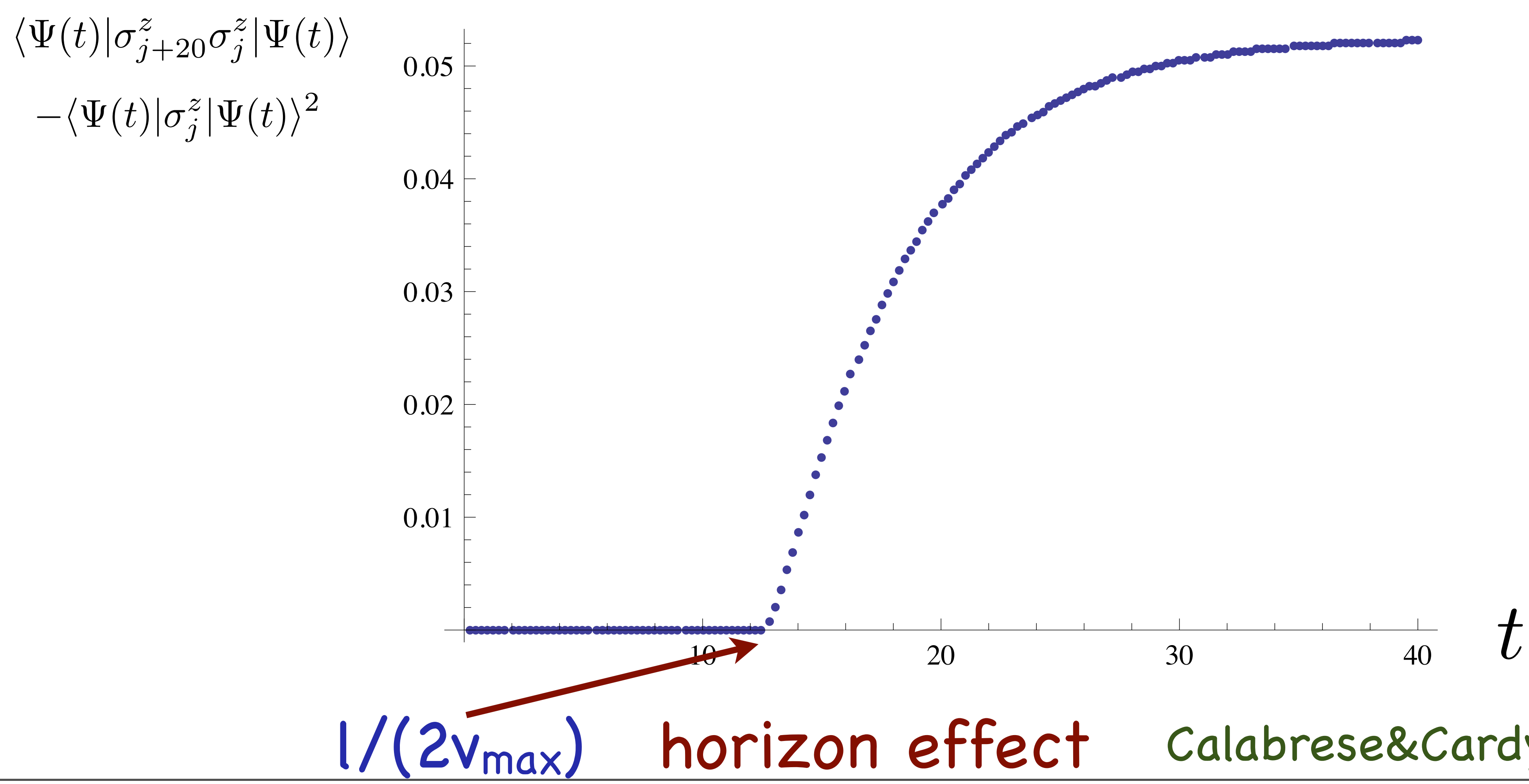

$h_0=0.2 \rightarrow h=0.8$

$l=20$ fixed

horizon effect

Calabrese&Cardy '05

Time evolution of order parameter correlators

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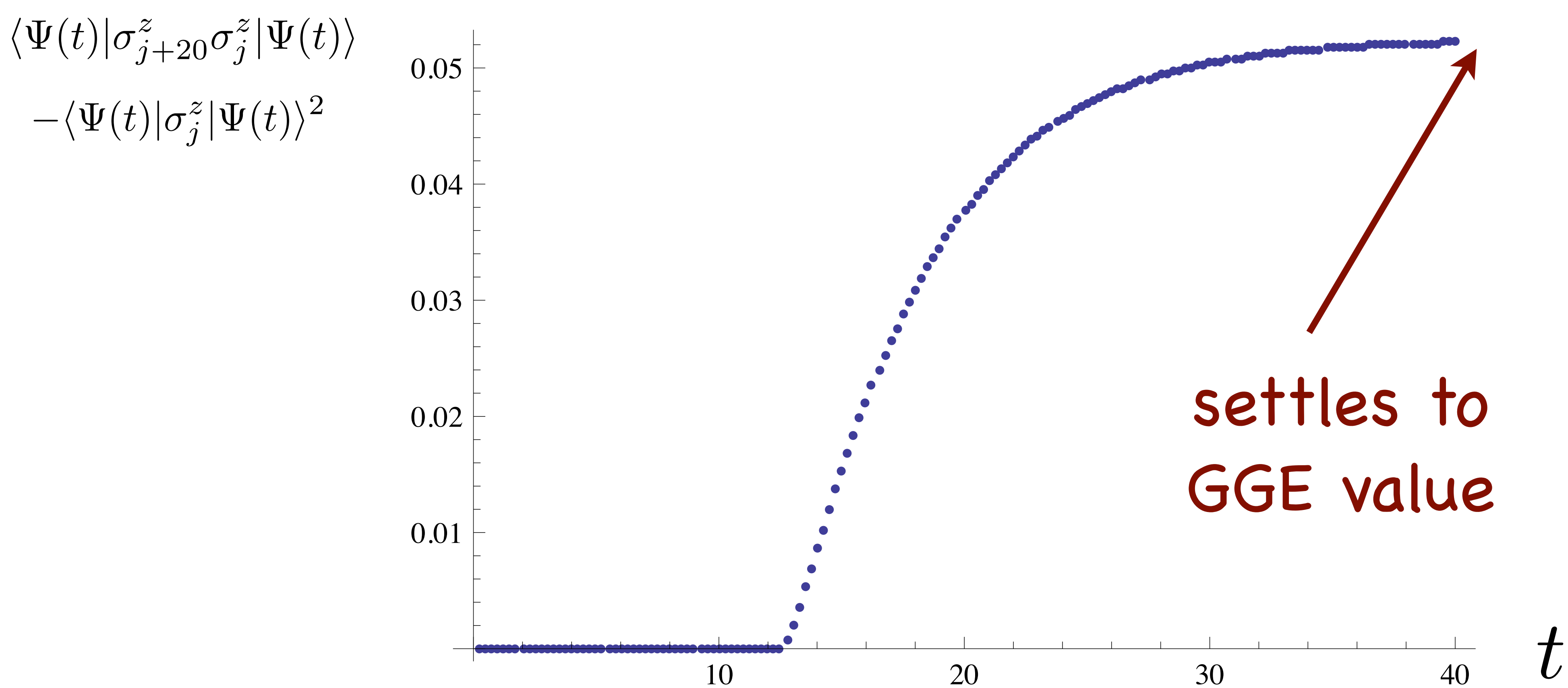


h_0=0.2 → h=0.8

l=20 fixed

Relaxation through ballistically moving domain walls

Time evolution of order parameter correlators

2. Two-point function for a quench within the disordered phase

$$\langle\Psi(t)|\sigma^z_{j+\ell}\sigma^z_j|\Psi(t)\rangle = \left[\mathcal{C}_{\rm PP}(\ell) + (h^2-1)^{\frac{1}{4}}\sqrt{4J^2h}\int_{-\pi}^{\pi}\frac{dk}{\pi}\frac{K(k)}{\epsilon_k}\sin(2t\epsilon_k - k\ell) + \dots\right]$$
$$\times\ \exp\left[-\int_0^{\pi}\frac{dp}{\pi}\ln\left[\frac{1+K^2(p)}{1-K^2(p)}\right]\left(\ell + \theta_H(\ell - 2t\epsilon'_p)[2t\epsilon'_p - \ell]\right)\right] + \dots$$

$\mathcal{C}_{\rm PP}(\ell)$ $K(k)$ known functions

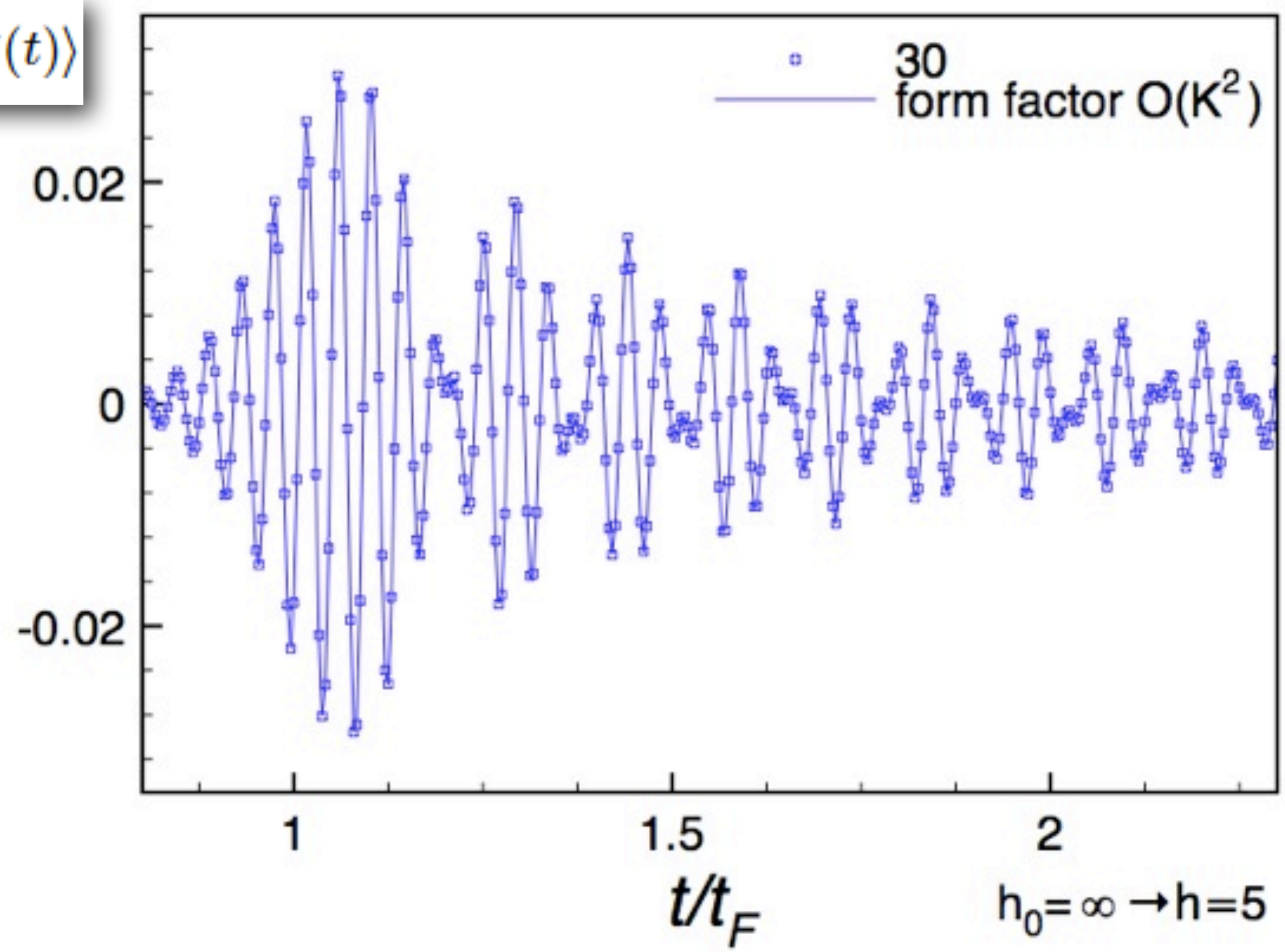
Low density expansion, breaks down at critical point.

Time evolution of order parameter correlators

2. Two-point function for a quench within the disordered phase

$\langle \Psi(t) | \sigma^z_{j+\ell} \sigma^z_j | \Psi(t) \rangle$

l=30 fixed



settles to GGE value

$$t_F = \frac{\ell}{2 v_{\max}}$$

Different relaxational mechanism involving magnon annihilation

Q4: What about dynamical response functions?

$$\langle \psi(t)|O(x,t_1)O(y,t_2)|\psi(t)\rangle \;=??$$

Q4: What about dynamical response functions?

- for $t \to \infty$ given by the GGE!

(FHLE, M. Fagotti & S. Evangelisti)

More generally: if $\lim_{t\to\infty} \langle \Psi_0(t)|\mathcal{O}_1 \dots \mathcal{O}_n|\Psi_0(t)\rangle = \mathrm{Tr}\big(\rho_{\rm stat}\mathcal{O}_1 \dots \mathcal{O}_n\big)$

then $\lim_{t\to\infty} \langle \Psi_0(t)|\mathcal{O}_1(t_1) \dots \mathcal{O}_n(t_n)|\Psi_0(t)\rangle = \mathrm{Tr}\big(\rho_{\rm stat}\mathcal{O}_1(t_1) \dots \mathcal{O}_n(t_n)\big)$

for fixed $t_1, t_2, \dots, t_n$

ultimately follows from

Lieb&Robinson '72
Bravyi, Hastings& Verstraete '06

Q4: What about dynamical response functions?

- for t→∞ given by the GGE!
- analytic results for 2-point functions

(FHLE, M. Fagotti & S. Evangelisti)

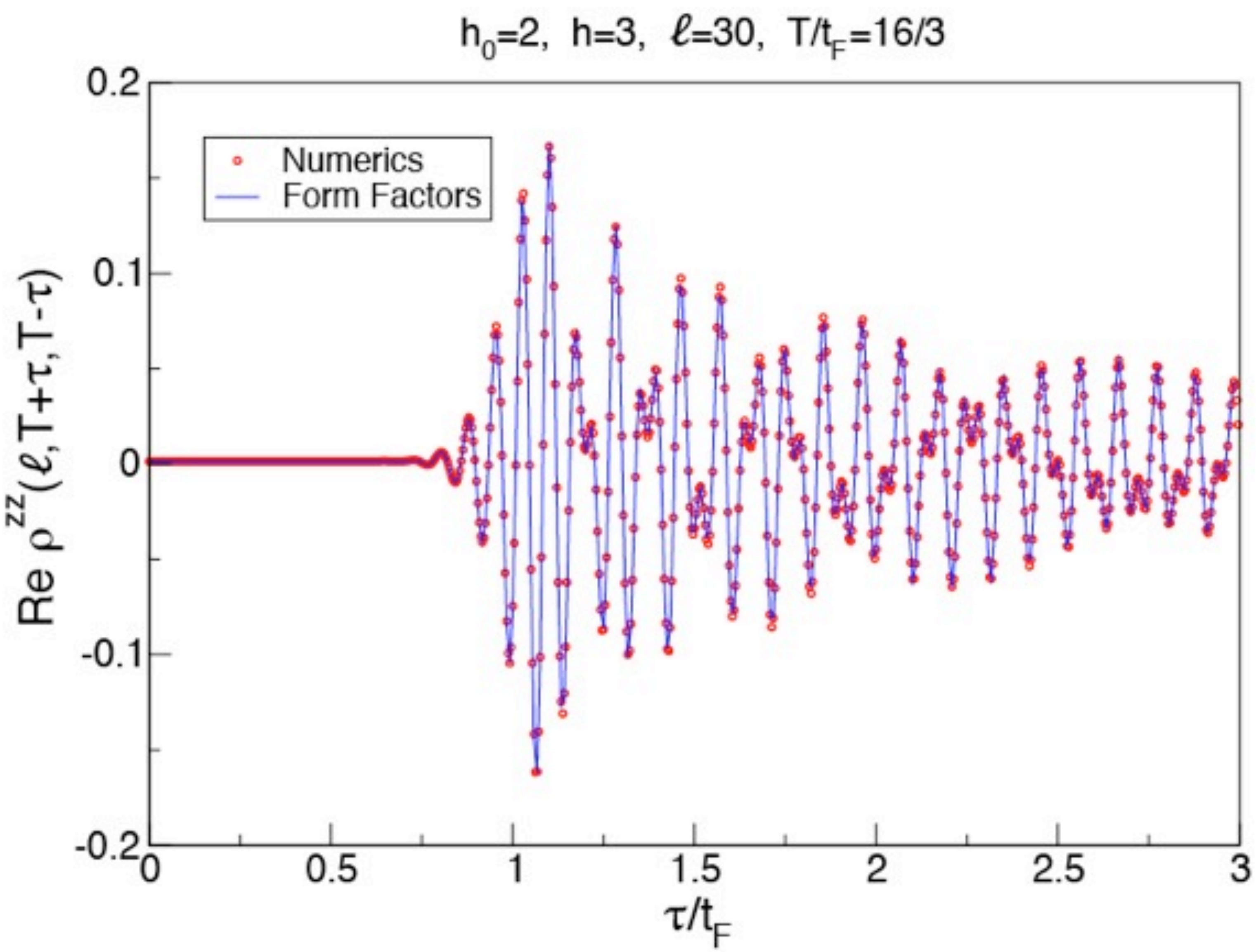
$$\rho^{zz}(t+t_1, t+t_2, \ell) = \langle \Psi(t) | \sigma^z_{\ell+j}(t_1) \sigma^z_j(t_2) | \Psi(t) \rangle$$

$h_0=2 \rightarrow h=3$

$$T = t + \frac{t_1 + t_2}{2}$$

$$\tau = \frac{t_1 - t_2}{2}$$

$$t_F = \frac{\ell}{2v_{\max}}$$



A Few Words about the methods:

Approach I: Block-Toeplitz Determinants

Express $\sigma_j^z(t)$ in terms of the "old" Bogoliubov fermions α_k

$$\langle 0|\sigma_j^z(t)\ \sigma_{j+n}^z(t)|0\rangle \xrightarrow{\text{Wick's thm}} \text{Pf}(T)$$

$$T_{ln} = \begin{pmatrix} f_{l-n} & -g_{n-l} \\ g_{l-n} & -f_{l-n} \end{pmatrix}$$ Block-Toeplitz matrix

$$f_l = i\int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{-ikl} \sin(\Delta_k)\ \sin(2\epsilon_h'(k)t)$$

$$g_l = \int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{-ik(l-1)} \left[\cos(\Delta_k) + i\sin(\Delta_k)\cos(2\epsilon_h'(k)t)\right]$$

Approach I: Block-Toeplitz Determinants

Express $\sigma_j^z(t)$ in terms of the "old" Bogoliubov fermions α_k

Wick's thm

$$\langle 0|\sigma_j^z(t)\ \sigma_{j+n}^z(t)|0\rangle \longrightarrow \text{Pf}(T)$$

t,n→∞, t/n fixed

Multi-dim stationary phase approx.

$$\exp\left(t \int_{2t\epsilon'_{h'}(k)<n} \frac{dk}{\pi} 2\epsilon'_{h'}(k) \ln\left[\cos(\Delta_k)\right] + n \int_{2t\epsilon'_{h'}(k)>n} \frac{dk}{\pi} \ln\left[\cos(\Delta_k)\right] \right)$$

c.f. (M. Fagotti & P. Calabrese '08)

Approach II: "Form-Factor" Sums

Consider a quench within the ordered phase h,h'<1

1. Go to large, finite volume L

2. initial state: must give symmetry broken ground state for L→∞

$$|0\rangle = \frac{1}{\sqrt{2}}\Big[|0\rangle_{\rm R} \pm |0\rangle_{\rm NS}\Big] \qquad \alpha_q|0\rangle_{\rm R} = 0 \qquad q_m = \frac{2\pi}{L}m\ , \quad m = -\frac{L}{2},\dots,\frac{L}{2}-1$$

periodic bc's on fermions

antiperiodic bc's on fermions

3. Express this in terms of the new Bogoliubov fermions

$$|0\rangle_{\rm NS} = \exp\left(i\sum_{p>0} K(q)\beta^\dagger_q\beta^\dagger_{-q}\right)|0'\rangle_{\rm NS}\ ,$$

$$|0\rangle_{\rm R} = \exp\left(i\sum_{q>0} K(q)\beta^\dagger_q\beta^\dagger_{-q}\right)|0'\rangle_{\rm R}\ .$$

$$K(q) = \tan\left[\frac{\theta_{h'}(q)-\theta_h(q)}{2}\right]$$

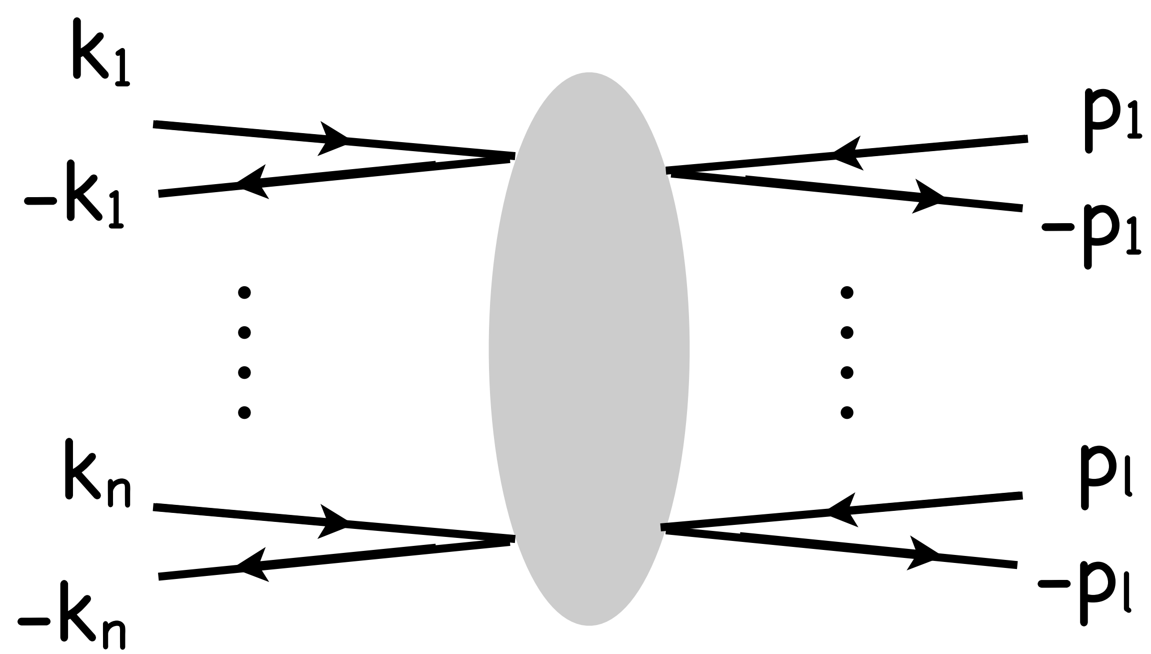
cf Rossini et al '10

4. Lehmann representation in terms of new Bogoliubov fermions

$$
{}_{\mathrm{NS}}\langle 0|\sigma^z_m(t)|0\rangle_{\mathrm{R}} = \sum_{l,n=0}^{\infty} \frac{1}{n!\, l!} \sum_{\substack{k_1,\dots,k_n \\ p_1,\dots,p_l}} \left[\prod_{j=1}^{n} K(k_j)\right] \left[\prod_{i=1}^{l} K(p_i)\right]
$$

$$
{}_{\mathrm{NS}}\langle -k_1, k_1, \dots, -k_n, k_n|\sigma^z_m(t)|p_1, -p_1, \dots, p_l, -p_l\rangle_{\mathrm{R}}
$$

$$
= \sum_{l,n=0}^{\infty} \frac{1}{n!\, l!}
$$



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$$

$$
{}_{\rm NS}\langle -k_1, k_1, \dots, -k_n, k_n|\sigma^z_m(t)|p_1, -p_1, \dots, p_l, -p_l\rangle_{\rm R}
$$

$$
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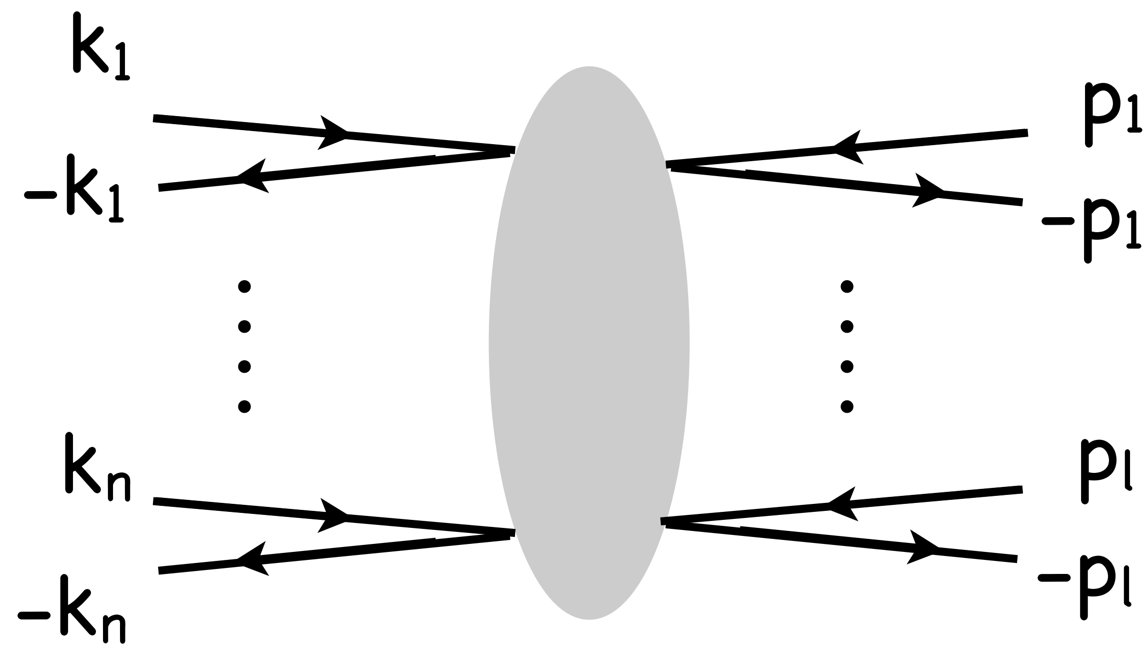
$$



form factors are known exactly for the lattice model

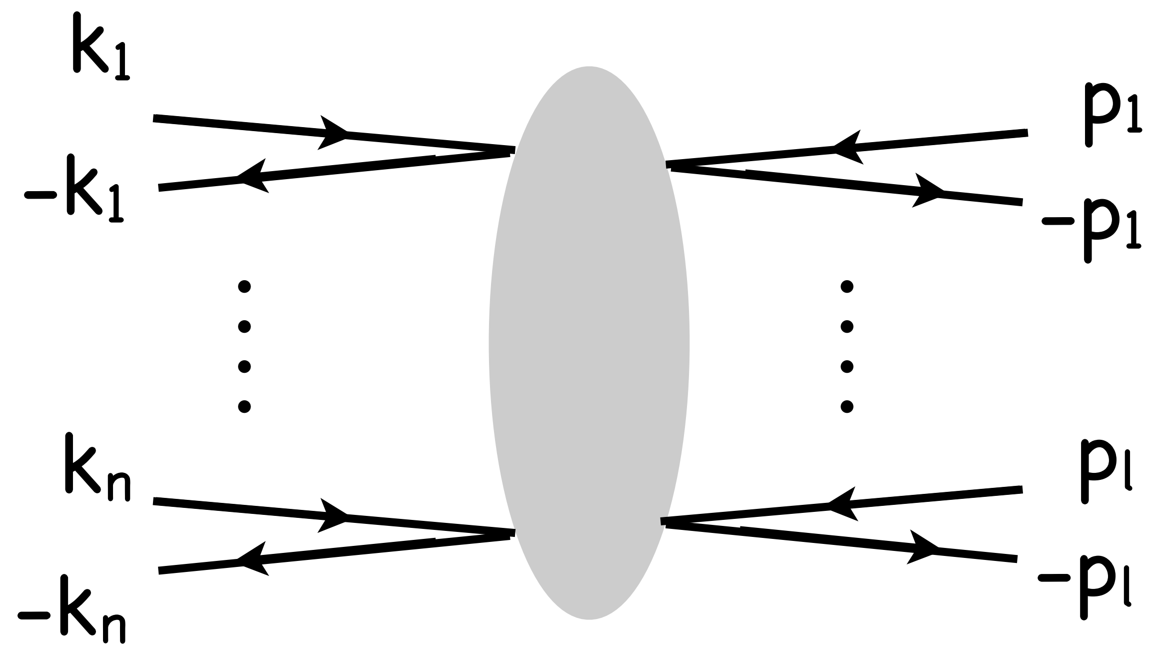
Vaidya&Tracy '78,
vonGehlen et al '08

4. Lehmann representation in terms of new Bogoliubov fermions

$$
{}_{\mathrm{NS}}\langle 0|\sigma^z_m(t)|0\rangle_{\mathrm{R}} = \sum_{l,n=0}^{\infty} \frac{1}{n!\ l!} \sum_{\substack{k_1,\dots,k_n \\ p_1,\dots,p_l}} \left[\prod_{j=1}^{n} K(k_j)\right] \left[\prod_{i=1}^{l} K(p_i)\right]
$$

$$
{}_{\mathrm{NS}}\langle -k_1, k_1, \dots, -k_n, k_n|\sigma^z_m(t)|p_1, -p_1, \dots, p_l, -p_l\rangle_{\mathrm{R}}
$$

$$
= \sum_{l,n=0}^{\infty} \frac{1}{n!\ l!}
$$



Idea: Consider K(q) as expansion parameter:

$$
n(q) = \frac{\langle 0|\beta^\dagger_q \beta_q|0\rangle}{\langle 0|0\rangle} = \frac{K^2(q)}{1+K^2(q)}
$$

density of excitations

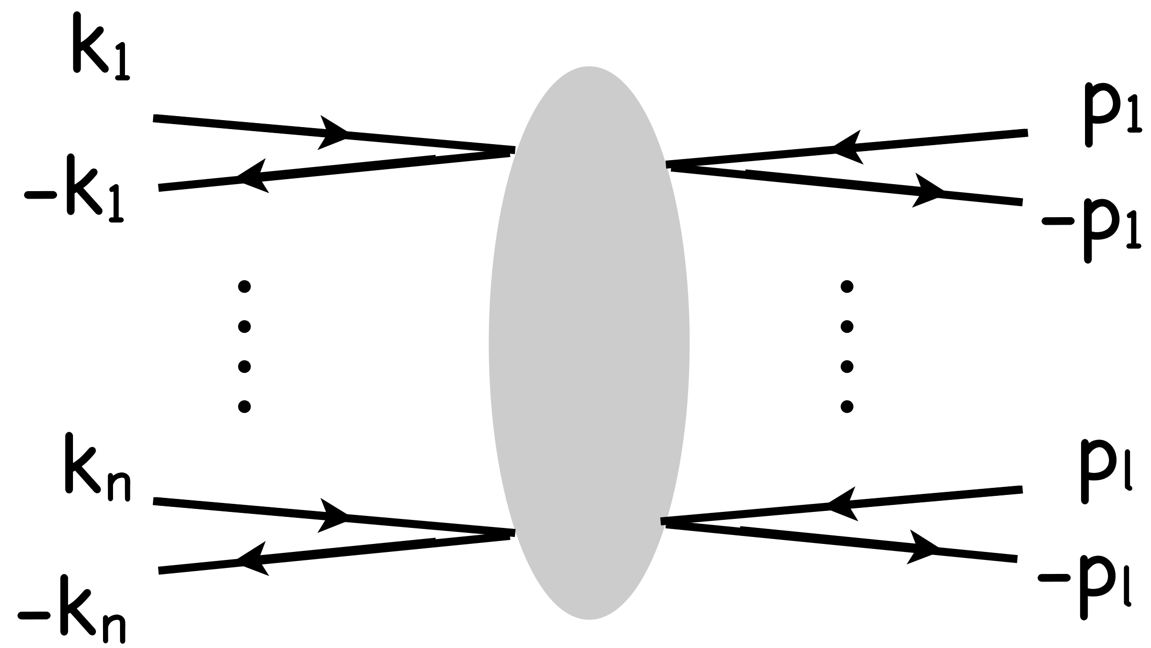
n(q) small ⇔ K(q) uniformly small in q

4. Lehmann representation in terms of new Bogoliubov fermions

$$ {}_{\rm NS}\langle 0|\sigma^z_m(t)|0\rangle_{\rm R} = \sum_{l,n=0}^{\infty} \frac{1}{n!\ l!} \sum_{\substack{k_1,\ldots,k_n \\ p_1,\ldots,p_l}} \left[\prod_{j=1}^{n} K(k_j)\right] \left[\prod_{i=1}^{l} K(p_i)\right] $$

$$ {}_{\rm NS}\langle -k_1, k_1, \ldots, -k_n, k_n|\sigma^z_m(t)|p_1, -p_1, \ldots, p_l, -p_l\rangle_{\rm R} $$

$$ = \sum_{l,n=0}^{\infty} \frac{1}{n!\ l!} $$



1. Dominant contributions from even orders $\mathbf{K^{2n}}$
2. Leading contributions **at order K^{2n}** from terms with n=l and $\{k_1,\ldots,k_n\}\approx\{p_1,\ldots,p_n\}$

"infrared singularities"

sum these to all orders ⇒

$$\frac{\langle 0|\sigma_m^z(t)|0\rangle}{\langle 0|0\rangle} \propto \exp\left(-t\int_0^\pi \frac{dk}{\pi}\left[K^2(k)+\mathcal{O}(K^6)\right]|2\epsilon'(k)|\right)$$

- **Low density expansion** of the full answer.
- Works well everywhere except very close to QCP.
- (dynamical) 2-point functions calculated similarly (but much more complicated).

Conclusions

1. Obtained **detailed analytic results** for the time evolution of (general) observables in the Ising case.

2. Noneq. evolution in integrable models appears to be **special.**

3. As proposed by Rigol et al, the stationary behaviour of **subsystems** is given by a GGE for the Ising chain.

4. The GGE gives both **static** and **dynamic** correlators at stationarity.

5. New methods can be applied to integrable QFTs (Ising field theory, sine-Gordon model)

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6. What happens for more general initial states (e.g. break translation invariance) ? ⇒ Ising chain.