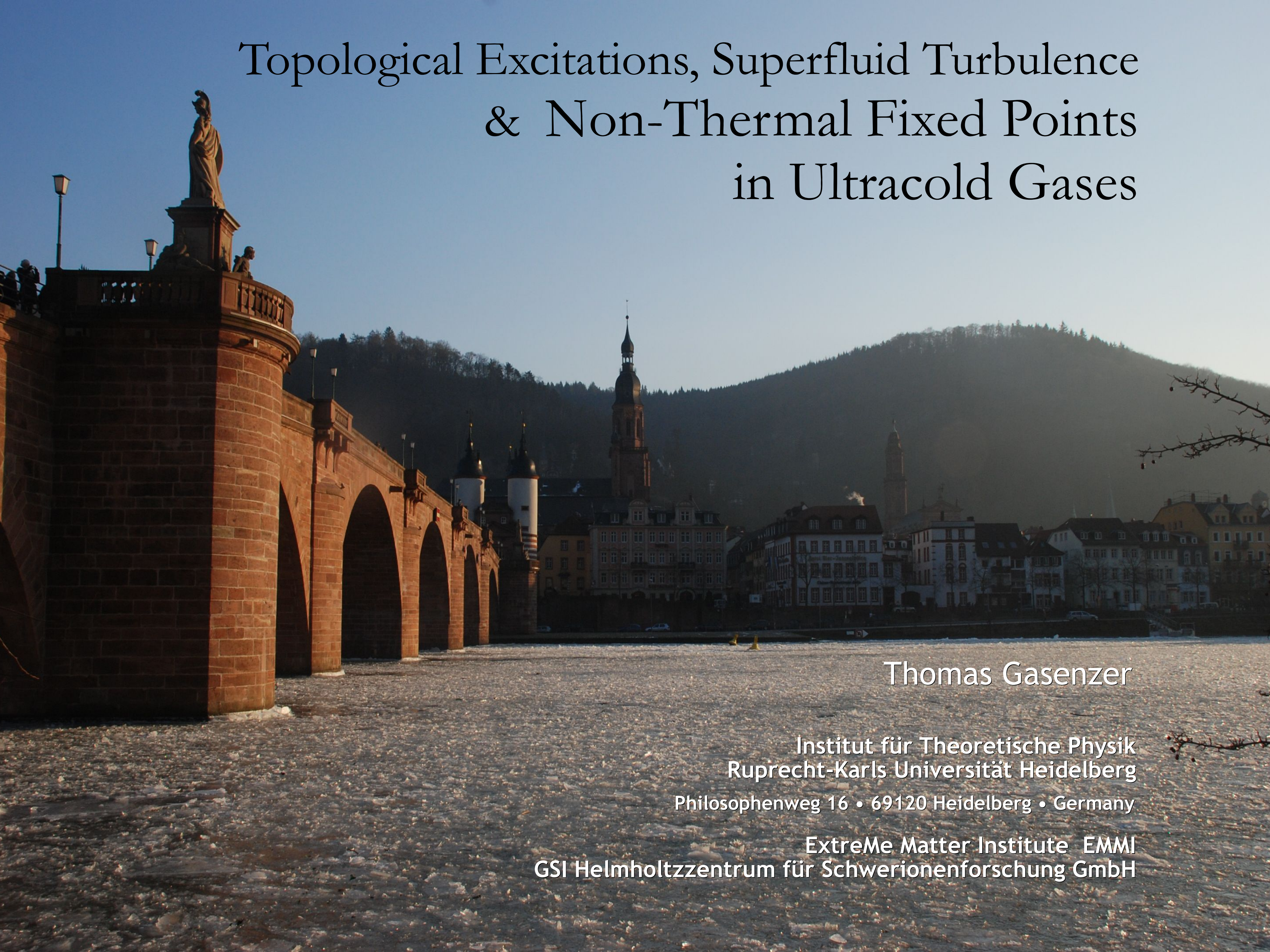
Topological Excitations, Superfluid Turbulence & Non-Thermal Fixed Points in Ultracold Gases

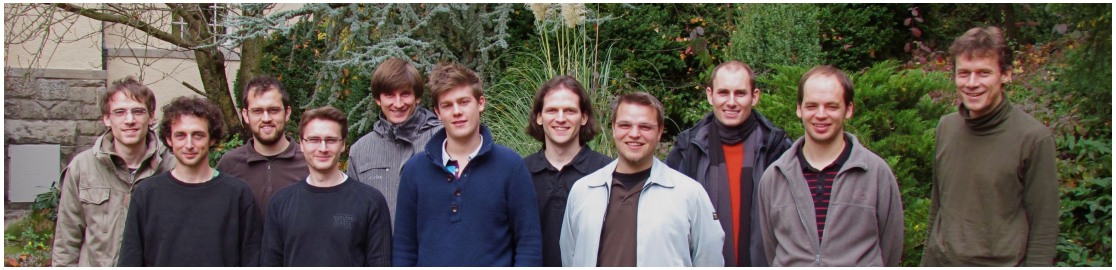


Thomas Gasenzer

Institut für Theoretische Physik
Ruprecht-Karls Universität Heidelberg
Philosophenweg 16 • 69120 Heidelberg • Germany

ExtreMe Matter Institute EMMI
GSI Helmholtzzentrum für Schwerionenforschung GmbH

Thanks & credits to...



...my work group in Heidelberg:

Sebastian Bock
Sebastian Erne
Martin Gärttner
Roman Hennig
Markus Karl
Steven Mathey
Boris Nowak
Nikolai Philipp
Dénes Sexty
Martin Trappe
Pascal Weckesser

...my former students:

Jan Schole (→ Heidelberg), Jan Zill (→ Queensland), Maximilian Schmidt (→ Jülich), Cédric Bodet (→ NEC), Alexander Branschädel (→ KIT Karlsruhe), Stefan Keßler (→ U Erlangen), Matthias Kronenwett (→ R. Berger), Christian Scheppach (→ Cambridge, UK), Philipp Struck (→ Konstanz), Kristan Temme (→ Vienna)

€€€...

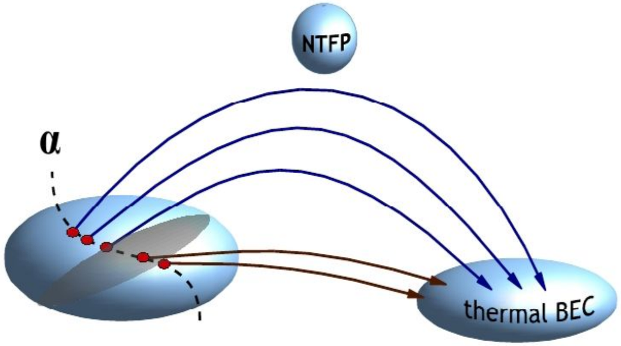
Alexander von Humboldt Stiftung / Foundation
DFG Heisenberg-Programm Deutsche Forschungsgemeinschaft
Ruprecht-Karls-Universität Heidelberg
LGFG BaWue
DAAD Deutscher Akademischer Austausch Dienst German Academic Exchange Service
Center for Quantum Dynamics
EMMI
Helmholtz Gemeinschaft

Equilibration

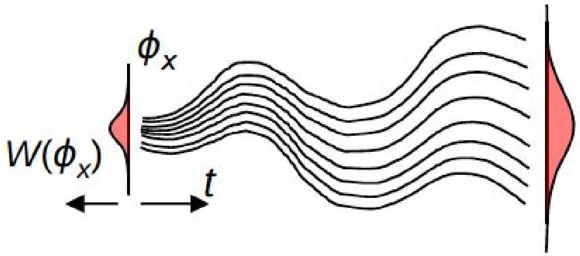
Initial state:
Far from equilibrium

Transient state,
e.g. Turbulence
Non-thermal fixed point

Final state:
Thermal equilibrium



Semi-classical Simulations



Classical field equation for ϕ(x,t):

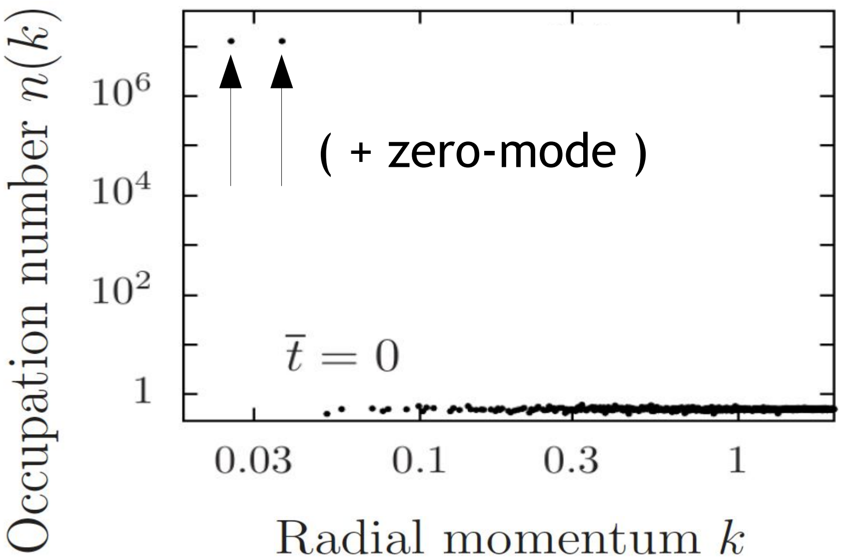
$$\mathrm{i}\partial_t \phi(\mathbf{x},t) = \left[-\frac{\nabla^2}{2m} + g|\phi(\mathbf{x},t)|^2\right]\phi(\mathbf{x},t)$$

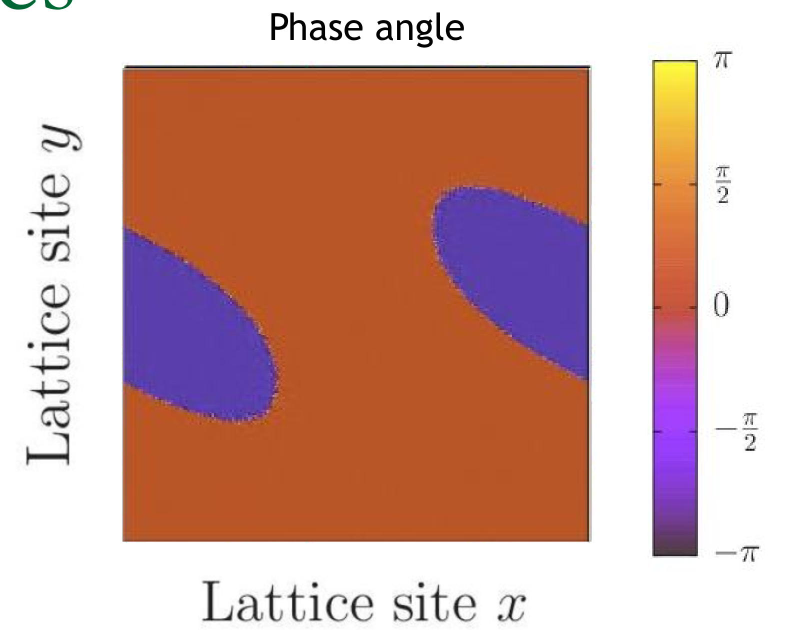
Observables: e. g. Momentum distribution

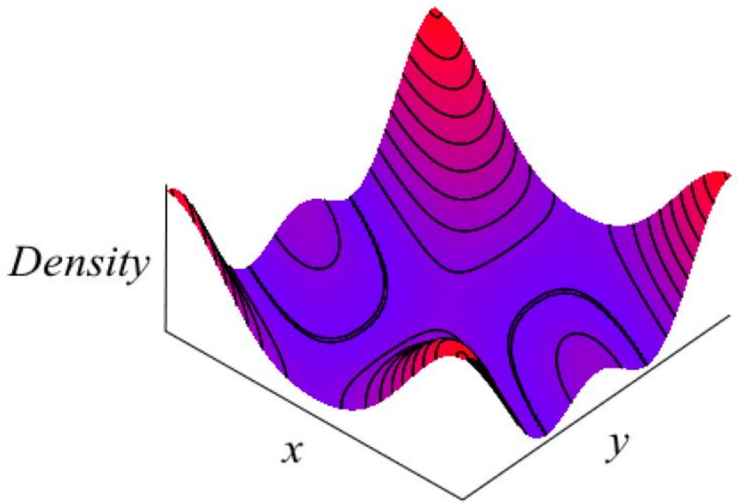
$$n(k) = \int \mathrm{d}^{d-1}\Omega_k \, \langle \phi^*(\mathbf{k})\phi(\mathbf{k})\rangle_{\mathrm{ensemble}}$$

2+1 D: Quench dynamics

B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)







2+1 D: Quench dynamics

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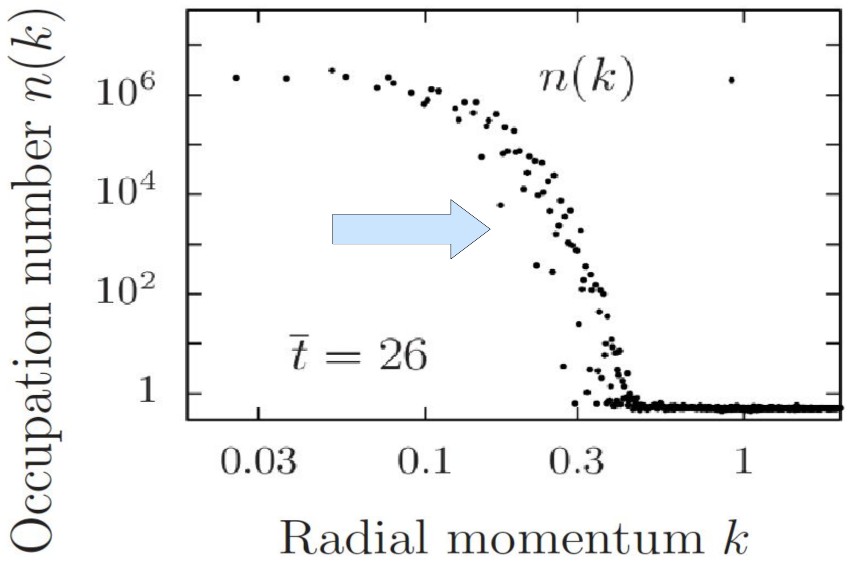

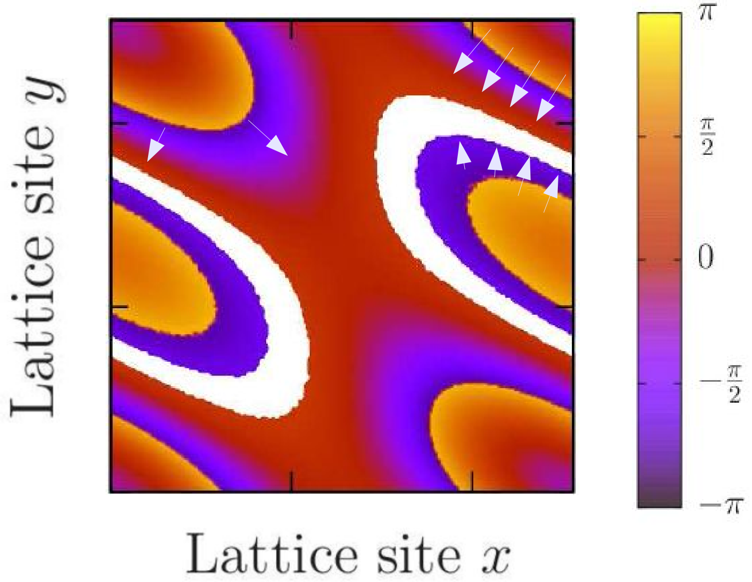

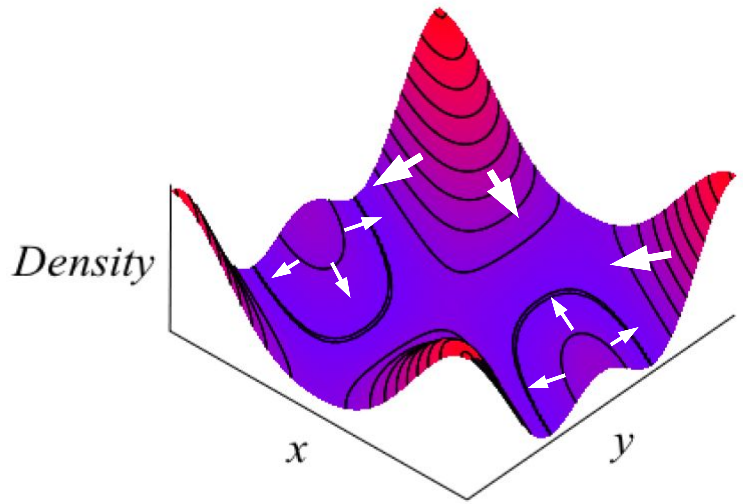

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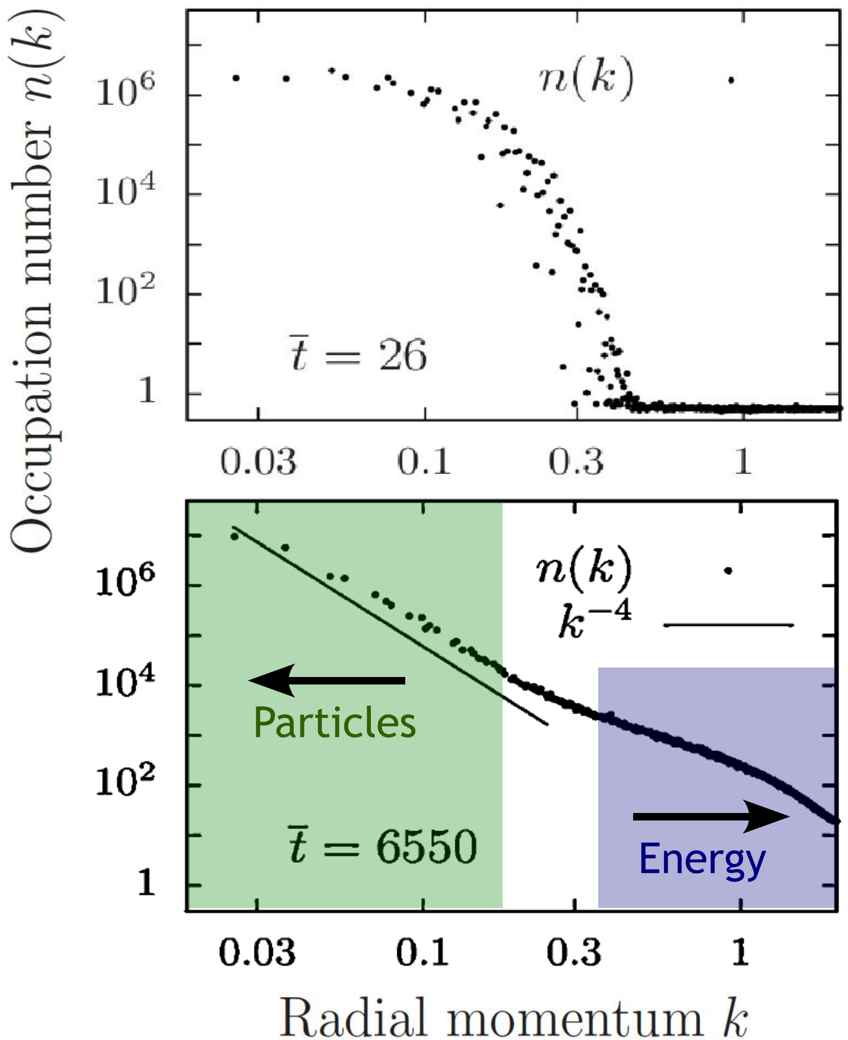

2+1 D: Quench dynamics

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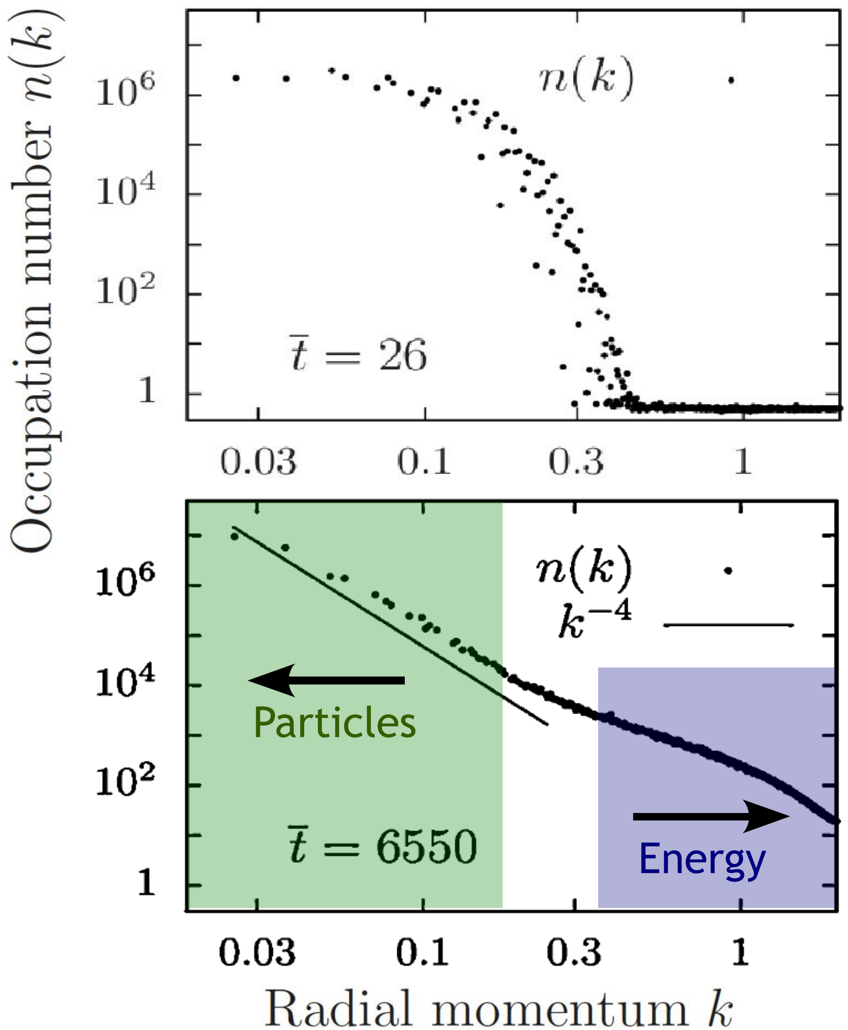
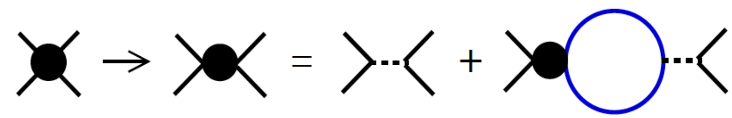
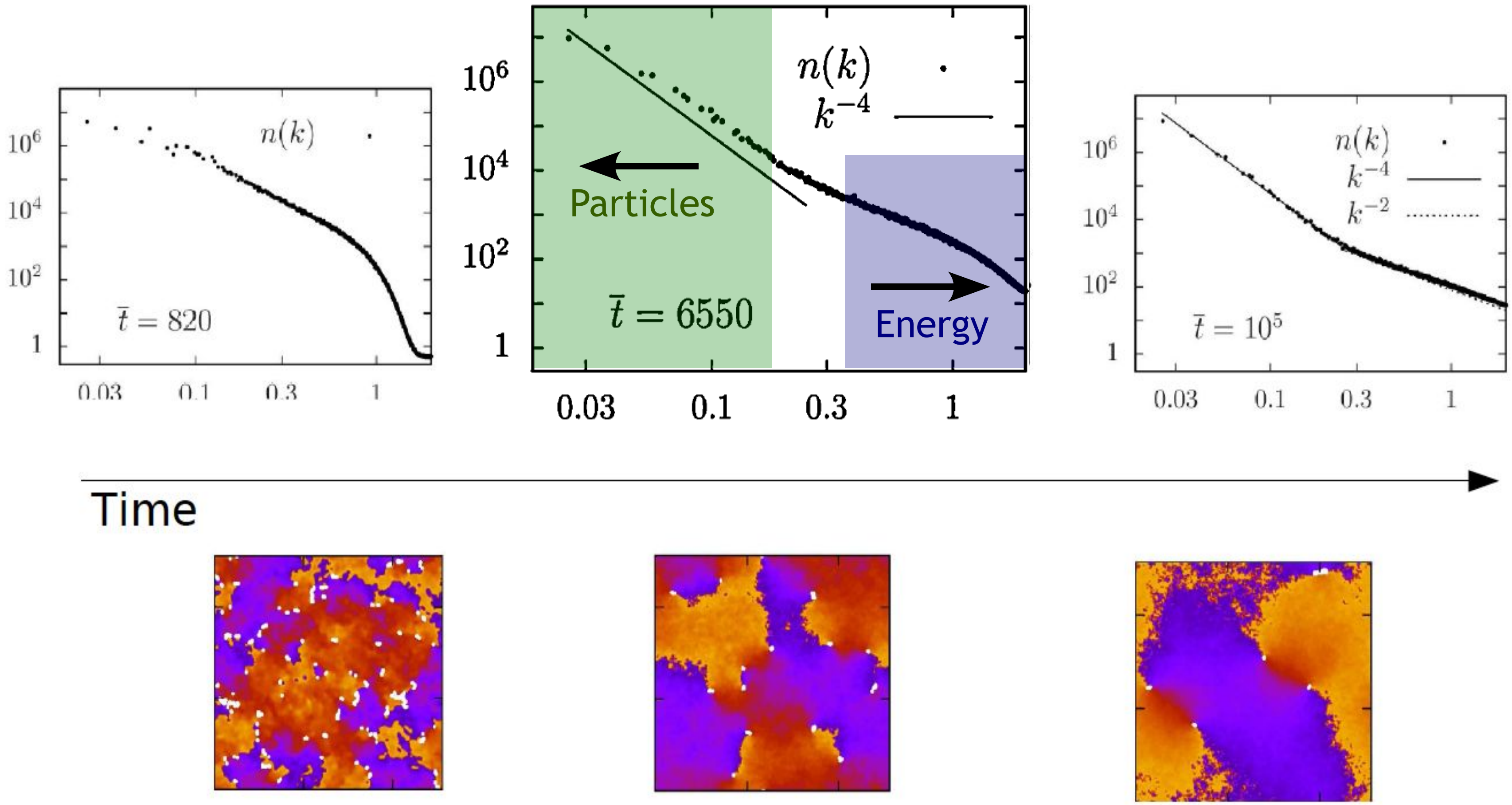




J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603,
J. Berges, G. Hoffmeister, NPB **813** (09) 383,
C. Scheppach, J. Berges, TG PRA **81** (10) 033611,

$$n^{\mathrm{IR}} \sim k^{-d-2}$$

2+1 D: Phase ordering dynamics



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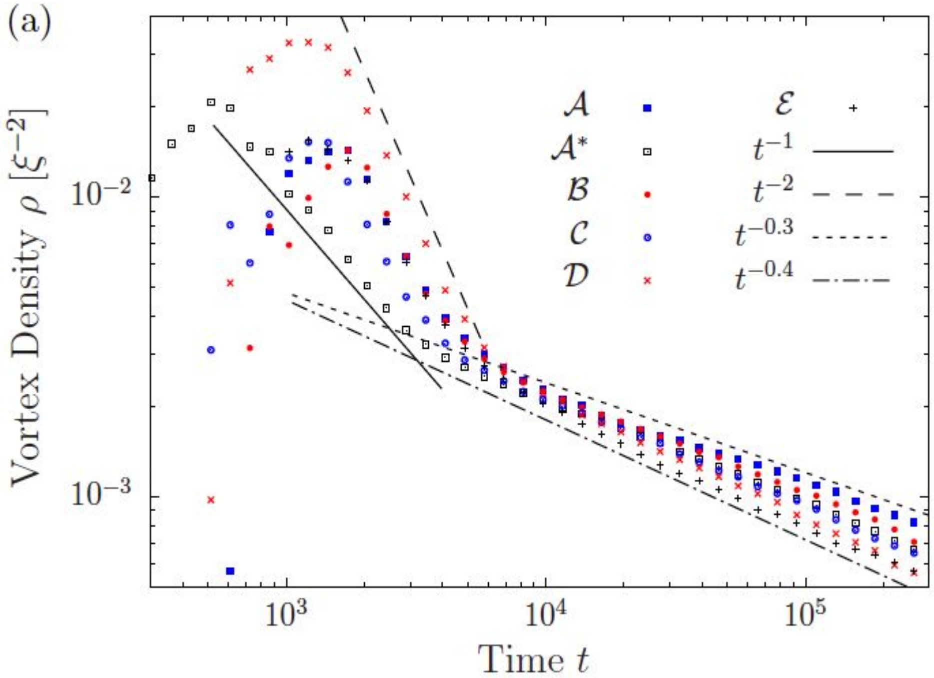
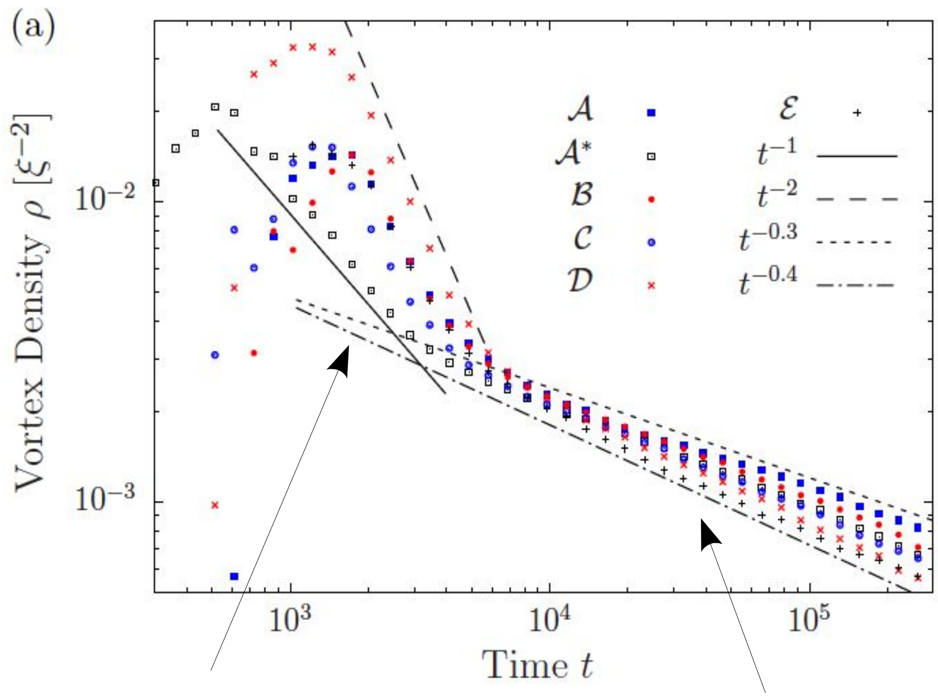
2+1 D: Phase ordering dynamics

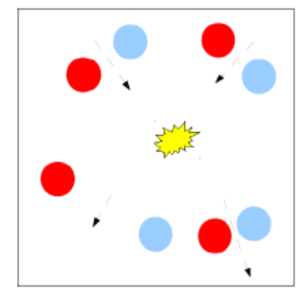


J. Schole, B. Nowak, TG, PRA **86** (12) 013624

2+1 D: Phase ordering dynamics



Scaling needs vortex unbinding



Non-universal decay law
(depends on initial vortex distribution)
Kinetic gas theory for dipoles

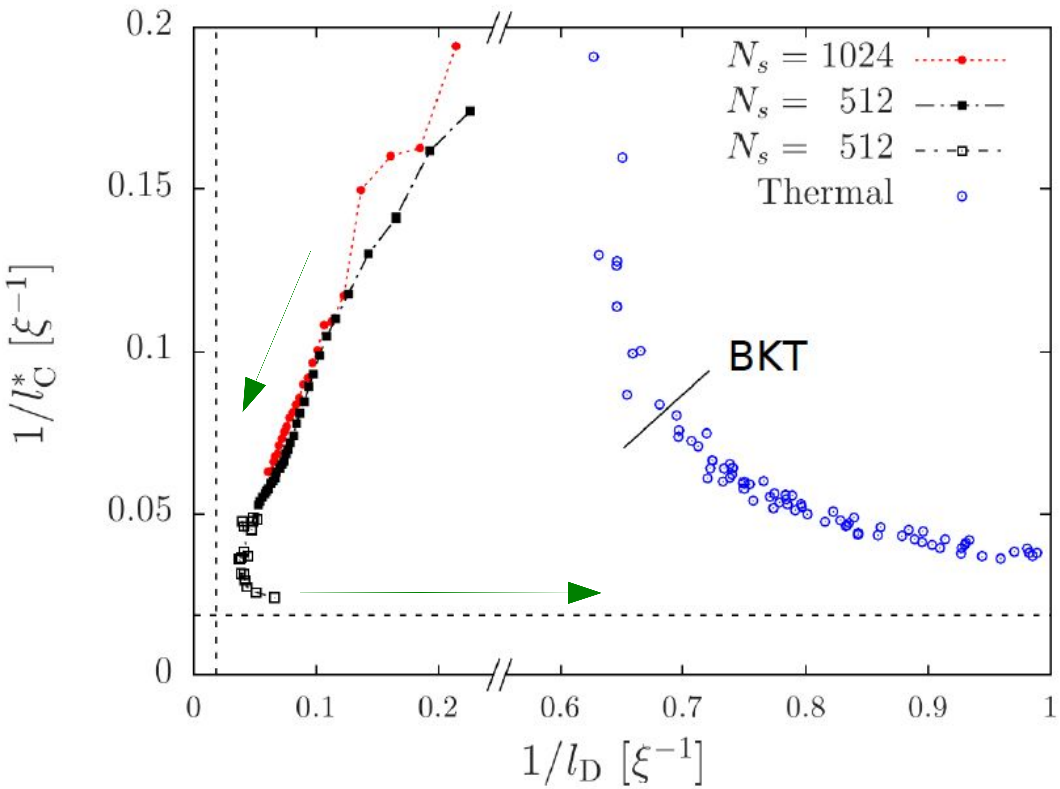
Universal decay regime
Strongly correlated dilute vortex gas
Scaling $n(k) \sim k^{-4}$

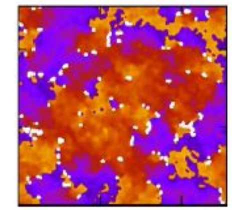
J. Schole, B. Nowak, TG, PRA **86** (12) 013624

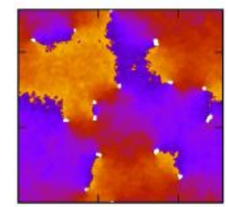
Approach of the NTFP

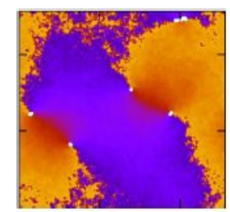
$l_C^* =$ Phase coherence length
$l_D =$ Vortex-Antivortex pair distance

more phase coherence









larger Vortex-Antivortex distances

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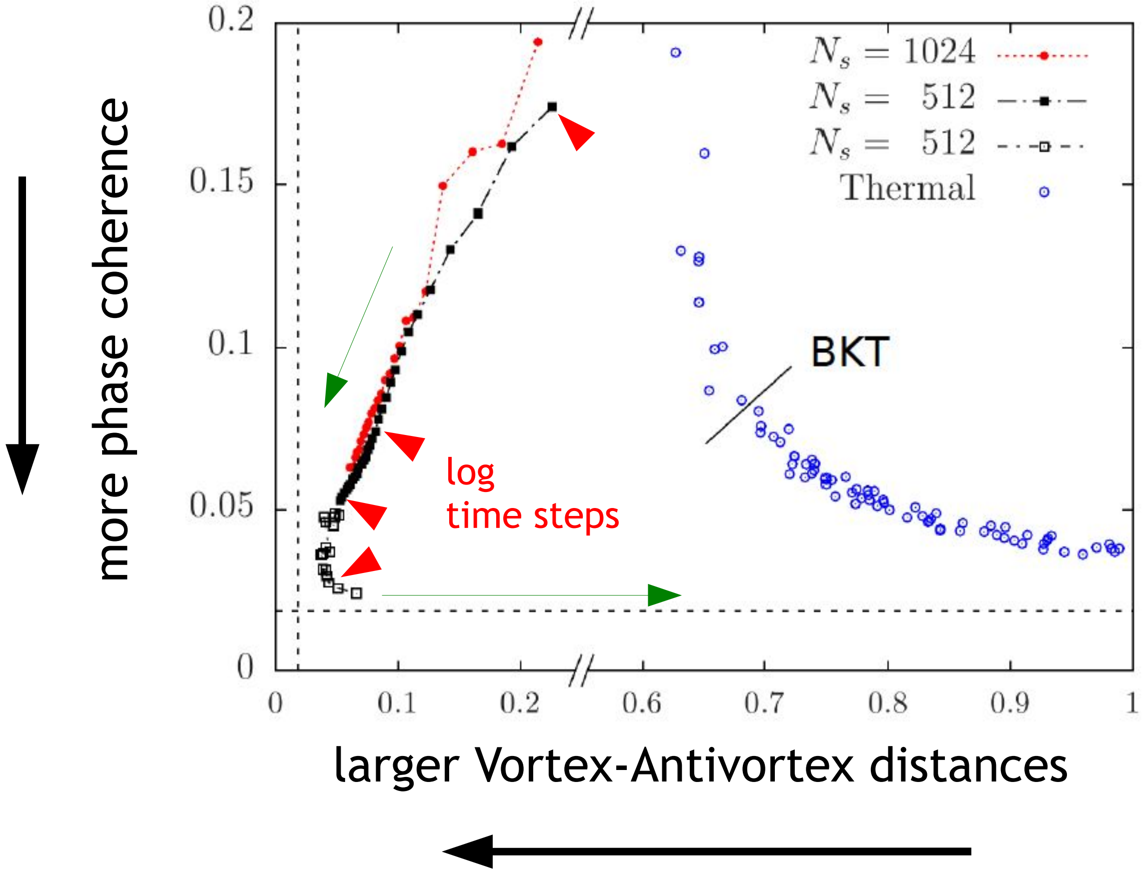
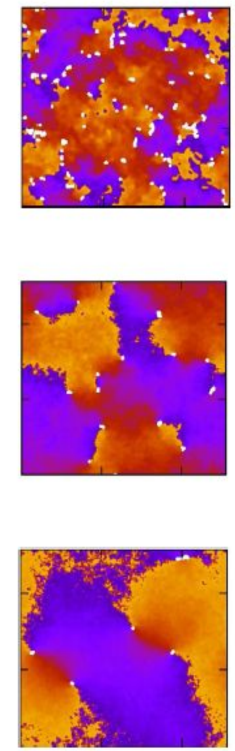
Approach of the NTFP





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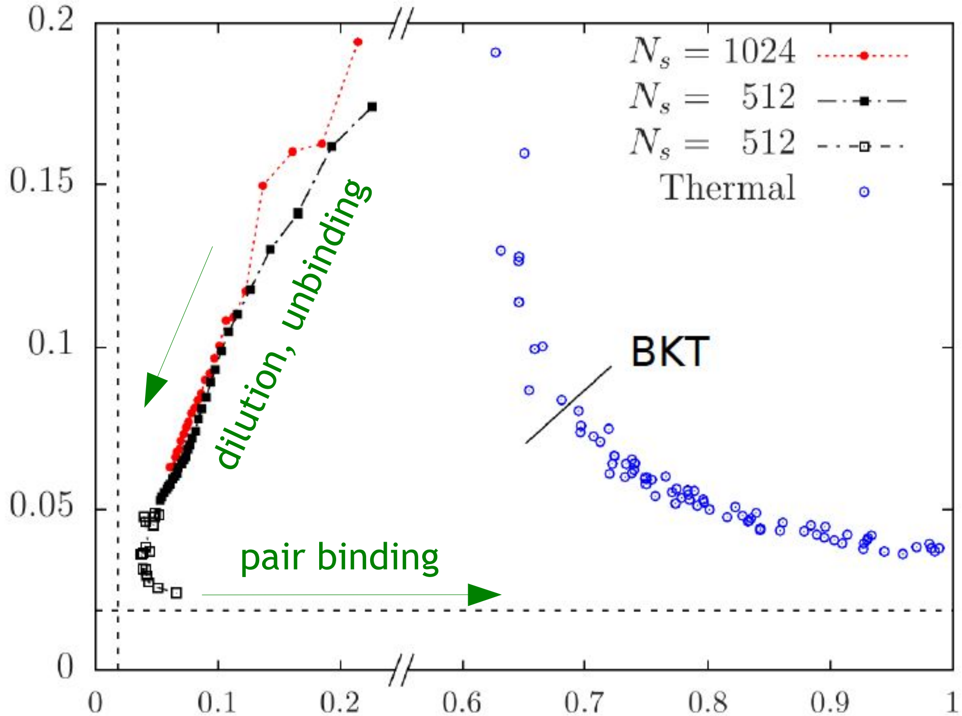
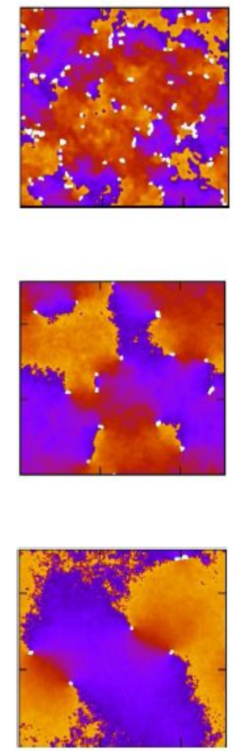
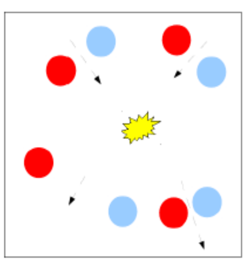
Approach of the NTFP

more phase coherence



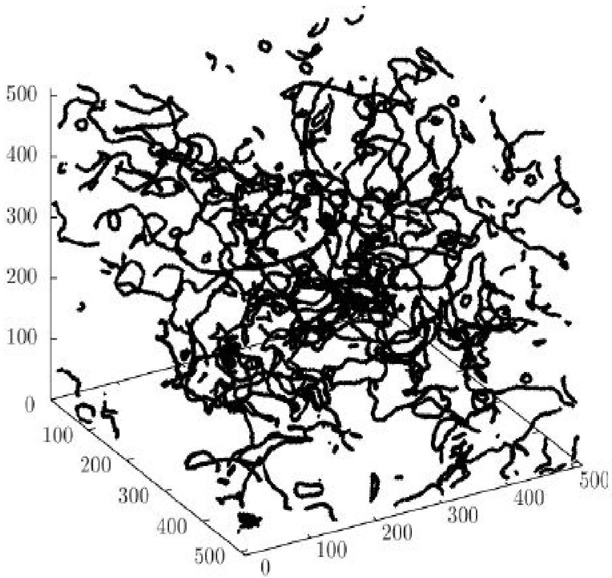


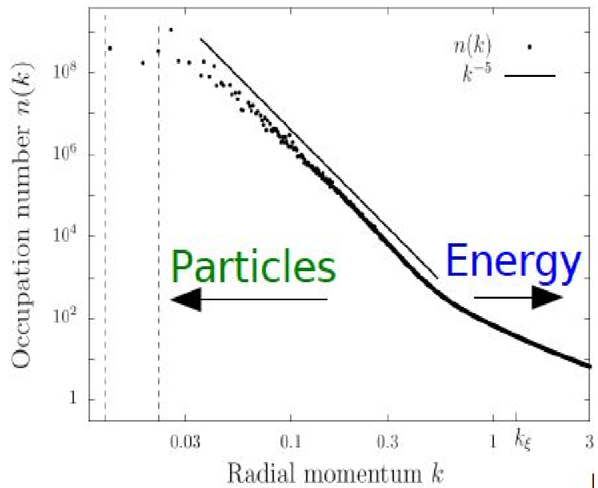
larger Vortex-Antivortex distances

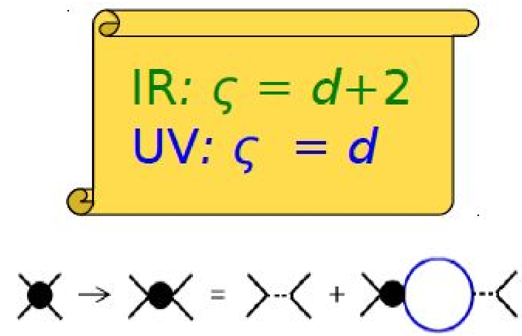


J. Schole, B. Nowak, TG, PRA **86** (12) 013624

3D Nonthermal Fixed Point







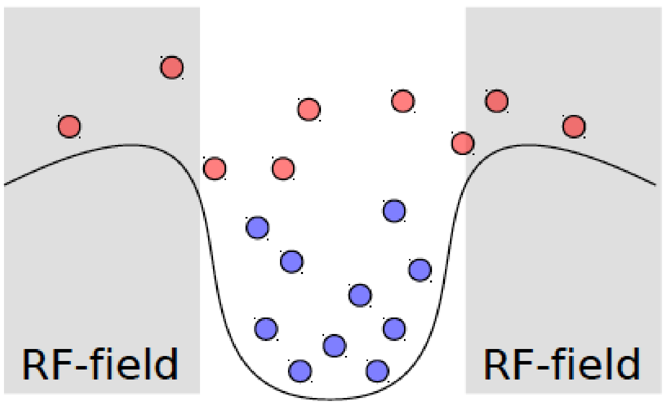
Vortices ← Spectrum → QFT

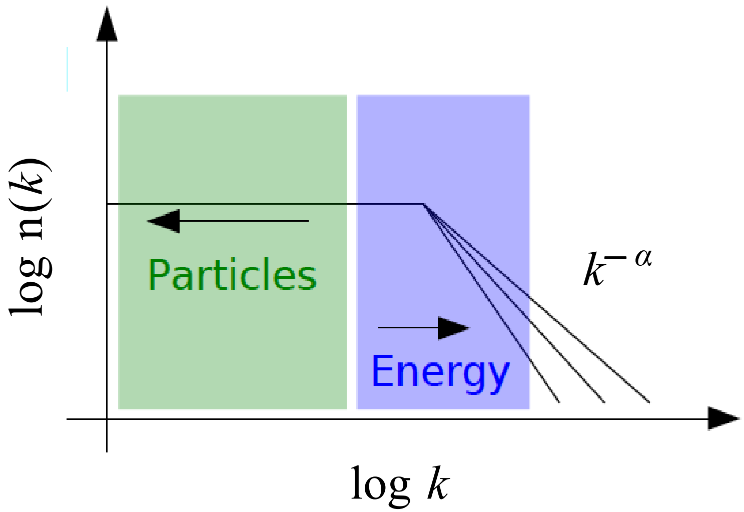
Quantum Turbulence

Strong & Weak Wave Turbulence

B. Nowak, D. Sexty, TG, PRB **84**(R) (11); B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)

3D: Bose Condensation





Experiments

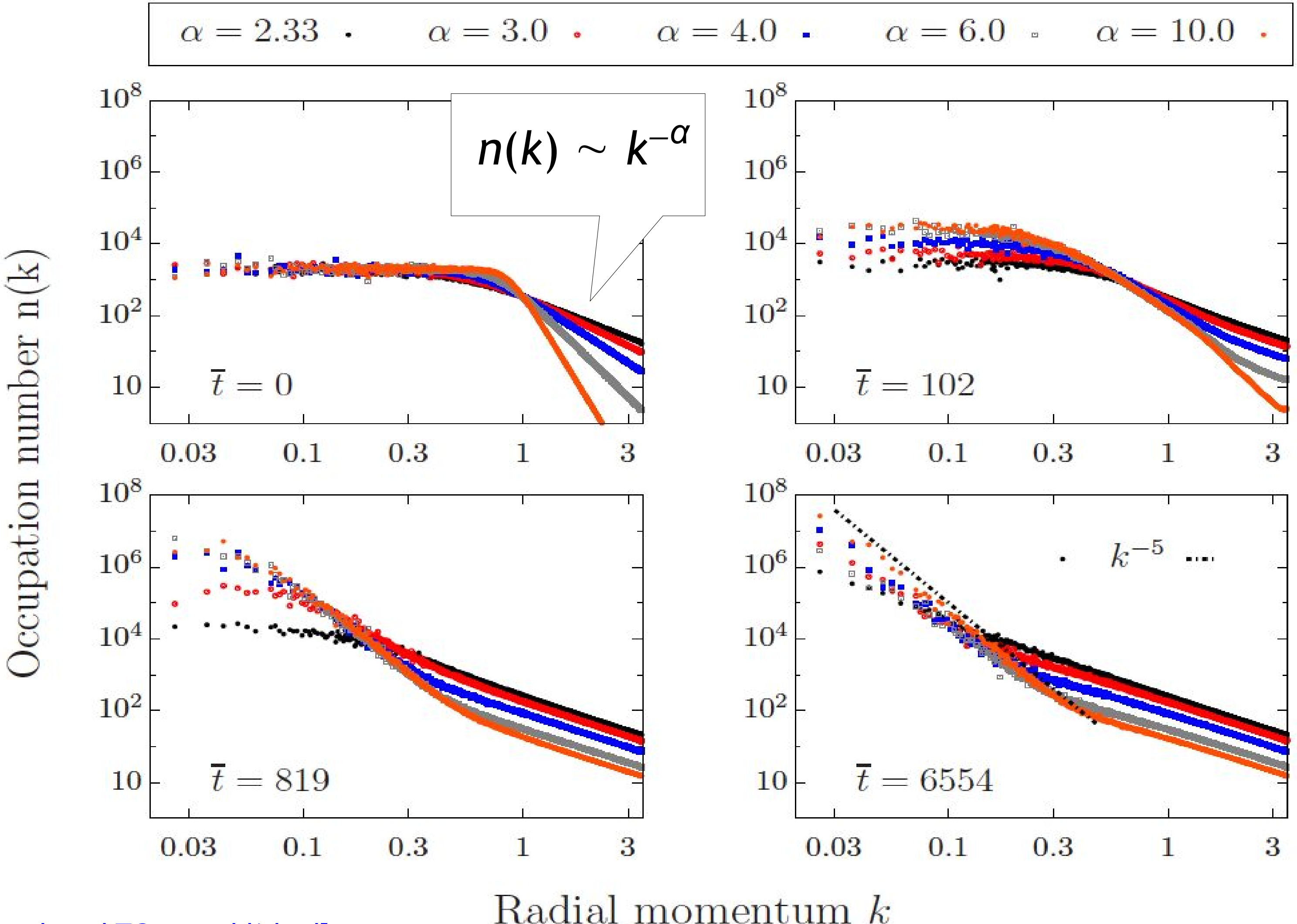
Hänsch, Esslinger (02)
Esslinger (07)
Hadzibabic (12)

Condensation Dynamics

Levich, Yakhot (70s);
Snoke, Wolfe (89);
Kagan, Svistunov, Shlyapnikov (91-94);
Damle, Sachdev (96)
Semikoz, Tkachev (95)
Berloff, Svistunov (02)
Anderson, Davis (08)
Blaizot, McLerran (12)
Berges, Sexty (12)

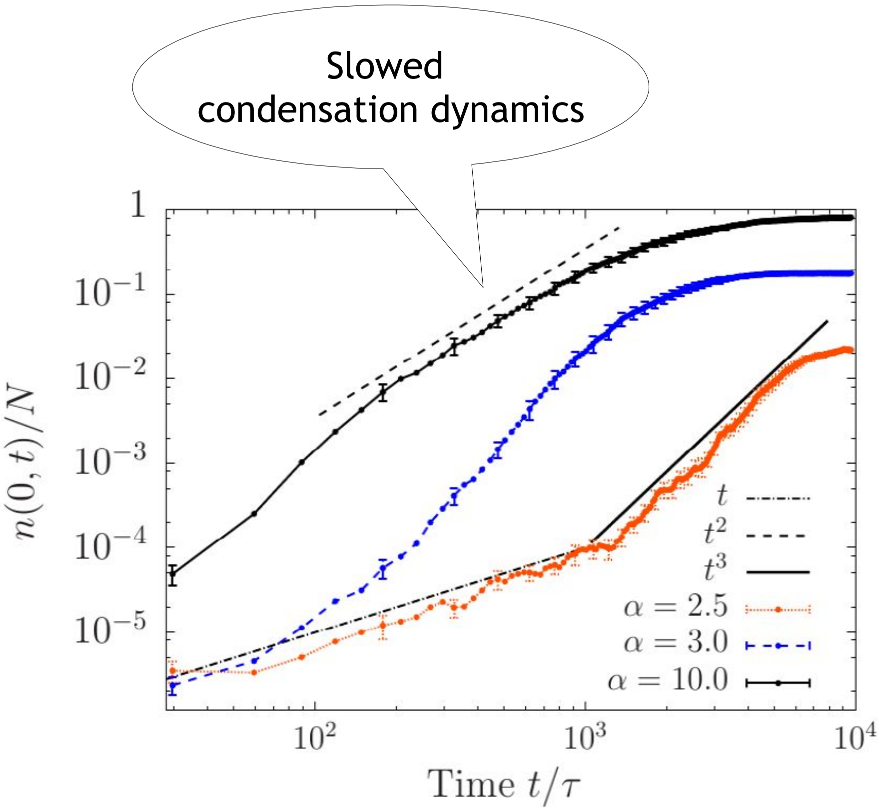
B. Nowak, TG, arXiv: 1206.3181 [cond-mat.quant-gas]

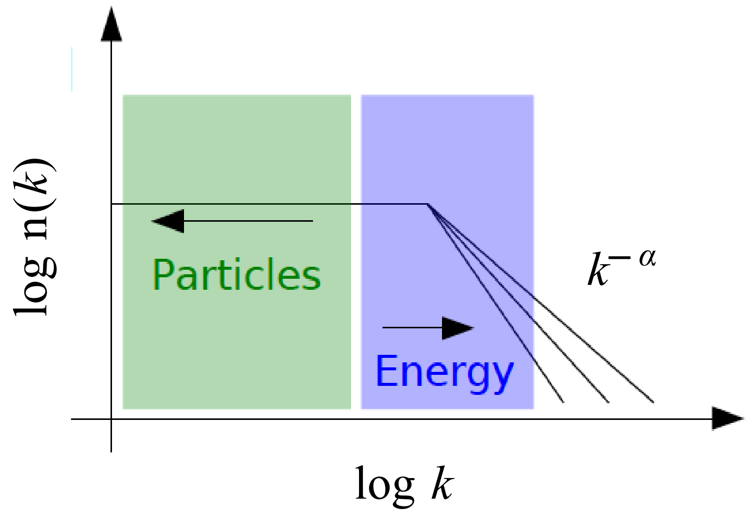
Hydrodynamic vs. kinetic Condensation



[B. Nowak and TG, unpublished]

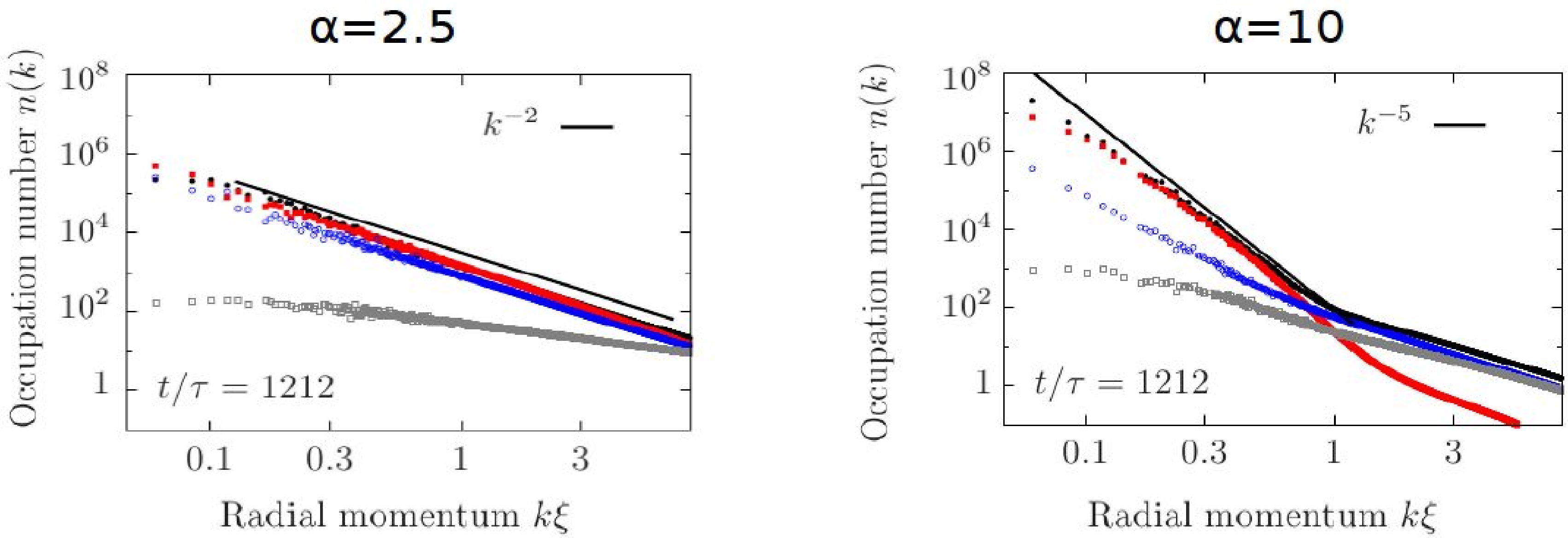
3D: Bose Condensation





B. Nowak, TG, arXiv: 1206.3181 [cond-mat.quant-gas]

3D: Bose Condensation



$n^i(k)$

solenoidal flow

$n^c(k)$

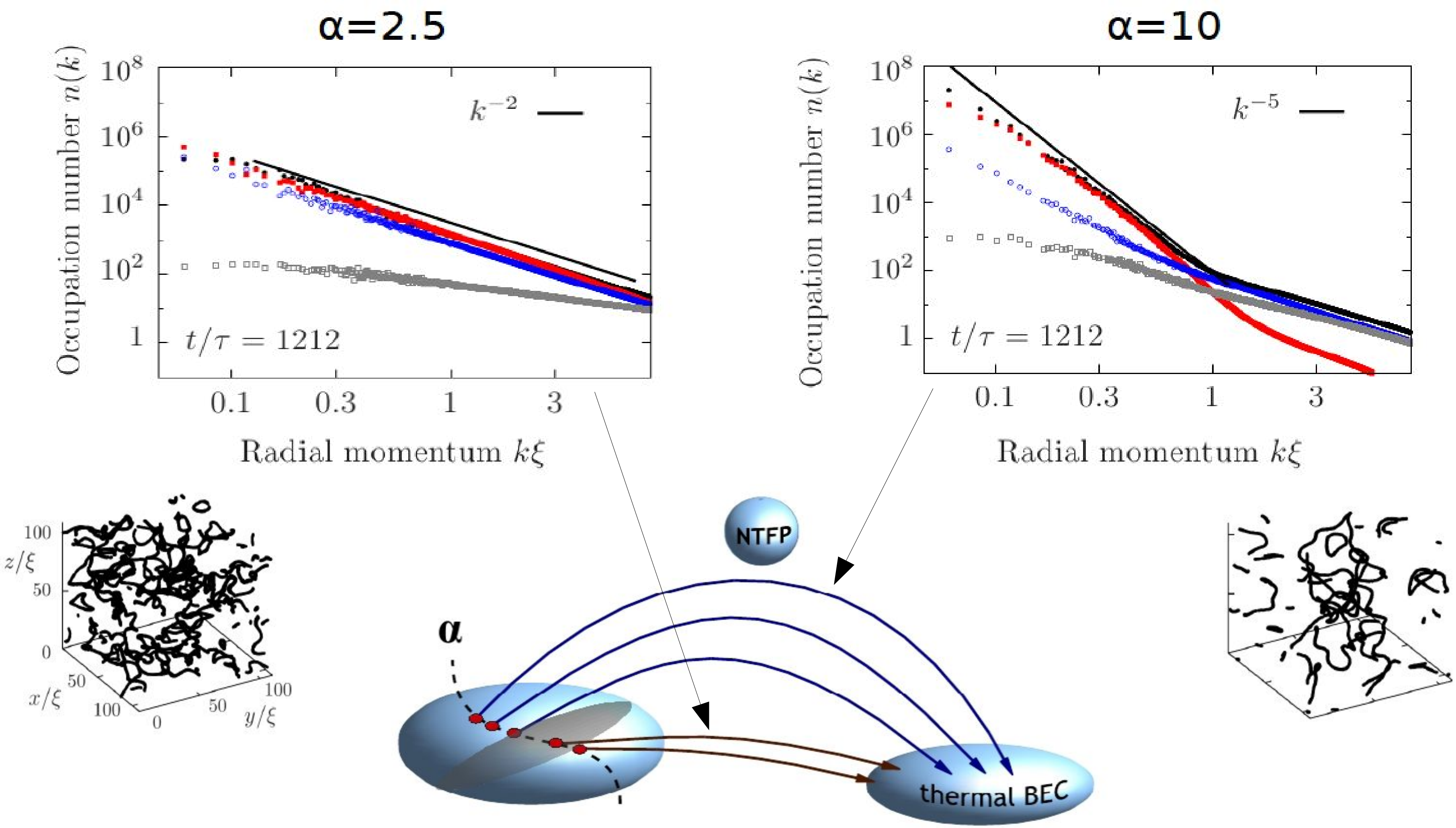
compressible component

$n^q(k)$

q pressure

B. Nowak, TG, arXiv: 1206.3181 [cond-mat.quant-gas]

3D: Bose Condensation



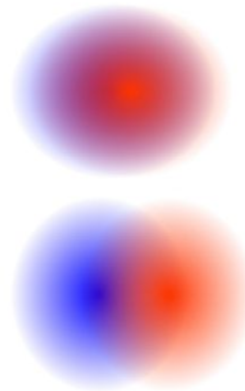
B. Nowak, TG, arXiv: 1206.3181 [cond-mat.quant-gas]

2-component BEC

Bose gas with internal two-level structure

miscible
$g_{12} < g$

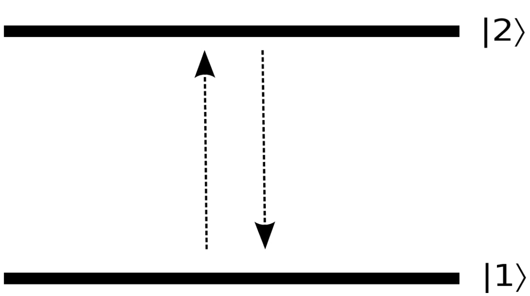
immiscible
$g_{12} > g$



non-linear interactions:

$$\hat{H}_{int} \sim \frac{g}{2}(\hat{n}_1^2 + \hat{n}_2^2) \quad |1\rangle \longleftrightarrow |1\rangle, \quad |2\rangle \longleftrightarrow |2\rangle$$

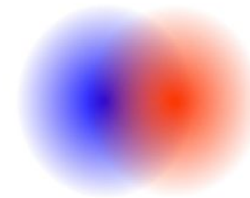
$$+ \; g_{12}\hat{n}_1\hat{n}_2 \quad |1\rangle \longleftrightarrow |2\rangle$$

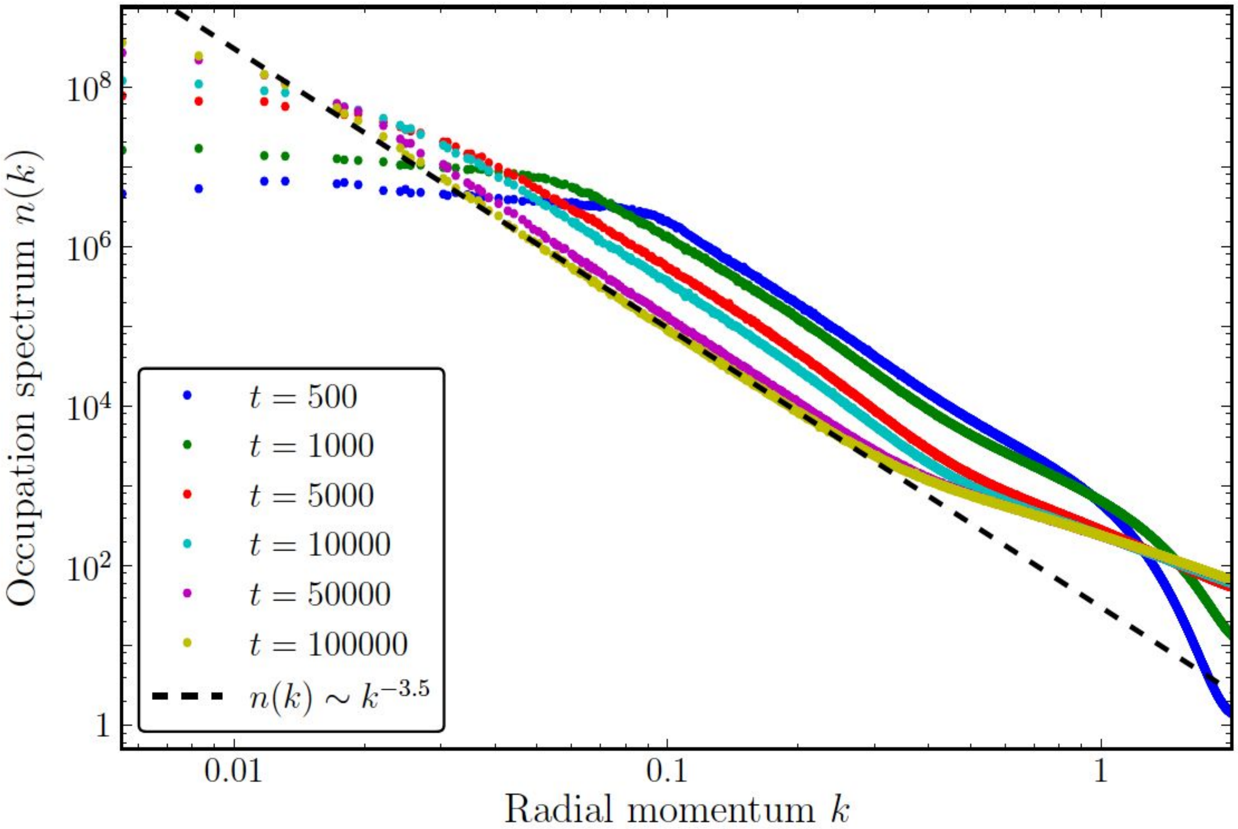


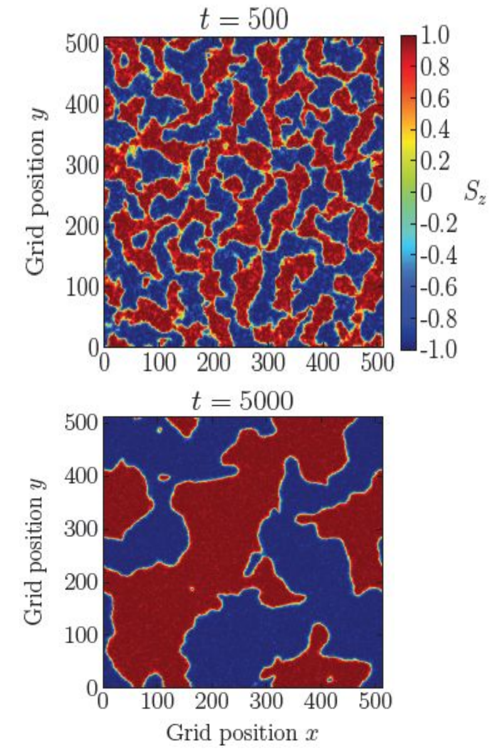
M. Karl, B. Nowak, TG, unpublished (12)

2-component BEC

immiscible
$g_{12} > g$







M. Karl, B. Nowak, TG, unpublished (12)

Decomposition of Energy for Spin system

$$E_{\rm kin} = \frac{1}{2}\int \mathrm{d}^2x\, \nabla\phi_j \nabla\phi_j$$
$$= \frac{1}{2}\int \mathrm{d}^2x \left[|\nabla\sqrt{\rho_T}|^2 + \frac{\rho_T}{4}\nabla S^a \nabla S^a + |\mathbf{w}|^2\right]$$

$$S^a = \phi_j \sigma^a_{ij} \phi_i$$

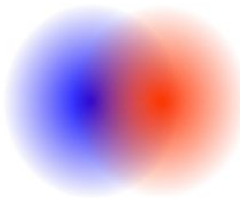
$$\rho_1 + \rho_2 \equiv \rho_T$$
$$\Theta_T = \varphi_1 + \varphi_2$$

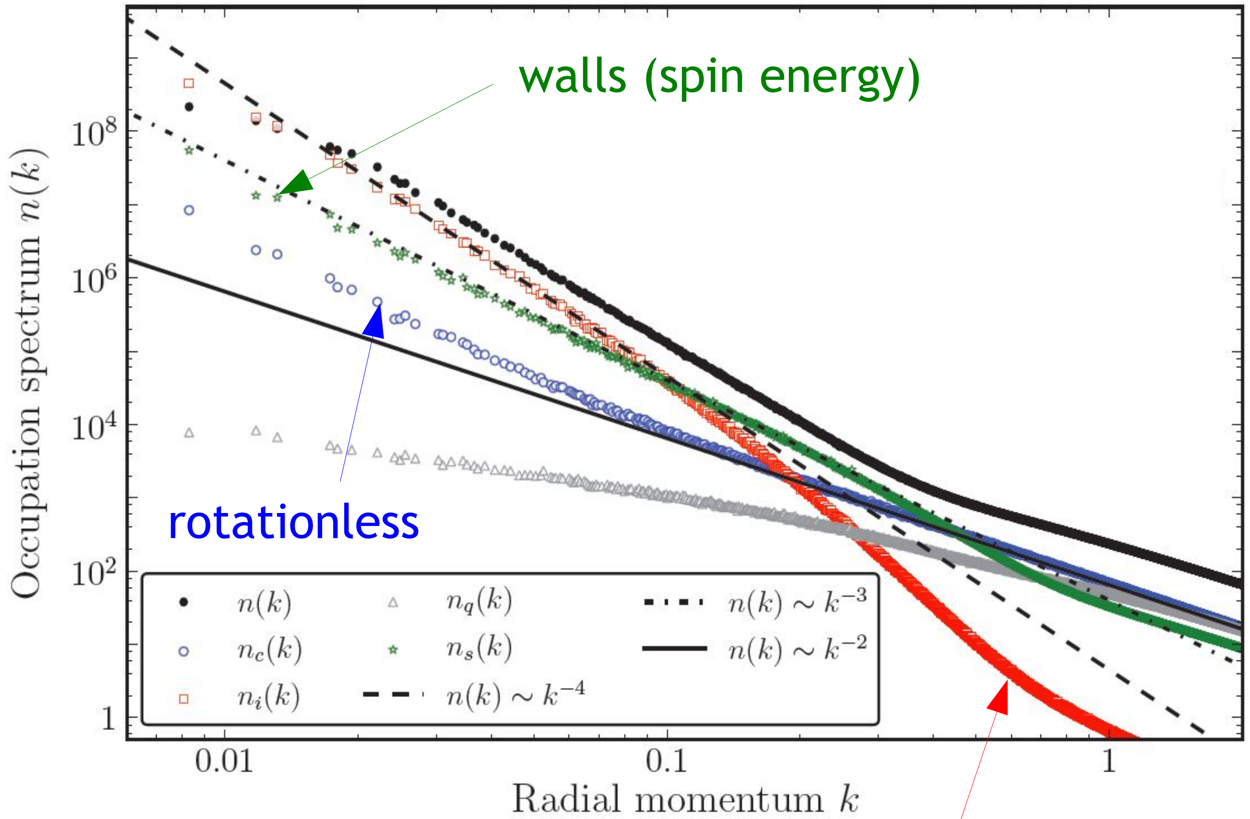
$$S^a \rightarrow \rho_T S^a$$

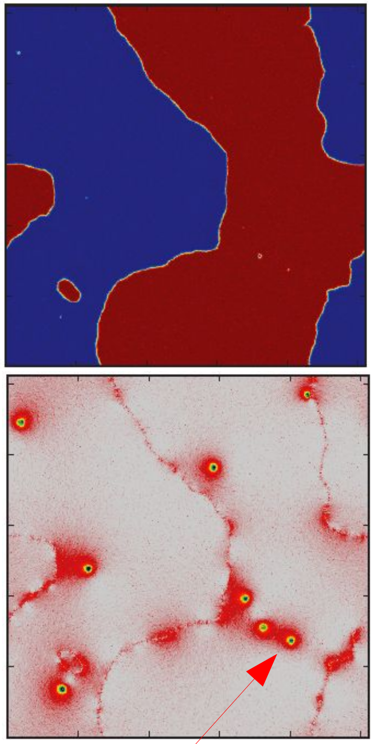
$$\mathbf{j}_T = \frac{1}{2}\rho_T \nabla\Theta_T + \frac{S^z \rho_T}{2\left[(S^x)^2 + (S^y)^2\right]}(S^y \nabla S^x - S^x \nabla S^y) = \rho_1 \nabla\varphi_1 + \rho_2 \nabla\varphi_2$$

2-component BEC

immiscible
$g_{12} > g$



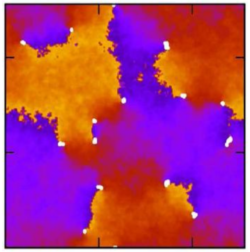


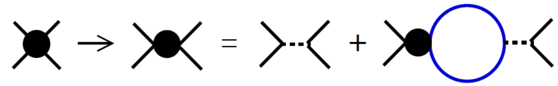


solenoidal ⇔ Skyrmions

M. Karl, B. Nowak, TG, unpublished (12)

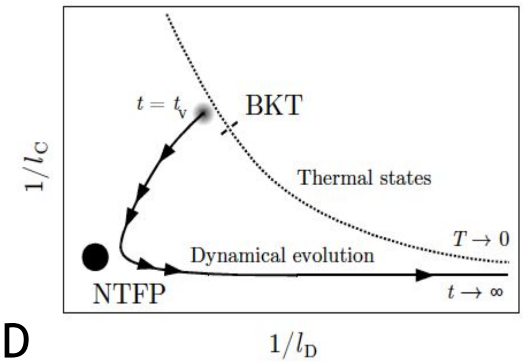
Summary



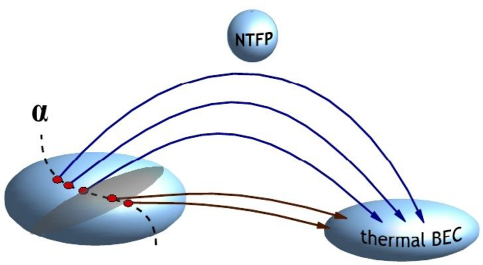


Non-Thermal Fixed Points

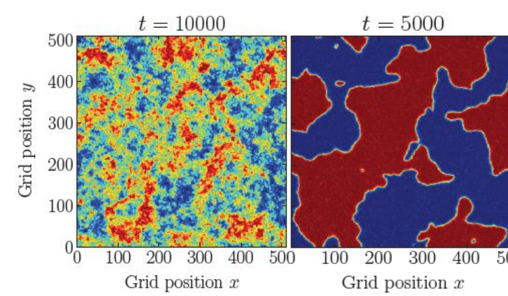
Superfluid Turbulence in 2D



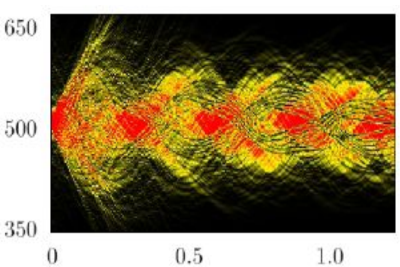
Approach of the NTFP in 2D



Dynamics of BE condensation



Charge separation/Pattern formation



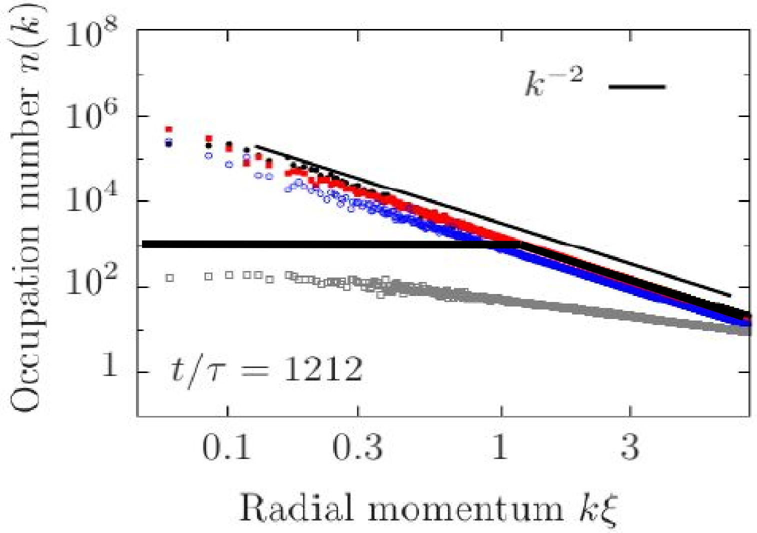
1D Soliton Gas as an NTFP

Supplementary slides

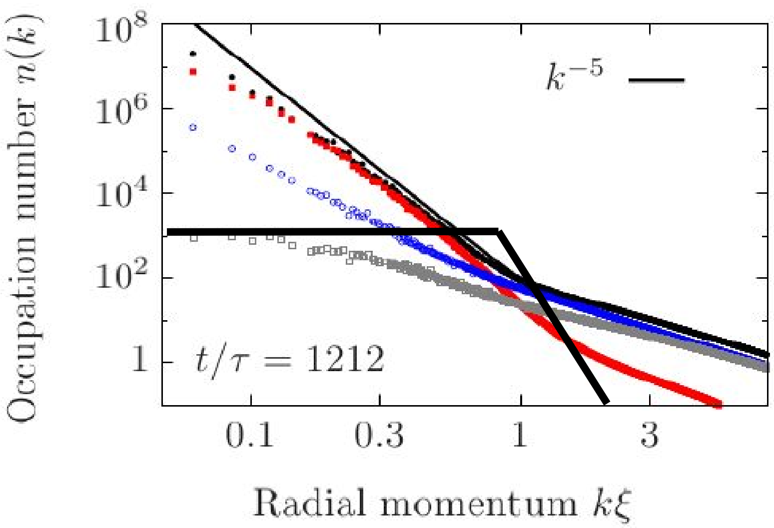
Bose-Einstein Condensation

3D: Bose Condensation

α=2.5



α=10



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solenoidal flow

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compressible component

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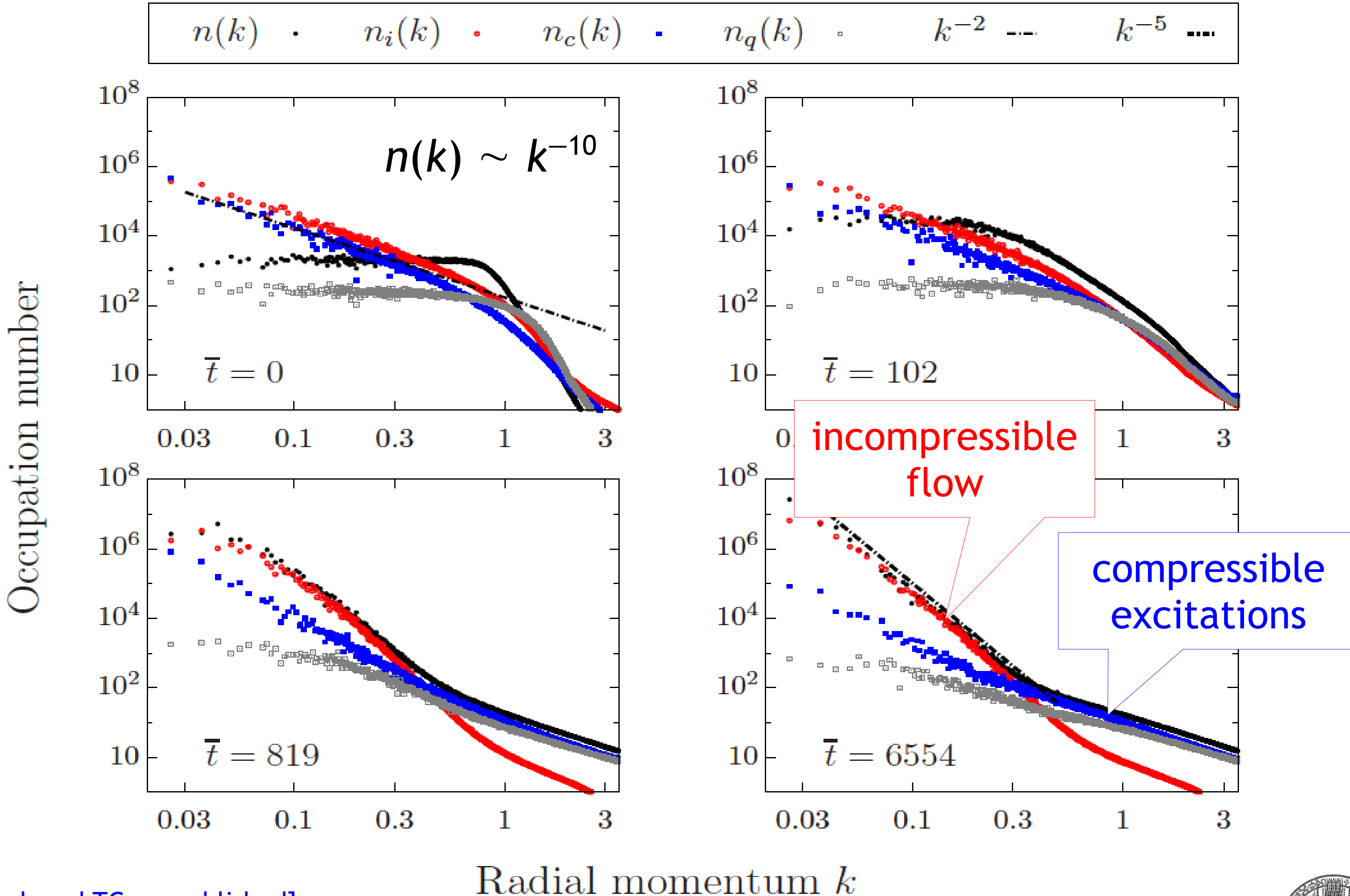
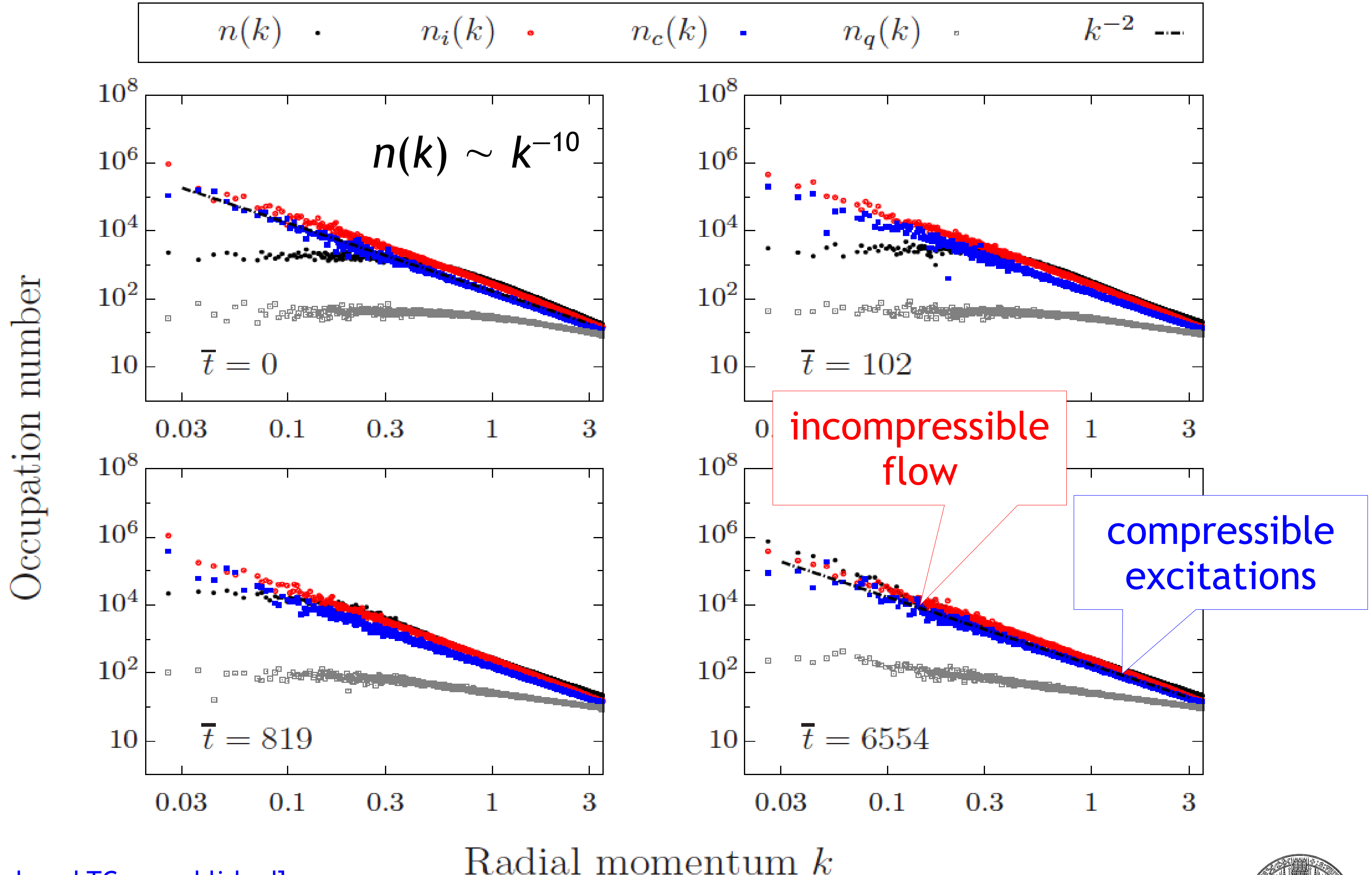
B. Nowak, TG, arXiv: 1206.3181 [cond-mat.quant-gas]

Hydrodynamic vs. kinetic Condensation



[B. Nowak and TG, unpublished]

Hydrodynamic vs. kinetic Condensation



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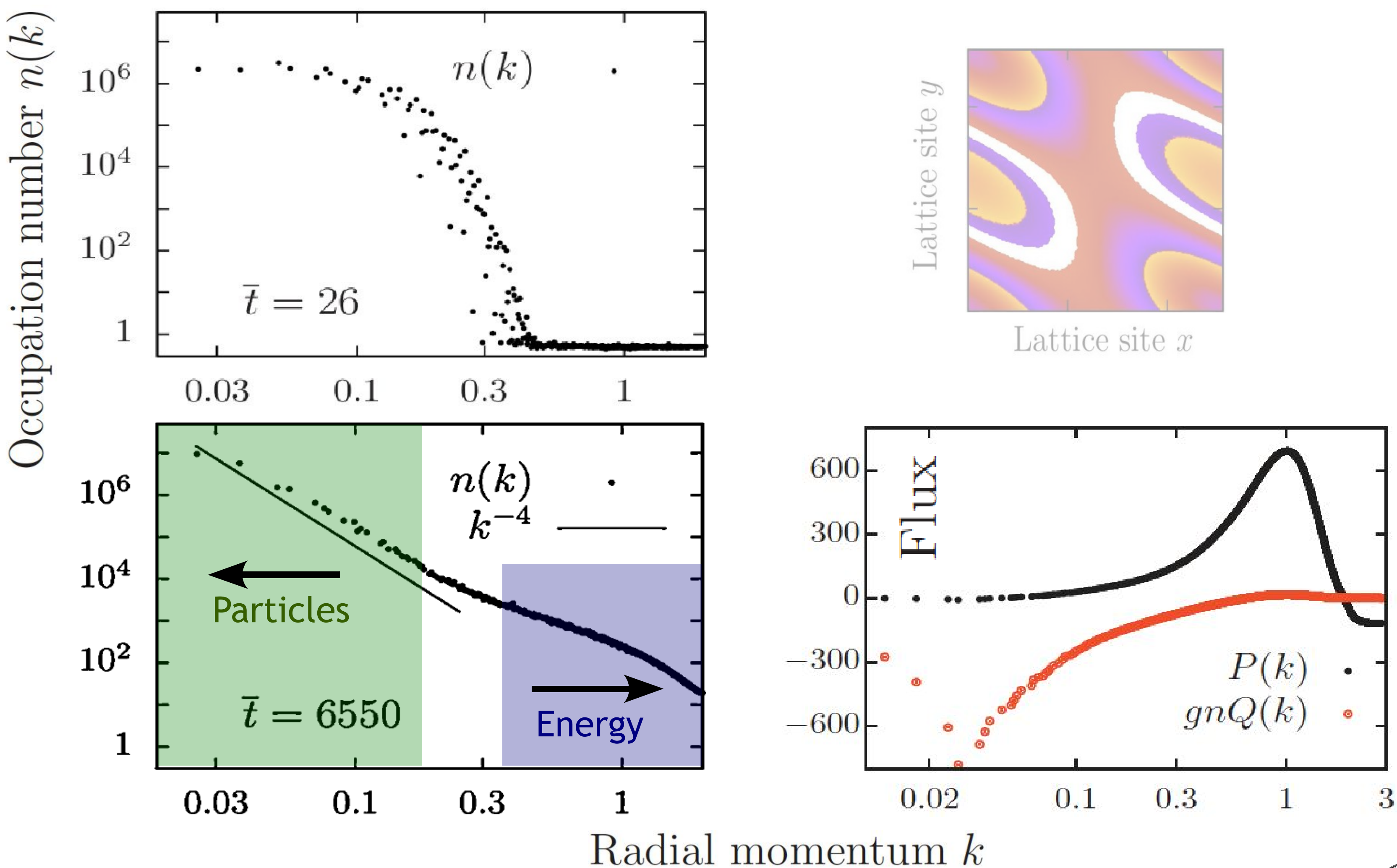
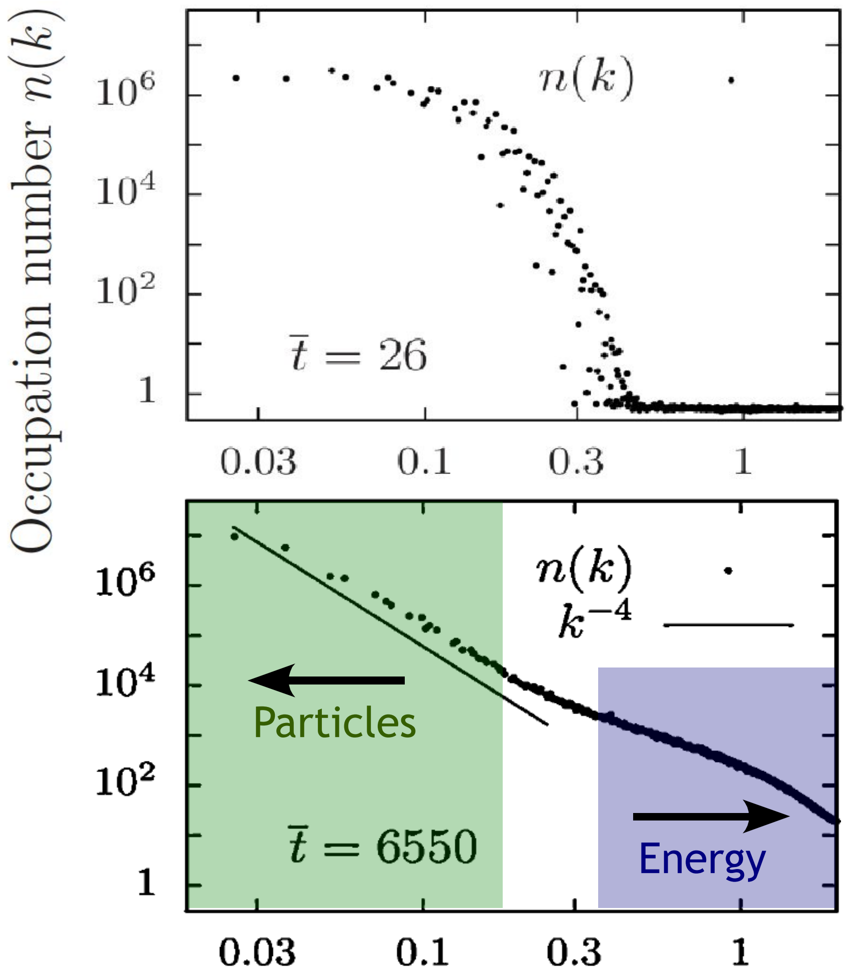
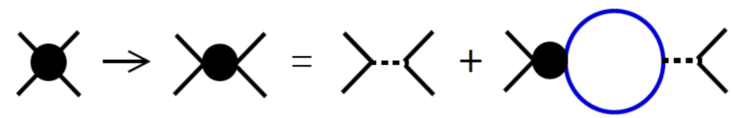
B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)



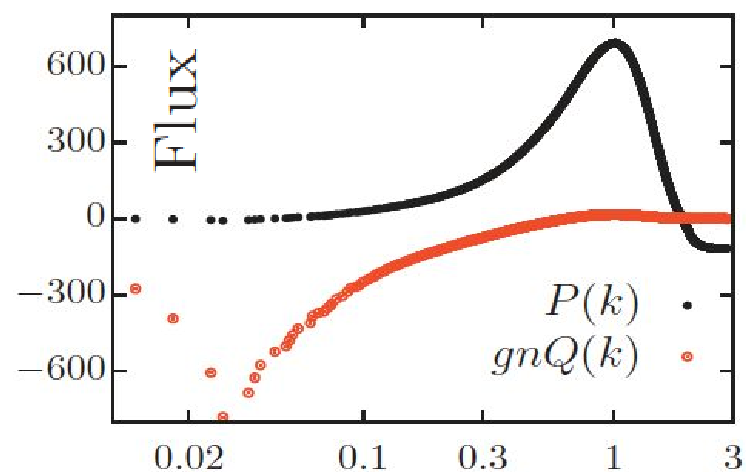
2+1 D: Quench dynamics

B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)





J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603,
J. Berges, G. Hoffmeister, NPB **813** (09) 383,
C. Scheppach, J. Berges, TG PRA **81** (10) 033611,



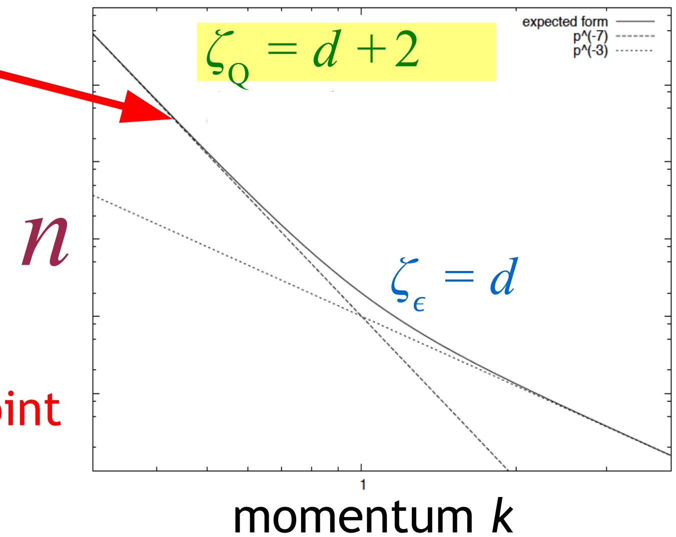
Radial momentum k

NTFP scaling in d dim^s

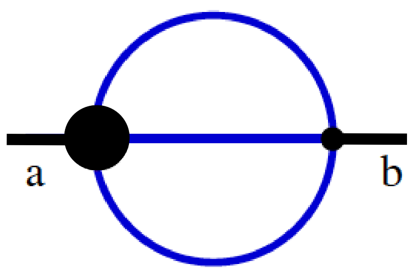
$$n \sim k^{-\zeta}$$

New exponent beyond Quantum Boltzmann!

@ Nonthermal Fixed Point

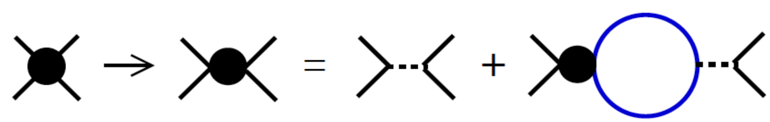


$\Sigma_{ab}(x,y) =$



Vertex bubble resummation:

(2PI to NLO in $1/N$)

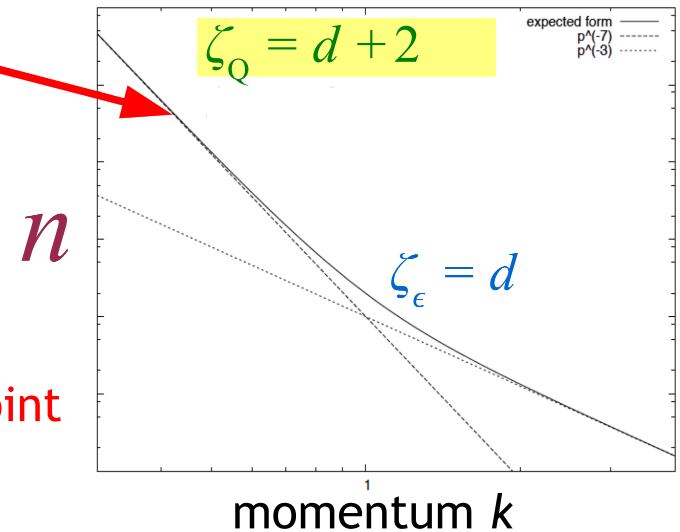


J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603, J. Berges, G. Hoffmeister, NPB **813** (09) 383
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

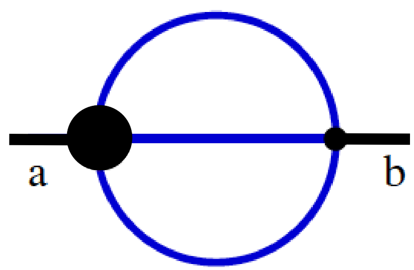
Bose gas in d spatial dimensions $n \sim k^{-\zeta}$

New exponent
beyond
Quantum Boltzmann!

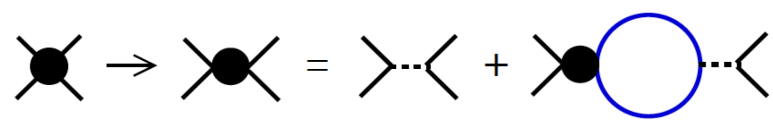
@ Nonthermal Fixed Point



$\Sigma_{ab}(x,y) =$



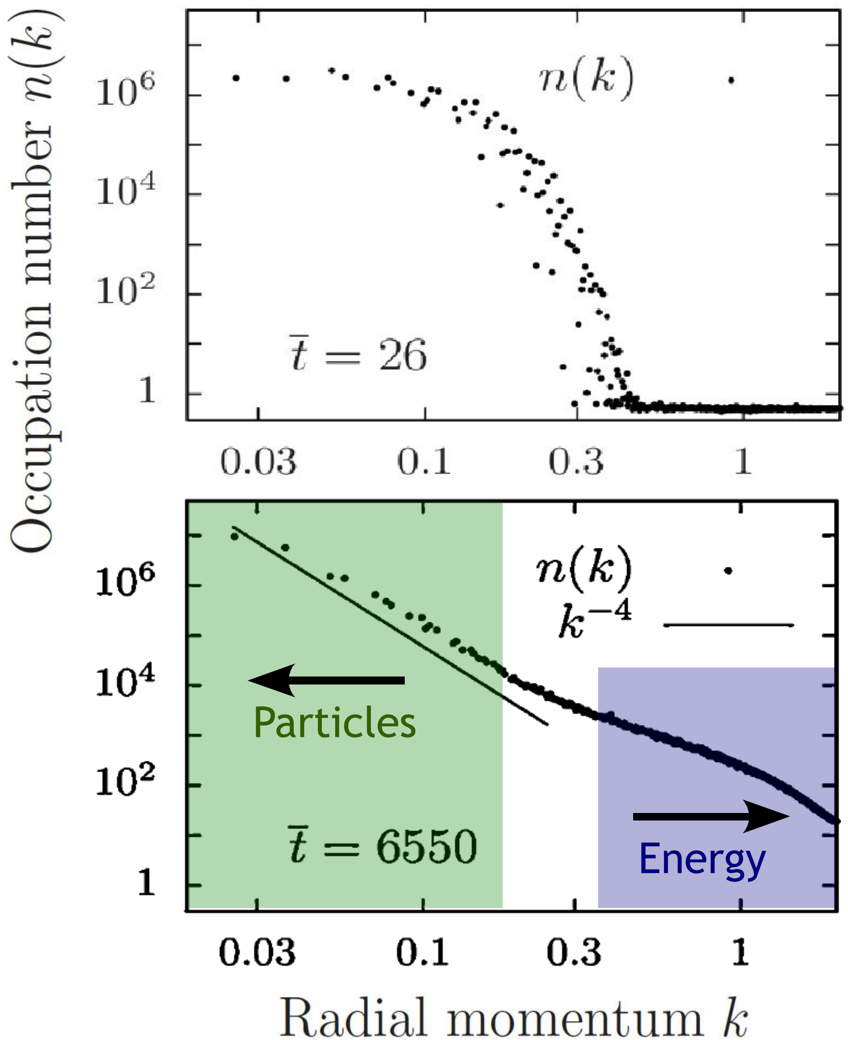
Vertex bubble resummation:
(2PI to NLO in $1/N$)

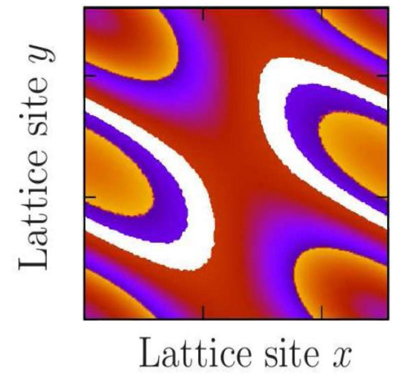


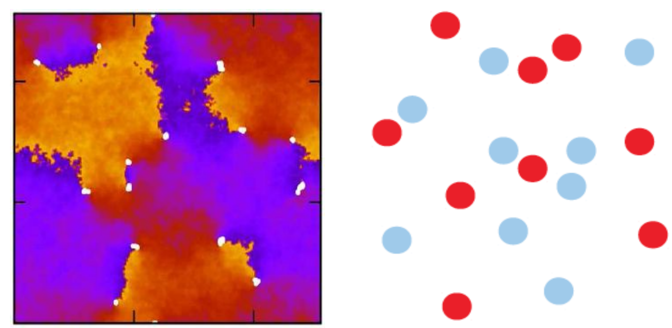
J. Berges, A. Rothkopf, J. Schmidt, PRL **101** (08) 041603, J. Berges, G. Hoffmeister, NPB **813** (09) 383
C. Scheppach, J. Berges, TG PRA **81** (10) 033611

2+1 D: Quench dynamics

B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)







$n(k) \sim k^{-4}$

$\Leftrightarrow E(k) \sim k^{-1}$

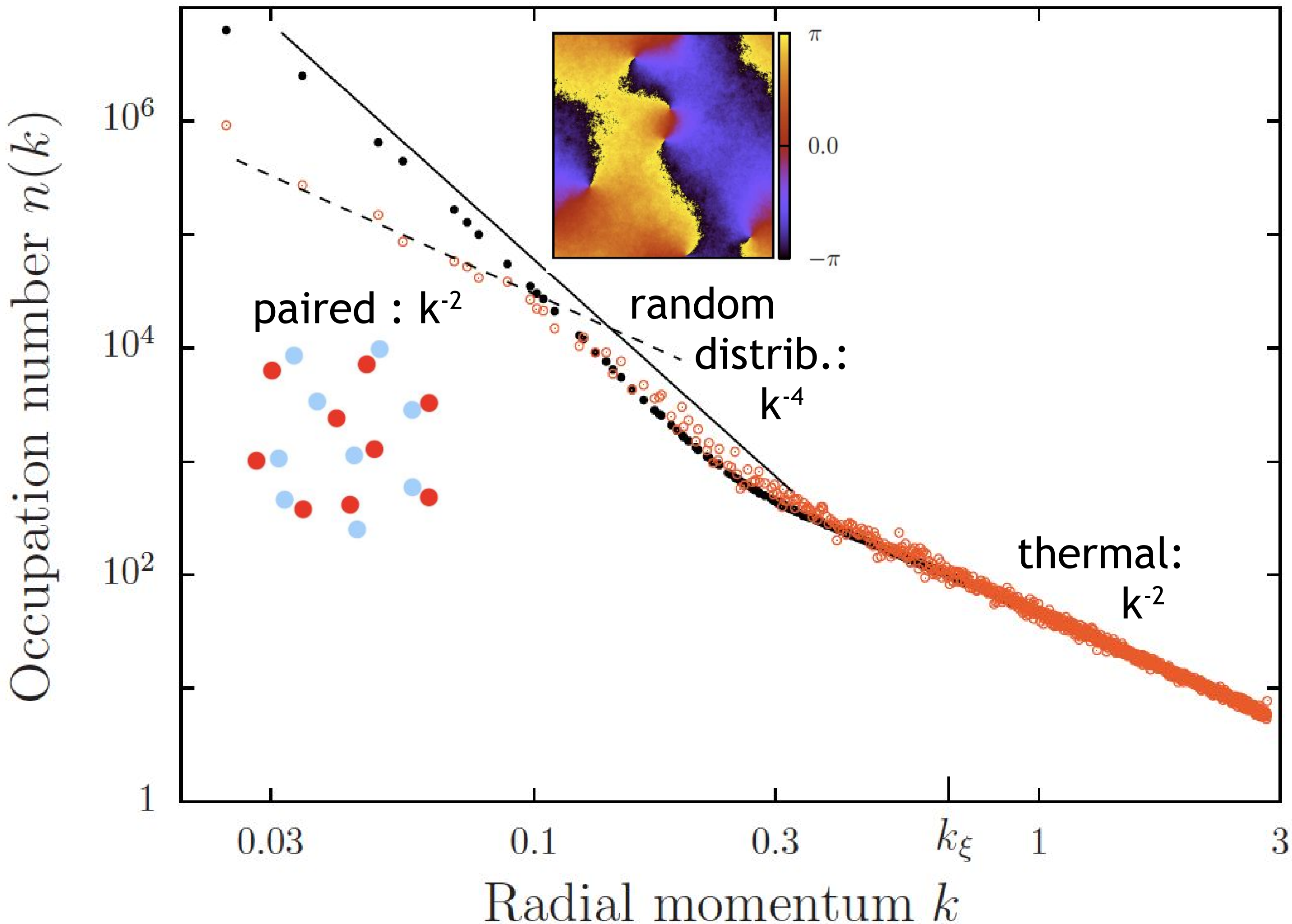
Point vortex model

Movie 2: Vortex “gas” & Spectrum

$$n(k) = \langle \Psi^*(\mathbf{k})\Psi(\mathbf{k})\rangle\big|_{\text{angle average}}$$

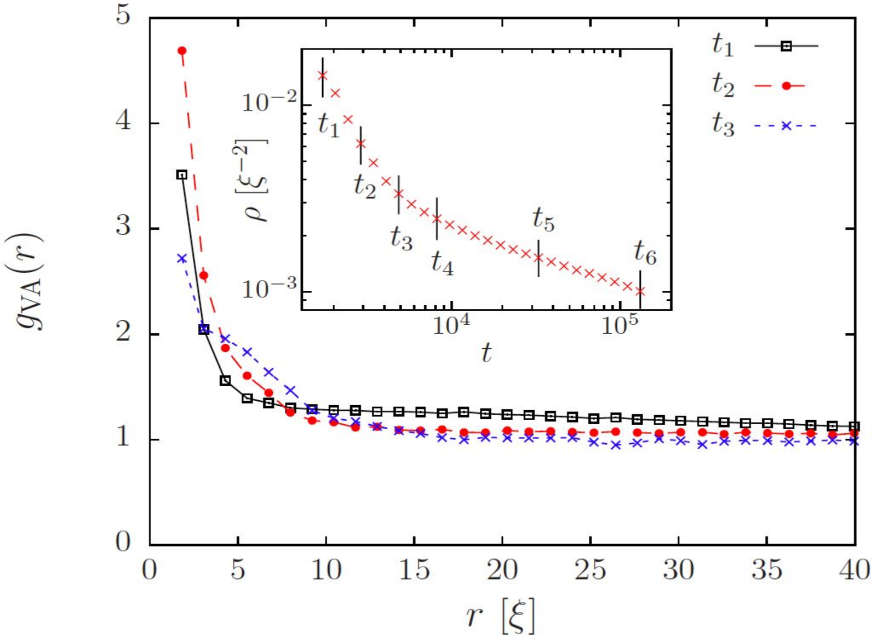
http://www.thphys.uni-heidelberg.de/~smp/gasenzer/videos/boseqt.html

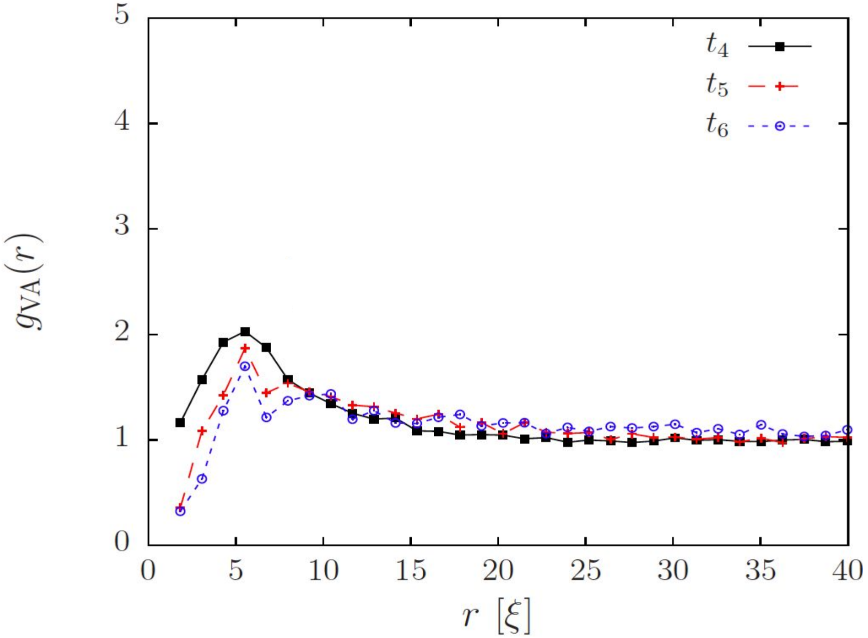
Scaling & Vortex correlations



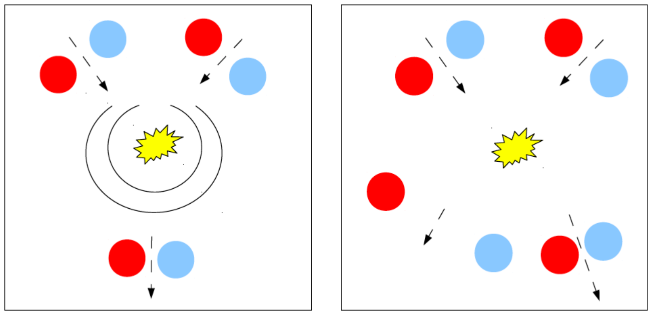
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.6127, PRA, to appear (12)

Dynamical vortex unpairing



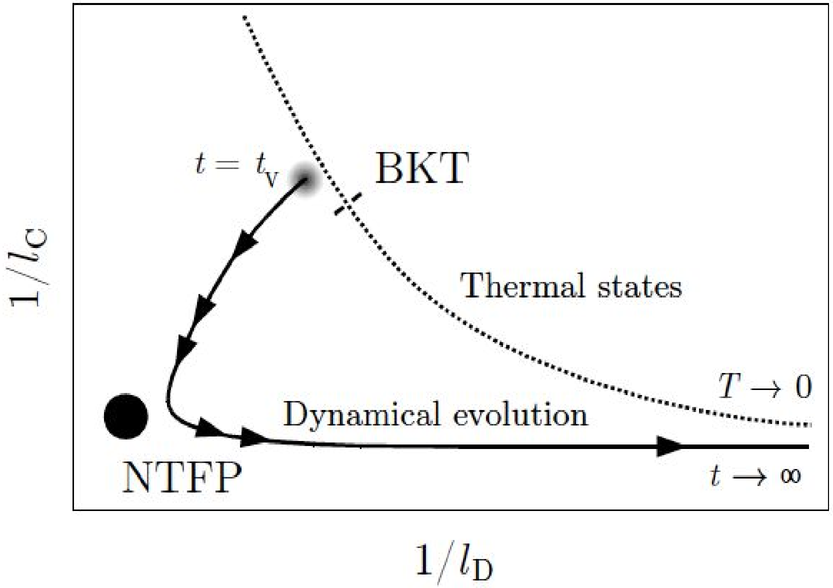


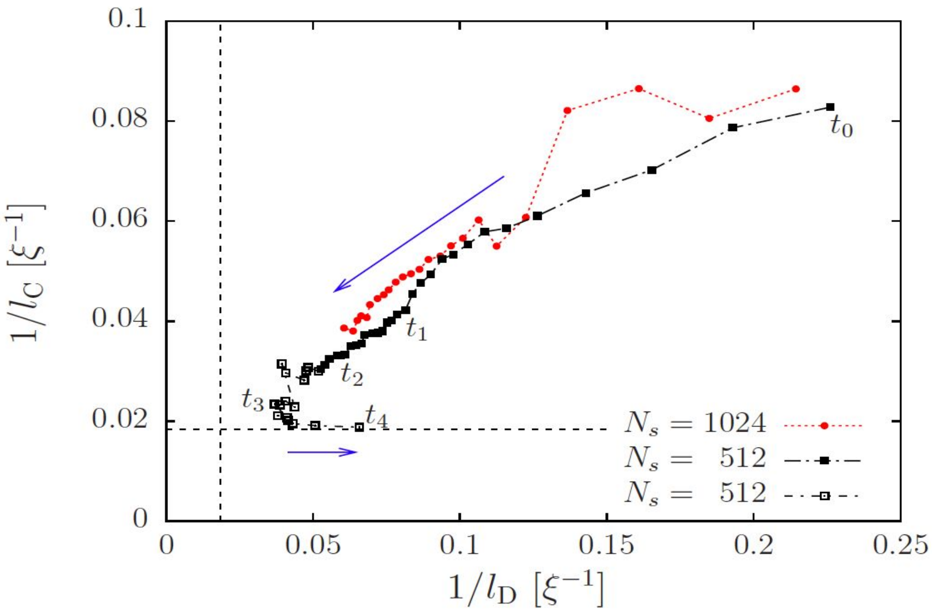
$$g_{\mathrm{VA}}(\mathbf{x},\mathbf{x}',t) = \frac{\langle \rho^{\mathrm{V}}(\mathbf{x},t)\rho^{\mathrm{A}}(\mathbf{x}',t)\rangle}{\langle \rho^{\mathrm{V}}(\mathbf{x},t)\rangle\langle \rho^{\mathrm{A}}(\mathbf{x}',t)\rangle}$$



J. Schole, B. Nowak, TG, arXiv:1204.2487 [cond-mat.quant-gas]

Nonthermal fixed point in 2D





J. Schole, B. Nowak, TG, arXiv:1204.2487 [cond-mat.quant-gas]

Perturbative RG for dyn. near BKT: Mathey & Polkovnikov, PRA **80**, 041601R (09), **81**, 033605 (10)
See also: Jelic & Cugliandolo, J. Stat. Mech. P02032 (11)

Spin Systems

Decomposition of Energy

$$E_{tot} = \int \left(\frac{1}{2} |\nabla \sqrt{n} e^{-i\varphi}|^2 + \frac{1}{2} g n^2 \right) d\boldsymbol{\rho}$$

$$= E_{kin} + E_q + E_{int}$$

rotationless

$$\nabla \times (\sqrt{n}\mathbf{u})^c = 0$$

$$E_{kin} = \frac{1}{2} \int |\sqrt{n}\mathbf{u}|^2 d\boldsymbol{\rho} \quad = E_{kin}^i + E_{kin}^c$$

$$\nabla \cdot (\sqrt{n}\mathbf{u})^i = 0$$

solenoidal

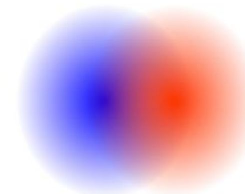
$$E_q = \frac{1}{2} \int (\nabla \sqrt{n})^2 d\boldsymbol{\rho}$$

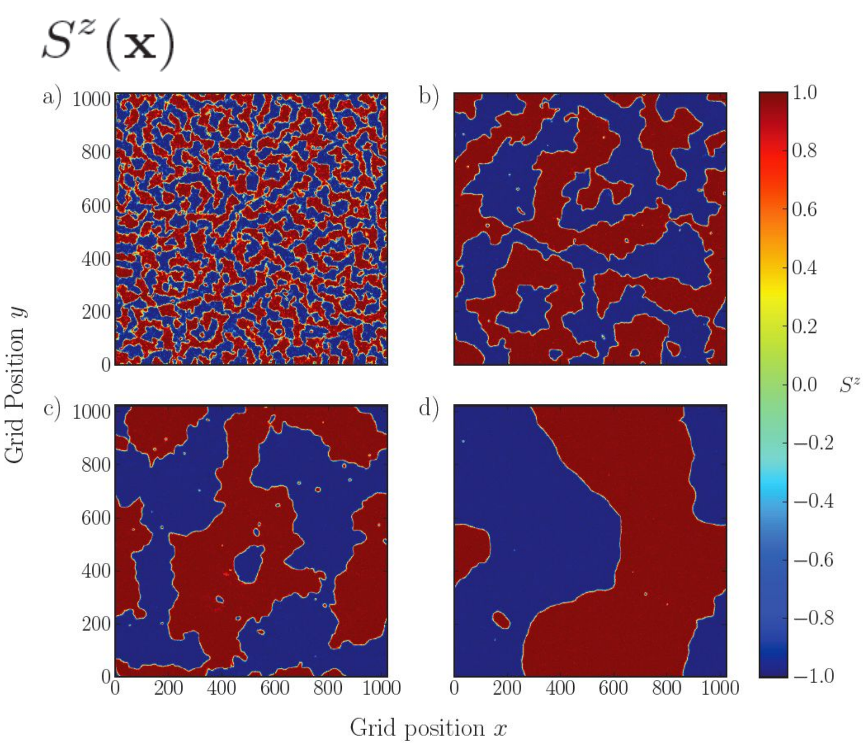
$$\mathbf{u}(\boldsymbol{\rho}, t) = \nabla \varphi(\boldsymbol{\rho}, t)$$

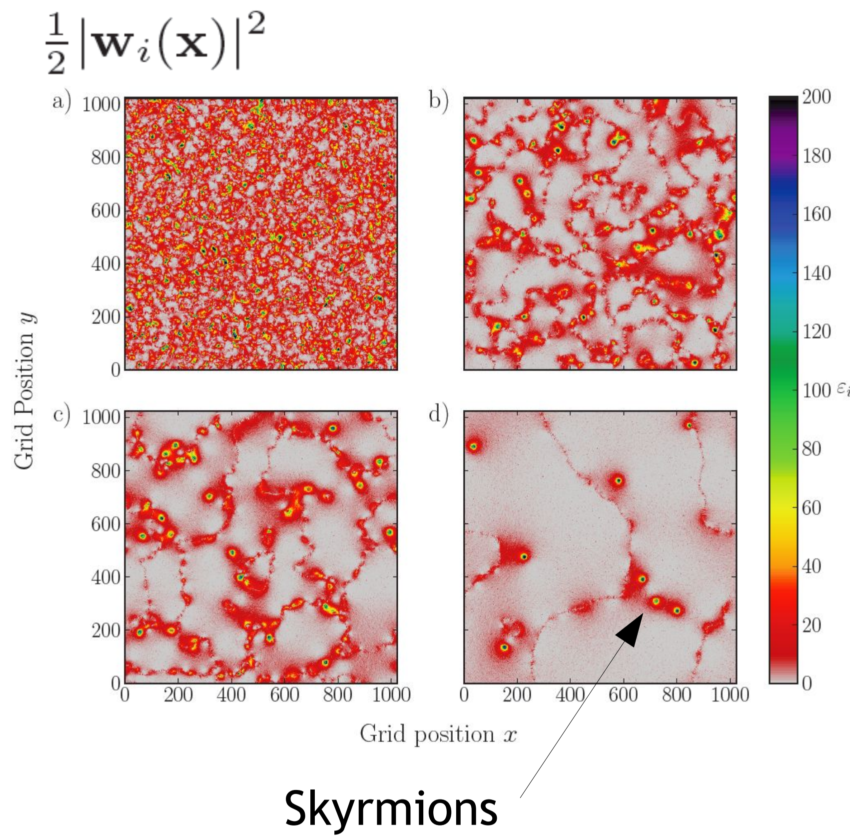
$$E_s(k) = \frac{1}{2} \int \mathrm{d}\Omega_k \, \langle \mathbf{w}_s^a(k) \cdot \mathbf{w}_s^a(k) \rangle \qquad \mathbf{w}_s^a = \frac{\sqrt{\rho_T}}{2} \nabla S^a$$

2-component BEC

immiscible
$g_{12} > g$

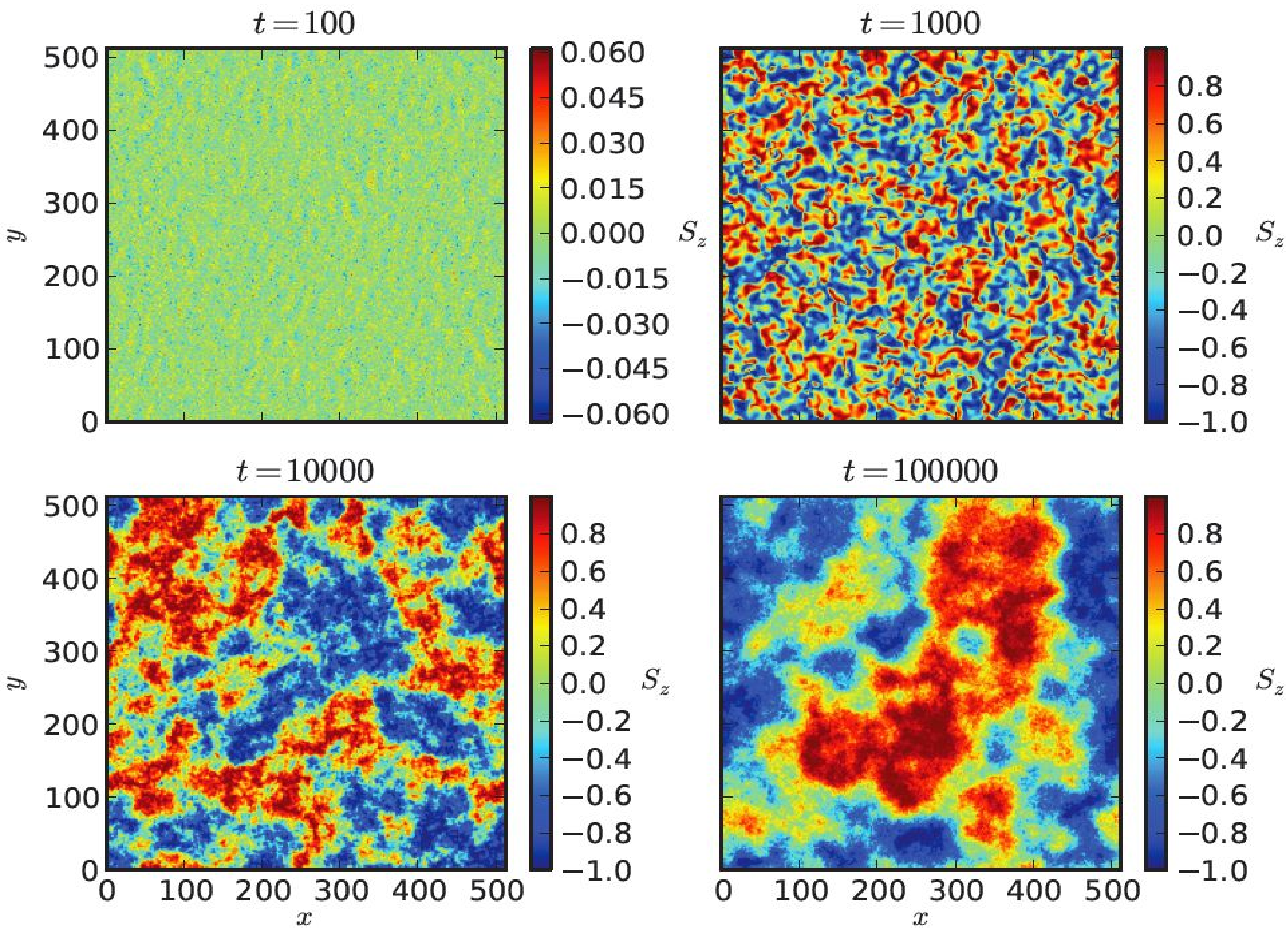






M. Karl, B. Nowak, TG, unpublished (12)

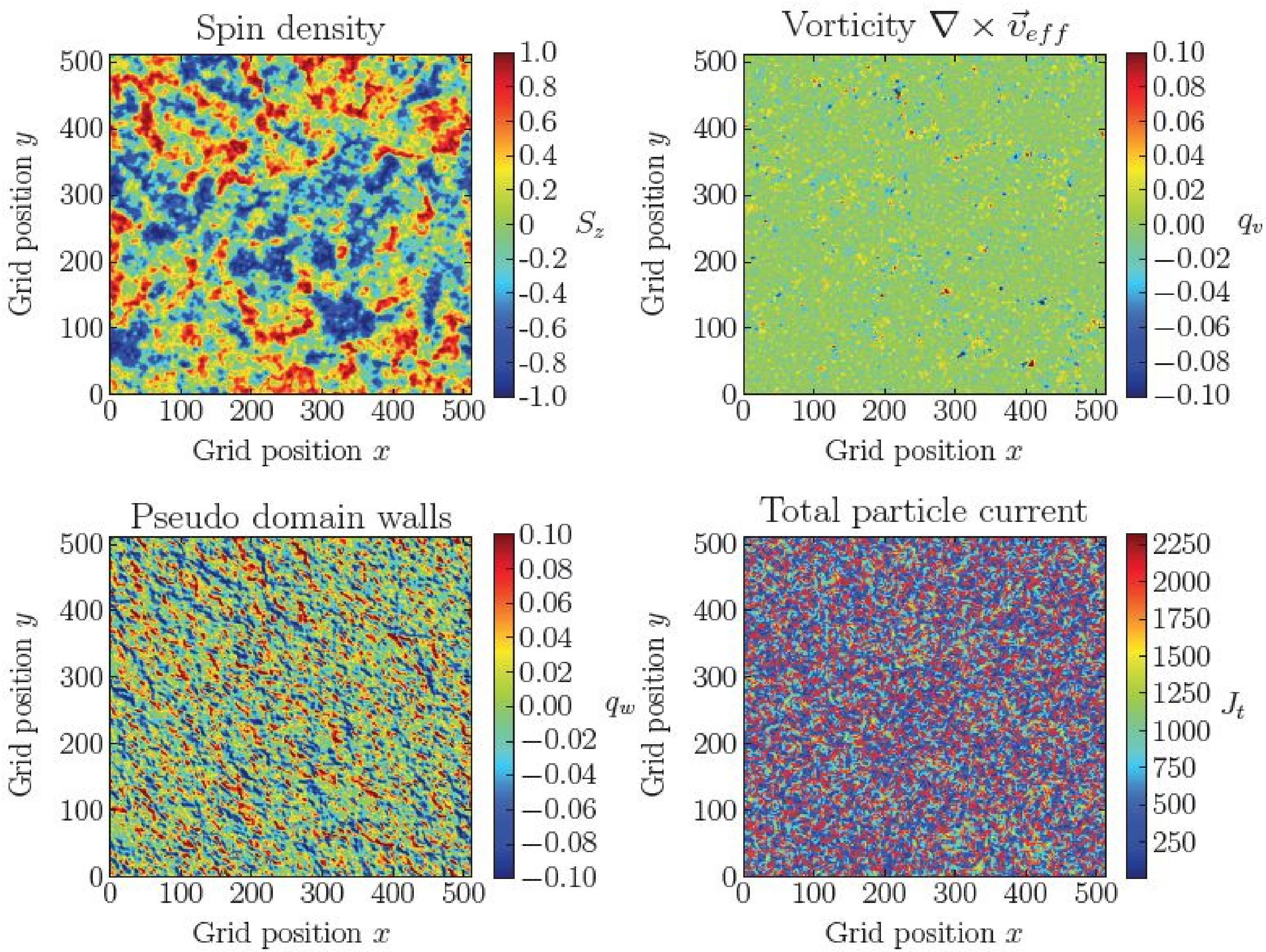
Domain formation in spin systems



[M. Karl, B. Nowak, and TG, unpublished]

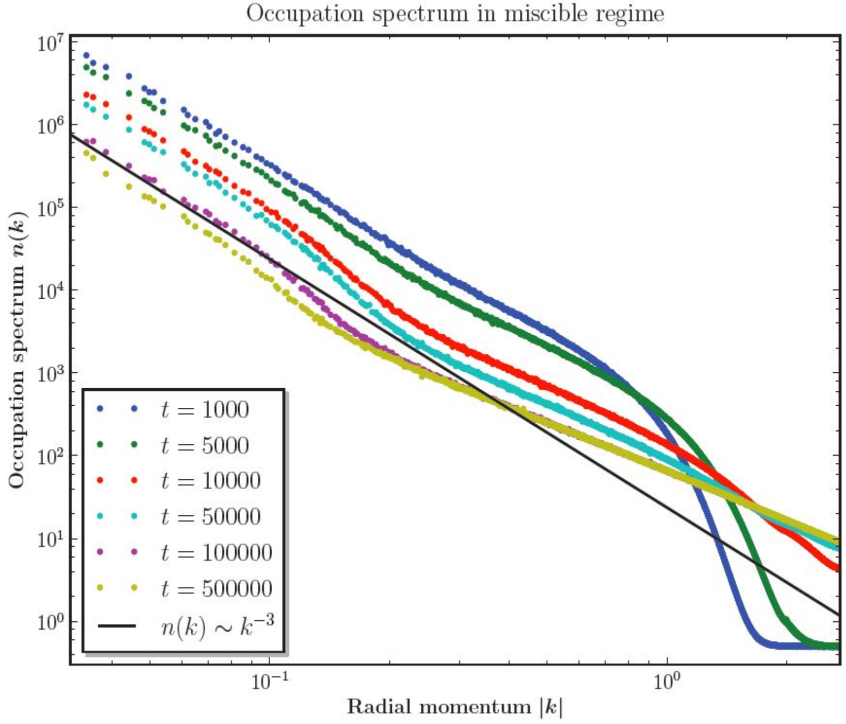
2-component BEC

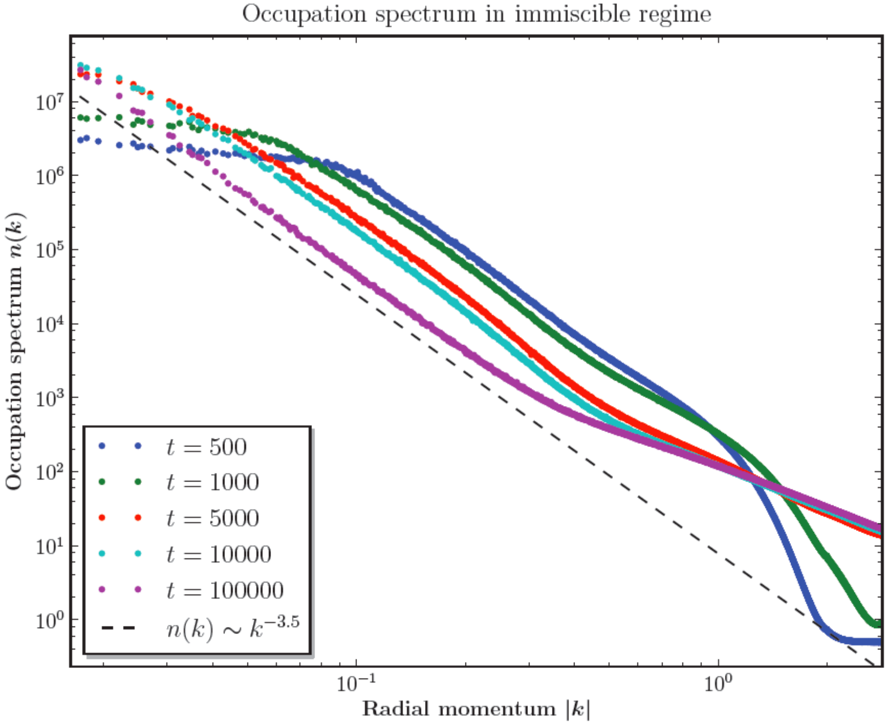
miscible
$g_{12} < g$



M. Karl, B. Nowak, TG, unpublished (12)

Domain formation in spin systems



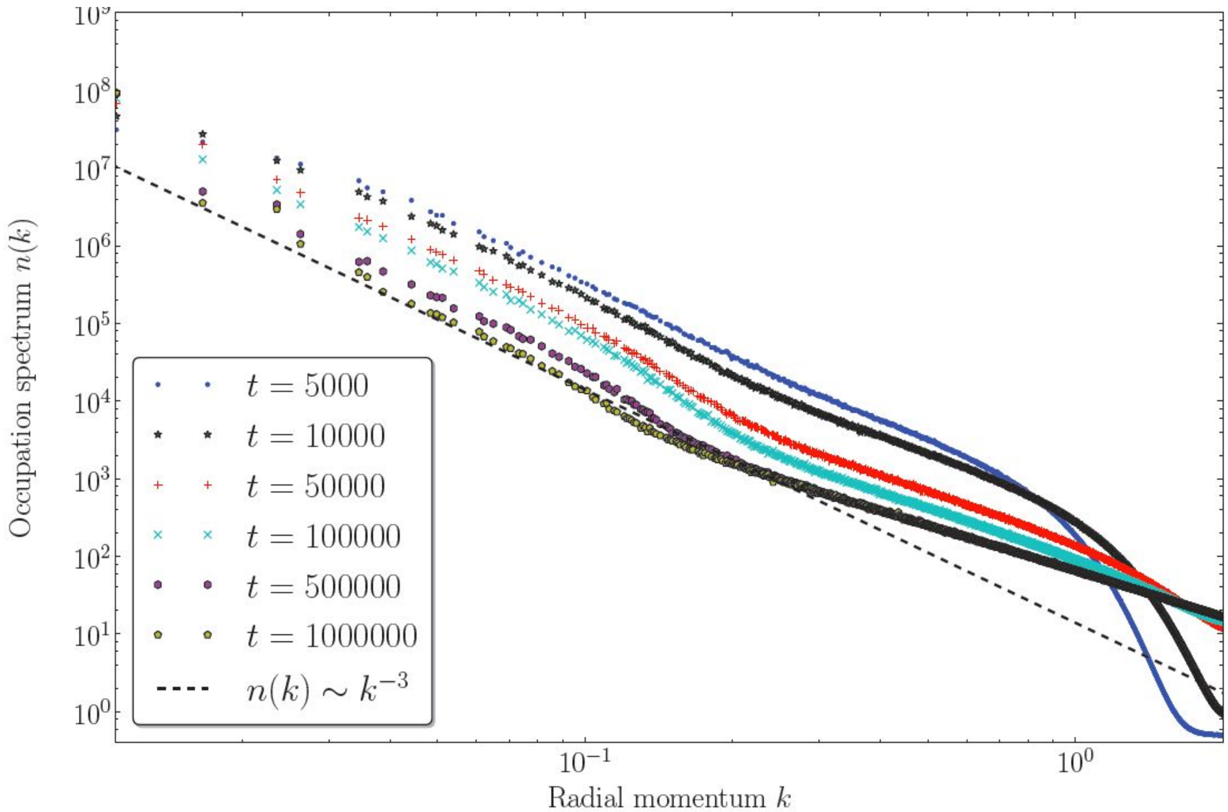


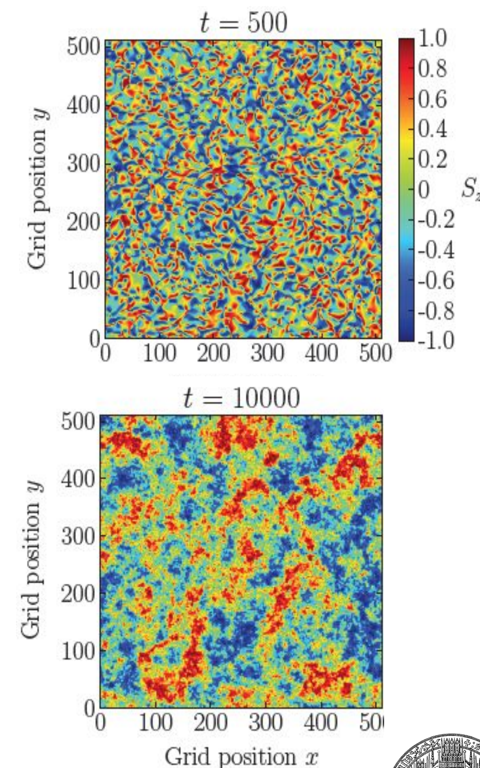
[M. Karl, B. Nowak, and T. Gasenzer, unpublished]

2-component BEC

miscible
$g_{12} = g$

immiscible
$g_{12} > g$

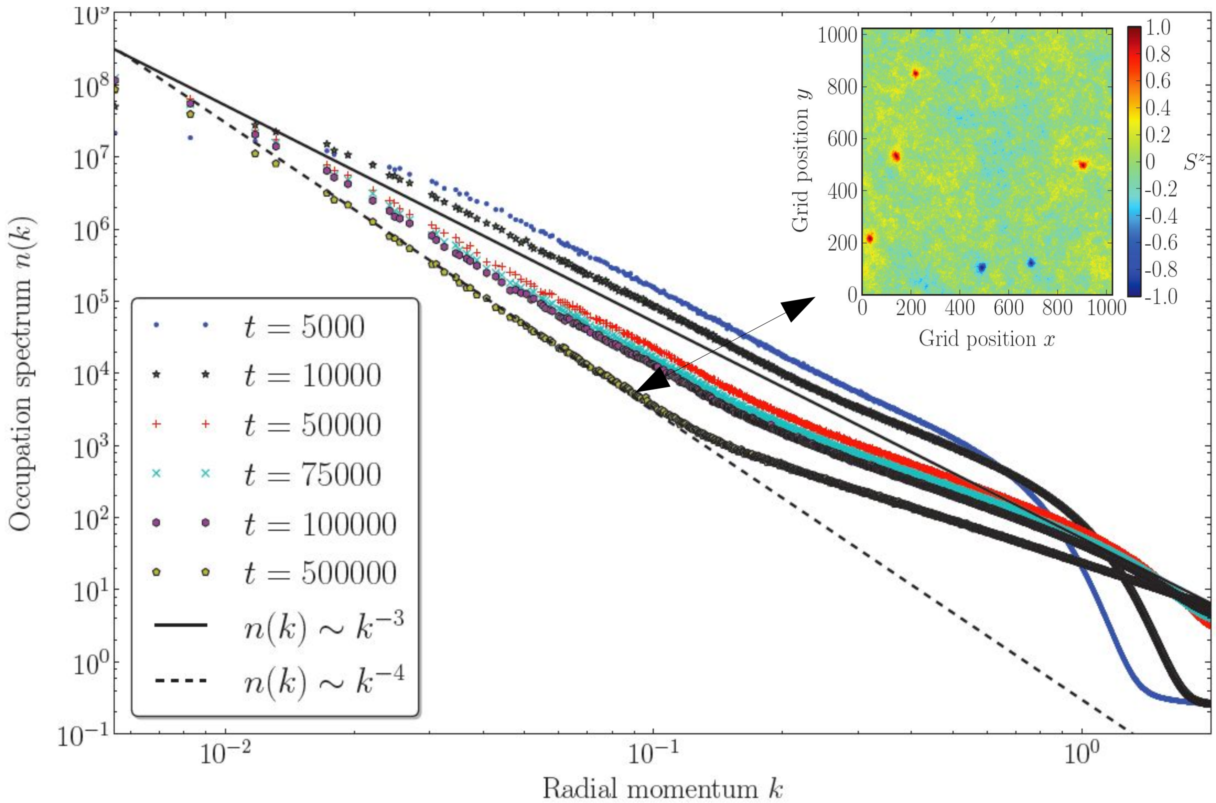


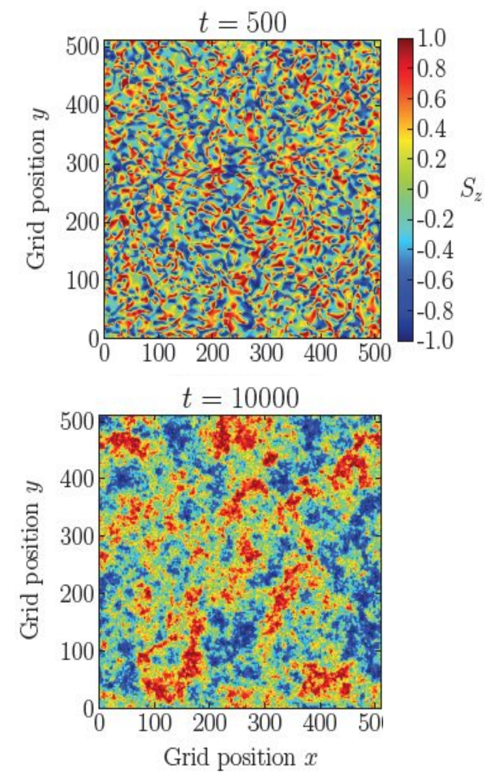


M. Karl, B. Nowak, TG, unpublished (12)

2-component BEC

miscible
$g_{12} < g$

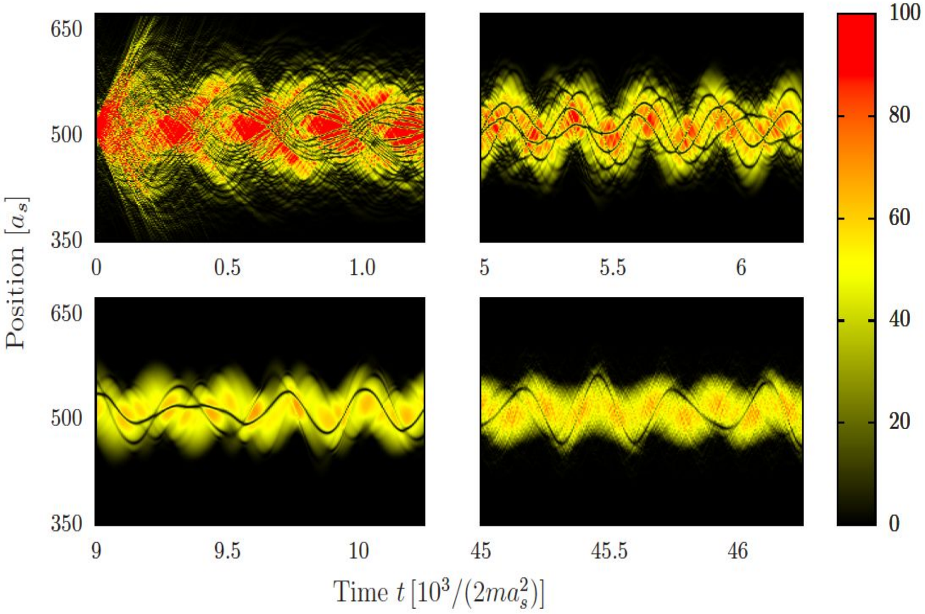


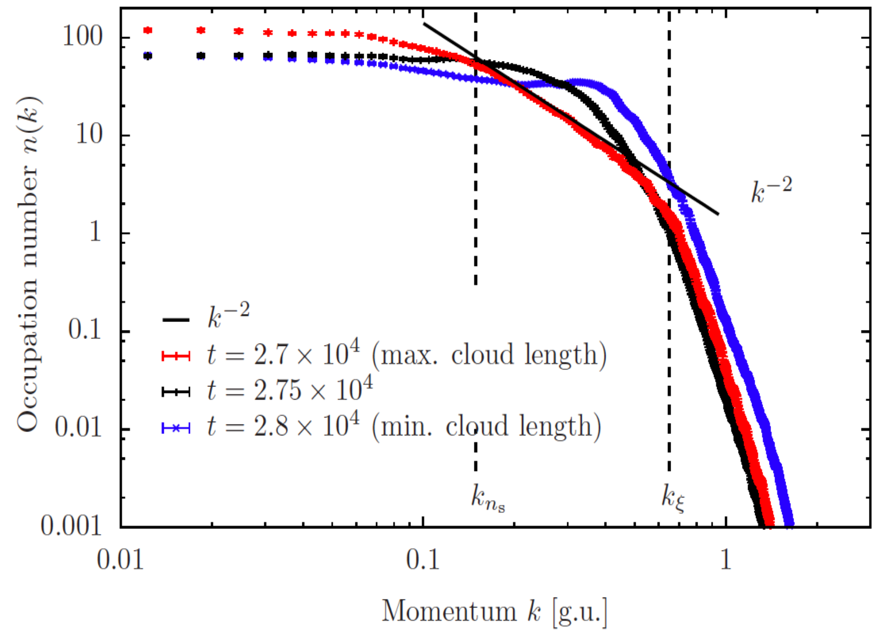


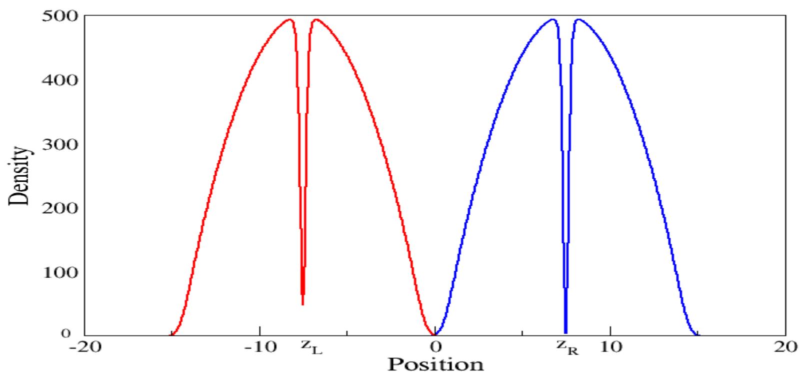
M. Karl, B. Nowak, TG, unpublished (12)

Solitons in 1D

Solitons in 1 spatial dimension







M. Schmidt, S. Erne, B. Nowak, D. Sexty, and TG, arXiv:1203.3651 [cond-mat.quant-gas]

Relativistic scalar field

Non-linear Klein-Gordon equation

O(2) symmetry

$$(\partial_t^2 - \partial_x^2)\varphi(x,t) + \lambda\varphi^3(x,t) = 0$$

Initial condition: Highly occupied zero mode, Unoccupied modes with k>0

(video)

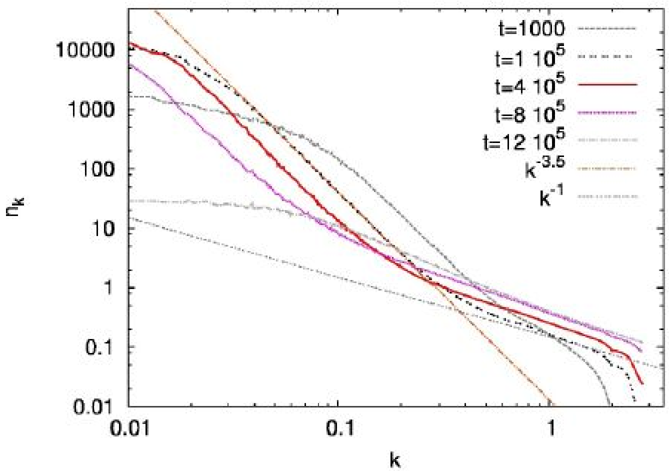
See also: http://www.thphys.uni-heidelberg.de/~sexty/videos

TG, B. Nowak, D. Sexty, arXiv:1108.0541 [hep-ph]

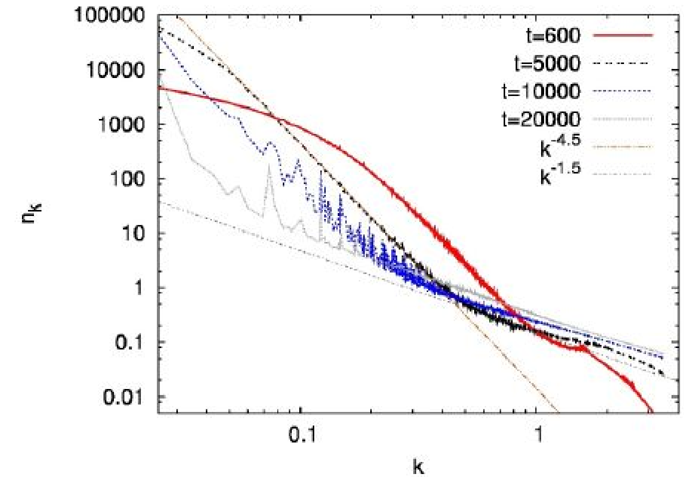
Relativistic simulations

Classical field equation: $$\left[\partial_t^2 - \Delta + \Phi^2\right]\Phi_a = 0$$

d=2



d=3



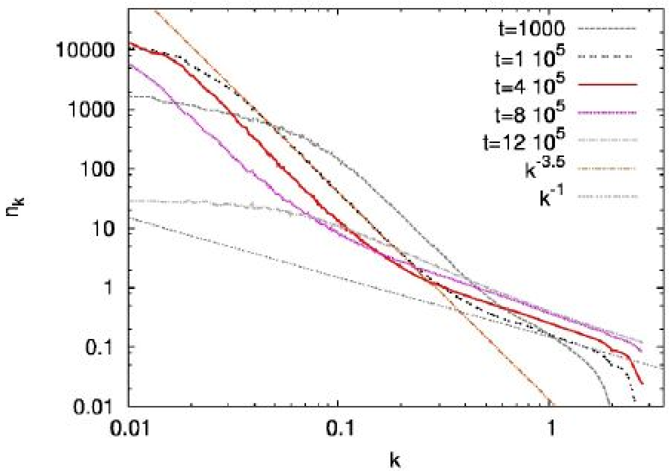
reheating after inflation

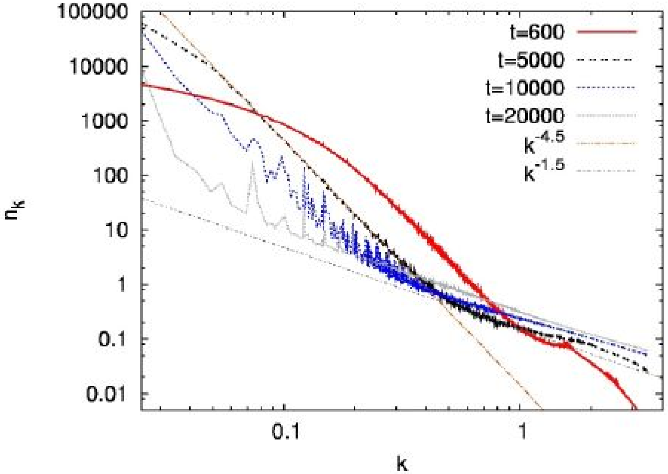
S. Khlebnikov, I. Tkachev, PRL (96)
R. Micha, I. Tkachev, PRD (04)
J. Berges, A. Rothkopf, J. Schmidt, PRL (08)
J. Berges, D. Sexty, PRD (11)

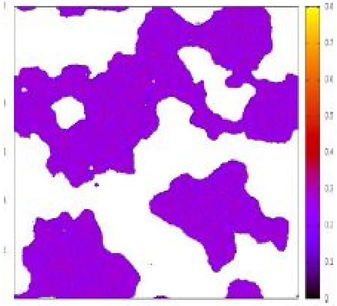
D. Sexty, B. Nowak, TG, PLB **710** (12)

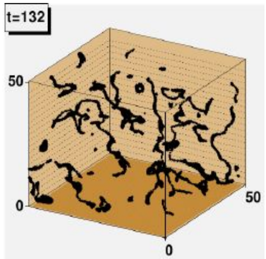
Relativistic simulations

Classical field equation: $\left[\partial_t^2 - \Delta + \Phi^2\right]\Phi_a = 0$

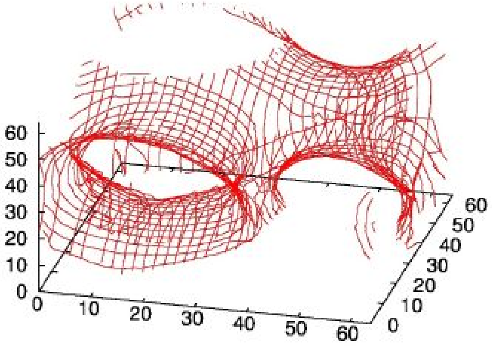








Tkachev (98)

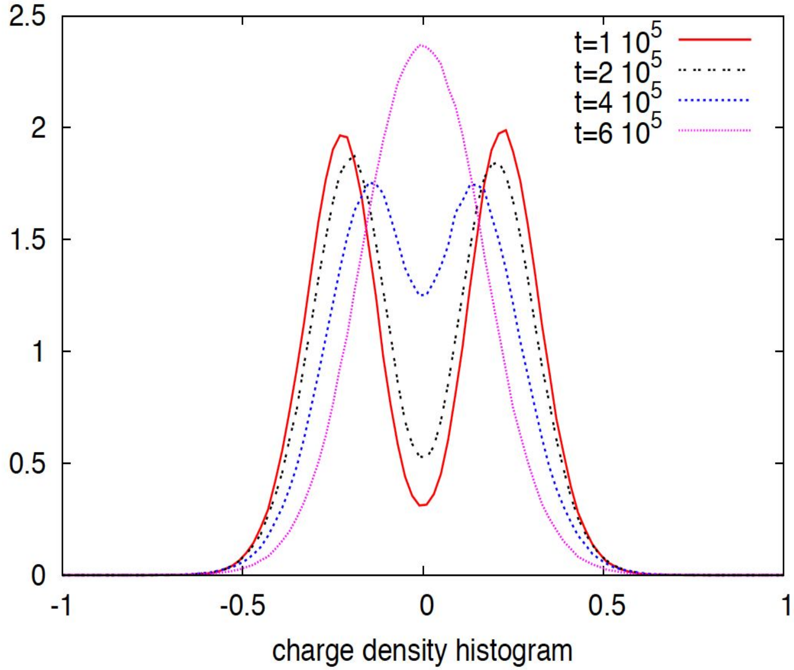


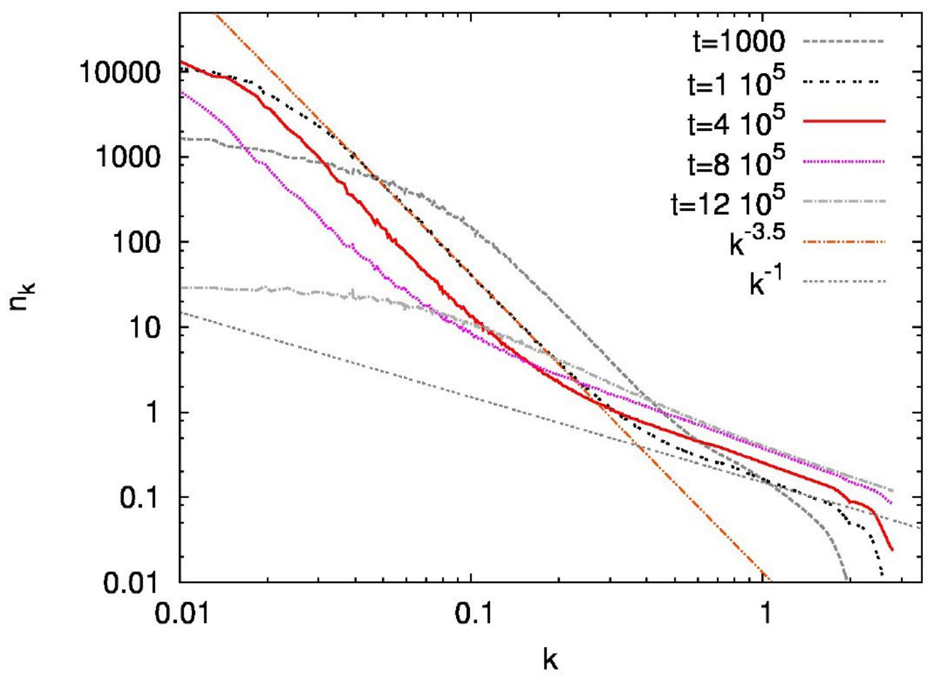
D. Sexty, B. Nowak, TG, PLB **710** (12)

Strong Turbulence = Charge Separation

Charge density distribution vs. power spectrum

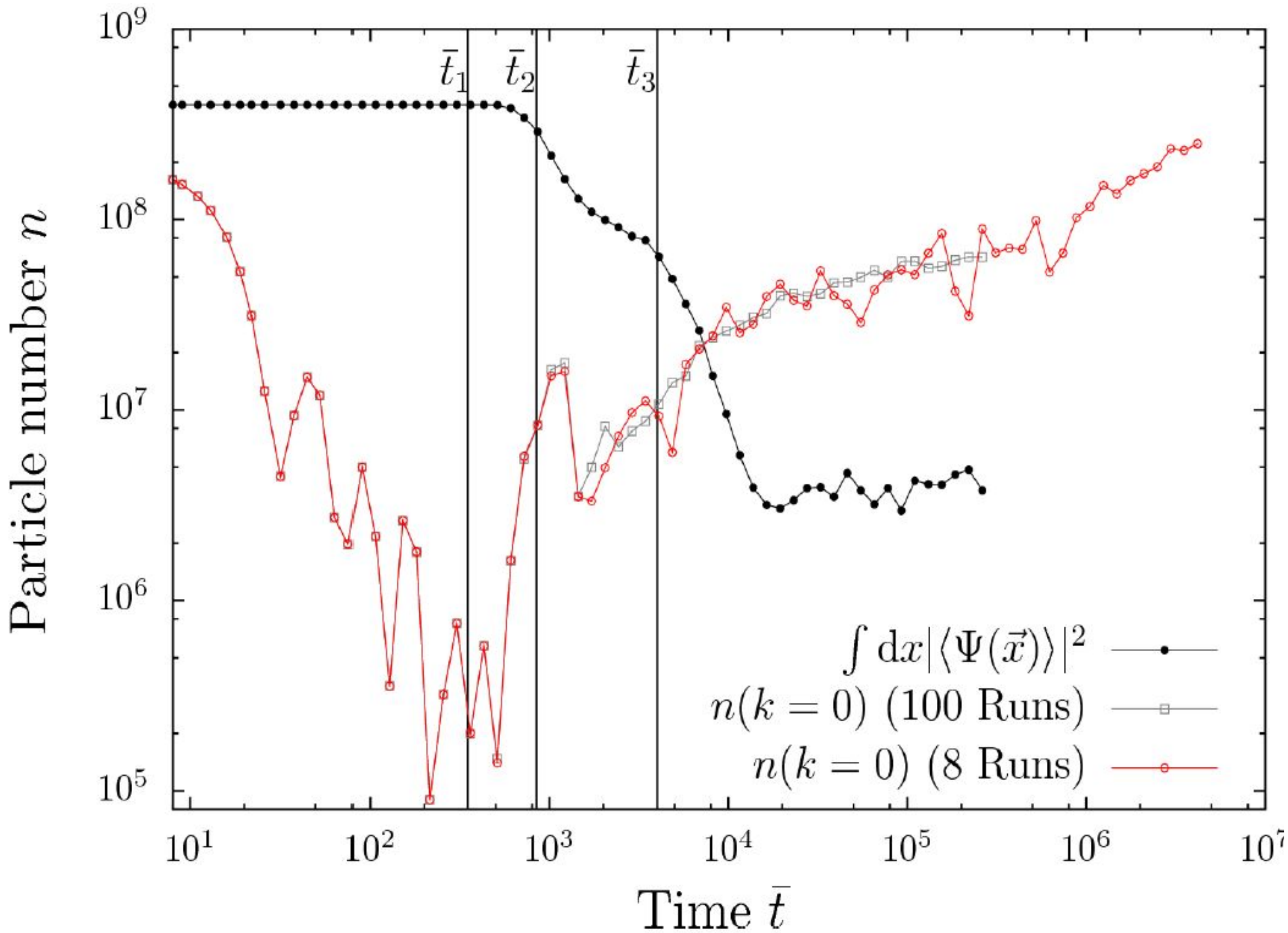
(d = 2, N = 2)





TG, B. Nowak, D. Sexty, arXiv:1108.0541 [hep-ph], PLB, to appear

Bose-Einstein condensation

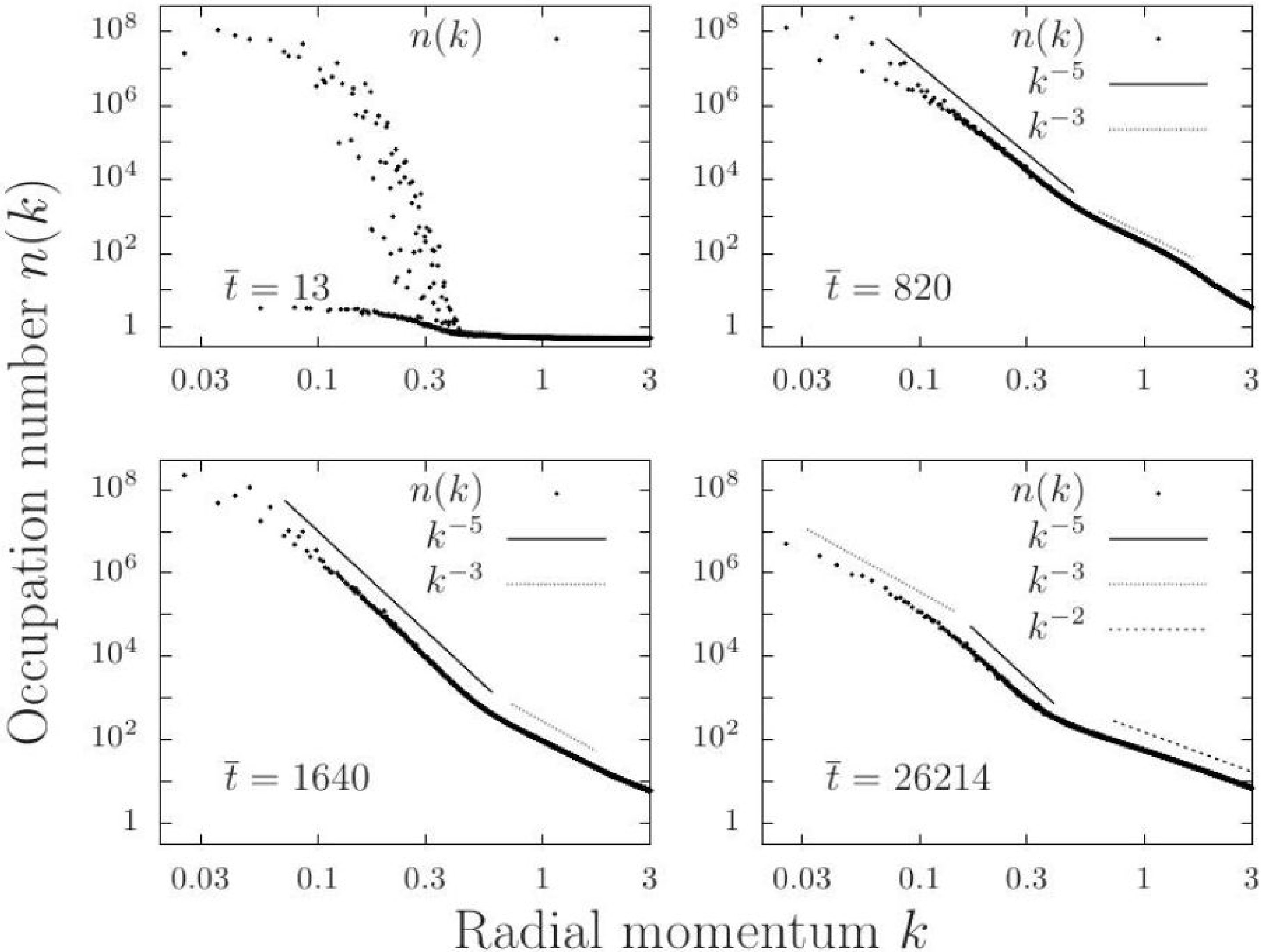


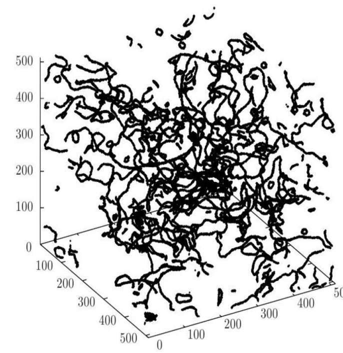
J. Schole, B. Nowak, D. Sexty, TG (unpublished)

For 3D see also N. Berloff & B. Svistunov, PRA 66 (02)

B. Nowak, D. Sexty, TG, PRB **84**: 020506(R), 2011

3+1 D simulations





Acoustic Turbulence

Decomposition of Energy

$$E_{tot} = \int \left(\frac{1}{2} |\nabla \sqrt{n} e^{-i\varphi}|^2 + \frac{1}{2} g n^2 \right) d\boldsymbol{\rho}$$

$$= E_{kin} + E_q + E_{int}$$

$$\mathbf{u}(\boldsymbol{\rho}, t) = \nabla \varphi(\boldsymbol{\rho}, t)$$

$$\nabla \times (\sqrt{n}\mathbf{u})^c = 0$$

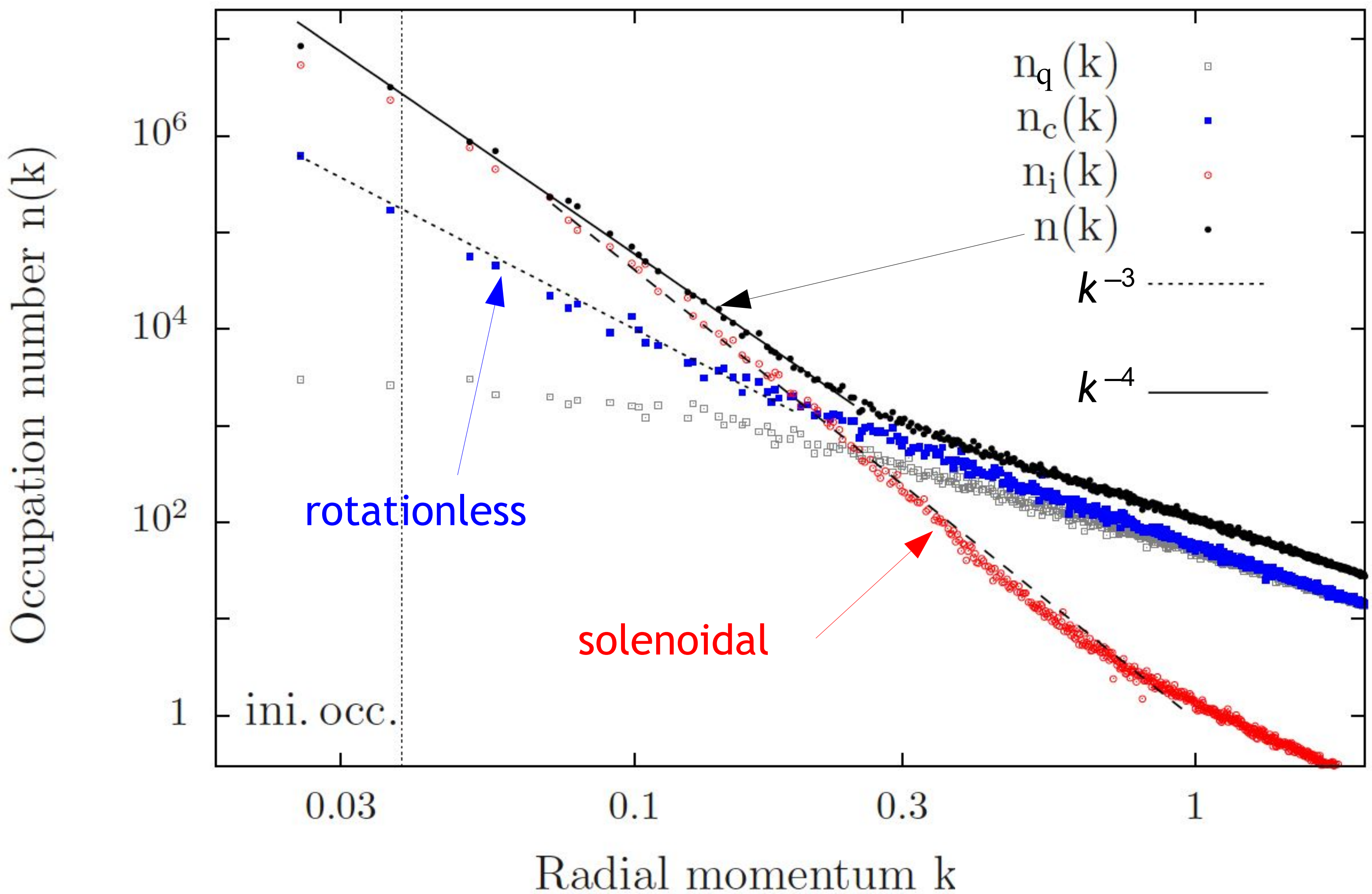
$$E_{kin} = \frac{1}{2} \int |\sqrt{n}\mathbf{u}|^2 d\boldsymbol{\rho} \quad = E_{kin}^i + E_{kin}^c$$

$$\nabla \cdot (\sqrt{n}\mathbf{u})^i = 0$$

$$E_q = \frac{1}{2} \int (\nabla \sqrt{n})^2 d\boldsymbol{\rho}$$

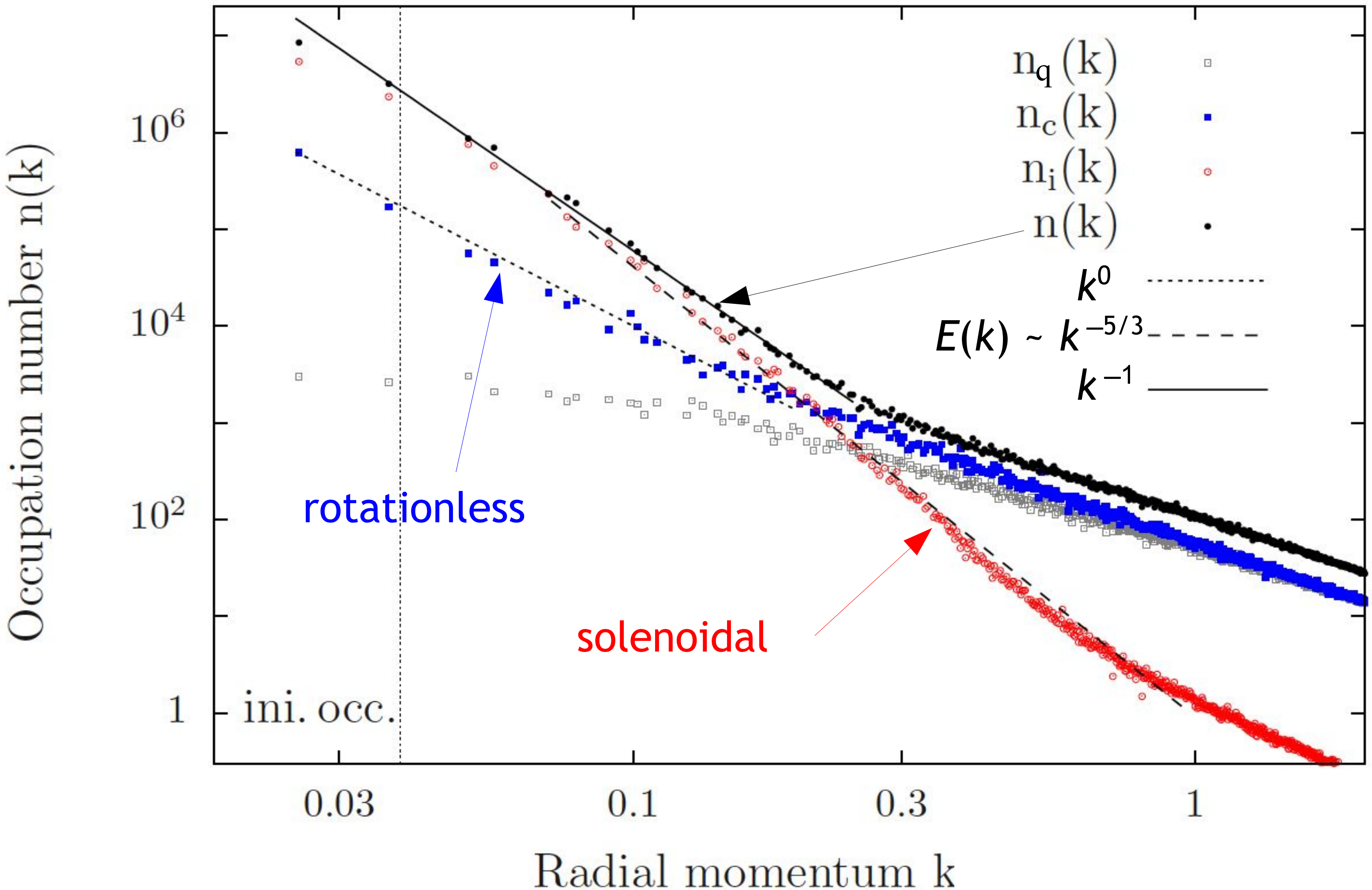
Decomposition of flow

B. Nowak, D. Sexty, TG, PRB **84**(R) (11);
B. Nowak, J. Schole, D. Sexty, TG, PRA **85** (12)



Simulations in 2+1 D

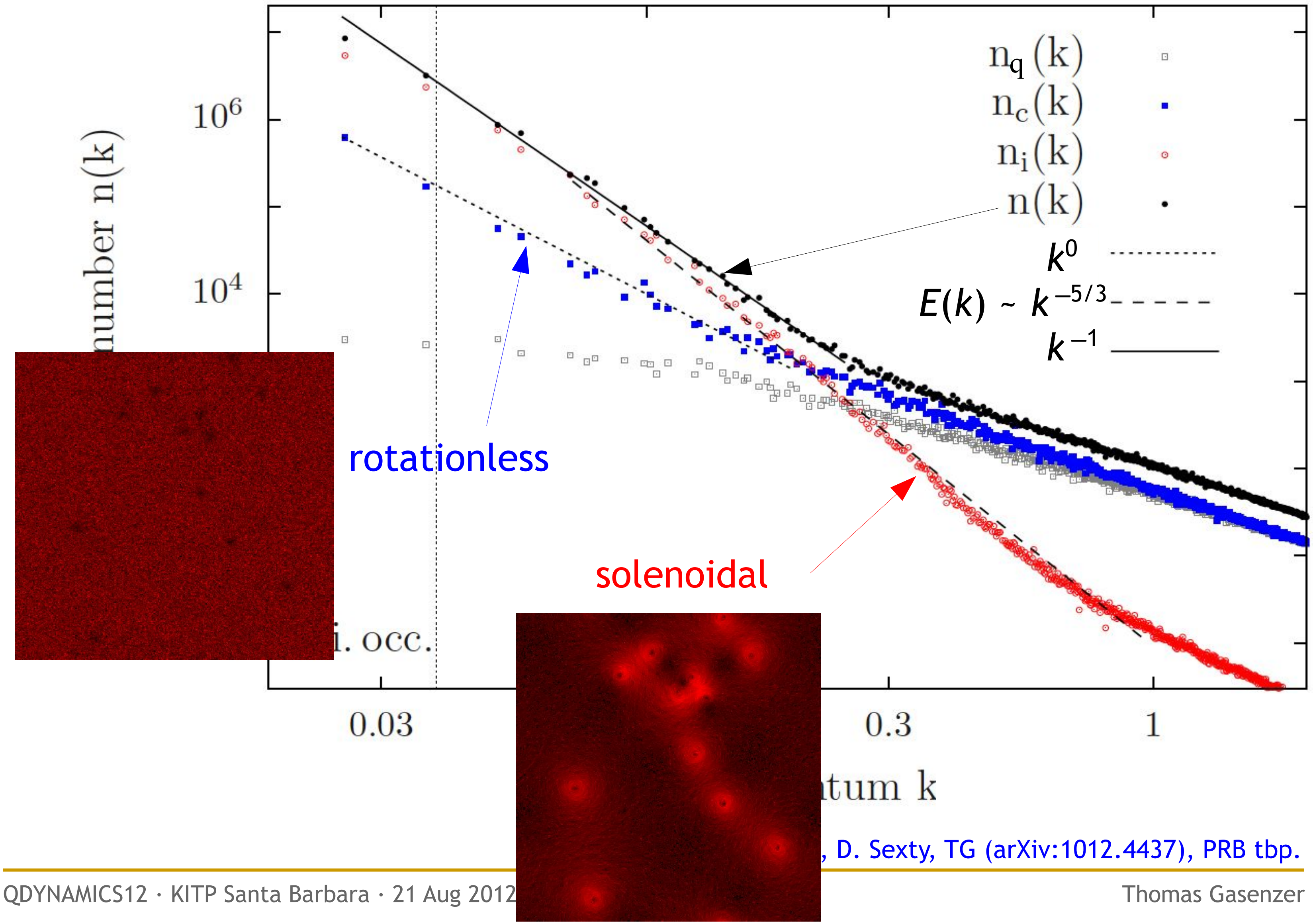
$$E(\mathrm{k}) = \omega(\mathrm{k})\mathrm{k}^{d-1}\mathrm{n}(\mathrm{k})$$



B. Nowak, D. Sexty, TG (arXiv:1012.4437), PRB tbp.

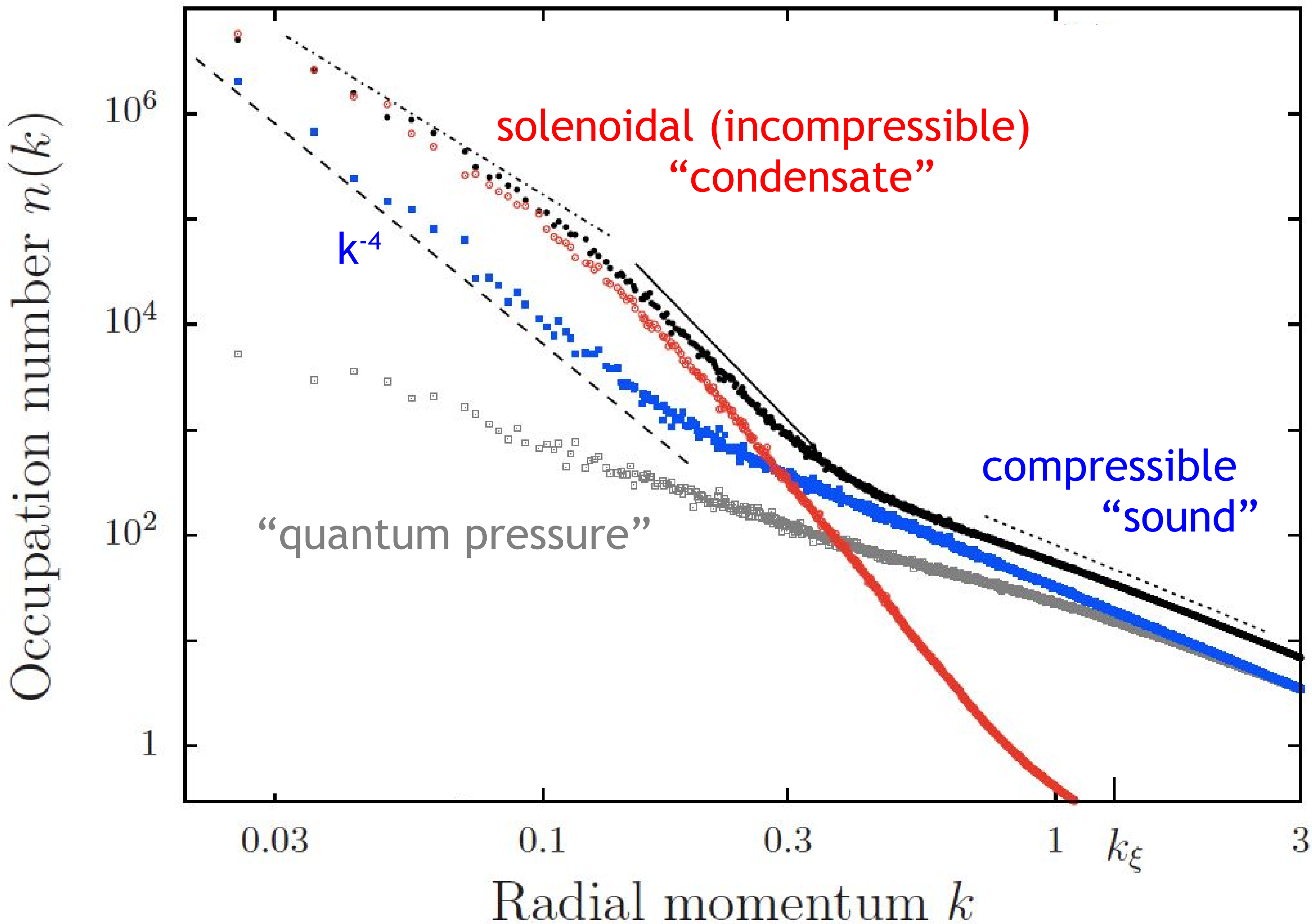
Simulations in 2+1 D

$$E(\mathrm{k}) = \omega(\mathrm{k})\mathrm{k}^{d-1}\mathrm{n}(\mathrm{k})$$

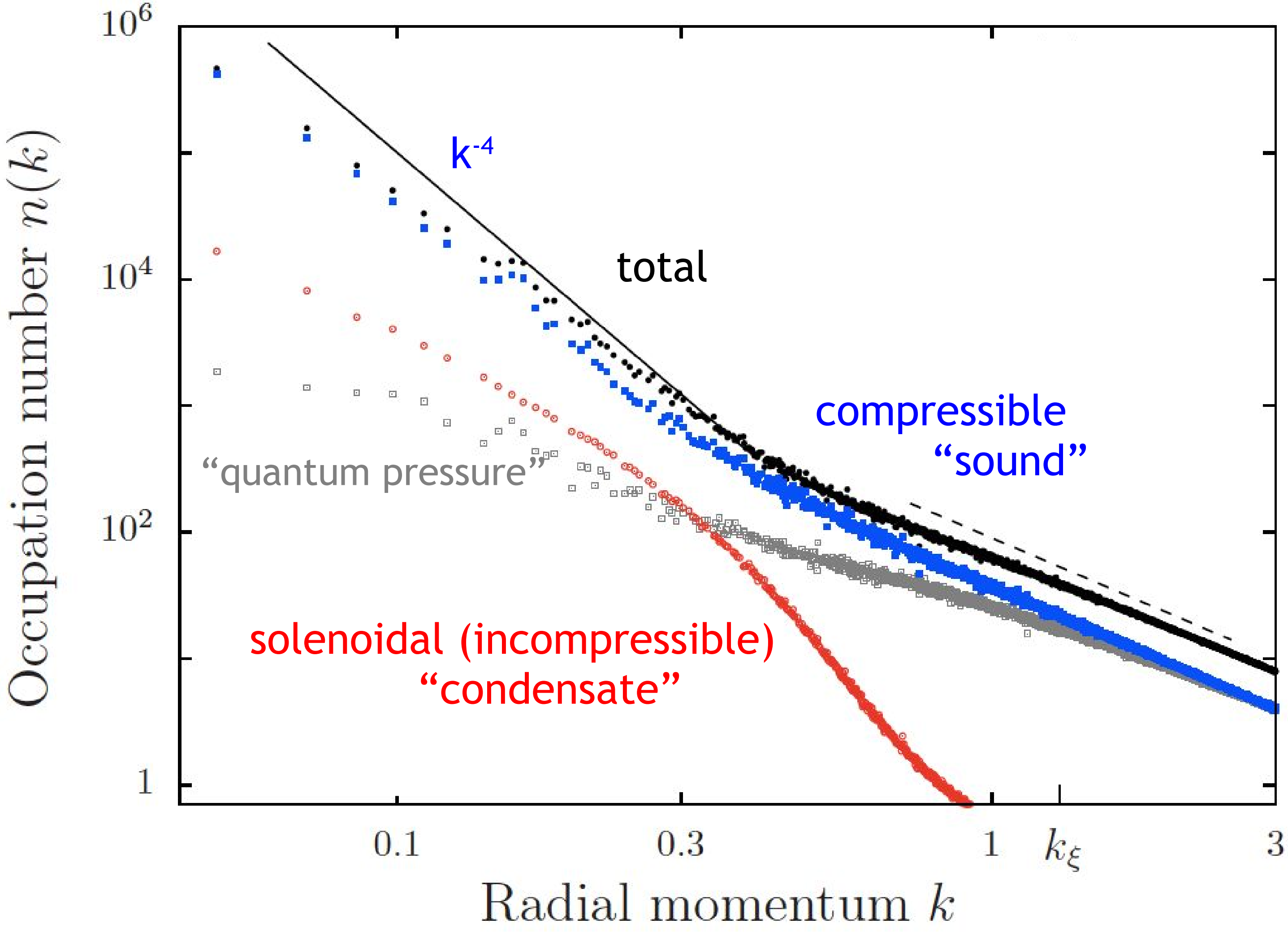


, D. Sexty, TG (arXiv:1012.4437), PRB tbp.

Decomposition of flow

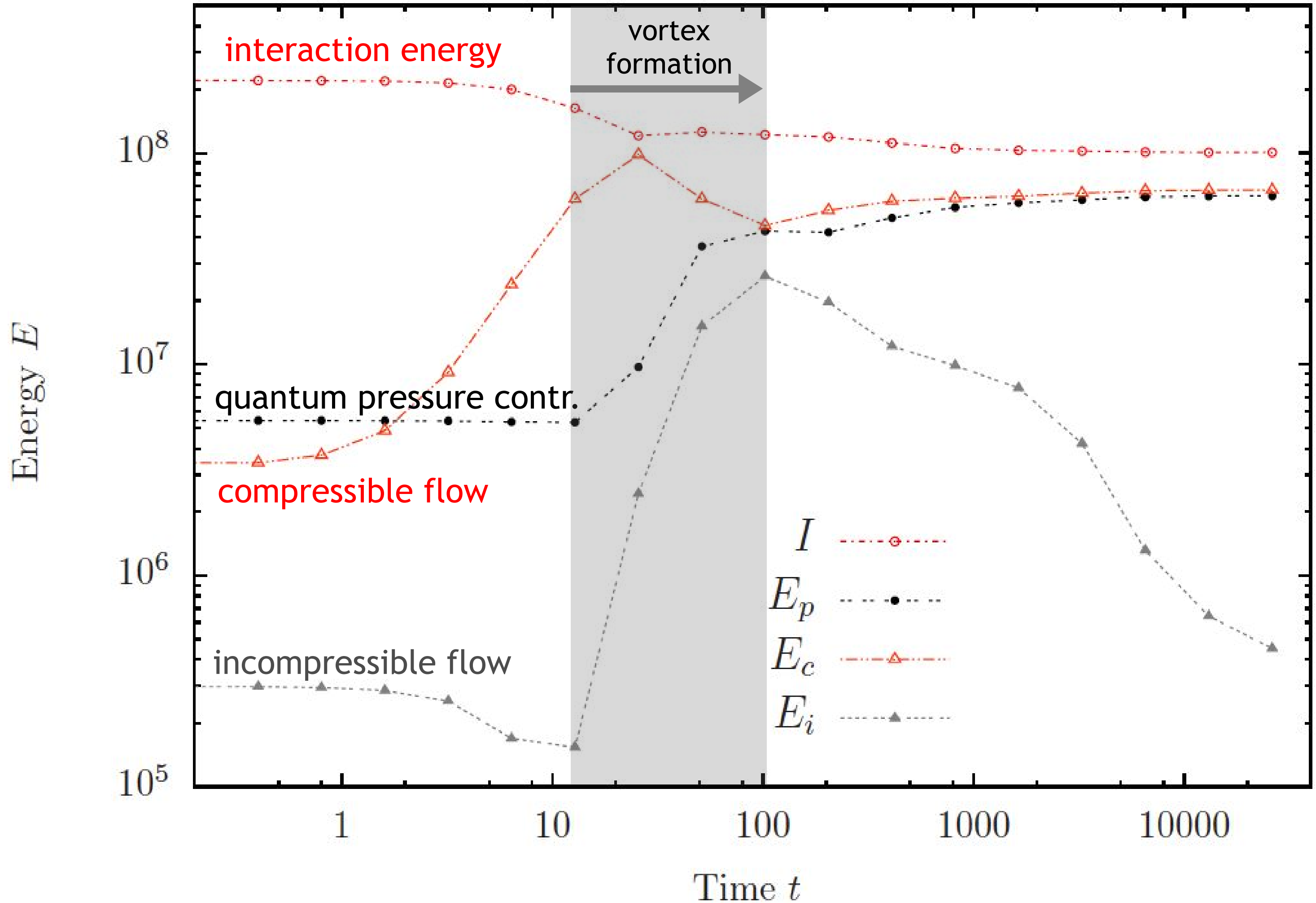


Acoustic turbulence



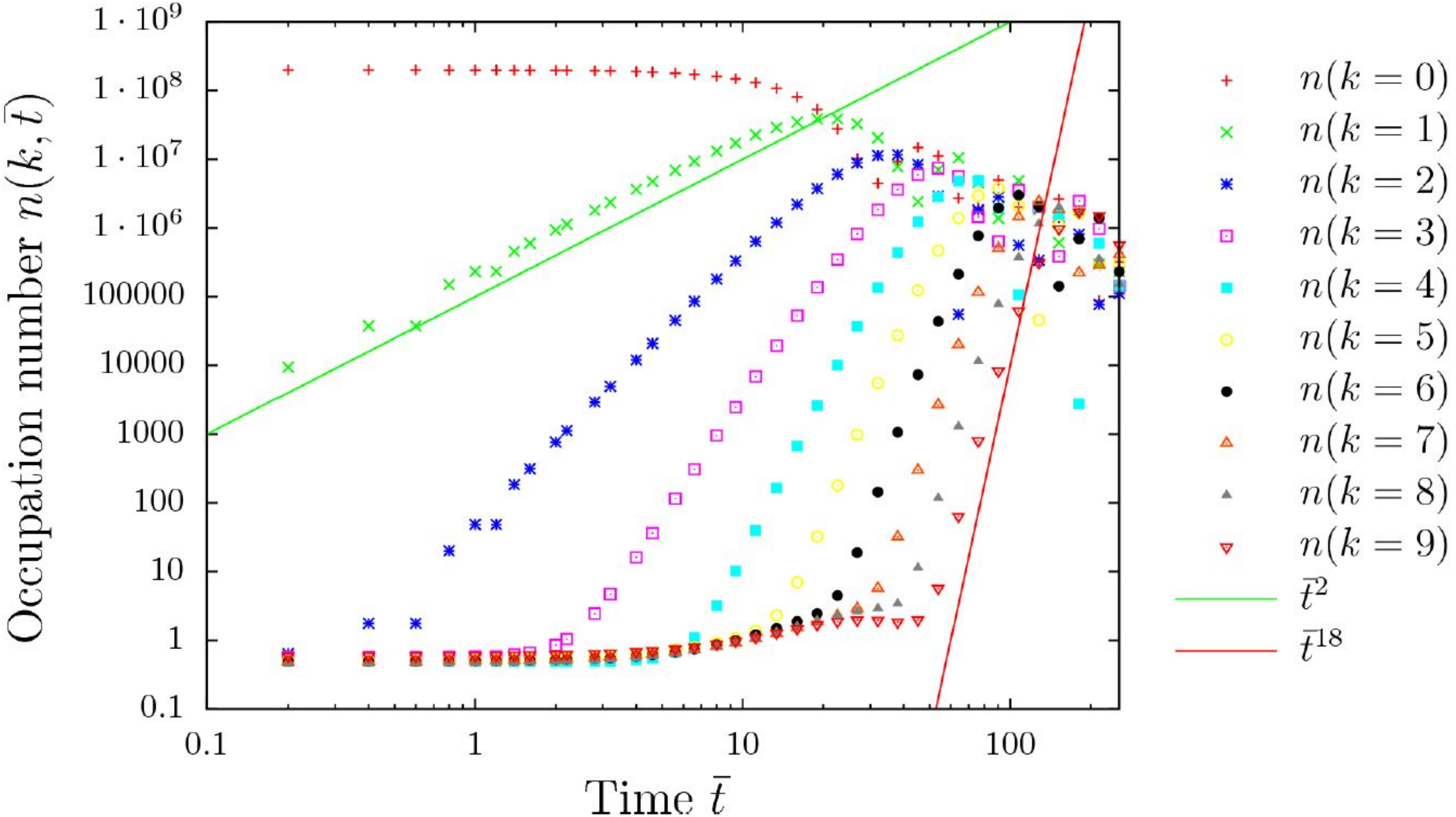
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.61XX [cond-mat.quant-gas]

Time evolution of Energy Components (3+1 D)



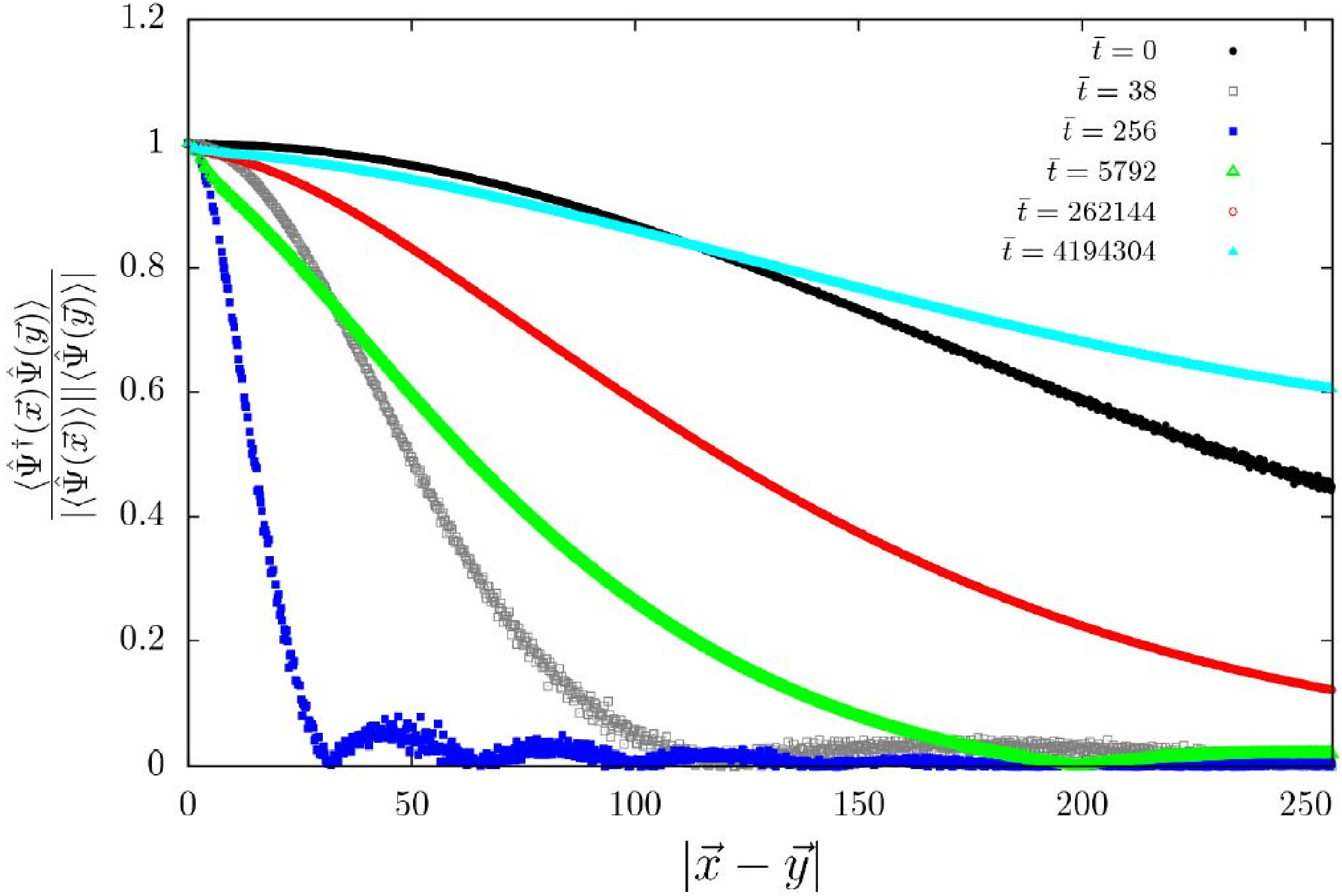
J. Schole, B. Nowak, D. Sexty, TG (unpublished)

Mode Occupations



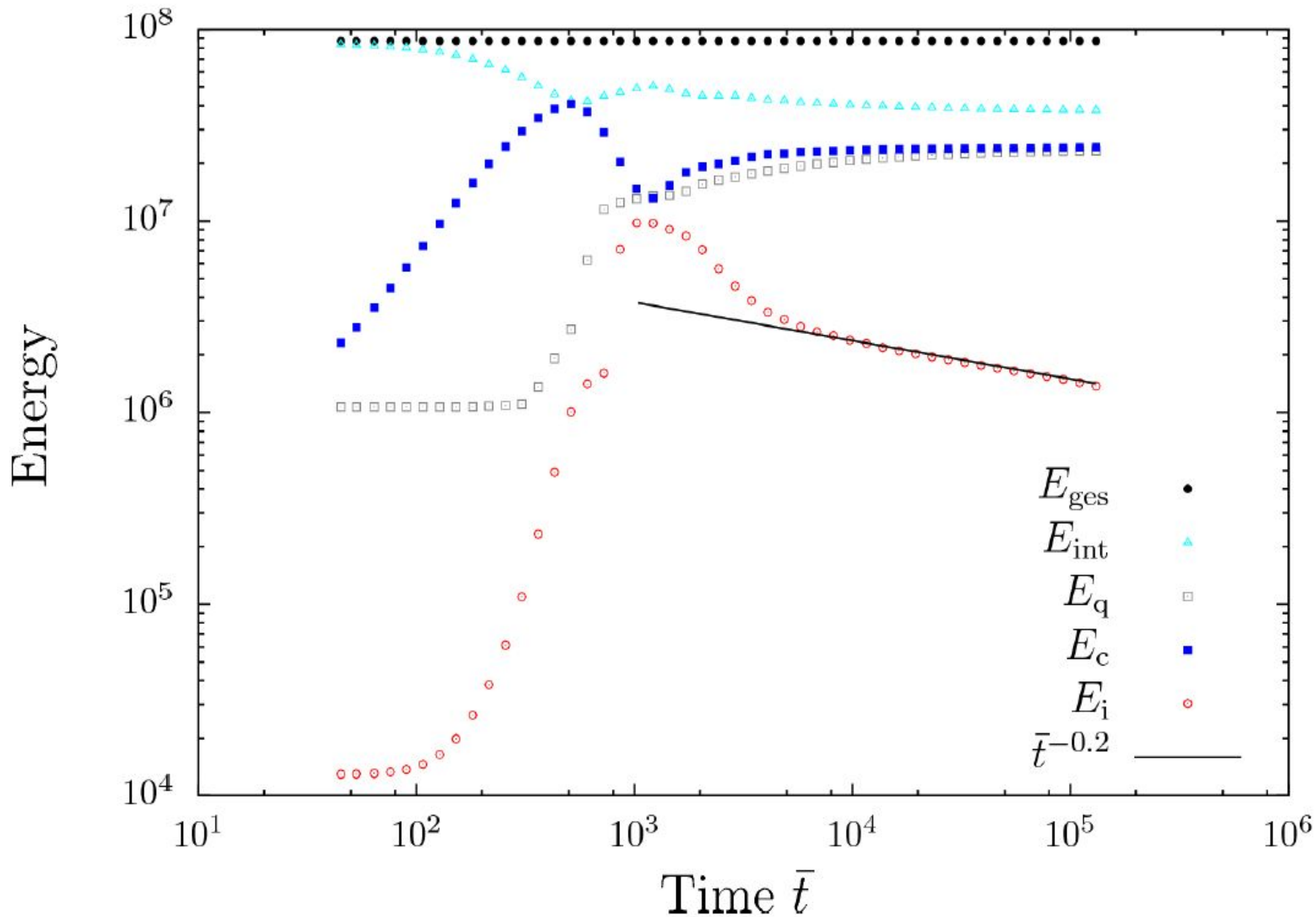
J. Schole, B. Nowak, D. Sexty, TG (unpublished)

1st-order Coherence



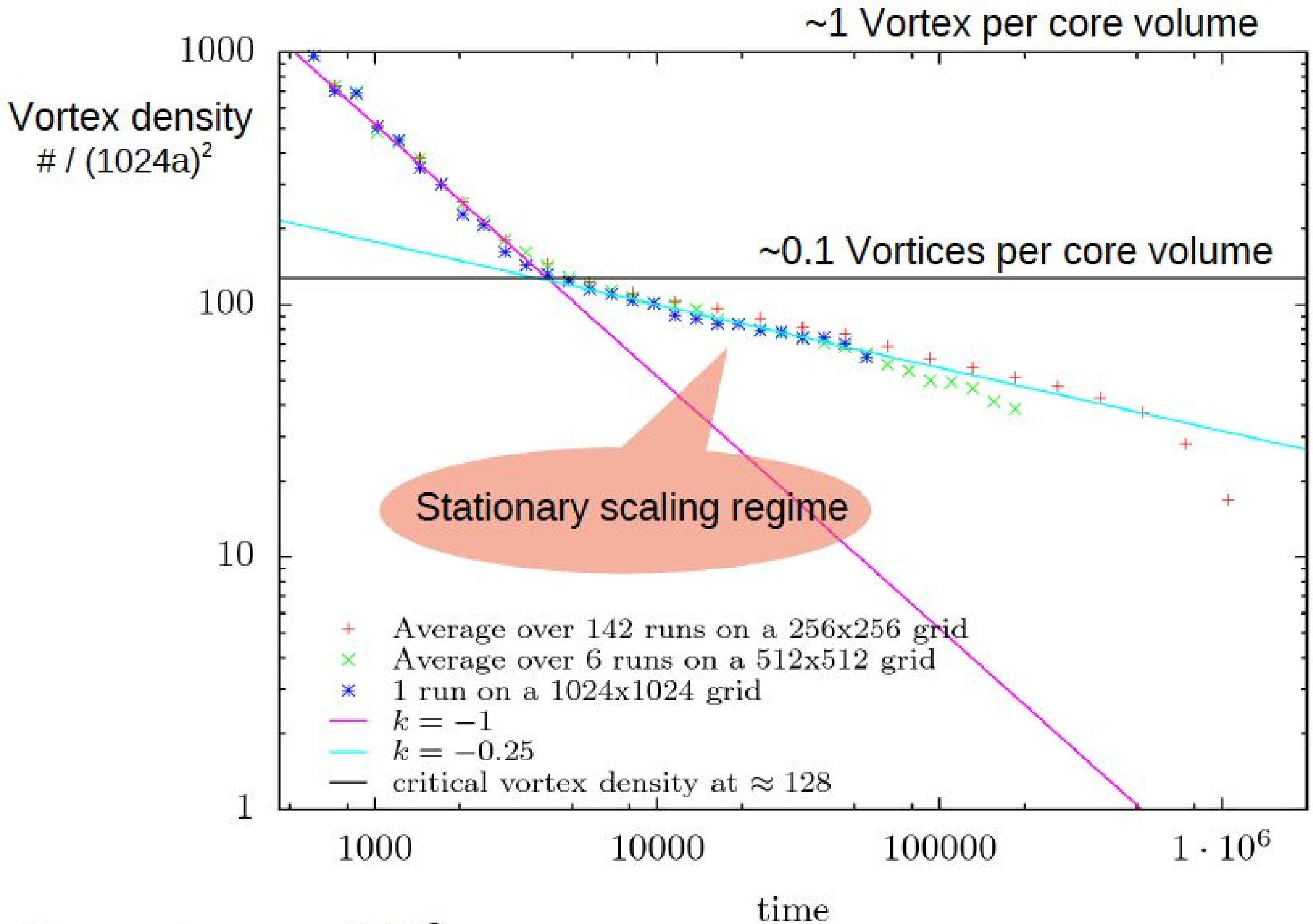
J. Schole, B. Nowak, D. Sexty, TG (unpublished)

Time-Evolution of Energy-components (2+1 D)



J. S., B. Nowak, D. Sexty, T. Gasenzer (unpublished)

Time evolution of vortex density



Core volume ~ $\pi(3\xi)^2$

J. Schole, B. Nowak, D. Sexty, TG (unpublished)

Enstrophy in classical turbulence

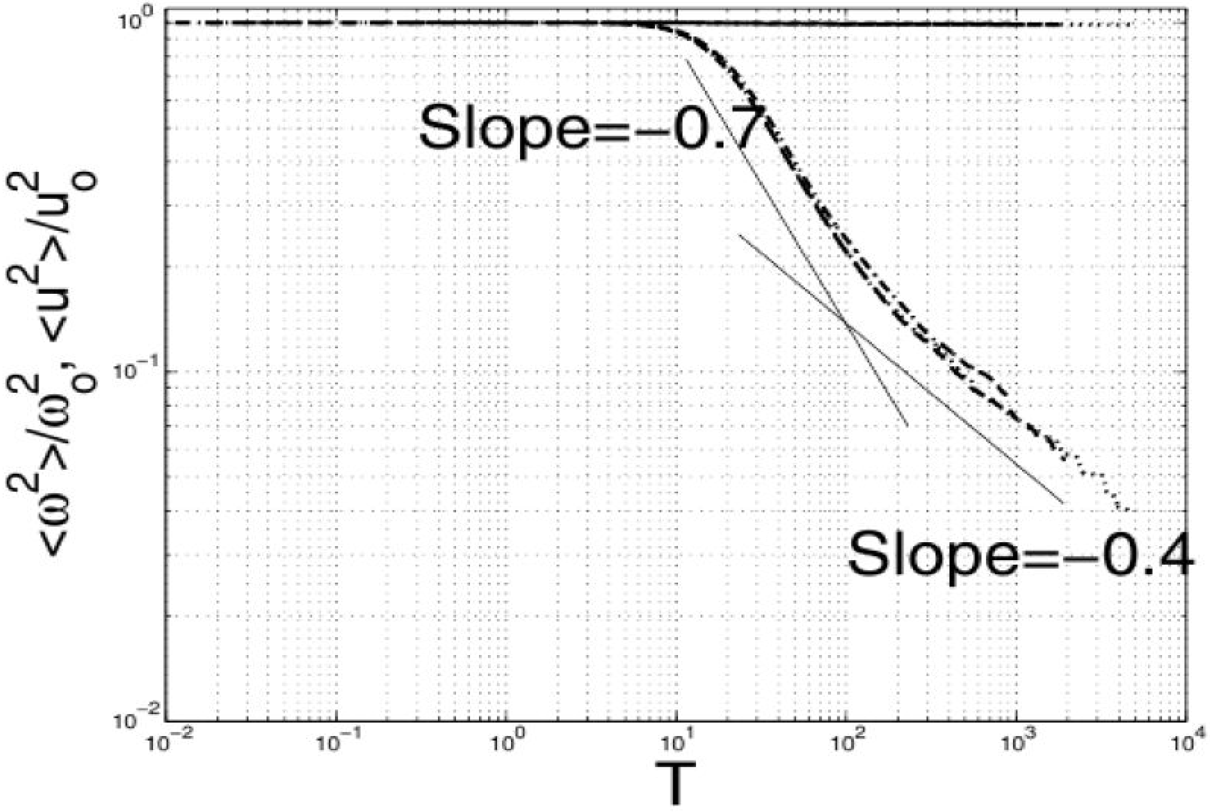
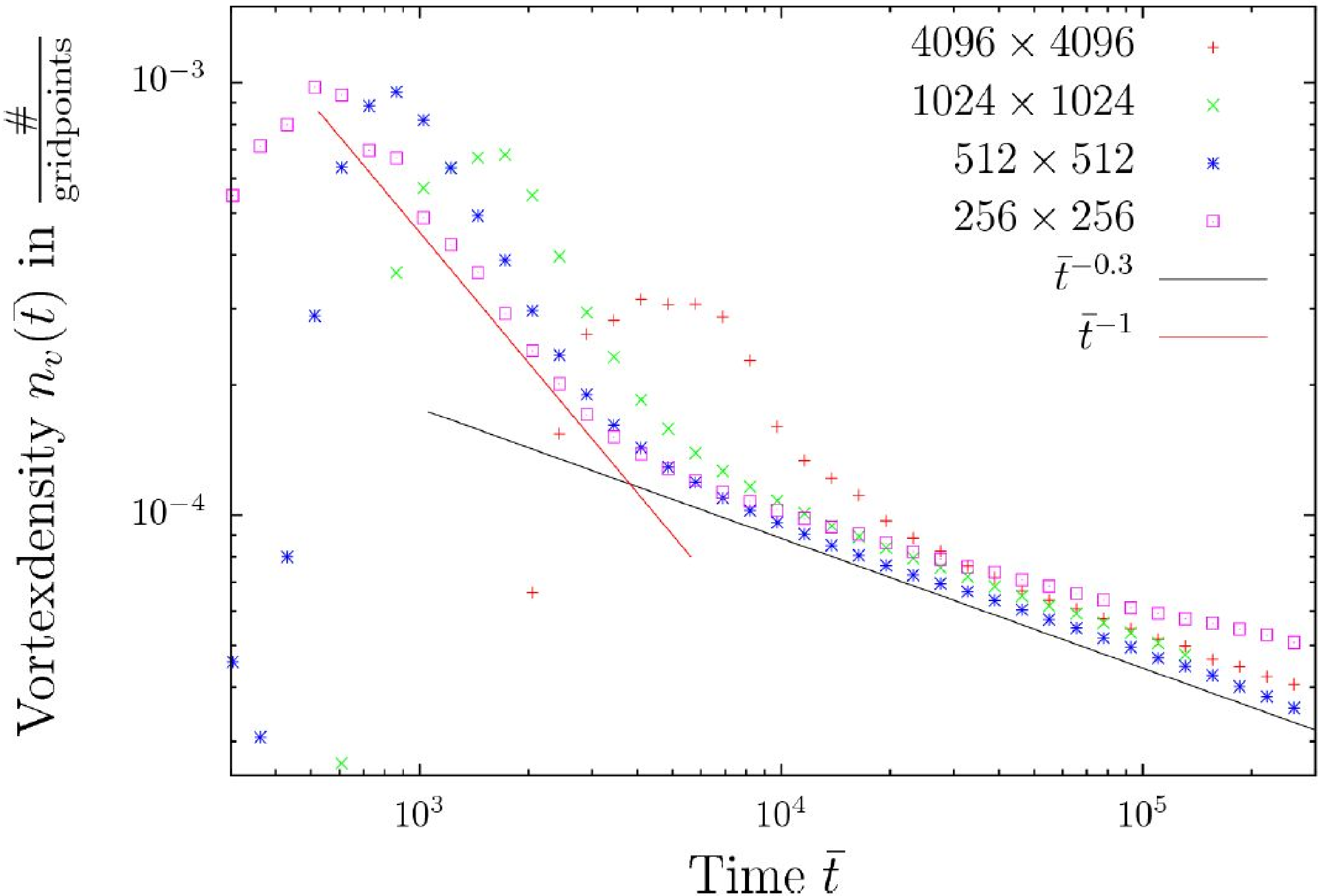


FIG. 1. Time evolution of energy (upper horizontal curve) and enstrophy. Resolution: dotted line (512^3); dashed line (1024^3); dash-dotted line (2048^3).

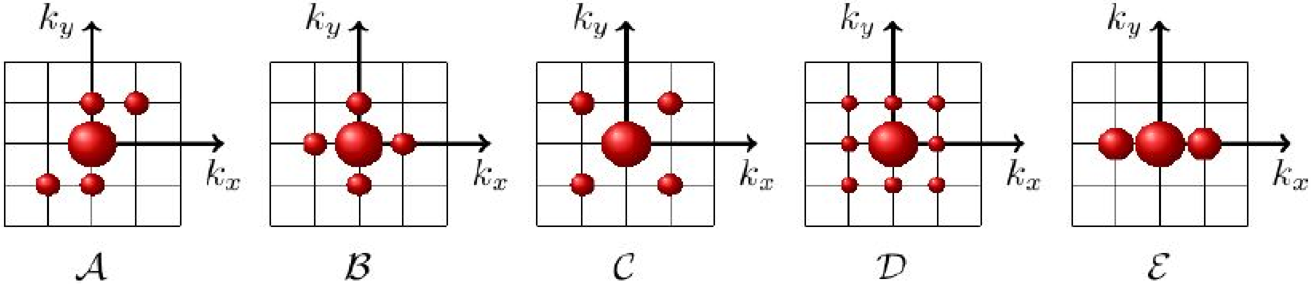
V. Yakhot, J. Wanderer, PRL 93:154502

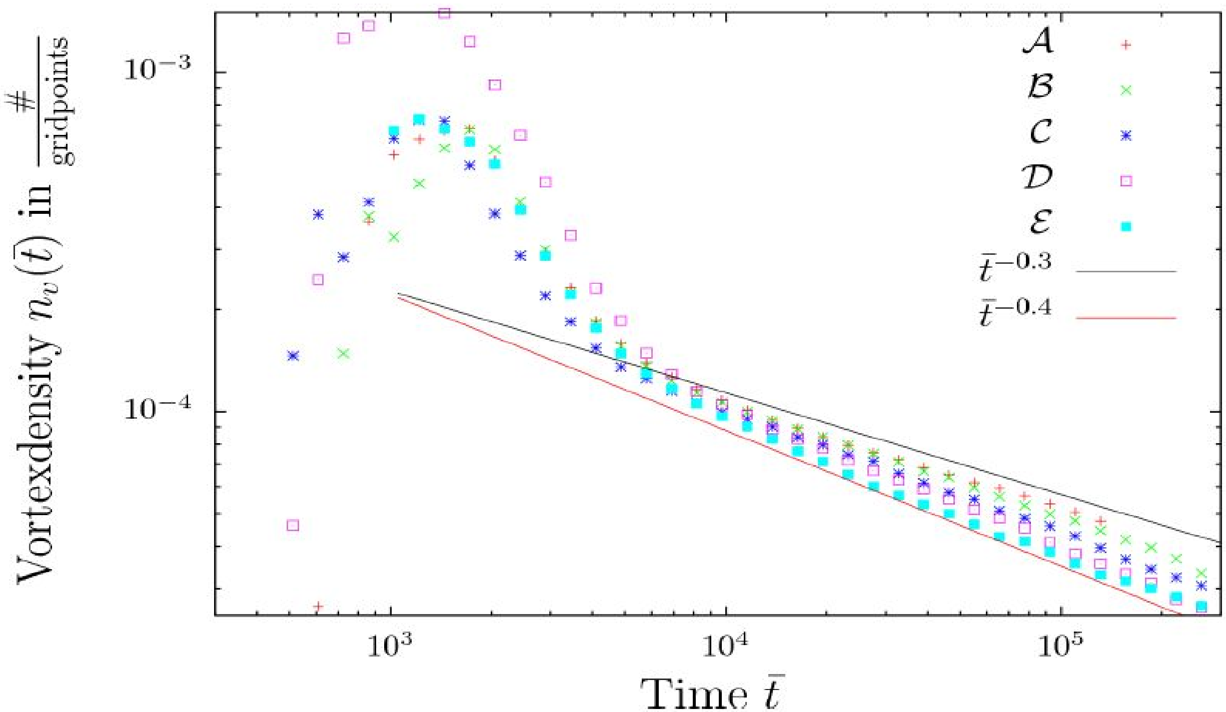
Vortex-Density Decay in 2d



J. S., B. Nowak, D. Sexty, T. Gasenzer (unpublished)

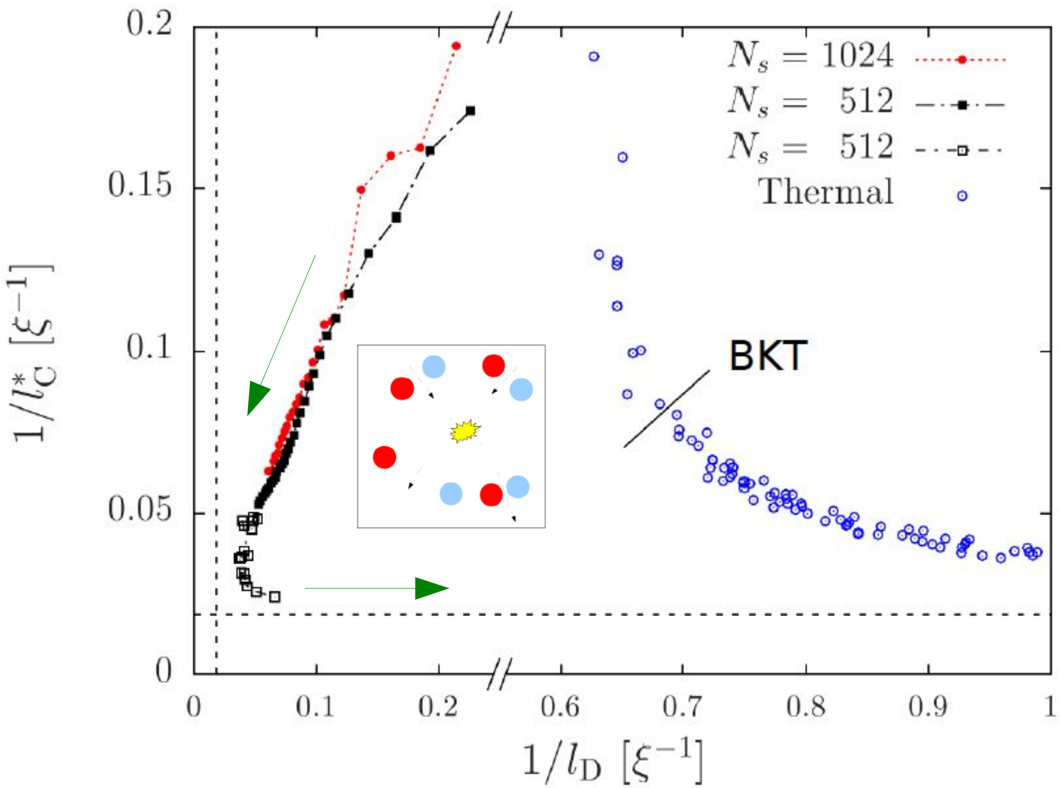
Vortex-Density Decay in 2d

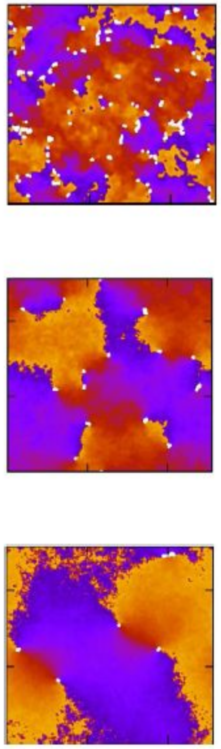




J. S., B. Nowak, D. Sexty, T. Gasenzer (unpublished)

Approach of the NTFP





l_C^* = Phase coherence length

l_D = Vortex-Antivortex pair distance

J. Schole, B. Nowak, TG, PRA **86** (12) 013624

Kinetic Theory

One of the power laws can be explained by a kinetic theory:

$$\partial_t n_v(t) = -\frac{n_{\rm dip}}{\tau_{\rm ann}}$$

$$n_{\rm dip} \sim n_v \qquad \sigma \sim d$$

$$\tau_{\rm ann} = \tau_{\rm coll}\,\alpha \qquad \bar{v} = \frac{1}{d}$$

$$\tau_{\rm coll} = \frac{l}{\bar{v}} \qquad d = \frac{1}{\sqrt{n_v}}$$

$$l \sim \frac{1}{n_v \sigma}$$

d : average pair distance

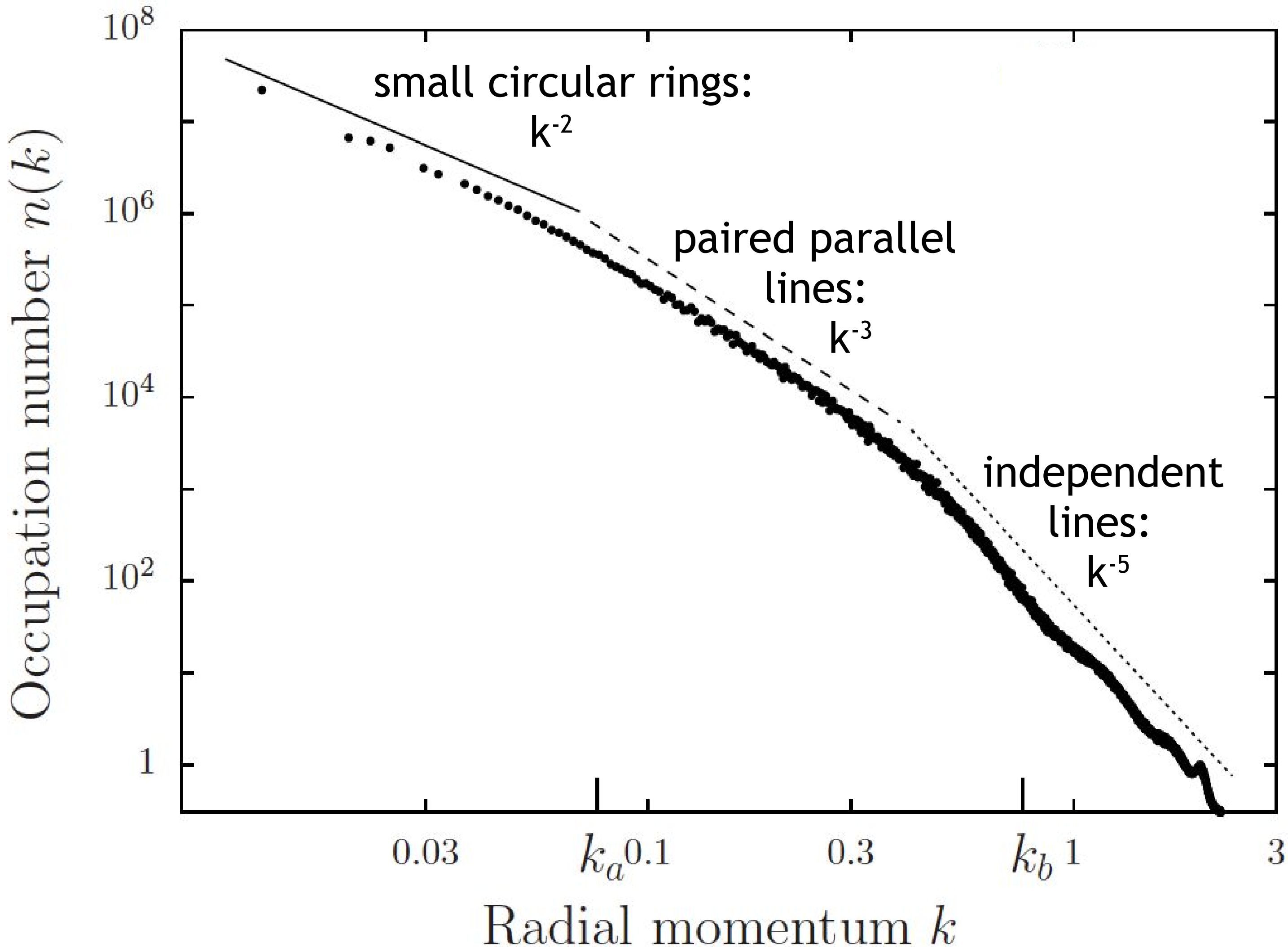
$\bar{v}$: average pair velocity

l : mean free path

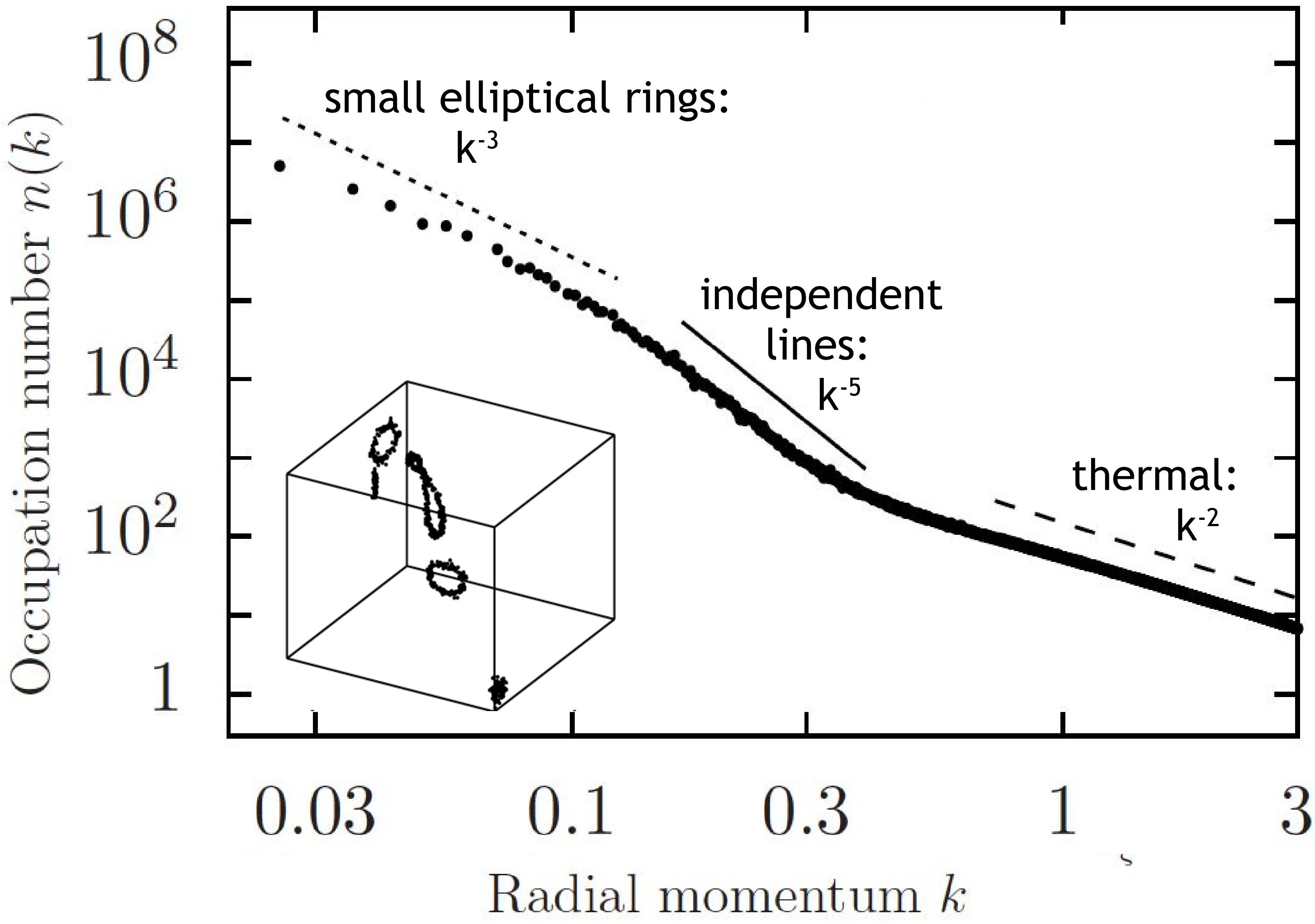
$$\Rightarrow \quad \partial_t n_v(t) \sim -n_v^2 \quad \Rightarrow \quad n_v(t) \sim t^{-1}$$

This result is valid under the assumption that the vortices are moving in pairs and that the pairs are homogeneously distributed.

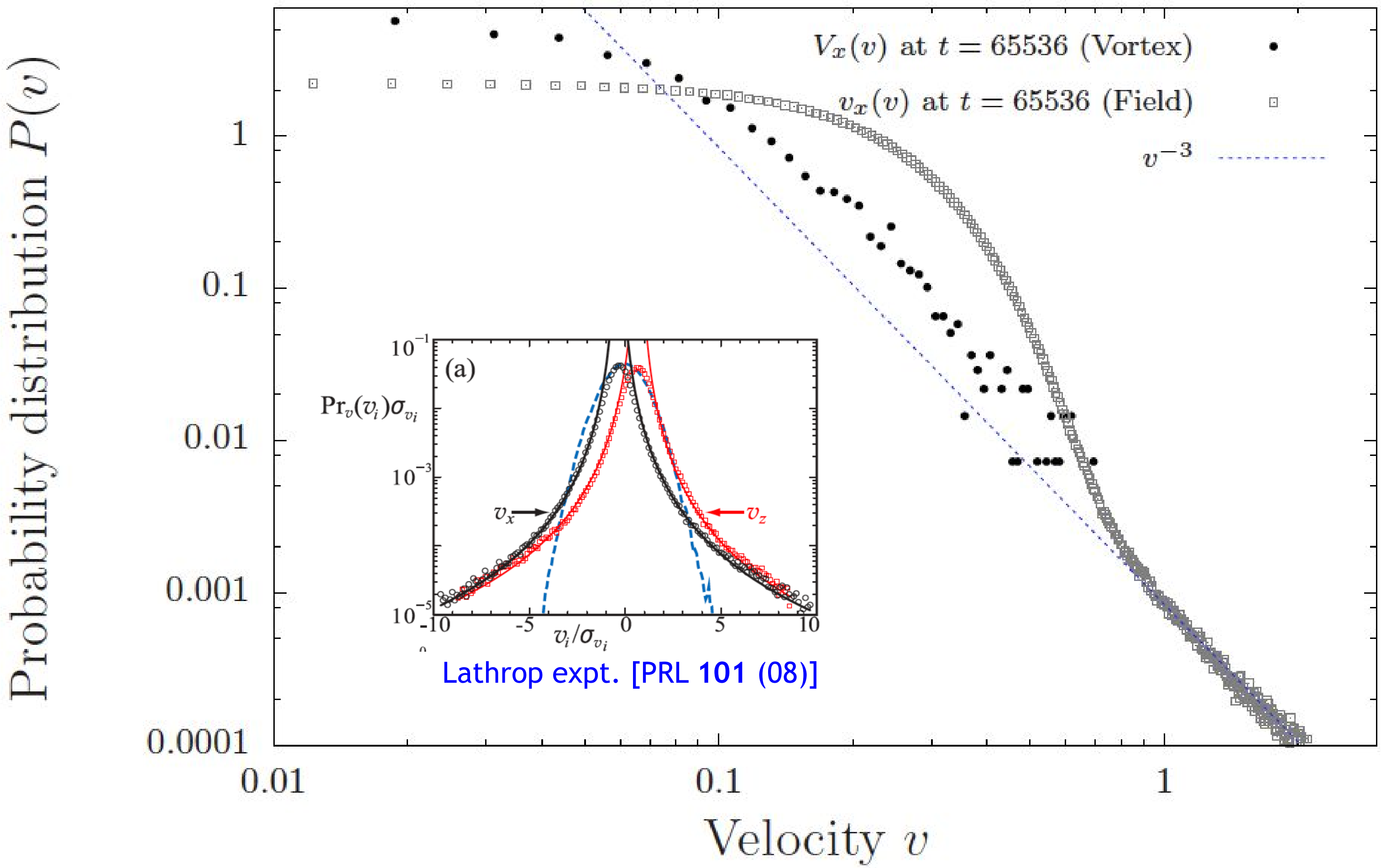
Line vortex model in 3+1 D



Simulations in 3+1 D



Vortex velocity distribution



J. Schole, B. Nowak, D. Sexty, TG (unpublished)
s. also C.F. White et al., PRL **104** (10); I.A. Min, Phys. Fluids **8** (96)

Velocity distributions

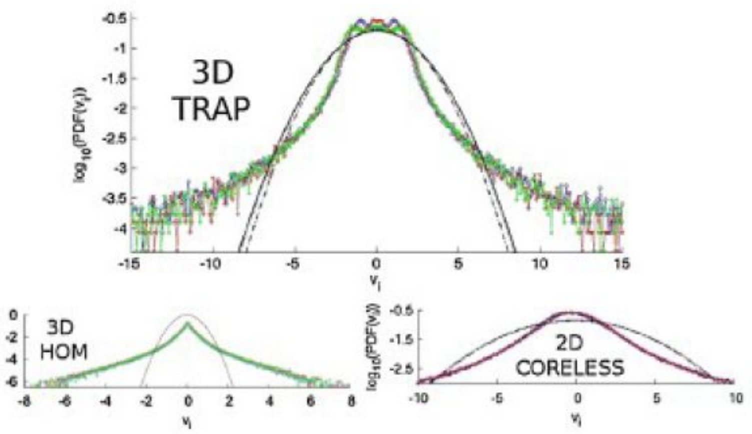
Paoletti et al. PRL 101, 154501 (2008):

Power law tails distinguish classical turbulence from classical turbulence.

Min et al. Phys. Fluids 8, 1169 (1996), White et al. PRL 104, 075301 (2010):

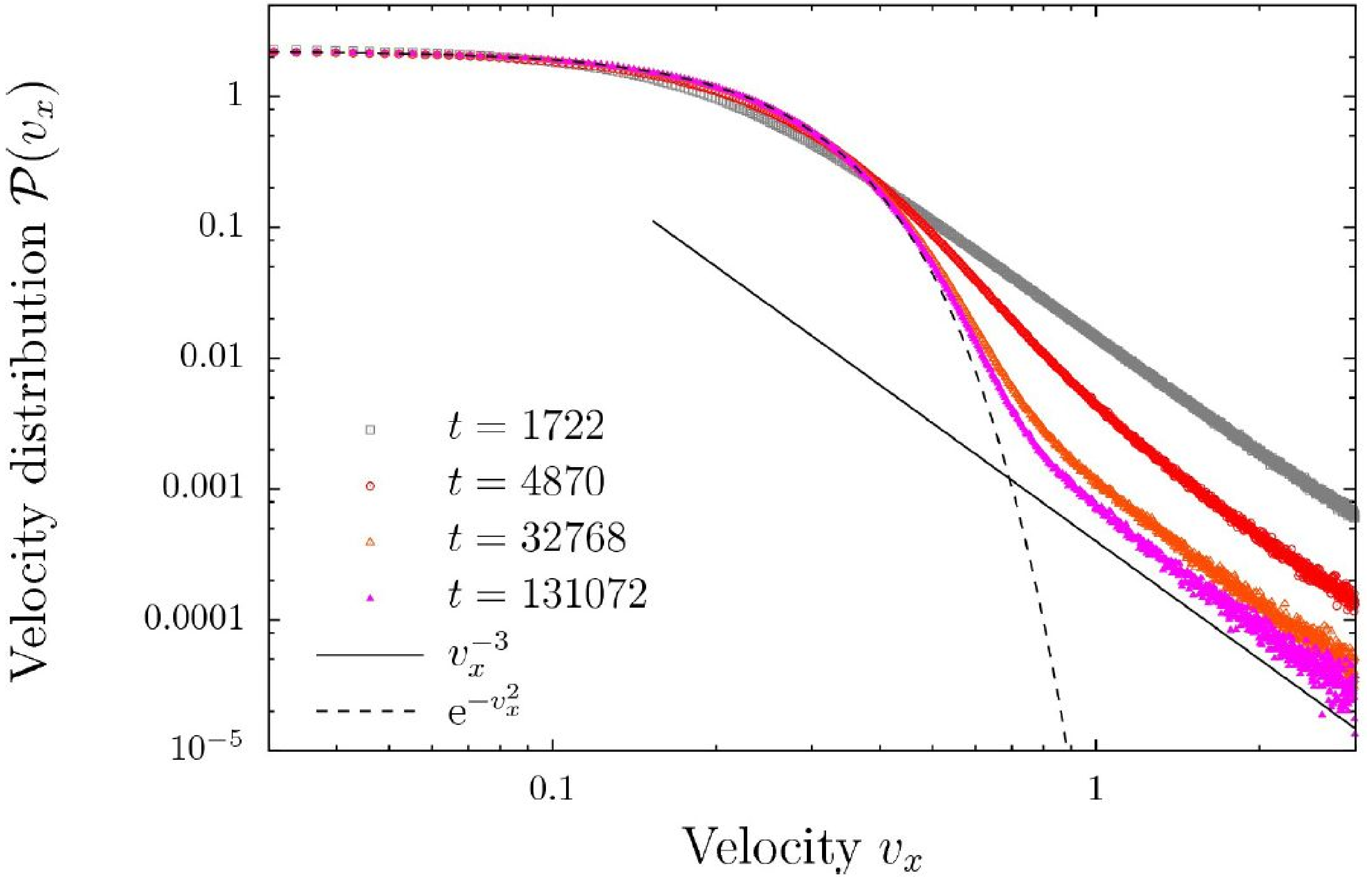
Point vortices: Power law tails

Vorticity patches: Gaussian distributions



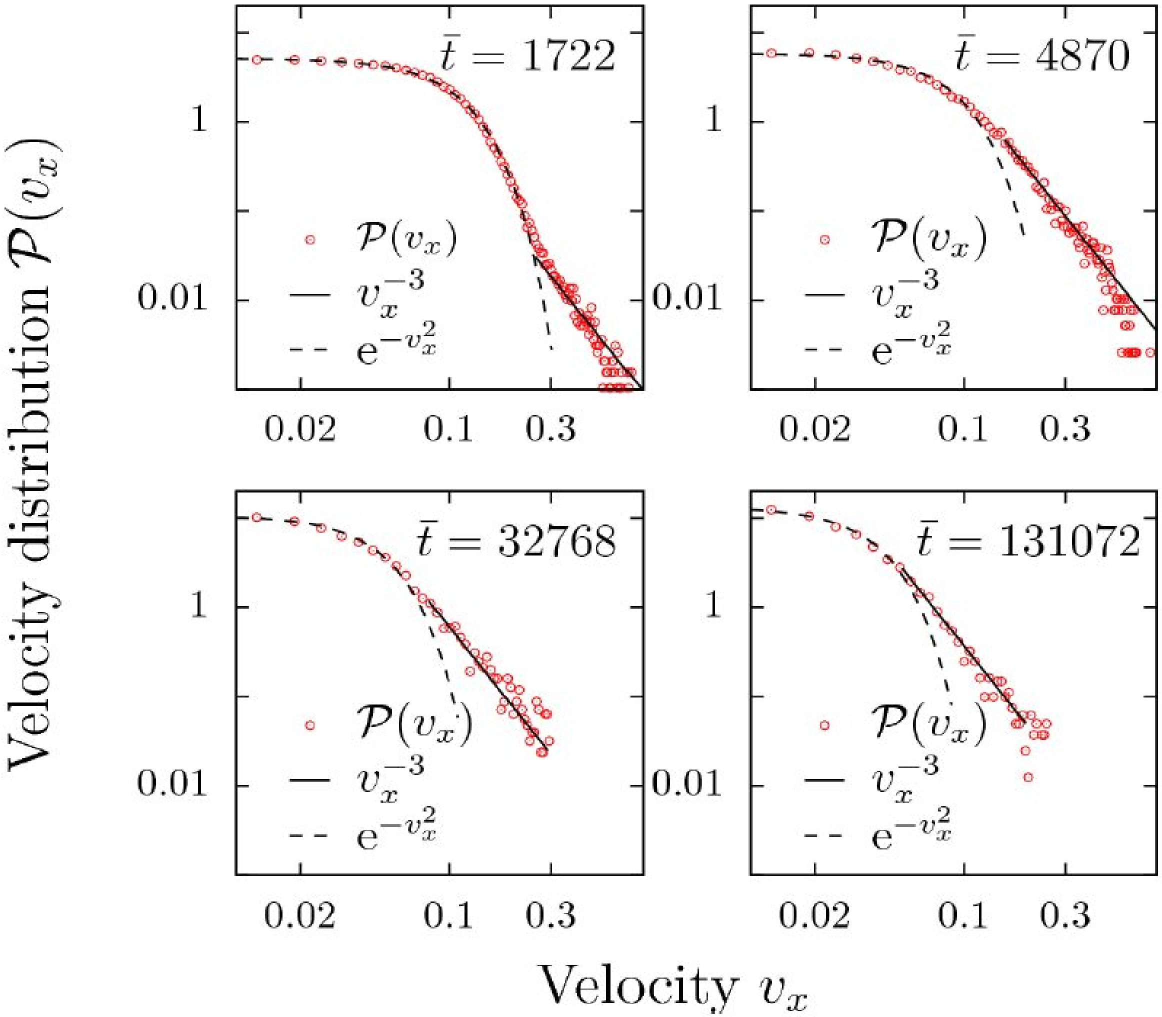
White et al. PRL 104, 075301 (2010)

Velocity distributions (Field)



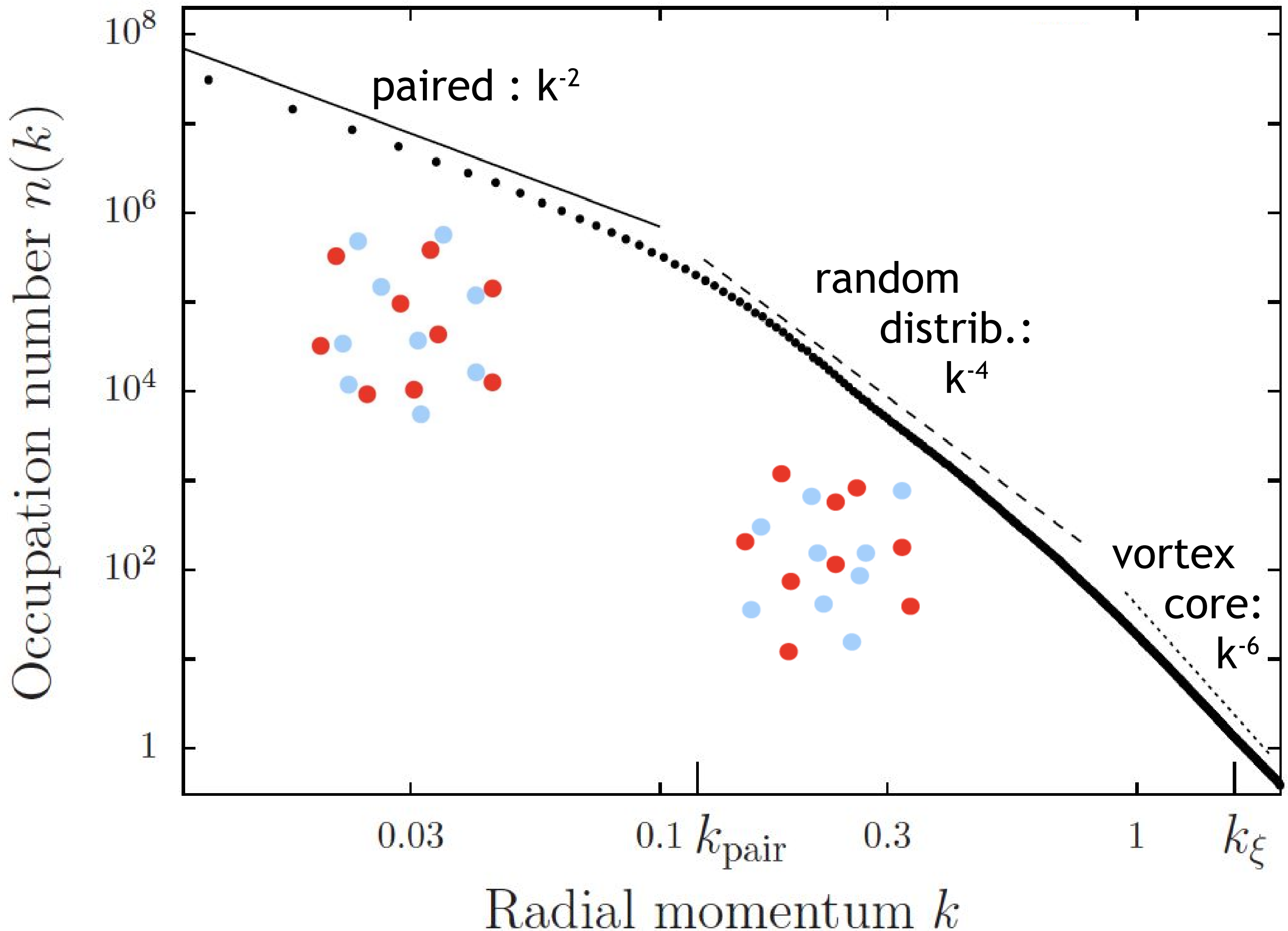
J. S., B. Nowak, D. Sexty, T. Gasenzer (unpublished)

Velocity distributions (Vortices)



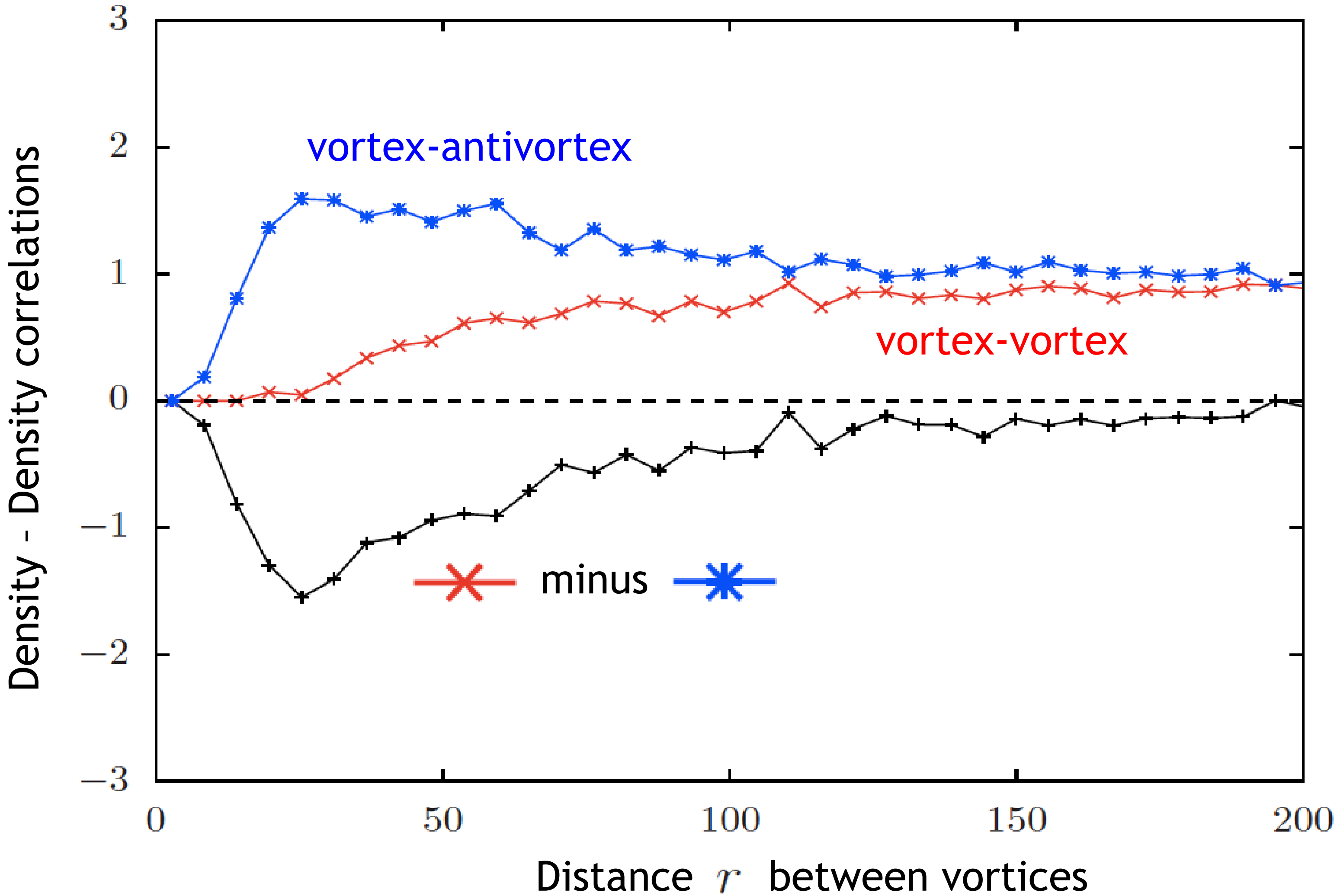
J. S., B. Nowak, D. Sexty, T. Gasenzer (unpublished)

Point vortex model in 2+1 D



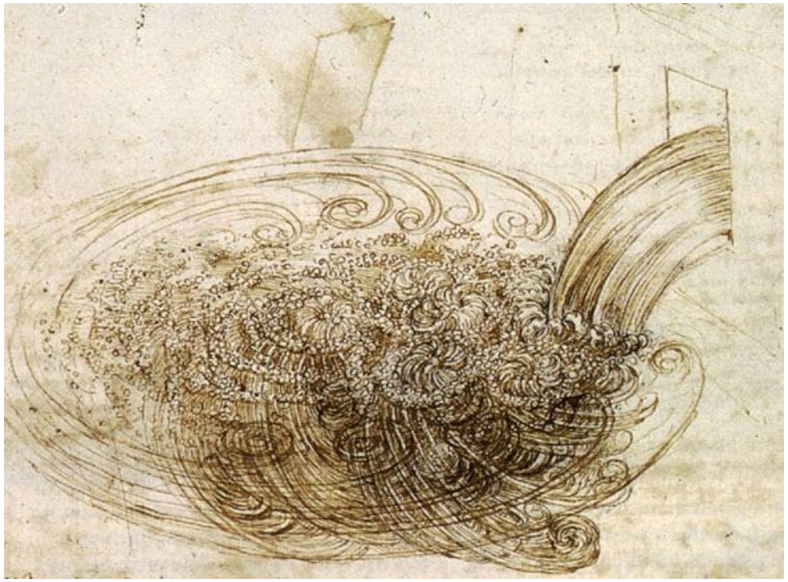
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.6127

Vortex position correlations



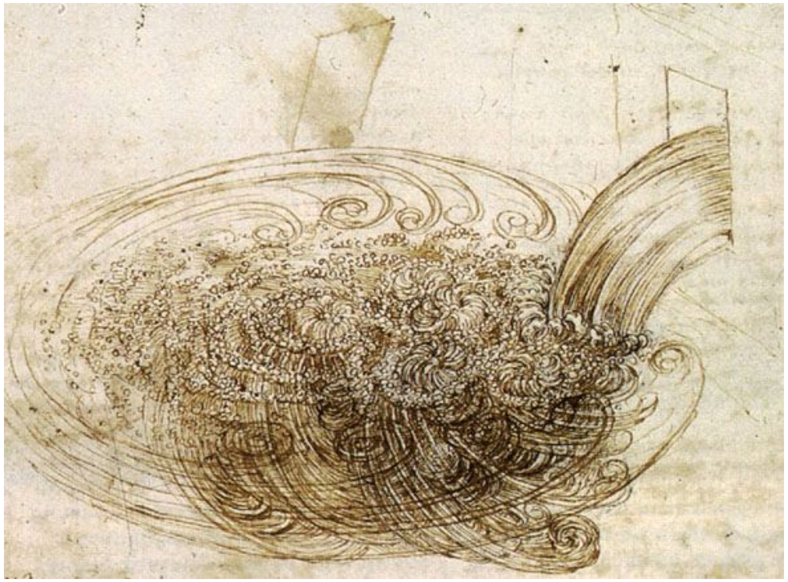
B. Nowak, J. Schole, D. Sexty, TG, arXiv:1111.6127

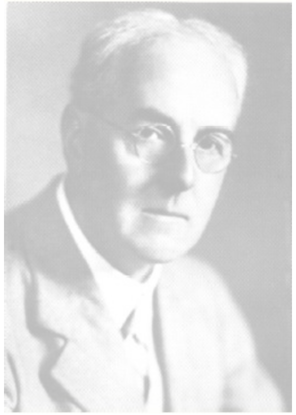
Classical Turbulence



Leonardo da Vinci
(1452-1519)

Classical Turbulence





Lewis F. Richardson
(1881-1953)

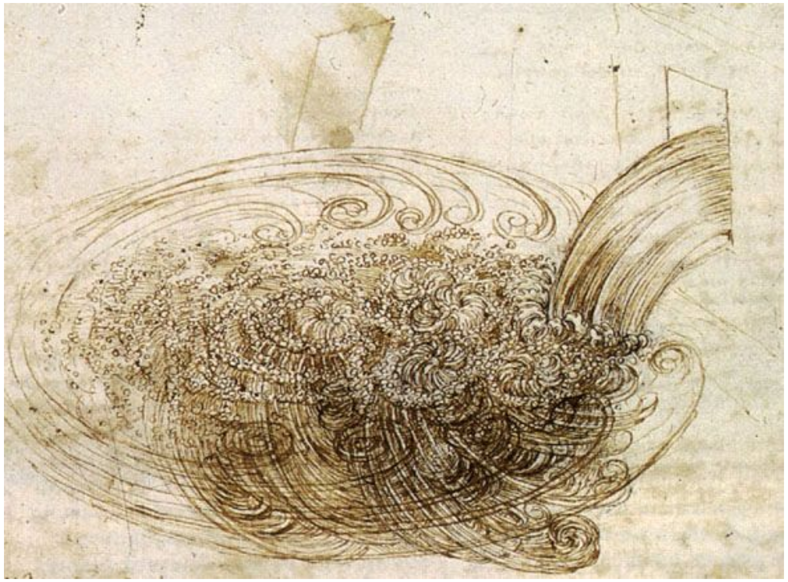
Richardson cascade

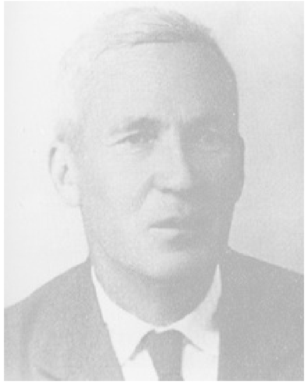
large scales (source)
→ small scales (sink)

*"Big whirls have little whirls that feed on their velocity,
and little whirls have lesser whirls and so on to viscosity."*

(Richardson, 1920)

Classical Turbulence





Andrey N. Kolmogorov
(1903-1987)

Richardson cascade

large scales (source)
$\rightarrow$ small scales (sink)

"Big whirls have little whirls that feed on their velocity, and little whirls have lesser whirls and so on to viscosity."

(Richardson, 1920)

Kolmogorov (1941) $E(k) \sim k^{-5/3}$ (for incompressible fluids)