Nonlinear Luttinger Liquid

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Yale University

Low-D Quantum Dynamics

Session in memory of Adilet Imambekov

KITP, August 23, 2012

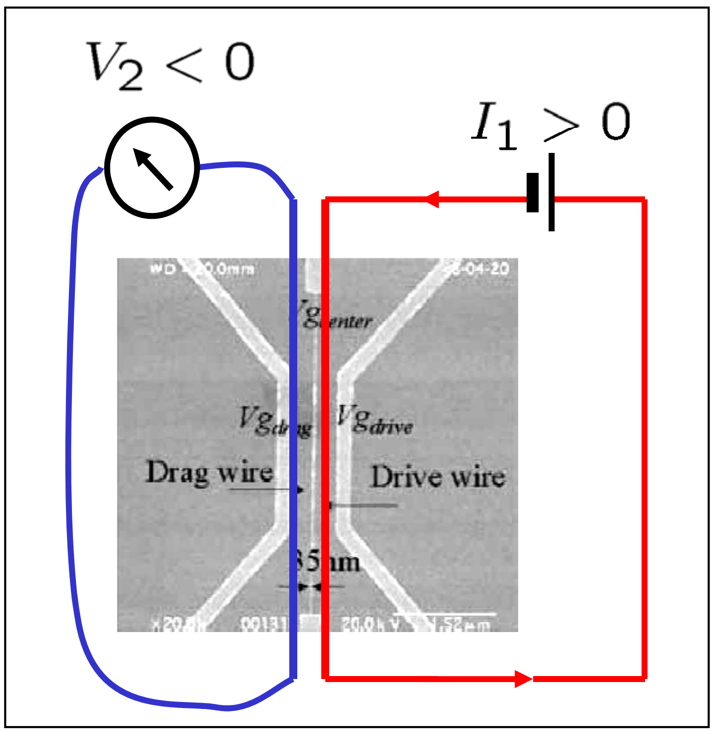
Outline

- Motivation: what is missed by linear Luttinger liquid
- The weak-interaction limit for a liquid of particles with generic spectrum
- The quantum impurity problem for interacting 1D fermions
- Universal theory of nonlinear Luttinger liquid (NLL)
- Phenomenology of the NLL
- Further developments: dynamics, kinetics, finite-size effects and integrable models

My own encounter with the limitations of the conventional Luttinger Liquid theory

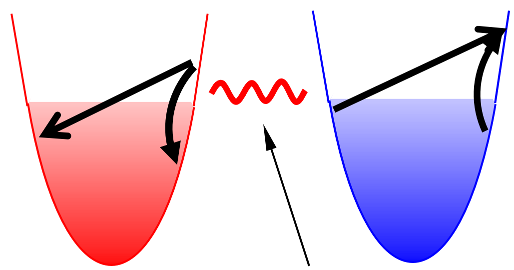
Coulomb drag effect

Tarucha group, 2001



No tunneling, only *e*-*e* scattering

$$R_{\rm drag} = -\frac{dV_2}{dI_1}$$



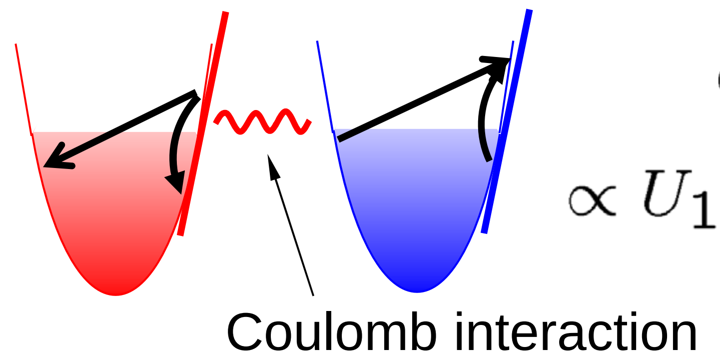
Coulomb interaction

$\sim 2k_F$ transfer

small-q transfer

Hazards of Linearization

$$R_{\mathrm{drag}} = -\frac{dV_2}{dI_1}$$



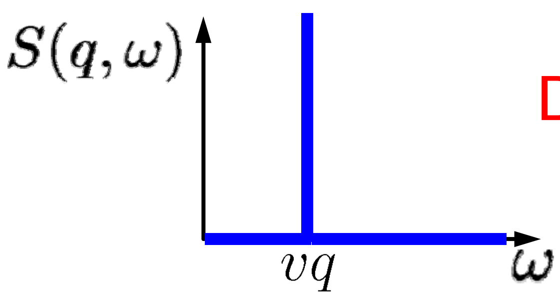
$$\propto U_{12}(2k_F)$$

$$\propto U_{12}(q \sim T/v_F)$$

$$\xi(k) = vk$$

velocity is *k* -- independent, same-branch momentum transfer does NOT affect current, no drag contribution

$$R_{\mathrm{drag}} \propto \int_0^\infty d\omega \int_0^\infty \frac{|U_{12}(q)|^2}{n_1 n_2} F_{\mathrm{thermal}}(\omega, T) S_1(q,\omega) S_2(q,\omega)$$



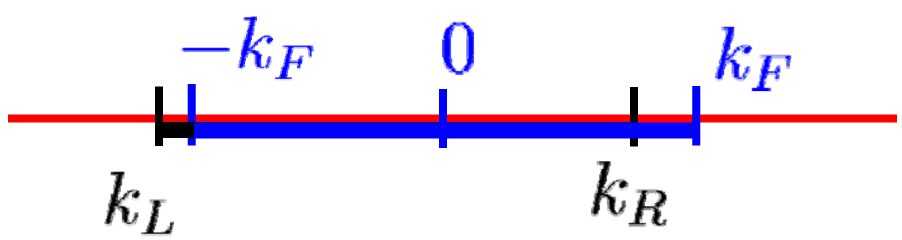
Dynamic structure factor is $\sim q\,\delta(\omega - vq)$

Bosonization (and Spectrum Curvature)

Haldane, 1983

$$k_{L,R}(x) \pm k_F \rightarrow \partial_x \varphi_{L,R}$$

excess number of left (*L*), right (*R*) movers



$$\xi_k = \pm v_F k + \frac{k^2}{2m}$$

$$H_{\mathrm{K}}(x) = \int_{k_L(x)}^{k_R(x)} \xi_k \, dk - \bar{E}_{\mathrm{K}} \rightarrow \frac{v_F}{2}(\partial_x \varphi_R)^2 + \frac{1}{m}(\partial_x \varphi_R)^3 + (R \leftrightarrow L)$$

$$H_{int}(x) = V_{LL}(\partial_x \varphi_L)^2 + V_{RR}(\partial_x \varphi_R)^2 + 2V_{LR}(\partial_x \varphi_L)(\partial_x \varphi_R)$$

$\varphi_{L,R} \leftrightarrow \varphi \pm \vartheta$ excess density ($n(x) = \partial_x \varphi$), momentum ($\propto \partial_x \vartheta$)

Harmonic Approximation, Quantized

$$\mathcal{H} = \frac{v}{2\pi}\int dx\left[\frac{1}{K}(\partial_x\varphi)^2 + K(\partial_x\vartheta)^2\right]$$

conjugate variable (momentum)

$[\varphi(x), \vartheta(y)] \propto \mathrm{sign}(x-y)$

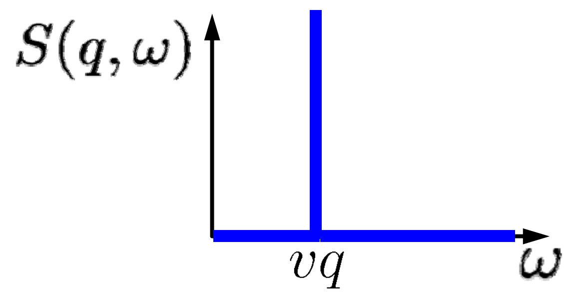
field of displacements, $n(x) = \partial_x\varphi$

Unintended consequences:

A. Dynamic structure factor is

$\sim q\,\delta(\omega - vq)$

at ANY interaction strength



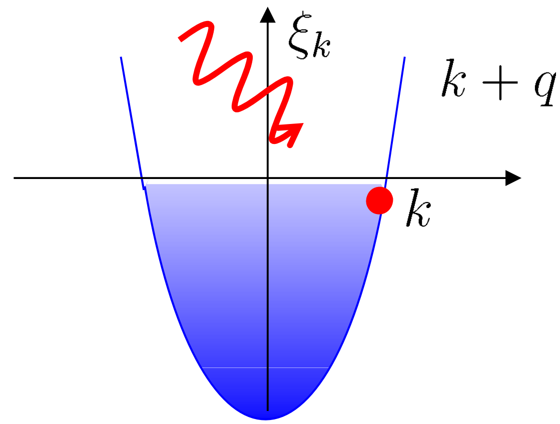
B. Harmonic system identical to free particles, no ANY kinetics

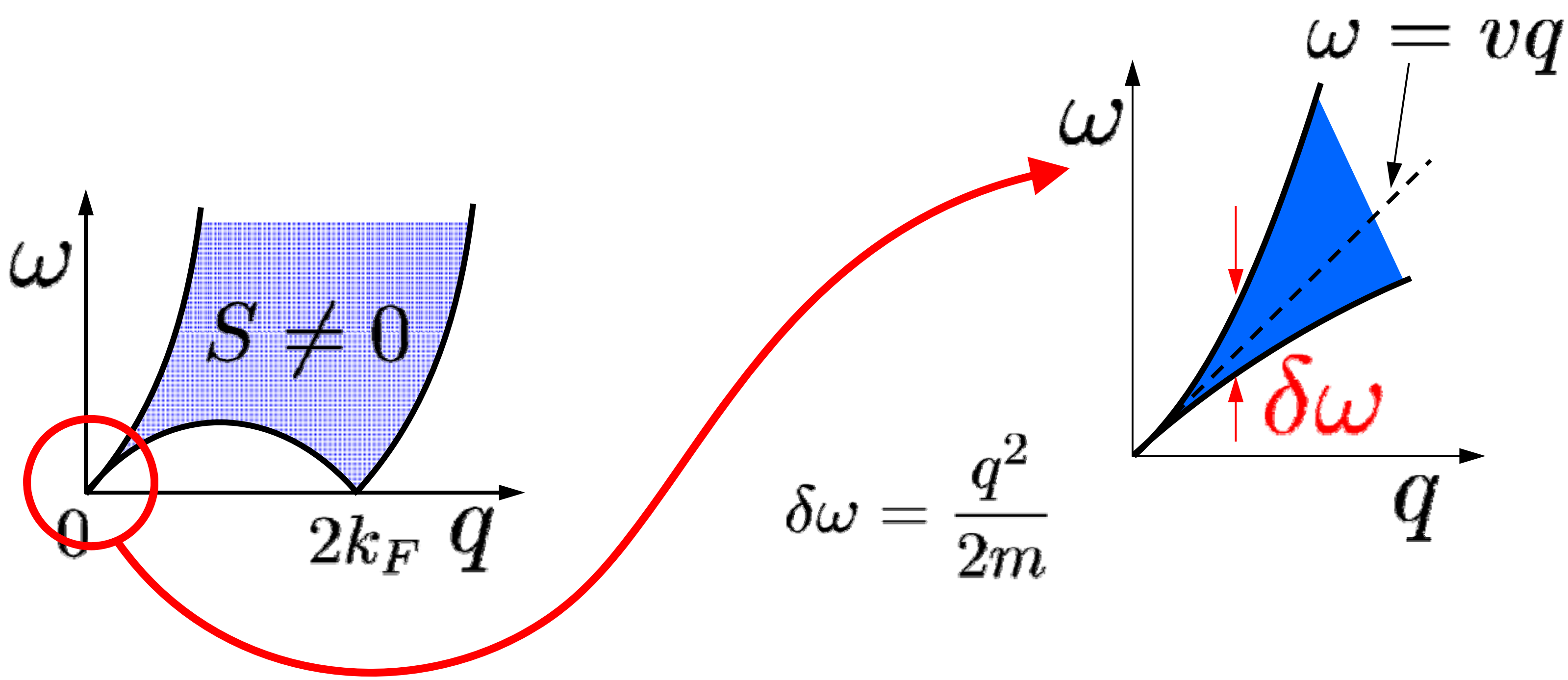
Can we account for the nonlinear dispersion AND interactions?

Back to free fermions

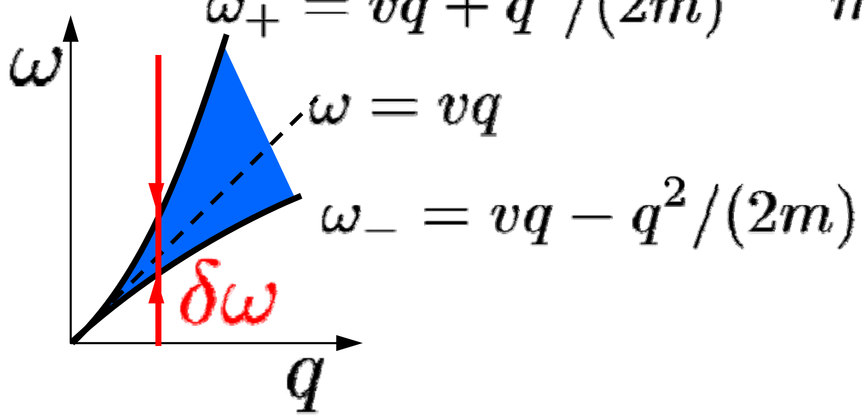
Lehmann (Golden rule – like) representation

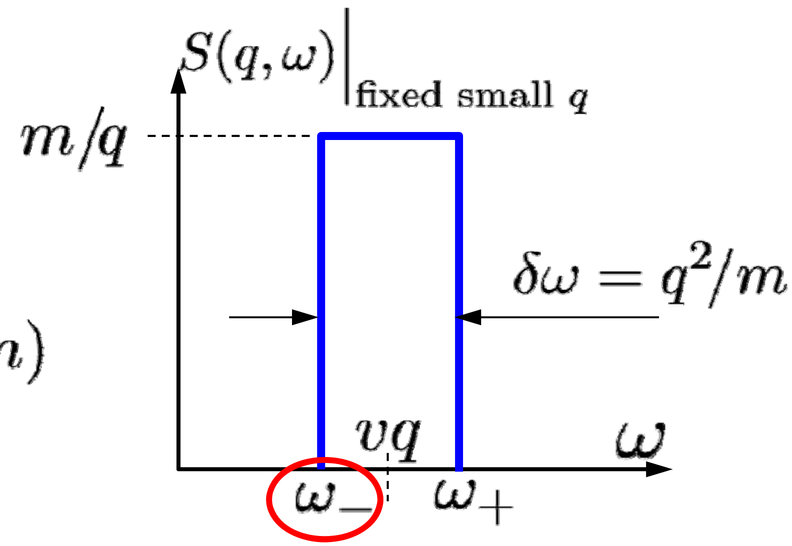
$$S(q,\omega) = 2\pi \sum_{k=k_F-q}^{k_F} \delta\left[\omega - (\xi_{k+q} - \xi_k)\right]$$





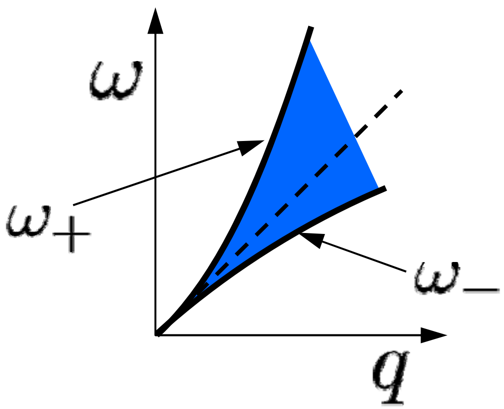
Curvature: free fermions perspective

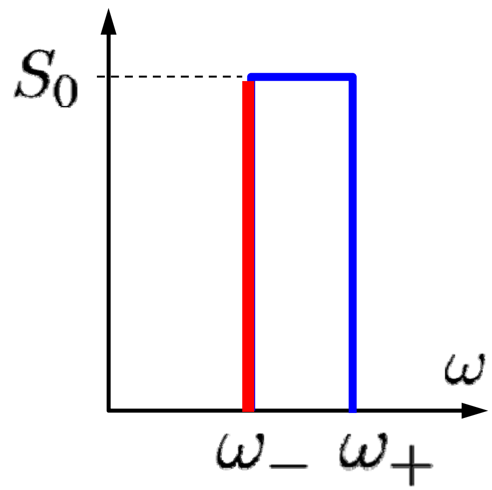


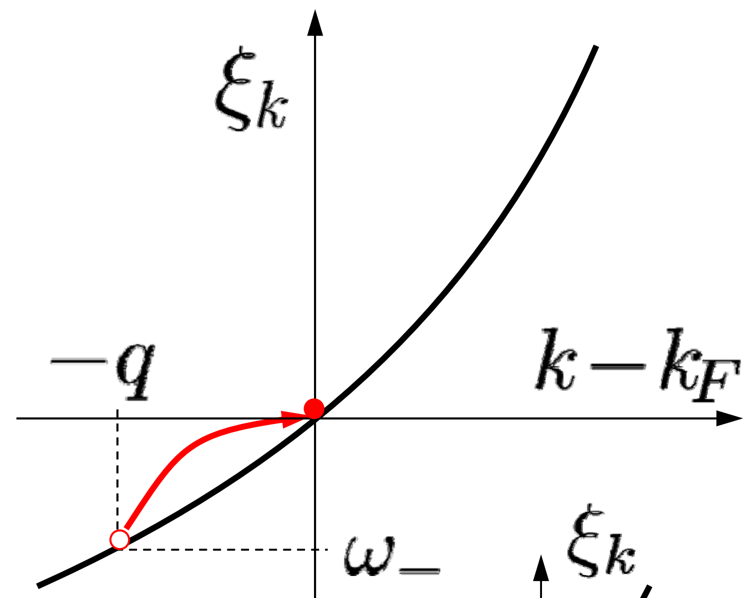


Effect of interaction, $\omega \to \omega_-$

free electrons

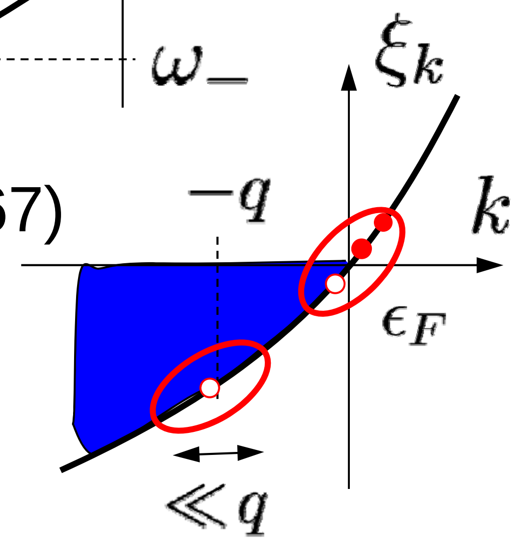






Analogy: Fermi-edge singularity (Mahan 1967)

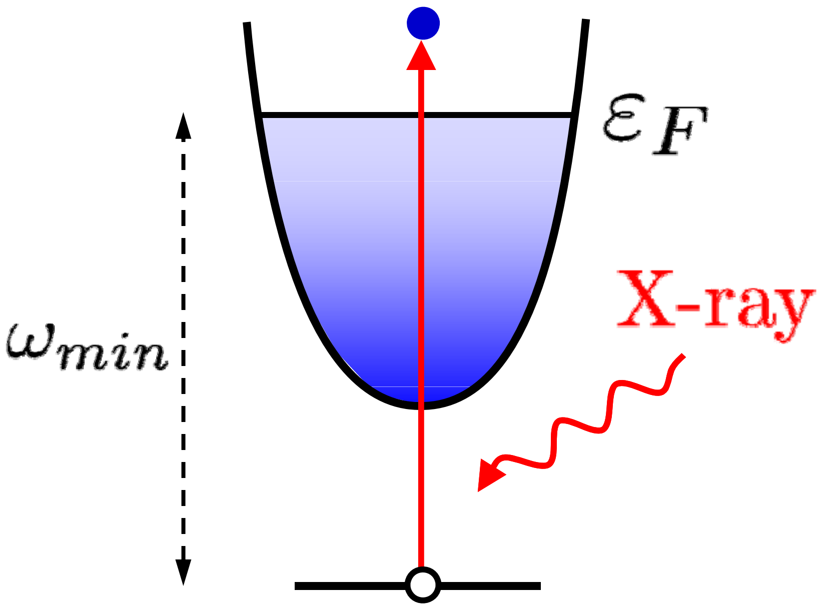
important states:

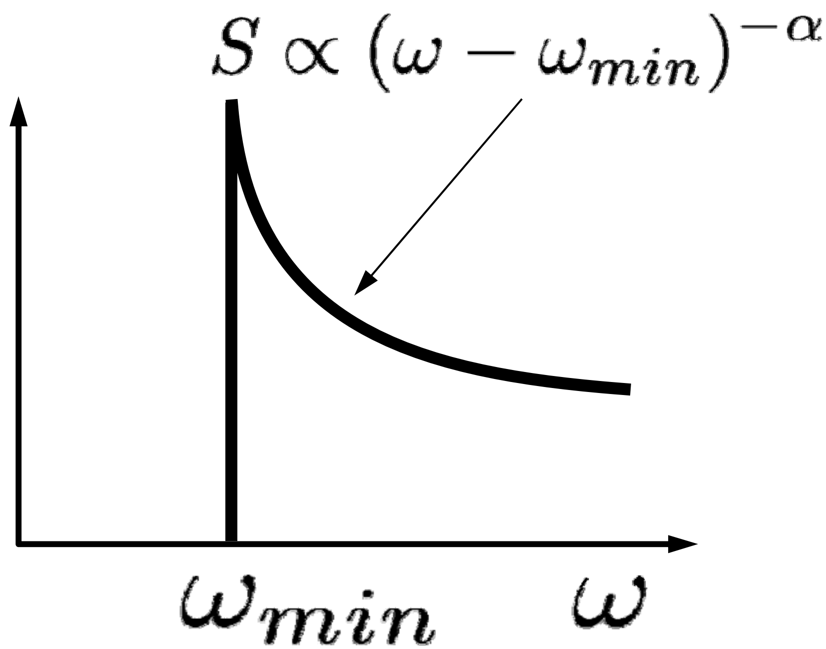


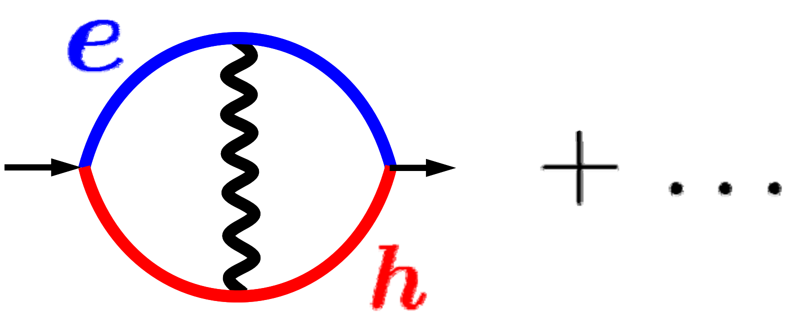
interaction with the “core hole”

singularity $[\ln(\omega - \omega_-)]^n$ in each order of perturb. theory in V

Fermi edge singularity in metals



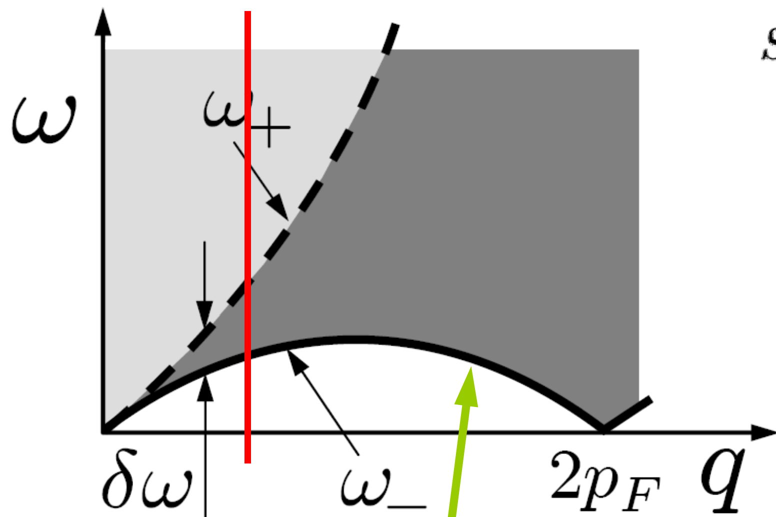




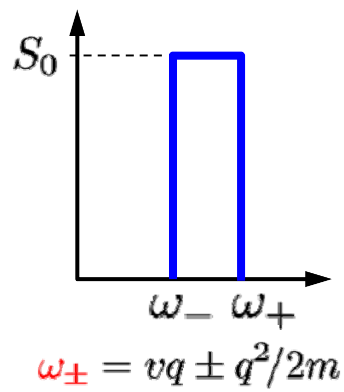
Mahan 67
Nozieres, DeDominicis 69

threshold + interactions = power law

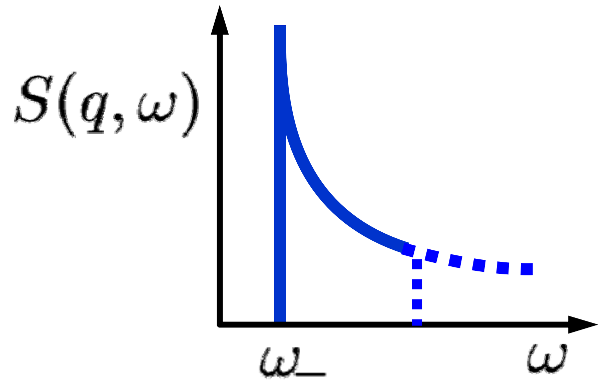
1D Fermions – Dynamic Structure Factor



No interaction:



Weak repulsion:



Power-law asymptote

$$\frac{S(q,\omega)}{S_0} = \left[\frac{\delta\omega}{\omega - \omega_-(q)}\right]^{\mu(q)} \propto \sum_l \frac{1}{l!}\left[(V_q - V_0)\ln\frac{\delta\omega}{\omega - \omega_-}\right]^l$$

$$\mu = (V_0 - V_q)m/\pi q \ll 1$$

“Leading logarithm” series

Winter 06-07: Adilet accepts the offer!!!

Subject: postdoc information
From: "Adilet Imambekov" <imambek@fas.harvard.edu>
Date: Fri, December 22, 2006 4:15 pm
To: "Leonid Glazman" <glazman@umn.edu>

```
Also, I wanted to know if there are any formal residency requirements to
stay in Yale during summers?
The reason I'm asking is that I'm seriously interested in
mountaineering, as much as in physics (as probably some other people you
know, like Ioffe). Going to mountaineering expeditions in summer is an
integral part of my life which I want to maintain while doing
postdoctorate studies.
```

From: "Adilet Imambekov" <imambek@cmt.harvard.edu>
Date: Fri, January 26, 2007 3:34 am
To: glazman@umn.edu

```
Leonid,
I officially accept the offer. I hope we will have great time doing
science together.
I checked with International office, it seems that it will be possible to
arrange
  my summer plans to be compatible with visa status.
                                 Adilet
```

From Bethe ansatz to Universality in under one year

27206 (2008) PHYSICAL REVIEW LETTERS W 18 JA

Exact Edge Singularities and Dynamical Correlations in Spin-1/2 Chains

Rodrigo G. Pereira,[1] Steven R. White,[2] and Ian Affleck[1]

[1]*Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, Canada V6T 1Z*

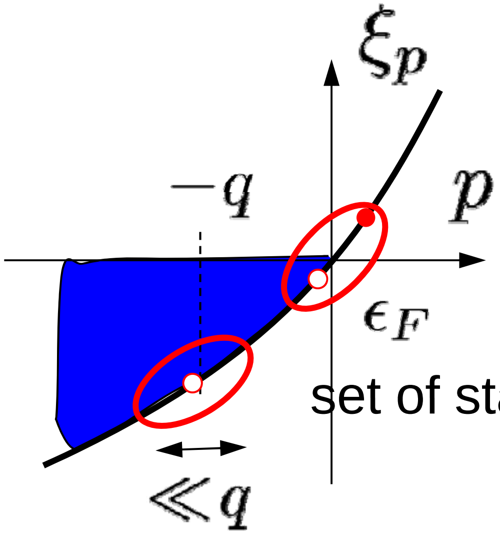
[2]*Department of Physics and Astronomy, University of California, Irvine, California 92697, USA*

(Received 6 September 2007; published 17 January 2008)

Exact formulas for the singularities of the dynamical structure factor, $S^{zz}(q, \omega)$, of the $S = 1/2$ xxz spin chain at all q and any anisotropy and magnetic field in the critical regime are derived, expressing the exponents in terms of the phase shifts which are known exactly from the Bethe ansatz solution.

Can we do the same for Lieb-Liniger?

Quantum impurity: a hint from perturbation theory



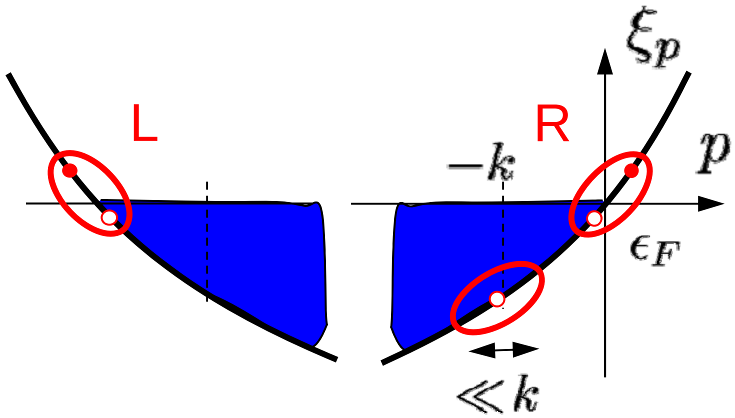
Fermi-edge singularity physics of responses

Shake-up pairs (narrow band containing Fermi level)

set of states admitting 1 hole $v_d \neq v_F$

$$H = \sum_p v_F p\, \psi^\dagger_{R,p}\psi_{R,p} + \sum_p \varepsilon_d(-q+p) d^\dagger_p d_p$$
$$+ [V(0) - V(q)] \sum_p \rho_{R,-p}\rho_{d,p}$$

Generalization on arbitrary interaction



Left and Right movers:

$$H_0 = \frac{v}{2\pi}\int dx\left(K(\nabla\vartheta)^2 + \frac{1}{K}(\nabla\varphi)^2\right)$$

$$K = \frac{\pi n_0}{mv}$$

d: $$H_d = \int dx\, d^\dagger(x)(\varepsilon(k) - iv_d\frac{\partial}{\partial x})d(x)$$

$$v_d = \partial\varepsilon(k)/\partial k$$

$$H_{int} = \int dx\left(V_\varphi \nabla\frac{\varphi}{2\pi} - V_\theta\nabla\frac{\vartheta}{2\pi}\right)d(x)d^\dagger(x)$$

Mapping on free chiral fermions

$$\varphi, \vartheta \to \tilde{\varphi} \pm \tilde{\vartheta} \to \tilde{\varphi}_{L,R}$$

$$H_0 = \frac{v}{2\pi} \int dx \left((\nabla \tilde{\varphi}_L)^2 + (\nabla \tilde{\varphi}_R)^2 \right)$$

Free chiral (L,R) fermions

$$H_d = \int dx \; d^\dagger(x)(\varepsilon(k) - i v_d \frac{\partial}{\partial x}) d(x)$$

impurity

$$H_{int} = \int dx \left(\tilde{V}_L \nabla \frac{\tilde{\varphi}_L}{2\pi} - \tilde{V}_R \nabla \frac{\tilde{\varphi}_R}{2\pi} \right) d(x) d^\dagger(x)$$

Forward-scattering of L and R fermions off impurity

Scattering phase shifts of L and R off impurity: $\frac{\delta_\pm}{2\pi} = \frac{\tilde{V}_{R,L}}{v \mp v_d}$

Scattering phases are provided by Bethe ansatz

100, 206805 (2008) PHYSICAL REVIEW LETTERS week end 23 MAY

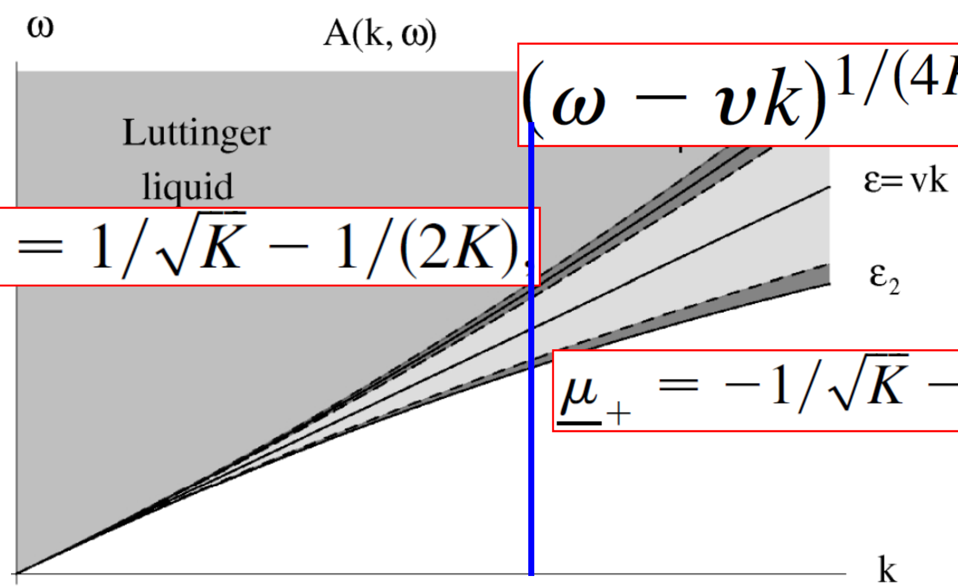
Exact Exponents of Edge Singularities in Dynamic Correlation Functions of 1D Bose Gas

Adilet Imambekov and Leonid I. Glazman

Department of Physics, Yale University, New Haven, Connecticut, USA, 06520

(Received 16 November 2007; published 21 May 2008)

The spectral function and dynamic structure factor of bosons interacting by contact repulsion and confined to one dimension exhibit power-law singularities along the dispersion curves of the collective modes. We find the corresponding exponents exactly, by relating them to the known Bethe ansatz solution of the Lieb-Liniger model. Remarkably, the Luttinger liquid theory predictions for the exponents fail even at low energies, once the immediate vicinities of the edges are considered.



$$(\omega - vk)^{1/(4K)-1}$$

linear Luttinger

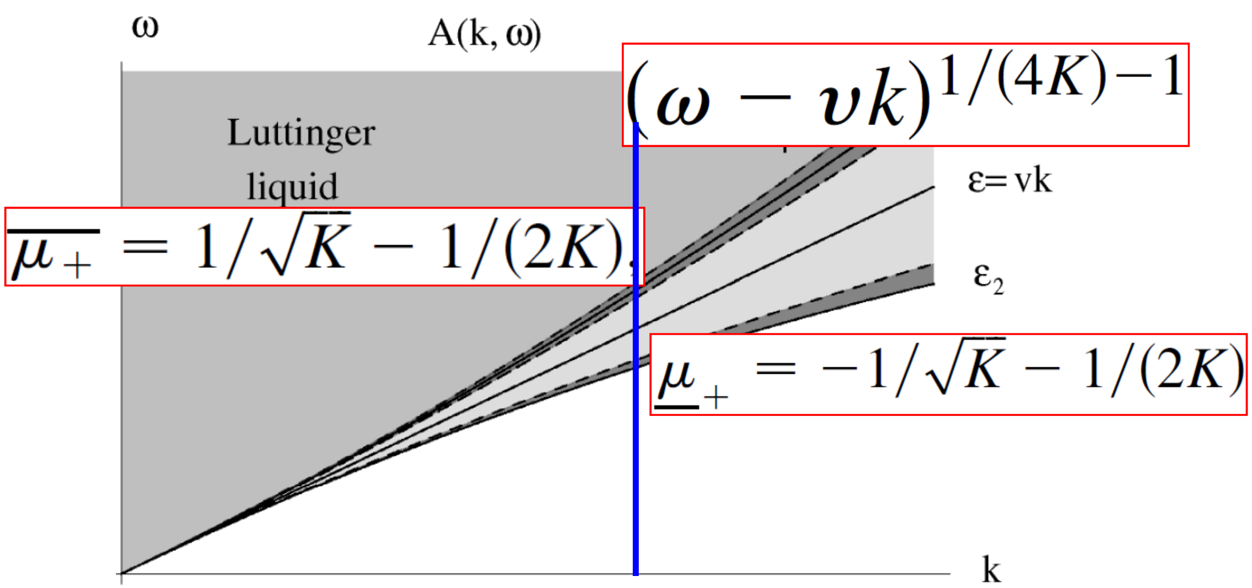
$$\overline{\mu_+} = 1/\sqrt{K} - 1/(2K)$$

$$\underline{\mu}_+ = -1/\sqrt{K} - 1/(2K)$$

The exponents at small wave vectors, are different from linear Luttinger, but depend only on *K*

Conjecture 1: Exponents are universal, NOT an artefact of Lieb-Liniger

Conjecture 2: The entire crossover between the TRUE edge behavior and linear Luttinger liquid is universal



linear Luttinger

"Rozhkovization"-1

1. Bosonization:

$$H_{\rm K}(x)=\int_{k_L(x)}^{k_R(x)}\xi_k\,dk-\bar{E}_{\rm K}\to\frac{v}{2}(\partial_x\varphi_R)^2+\frac{1}{m}(\partial_x\varphi_R)^3+(R\leftrightarrow L)$$

$$H_{int}(x)=V_{LL}(\partial_x\varphi_L)^2+V_{RR}(\partial_x\varphi_R)^2+2V_{LR}(\partial_x\varphi_L)(\partial_x\varphi_R)$$

2. Rotation, re-scaling: $\varphi_L,\varphi_R\to\varphi,\vartheta\to\tilde{\varphi}\pm\tilde{\vartheta}\to\tilde{\varphi}_{L,R}$

$$H_0=\frac{v}{2\pi}\int dx\,\left((\nabla\tilde{\varphi}_L)^2+(\nabla\tilde{\varphi}_R)^2\right)+[\text{qubic terms}]$$

3. Re-fermionization

“Rozhkovization”-2

3. Re-fermionization

$$H_0 = \frac{v}{2\pi}\int dx\,\left((\nabla\tilde{\varphi}_L)^2 + (\nabla\tilde{\varphi}_R)^2\right) + [\text{qubic terms}]$$

$$\tilde{H}_1 = \mathrm{i}v\int dx\,\Big(:\tilde{\Psi}^\dagger_{\mathrm{L}}(x)\nabla\tilde{\Psi}_{\mathrm{L}}(x): - :\tilde{\Psi}^\dagger_{\mathrm{R}}(x)\nabla\tilde{\Psi}_{\mathrm{R}}(x):\Big)$$

$$\tilde{H}_2 = \frac{1}{2m_*}\int dx\,\Big(:(\nabla\tilde{\Psi}^\dagger_{\mathrm{L}})(\nabla\tilde{\Psi}_{\mathrm{L}}): + :(\nabla\tilde{\Psi}^\dagger_{\mathrm{R}})(\nabla\tilde{\Psi}_{\mathrm{R}}):\Big)$$

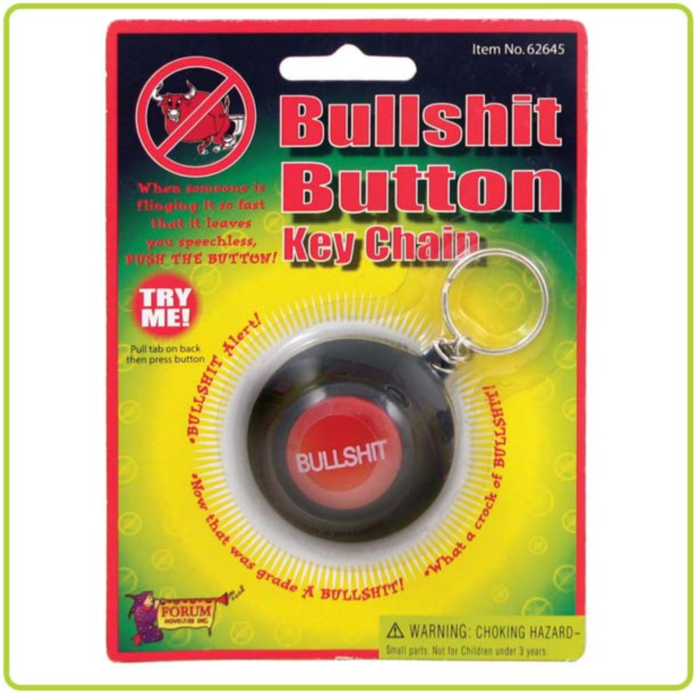
$$\tilde{H}_{int} = i\tilde{V}\int dx\,\Big(:\tilde{\Psi}^\dagger_R\tilde{\Psi}_R::\tilde{\Psi}^\dagger_L\nabla\tilde{\Psi}_L: - L\leftrightarrow R\Big)$$

Extra gradient, vanishes in the long-wavelength limit

Perturbation theory and the button

$$\tilde{H}_{int} = i\tilde{V}\int dx\left(:\tilde{\Psi}_R^{\dagger}\tilde{\Psi}_R::\tilde{\Psi}_L^{\dagger}\nabla\tilde{\Psi}_L: - L \leftrightarrow R\right)$$

amenable to perturbative treatment



Relation of "old" fermions to "new" ones

$$\Psi^{\dagger}_{\mathrm{R}}(x) = \tilde{F}^{\dagger}_{\mathrm{R}}(x)\tilde{\Psi}^{\dagger}_{\mathrm{R}}(x)$$

$$\tilde{F}^{\dagger}_{R}(x) = \exp\left[-i\int^{x} dy\left(\frac{\delta_{+}}{2\pi}\tilde{\rho}_{R}(y) + \frac{\delta_{-}}{2\pi}\tilde{\rho}_{L}(y)\right)\right]$$

Evaluation of $\langle\Psi_{R}(x,t)\Psi^{\dagger}_{R}(0,0)\rangle$ is a free-fermion problem,

$$\tilde{H}_1 = \mathrm{i}v\int dx\left(:\tilde{\Psi}^{\dagger}_{\mathrm{L}}(x)\nabla\tilde{\Psi}_{\mathrm{L}}(x): - :\tilde{\Psi}^{\dagger}_{\mathrm{R}}(x)\nabla\tilde{\Psi}_{\mathrm{R}}(x):\right)$$

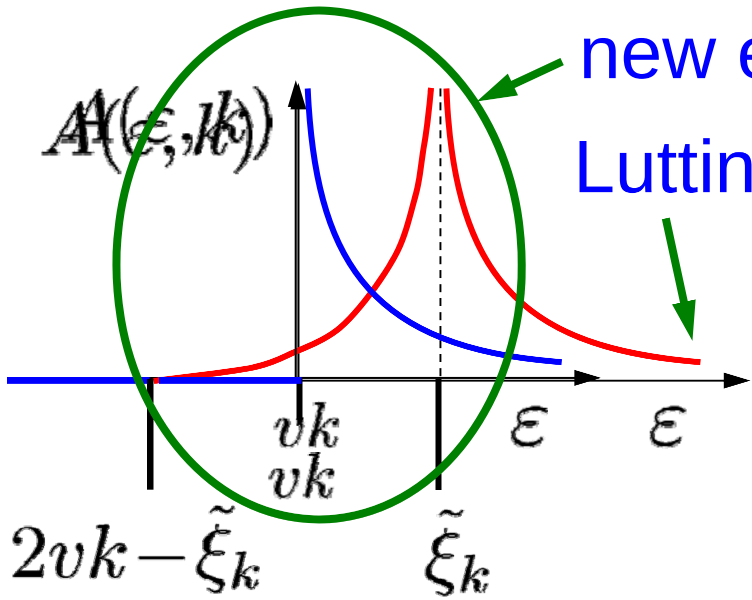
$$\tilde{H}_2 = \frac{1}{2m_*}\int dx\left(:(\nabla\tilde{\Psi}^{\dagger}_{\mathrm{L}})(\nabla\tilde{\Psi}_{\mathrm{L}}): + :(\nabla\tilde{\Psi}^{\dagger}_{\mathrm{R}})(\nabla\tilde{\Psi}_{\mathrm{R}}):\right)$$

it reduces to evaluation of "fermion determinants"

Spectral function threshold at $p > k_F$

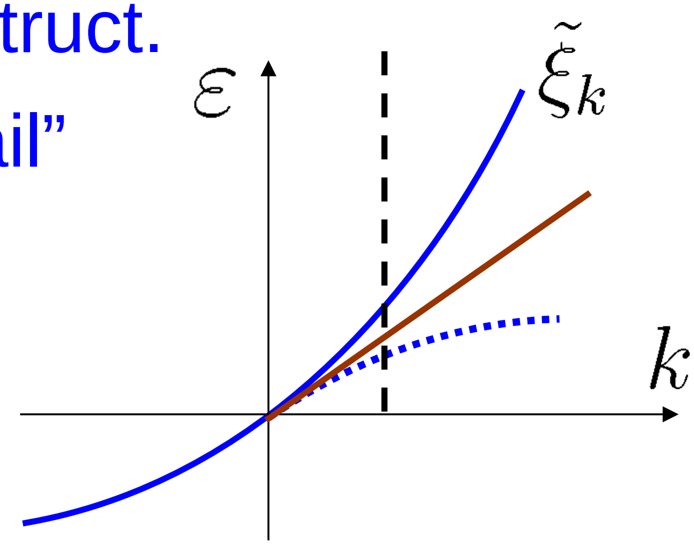
$|\varepsilon - vk| \ll k\,,\ \ k/k_F \to 0$ (here *k* is measured from k_F)

Finite mass of fermion – new energy scale $\delta\omega = \dfrac{k^2}{2m_*}$



new edge struct.

Luttinger “tail”



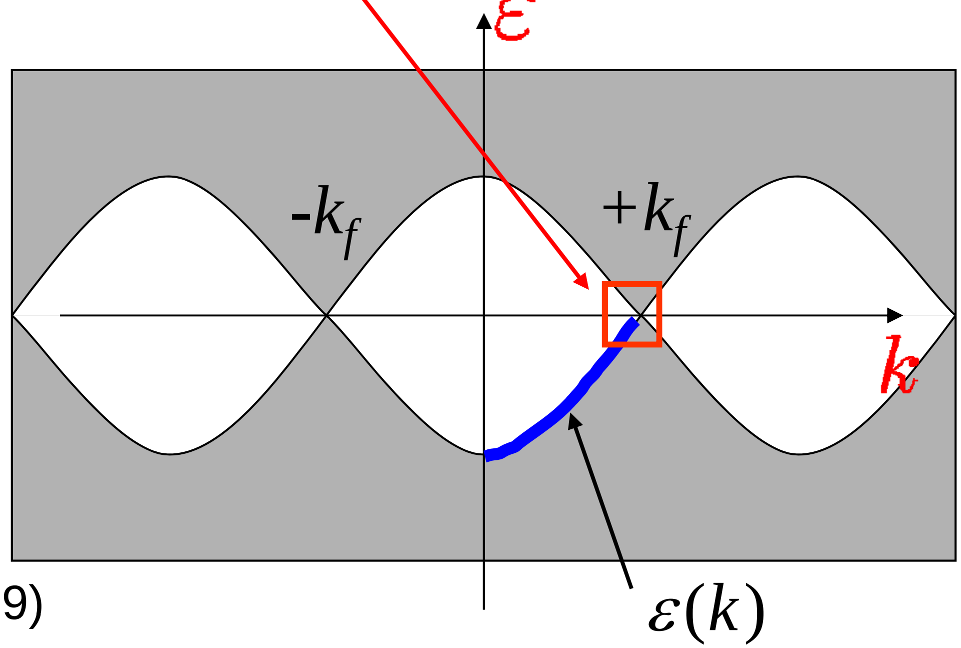
Universal crossover function

$$A(\varepsilon, k) = A\left(\frac{\varepsilon - vk}{\delta\omega}\right)$$

Phenomenology away from Fermi points

Universal behavior

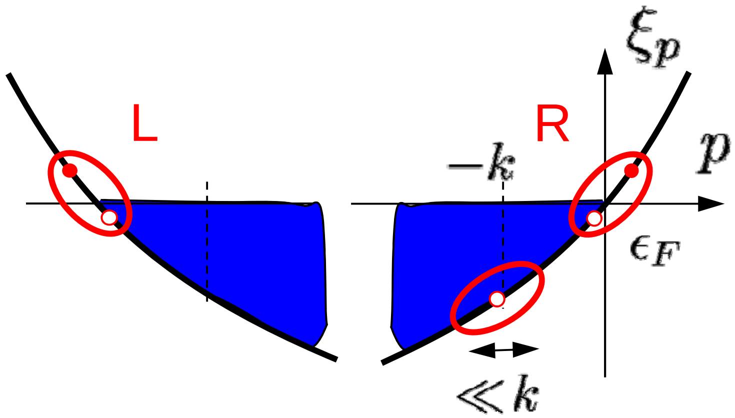
Imambekov, LG,
Science **323**, 228 (2009)



Imambekov, LG,
Phys. Rev. Lett. **102**, 126405 (2009)

Function $\varepsilon(k,n)$ completely defines the singularities!

Phenomenology, arbitrary momentum



Left and Right movers:

$$H_0 = \frac{v}{2\pi}\int dx \left(K(\nabla\vartheta)^2 + \frac{1}{K}(\nabla\varphi)^2 \right)$$

d: $$H_d = \int dx\, d^\dagger(x)(\varepsilon(k) - iv_d \frac{\partial}{\partial x})d(x)$$ $$v_d = \partial\varepsilon(k)/\partial k$$

$$H_{int} = \int dx \left(V_\varphi \nabla \frac{\varphi}{2\pi} - V_\theta \nabla \frac{\vartheta}{2\pi} \right) d(x) d^\dagger(x)$$

$$\frac{1}{2}V_\theta = \frac{\partial\varepsilon(k,n)}{\partial k} - \frac{k}{m}$$

use Galilean invariance, Baym&Ebner, 1967

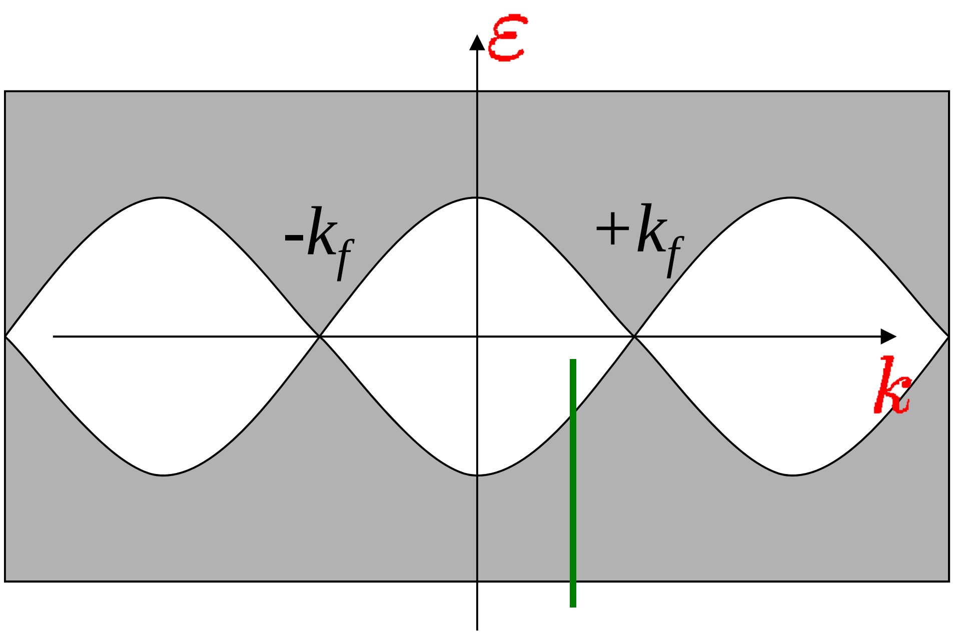
$$\frac{1}{2}V_\varphi = \frac{\partial\varepsilon(k,n)}{\partial n} + \frac{\pi v}{K}$$

Imambekov, 2008

Observables: Spectral function $A(k,\omega)$

$$A(k,\omega) \propto \theta(\varepsilon(k)-\omega)\left|\frac{1}{\varepsilon(k)-\omega}\right|^{1-\left(\frac{\delta_+(k)}{2\pi}\right)^2-\left(\frac{\delta_-(k)}{2\pi}\right)^2}$$

$$\delta_+,\delta_- \leftrightarrow V_\varphi, V_\theta$$



$$\frac{1}{2}V_\varphi = \frac{\partial\varepsilon(k)}{\partial\rho} + \frac{\pi v}{K}$$

$$\frac{1}{2}V_\theta = \frac{\partial\varepsilon(k)}{\partial k} - \frac{k}{m}$$

Jolly Good Days

From Adilet to Leonid, in memoriam of the expropriated book and jolly good days of 2007-2009, which included:

Леониду от Адилета в память об экспроприированной книжке и о веселых деньках 2007-2009, которые включали в себя:

…

Extensions, “Moratorium”, Houston

Extensions:

phenomenology of spin-1/2 fermions

[Schmidt, Imambekov, LG Phys. Rev. Lett. **104**, 116403 (2010)+PRB(2010)];

theory of pre-factors in dynamic responses

[Shashi, LG, J-S Caux, Imambekov, Phys. Rev. B. **84**, 045408 (2011)]

Rev. Mod. Phys., Aspen, Open Problems, Florence

- One-dimensional quantum liquids: Beyond the Luttinger liquid paradigm
  Adilet Imambekov, Thomas L. Schmidt, and Leonid I. Glazman
  Accepted Thursday Feb 23, 2012 Rev. Mod. Phys.; arXiv:1110.1374

Subject: you are a good theorist:-)
From: Adilet Imambekov <adilet@rice.edu>
Date: 8/18/2011 9:58 PM
To: Leonid Glazman <leonid.glazman@yale.edu>

In the evening the time gets better, 17:52.

Most intriguing open questions:
kinetics of weakly-non-integrable systems;
quantum quench of a nonlinear Luttinger liquid