Overview of $(2,0)$ theories

Michael R. Douglas

Simons Center / Stony Brook University

Quantum fields beyond perturbation theory

Abstract

We survey the $(2,0)$ superconformal field theory in six dimensions, including its relation to $D = 5$ maximally supersymmetric Yang-Mills theory (MRD arXiv:1012.2880 and LPS Lambert-Papageorgakis-Schmidt-Sommerfeld arXiv:1012.2882), and other approaches to its formulation.

Outline

Inspired by string/M theory and AdS/CFT, many superconformal field theories have been discovered and studied. The most mysterious and arguably the most important is the $(2,0)$ theory in six dimensions (Witten, 1995). There is a free or "abelian" $(2,0)$ theory, a field theory of a self-dual tensor (3 physical modes), 5 scalars and 8 fermions. There are also "nonabelian" $(2,0)$ theories, with various indirect definitions and relations to other superconformal theories:

- Type IIb string theory on a four-dimensional ADE singularity, leading to an ADE classification of these theories.
- N coincident M5-branes, leading to the A_{N-1} theory.
- The large N limit is dual to M theory on $AdS_7 \times S^4$.
- After compactification on T^2, one gets $N = 4$ maximally supersymmetric Yang-Mills theory, with complex gauge coupling τ equal to the complex modulus of T^2. This gives S-duality a geometric explanation.
- Twisted compactifications on manifolds with $d = 2, 3, 4$ lead to new superconformal theories in $D = 4, 3, 2$.

However there is no satisfactory Lagrangian, and since the theory has no dimensionless coupling, no argument that there must be one, nor is any other usable microscopic definition known.

Here are some of the other ideas which have been tried:

- On the Coulomb branch there are BPS strings, with tension proportional to differences of scalar vevs $\phi_i - \phi_j$. In the unbroken limit $\phi_i \to \phi_j$, these might lead to "tensionless strings" as fundamental degrees of freedom.
- One can conjecturally define M theory in the light-cone frame as the large N limit of D0-branes in IIa theory – BFSS "Matrix theory." This idea also leads to a definition of the A_N and D_N $(2, 0)$ theories as large N limits of D0-D4 systems (Aharony *et al* '97).
- One might start with the theory compactified on S^1, T^2 or possibly other manifolds, add in any extra degrees of freedom of the $D = 6$ theory (such as Kaluza-Klein states), and take the large volume limit. The simplest starting point would be $D = 5$ MSYM, if we can make sense if it (it is perrturbatively nonrenormalizable).

More ideas:

- There is an argument called “deconstruction” that the A_N quiver $D = 4$, $N = 2$ SYM becomes $(2, 0)$ theory in an appropriate limit. Essentially the quiver is a lattice approximation to the fifth dimension. While it is sensible, the problem is that the gauge theories are strongly coupled in this limit.
- There is some work on $D = 6$ conformal blocks, which might allow working with the bootstrap.
- One can construct $(2, 0)$ theory on a curved background by decoupling the degrees of freedom at a singularity in IIb string theory on $AdS_5 \times S^5/\mathbb{Z}_N$. Using dualities this can lead to a realization in terms of $D = 4$ SCFT’s (Aharony, Berkooz and Rey, to appear).

In this talk we will survey these approaches as well as other recent results:

- Computation of the superconformal index by localization methods ($\mathrm{Kim}^3+\mathrm{Lee}$ 1307.7660).
- Calculations reproducing the free energy with its N^3 scaling (Maxfield and Sethi 1204.2002; Källen *et al* 1207.3763).
- Constraints from superconformal invariance on the S-matrix (Czech, Huang and Rozali 1110.2791).

Finally there are some more speculative ideas that should be pursued:

- Explicitly adding extra states to $D = 5$ MSYM.
- Nonlocal actions in six dimensions.
- Reformulation in terms of $D = 6$ twistors.
- Wilson surface computations.

Basic facts about $(2,0)$ theory

- It is a local field theory, with local correlation functions, which can be defined in any (nonsingular) space-time with a fixed metric.
  One argument for this is the reduction to $D < 6$ gauge theory.
  Another is that it has a gravity dual.
  But while we will assume it, it is not beyond all doubt.
- It has no dimensionless parameters, and no relevant operators which preserve $(2,0)$ supersymmetry.
- The A_N and D_N theories have $O(N^3)$ degrees of freedom, as measured by the free energy, or by the Weyl anomaly. Later we will discuss recent computations which reproduce this.
- The theory involves extended objects – not only do they exist, but if we remove them the theory becomes non-interacting.
- There is a moduli space $\mathbb{R}^{5N}/W_N$, just like $D = 5$ MSYM. When the Weyl symmetry W_N is completely broken, the low energy limit is the product of N abelian theories – this is the Coulomb phase.

As usual in superconformal gauge theory, the observables are very different in the Coulomb and the unbroken phases.

Coulomb:
- Has an S-matrix, and a rich spectrum of charged BPS states.

Unbroken:
- Has superconformal symmetry with bosonic part $Spin(6,2) \times Spin(4)_R$.
- S-matrix is very subtle to define because of IR singularities, and may not make sense at all.
- Probably has "Wilson surfaces," extended observables analogous to Wilson loops.

In $D = 4$ MSYM, we have some understanding of the relation between the two phases, mostly from perturbation theory and the analogy to QCD (factorization theorems etc.) – this is why people talk about the "S-matrix of $N = 4$ SYM" without belaboring this point. We do not understand this relation for $(2,0)$ theory. But, there is no known reason why the S-matrix should not exist in the Coulomb phase.

Any quantum theory, including $(2, 0)$ theory, must have a Hilbert space (for a choice of spacelike surface), a Hamiltonian and other local operators. For local QFT and string theories we know how to define all of this in terms of fields, operators which are functions of a point (for QFT) or a loop (for strings). This does not require that perturbation theory be convergent or even exist.

More generally we can imagine a field theory in which the operators are functions on some other space of geometric objects, perhaps higher dimensional manifolds, perhaps more general collections of points (such as measures), or noncommutative objects such as particular large N limits of matrices. If the objects have any relation to space-time, so that fields cannot naturally be associated to single points, then they are "extended objects."

Every example we understand of an interacting quantum theory in more than four dimensions involves extended objects. This includes cases we only partly understand such as BFSS Matrix theory. This are also general arguments that interacting local point field theories cannot exist in more than four space-time dimensions.

On the other hand, the usual state-local operator correspondence of CFT tells us that we should be able to understand the unbroken phase entirely in terms of local operators and their correlation functions. Furthermore, extended objects in a conformal theory can have no preferred size, and this implies that they do not have the "stationary states" we are familiar with for fundamental strings or quantum solitons.

Let us suppose they are described by some sort of wave function on a configuration space, call it

$$\Psi(X, \rho, \ldots), \tag{1}$$

where ρ is a measure of the size of the object – the extent of a string, the scale size of an instanton, or whatever. It is hard to see how such a wavefunction can be both normalizable in Ψ, and stationary under time evolution.

The $(2, 0)$ supersymmetry algebra is

$$\{Q, Q\} = P_m \Gamma^m + Z^I_m \Gamma^m \Gamma_I + Z^{IJ}_{mnp} \Gamma^{mnp} \Gamma_{IJ} \tag{2}$$

where $m = 0, \ldots, 5$ are space=time indices, $I = 6, \ldots 10$ index an $SO(5)_R$ vector, and Z^{IJ}_{mnp} is self-dual in space-time. Objects which source a single central charge are $1/2$ BPS, including the basic self-dual tensor multiplets.

In brane terms, the central charge Z^I_m with $m \neq 0$ is sourced by an M2 ending on the M5, stretching along m and extending out in the I direction. In six dimensions this looks like a self-dual string charged under the tensor gauge potential.

An M2 stretching between M5's is a self-dual string with tension $|\vec{\phi}_i - \vec{\phi}_j|$ (note that $D = 6$ scalars naturally have mass dimension 2). On T^n compactification they reduce to the monopoles and W bosons of the Yang-Mills theory Coulomb phase.

The central charge Z^{IJ}_{mnp} is sourced by an M5 branching (an embedding like $z_1 z_2 = \epsilon$).

Since there are strings in the Coulomb phase, whose tension goes to zero in the unbroken phase (at least formally), it is reasonable to hypothesize that they are the fundamental extended objects. There are actions for tensionless strings, starting with Schild 1977, and a modest literature which began before the proposal of $(2,0)$ theory, in good part motivated by the desire to understand the high energy limit of string scattering amplitudes, which has many interesting properties (Gross and Mende 1988; Amati, Ciafaloni, and Veneziano 1988; Sundborg 1988; ...). The simplest action (Lindström Sundborg Theodoridis 1991) is

$$S = \int d^2\sigma \; V^\alpha V^\beta D_\alpha X \cdot D_\beta X + \lambda X^2 \tag{3}$$

where X embeds the string into the lightcone of $\mathbb{R}^{D,2}$, there is a gauged scale invariance, and V is a 2d vector (the world-sheet metric is rank 1).

So far most work is inconclusive, while the few conclusive works are negative – for example Gustafsson *et al* hep-th/9410143 showed that the action above can only be quantized in $D = 2$. The evidence so far suggests that tensionless strings do not exist (J. Schwarz, private communication).

Any description in terms of fundamental strings would also have to address the problem that strings normally do not allow defining local correlation functions.

In addition, we repeat the general point that extended objects in a conformal theory cannot have a preferred scale. Although a particular object can certainly have an average expected size, there is no reason why this should not run off to zero or infinity under time evolution. So the picture of tensionless strings moving in a background six-dimensional space-time, with no other dynamical variables needed to describe their free motion, is certainly wrong.

To compactify the abelian theory on S^1, note that a self-dual tensor reduces to an ordinary vector boson in $D = 5$, as the extra components H_{pqr} are dual to $H_{5mn} \equiv F_{mn}$.
Clearly the postulated relation between $(2, 0)$ and $D = 4$ MSYM is simplest if the nonabelian version of this is $D = 5$ MSYM. This also follows from the stringy arguments that an M5 reduces to a D4. The off-diagonal gauge bosons come from self-dual strings wound around x_5.
Going in reverse, the KK momentum is then

$$P_5 = \int dx^5 T_{05} \quad \propto \quad \int dx^5 \sum H_{0mn} H_{5mn} \tag{4}$$

$$\rightarrow \int dx^4 \sum (*F)_{mn} F_{mn} \quad \rightarrow \quad \int \mathrm{dx}^4 \mathrm{Tr\ F} \wedge \mathrm{F} \tag{5}$$

Thus the KK momentum is identified with the instanton charge. Since the latter is quantized, this corresponds to a compact extra dimension.

In the unbroken phase, the self-dual solution, which in $D = 4$ would have been an instanton, leads to an "instantonic particle" in $D = 5$. Identifying the one-instanton sector with the minimal KK momentum, and using the fact that instanton number sources a central charge, one finds

$$\frac{1}{R_5} = \frac{4\pi^2}{g^2_{YM5}}. \tag{6}$$

This is $1/2$ BPS (more specifically, $(2, 0)$ BPS in the notation of Hull 0004086) which is correct for a KK mode of a self-dual tensor multiplet. There is a similar "dyonic instanton" in the Coulomb phase (Lambert and Tong; LPS). However it is $1/4$ BPS and it is not immediately evident that its multiplet structure matches up with $(2, 0)$ theory. However there is now evidence for this from localization calculations (Kim^3+Lee 1307.7660). These computations start from various reductions of $(2, 0)$ – on $S^5 \times S^1$; on $S^5/\mathbb{Z}_K \times S^1$, and the AdS$_7$ dual, and find a consistent superconformal index which includes $1/4$ BPS states.

As for the non-winding self-dual string, it is the $D = 5$ 't Hooft-Polyakov monopole.
The upshot is that all of the $(2,0)$ BPS states can already be seen in the $D = 5$ theory. This would strongly suggest that the $D = 5$ theory is already a complete theory, except that it is perturbatively nonrenormalizable. The perturbative expansion is controlled by the effective dimensionless coupling $g_5^2 E$, where E is the energy scale of a process. At low energies, the expansion should be good, while for $E \sim 1/g_5^2$ it will break down. This is signaled by UV divergences which first appear at six loops (Bern *et al* 1210.7709).
A sensible physical theory does not have UV divergences and thus the first question is to determine the scale Λ at which the effective theory breaks down. In the case at hand this follows from the fact that $(2,0)$ theory has no dimensionless parameters. Thus, since we already have a dimensionful parameter $g_{YM5}^2 \propto R_5$, the only energy scale that it can be is $\Lambda \propto 1/R_5$.

In all known quantum theories, UV divergences are regulated either by new states at the energy scale Λ, or by the fact that the fundamental degrees of freedom are extended objects. Normally there is not a sharp distinction between these options as the extended object usually has "modes" or stationary states whose spectrum starts at Λ. However note that in CFT we argued that the extended objects would not have stationary states, so perhaps this intuition is misleading in this case.

If we grant that new degrees of freedom must come in at the scale $\Lambda \propto 1/R_5$, it seems that they must be the instantonic particles that we discussed. Thus, one can hypothesize that while $D = 5$ MSYM is perturbatively nonrenormalizable, if we could also include the quantum effects of these instantonic particles, it would be finite.

The simplest way that this could work would be that we could retain the structure of $D = 5$ perturbation theory, but add the additional instantonic particles as propagating states. Some support for this idea can be found by considering the further reduction to $D = 4$ by compactification on T^2. This defines not $D = 4$ MSYM, but a theory that we might call "extended $D = 4$ MSYM" as it contains nonrenormalizable coupings controlled by the size of the T^2, and additional states from the unwrapped self-dual string.

The usual argument relating $D = 6$ and $D = 4$ is that a self-dual string winding an A or B cycle of T^2 becomes a W or monopole. One might ask what happened to the unwound string. In the MSYM limit it goes to infinite tension – consider the parameters which determine the W and monopole masses in $D = 4$ and $D = 6$, these are $M_W = g_4\phi = R_5\hat{\phi}$ and $M_M = \frac{\phi}{g_4} = R_4\hat{\phi}$, where $\hat{\phi}$ is the $D = 6$ scalar vev (and the string tension) while ϕ is the $D = 4$ scalar vev. We see that $\phi^2 = \hat{\phi}^2 \cdot vol\,(T^2)$, and the limit $vol\,(T^2) \to 0$ must be taken with $\hat{\phi}^2 \to \infty$. However at finite $vol\,(T^2)$ it survives.

Before taking the idea that $D = 5$ MSYM has all the $(2, 0)$ degrees of freedom too seriously, we should explain the N^3 behavior of its free energy this way. Various explanations have been offered involving complicated bound states (3-prong string junctions, other $1/4$ BPS bound states, e.g. see Bolognesi and Lee 1105.5073) and/or new quasiparticles, as in the proposal of Collie and Tong 0905.2267 that the instanton particles fractionate. Their common feature is that we need to understand complicated dynamics.

In recent years these problems have been attacked using localization techniques along the lines of Nekrasov, Pestun, Kapustin *et al* and many others. The $N^{3/2}$ behavior of $D = 3$ maximal superconformal theory (ABJM theory) was obtained this way by Drukker, Mariño and Putrov 1007.3837. It has been applied to $D = 5$ gauge theory by several groups: Kim *et al*; Hosomichi, Seong and Terashima; and Källen, Minahan, Nedelin and Zabzine, whose 1304.1016 finds agreement with the gravity result, including the leading coefficient. This involves numerous subtleties, especially in taking the Euclidean continuation.

The localization calculation is a long story, but the main idea is that one computes an index which (in these theories with extended supersymmetry) can be related to the free energy, but for which the action admits Q-exact deformations. This allows concentrating the functional integral on points in space-time, and reduces the gauge theory calculation to a matrix model.

The basic result in $D = 3$, and also in $D = 6$, is then a free energy

$$F = cN^2 f(\lambda) \tag{7}$$

where $\lambda \sim g^2_{YM} N$ is the 't Hooft coupling. Thus it formally has the usual N^2 scaling, but unlike $D = 4$ the function $f(\lambda)$ has a non-trivial power-like behavjor at large λ, $f \sim \lambda^{-1/2}$ in $D = 3$ and $f \sim \lambda$ in $D = 6$.
Using the gauge-gravity dictionary, one finds the leading supergravity term has the predicted power in g^2_{YM}, leaving the remaining power of N which corrects the result to the expected power of N.
While in $D = 3$ this is a nontrivial strong coupling effect, in $D = 6$ the correction λ is simply the result of doing a two-loop diagram (Kim and Kim, 1206.6339). Furthermore the required power of g^2_{YM5} is simply the R_5 factor expected for a $D = 6$ free energy. Thus the answer to the puzzle is surprisingly prosaic.
It is still nontrivial that this two-loop diagram dominates. The localization computation makes many assumptions, in particular about the treatment of singularities of instanton moduli space. But it is based on the degrees of freedom of $D = 5$ MSYM.

Having better justified the idea that $D = 5$ MSYM contains much and perhaps all of the $(2,0)$ theory, and even the idea that perturbation theory has something to teach us, let us continue.
If we follow the chain $(2,0) \to D = 5 \to D = 4$, we can use these properties to try to constrain $D = 5$ MSYM. The perturbative argument is straightforward: compactify $D = 5$ MSYM on a circle of radius R_4, then

$$g_5^2 \int dp_4 \; f(p_4) \to \frac{R_5}{R_4} \sum_n f\left(\frac{n}{R_4}\right)$$

and the perturbative expansion is a series in $g_4^2 = R_5/R_4$.

Furthermore, the limit $L \to 0$ takes $R_4, R_5 \propto L \to 0$, so the 5d KK states go to infinite energy and can be dropped. This is the usual argument for the relation between $(2,0)$ theory, $D = 5$ and $D = 4$ MSYM.

$$g_5^2 \int dp_4 \; f(p_4) \rightarrow \frac{R_5}{R_4} \sum_n f\left(\frac{n}{R_4}\right)$$

However, UV divergences in $D = 5$ can potentially spoil this argument. These clearly come from states with nonzero p_4 (since the $D = 4$ theory was finite) and start to appear at energies $E \sim 1/R_4$. Such KK modes look like $D = 4$ particles with mass $M \sim n/R_4$.

Because the underlying theory is $(2, 0)$ theory, there are no actual UV divergences – an apparent $D = 5$ UV divergence, is cutoff at the scale $\Lambda \sim 1/R_5$. Compared to pure $D = 4$ MSYM, one gets finite quantum corrections from states with $1/R_4 \leq E \leq 1/R_5$.

Thus, an apparent $D = 5$ divergence Λ^n, produces a correction

$$\left(\frac{\Lambda}{M}\right)^n \sim \left(\frac{R_4}{R_5}\right)^n \sim \tau^n,$$

or $\log \tau$ for a log divergence. Note that L has cancelled out, and these corrections need not disappear as $L \rightarrow 0$.

It is hard to know what properties we should look for in $D = 5$ amplitudes, but we do know a fair amount about the $D = 4$ amplitudes obtained by compactification on T^2. Let the volume of the T^2 be L^2, then as $L \to 0$ we recover $N = 4$ SYM, while we expect corrections to come with positive powers of L. For example,

$$S_{eff} = \int \frac{1}{g_4^2} tr\ F^2 + \theta tr\ F \wedge F + cL^4 tr\ F^4 + \ldots.$$

The coefficients c should be computable functions of $\tau = 4\pi/g_4^2 + i\theta$, and by the geometric origin of τ, should satisfy S-duality.

Taken at face value, this argument suggests that $D = 5$ MSYM must be *UV finite*, to prevent these problematic corrections. However this would be too quick:

- There might be other positive powers of L in front of these corrections, and
- There is a problem with taking $g_4 \to 0$ at fixed L.

To explain the second point, since

$$R_4 = \frac{L}{g_4}, \qquad R_5 = g_4 L,$$

taking $g_4 \to 0$ at fixed L decompactifies the fifth dimension. We need to take $L \to 0$ faster than $g_4 \to 0$ to keep the four dimensional interpretation.

Thus, compactified $D = 5 \rightarrow D = 4$ results are only guaranteed to have a clear interpretation, if we can compute them at finite g_4. There is one case where we can unambiguously do this, namely the $tr\ F^4$ term, which receives no higher loop corrections. Summing the one-loop contributions of KK modes with $p_4 \neq 0$, we find a coefficient

$$\sum_{p_4 \neq 0} \frac{g_4^4}{p_4^4} = \zeta(4) L^4$$

since the g_4 dependence of p_4 cancels that of the numerator.

This result is not S-dual. It can be promoted to an S-dual result by the ansatz of summing over both p_4 and p_5, leading to the coefficient

$$\sum_{(m,n) \neq (0,0)} \left(\frac{(Im\ \tau)^2}{|m\tau + n|^2} \right)^2 = \zeta(4) E(\tau, 2) L^4,$$

where $E(\tau, 2)$ is a nonholomorphic Eisenstein series. As a protected amplitude, this should be checkable in string theory, perhaps IIa little string theory (the throat region of NS 5-branes) on T^2.

One could try a similar approach to higher loop amplitudes, starting with $D = 5$ on $\mathbb{R}^4 \times S^1$, and writing the loop amplitudes as explicit sums of $D = 4$ MSYM loops with KK modes running around the loops. Since $N = 4$ is finite, all the UV divergences will be postponed until we actually do the sum.

We can then promote the sums over internal KK modes, to doubles sums over $(2, 0)$ KK modes which are indexed by (p_4, p_5). Although logically we do not need to assume that the extra states are present in $D = 5$ MSYM, as we discussed this assumption seems to work and would allow deriving the couplings to the new states from soliton calculations in $D = 5$. Papageorgakis and Roysten have looked into this (see their talk at String-Math 2013) and noted that the usual exponential form factor of a soliton need not be present for the instantonic particles of $D = 5$, as it it is set by the scale size, but this is a collective coordinate which would be integrated down to zero.

It is not at all clear how to do such calculations. One needs to integrate against a wave function for the soliton, but although we can work out the moduli space and Hamiltonian, they have singularities at zero size, and the properties of the states we are interested in depend on how we treat the singularity. Consider the simplest $SU(2)$ one-instanton case – this bosonic moduli space is a product of the center of mass with $\mathbb{R}^4/\mathbb{Z}_2$. The right answer is that this QM has one BPS bound state, and this will follow if we resolve the singularity, for example by a noncommutative deformation of the instanton problem. But this introduces a scale and is surely not the right way to think about unbroken $(2,0)$ theory.

A simpler option is just to make an ansatz for the spectrum and coupling of the extra states. As we saw, imposing S-duality in $D = 4$ is a significant constraint.
Then, the ansatz for an ℓ-loop perturbative amplitude in $D = 6$ will be a sum over modified $D = 4$ amplitudes,

$$\mathcal{A}_\ell = \sum_{p_4^{(1)}, p_5^{(1)}} \ldots \sum_{p_4^{(\ell)}, p_5^{(\ell)}} \mathcal{A}_{D=4}(\vec{p}_4, \vec{p}_5; s, t, u, \ldots). \tag{8}$$

This will *a priori* be more divergent than standard $D = 5$, and in fact power counting would predict a divergence at $\ell = 3$. However, if the new states come in with appropriate couplings, it is possible that this divergence will cancel, and that higher loop divergences might cancel. This is a more concrete form of the hypothesis that the nonperturbative states of $D = 5$ MSYM cut off its perturbative divergences and that properly incorporating them would lead to the $(2, 0)$ theory.

Of course, if it really turned out that such an ansatz led to all-orders finiteness, surely this would have some simple explanation, probably that the enlarged system had additional supersymmetry. Thus a more straightforward way to work on this proposal would be to hypothesize such a supersymmetry and look for an action (or other description) which realized it.

While this sounds very much like the project of finding an action for $(2,0)$ theory, the biggest difference is that there is no *a priori* reason why this action should be local in six dimensions, since it is describing the couplings to solitonic objects. This immediately evades most of the no-go results and in fact what success there has been in writing actions starts by giving up on locality from the start.

This is beause of a paradox pointed out in the first papers by Witten. The gauge coupling g_5^2 in $D=5$ has dimensions of length. Thus dimensional reduction from 6 to 5 leads to a paradox: how can

$$\int_0^{2\pi R_5} dx_5 \int d^5x \mathcal{L}_6 \rightarrow \frac{2\pi}{R_5} \int d^5x \mathcal{L}_5? \tag{9}$$

Even the most basic points about $(2,0)$ theory are confusing. The starting point for SYM, after gauge invariance, is the observation that there is a cubic interaction $f^{abc}A_aA_b\partial A_c$. This leads to a cubic S-matrix element, which is the first example given in string textbooks.

It is not obvious how to define a nonabelian gauge symmetry which acts on tensors (self-dual or not). The evident thing to try is to introduce an auxiliary vector potential under which the tensor is charged in the standard way. This has been tried many times over the years. For example, a $D=6$ $(1,0)$ supersymmetric action which couples a self-dual tensor multiplet to a Yang-Mills multiplet was given in Bergshoeff *et al* 9605087). This was extended to an interacting theory of vectors and tensors by Samtleben, Sezgin and Wimmer in 1108.4060.

There is a puzzle about the S-matrix in the Coulomb phase, pointed out by Czech, Huang and Rozali 1110.2791. They work out the constraints of $(2,0)$ superconformal invariance, both in $D=6$ and in the compactification on S^1. One of their claims is that there is no 3-point scattering element for self-dual tensor multiplets in $D=6$. This would seem to immediately rule out the type of action we are looking for. However, as they point out, there are subtleties having to do with nonlinear gauge invariance and they also exclude the action of Samtleben *et al*, so this needs further work. They do get interesting results in $D=5$ which could be used in the extra state approach I described earlier.

Although there is no candidate nonlocal $(2,0)$ action as yet, let us look at a proposed bosonic action in Chu and Ko 1203.4224 (see also Ho, Huang and Matsuo 1104.4040; Samtleben, Sezgin and Wimmer, 1108.4060; and Bonetti, Grimm and Hohenegger 1209.3017).

This is based on an abelian action written by Perry and Schwarz, for a 5×5 antisymmetric field B_{mn}. It is

$$S = \frac{1}{2} \int d^6x \left(-\tilde{H}^{mn}\tilde{H}_{mn} + \tilde{H}^{mn}\partial_5 B_{mn} \right). \qquad (10)$$

where $\tilde{H}_{mn} = \epsilon_{mnpqr}(dB)^{pqr}$. There is a gauge symmetry $\delta B_{mn} = \Lambda_{mn}$ and after gauge fixing the equaton of motion becomes self-duality $H = *H$.

The Chu-Ho action takes B_{mn} in the adjoint and introduces an auxiliary gauge field A_m, so that now $H = DB = dB + [A, B]$. It is determined by a constraint

$$F_{mn} = \int dx_5 \tilde{H}_{mn}. \qquad (11)$$

Chu and Ho claim that this can be implemented by an auxiliary field, which contributes no new degrees of freedom.

This action solves the paradox about dimensional reduction on S^1. The integral in the constraint gives a factor of R_5, which on solving for H leads to the $D = 5$ YM action for F with coupling $g^2 = R_5$.

Wilson surfaces

The large N limit of $(2, 0)$ is not so different from that of $D = 4$ MSYM. Even at finite N the analogy probably goes much farther?

What is the analog of the Wilson loop? Is there a Wilson surface defined by carrying around an M2 which ends on the M5? Can we compute it at strong coupling as a membrane amplitude? There are a few papers, e.g. by Gustavsson.

Analog of dual superconformal invariance? Δx for Wilson loop with corners is related to Δk for gluon scattering.

Singularities of the Wilson loop are related to the IR subtleties of $N = 4$ SYM. The new singularity of the Wilson surface is a conical singularity – how does it behave ?

Given a lightlike surface in $D = 6$ with conical singularities, what are its conformal invariants?

Perhaps easier is a surface with line singularities, which bounds a 3d manifold with boundary kinks. We can combine A and B cycles and other states.

Conformal bootstrap

The only work on this I know of is Arutyunov and Sokatchev 0201145, which expresses the four-point function of $(2,0)$ stress tensors in terms of an unknown function, and obtains this from the gravity dual.
We have many if not all of the ingredients needed to work on it:
The $D = 6$ representation theory is known.
The spectrum of $1/2$ and $1/4$ BPS operators can be obtained from the localization computations.
The $D = 6$ superconformal Ward identities are known and the superconformal blocks could be worked out (see Dolan and Osborn 0309180 for ordinary conformal blocks).