# Progress in EFT

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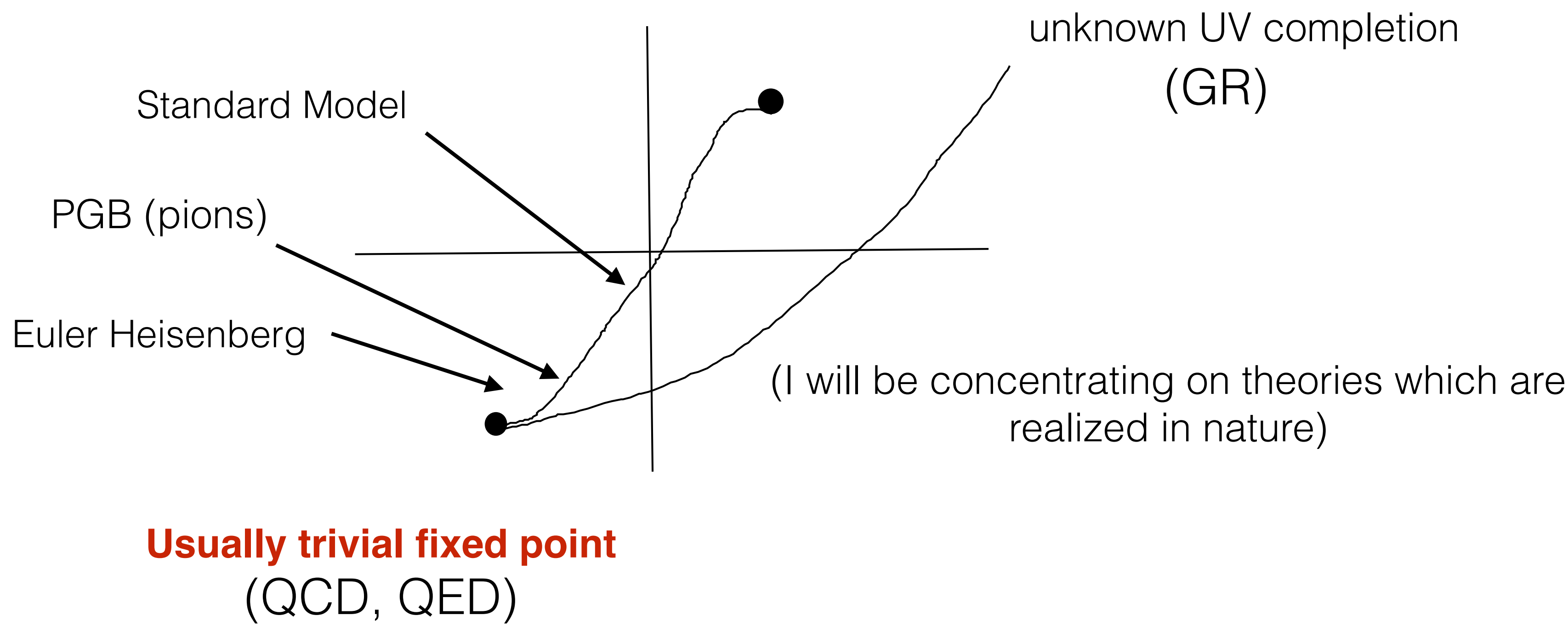
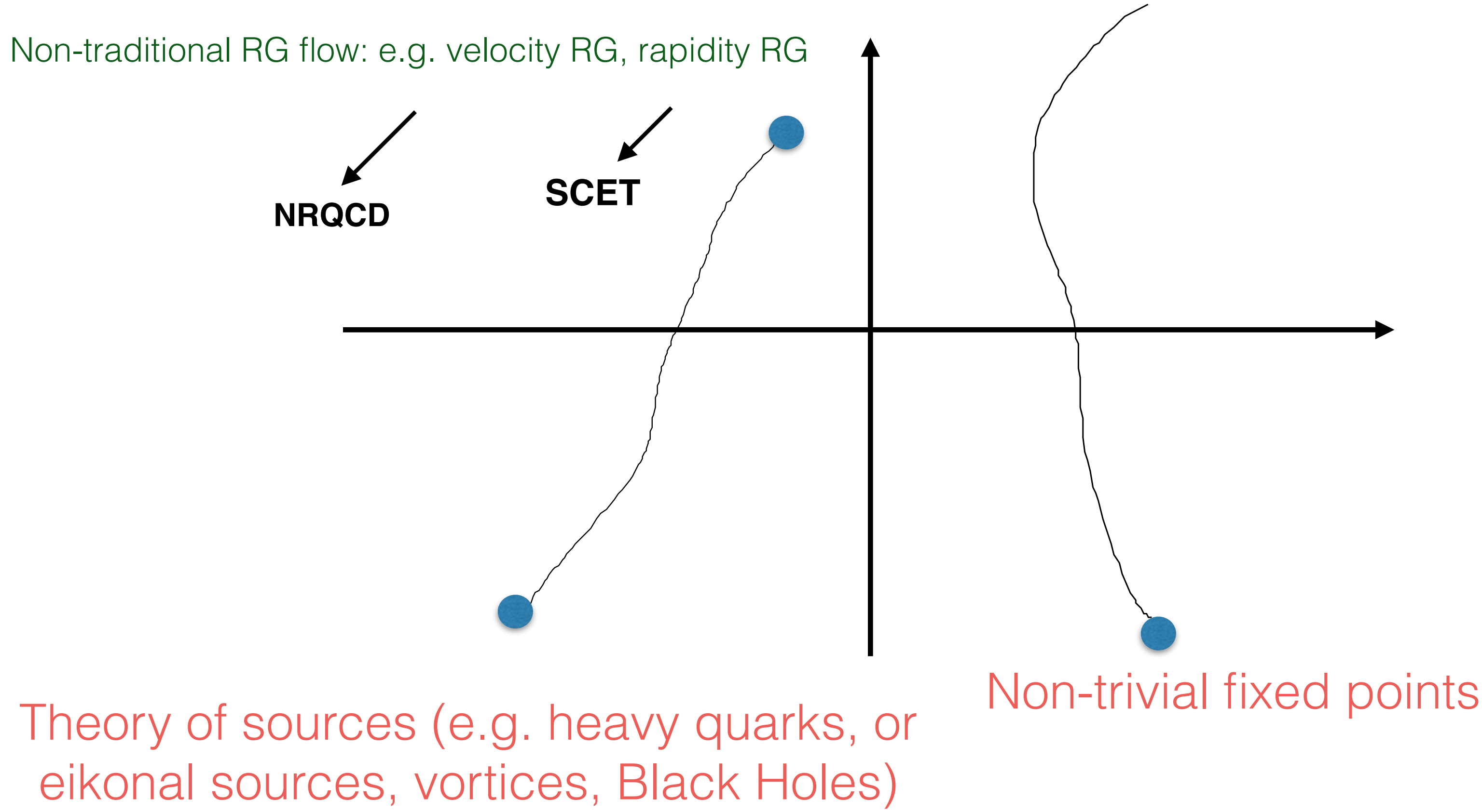
**Quantum Fields Beyond Perturbation Theory**
**KITP 1/28/14**

$$EFT = QFT - CFT$$

# Theory Space of Canonical EFTs




To get something interesting we must consider non-trivial backgrounds




- Non-Fermi Liquids
- Fermions at Unitarity

- Heavy Quark EFT
- NRQCD
- Soft Collinear EFT (SCET)
- NRGR (Non-Relativistic GR)
- Large Scale Structure (see Senatore)
- Hydrodynamics

Crucial distinction between EFT with/out non-trivial backgrounds is that the explicit symmetry breaking can lead to hierarchies of scales which would otherwise not be present. These scales can be explicit (e), dynamically generated (dg), or induced by the measurement process (m).

HQET: $m_Q \quad (e), \Lambda_{QCD}(dg)$

NRQCD: $m_Q \quad (e), mv \quad (dg), \quad mv^2 \quad (dg), \Lambda_{QCD} \quad (dg)$

SCET: $Q \quad (e), (Q(1-x), p_T, Q\tau, Qe) \quad (m), \Lambda_{QCD}(dg)$

## These scales can in general introduce two novel effects:

- Modal Field Decompositions

**Fields are split into modes which have differing momentum scalings. The necessary modes are determined by matching cut structure of the full theory. This must be done in a way which is consistent with gauge invariance and care must be taken not to double count.**

- Non-Wilsonian Renormalization group.

**Large Logs arise as a consequence of ratios which are unrelated to invariant masses.**

Any well defined EFT must have an action in which each term scales homogeneously in the relevant expansion parameter in order to preserve the systematics

Moreover, it often the case that such actions lead to mode factorization, which is crucial in the case of QCD for predictive power.

# Modes needed to reproduce non-analyticities fixedby Landau conditions,or more physically Colemen-Norton Theorem

## SCET: Effective theory of highly energy particles

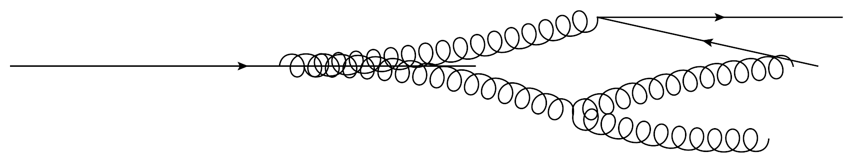
(Bauer,Luke,Fleming, Pirjol,Stewart)

$p^\mu_{\bar{n}} \sim (1, \lambda^2, 1)$

$p^\mu_{\bar{n}} \sim (\lambda^2, 1, \lambda)$ $\quad (\lambda = \frac{p_{IR}}{Q})$

Collinear modes

Reproduce NA structure of Jets



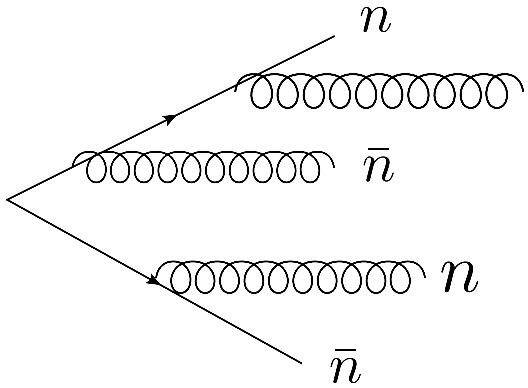
$$\psi(x) = \sum_{p^+} e^{ip_+ x_-} \xi_{p_+}(x)$$

Remove large momentum from the field (reminiscent of EFT of Fermi surface)

$(p_\mu/Q \ll 1)$

In addition we have SOFT modes which could in principle talk between jets

Naively modes these mode do not decouple as their interactions are LO in power counting




$$\bar{\xi}_n W_n^\dagger W_{\bar{n}} \xi_{\bar{n}}$$

Manifest $SU(3)_n \otimes SU(3)_{\bar{n}}$ Gauge Symmetry

$$L = L_n + L_{\bar{n}}$$

$$T_{\mu\nu} = T^n_{\mu\nu} + T^{\bar{n}}_{\mu\nu} + O(1/Q)$$

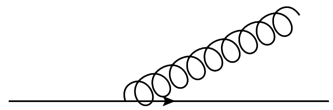
How do soft modes affect this picture?

# Nature of the Soft Mode depends upon the choice of observable

| SCETI | SCETII |
|---|---|
| $p^\mu \sim (\lambda^2, \lambda^2, \lambda^2)$ | $p^\mu \sim (\lambda, \lambda, \lambda)$ |
| Observable insensitive to soft recoil | Observable sensitive to soft recoil |
| DIS x->1 | Sudakov Form Factor |
| Drell-Yan at Threshold | Transverse Momentum Distributions |
| Jet Thrust | Jet Broadening |

**How does factorization Persist?**

## SCETI



US interactions allowed at level of action

$$S = \sum_n \int d^4x \; \bar{\xi}_n \left( in\cdot D + \not{D}_c^{\perp} \frac{1}{\bar{n}\cdot D_c} \not{D}_c^{\perp} \right) \frac{\not{\bar{n}}}{2} \xi_n$$

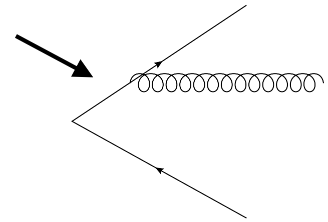
US gauge field acts as background field, factorization is made manifest by BPS field redefinition

$$\xi \to Y\xi \qquad Y = P e^{i\int_0^{\infty} n\cdot A(n\lambda + x)d\lambda}$$

Wilson lines appear in operators

## SCETII

off shell state



Soft interactions only allowed at level of operators

Integrating out off shell modes generates soft Wilson lines

$$O_{SFF} = \bar{\xi}_n W_n S_n^{\dagger} \gamma_{\mu}^{\perp} S_{\bar{n}} W_{\bar{n}}^{\dagger} \xi_{\bar{n}}$$

$$\langle p_n \mid O_{SFF} \mid p_{\bar{n}} \rangle = J_n J_{\bar{n}} S$$

$$S = \langle 0 \mid S_n^{\dagger} S_{\bar{n}} \mid 0 \rangle \qquad J_n = \langle p_n \mid \bar{\xi}_n W_n^{\dagger} \mid 0 \rangle$$

In both cases factorization is manifest at level of the action and symmetry group is

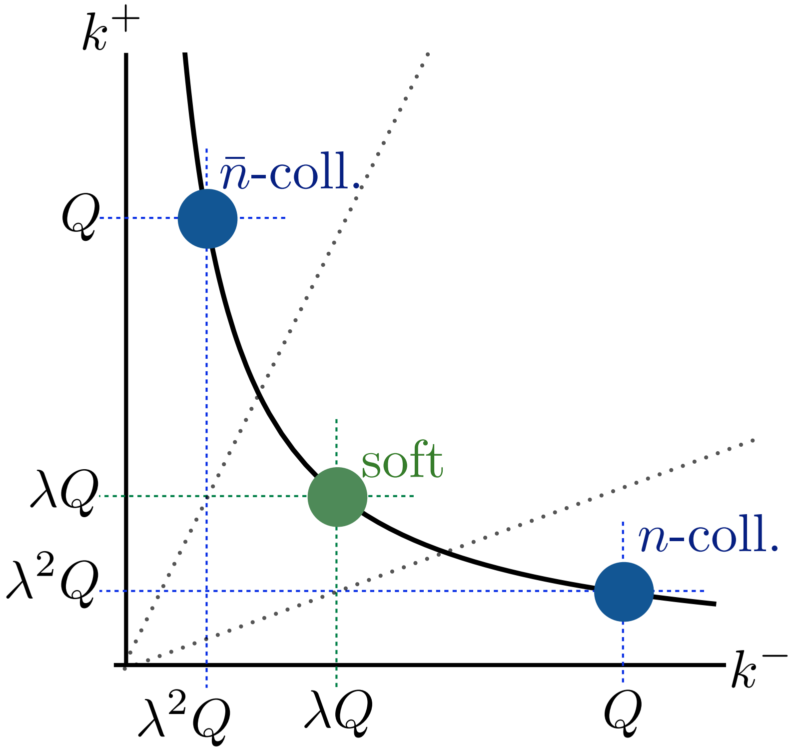
$$SU(3)_n \otimes SU(3)_{\bar{n}} \otimes SU(3)_{S(US)}$$

factorization $\quad L = L_n + L_{\bar{n}} + L_{S,US}$

$$\langle p_n p_{\bar{n}} \mid O_n O_{\bar{n}} O_{S,US} \mid p_n p_{\bar{n}} \rangle = \langle p_n \mid O_n \mid p_n \rangle \otimes \langle p_{\bar{n}} \mid O_{\bar{n}} \mid p_{\bar{n}} \rangle \otimes \langle 0 \mid O_{S,US} \mid 0 \rangle$$

Matrix Factorizes to all orders

# Crucial Distinction Between SCETI and SCETII




SCETII involves modes that sit on same rapidity hyperbola. This leads to the need for a factorization scale, which arises in the form of a new set of divergences which are not regulated by dim. reg.

Manifest itself in the form of rapidity divergences which do not cancel sector by sector

Introduce a rapidity scale $\nu$ which separates modes

$$I = \int \frac{dk_+}{k_+} \mid k_+/\nu \mid^{-\eta}$$

$\exists$ Gauge invariant prescription

$$d\sigma = S(\nu,\mu) J_n(\mu,\nu) J_{\bar{n}}(\mu,\nu)$$

# Rapidity Renormalization Group

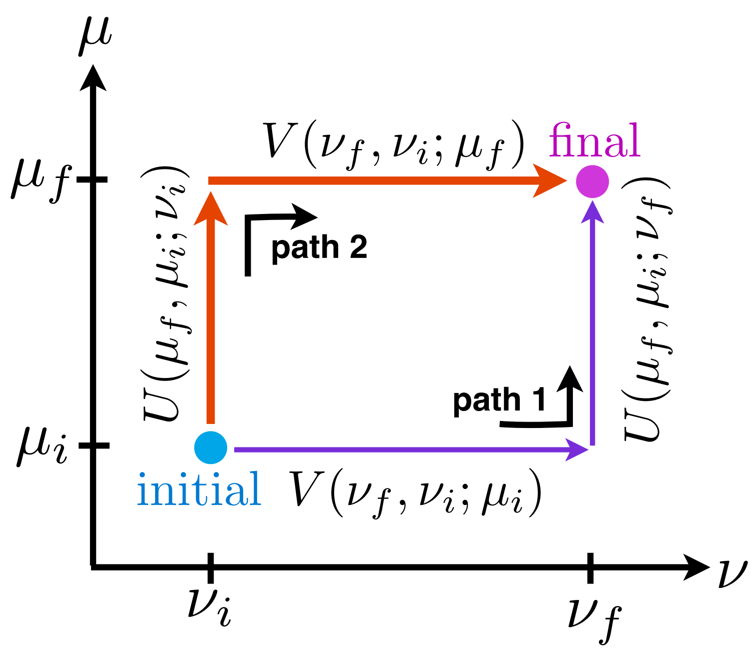
$\nu \frac{d}{d\nu} S = \gamma_S^\nu S \qquad \nu \frac{d}{d\nu} J_n = \gamma_J^\nu J_n$

(Chiu, Jain,Neill,IZR )

(Also see earlier work by Balitsky)

$$[\frac{d}{d\log\nu}, \frac{d}{d\log\mu}] = 0$$

$$(\frac{\partial}{\partial \ln[\mu]} + \beta \frac{\partial}{\partial g})\gamma_\nu = \frac{d}{d\ln[\nu]}\gamma_\mu = \mathbb{Z}\Gamma_{\rm cusp} \,,$$




Allows for systematic resummation of rapidity logs along with control of scale dependence

Phenomenlogical Implications

# (Higgs) Transverse Momentum Distribution

(Chiu,Jain,Neill,IZR)

$$\frac{d\sigma}{dp_\perp^2\, dy} = \frac{C_t^2}{8v^2 S(N_c^2-1)} \int \frac{d^4 p_h}{(2\pi)^4}(2\pi)\delta^+(p_h^2 - m_h^2)\,\delta\!\left(y - \frac{1}{2}\ln\frac{p_h^+}{p_h^-}\right)\delta(p_\perp^2 - |\vec{p}_{h\perp}|^2)$$

$$4(2\pi)^8 \int d^4x\, e^{-ix\cdot p_h} H(m_h) f^{\mu\nu}_{\perp\, g/P}(0, x^+, \vec{x}_\perp) f_{\perp\, g/P\, \mu\nu}(x^-, 0, \vec{x}_\perp)\mathcal{S}(0,0,\vec{x}_\perp)$$

$$\mathcal{S}(0,0,\vec{x}_\perp) = \frac{1}{(2\pi)^2(N_c^2-1)}\langle 0|S_n^{ac}(x)S_{\bar{n}}^{ad}(x)S_n^{bc}(0)S_{\bar{n}}^{bd}(0)|0\rangle\,,$$

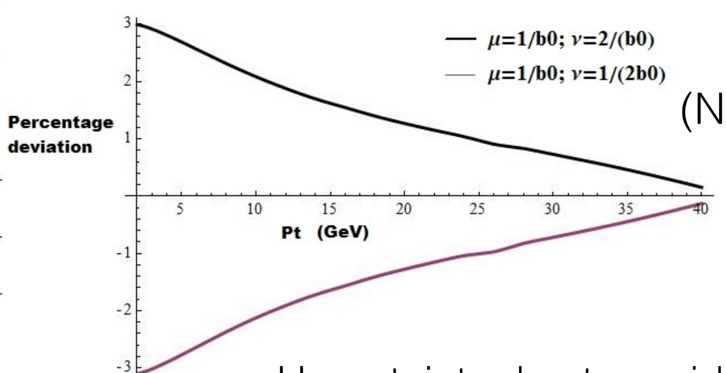
$$f^{\mu\nu}_{\perp\, g/P}(0,x^+,\vec{x}_\perp) = \frac{1}{2(2\pi)^3}\langle p_n|[B^{A\mu}_{n\perp}(x^+,\vec{x}_\perp)B^{A\nu}_{n\perp}(0)]|p_n\rangle\,,$$

$$f^{\mu\nu}_{\perp\, g/P}(x^-,0,\vec{x}_\perp) = \frac{1}{2(2\pi)^3}\langle p_{\bar{n}}|[B^{A\mu}_{\bar{n}\perp}(x^-,\vec{x}_\perp)B^{A\nu}_{\bar{n}\perp}(0)]|p_{\bar{n}}\rangle$$

TMPDF's match onto PDF at the scale pt

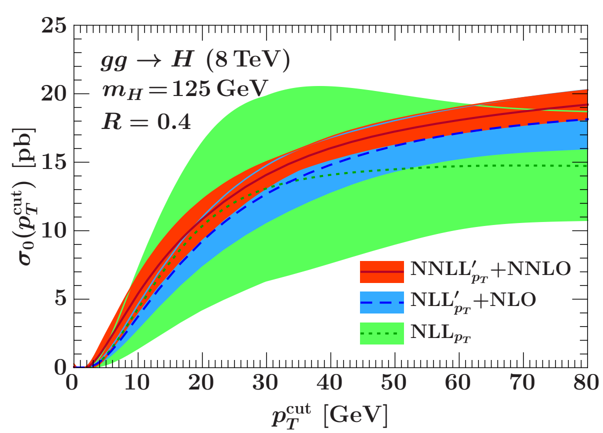
$$f_\perp \sim f_\perp(\mu = p_t, \nu = m_H) \qquad S \sim S(\mu = p_t, \nu = p_T) \qquad H \sim H(\mu = m_H)$$

Working in P.T. implies both canonical scale as well as rapidity scale dependence




(Neill, IZR, Vaidya)

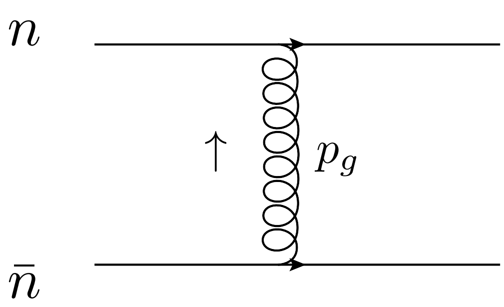
Uncertainty due to rapidity scale




(Stewart, Tackmann Walsh, Zuberi)

# SCET formalism is lacking a treatment of a nettlesome mode

(Work in progress with I. Stewart)

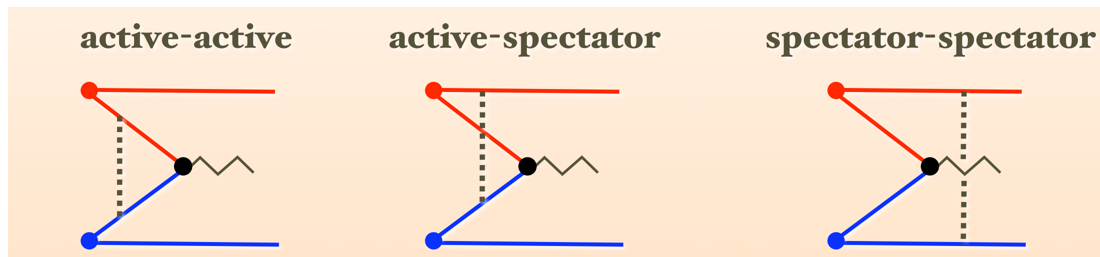



$$O_g = \frac{g^2}{p_T^2}(\bar{\xi}_n W_n T^a \frac{\bar{\not{n}}}{2} W_n^\dagger \xi_n)(\bar{\xi}_{\bar{n}} W_{\bar{n}} T^a \frac{\not{n}}{2} W_{\bar{n}}^\dagger \xi_{\bar{n}})$$

## The Glauber mode

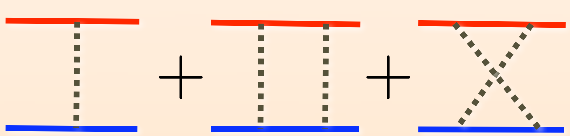
$$p_g^\mu \sim (\lambda^2, \lambda^2, \lambda)$$

Contributes at leading order to action, threatens factorization.



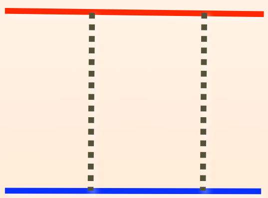

If there were no hard interaction then Glauber is responsible for forward scattering, so Glaubers form a phases in hard collisions

- Abelian Eikonal Phase



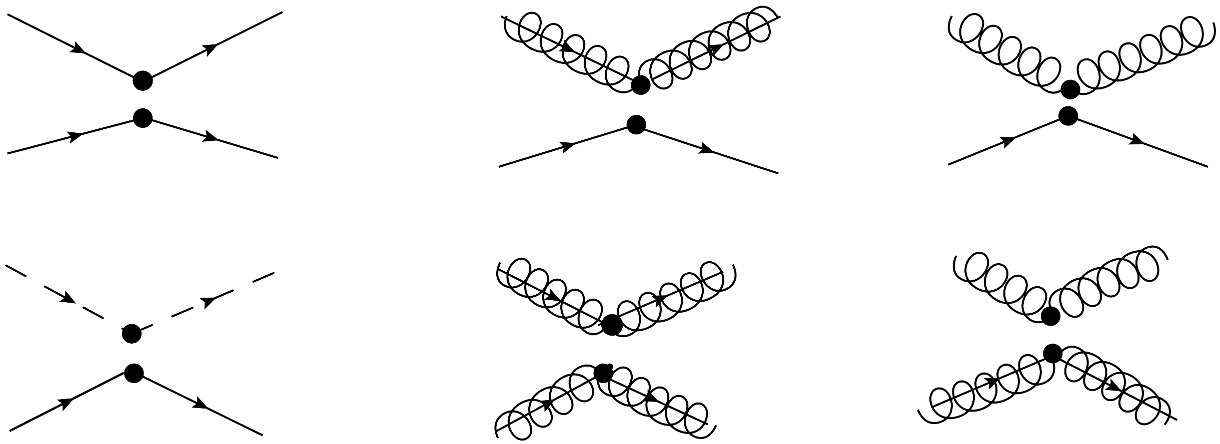
$$+ \ldots = \sum_{m=0}^{\infty} \frac{1}{(m+1)!}(i\tilde{\phi}_G)^{m+1} = e^{i\tilde{\phi}_G} - 1$$

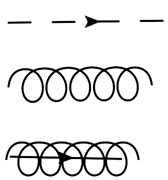
# Note: to make sense of integrals in EFT need rapidity regulator



$$\int \bar{d}^d k \, \frac{|\, k_3/\nu \,|^{-\eta} \qquad 1}{k_\perp^2 (k_\perp - q_\perp)^2 \left[k^+ + p^+ - \frac{(p_\perp + k_\perp)^2}{p^-} + i0\right]\left[-k^- + p'^- - \frac{(p'_\perp - k_\perp)^2}{p'^-} + i0\right]}$$

This had to be the case since Glauber shares a rapidity hyperbola with collinears, need rapidity factorization (non-trivial RRG?)

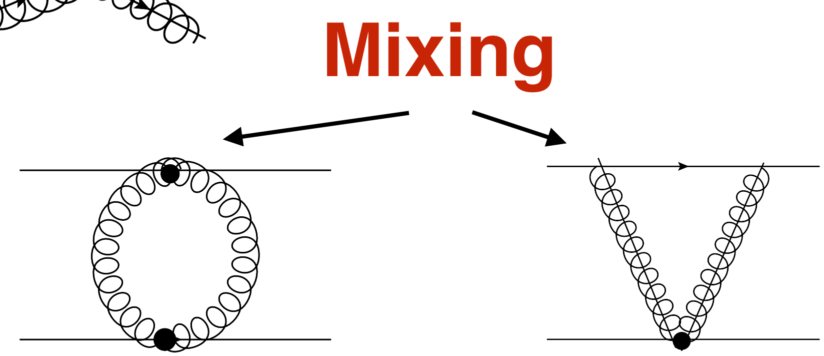




soft quark

soft gluon

collinear gluon



# Mixing induces both RG as well as RRG running

If we write four body operators as product of bi-linears (allowing for identity operator) then the problem is reduced to mixing of bilinear and time ordered products

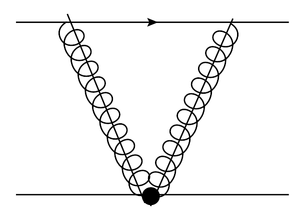
$$O_4 \equiv O_n O_{\bar{n}} O_S$$

$$O_n \equiv O_n(\mu = \sqrt{t}, \nu = \sqrt{s})$$
$$O_S \equiv O_S(\nu = \sqrt{t})$$

natural scales

**Let us focus on RRG**

To eliminate resum Log(s) let us run the collinear sector in nu from s down to t.



$(\bar{\xi}\xi, BB)$ basis

$$\nu \frac{d}{d\nu} \xi_i = A_{ij} \xi_j \qquad A = \begin{pmatrix} 0 & y \\ 0 & x \end{pmatrix} \qquad y = x = \frac{\alpha(\mu) C_A}{2\pi} \log(\mu^2/t)$$

# Eigensystem

$$[\lambda_1 = 0, \rho_1 = (1,0)\ ;\ \lambda_2 = x, \rho_2 = (1,1)]$$

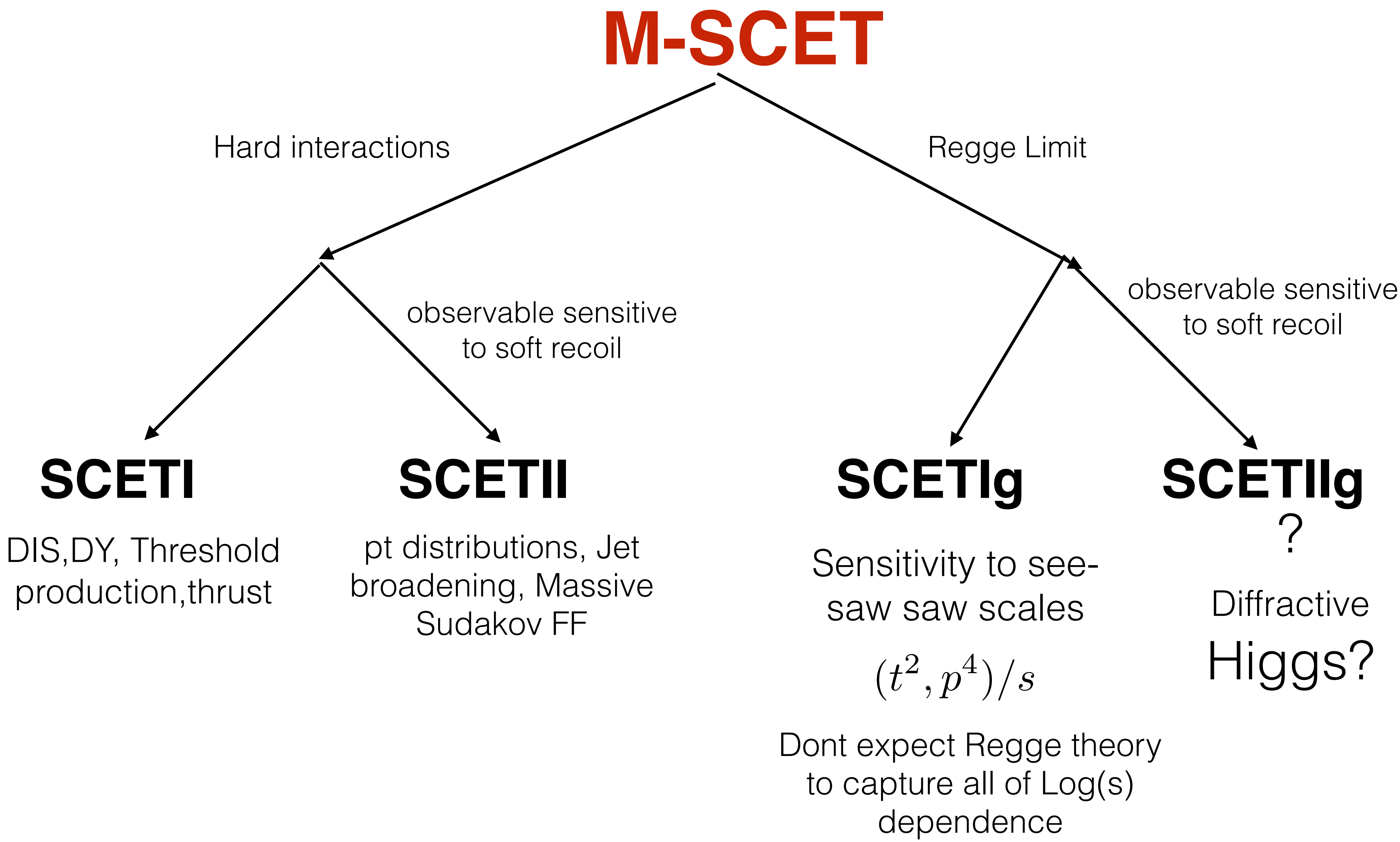
$$\bar{\xi}_n \not{\bar{n}} \xi_n(\nu = \sqrt{s}) = \bar{\xi}_n \not{\bar{n}} \xi_n(\nu = \sqrt{t})$$

$$(BB_n + \bar{\xi}_n \not{\bar{n}} \xi_n)\xi_n(\nu = \sqrt{s}) = [(BB_n + \bar{\xi}_n \not{\bar{n}} \xi_n)(\nu = \sqrt{t})](\sqrt{s}/\sqrt{t})^{-x}$$

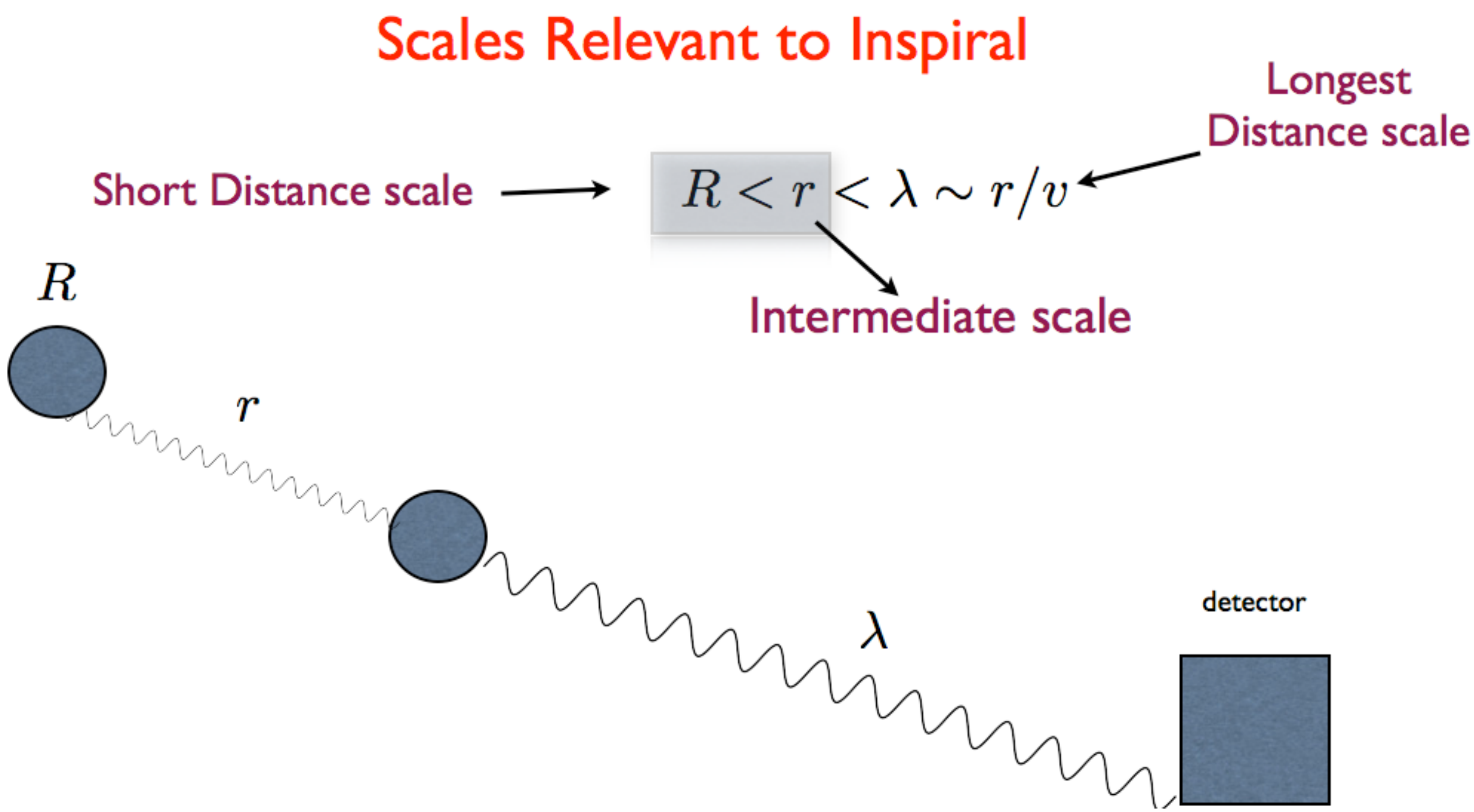
$$BB_n(\nu = \sqrt{t}) = [(\sqrt{t}/\sqrt{s})^x - 1]\bar{\xi}_n \not{\bar{n}} \xi_n(\nu = \sqrt{s}) + (\sqrt{t}/\sqrt{s})^x BB_n(\nu = \sqrt{s})$$

Gluon Reggeization

- Exponent is IR finite to all orders
- Anomalous dimensions leads to universality of Reggeization
- There can be additional Log(s) dependence depending upon the choice of PHYSICAL observable. e.g. hemisphere masses. (need to match onto next theory)


M-SCET
Hard interactions
Regge Limit
observable sensitive to soft recoil
observable sensitive to soft recoil
SCETI
DIS,DY, Threshold production,thrust
SCETII
pt distributions, Jet broadening, Massive Sudakov FF
SCETIg
Sensitivity to see-saw saw scales
$(t^2, p^4)/s$
Dont expect Regge theory to capture all of Log(s) dependence
SCETIIg
?
Diffractive
Higgs?

# Gravitational EFT for Compact Bodies (Walter Goldberger, IR)




Interested in calculating gravitational wave form with high precision (LIGO)

## This is a modal theory which share many similarities (when working in PN approximation) with NRQCD

- Modes which generate internal dynamics of compact bodies.
- Potential $p^\mu = (v/r, 1/r)$
- Radiation (only IR modes in theory) $p^\mu = (v/r, v/r)$

## Two Stage Theory

- Integrate out short distance modes match on to theory of point particleS
- Integrate out potential mode leaving an effective theory of multipole moment coupling to radiation field

# 1) treat constituents as point particles

$$S_M = -m\int ds \qquad S = \int -2M_{pl}^2\sqrt{g}Rd^4x$$

$$S_{FS}^{LO} = c_E\int d\tau E_{\mu\nu}E^{\mu\nu} + c_B\int d\tau B_{\mu\nu}B^{\mu\nu}$$

$$+C_R\int d\tau R + C_v\int d\tau v_\mu v_\nu R^{\mu\nu} + \ldots .$$

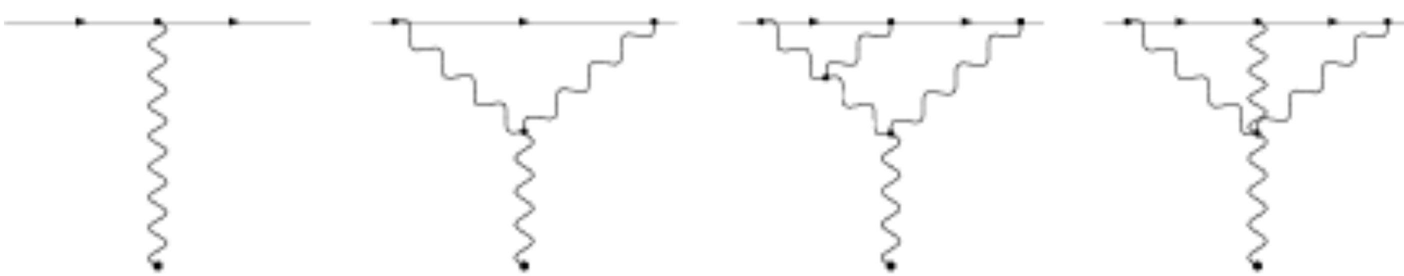
Removable by field redefs
(Birkoffs Thm)

more on
these later

$C_E, C_B \sim R^5$

$C_R, C_v \sim R^3$

This theory is applicable to either EMRI or PN at this point.
One point function is UV log divergent absorbable into $C_v$ $C_R$



(a) (b) (c) (d)

## 1) Integrate out short distance potential mode

## 2) Match onto a theory of long wavelength radiation gravitons coupling to multipole moments of system.

Radiation treated as background field maintains diff inv.

Radiation $\sim v^{5/2}$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{pl}} \equiv \eta_{\mu\nu} + \frac{H_{\mu\nu}}{M_{pl}} + \frac{\bar{h}_{\mu\nu}}{M_{pl}}.$$

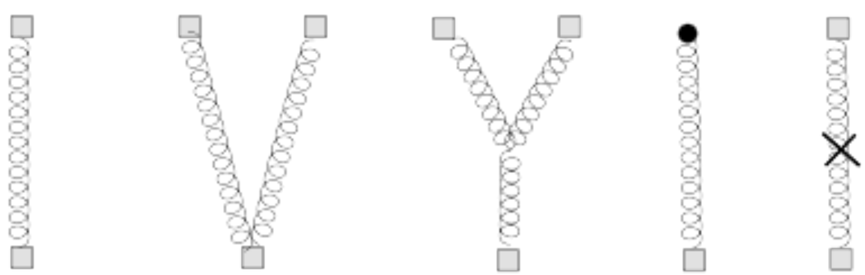
Potential $\sim v^2$

$$Z[x_i, v_i] = \int (DH)(D\bar{h}) e^{iS(x,v,\bar{h},H)}$$

Every term in S scales homogenously in v

$$= \int (D\bar{h}) e^{iS_{eff}(V_i, L, Q, \bar{h},)}$$

Calculate to some fixed order in PN



Quantum corrections suppressed by 1/L

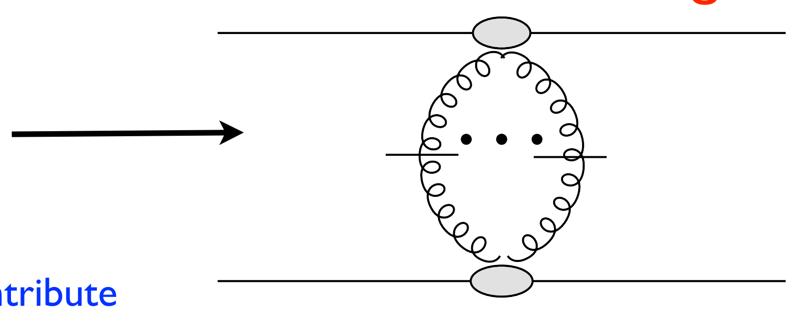
potential calculated at $3PN\ and\ O(G^2v^4)\ 4PN$ (Foffa and Sturani)

**100+Diagrams usual story, however we can use modern unitarity +BCFW methods to reduce the workload** (D. Neill, IZR)

Calculate tree level S-matrix for scalar-graviton scattering via BCFW

Unitarity (Bern, Dixon, Kosower)

(S-channel cuts don't contribute to classical potential)



Sew together to tree level S-matrix elements

Eliminates the need to calculate all the graviton Feynman diagrams

Match onto a theory of massive scalar interacting via a set of potentials

$$L = \sum_i \int d^3p d^3q\ V_i(q,p)\ \phi^\dagger(p+q/2)\phi(p-q/2)\phi^\dagger(-p-q/2)\phi(-p+q/2)$$

Given potentials we can also go to probe limit and extract metric, thus generating classical space-time forgoing GR. Only assumption is the existence of a spin two massless field, the rest follows from Lorentz invariance, unitarity and locality.

# Radiation Theory

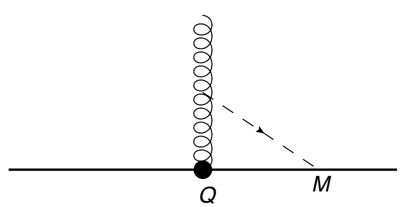
One we have integrated out the potentials we match onto another point particle theory, endowed with moments of binary.

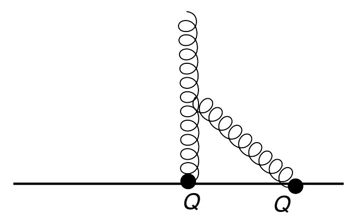
$$S = -\int M d\tau - \frac{1}{2}\int dx^{\mu}\omega_{\mu}^{ab}L_{ab} + \int d\tau(\frac{1}{2}Q_{ab}E^{ab} - \frac{4}{3}J^{ab}B_{ab} + \frac{1}{3}O^{abc}\nabla_{c}E_{ab} + \ldots..)$$

source moments (worked out to all orders (Ross))

Power Loss can be calculated via in-out S matrix elements $A_h(k) =_{out} \langle \epsilon(k) \mid 0 \rangle_{in}$

note that higher order effects involving calculation within this final theory: e.g. tail and memory effects




``radiative moments``

Tail Effect

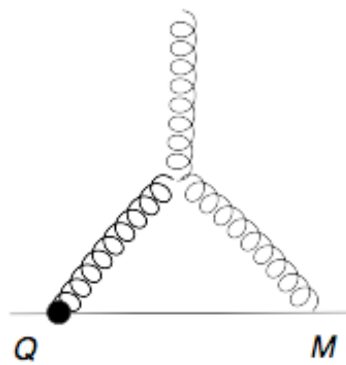
Memory Effect

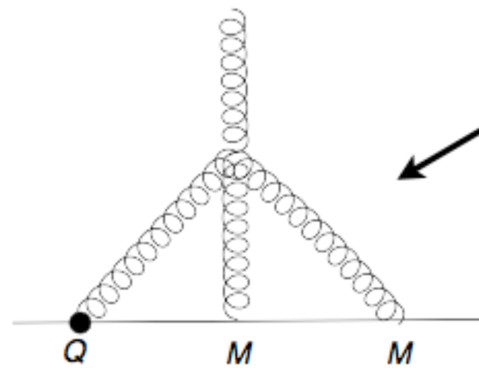
# Renormalization of the Radiation Theory and Log Resummation

Quadrapole renormalization (Goldberger and Ross)

## Quadrapole moments are scale dependent via

IR div. Coulomb phase (cancels in any physical observable)




UV div. physical log

$$\frac{1}{\epsilon_{UV}} + ALog(\omega^2/\mu^2)$$

+....

Divergence gets absorbed into renormalized quadrapole $Q^R_{ij} = Z^{-1}(\omega,\mu) Q^B_{ij}$

$$Z^{\bar{M}S} = 1 + \frac{107}{105}(Gm\omega)^2\left(\frac{1}{\epsilon_{UV}} + \gamma_E + Log(4\pi)\right)$$

$$\mu\frac{d}{d\mu}Q^B = 0$$

$$\mu\frac{d}{d\mu}Q^R = -\frac{214}{105}(Gm\omega)^2 Q^R$$

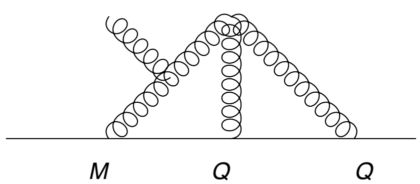
By Choosing $\mu = \omega$ we eliminate the logs in the amplitude.

$$Q^R(\omega,\mu) = (\mu/\mu_0)^{(-214/105(Gm\omega)^2)} Q(\omega,\mu_0)$$

Infinite sum of log enhanced terms $\sum_n C_n (Gm\omega)^{2n} Log^n(r\omega)$

$$-\frac{39201376}{3472875}(Gm\omega)^6 Log^3(\omega r) \sim v^{18}$$ checked in test mass limit (Fujita)

# Mass Renormalization (Goldberger, Ross, IZR)




+...

$\beta_Q = -214/105$

Lagrangian mass parameter is asymptotically free
**and time dependent**

$$\mu \frac{d}{d\mu}\bar{m} = -2G^2 \langle Q^{(3)}_{ij} Q^{(3)}_{ij} \rangle$$

$$\frac{\bar{m}(\mu)}{\bar{m}(\mu_0)} = exp\left[\frac{\langle Q^{(2)}_{ij} Q^{(2)}_{ij}\rangle_{\mu_0} - \langle Q^{(2)}_{ij} Q^{(2)}_{ij}\rangle_{\mu}}{\beta_Q \bar{m}_0^2}\right] = 1 - \frac{1}{2}\frac{\langle Q^{(3)}_{ij} Q^{(3)}_{ij}\rangle}{\bar{m}_0^2} r_s^2 Ln(v) + \frac{107}{420}\frac{\langle Q^{(4)}_{ij} Q^{(4)}_{ij}\rangle}{\bar{m}_0^2} r_s^4 Ln^2(v) - \frac{11449}{132300}\frac{\langle Q^{(5)}_{ij} Q^{(5)}_{ij}\rangle}{\bar{m}_0^2} r_s^6 Ln^3(v) + \ldots.$$

$$E(\Omega) = -\frac{\mu}{2}\frac{448}{15}\nu x^5 \ln x + \ldots$$ Agrees with (Blanchet,Detweiler, Le Tiec and Whitting)

EFT methods have been used be to reach state of the art calculations. Some of these results have yet to be calculated using traditional methods: e.g. 3PN multipole moment for spinning holes (Porto,Ross,IZR).

# Finite Size Effects

$$S^{LO}_{FS} = c_E \int d\tau E_{\mu\nu} E^{\mu\nu} + c_B \int d\tau B_{\mu\nu} B^{\mu\nu} \;\; \sim v^{10}$$

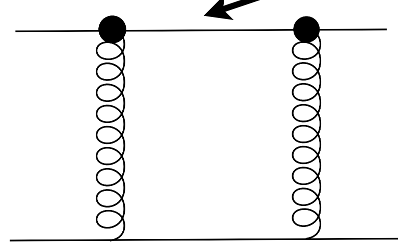
However, in addition there are dissipative effects which can not be accounted for by local operators, we add degrees of freedom to world line.

(W. Goldberger,IZR)

$$S_{dis} = \int d\tau (Q_{ab} E^{ab} + M_{ab} B^{ab})$$

Dynamical evolution of non-gapped DOF

Absorptive potential



$$q_{ij}(t) \;=\; \partial_i \partial_j \frac{1}{|\vec{x}_1 - \vec{x}_2|}.$$

$$-iVT = \frac{M_2^2}{1024\pi^2 M_{pl}^4} \int d\tau d\tau' \langle 0 \mid T(Q_1^{ij}(\tau) Q_1^{kl}(\tau')) \mid 0 \rangle q_{ij} q_{kl} + (1 \leftrightarrow 2)$$

$$\langle 0 \mid T(Q_1^{ij}(\tau) Q_1^{kl}(\tau')) \mid 0 \rangle = A(\tau - \tau')(\frac{2}{3}\delta^{ij}\delta^{kl} - \delta^{ik}\delta^{jl} - \delta^{il}\delta^{jk}).$$

## The imaginary part of the correlator can be matched via the optical theorem

$$\sigma_{abs} = \frac{\omega^3}{2M_{pl}^2} Im(i\tilde{A}(\omega))$$

$$\frac{dP}{d\omega} = -\frac{1}{T}\frac{G_N}{64\pi^2}\sum_{a\neq b}\frac{\sigma_{abs}^{(b)}}{\omega^2}M_{(a)}^2 \mid q_{ij}^{(a)}(\omega)\mid^2 .$$

Also spinning of this version (Porto)

Valid for any compact object, in black hole case

$$P = \frac{32}{5}G^7(M_1^6M_2^2 + M_2^6M_1^2)\left(2\frac{\dot{r}^2}{r^8} + \frac{\dot{\vec{x}}^2}{r^8}\right)$$

(poisson)

The real part has a Taylor expansion has coefficients which correspond to the Coefficients of the finite size local operators.

# Vanishing of $c_E$ for BHs

(Damour, and Nagar; Binnington and Poisson)

(Kol and Smolkin; Chakrabarti, Delsate and Steinhoff (CDS))

$$A(\omega) \sim \frac{i2MG\omega}{45} + (2MG\omega)^2 \left(\frac{3486611}{54096525} - \frac{1}{45} Log(\omega/\mu) + \ldots\right.$$ (CDS)

mu dependence cancelled by mu dependence of $C_{\dot{E}^2}(\mu)$

$$S_{FS} = C_{\dot{E}^2}(\mu) \int d\tau \dot{E}^2$$

*Absence of constant term implies that* $c_E = 0$

(hidden symmetry?)

# This is a remarkable power law fine-tuning as there exist diagrams which renormalize this operator

(In progress with W. Goldberger)

$$G_{abcd}(\omega) = \int d\tau e^{i\omega\tau}\theta(\tau)\langle\Omega \mid [Q_{ab}(\tau), Q_{cd}(0)] \mid \omega\rangle$$

$$G_{abcd}(\omega) = (-\frac{2}{3}\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})(a_R + ia_I + \omega(b_R + ib_I) + \omega^2(c_R + ic_I) + \ldots)$$

$$Q^{ab}(\omega) = -\frac{1}{2}E^{ab}_{BG}(\omega)F(\omega) \qquad F(\omega) = (a_R + ia_I + \omega(ib_I) + \omega^2(c_R + ic_I) + \ldots)$$

$$ReF(\omega) = P\sum_m \frac{\mid \langle\Omega \mid Q_{ab} \mid m\rangle \mid^2}{E_\Omega - E_m - \omega}$$

$$\sum_m \frac{\mid \langle\Omega \mid Q_{ab} \mid m\rangle \mid^2}{E_\Omega - E_m} = 0.$$

**Not a pure state**

$$\sum_{m,n} e^{-\beta(E_n)} \frac{\langle n \mid Q_{ab} \mid m\rangle\langle m \mid Q_{ab} \mid n\rangle}{E_n - E_m} = 0$$

**Suppose it is thermal**

# Other applications of world line EFT

- Caged Black Holes (Chu,Golberger, IZR), (Kol, Smolkin), (Gilmore, Smolkin, Ross)
- EMRI (Galley, Galley and Porto)
- Fluctuation Forces on membranes (Deserno, IZR, Yolcu)
- Casimir Cogs (Vaidya)