Noise-resilient quantum circuits

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Why don't we have a large-scale quantum computer?

- Classical
  - Noise rate : $\sim 10^{-15}$.
  - Primary source of error : Neutrons from the cosmic rays.
- Quantum
  - Noise rate : $\sim 10^{-2} - 10^{-3}$.

Fighting noise

The theory of fault tolerance comes to a rescue! But...

- Can you fit millions of qubits in a single dilution fridge?
- Gate speed problem
  - Too slow : Quantum computer with clock speed of 1Hz?
  - Too fast : Decoding can't keep up.
- $+$ all the problems that are yet to be discovered.

We're not going to have a large scale quantum computer tomorrow, but it is very likely that we will have a noisy quantum computer consisting of say, 100 qubits in near term. What should we do with them?

Focusing on large-depth quantum circuits

- There seems to be a consensus that large-depth quantum circuits, without error correction, will be useless.
  1. Without error correction, error keeps accumulating.
  2. In the long run, the effect of error cannot be bounded.
  3. Therefore, such circuits will be completely useless without error correction.

Not true

Noise resilient circuits

Definition

A family of circuits $\{\mathcal{C}_n\}$ is *k-noise-resilient* if for every bounded k-local operator O with $k = O(1)$,

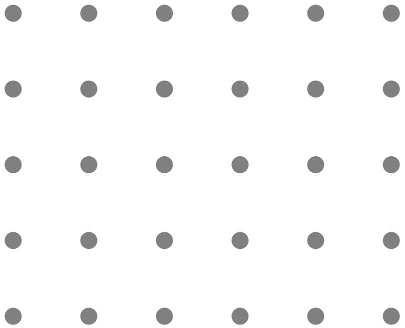
$$|\langle O \rangle_0 - \langle O \rangle_\epsilon| \leq c\epsilon.$$

- $\langle O \rangle_0$: E.v. of O, over a state $\mathcal{C}_n |\psi\rangle$
- $\langle O \rangle_\epsilon$: E.v. of O when every gate/prep/measurement becomes noisy. (ϵ: noise rate)

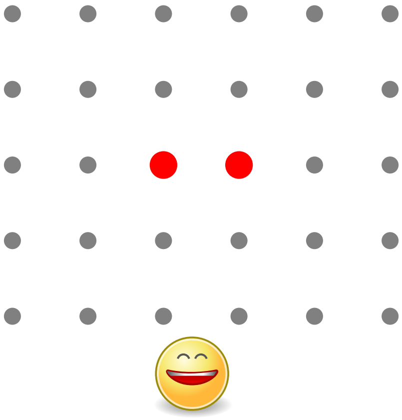
for every product state $|\psi\rangle$, for all $n < \infty$, for some $c = O(1)$.

* There are noise-resilient circuits that are not short depth.

Noise resilient circuits



Noise resilient circuits



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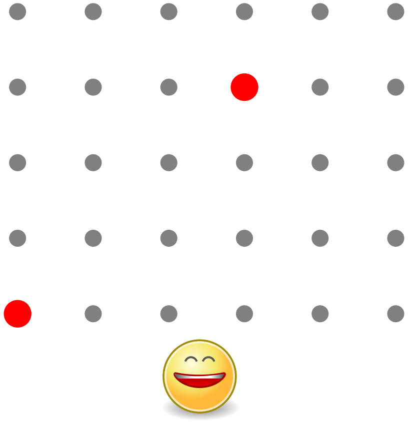

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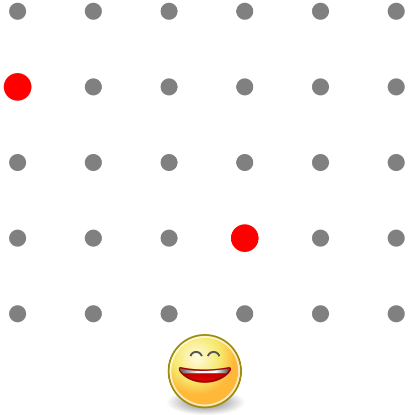

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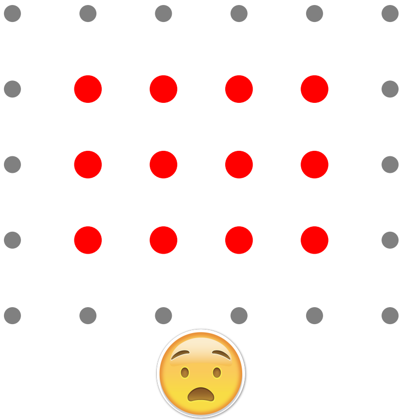

Agendas

1. Why noise resilience is important.
2. Intuition behind noise resilience.
3. Abundance of noise-resilient circuits.
4. How to use noise-resilient circuits.

1/poly(n) vs constant error

If you can estimate everything down to $1/\text{poly}(n)$ precision, that's great! But not every bit of useful information requires such precision. For instance, look at any averaged observables:

$$|\sum_{i=1}^{N} \frac{\langle h_i \rangle_0}{N} - \sum_{i=1}^{N} \frac{\langle h_i \rangle_\epsilon}{N}| \leq c\epsilon \max_i \|h_i\|.$$

- Energy/site is *the* figure of merit for comparing different numerical methods.
- For macroscopic observables, e.g., magnetization/site, its poly(n)th digit provides little useful information.

Name of the game

In quantum many-body physics,

- Lower the energy/site is, the better.
- Differences of energy/site between different method is $O(1)$, not $O(1/\text{poly}(n))$.

Question : Can a near-term quantum device participate in this game and win against all classical methods?

Outperforming classical computers

Question : Can a near-term quantum device participate in this game and win against **all** classical methods?

- The space of all classical methods is a huge space. Even if it can, it will be difficult to prove.
- However, there are models that have been studied intensively, AF Heisenberg on Kagome lattice, Fermi-Hubbard on square lattice.
- Physicists worked hard on beating each other's energy for decades.
- If a noisy quantum computer can beat the lowest known energy/site, that would be an indication that a noisy quantum computer can outperform classical computer for certain tasks.

Moreover, it will actually be useful.

- Averaged quantities : Energy per site/ magnetization per site, etc.

$$\langle \bar{O} \rangle = \langle \frac{\sum_i O_i}{N} \rangle$$

- Response functions : Magnetic susceptibility, bulk modulus, etc.

$$\alpha = -\frac{1}{V}\frac{\partial V}{\partial \lambda}$$

* V : extensive quantities

Agendas

1. Why noise resilience is important.
2. **Intuition behind noise resilience.**
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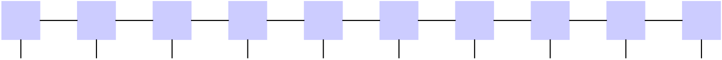
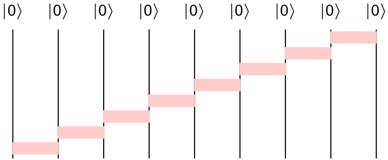
Guiding example : MPS



1. Generically, a unitary circuit that prepares a MPS is large-depth, but noise-resilient.
2. Running such a circuit on a quantum computer is ill-motivated, but we can learn some lessons.

Guiding example : MPS



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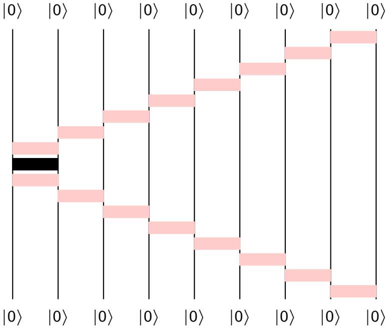

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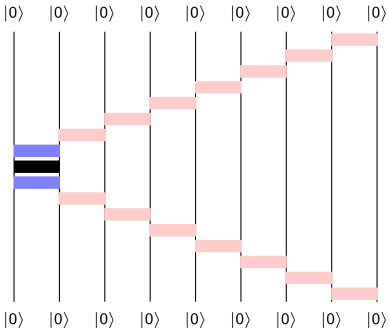

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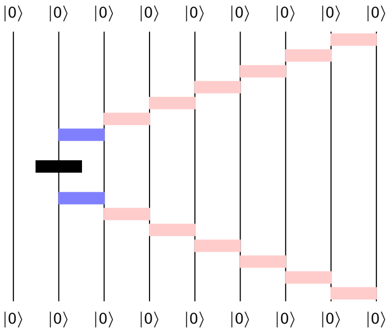

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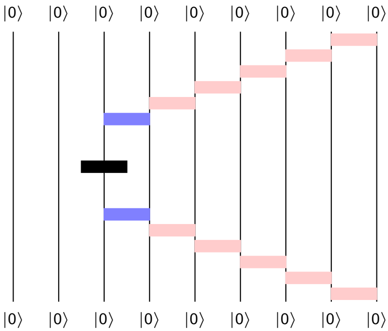

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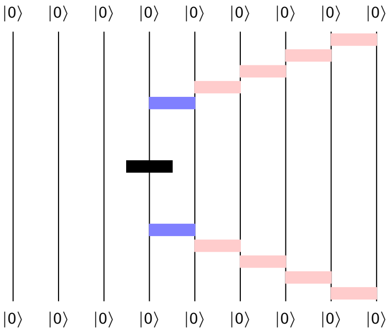

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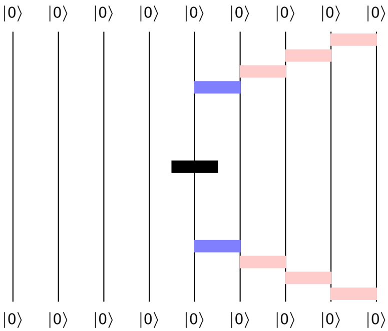

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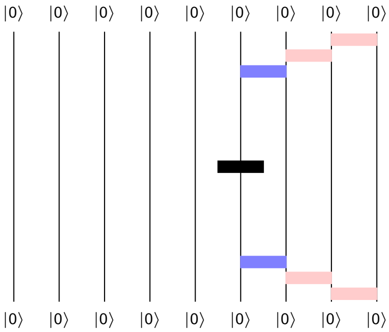

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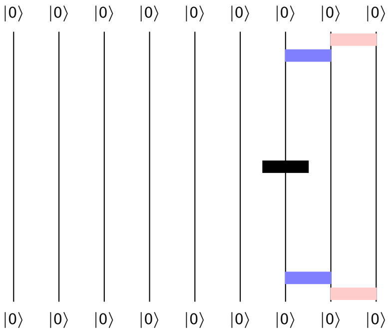

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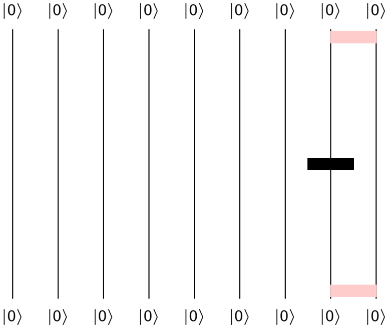

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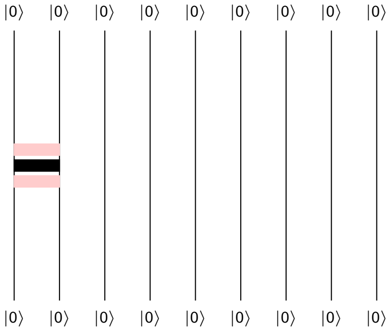

At each time, a fresh ancilla is introduced, interacts with the system, and the system is discarded. The ancilla becomes the new system.

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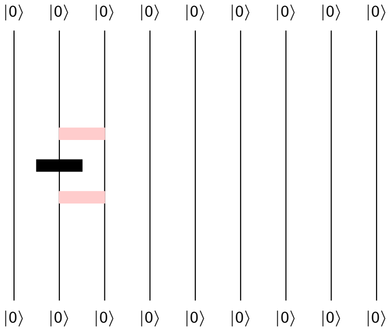

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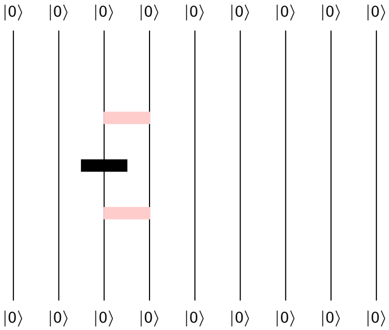

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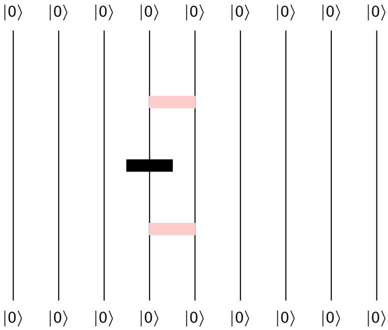

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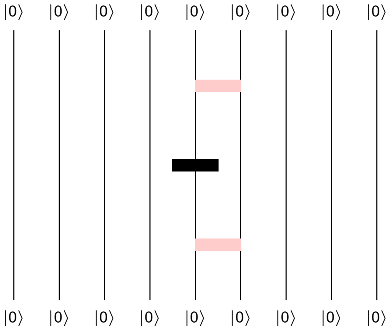

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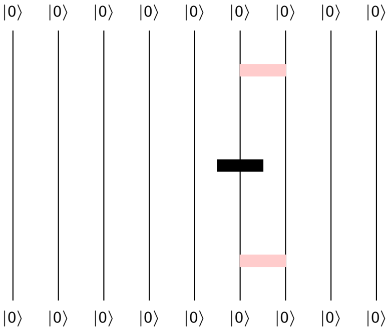

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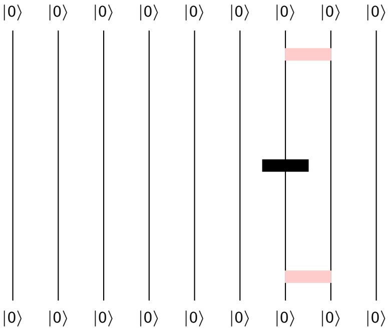

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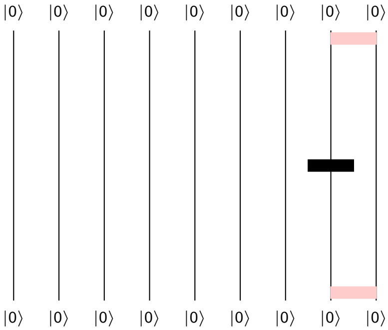

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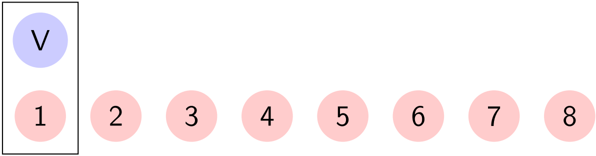

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: Virtual qubit
-

: ith physical qubit

MPS can be prepared by sequentially applying a unitary over V and i, from $i = 1$ to N. [Fannes et al. (1992)]

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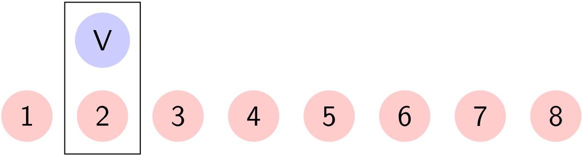

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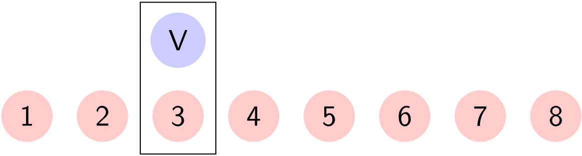

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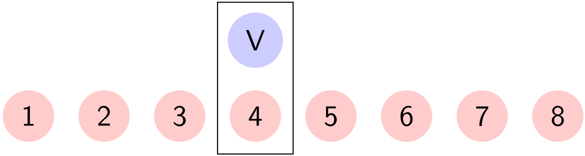

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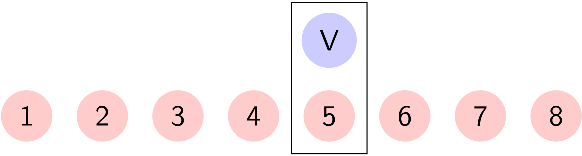

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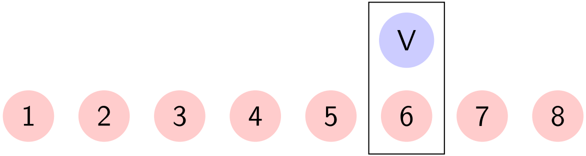

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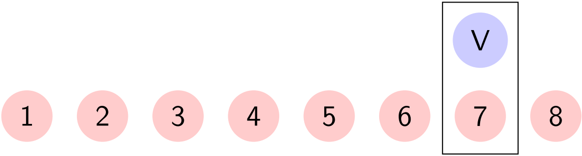

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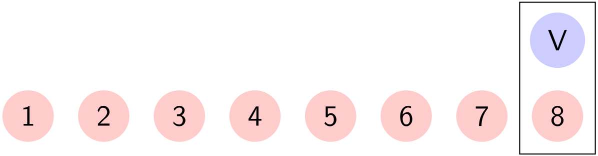

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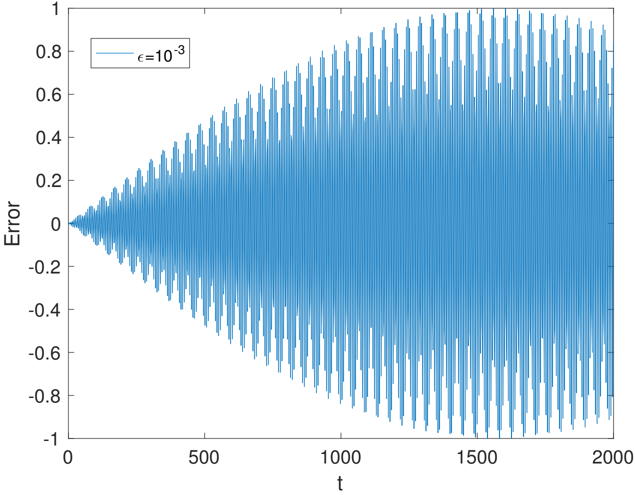
Guiding example : MPS

$$\langle O \rangle_{\mathsf{MPS}} = \mathrm{Tr}(\rho \Phi_n \circ \cdots \circ \Phi_2 \circ \Phi_1(O))$$

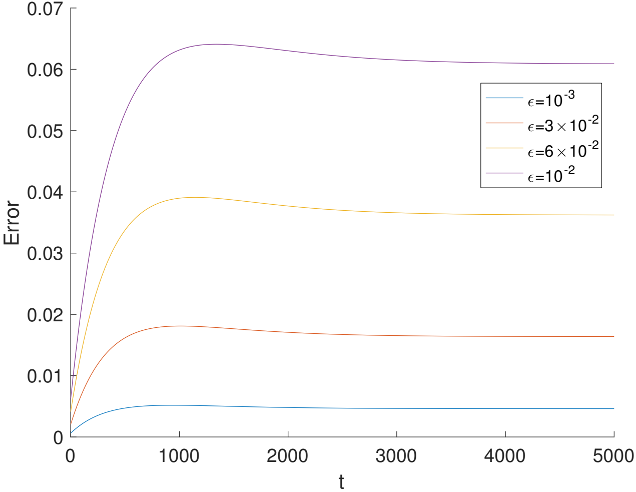
- Φ_i : Transfer operator
  - Norm-nonincreasing
  - Eigenoperator with eigenvalue λ
    - $|\lambda| = 1$: Oscillating mode
    - $|\lambda| < 1$: Decaying mode

* Generically, the identity operator is the only eigenoperator with $|\lambda| = 1$.

Oscillating modes are unstable



Decaying modes are stable



- Short-time : Little accumulation of noise
- Long-time : Noise in early time is killed off if *every* mode decays.

Decaying modes are stable

A state at time T is influenced by every perturbation at time $t \leq T$

$$\delta(t) \approx \Phi_{t\to T}(V_t)$$

- $\Phi_{t\to T}$: Transition from t to T.
- V_t : Perturbation at time t. (Bounded by ϵ)
- $\Phi_{t\to T}(V_t)$ decays exponentially in $T - t$.
  - In particular, $\sum_{t=0}^{T} \Phi_{t\to T}(V_t) = O(\epsilon)$ for all T.

Comments

1. MPS is useful : It can describe gapped 1D systems efficiently. [Hastings (2006)]
2. Contraction of MPS is noise-resilient, even though the circuit depth scales with the system size.
3. But we can already contract MPS efficiently on a classical computer, so there's not much point of implementing the circuit on a quantum computer.

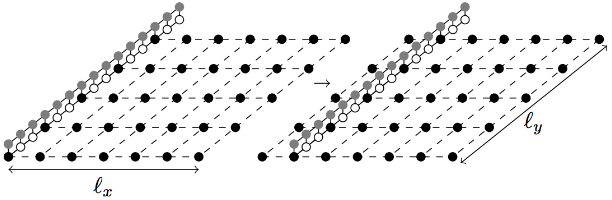
Agendas

1. Why noise resilience is important.
2. Intuition behind noise resilience.
3. **Abundance of noise-resilient circuits.**
4. How to use noise-resilient circuits.

Design principles

1. Nontriviality : The circuit must do something that physicists care about.
    - A noiseless circuit, applied to a simple-to-prepare states(e.g., product state) outputs k-point functions of states that are of interests to physicists.
2. Advantage : There must be an advantage in running the circuit over simulating it.
3. Noise-resilience : The circuit must be noise-resilient.
    - A reduced circuit for every k-local observable only consists of fastly decaying modes.

Example 1 : A subclass of PEPS



MPS vs PEPS

- Virtual qubit vs Rows of virtual qubits
- Physical qubit vs A row of physcal qubits
- Arbitrary unitary vs Finite-depth unitary

1. Nontriviality : The ansatz describes any Levin-Wen/quantum double [K (2017)]
2. Advantage : $O(\ell_x)$ time.
3. Noise resilience : Holds under a physical condition. [K (2017)]

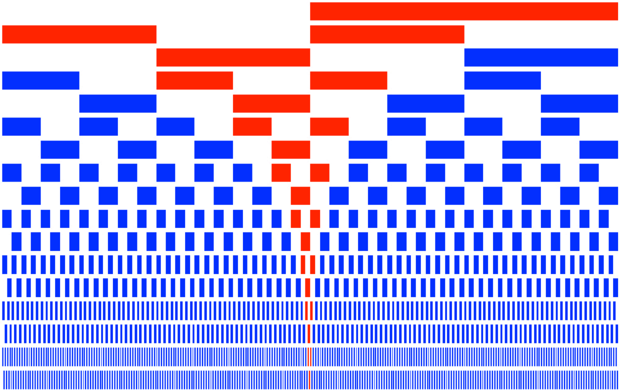
Example 2 : MERA-like circuits

- MERA
  - Introduced by Vidal(2006).
  - MERA is formally a quantum circuit consisting of $O(\log N)$ layers.
  - Each layers consist of depth-2 locality-preserving quantum circuits.
    - One layer of isometry, one layer of disentangler
  - The dimension of the circuit elements are $O(\chi^n)$.
- Deep MERA [K and Swingle, in prep.]
  - Decompose χ-dimensional Hilbert space into $O(\log \chi)$ qubits.
  - To be experiment-friendly, decompose the circuit into 2-qubit gates.
  - Each layers consist of depth-D locality-preserving quantum circuits.
  - Ground states of various physical models can be described with $D = O(\log \frac{1}{\delta})$ for precision δ. [Haegeman et al. (2017)]

* N : Number of qubits

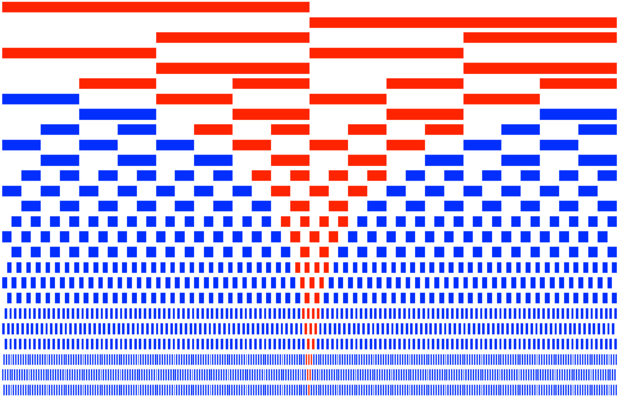
* n : Fixed constant, e.g., 4.

Example 2 : MERA-like circuits



N=512, D=2

Example 2 : MERA-like circuits



N=512, D=3

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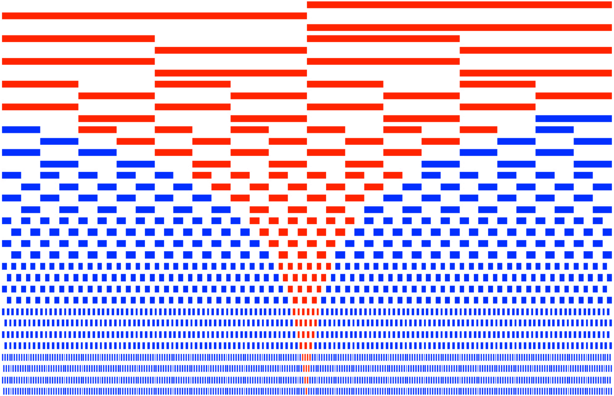
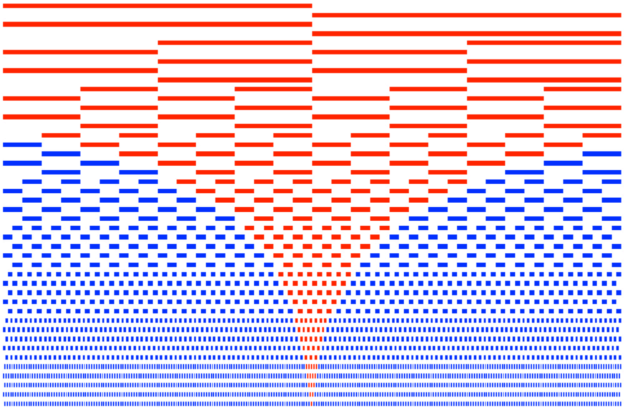


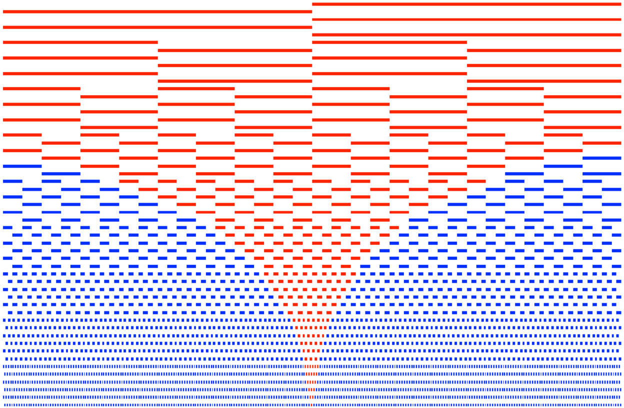
N=512, D=4

Example 2 : MERA-like circuits



N=512, D=5

Example 2 : MERA-like circuits



N=512, D=6

Example 2 : MERA-like circuits

$$\langle O \rangle = \mathrm{Tr}(\Phi_{\log N} \circ \cdots \Phi_2 \circ \Phi_1(\rho) O)$$

[Giovanetti et al. (2008), Evenbly and Vidal (2009)]

1. $\langle O \rangle$ can be computed efficiently both classically and quantumly. However, for deep MERA with depth D,
   - Classical computer seems to need $O(2^{cD} \log N)$ time.(Matrix-Matrix multiplication)
     - This is in 1D. In $d-$dimension, it scales as $O(2^{c' D^d} \log N)$.
   - Quantum computer can do the same job in $O(D \log N)$ time.
2. Noise resilient iff lowest scaling dimension > 0.

Error analysis

For a deep MERA in d-spatial dimensions with depth D, with noise rate ϵ

1. Gate + measurement + preparation error : $\sim D^{d+1}\epsilon$ (Generically)
2. Approximaton error : $\sim e^{-cD}$ (For known models) [Haegeman et al. (2017)]

Optimizing D,

$$O(\epsilon(\log^{d+1}(1/\epsilon))$$

error can be achieved, using $O(\log^{d}(1/\epsilon))$ qubits.

Numerical experiment

Setup

1. Place random $SU(4)$ in each circuit locations.
2. Compute nearest neighbor reduced density matrix with and without noise.
   - Noise model : Apply depolarizing noise with $p = 0.001$ after $SU(4)$.
3. Compute the trace distance.
4. Repeat 100 times for different choices of D.

Result

D	Errors	Average	Std	Min	Max
2	624	7.2×10^{-3}	1.2×10^{-3}	5.8×10^{-3}	10.7×10^{-3}
3	1656	7.3×10^{-3}	1.0×10^{-3}	5.1×10^{-3}	9.9×10^{-3}
4	3048	1.4×10^{-2}	1.6×10^{-3}	1.1×10^{-2}	1.8×10^{-2}
5	4680	1.9×10^{-2}	1.5×10^{-3}	1.6×10^{-2}	2.3×10^{-2}

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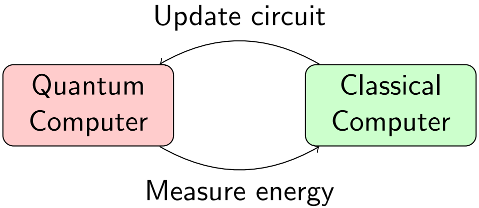
1. Why noise resilience is important.
2. Intuition behind noise resilience.
3. Abundance of noise-resilient circuits.
4. **How to use noise-resilient circuits.**

Quantum variational method



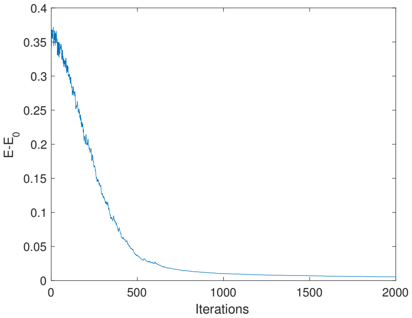
Quantum-classical feedback loop

1. Quantum processor runs a circuit and measures the energy. (Difficult classically)
2. Classical computer updates the circuit to lower the energy. (zeroth order classical optimization methods)
3. Repeat until convergence.
4. Measure physical observables, e.g., magnetization.

* Inspired from variational quantum eigensolver [Peruzzo et al. (2013)]

Classical optimization

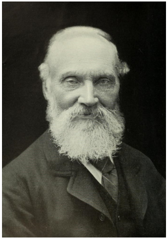
1. Decompose the circuit into $SU(4)$.
2. Sample energy.
3. Sequentially optimize each $SU(4)$.
   - Employ classical zeroth-order optimization with noisy measurement, e.g., SPSA.



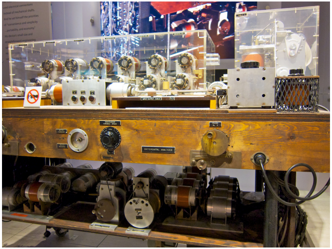
Comments/Speculations

- Quantum computer is really good at contracting tensor networks.
- Contraction of "physical" tensor networks are resilient to noise.
- RG kills your mistakes so that all the imperfections, even if added together, does not blow up.
- The trick is to change the "scale" to "time."
- A universal structure of ground state entanglement (tensor network structure) seems to guarantee the robustness of correlation functions. That's great because we care about correlation functions.
- **Tensor networks are "efficient," but not all of them are practical yet. A near-term quantum computer can assist these calculations, even if it is noisy.**

Outlook



Lord Kelvin



Differential Analyzer

- Lord Kelvin proposed an analog machine that can predict the flow of sea tides. (19th century)
- The machine essentially solved a linear differential equation. Such machines were built in the early 20th century and people actually used it!(Bush, Hartree)
- These devices were used for scientific purposes, e.g., studies of heat flow, explosive detonations, and transmission lines, until they were eventually replaced by digital computers.

Noisy quantum computer = Differential analyzer of the 21st century?