Spin-bath decoherence: Old results and new surprises

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Collaborators: Jan Fischer, Daniel Klauser, Daniel Loss (Basel)

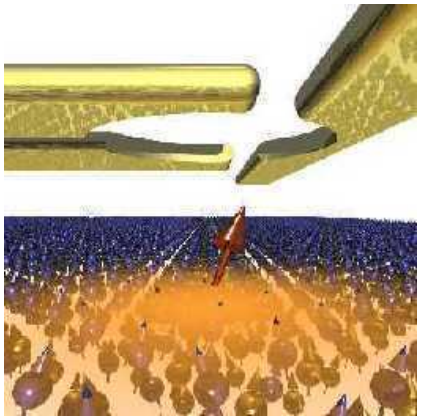
arXiv:0911.4149

(KITP, UC Santa Barbara, 2009 Dec. 3)

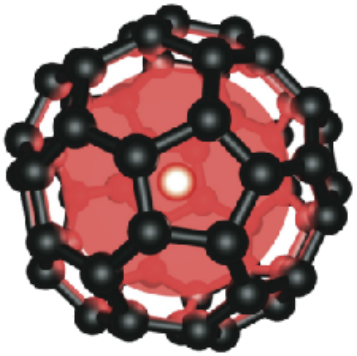
Nuclear spins are (almost) everywhere...

NV centers in diamond

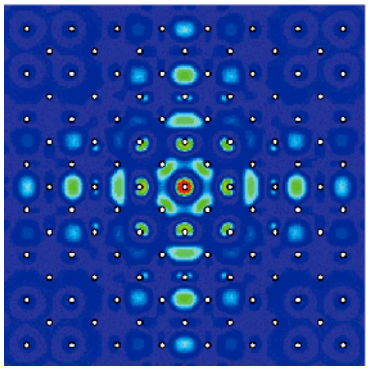
Quantum dots



$N@C_{60}$

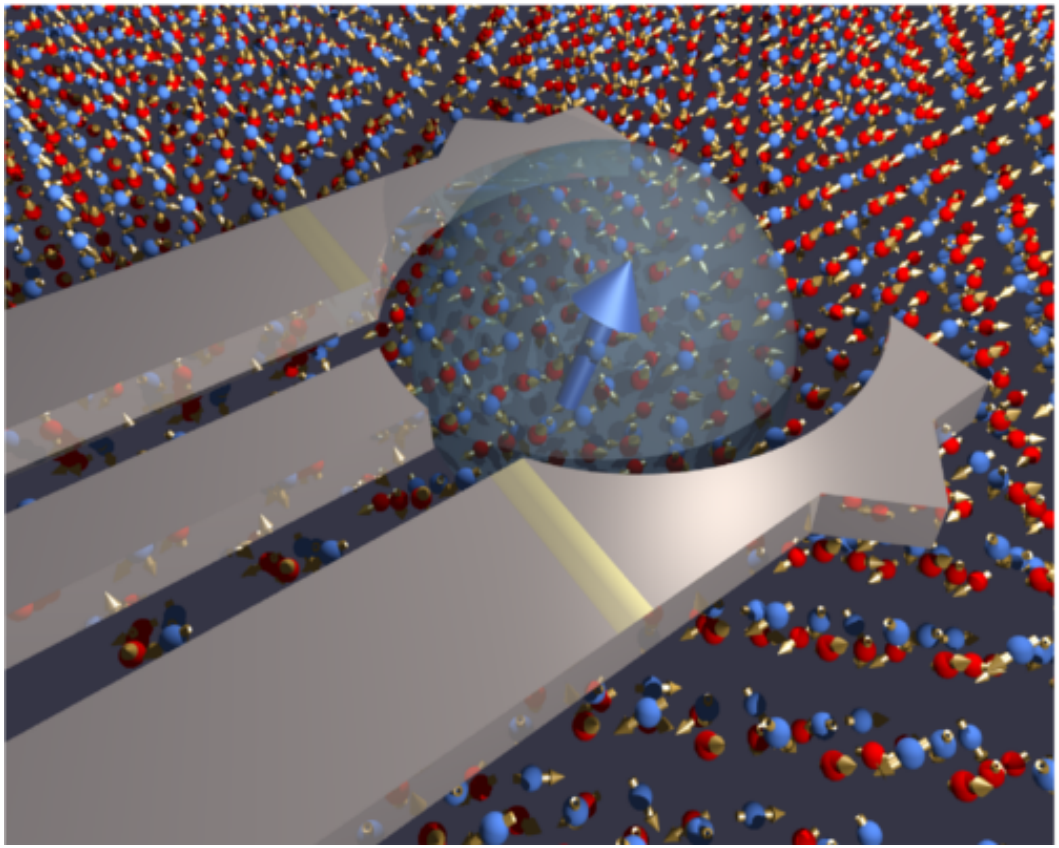


Phosphorus donors



Coherence

Problem: One spin sees many

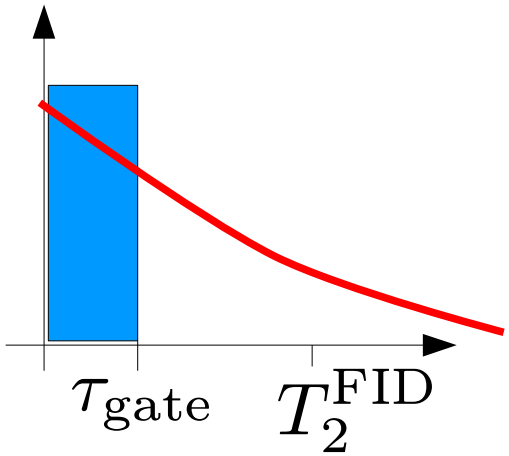


$N \sim 10^6$

nuclei

WAC and J. Baugh, `Nuclear spins in nanostructures',
Phys. Stat. Solidi B (2009)

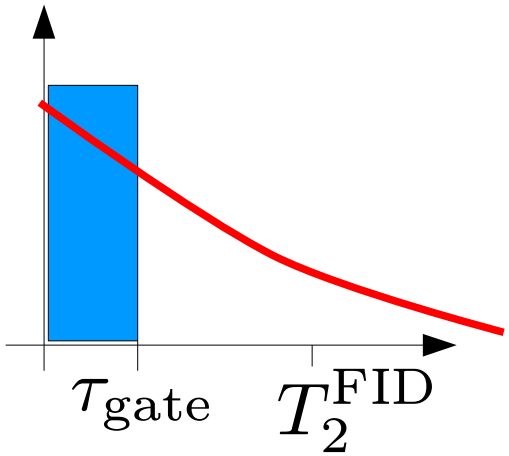
Free-induction vs. Echoes



Free-induction decay - approximate error rate?:

$$\eta \sim \tau_{\mathrm{gate}}/T_2^{\mathrm{FID}} \qquad S_x(t) \propto e^{-t/T_2}$$

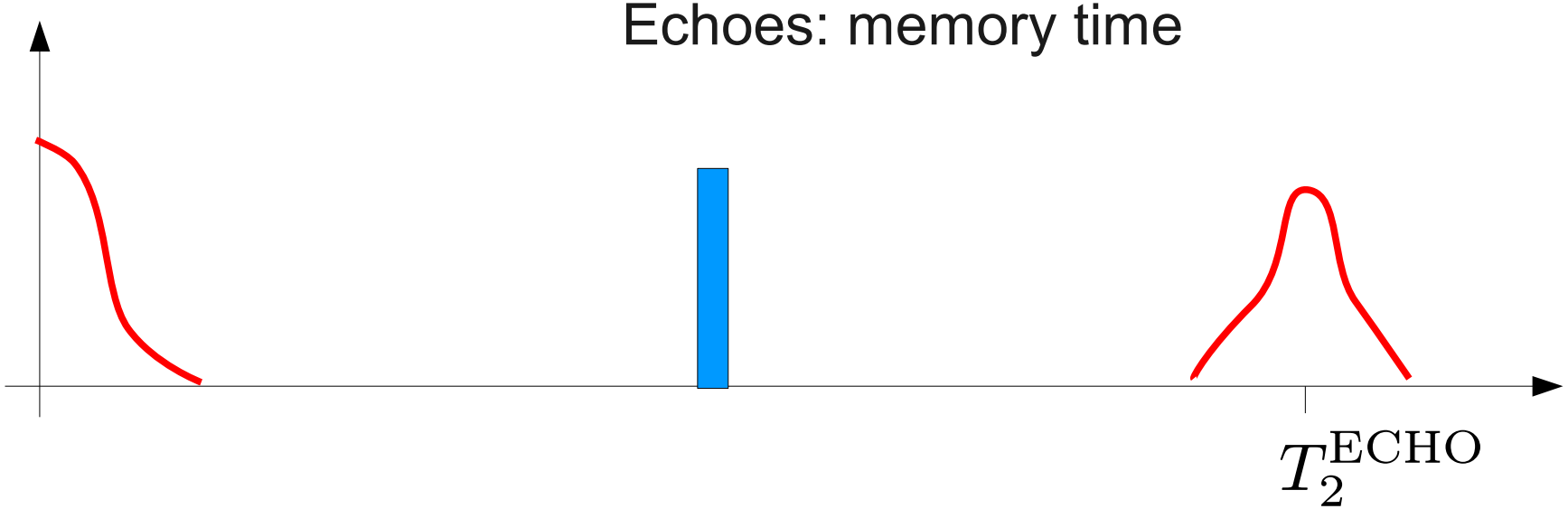
Free-induction vs. Echoes



Free-induction decay - approximate error rate?:

$$\eta \sim \tau_{\mathrm{gate}}/T_2^{\mathrm{FID}} \qquad S_x(t) \propto e^{-t/T_2}$$

Echoes: memory time



In general, even for a single spin: $T_2^{\mathrm{FID}} \neq T_2^{\mathrm{ECHO}}$

Hyperfine Hamiltonian

$$H_{\mathrm{hf}} = bS^z + \mathsf{h} \cdot \mathsf{S}$$

Electron Zeeman energy

Coupling to nuclear field

$$\mathsf{h} = \sum_k A_k \mathsf{I}_k$$

$$A = \sum_k A_k$$

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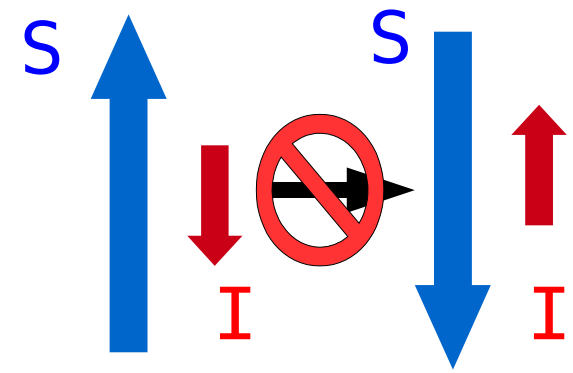
$$\mathsf{h} \cdot \mathsf{S} = h^z S^z + \underbrace{\frac{1}{2}\left(h^+ S^- + h^- S^+\right)}_{V_{\mathrm{ff}}}$$

V_{ff} does not conserve energy for large b



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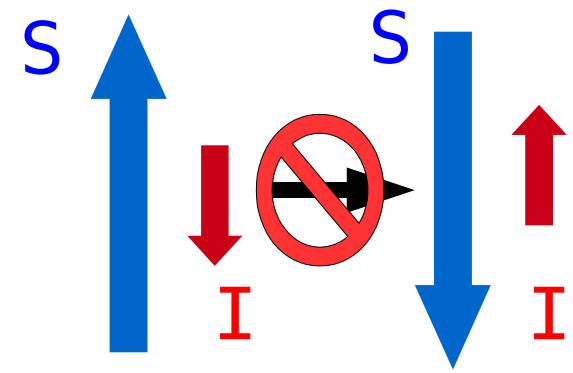
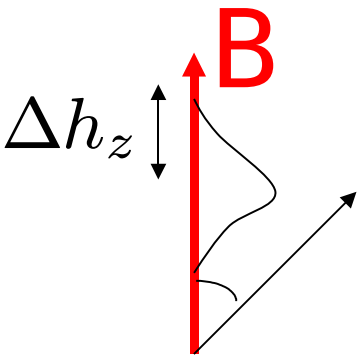


V_{ff} does not conserve energy for large b

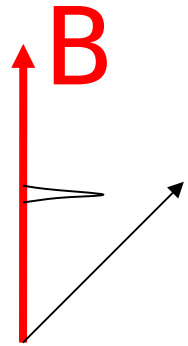
Perturbation theory in $\dfrac{A}{b} \ll ۱$ $\quad b/g^*\mu_B \gtrsim ۳.۵\,\mathrm{T}$ (GaAs)

Nuclear-spin bath preparation



$$\Rightarrow \langle S_x \rangle_t \propto e^{-(t/\tau)^2} \quad \tau \sim \mathrm{ns}$$

measurement



$$\Rightarrow \langle S_x \rangle_t \propto e^{i\omega t}$$

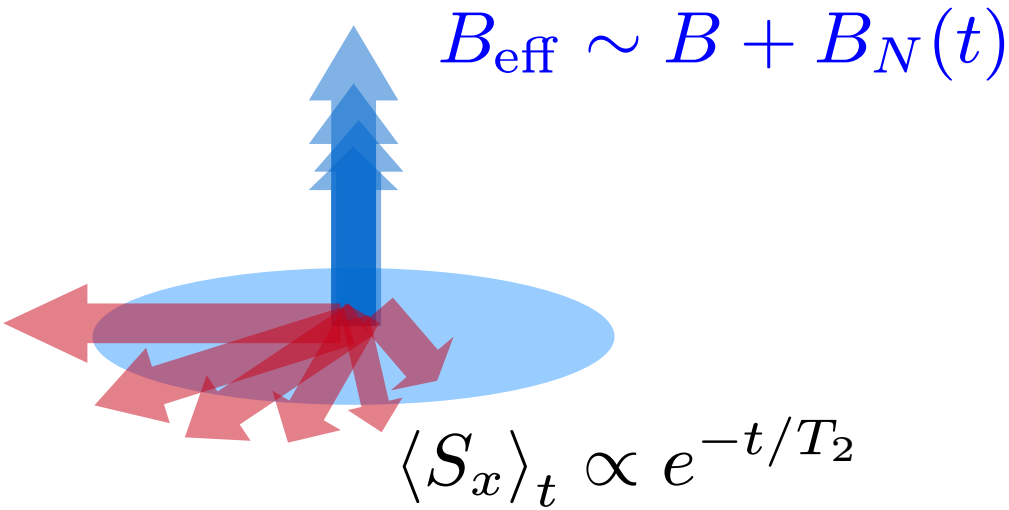
(narrowed state)

Theory: WAC and Loss, PRB (2004), Klauser, WAC and Loss, PRB (2006,2008), Stepanenko et al., PRL (2006), Giedke et al., PRA (2006), Ribeiro and Burkard, PRL (2009),

Expt.: Greilich et al., Science (2006), (2007), Reilly et al., Science (2008), Xu et al., Nature (2009), Vink et al., Nat. Phys. (2009), Latta et al., Nat. Phys. (2009)

After Narrowing...

Dynamics in nuclear-spin system lead to decay



PHYSICAL REVIEW VOLUME 123, NUMBER 3 FEBRUARY 1, 1961

Spectral Diffusion Decay in Spin Resonance Experiments

J. R. KLAUDER AND P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey
(Received September 1, 1961)

While some progress has been made in solving, under rather restricted circumstances and with assumptions which are not by any means always valid, the exact quantum-mechanical equations of motion,[2] there is little hope of real progress in that direction on such immensely complicated questions as spectral diffusion.

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Not an 'easy' problem!

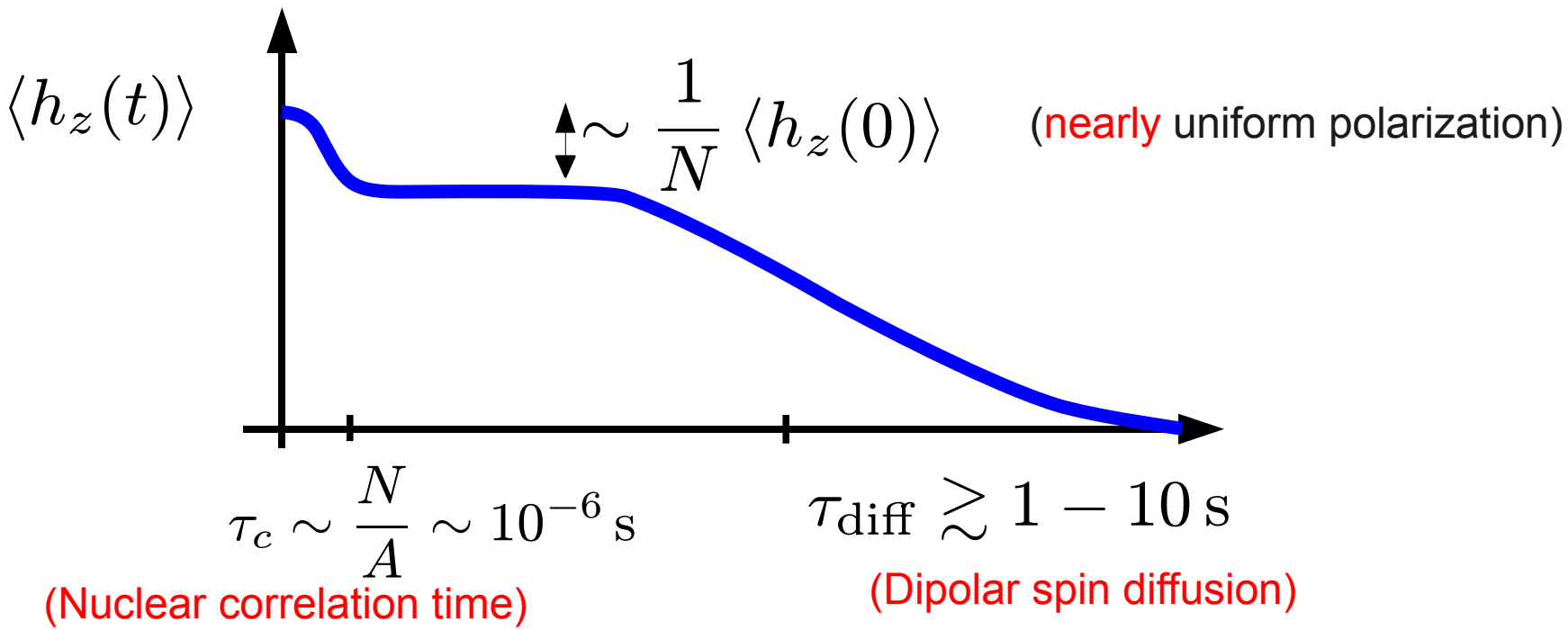
Nuclear-spin dynamics

D. Klauser, WAC, D. Loss, PRB (2008)

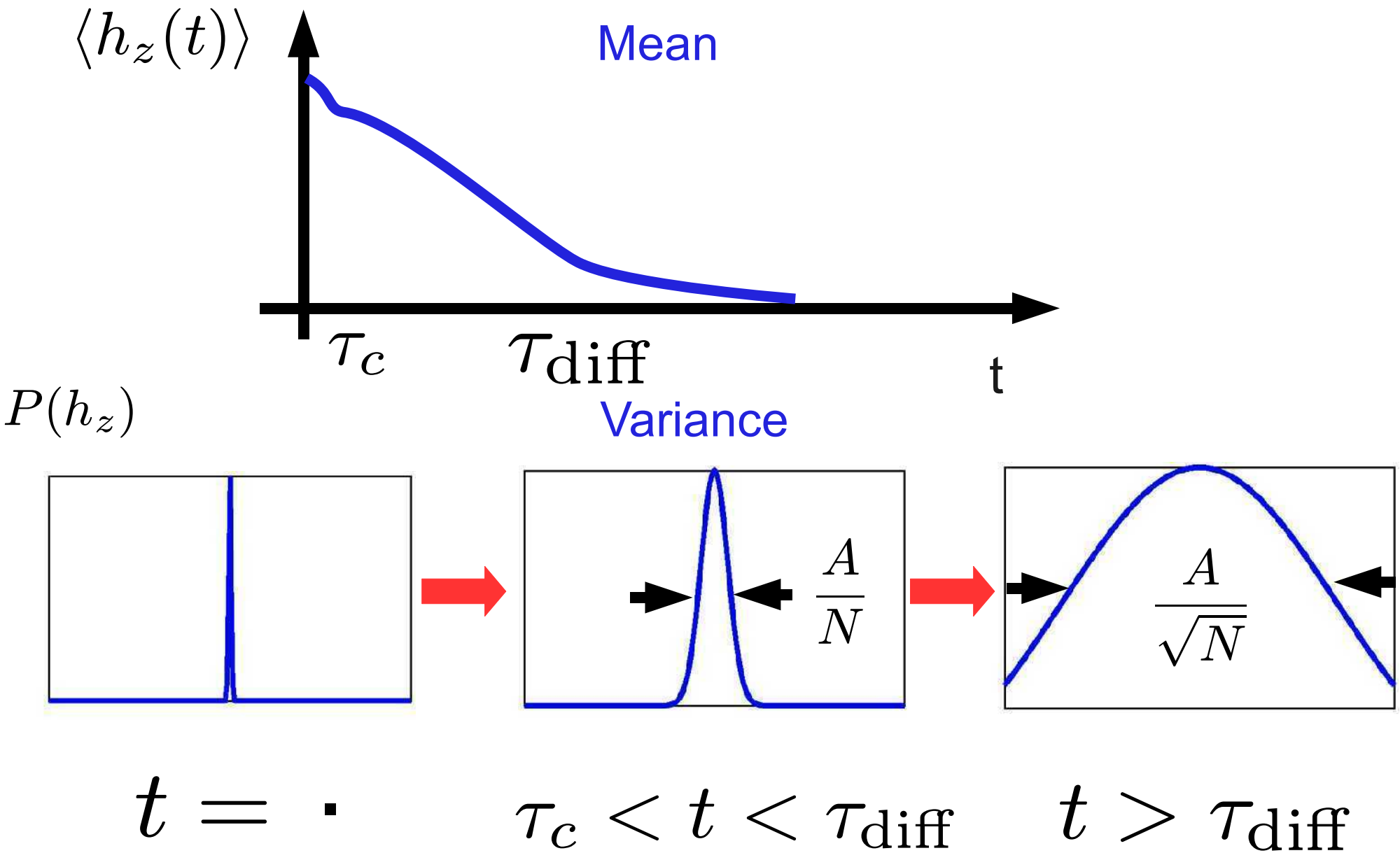
Short time:

$$\langle h_z(t)\rangle \simeq \langle h_z(0)\rangle \left(1 - \left(\frac{t}{\tau_n}\right)^2 + \mathcal{O}(t^3)\right) \qquad \tau_n \sim \frac{N^{3/2} b}{A^2} \sim 10^{-4}\,\mathrm{s}$$

Beyond short time (generalized master equation):



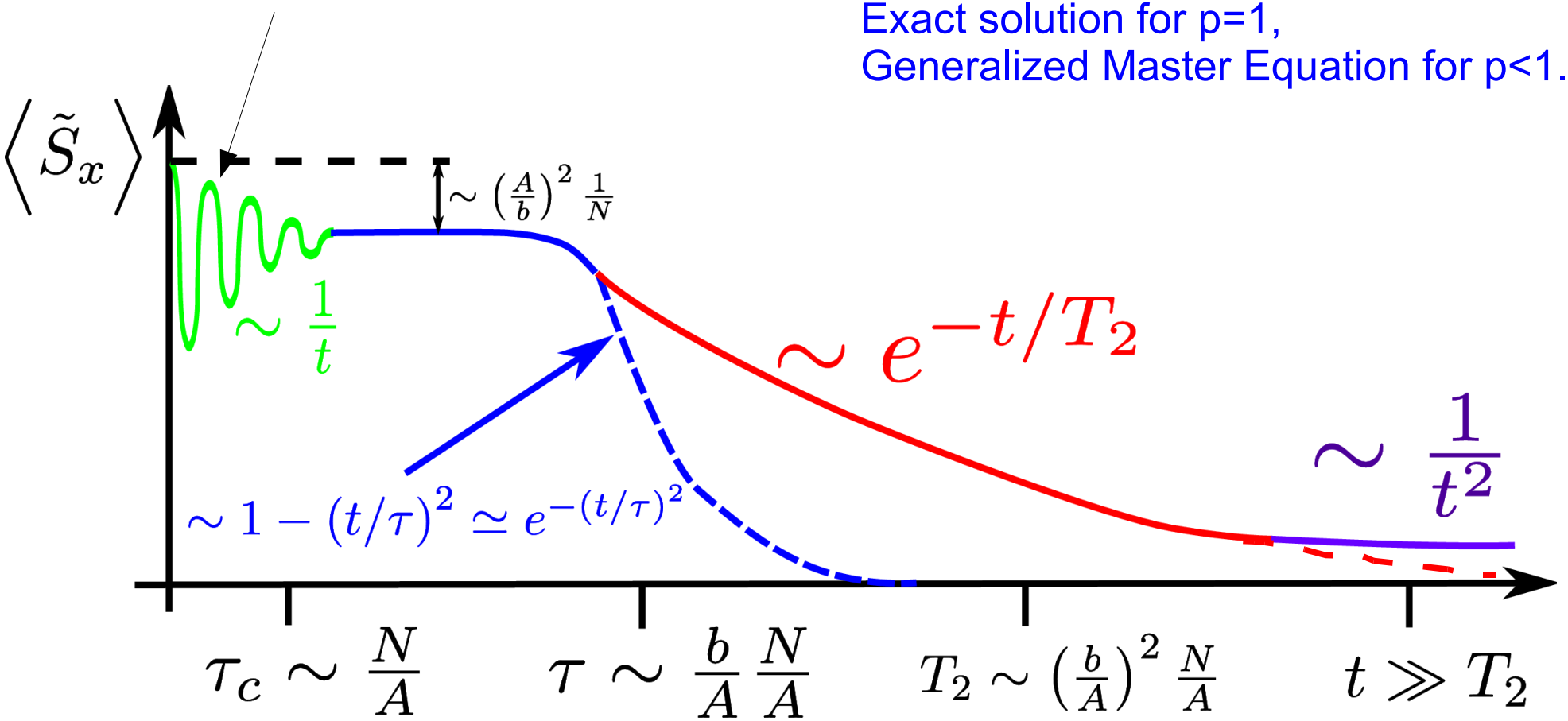
Mean and variance



Free-induction decay: history

Khaetskii, Loss, Glazman, PRL (2002), PRB (2003)
WAC and Loss, PRB (2004)

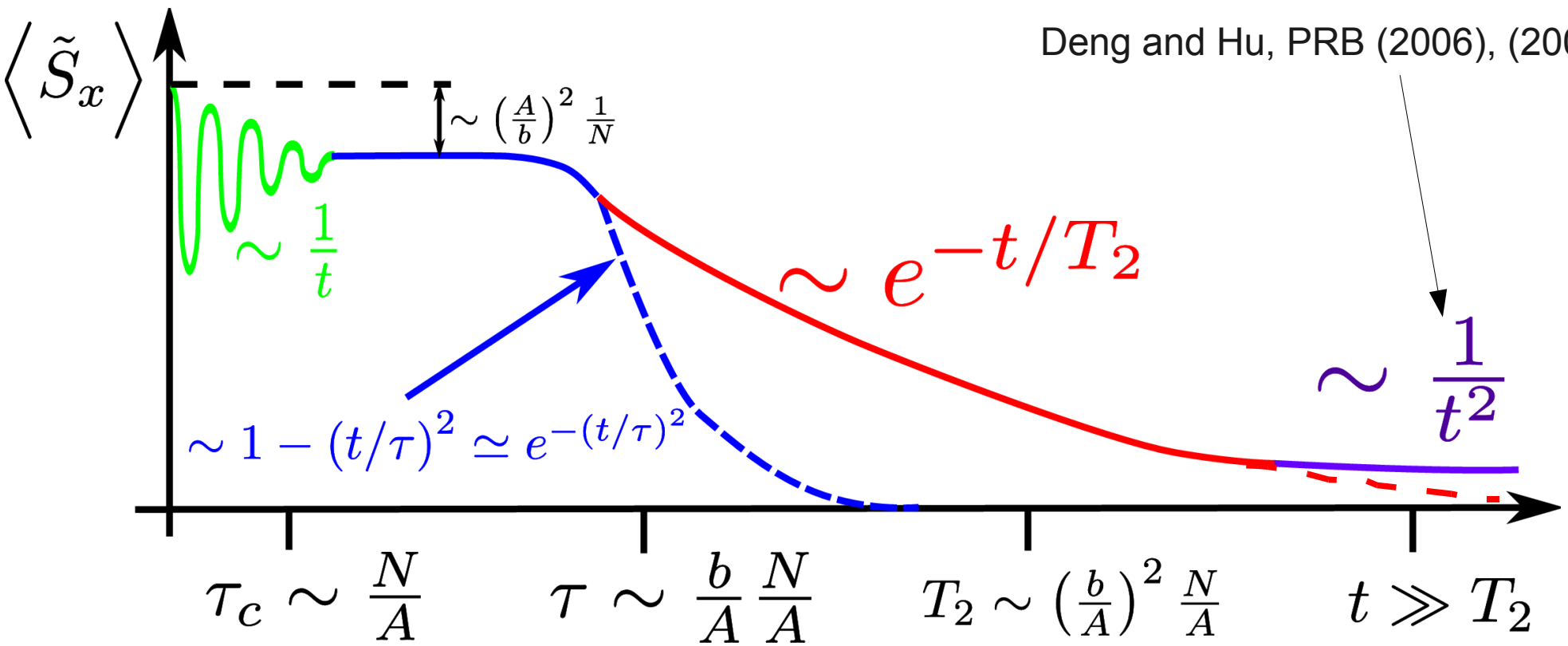
Exact solution for p=1,
Generalized Master Equation for p<1.



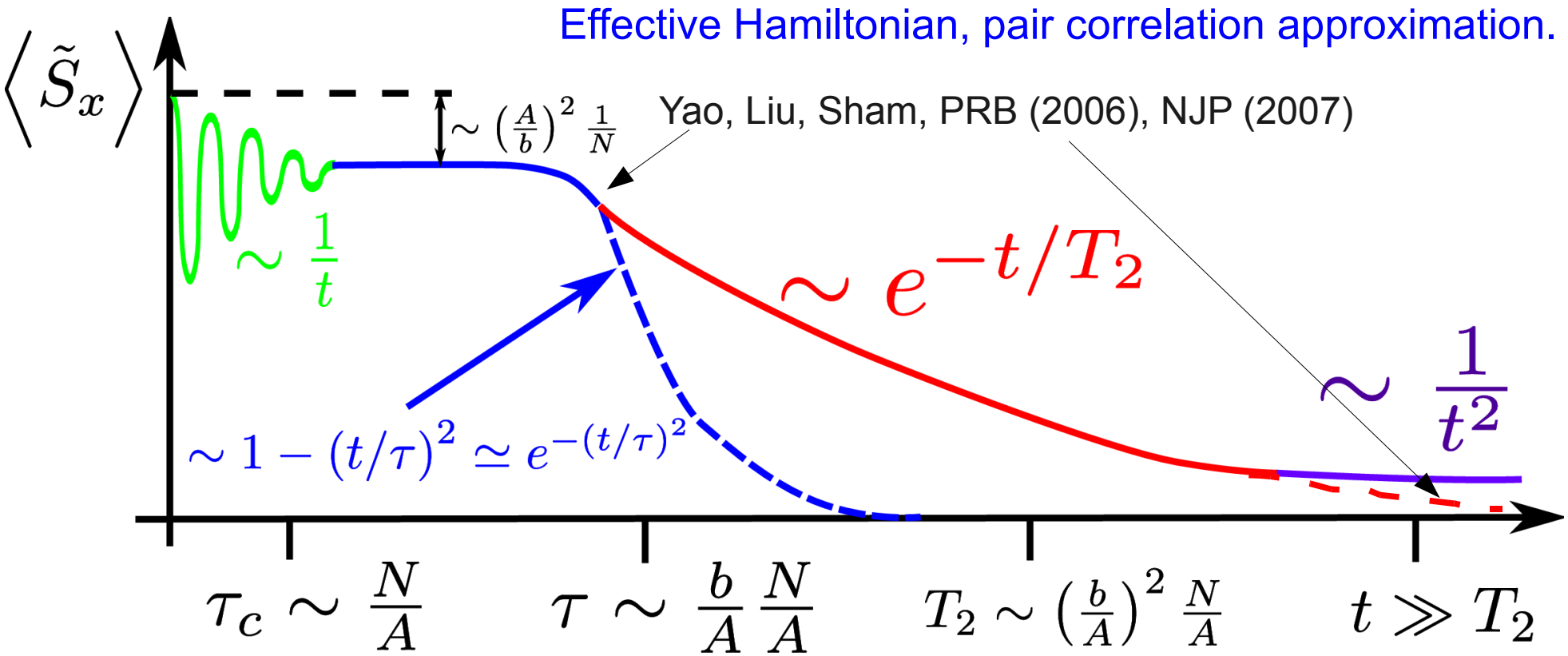
Free-induction decay: history

Equation-of-motion method.

Deng and Hu, PRB (2006), (2008)

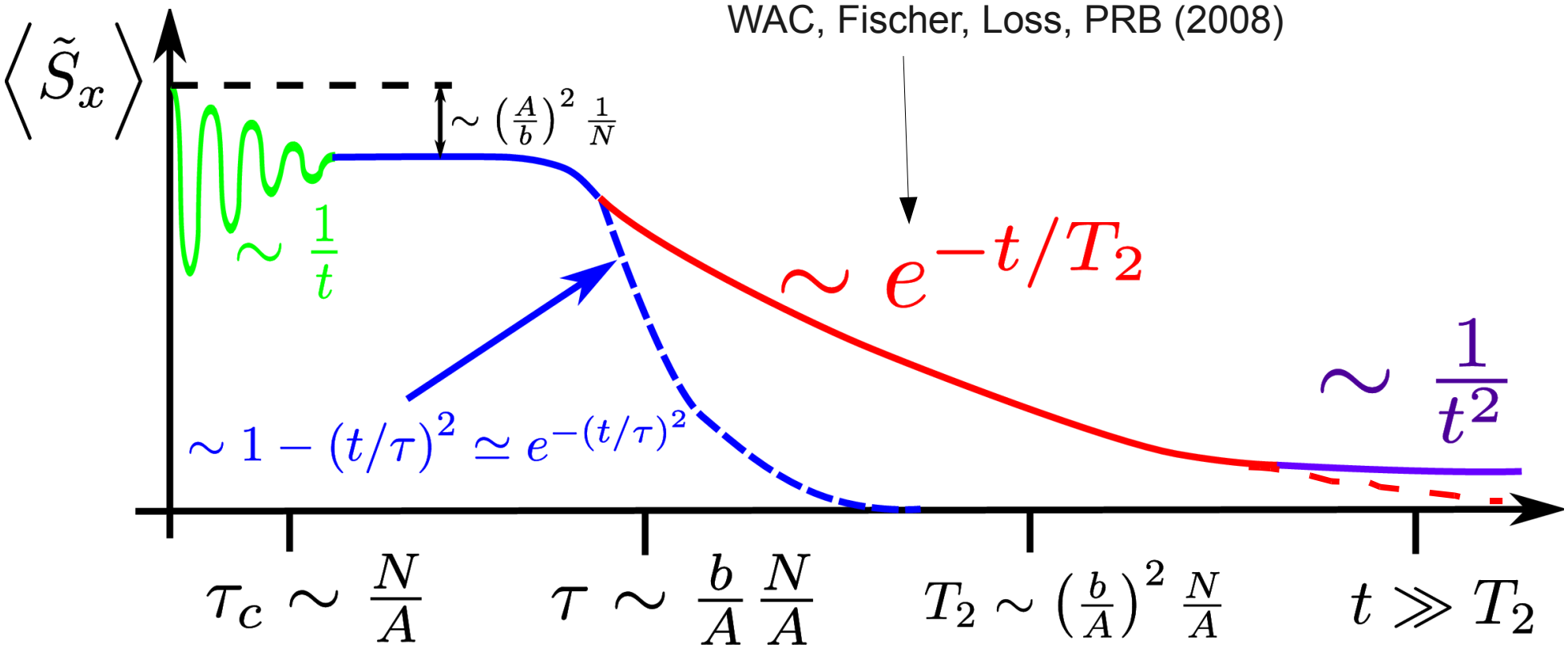


Free-induction decay: history



Free-induction decay: history

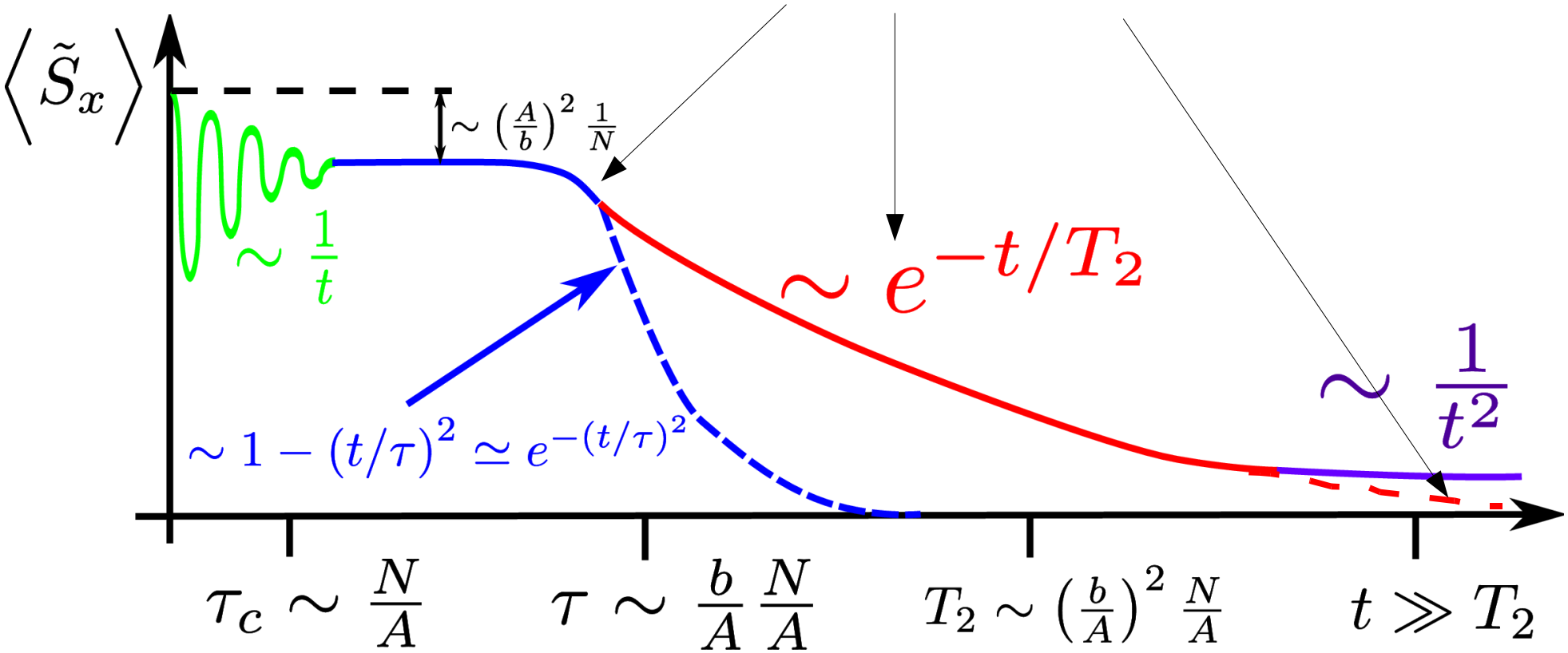
Effective Hamiltonian,
Born-Markov approximation.
WAC, Fischer, Loss, PRB (2008)



Free-induction decay: history

Effective Hamiltonian,
High-order resummation, low b-field.

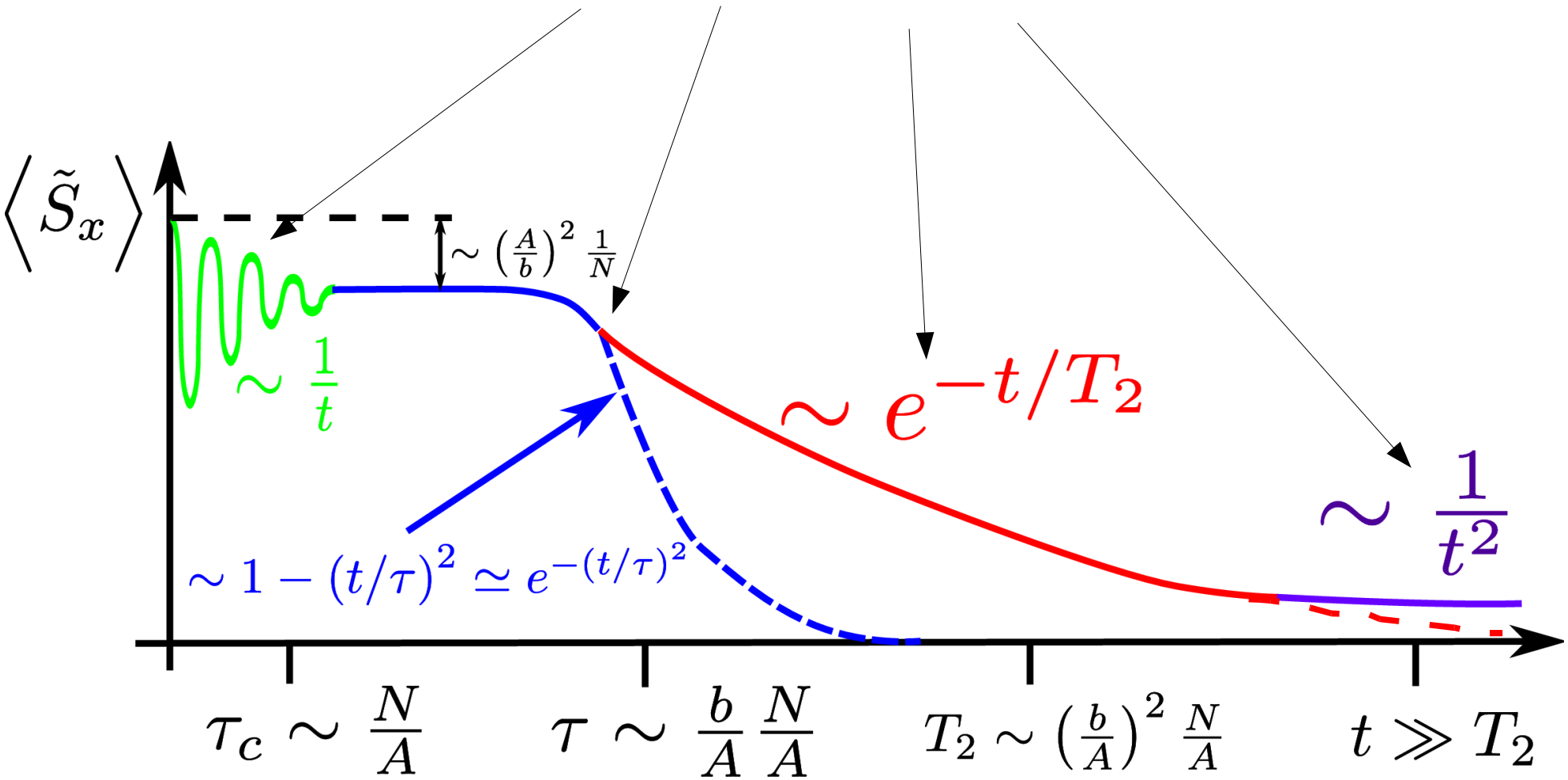
Cywinski, Witzel, Das Sarma, PRL (2009), PRB (2009)



Free-induction decay: history

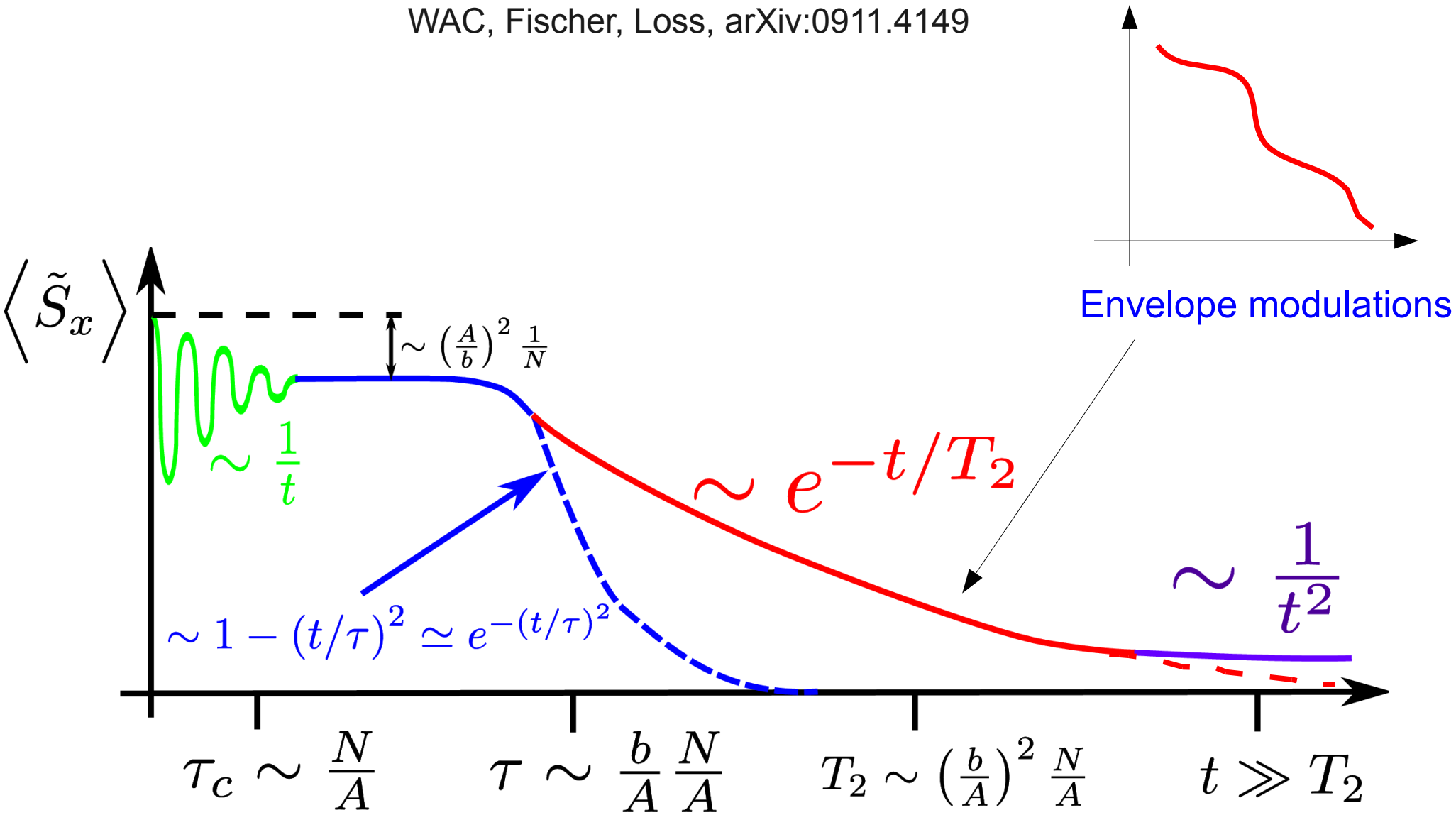
Generalized Master Equation, Higher order.

WAC, Fischer, Loss, arXiv:0911.4149



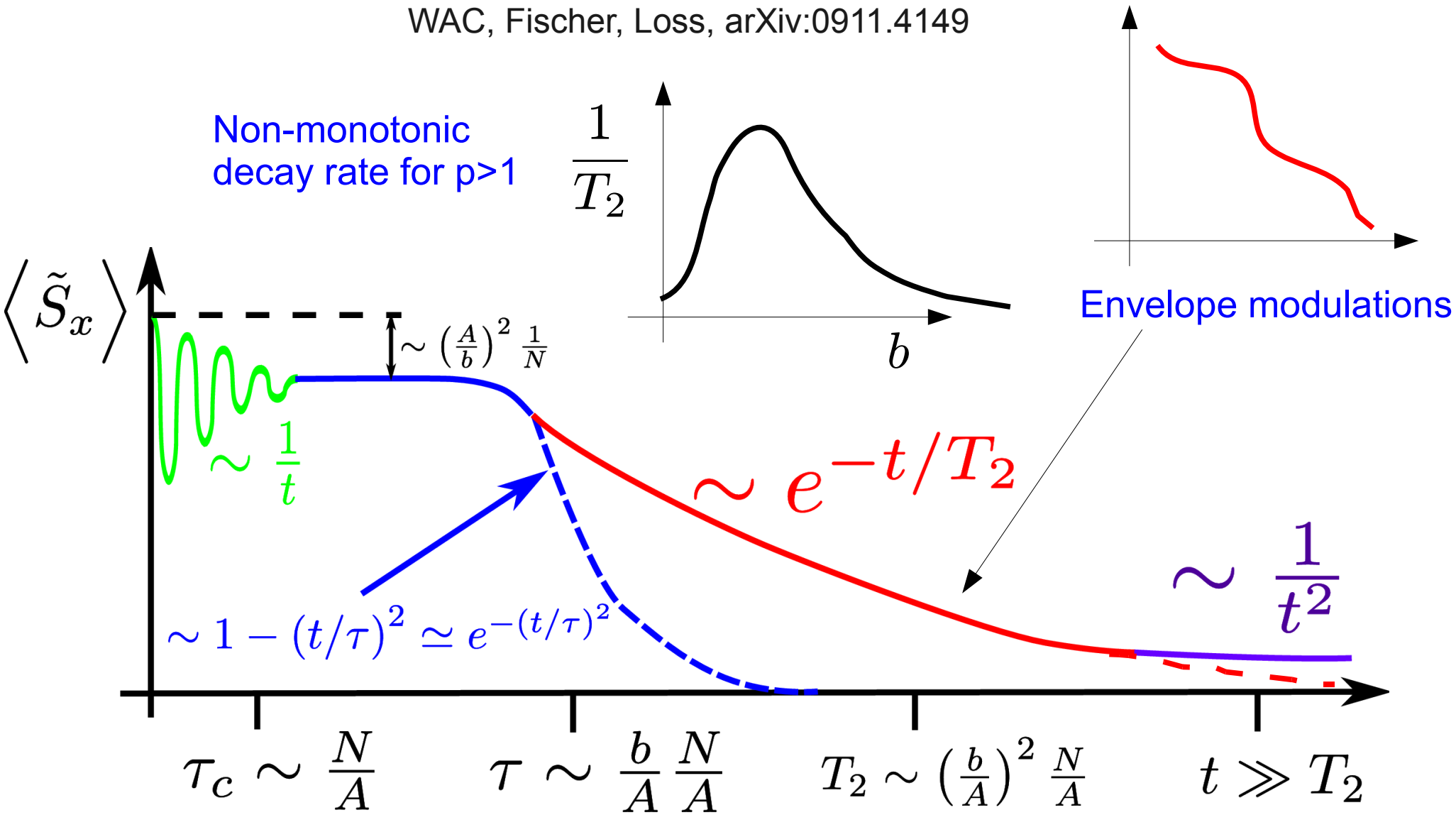
Free-induction decay: history

WAC, Fischer, Loss, arXiv:0911.4149



Free-induction decay: history

WAC, Fischer, Loss, arXiv:0911.4149



Solve the problem in two ways:

$$\langle \mathcal{O} \rangle_t = \langle \psi(0) | \, e^{iHt} \mathcal{O} e^{-iHt} \, | \psi(0) \rangle$$

$$H = H_0 + V_{\text{ff}}$$

(1) Effective Hamiltonian

$$\tilde{H} = e^S H e^{-S} = H_0 + V_{\text{eff}} + O\left(V_{\text{ff}}^3\right)$$

neglected

$$\left| \tilde{\psi}(0) \right\rangle = e^S \, |\psi(0)\rangle = |\psi(0)\rangle + O\left(V_{\text{ff}}\right)$$

Expand in powers of $V_{\text{eff}} \sim O\left(V_{\text{ff}}^2\right) \sim O\left(\frac{A}{b}\right)$

(2) Work directly with the 'real' Hamiltonian

Expand in powers of V_{ff}

Initial conditions

Fast initialization:

$$\rho(0) = \rho_S(0) \otimes \rho_I(0)$$

Sufficient condition: $\tau_{\rm init} \lesssim 1/A \simeq 50\,{\rm ps}$

Narrowed bath:

$$\rho_I(0) = \sum_i \rho_{ii} \left|n_i\right\rangle \left\langle n_i\right| \qquad \omega \left|n_i\right\rangle = \omega_n \left|n_i\right\rangle$$

Generalized Master Equation (GME)

Coherence factor: $$x_t = 2e^{-i(\omega_n + \Delta\omega)t} \langle S_+ \rangle_t$$

GME: $$\dot{x}_t = -i\Delta\omega x_t - i \int_0^t dt' \tilde{\Sigma}(t - t') x_{t'}$$

Lamb shift: $$\Delta\omega = -\mathrm{Re} \int_0^\infty dt \tilde{\Sigma}(t)$$

Markov: $$\frac{1}{T_2} = -\mathrm{Im} \int_0^\infty dt \tilde{\Sigma}(t)$$

$$x_t \simeq x_0 e^{-t/T_2}$$

Direct expansion vs. effective H

$$\Sigma(s) = \int_0^\infty e^{-st}\Sigma(t)$$

Expanding in $V_{\rm ff}$

$$\tilde{\Sigma} \simeq \tilde{\Sigma}^{(2)} + \tilde{\Sigma}^{(4)} + O\left(V_{\rm ff}^6\right)$$

$$\Delta\omega \simeq -{\rm Re}\tilde{\Sigma}^{(2)}(s=0^+) = O(V_{\rm ff}^2)$$

$$\frac{1}{T_2} \simeq -{\rm Im}\tilde{\Sigma}^{(4)}(s=0^+)$$

Expanding in $V_{\rm eff} \sim V_{\rm ff}^2$

$$\tilde{\Sigma}_{\rm eff} = \tilde{\Sigma}_{\rm eff}^{(2)} + O\left(V_{\rm ff}^8\right)$$

$$\Delta\omega_{\rm eff} \simeq -{\rm Re}\tilde{\Sigma}_{\rm eff}^{(2)}(s=0^+) = O(V_{\rm ff}^4)$$

$$\frac{1}{T_2} \simeq -{\rm Im}\tilde{\Sigma}_{\rm eff}^{(2)}(s=0^+)$$

For one isotope: $\tilde{\Sigma}^{(4)} = \tilde{\Sigma}_{\rm eff}^{(2)}$ (with 1/N corrections)

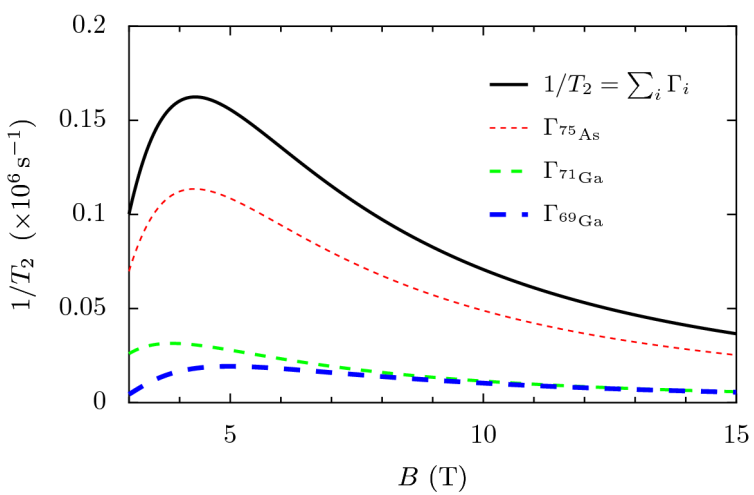
Multiple isotopes: $\tilde{\Sigma}^{(4)} \neq \tilde{\Sigma}_{\rm eff}^{(2)}$

Non-monotonic decoherence Rate!

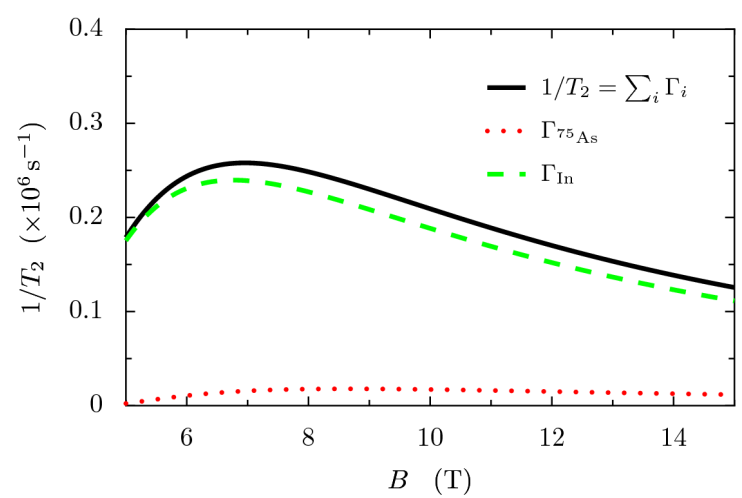
$$\frac{1}{T_2} \simeq -\mathrm{Im}\tilde{\Sigma}^{(4)}(s=0^+) \propto \frac{1}{b^2}\sum_{k,k'} A_k^2 A_{k'}^2 \delta(A_k - A_{k'} - \Delta\omega)$$

$$A_k \leq A/N \qquad \Delta\omega \propto \frac{1}{b}$$

GaAs



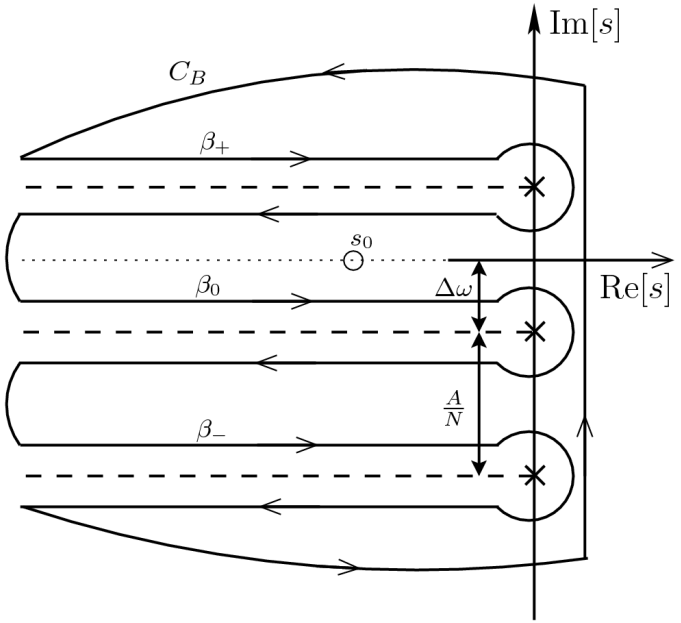
InGaAs



Qualitative behavior (maximum) is controlled by $\left(1-p^2\right)\frac{A}{b} < 1$

Full non-Markovian time dependence

$$x(s) = \frac{x_0}{s - i\Delta\omega - i\tilde{\Sigma}(s)}$$



$$x_t = \lim_{\gamma \to 0} \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{st} x(s)$$

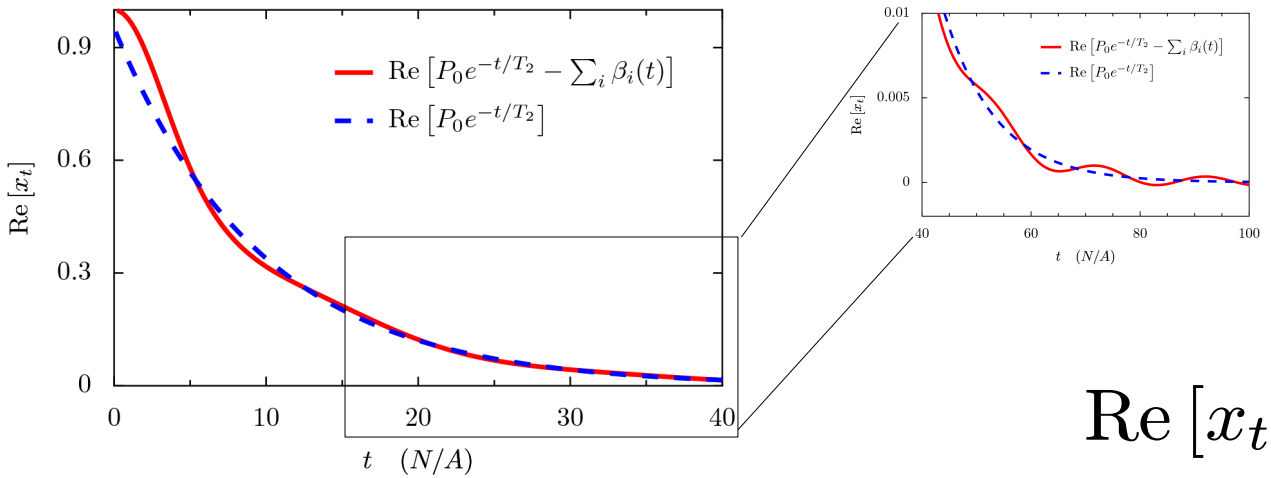
$$= \sum_i \mathrm{Res}[e^{st} x(s), x = s_i] - \sum_\alpha \beta_\alpha(t)$$

Exponential decay or sustained oscillations

Power-law decay

See also, e.g., DiVincenzo and Loss, PRB (2005) (spin-boson model)

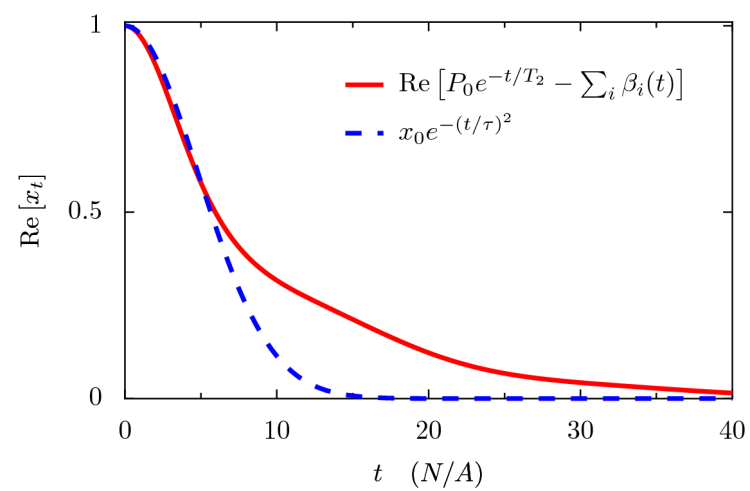
$A/b = \frac{۱}{۳}$ Envelope modulations!



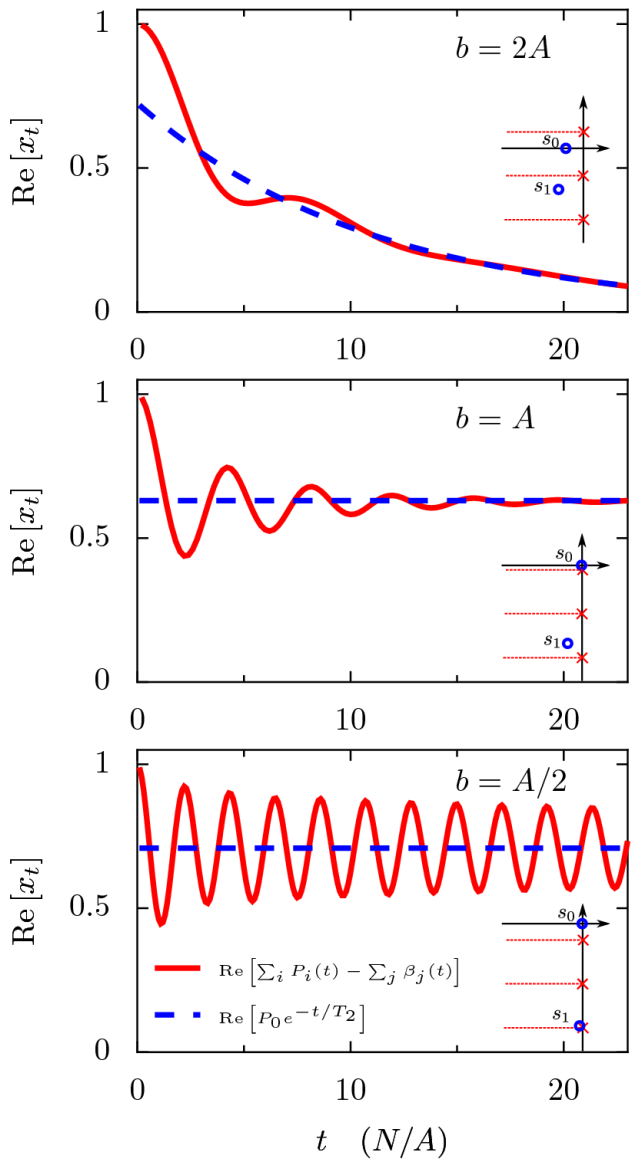
$$t \gg \frac{۱}{\Delta\omega}$$

$$\mathrm{Re}\,[x_t] \sim \frac{C\cos\left(\Delta\omega t + \phi\right)}{t^{۲}}$$

Short time: $t < \tau$



Non-perturbative regime b~A



Biexponential decay, strong modulations

Higher-order corrections needed

Conclusions

New envelope modulations of the free-induction decay envelope (distinct from ESEEM)

In general, non-monotonic dependence of $1/T_2$ on magnetic field (reaches a maximum!)

Neither of these result is recovered correctly from the leading-order effective Hamiltonian.