Quantum Corrections to Holographic Entanglement

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[XD & Lewkowycz, 1705.08453]
[XD, Harlow & Wall, 1601.05416]

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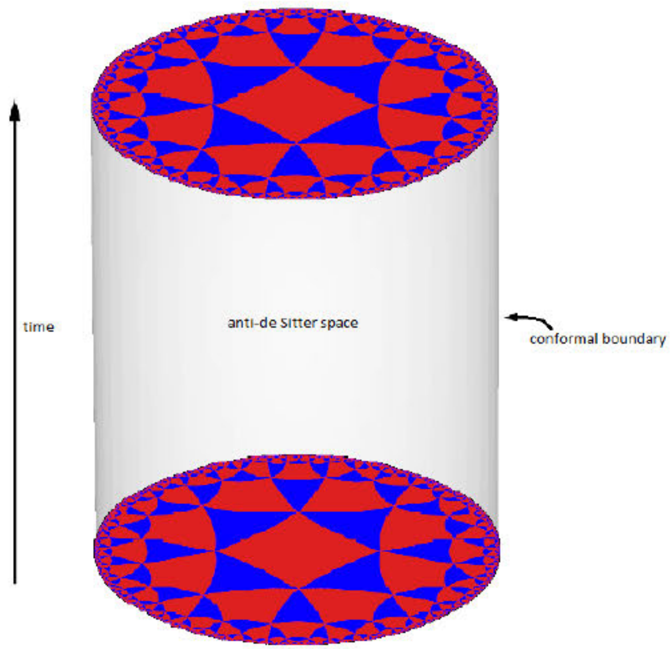
- One of the most surprising features of quantum gravity is that gravitational entropy is geometrized as a surface area.
- In this talk I will focus on understanding and applying this feature in our best-understood model of quantum gravity: the AdS/CFT correspondence.



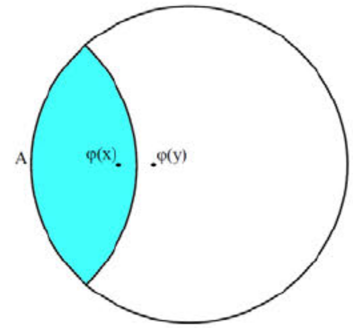
Quantum gravity in AdS_{d+1}	Holographic CFTs on ∂AdS_{d+1}
Isometry group $O(d,2)$	Conformal group $O(d,2)$
Black hole states	Thermal states
Gauge symmetry	Global symmetry
Expansion in G_N	Expansion in $1/N$

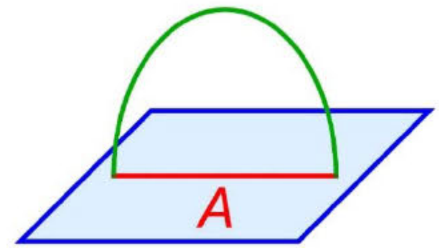
Holographic Entanglement Entropy

In this context, entanglement entropy is given by the area of a dual surface:

[Ryu & Takayanagi '06]

$$S = \min \frac{\text{Area}}{4G_N}$$





- Practically useful for understanding entanglement in strongly-coupled systems.

[Huijse, Sachdev & Swingle '11]
[XD, Harrison, Kachru, Torroba & Wang '12; …]

- Conceptually important for understanding the emergence of spacetime from entanglement.

[Van Raamsdonk '10; Maldacena & Susskind '13; …]

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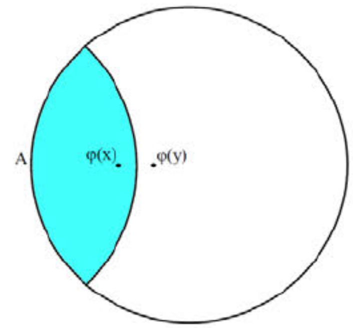

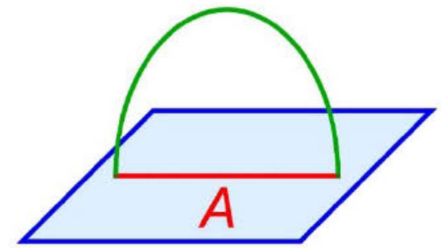

- The minimal surface defines important concepts such as the entanglement wedge and subregion duality.

[Czech, Karczmarek, Nogueira & Van Raamsdonk '12; Almheiri, XD & Harlow '14; XD, Harlow & Wall '16; ...]

- The Ryu-Takayanagi (RT) formula can be derived from AdS/CFT.

[Lewkowycz & Maldacena '13]

- Conversely, RT leads to Einstein's equations.

[Lashkari, McDermott & Van Raamsdonk] [Faulkner, Guica, Hartman, Myers & Van Raamsdonk] [XD & Lewkowycz 1705.08453; ...]

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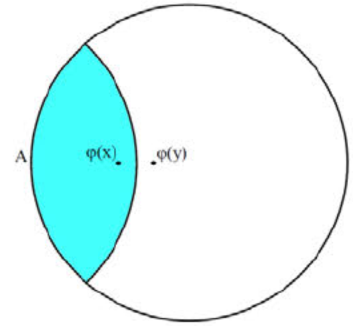
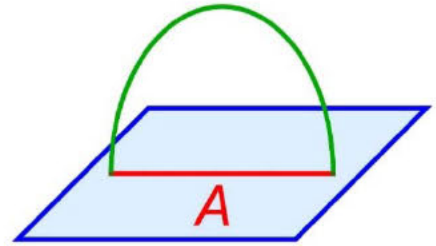




- For time-dependent states, we have a covariant generalization (HRT):

[Hubeny, Rangamani & Takayanagi '07]

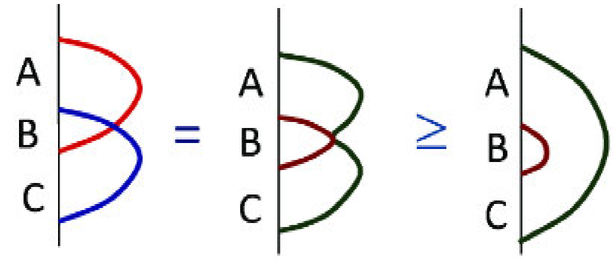
$$S = \mathrm{ext}\frac{\mathrm{Area}}{4G_N}$$

- Powerful tool for studying time-dependent physics such as quantum quenches.

- Can also be derived from AdS/CFT. [XD, Lewkowycz & Rangamani '16]

Holographic Entropy Cone

- RT satisfies strong subadditivity:



[Headrick & Takayanagi]

$$\Rightarrow \quad S_{A\cup B} + S_{B\cup C} \geq S_{A\cup B\cup C} + S_B$$

- Also satisfies other inequalities such as monogamy of mutual information

$$S_{AB} + S_{BC} + S_{CA} \geq S_{ABC} + S_A + S_B + S_C$$

[Hayden, Headrick & Maloney]

and

$$S_{ABC} + S_{BCD} + S_{CDE} + S_{DEA} + S_{EAB} \geq S_{ABCDE} + S_{AB} + S_{BC} + S_{CD} + S_{DE} + S_{EA}$$

[Bao, Nezami, Ooguri, Stoica, Sully & Walter]

- Provide nontrivial conditions for a theory to have a gravity dual.
- Holographic entropy cone for time-dependent states?
- AdS_3/CFT_2: same as the static case. [XD & Czech, in progress]

Corrections to Ryu-Takayanagi

RT has been refined by higher derivative corrections and quantum corrections.

- Higher derivative corrections (α'):

$$S = \text{ext} \frac{A_{\text{gen}}}{4G_N}$$

[XD '13; XD & Lewkowycz, 1705.08453; ...]

- Example:

$$L = -\frac{1}{16\pi G_N}\left(R + \lambda R_{\mu\nu}R^{\mu\nu}\right) \xrightarrow{\text{yields}} A_{\text{gen}} = \int_X 1 + \lambda\left(R_a^a - \frac{1}{2}K_a K^a\right)$$

- Applies to (dynamical) black holes and shown to obey the Second Law.

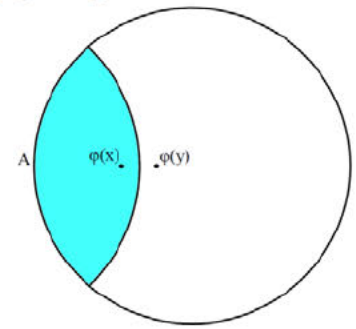
[Bhattacharjee, Sarkar & Wall '15] [Wall '15]

Corrections to Ryu-Takayanagi

I will focus on quantum corrections ($G_N \sim 1/N^2$) which come from matter fields and gravitons.

- The prescription is surprisingly simple:

$$S = \text{ext}\left(\frac{\langle A \rangle}{4G_N} + S_{\text{bulk}}\right)$$



[Engelhardt & Wall '14; XD & Lewkowycz, 1705.08453]

- Quantum extremal surface.
- Valid to all orders in G_N.
- Conjectured in [Engelhardt & Wall '14].
- Natural: invariant under bulk RG flow.
- Matches one-loop FLM result. [Faulkner, Lewkowycz & Maldacena '13]

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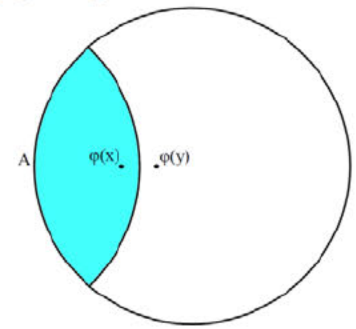


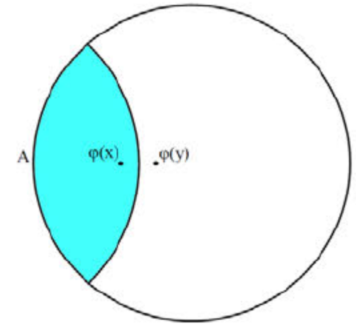
- Can be derived from AdS/CFT. [XD & Lewkowycz 1705.08453]
- Example: 2d dilaton gravity with N_f matter fields. [Russo, Susskind & Thorlacius '92]

➢Quantum effects generate nonlocal effective action:

$$L = -\frac{1}{2\pi}\left[e^{-2\phi}(R + 4\lambda^2) + \frac{N_f}{96\pi} R \frac{1}{\nabla^2} R\right]$$

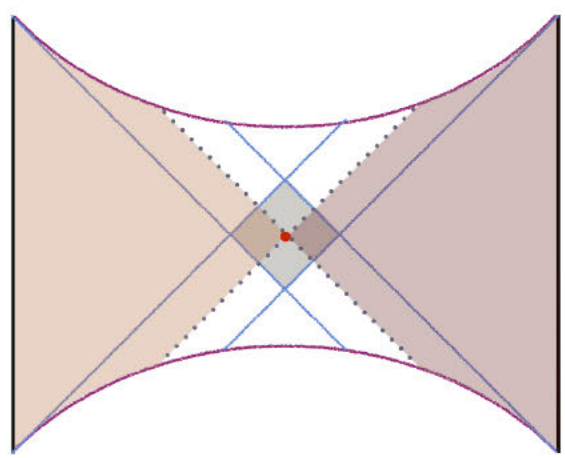
➢Appears local in conformal gauge. Can check quantum extremality.

$$S = \text{ext}\left(\frac{\langle A \rangle}{4G_N} + S_{\text{bulk}}\right)$$



What can we learn from these quantum corrections?

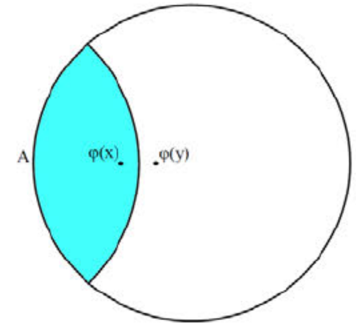
Physics behind black hole horizons!



AdS/CFT as a Quantum Error Correcting Code

Bulk Gravity	**Quantum Error Correction**
• Low-energy bulk states	• States in the code subspace
• Different CFT representations of a bulk operator	• Redundant implementation of the same logical operation
• Algebra of bulk operators	• Algebra of protected operators acting on the code subspace
• Radial distance	• Level of protection

$$S = \text{ext}\left(\frac{\langle A \rangle}{4G_N} + S_{\text{bulk}}\right)$$



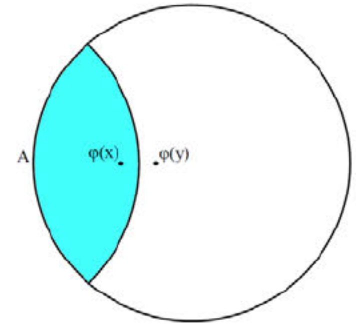
What can we learn from these quantum corrections?

Any bulk operator in the entanglement wedge of region A may be represented as a CFT operator on A.

In other words:

$\forall\ O$ in the entanglement wedge of A, $\exists\ O_A$ on A, s.t. $O|\phi\rangle = O_A|\phi\rangle$ holds for $\forall\ |\phi\rangle \in H_{code}$.

$$S = \text{ext}\left(\frac{\langle A \rangle}{4G_N} + S_{\text{bulk}}\right)$$



What can we learn from these quantum corrections?

Any bulk operator in the entanglement wedge of region A may be represented as a CFT operator on A.

This new form of subregion duality goes beyond the old "causal wedge reconstruction": entanglement wedge can reach behind black hole horizons.

Valid to all orders in G_N.

Comments

- Can also derive (quantum-corrected) RT formula from quantum error correction with complementary recovery. [Harlow '16]

- Can study Rényi entropies from the perspective of a quantum error correcting code and compare to nontrivial predictions from AdS/CFT (in terms of cosmic branes). [XD, Harlow & Marolf, in progress]

➢ Implies additional conditions for a quantum error correcting code to have a gravity dual.

➢ Can we now find better toy models of holography?

Comments

- Quantum-corrected RT formula leads to nontrivial relations for the modular Hamiltonian and relative entropy between the bulk and boundary, valid to all orders in G_N.

➢ Can we now find better representations of bulk operators in the entanglement wedge?

➢ How can we enjoy these results in the context of the black hole information problem or cosmology?

Thank you!