Quantum advantage with shallow circuits

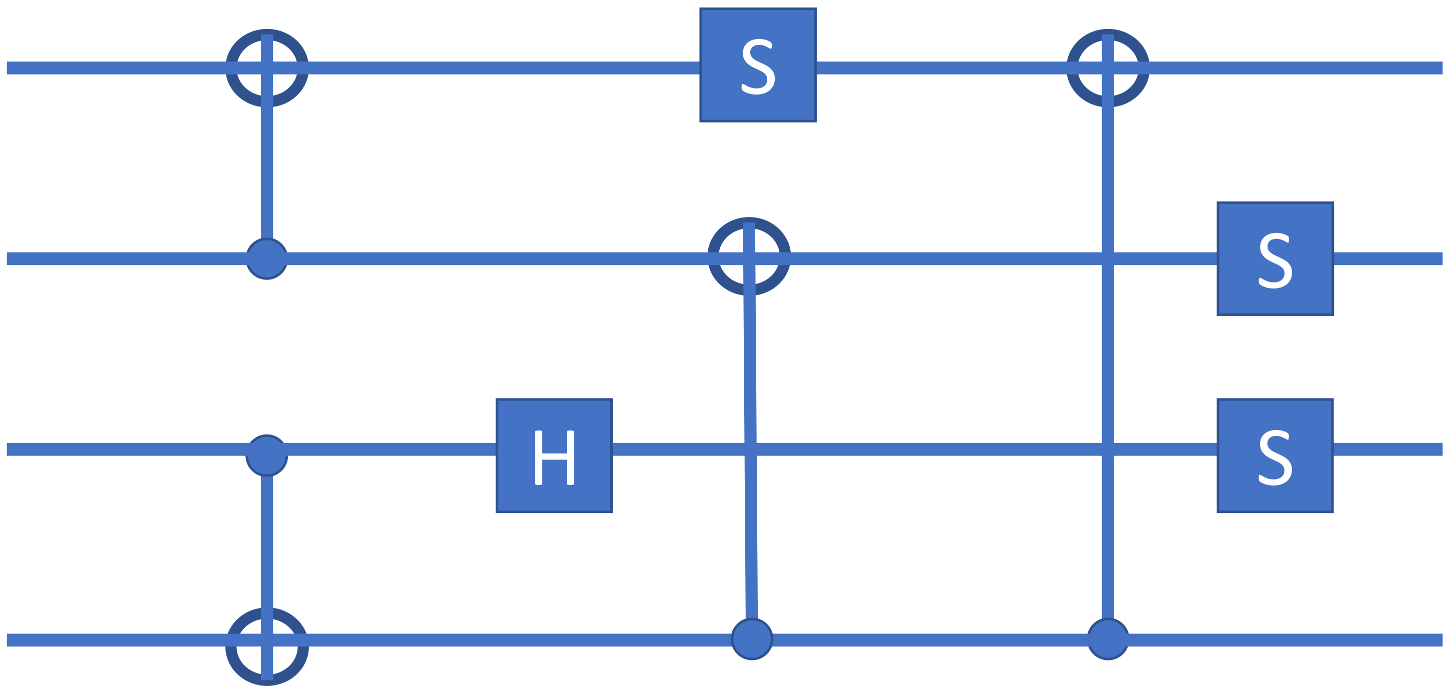
arXiv:1704.00690

Sergey Bravyi (IBM)
David Gosset (IBM)
Robert Koenig (Munich)

In this talk I will describe a **provable**, **non-oracular**, quantum speedup which is attained by constant-depth quantum circuits in a 2D architecture.

I. Overview

A **depth-d quantum circuit** consists of d time steps.
Each time step contains one- and two-qubit gates acting on disjoint qubits.



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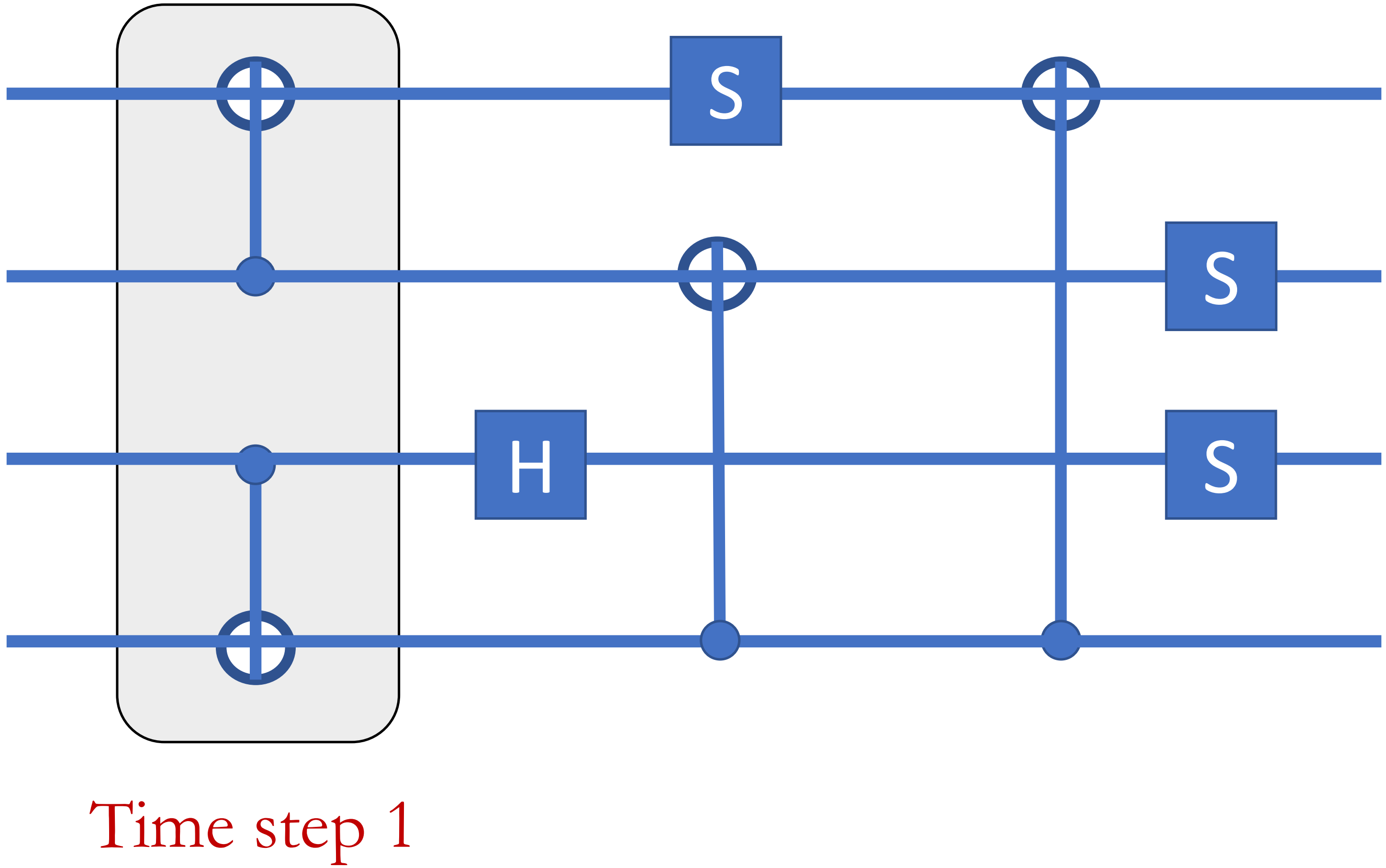

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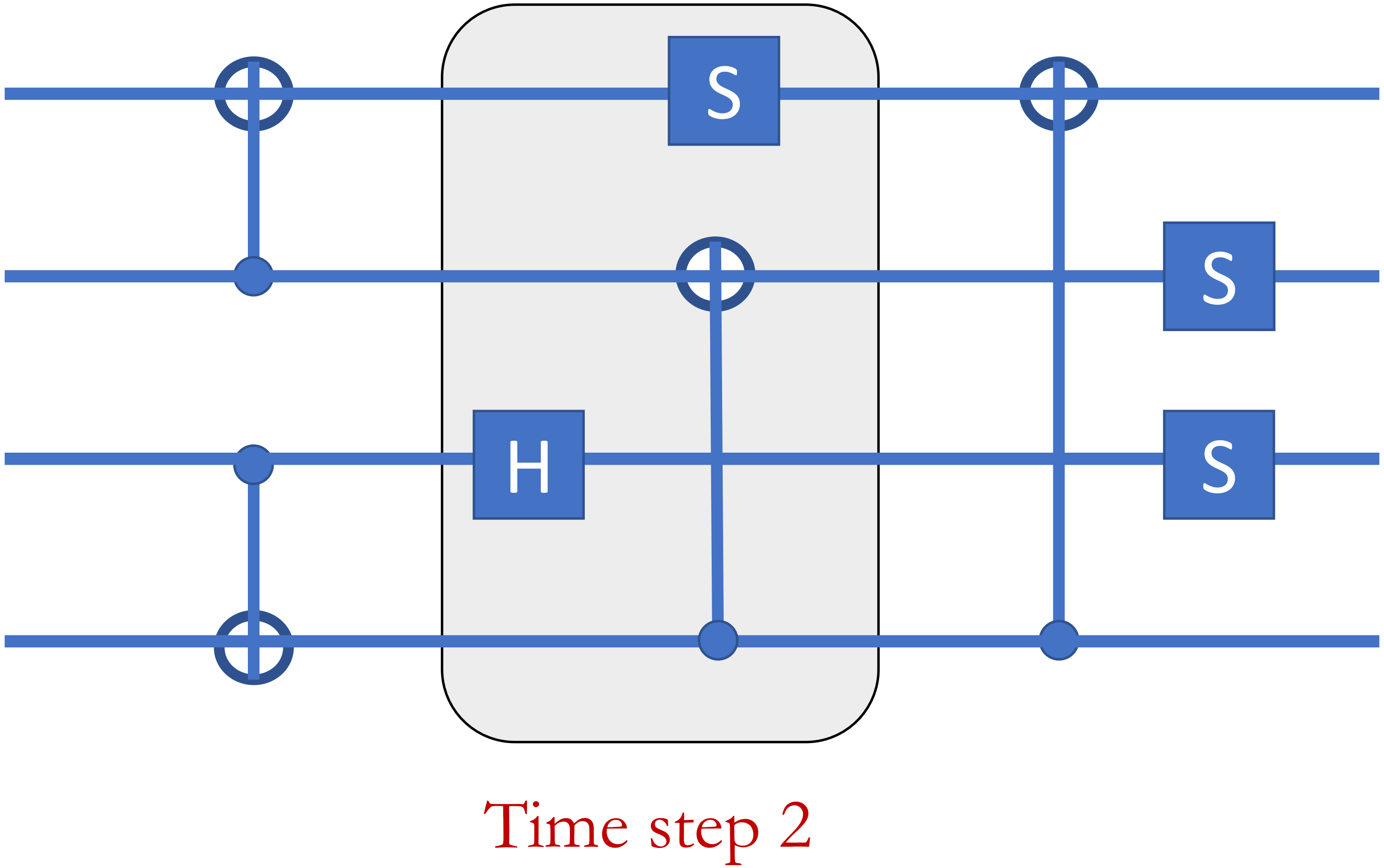

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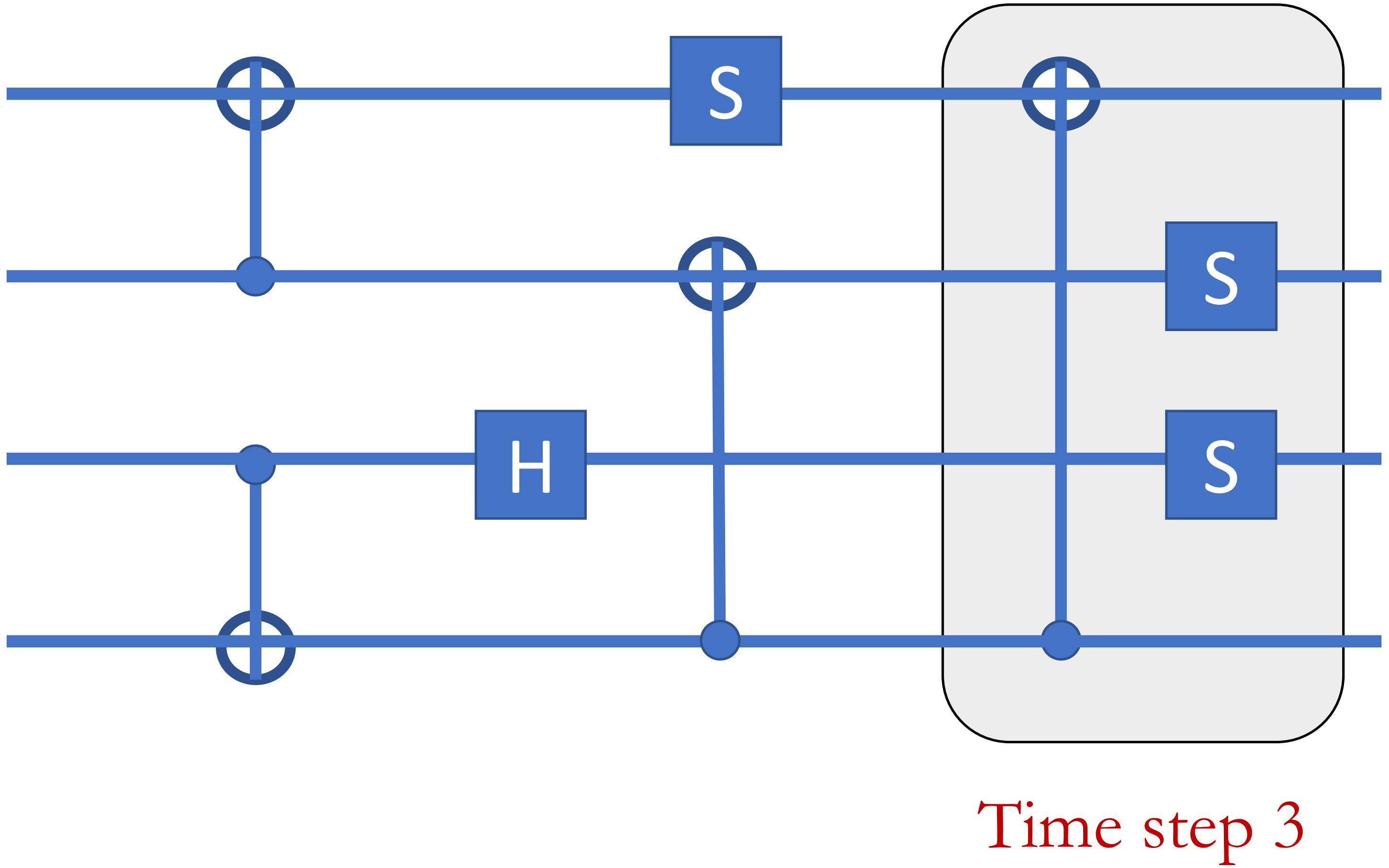

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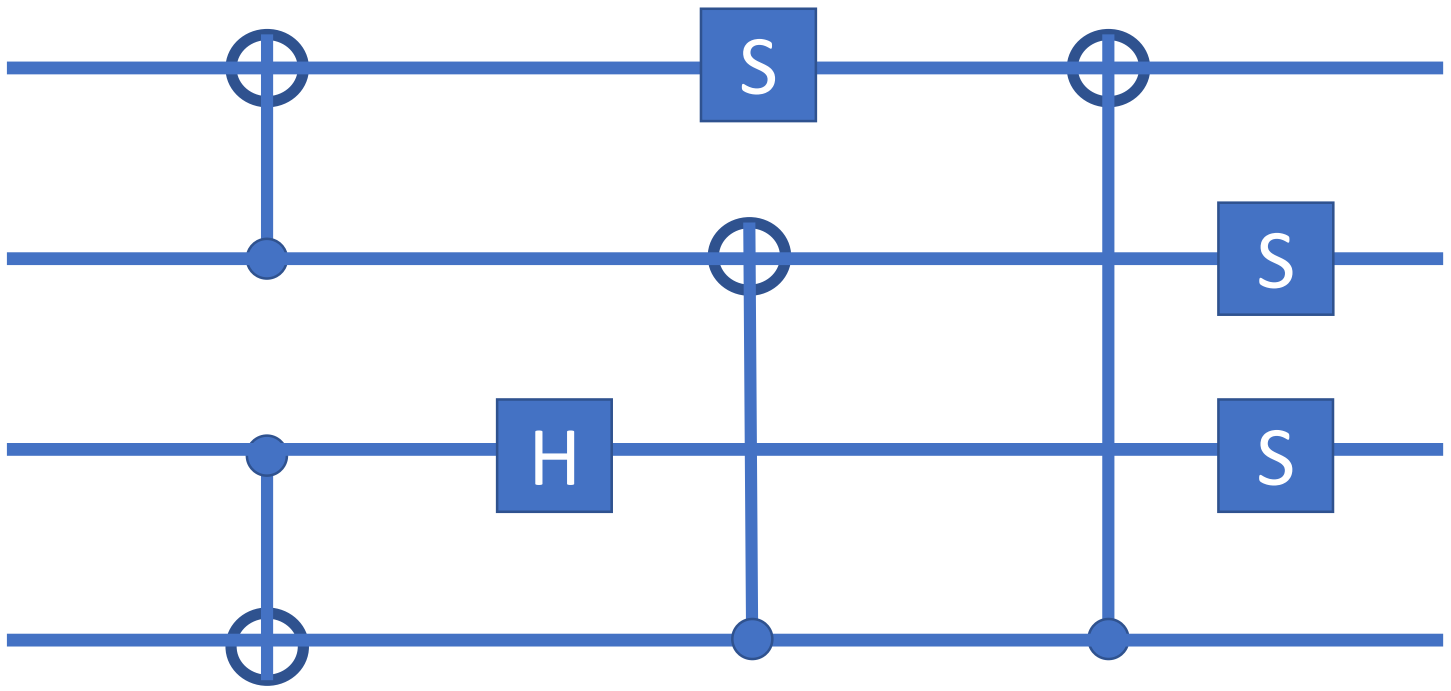

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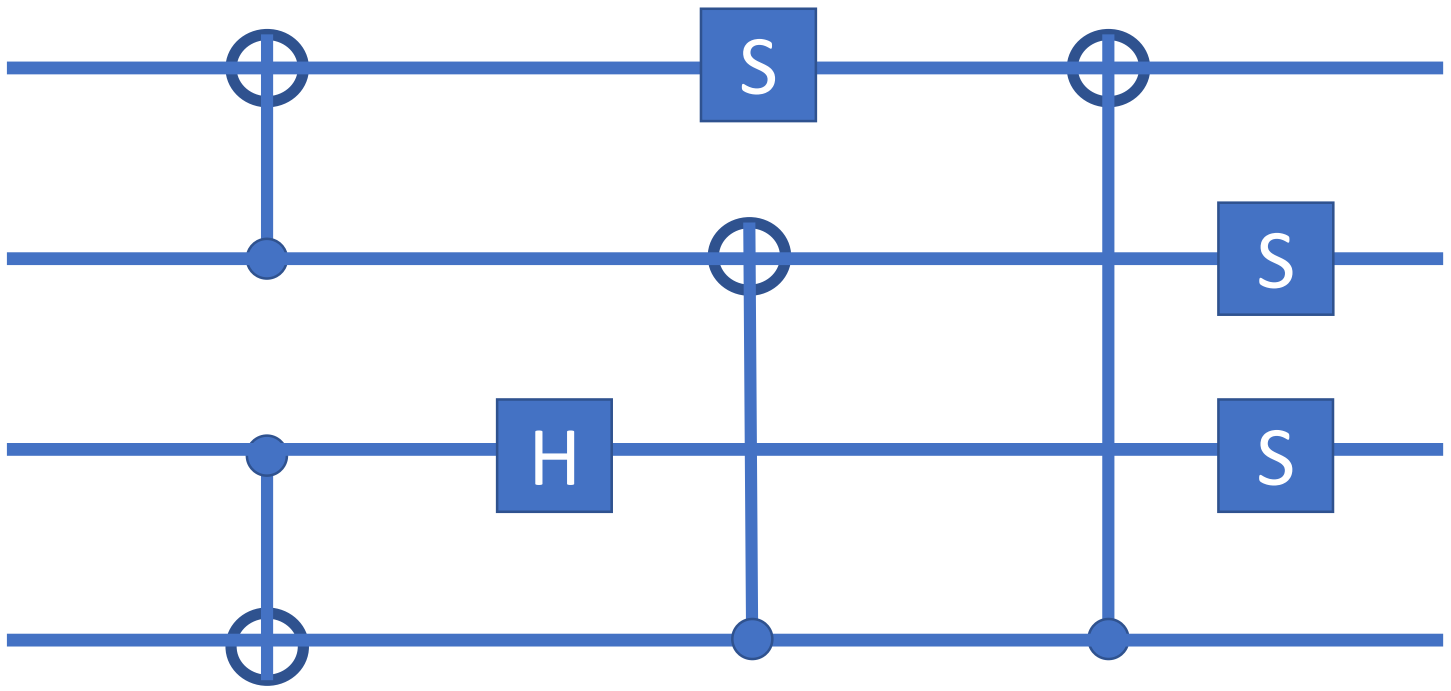
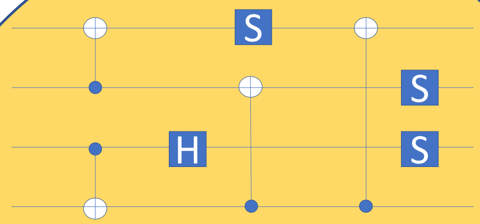


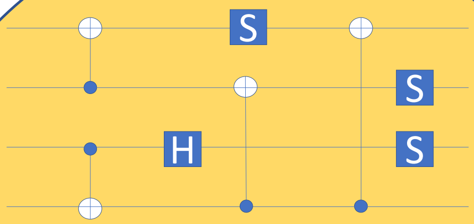
We are interested in **constant-depth quantum circuits**, for which $d = O(1)$.



Constant-depth quantum circuits

Constant-time quantum computation

How much does parallelism buy us if we only have a fixed computation time?



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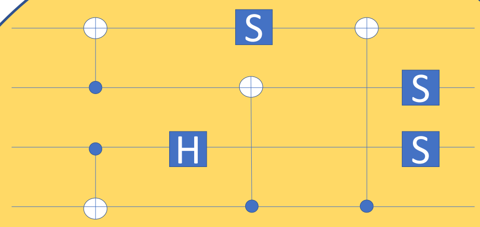
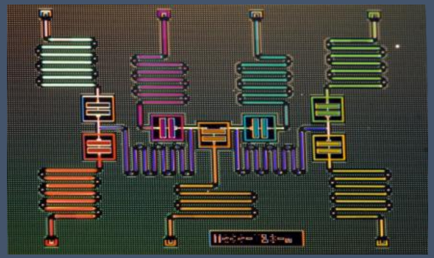


Constant-depth quantum circuits

Quantum algorithms for small quantum computers



Constant-time quantum computation

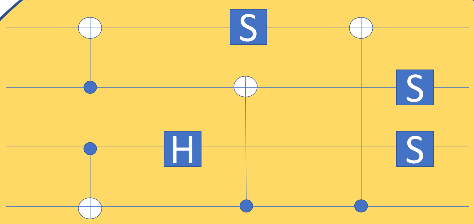
How much does parallelism buy us if we only have a fixed computation time?

Structure/Simulation

Cannot prepare codewords of good quantum codes [Eldar, Harrow 2015]

Efficient classical simulation of depth-2 circuits [Terhal, Divincenzo 2002]

General simulation algorithms (superpolynomial) [Aaronson, Chen 2016]



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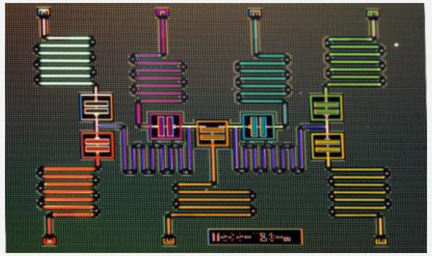

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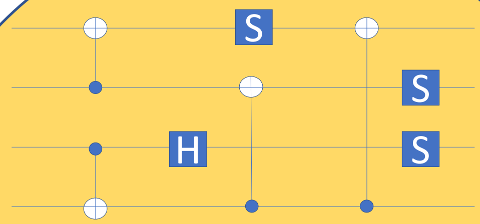

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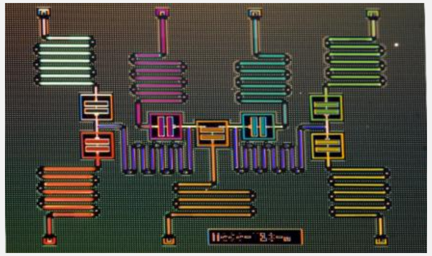
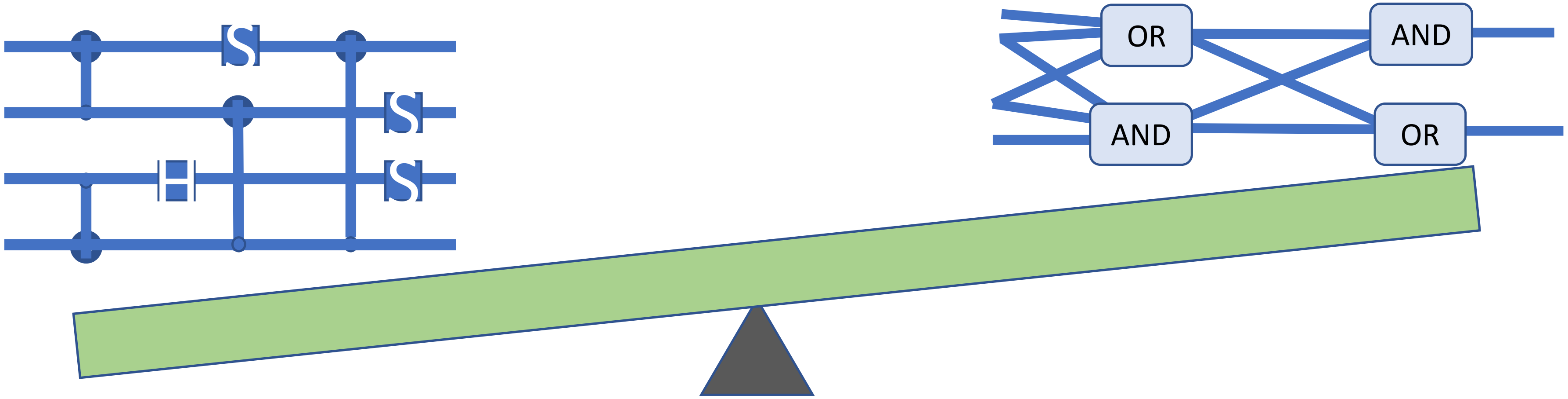


Beat poly-time classical computation?

Constant-depth unlikely to be classically simulable.
[Terhal, Divincenzo 02]

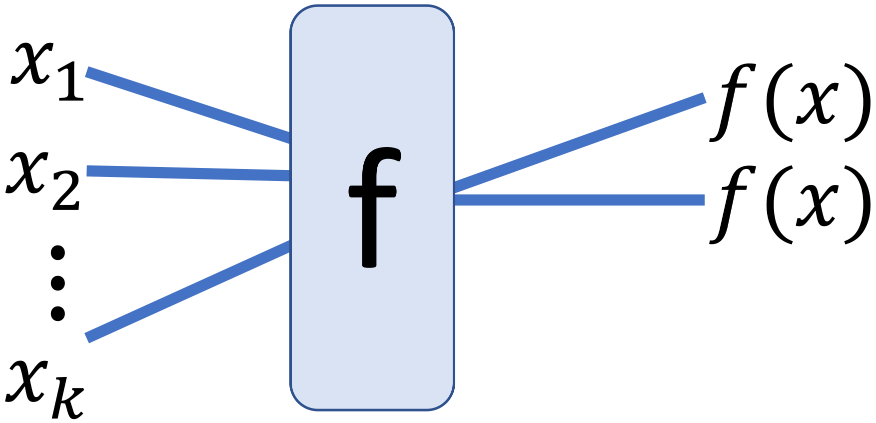
Beat the best classical computers for some task?
[Gao et al. 17]
[Bermejo-Vega et al. 17]
…uses IQP results…
[Bremner, Montanaro, Shepherd 16]



This talk: Are constant-depth quantum circuits more powerful than constant-depth classical circuits?

Classical circuits

A classical gate computes a boolean function $f: \{0,1\}^k \rightarrow \{0,1\}$



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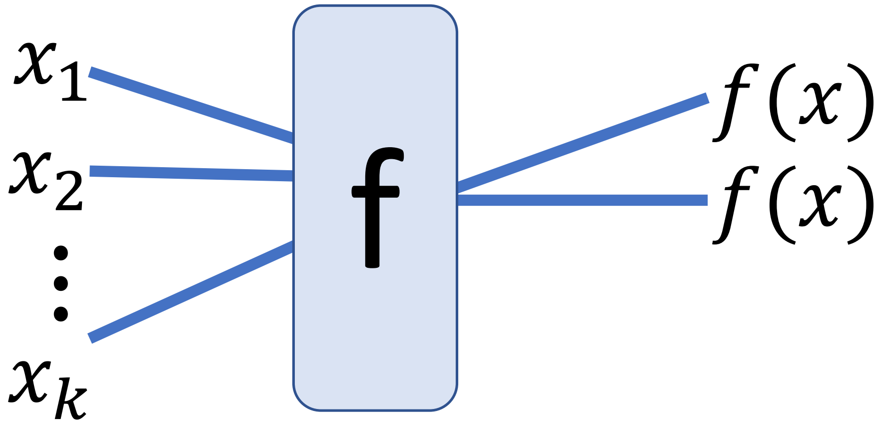
Number of input bits k is called the **fan-in**



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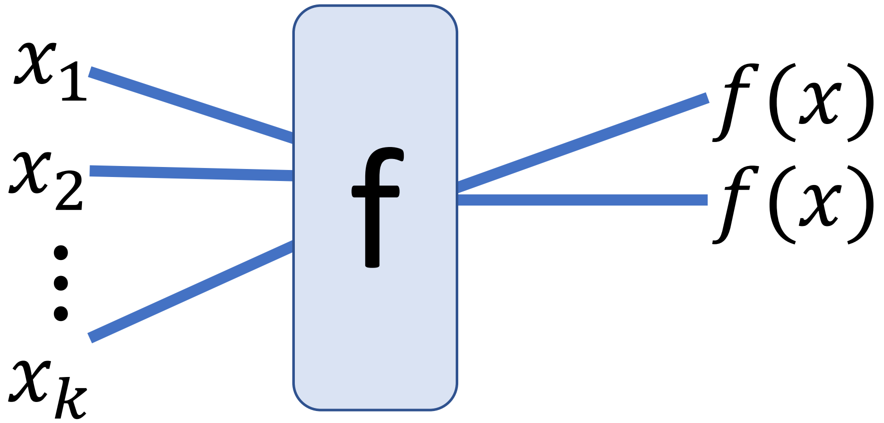

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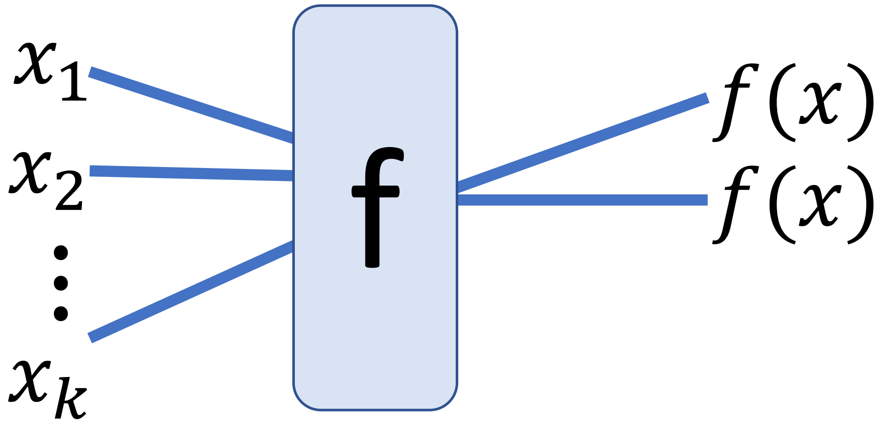

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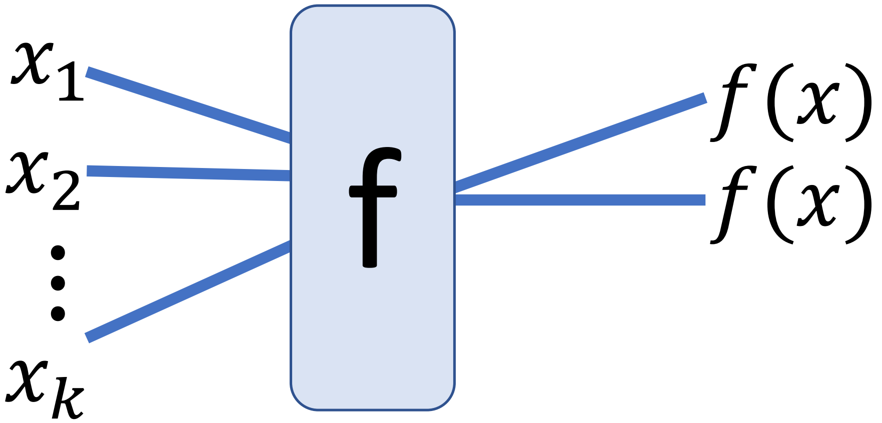

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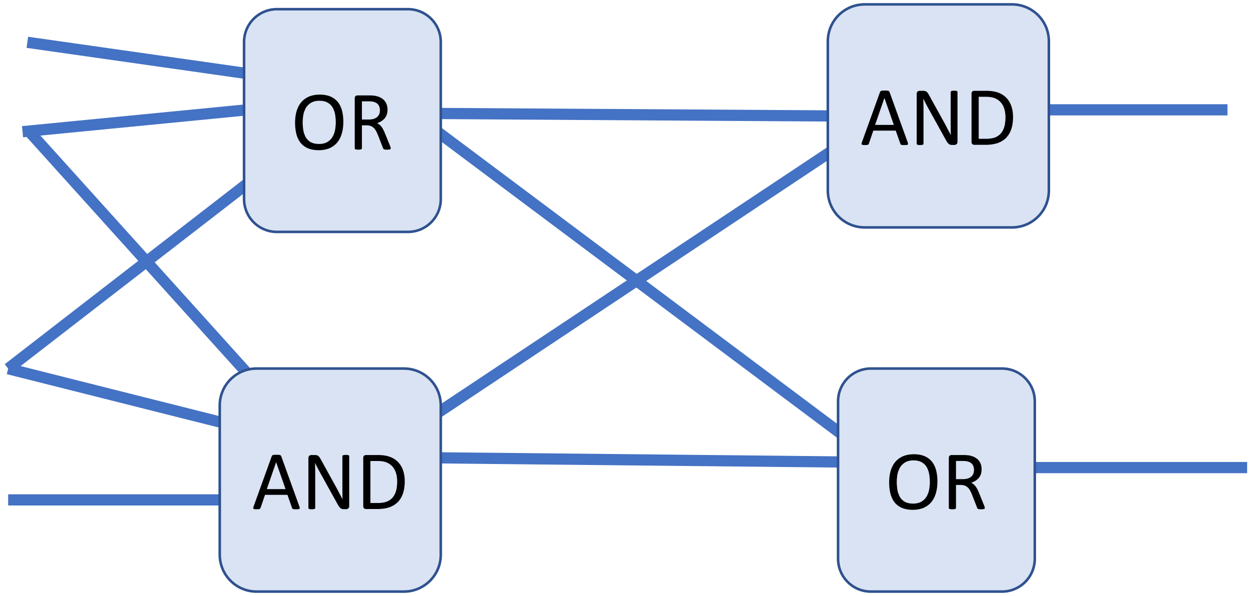
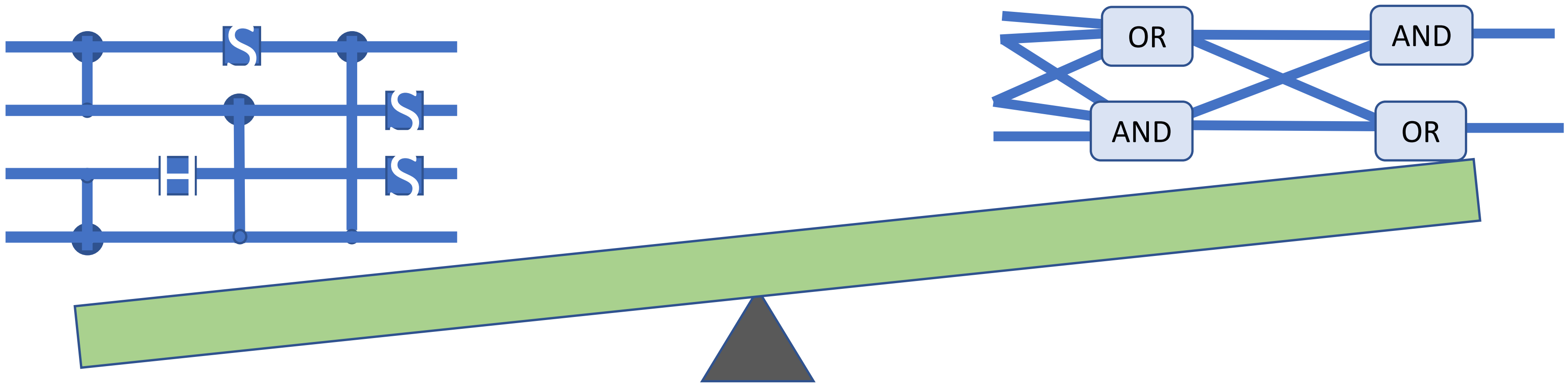
Constant-depth classical circuits

A depth-d classical circuit consists of d layers (time steps) of gates.



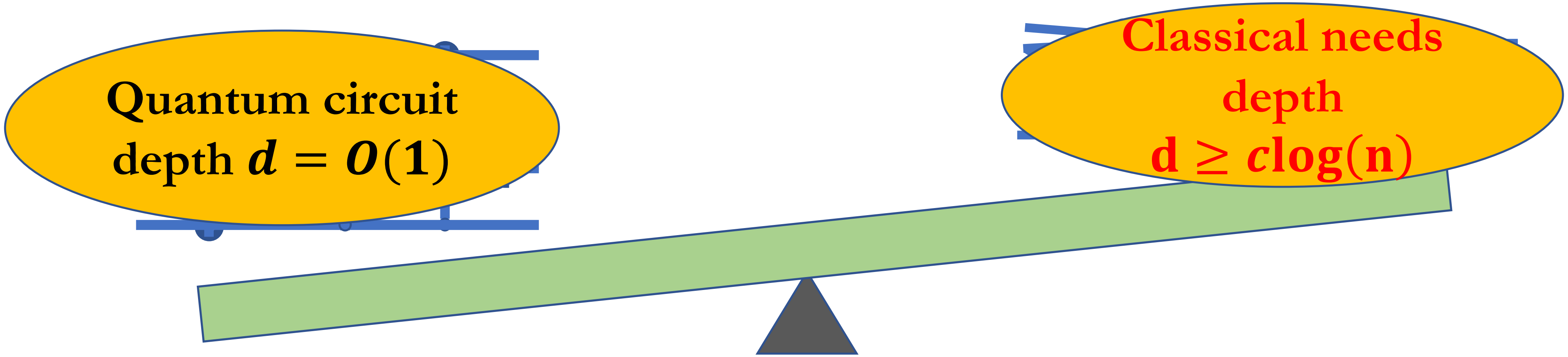
We consider constant-depth circuits composed of bounded fan-in gates.
This class of circuits is known as NC^0.
We also allow the circuit to be probabilistic (random input bits are provided).



Our result:

We describe a computational problem that is solved with certainty by a constant-depth quantum circuit.

We prove that any classical circuit which solves the problem with high probability must have depth increasing logarithmically with input size.



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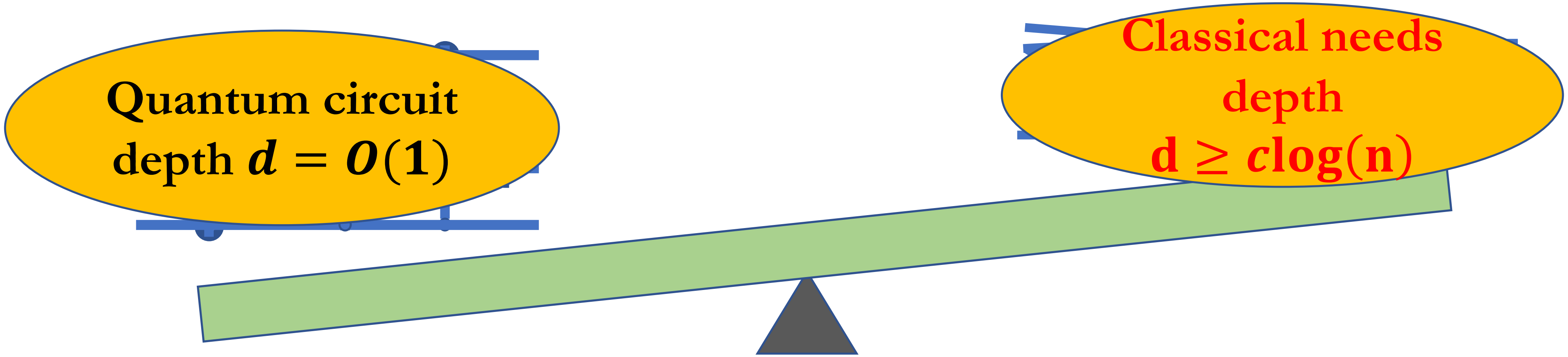

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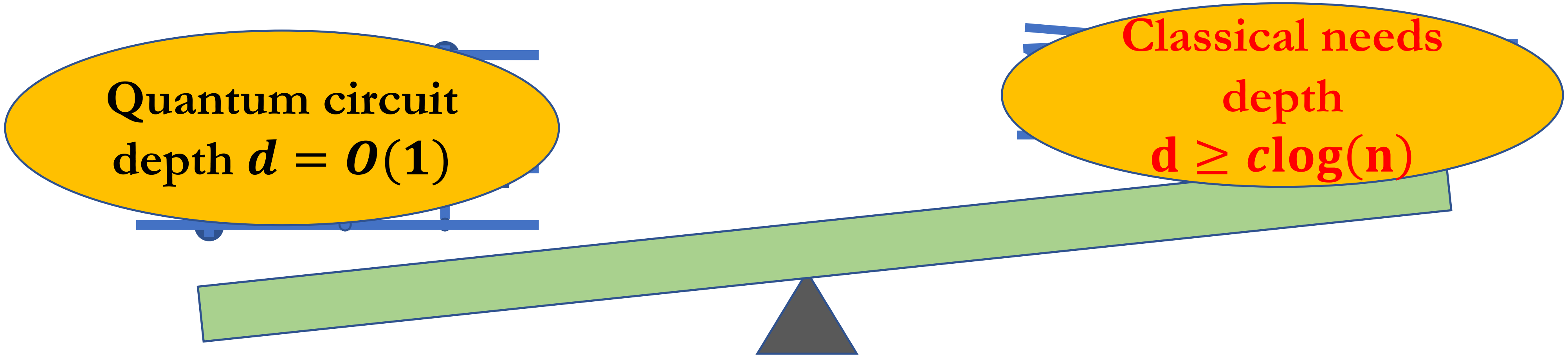

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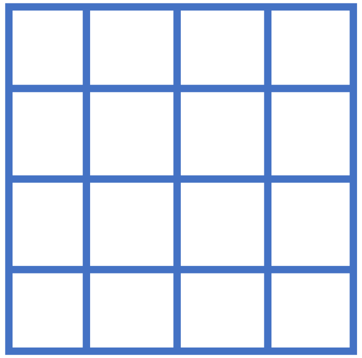
The quantum speedup is unconditional
(does not rely on complexity-theoretic conjectures and is non-oracular)

II. The 2D Hidden Linear Function Problem

Quadratic form on a grid

Let $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ be an $\mathrm{N} \times \mathrm{N}$ grid graph. Write $n = N^2 = |V|$



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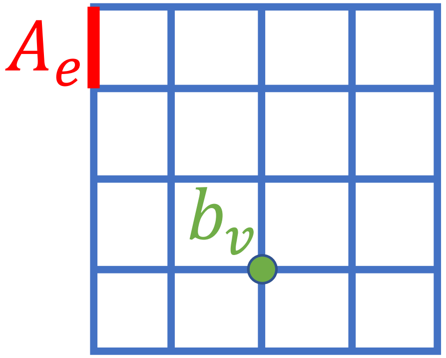

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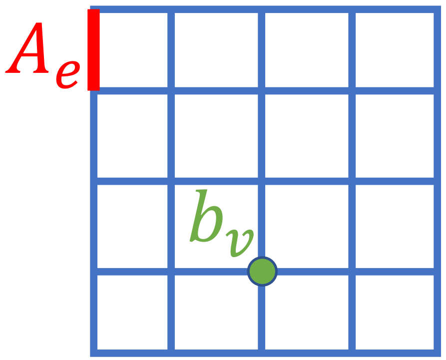

Choose coefficients $A_e \in \{0,1\}$ for each edge and $b_v \in \{0,1\}$ for each vertex.

Any choice of coefficients defines a quadratic form $q\colon \{0,1\}^n \to \mathbb{Z}_4$

$$q(x) = \sum_{e=(v,w)\in E} 2A_e x_v x_w - \sum_{v\in V} b_v x_v$$

The quadratic form hides a linear function

Define a set

$$\mathcal{L}_q = \{x \in \mathbb{F}_2^n\colon\ q(x \oplus y) = q(x) + q(y)\ \text{ for all } y \in \mathbb{F}_2^n\}$$

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Lemma
The set $\mathcal{L}_q$ is a linear subspace of $\mathbb{F}_2^n$. Furthermore, there is a "secret" bit string $z \in \{0,1\}^n$ such that

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Define a computational problem where the goal is to find a secret bit string…

The 2D Hidden Linear Function Problem

Input: Coefficients $A \in \{0,1\}^{|E|}$ and $b \in \{0,1\}^{|V|}$.

Specifies a quadratic form $\boldsymbol{q(x)}$ and a subspace $\boldsymbol{\mathcal{L}_q \subseteq \mathbb{F}_2^n}$

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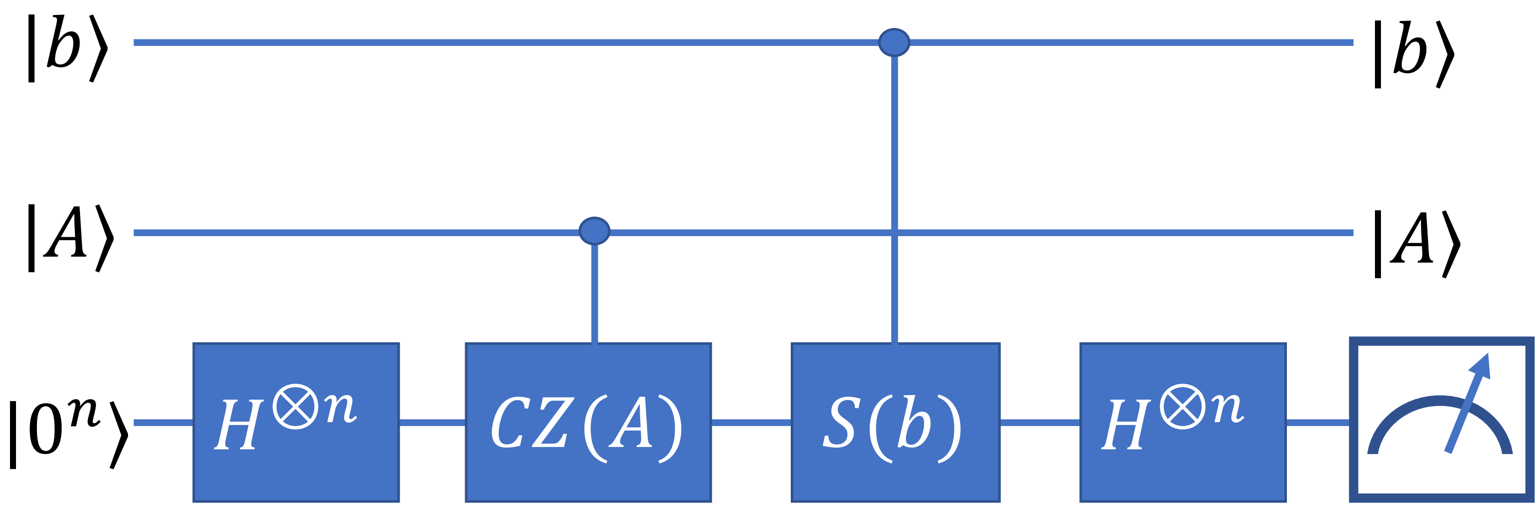
Can be viewed as a non-oracular version of the Bernstein-Vazirani problem.
[Bernstein Vazirani 1993]

In general each instance of the 2D HLF has many valid solutions z .

Quantum algorithm

A qubit for each vertex of the $N \times N$ grid



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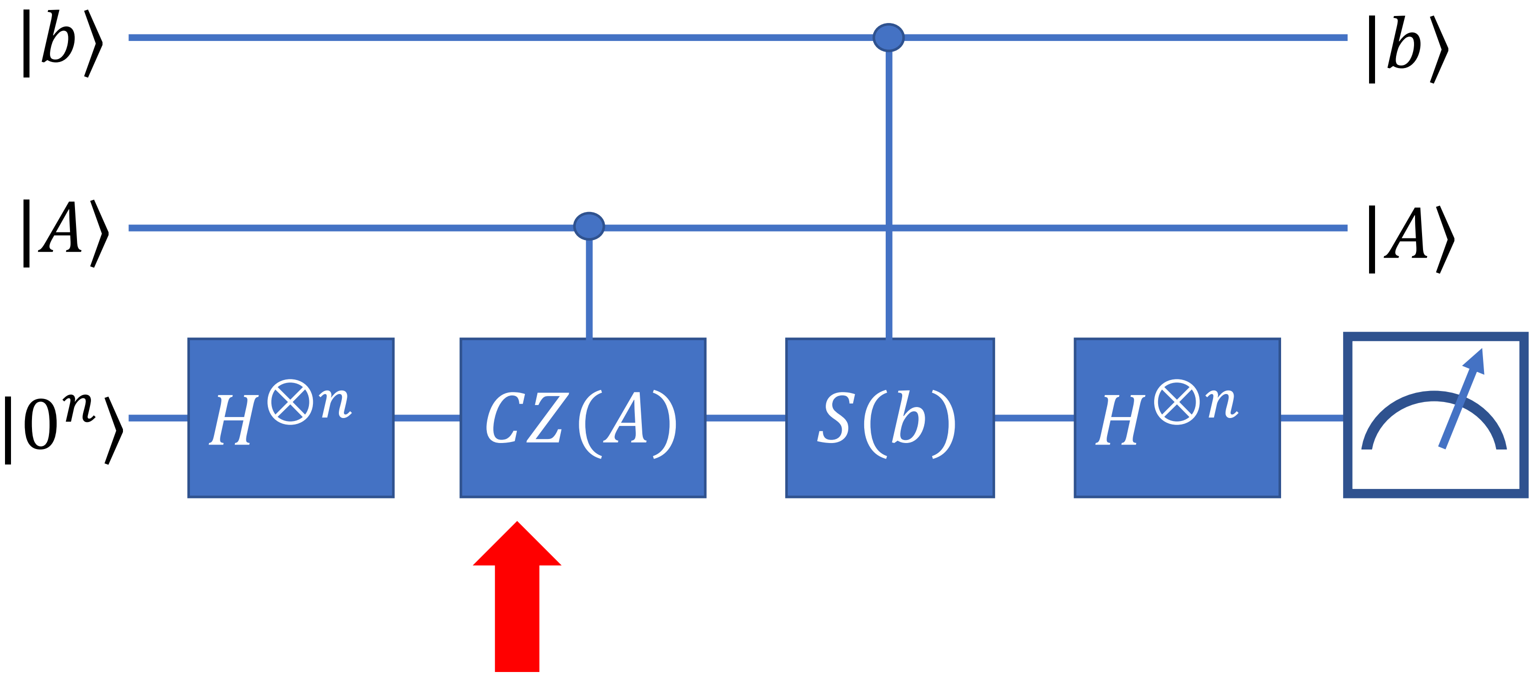

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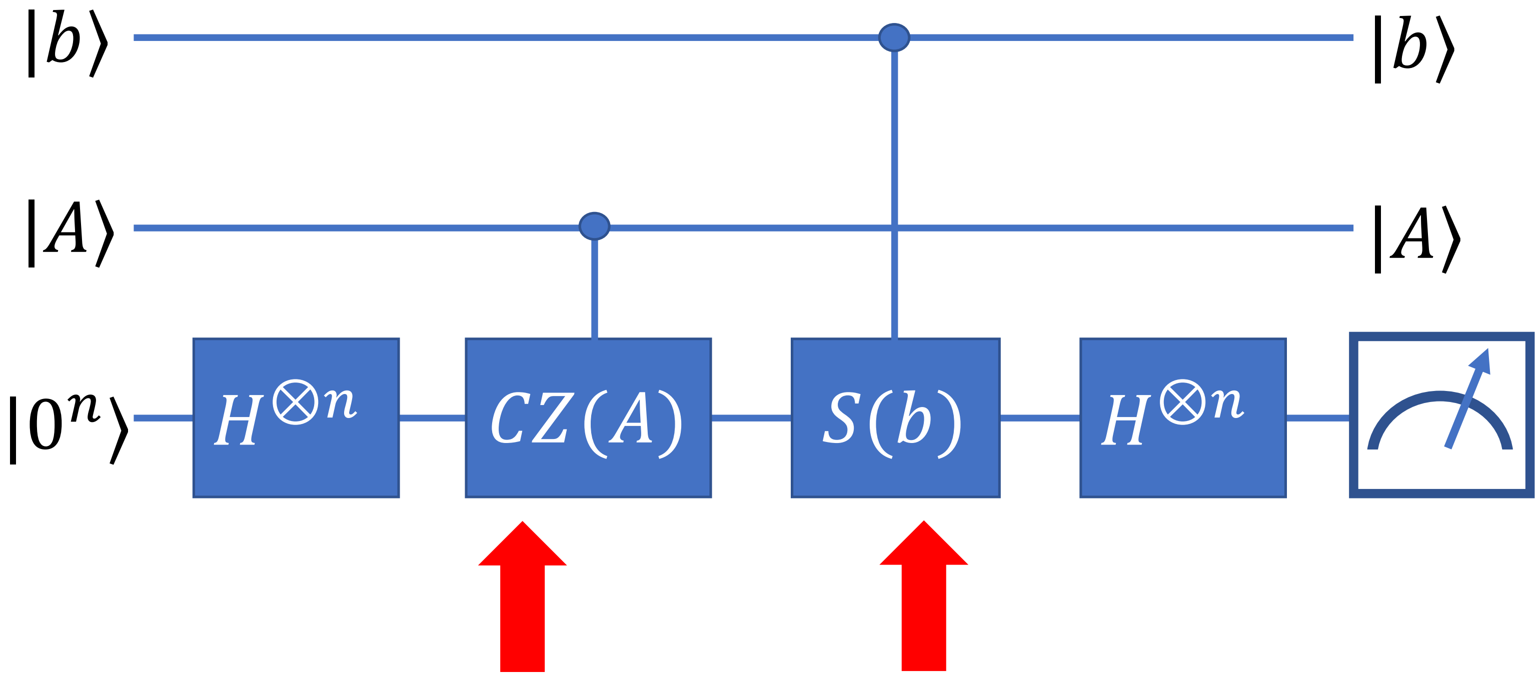

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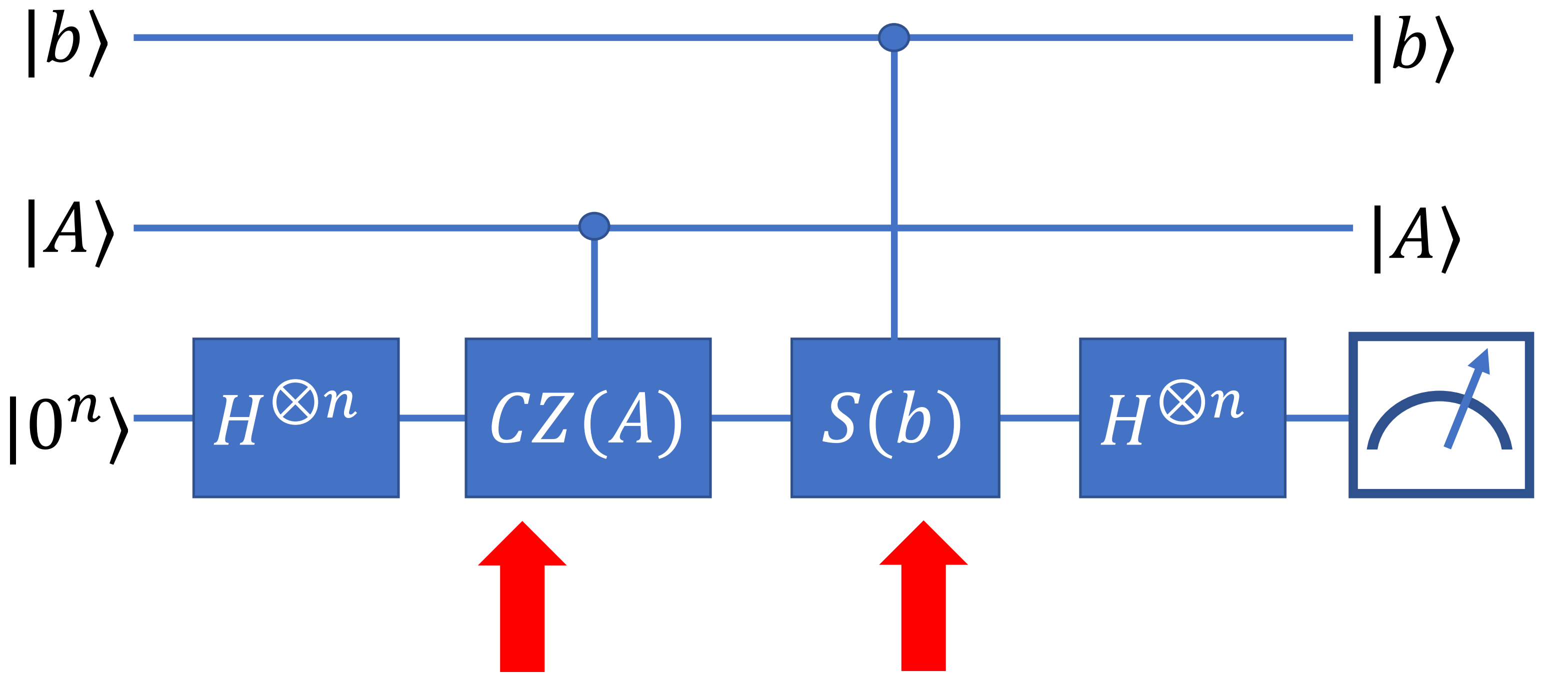

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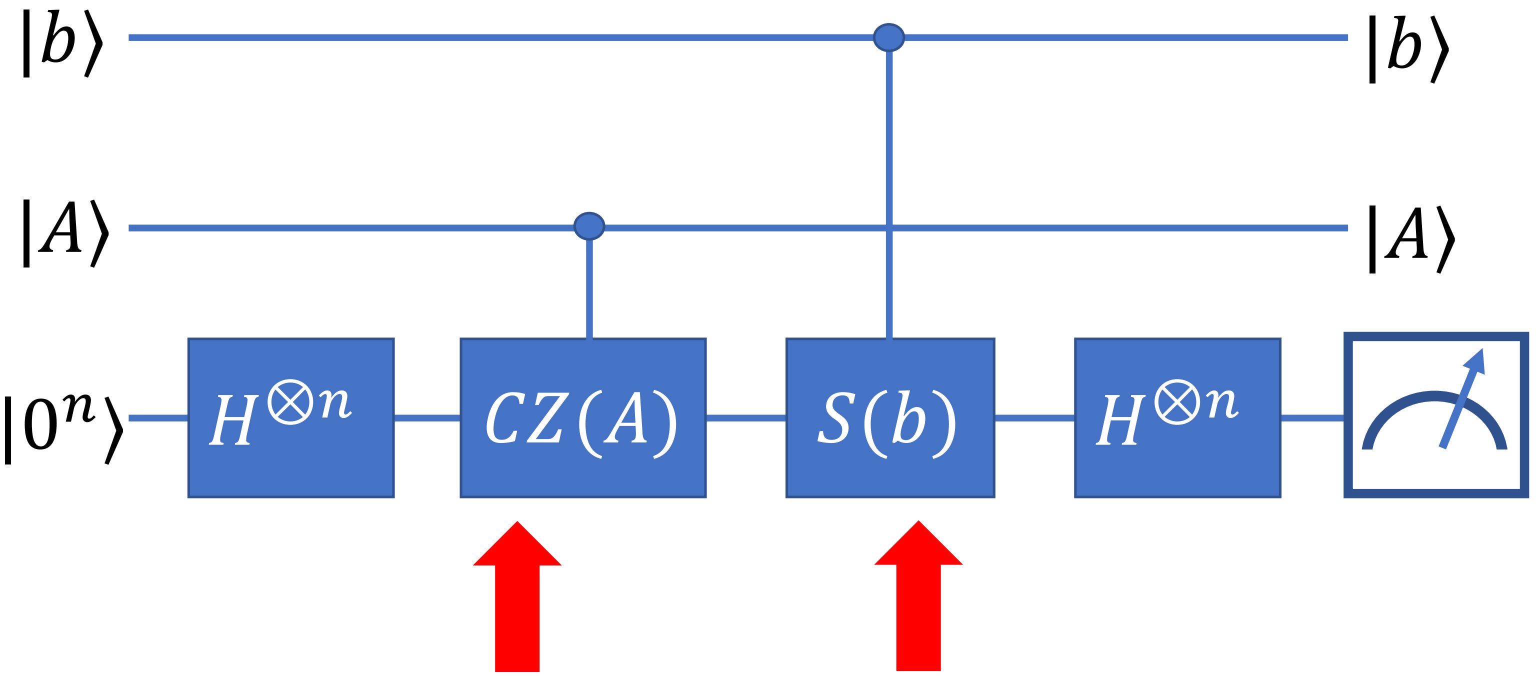

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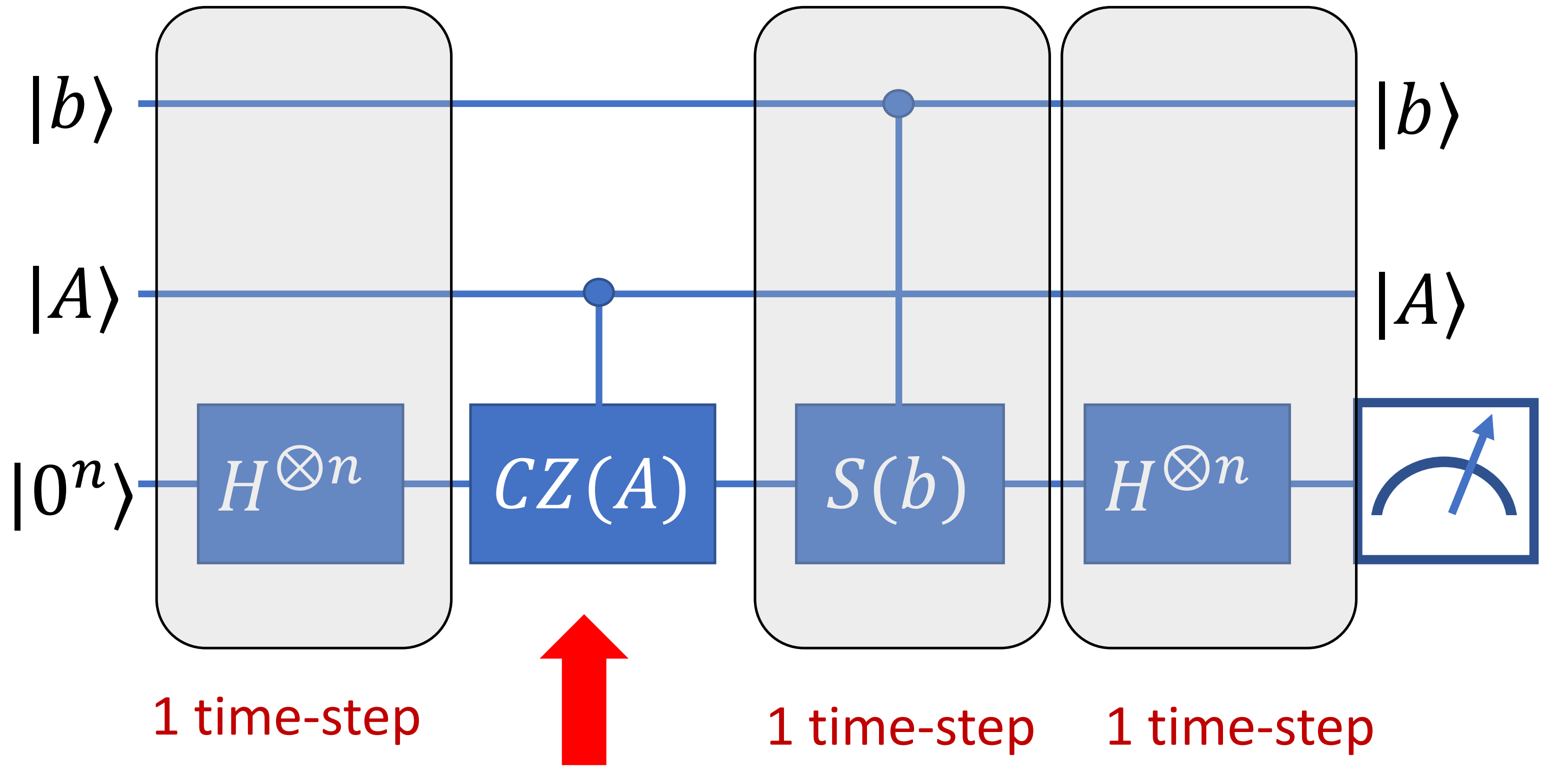
Fact: The output z is a uniformly random solution to the 2D HLF Problem.

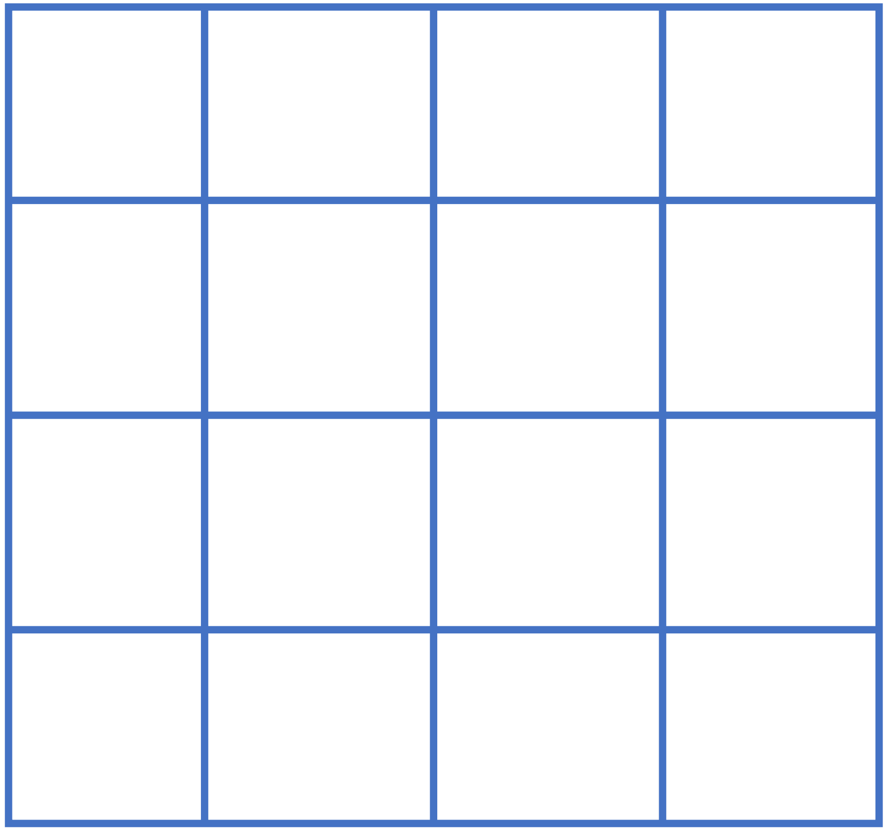
The algorithm can be implemented in constant-depth



Four layers of **CCZ** gates.
(even/odd vertical/horizontal edges)
Decompose **CCZ** gates into 1- and 2-qubit gates.

…it only requires classically controlled Clifford gates between nearest neighbor qubits on a 2D grid.

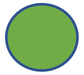
Example:



Place a qubit at each vertex

Place input bits on vertices and edges:

 : Edge with $A_e = 1$

 : Vertex with $b_v = 1$

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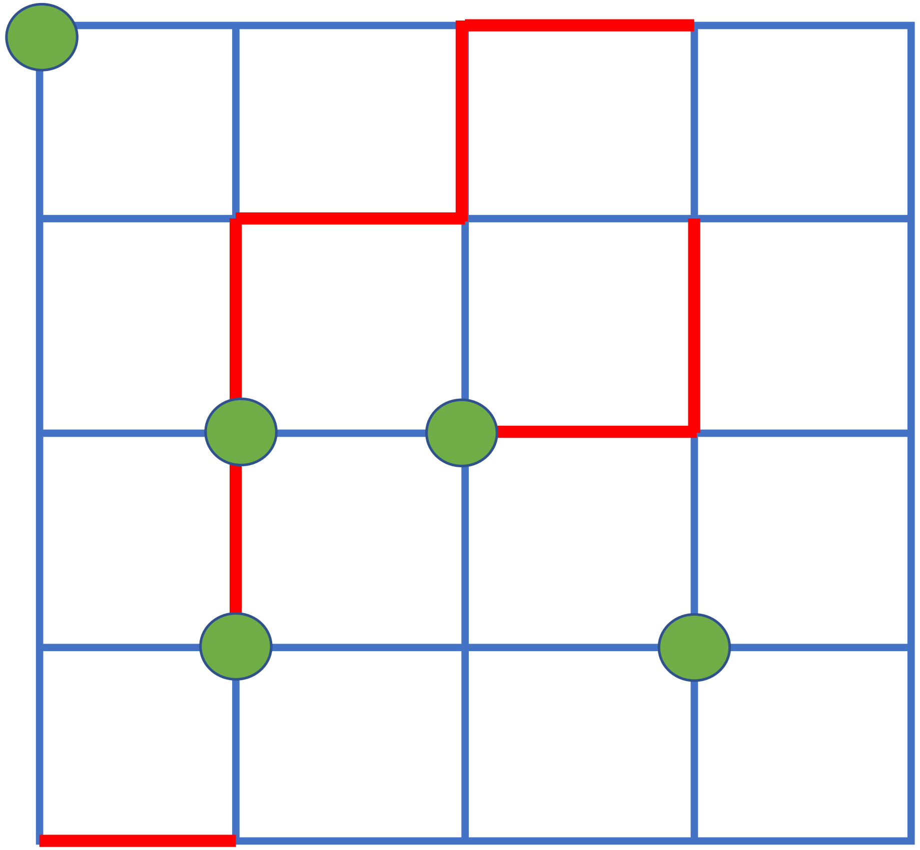

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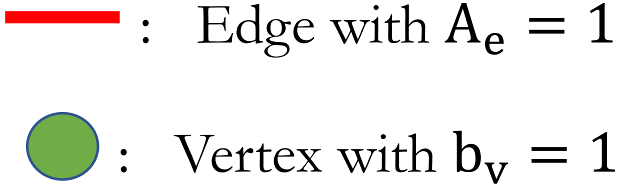

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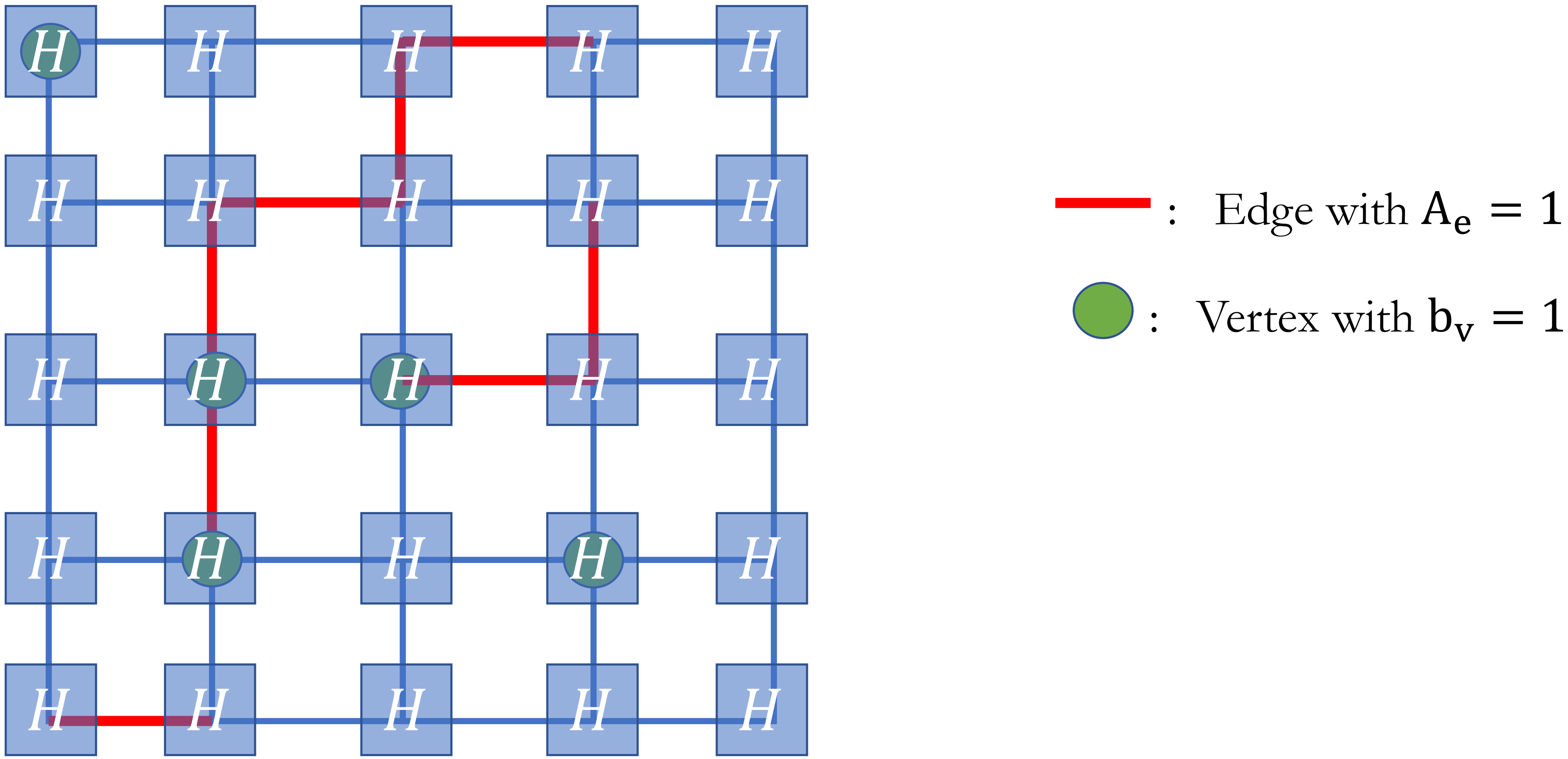

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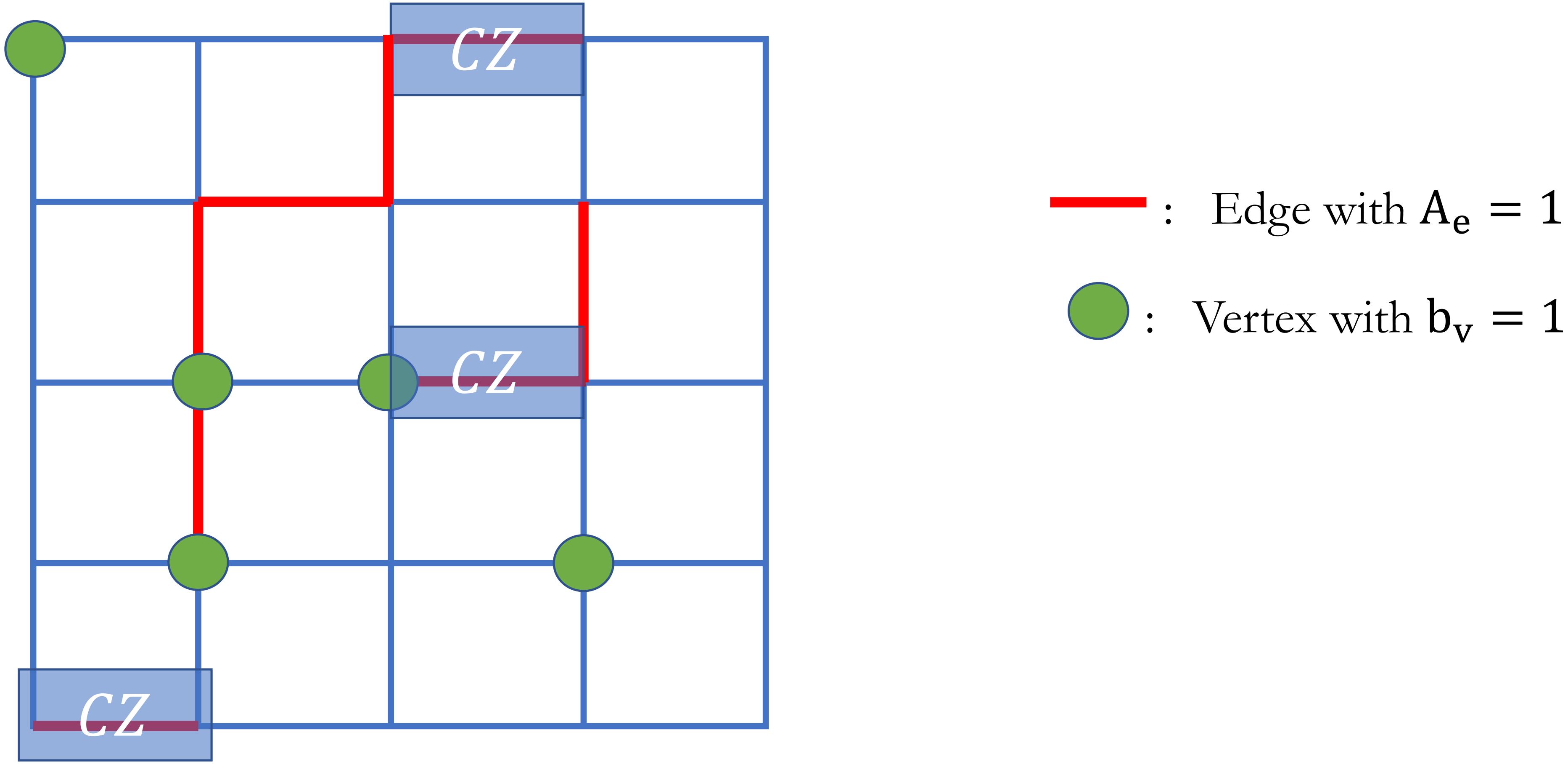

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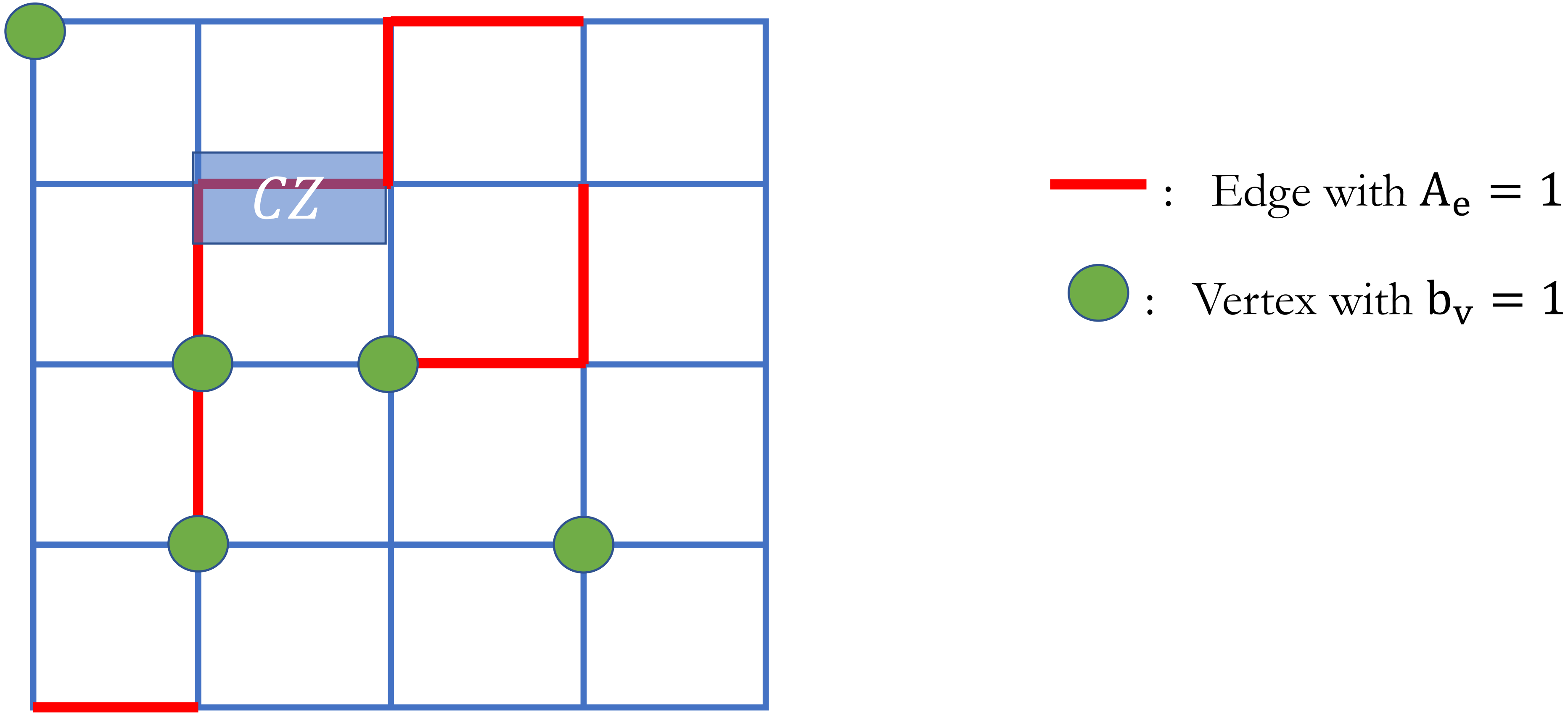

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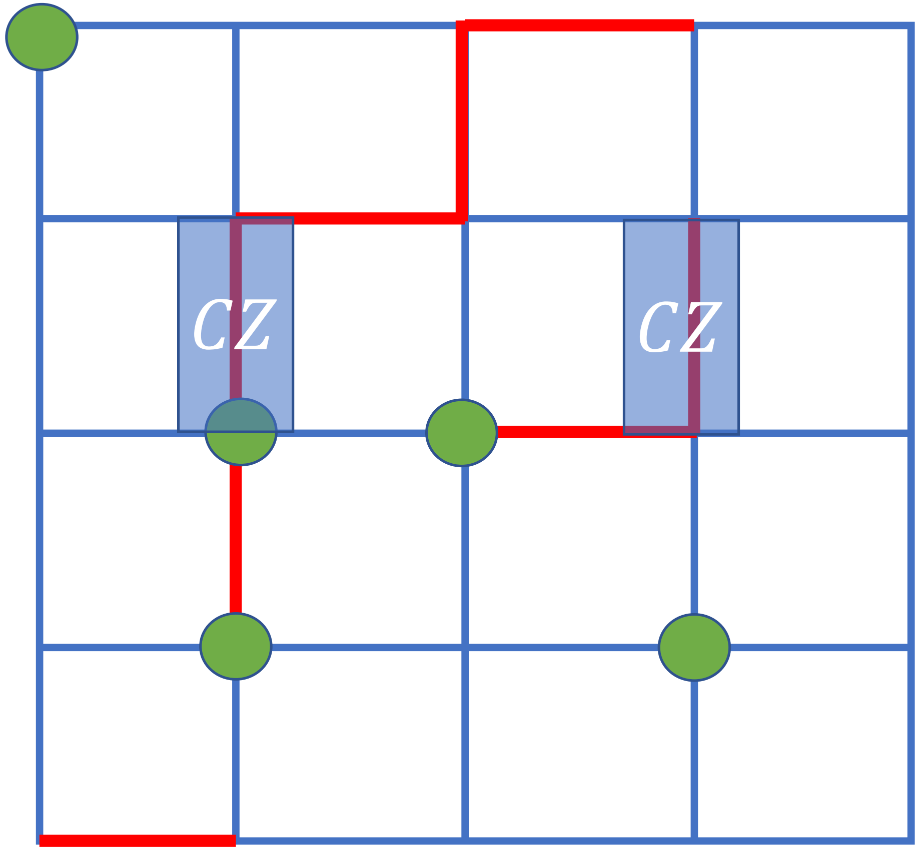

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: Vertex with $b_v = 1$

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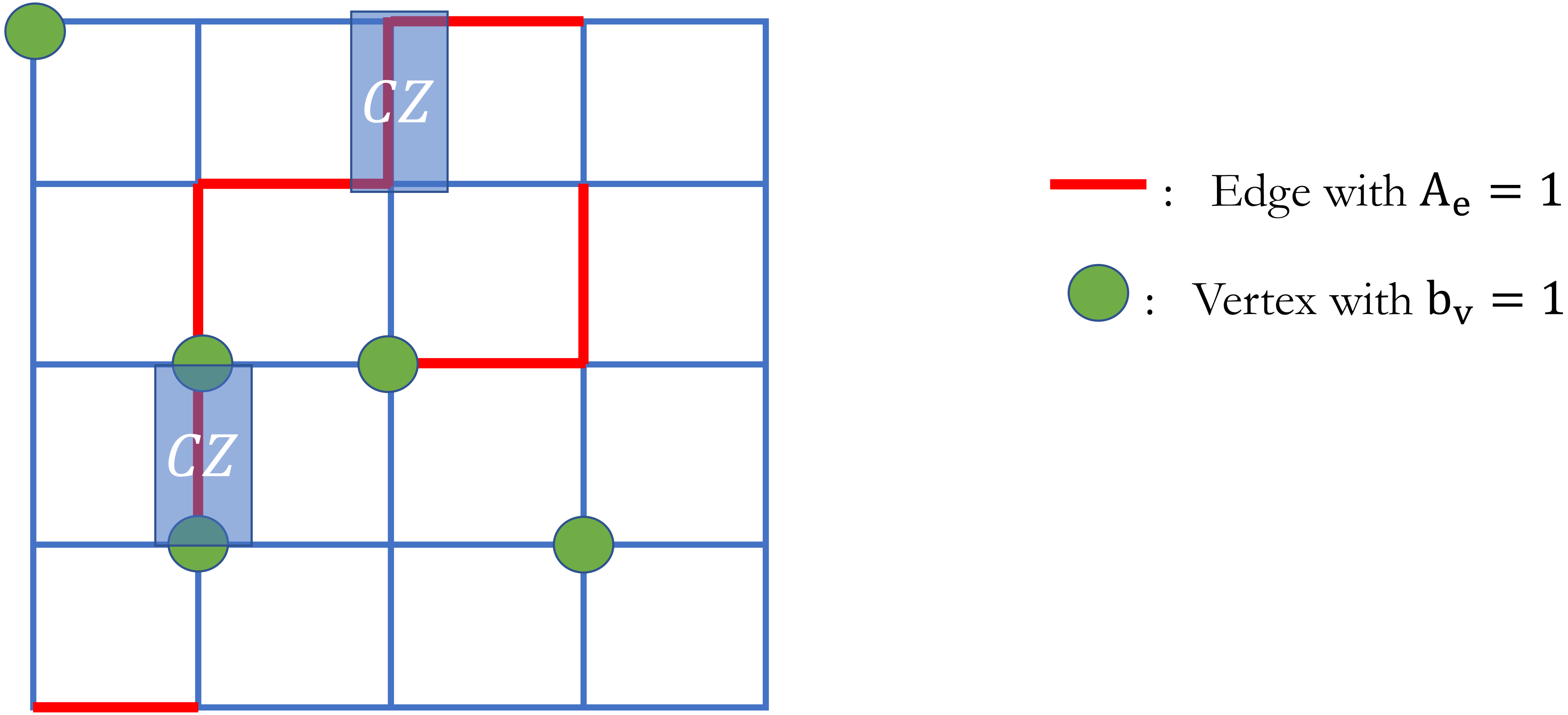

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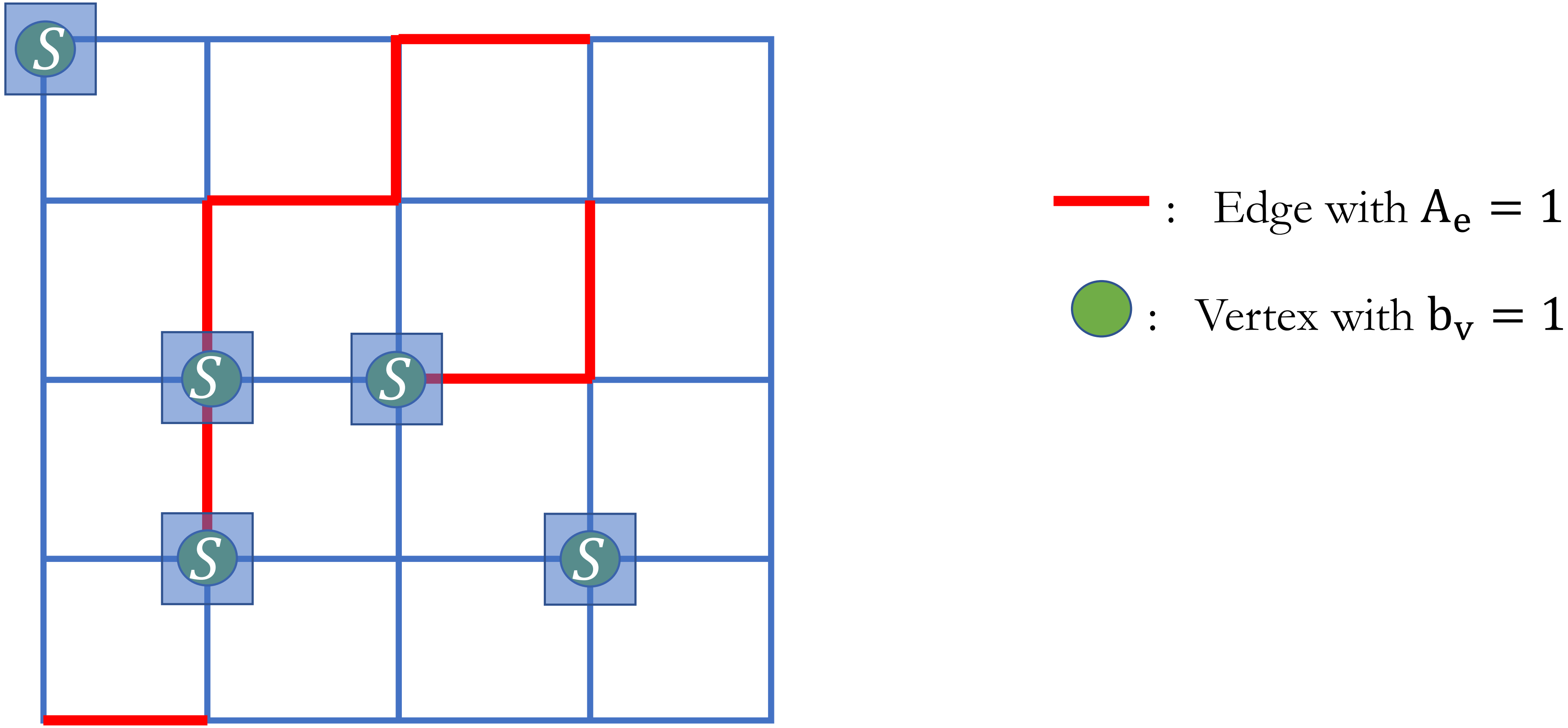

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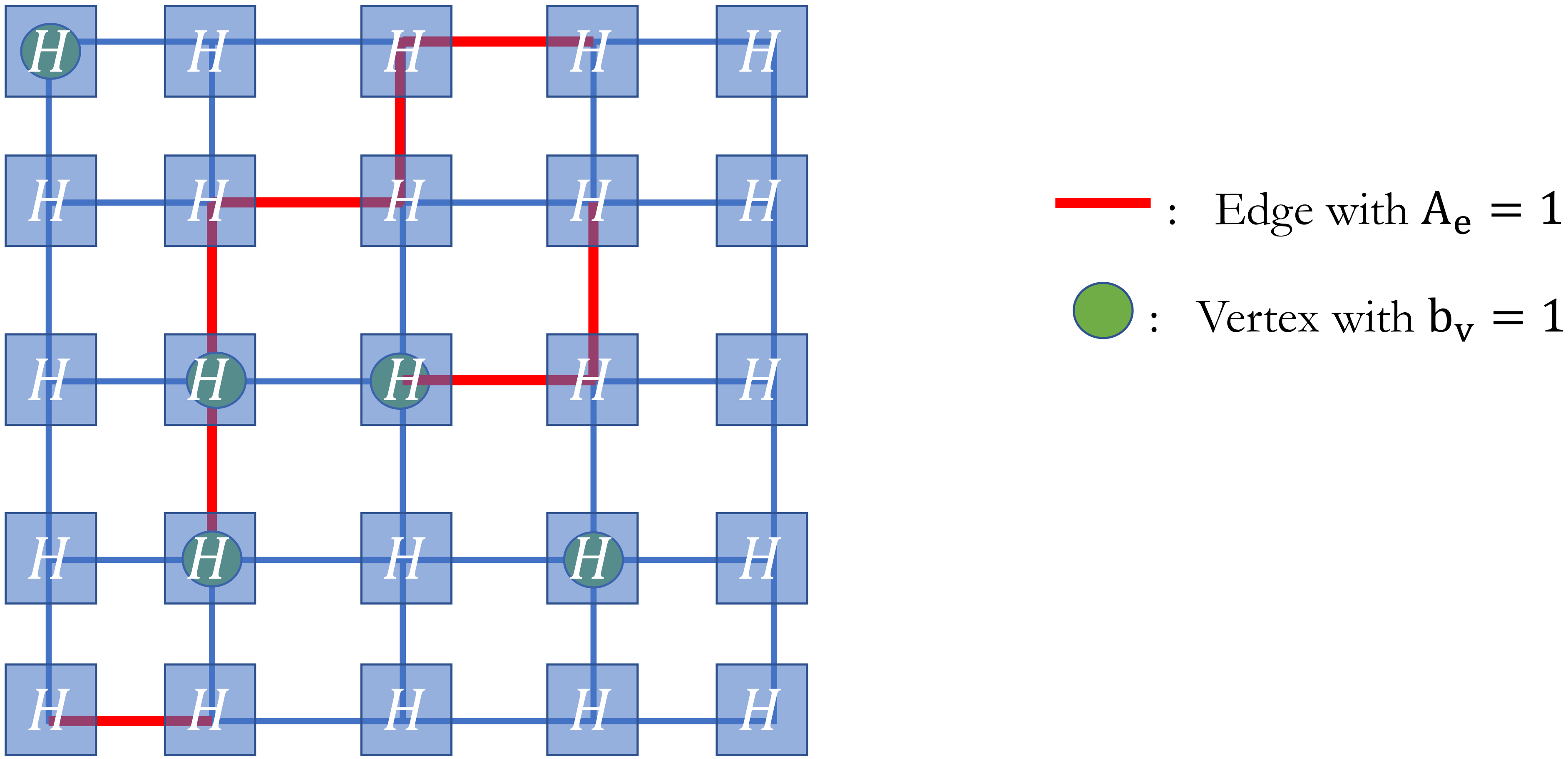

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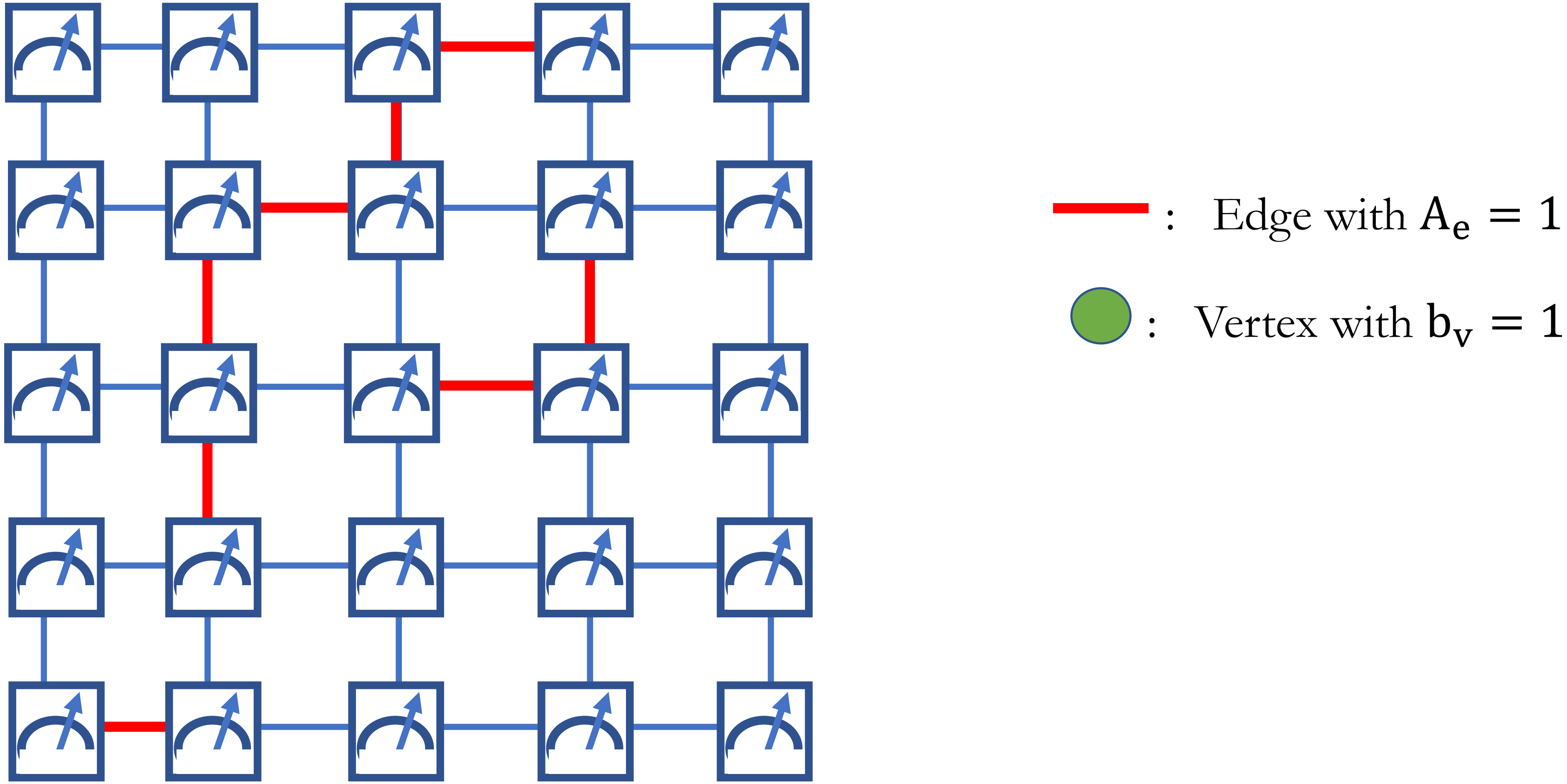


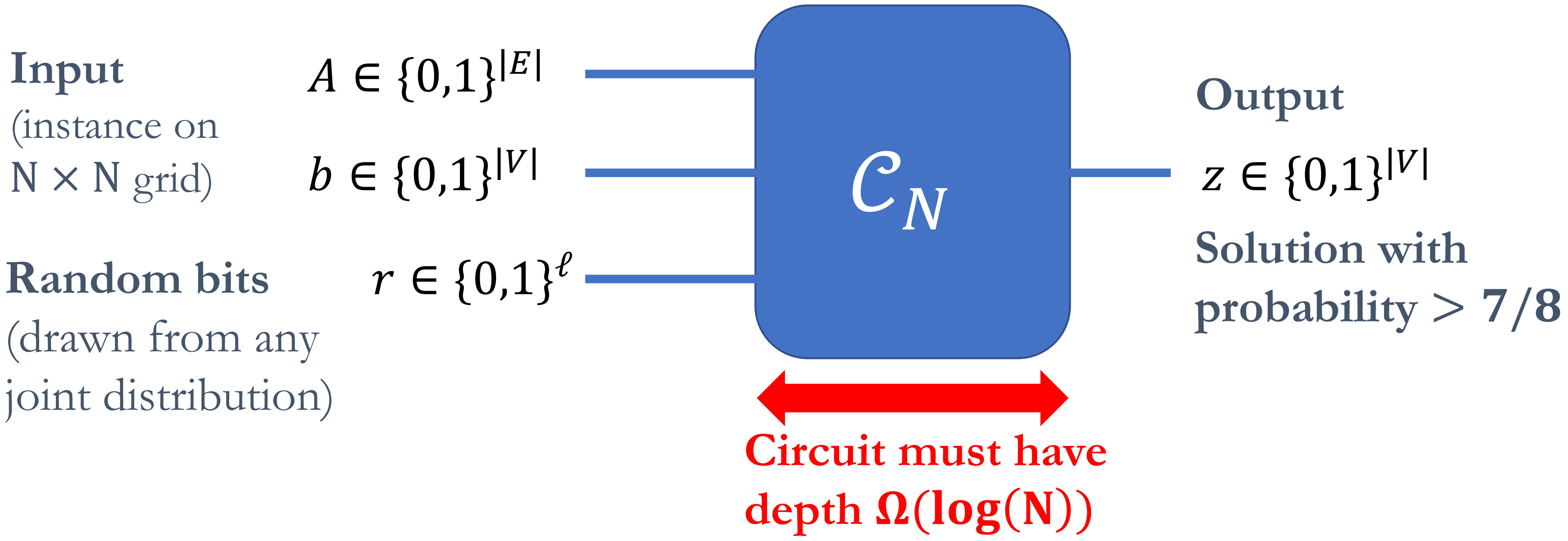
The 2D HLF problem is solved by a constant-depth quantum circuit with gates acting locally in 2D.

Next we show that it cannot be solved by a constant-depth classical circuit…

III. Classical lower bound

Theorem: The following holds for all sufficiently large N. Let $\mathcal{C}_N$ be a classical probabilistic circuit composed of gates of fan-in $\leq K$ which solves size-N instances of the 2D HLF Problem with probability greater than $7/8$. Then

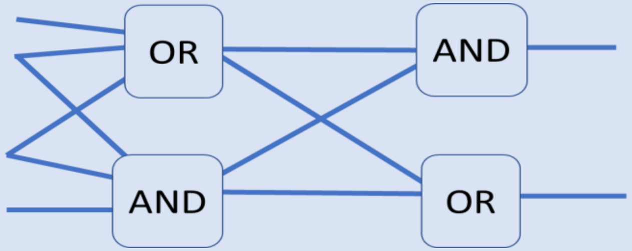
$$\mathrm{depth}(\mathcal{C}_N) \geq \frac{\log(N)}{8\log(K)}$$


Input
(instance on
N × N grid)
$A \in \{0,1\}^{|E|}$
$b \in \{0,1\}^{|V|}$
Random bits
(drawn from any
joint distribution)
$r \in \{0,1\}^{\ell}$
$\mathcal{C}_N$
Output
$z \in \{0,1\}^{|V|}$
Solution with
probability > 7/8
Circuit must have
depth $\Omega(\log(N))$

Proof Ideas

Locality in shallow classical circuits

Each output bit can only depend on $O(1)$ input bits.

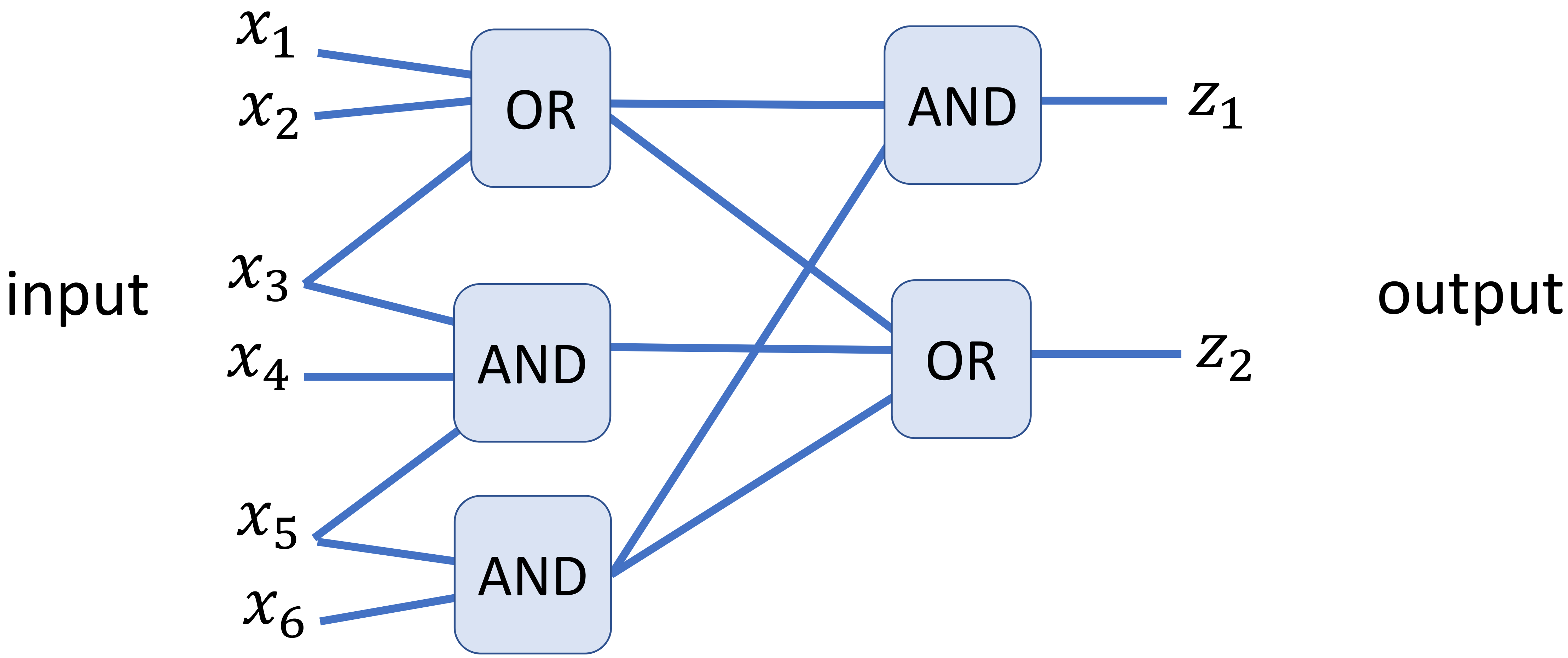


Vs.

Quantum nonlocality

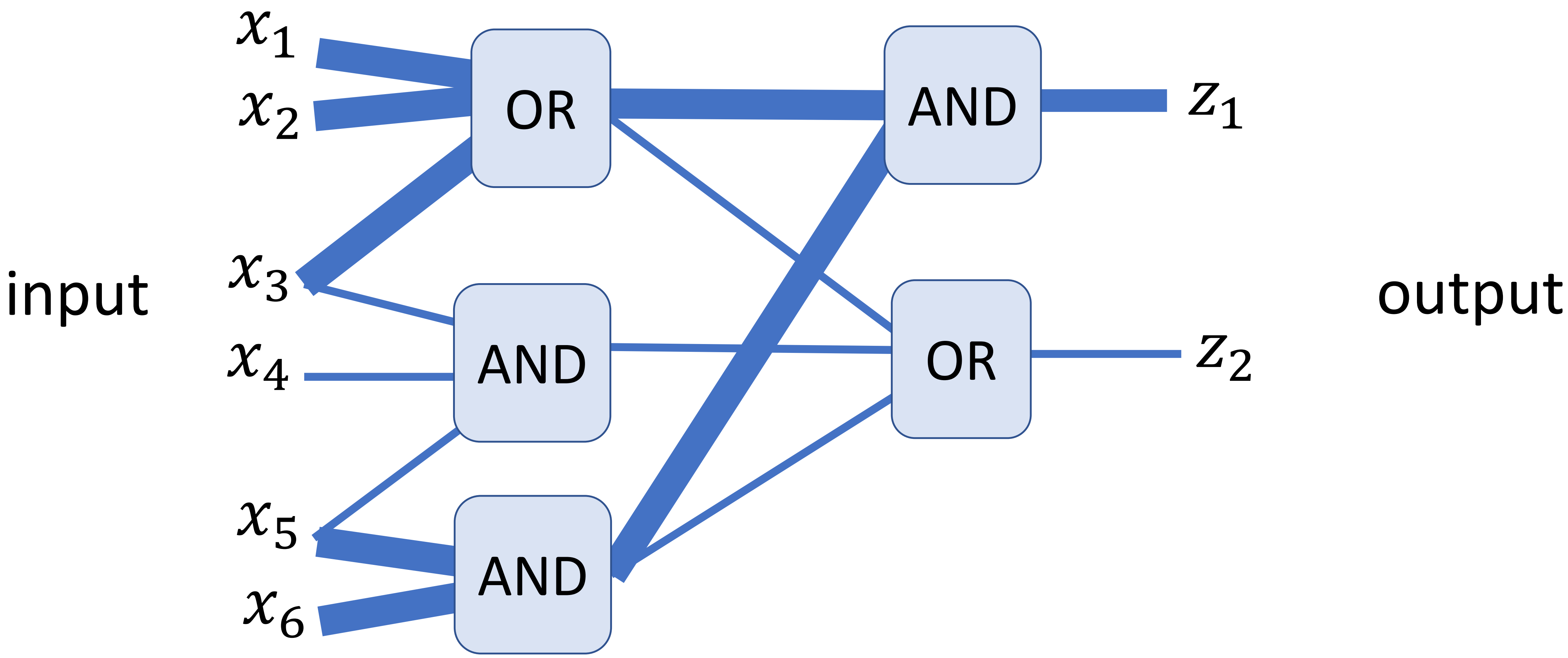
Measurement statistics of entangled quantum states cannot be reproduced by local hidden variable models

Locality in classical circuits



The **lightcone** $L(z_k)$ of an output bit z_k is the set of input bits x_i that are causally connected to z_k.

Locality in classical circuits



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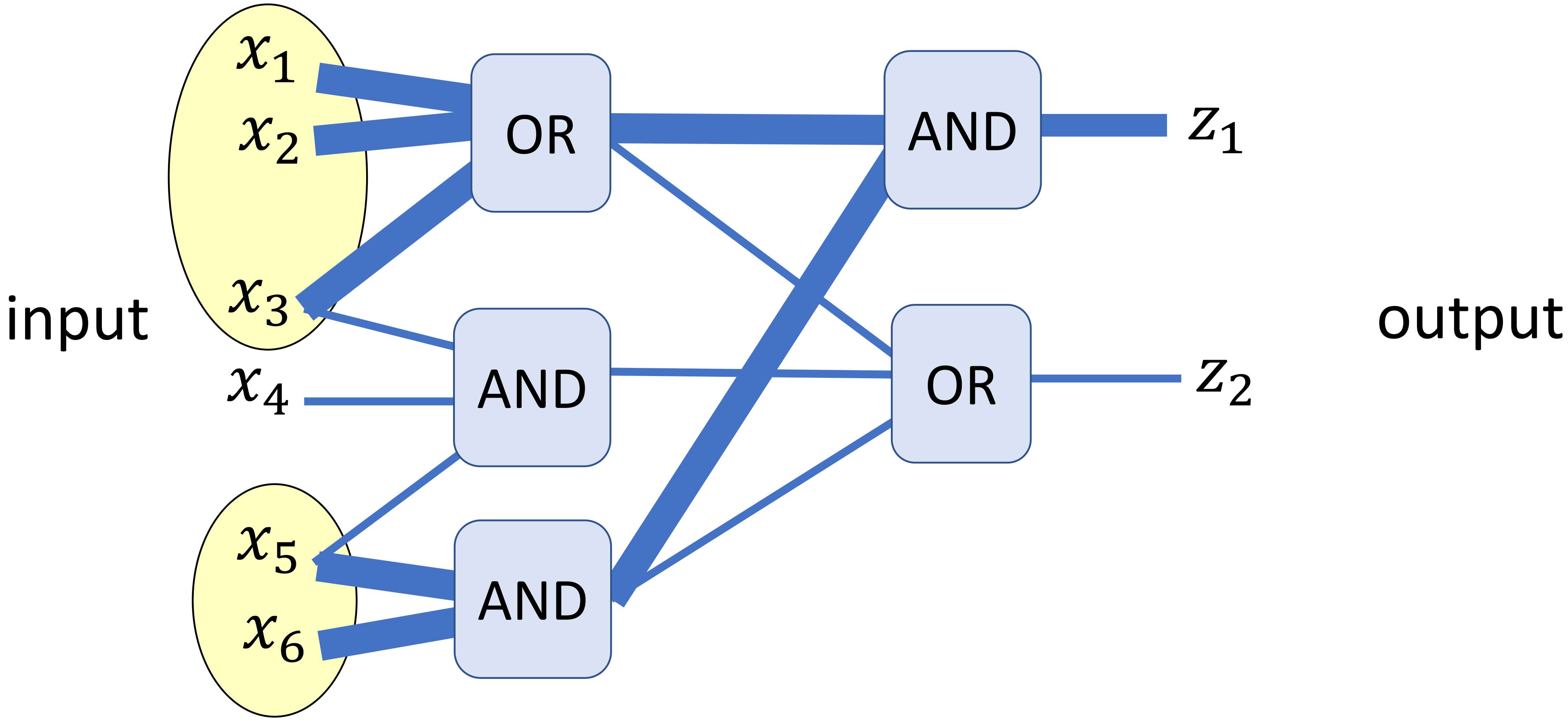

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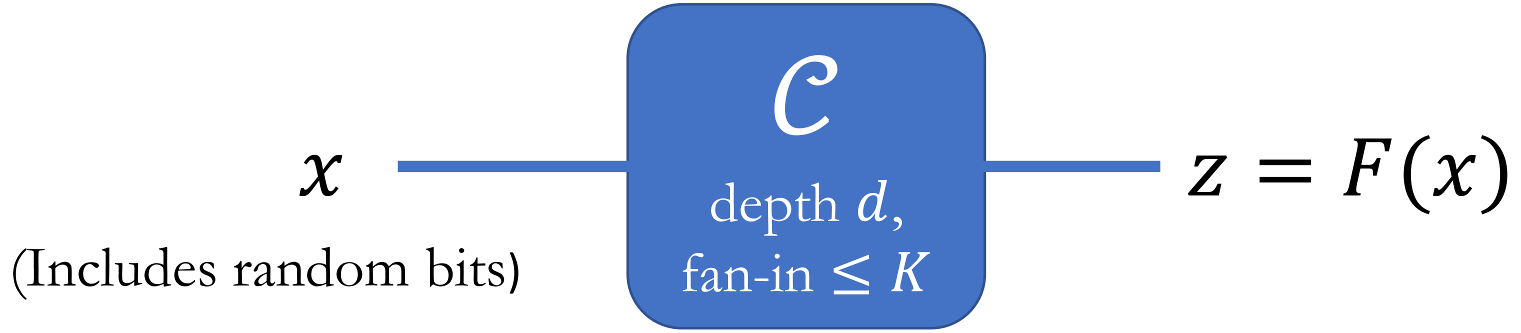

"Constant-depth locality": Lightcones of output bits have constant size

$$|L(z_k)| \leq K^d$$

We'll see that the 2D Hidden Linear Function problem cannot be solved by "constant-depth local" circuits. First consider simpler forms of locality…

Quantum nonlocality beats completely local circuits

[Greenburger et al. 1990][Mermin 1990]

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

satisfies:

$$\begin{cases} P|GHZ\rangle = |GHZ\rangle \\ P \in \{XXX, -XYY, -YXY, -YYX\} \end{cases}$$

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Choose bits b_1, b_2, b_3 and then measure each qubit of $|\mathrm{GHZ}\rangle$ in either the X basis (if $b_j = 0$) or the Y basis (if $b_j = 1$). Outcomes $z_j \in \{-1, +1\}$ satisfy:

$$i^{b_1+b_2+b_3} z_1 z_2 z_3 = 1 \quad \textbf{whenever} \quad b_1 \oplus b_2 \oplus b_3 = 0$$

"GHZ relation"

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"GHZ relation"

The GHZ relation cannot be satisfied by a completely local classical probabilistic circuit where each output bit z_j is correlated with at most one of the input bits b_k.

Quantum nonlocality beats geometrically local circuits

[Barrett et al. 2007]

Graph state on an M-cycle (M even): $|\Phi_M\rangle = \left(\prod_{j=1}^{M} CZ_{j,j+1}\right) H^{\otimes M} |0^M\rangle$

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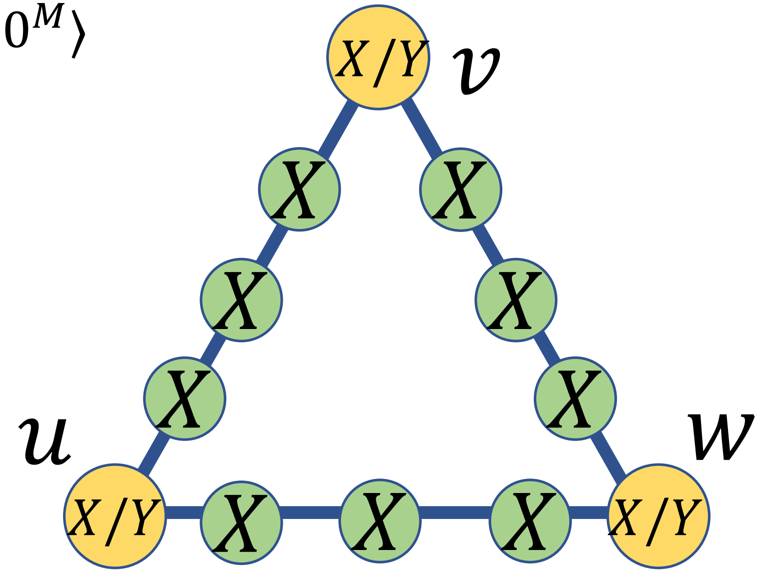

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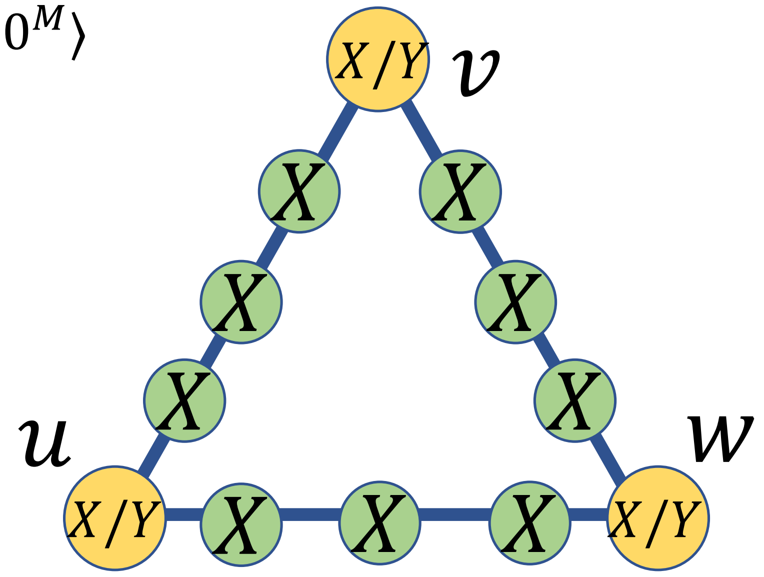

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$b_u, b_v, b_w \in \{0,1\}$

Measurement bases

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$z \in \{0,1\}^M$

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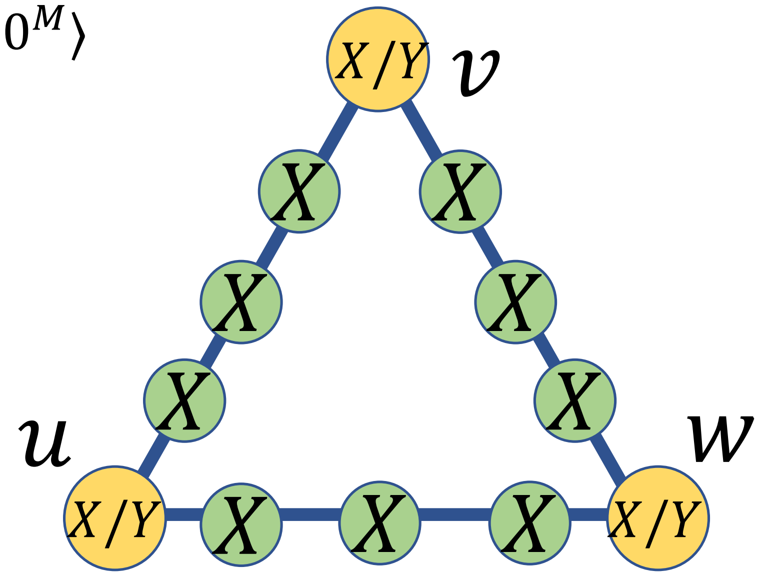

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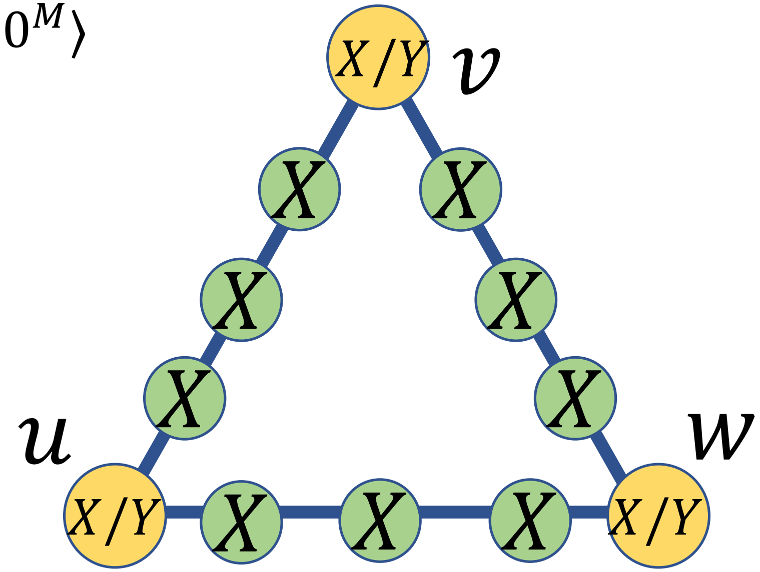

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Fact: Input/output satisfy a **"cycle relation"** $R(b_u, b_v, b_w, z) = 1$ similar to the GHZ relation.

Lemma: Suppose a classical circuit satisfies the cycle relation with probability $> 7/8$. Then some output bit z_k is correlated with a **distant** input bit b_u, b_v or b_w.
(this means it is not the nearest vertex of the triangle)

…How is this related to the 2D Hidden Linear Function Problem?

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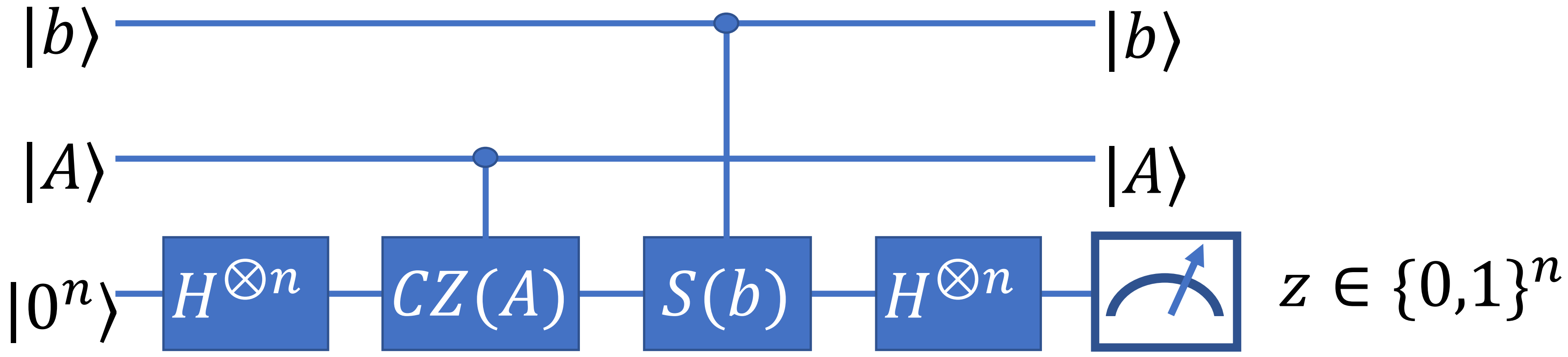

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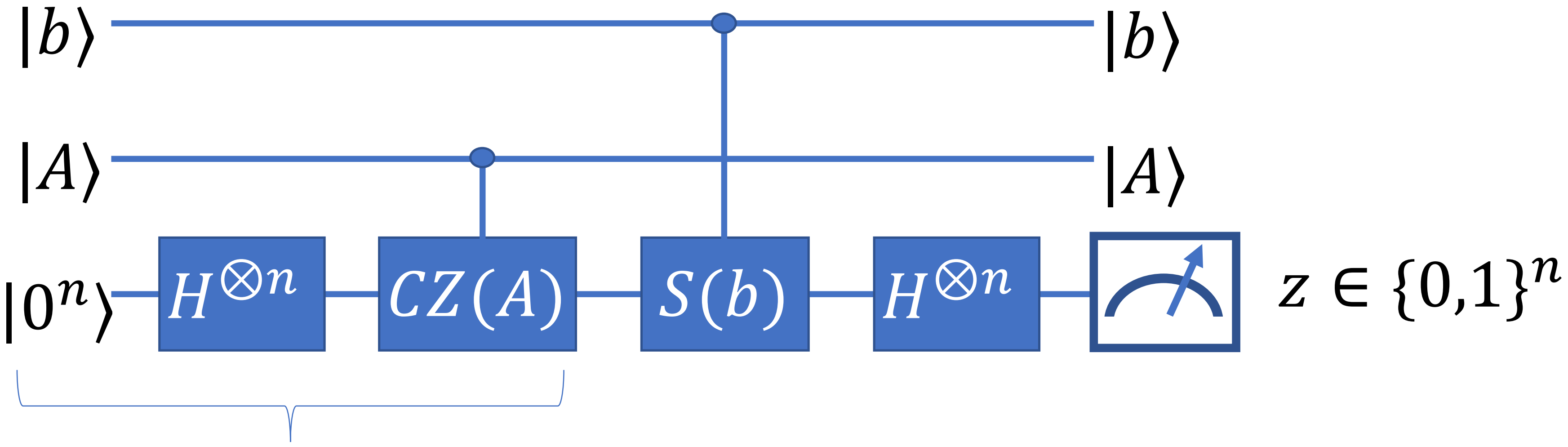

Prepare graph state for graph with adjacency matrix A

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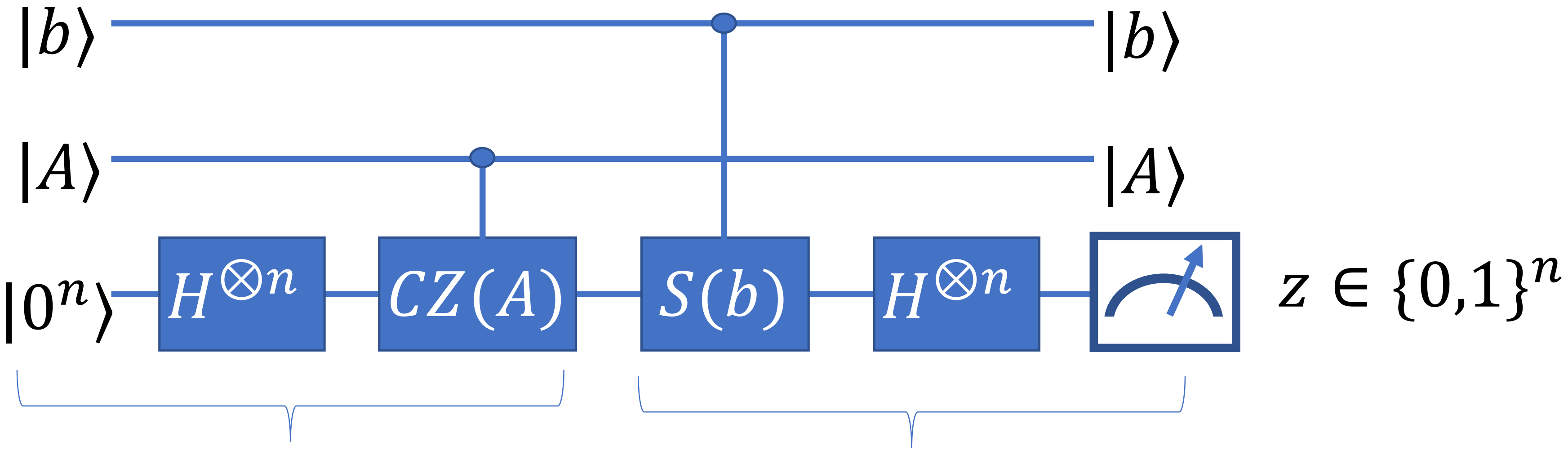

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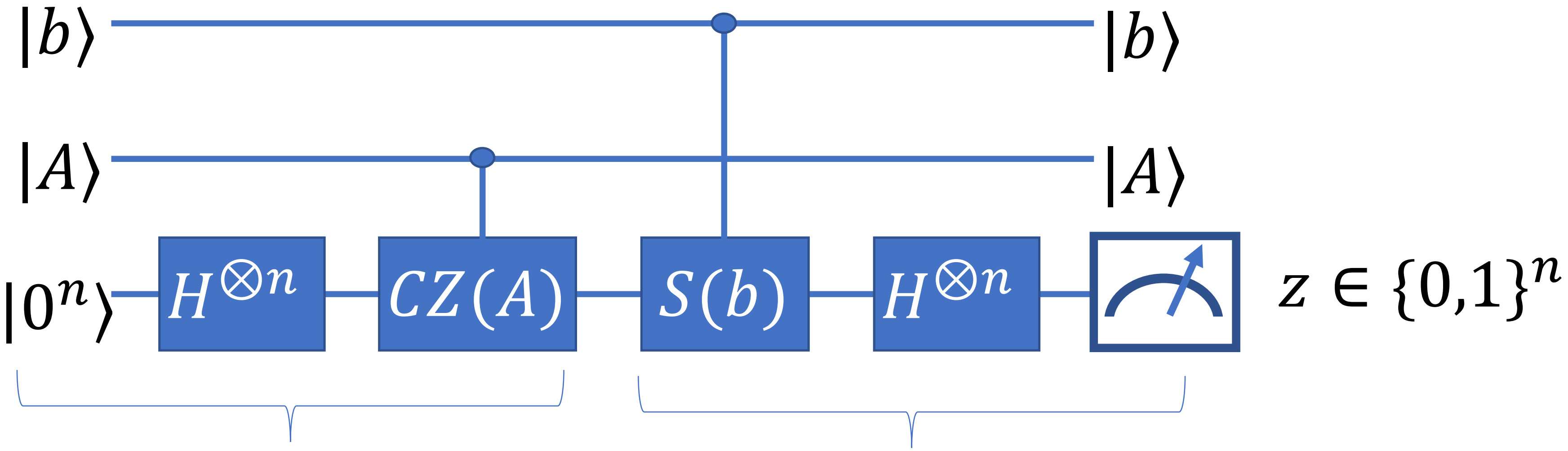

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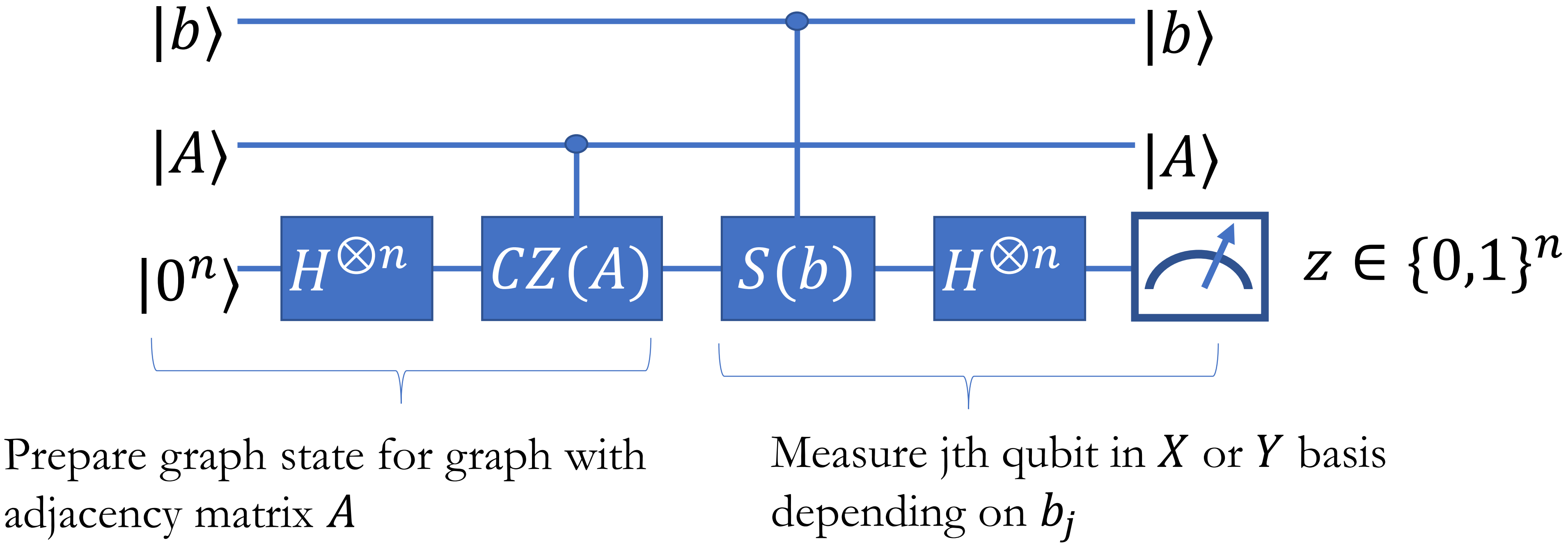

Choosing *A* to describe the adjacency matrix of a cycle and choosing *b* appropriately we infer (from Barrett et al.) a cycle relation satisfied by input/output.

A classical circuit which solves the 2D HLF problem must also satisfy all such cycle relations….

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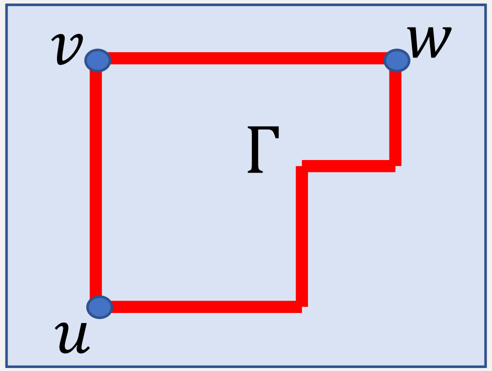
Lemma: Suppose a classical circuit has depth less than $\frac{\log(N)}{8\log(K)}$.

Then we can find 3 vertices u, v, w on the even sublattice of the $N \times N$ grid and a cycle Γ which passes through them, such that **input bits $\boldsymbol{b_u}, \boldsymbol{b_v}, \boldsymbol{b_w}$ are not correlated with any distant output bits on $\boldsymbol{\Gamma}$.**



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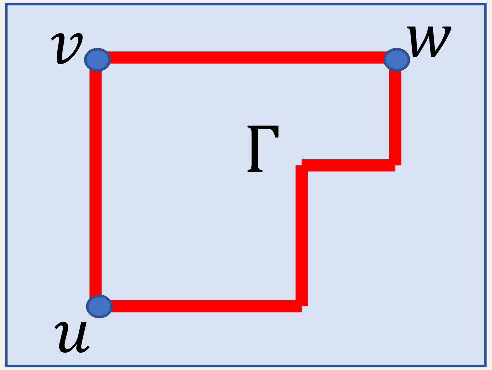

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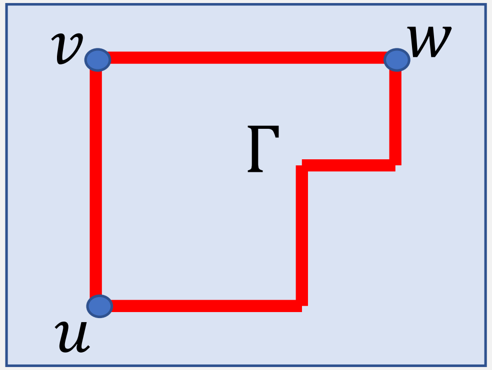

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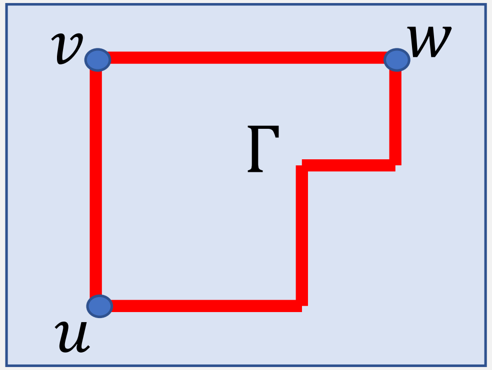

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This provides our lower bound on the depth of any classical circuit which solves the 2D HLF problem with probability greater than 7/8.

Open problems

Recursive HLF problems? The recursive version of Bernstein-Vazirani gives a superpolynomial speedup in query complexity.

Noisy constant-depth quantum circuits vs noiseless constant-depth classical circuits ?

Sampling problems? Can constant-depth quantum circuits sample from a distribution that can't be sampled by classical constant depth circuits? A recent characterization of distributions sampled by NC^0 circuits might be useful [Viola 2014].

Polynomial speed-up ? Constant-depth quantum algorithm solves the 2D HLF Problem in linear time. Best known classical algorithm takes time $O(n^2)$.