Wormholes and quantum teleportation

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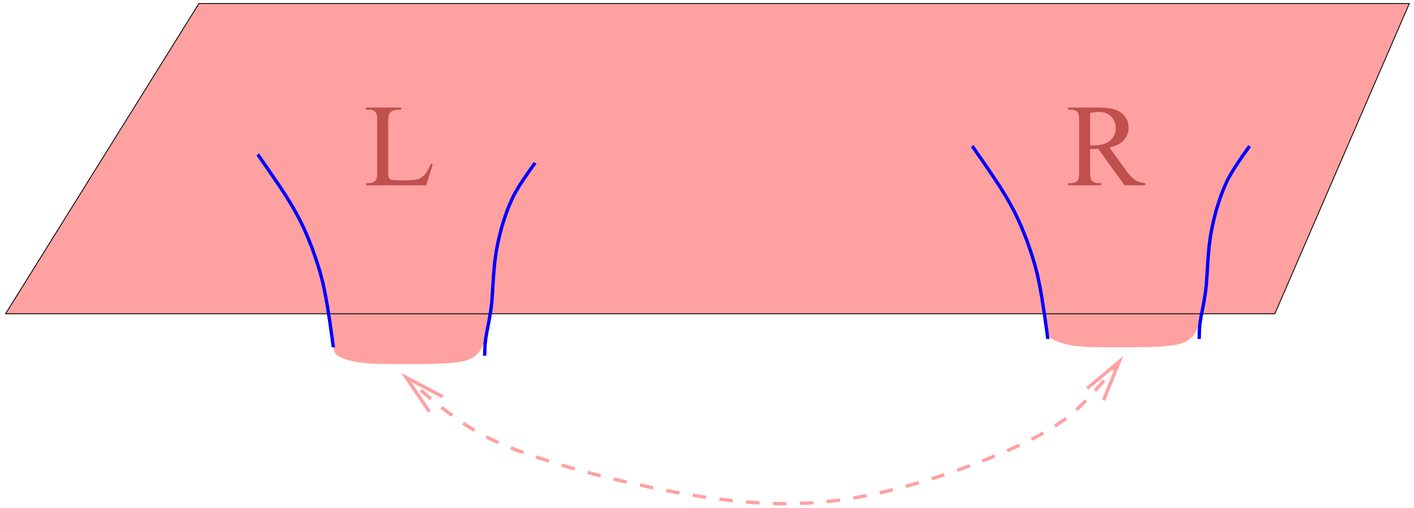
Based on - Gao, Jafferis and Wall
- J.M. , Stanford, Yang.

KITP, 2017

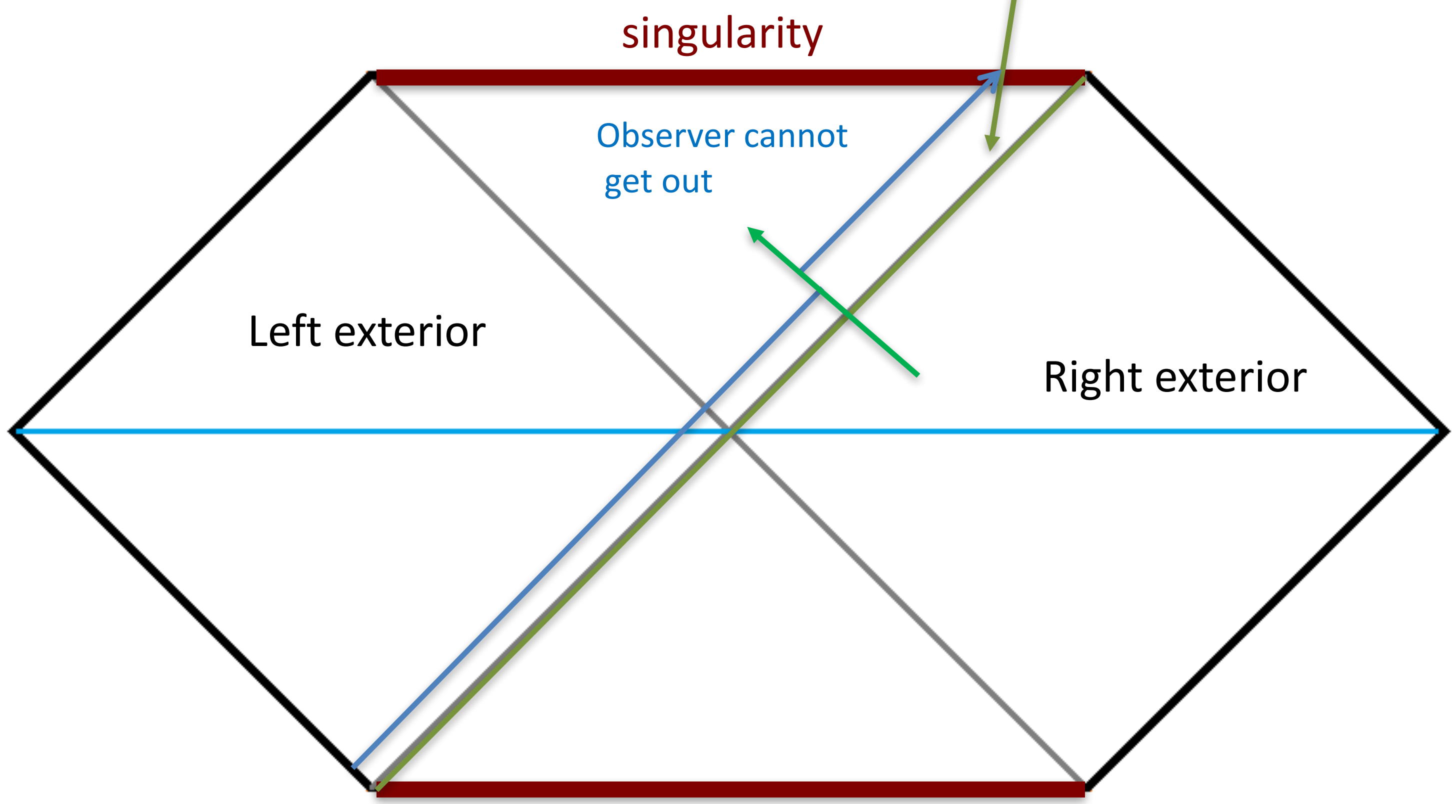
- General relativity has wormhole-like solutions.
- Simplest version is the maximally extended Schwarzschild solution.



These are not traversable. Even quantum mechanically → Integrated null energy condition

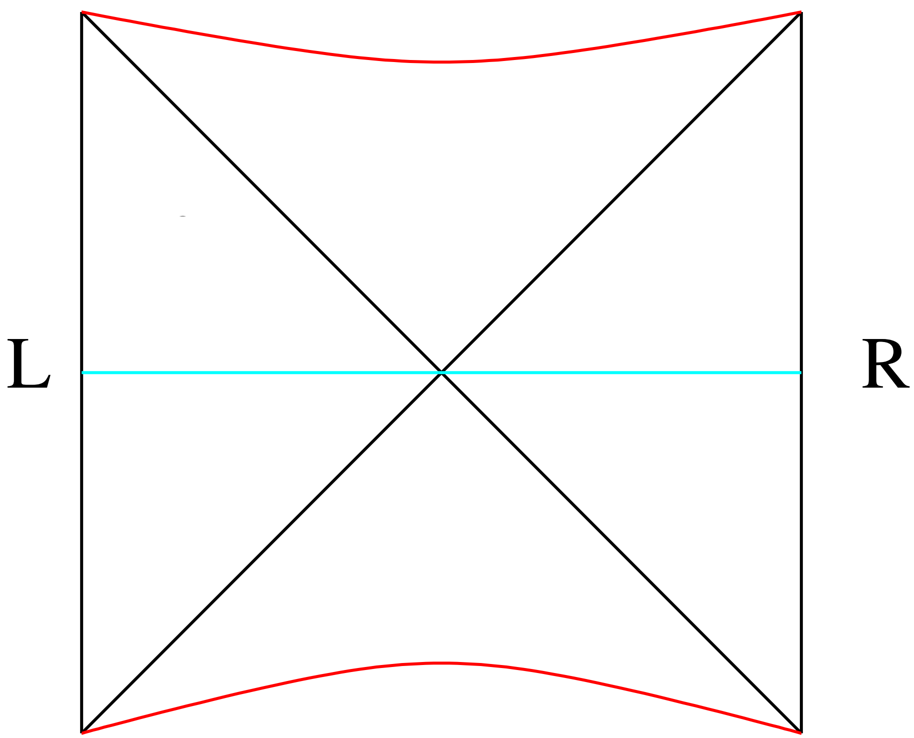
$$\int dx^+ T_{++} \geq 0$$

Faulkner, Leigh, Parrikar, Wang
Hartman, Kundu, Tajdini



- This is good, otherwise general relativity would lead to violations of the principle on which it is based: a maximum propagation speed for signals.
- We will talk about some special situations where it makes sense to talk about traversable wormholes. These do not violate any of the above principles. But they tell us interesting things about black holes.

Kruskal-Schwarzschild-AdS black hole



Geometric connection from entanglement

Entangled state in two non-interacting quantum systems.

Israel
JM

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R$$

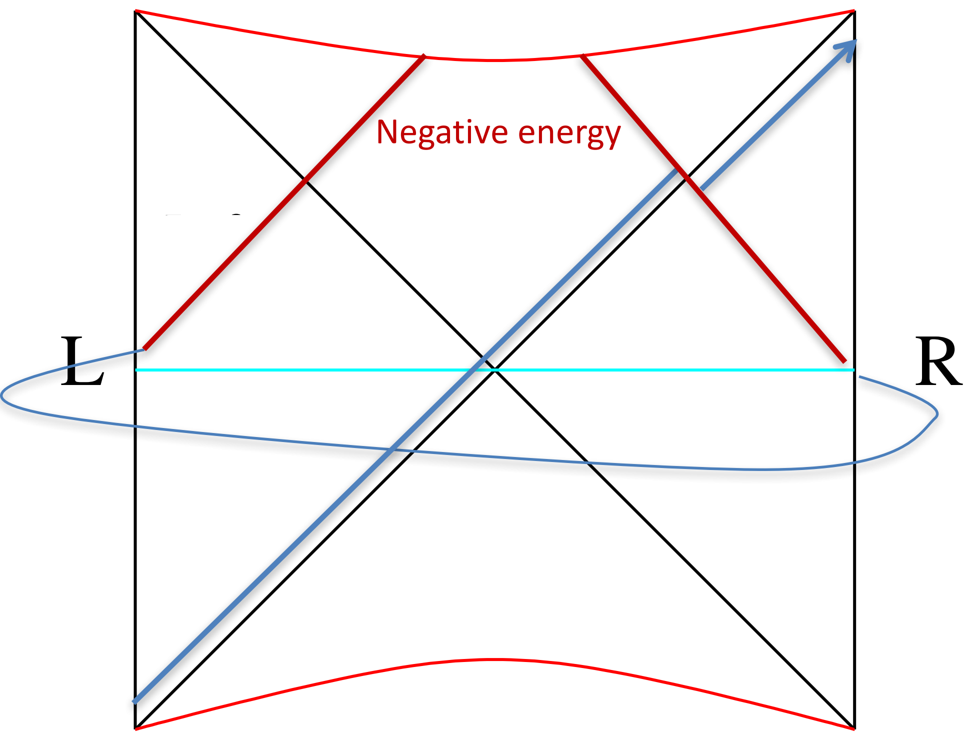
ER= EPR
JM Susskind

What if we coupled these two decoupled quantum systems ?

Traversable wormholes

Gao, Jafferis, Wall

(Susskind)



Couple the two quantum systems.
Direct interaction between fields near each boundary.

$$S_{int} = g\phi_L(0)\phi_R(0)$$

Can create negative energy in the interior →

Gravitational scattering pushes the particle through.

- We will study this phenomenon in more detail for the particular case of nearly-AdS_2 , where the effect is particularly simple
- We will show that the SYK quantum mechanical theory displays the same phenomenon.
- (Our interest is mainly in the construction of the black hole interior.)

Nearly AdS_2 = near extremal black holes

Keep the leading effects that perturb away from AdS_2

Jackiw Teitelboim
Almheiri Polchinski

$$\phi_0 \int d^2x\sqrt{g}R + \int d^2x\sqrt{g}\phi(R+2)$$

Ground state entropy

Comes from the area of the additional dimensions, if we are getting this from 4 d gravity for a near extremal black hole. General action for any situation with an AdS_2 region.

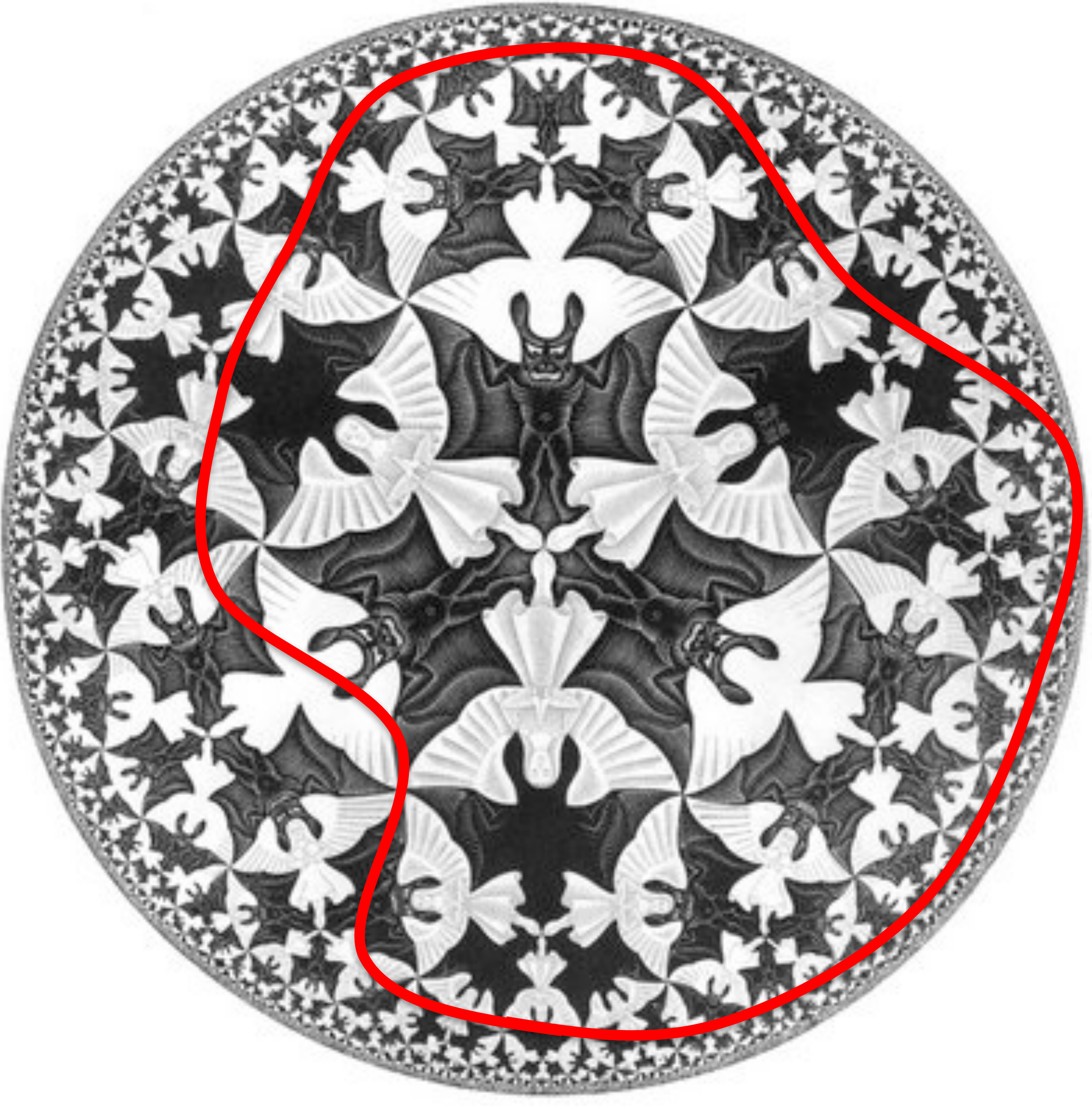
$$\int_{\mathrm{Bulk}} \sqrt{g}\phi(R+2) + \phi_b \int_{\mathrm{Bdy}} K + S_{\mathrm{matter}}[g,\chi]$$

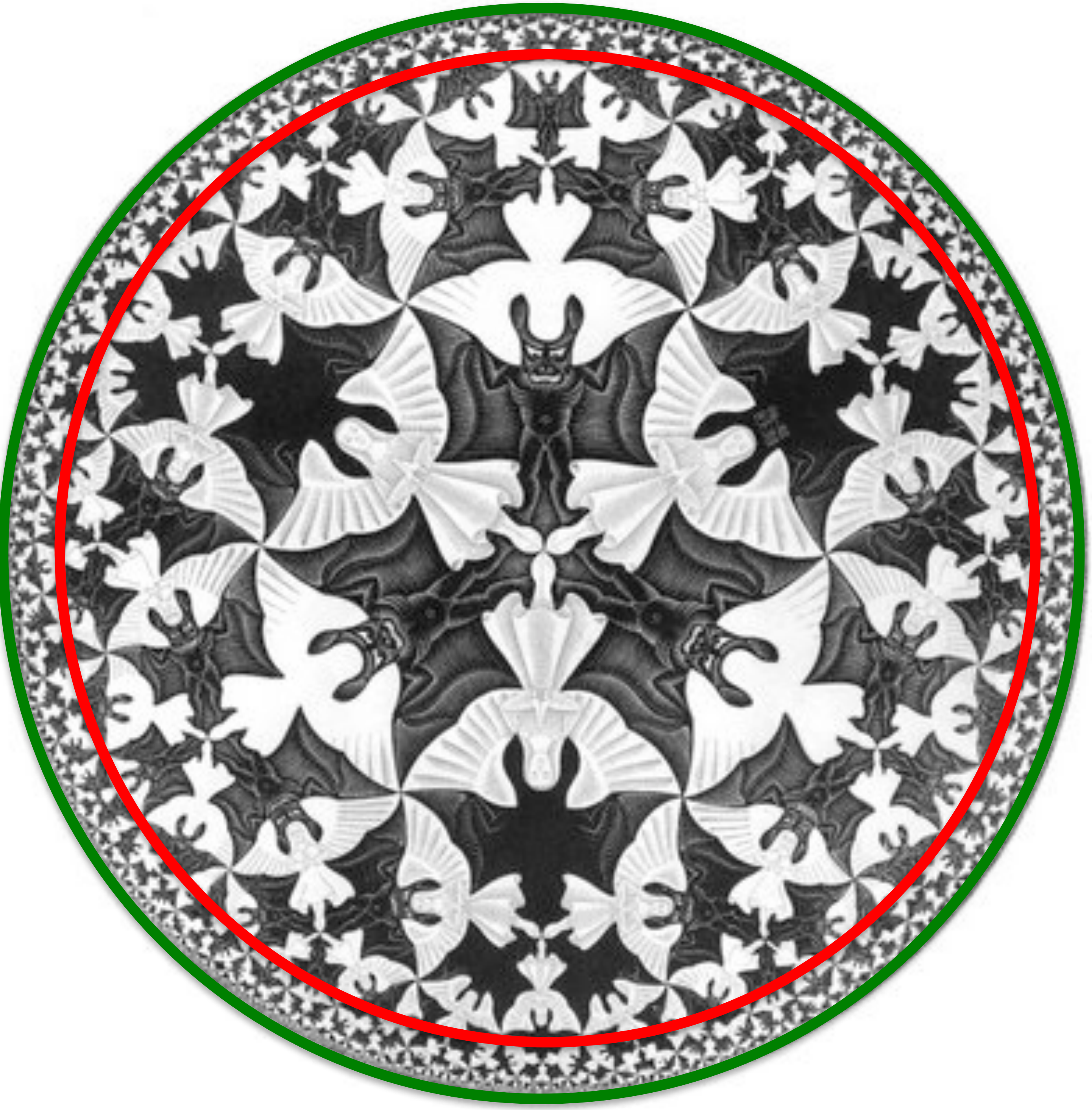
Equation of motion for ϕ → metric is AdS_2 . Rigid geometry !

Only dynamical information → location of the boundary.

$$\phi_b \int_{\mathrm{Bdy}} K$$

Local action on the boundary

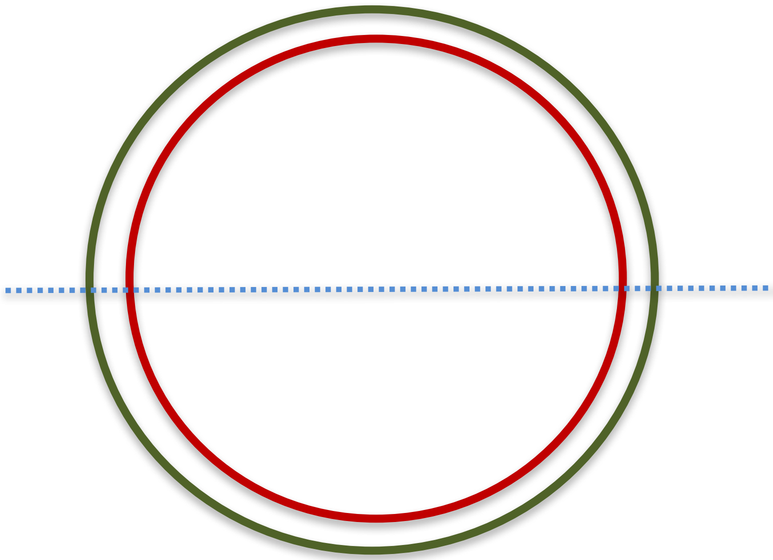




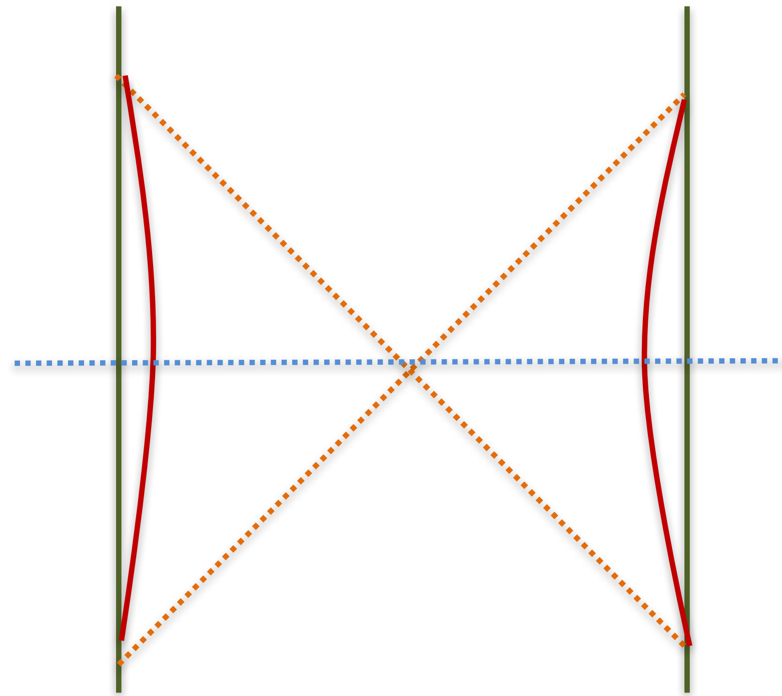
One solution

ADM mass → size

Rest of solutions
Related by AdS_2
isometries



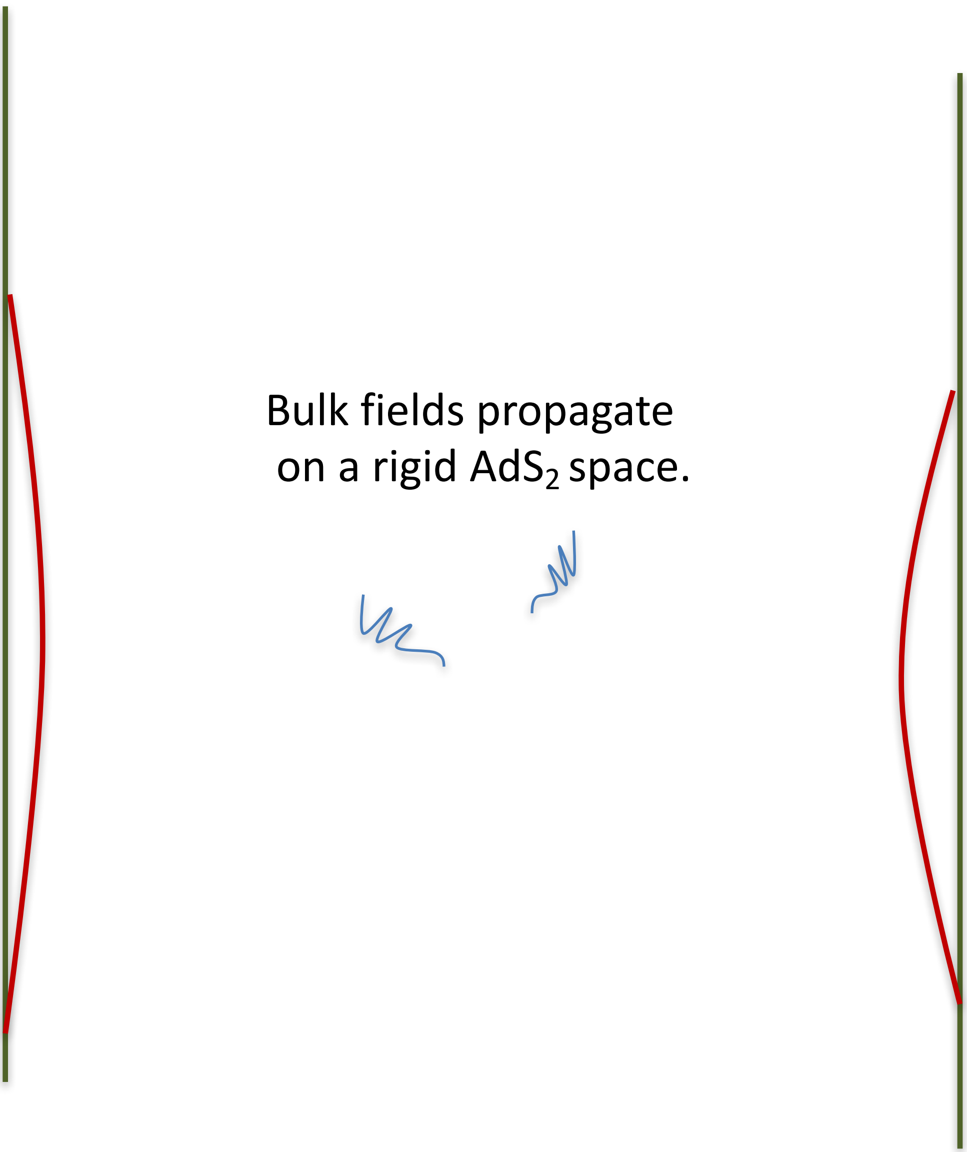
Euclidean black hole



Kruskal Schwarzschild AdS_2 wormhole

Very simple gravitational dynamics

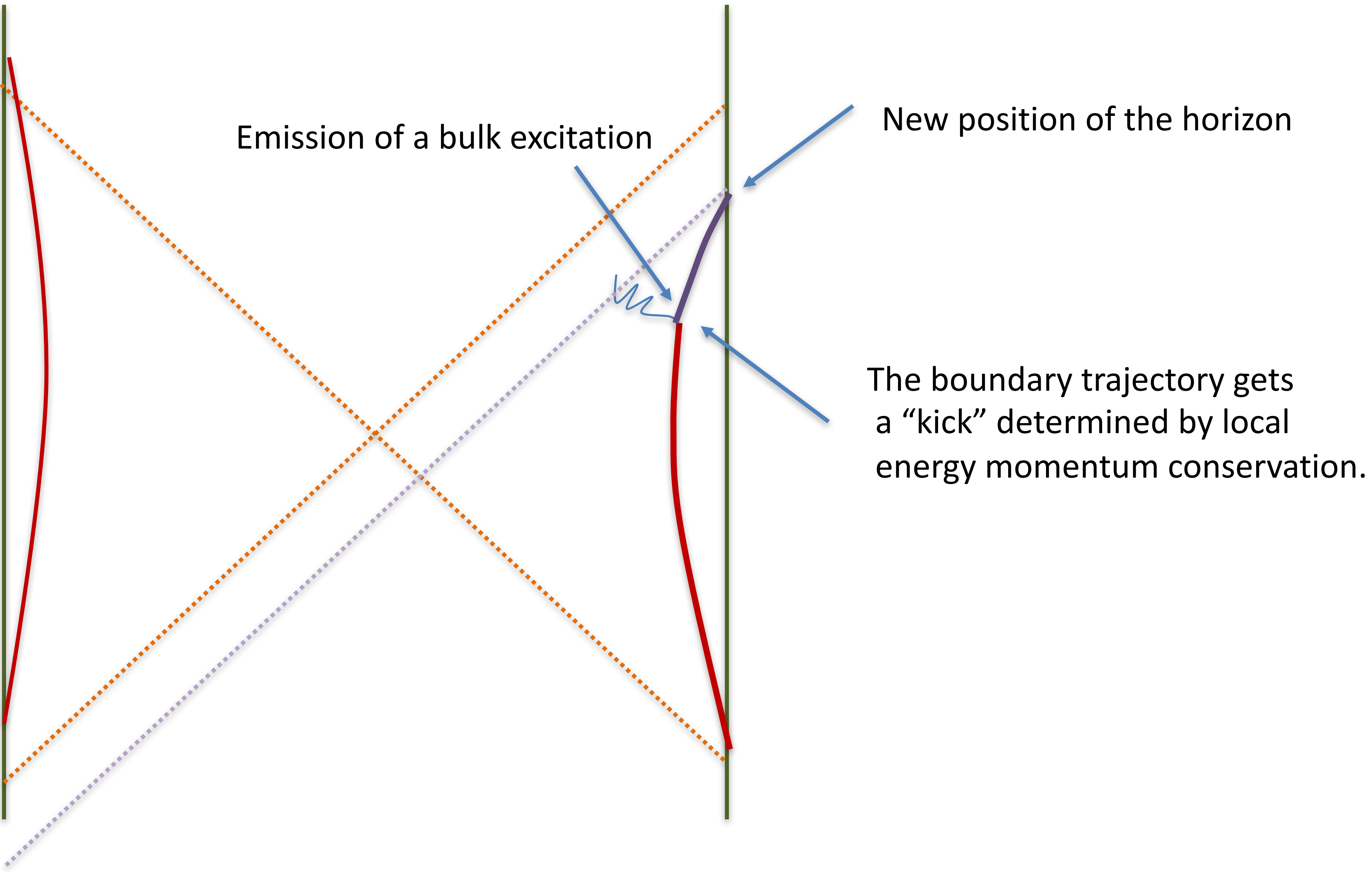
Dynamics



Bulk fields propagate on a rigid AdS_2 space.

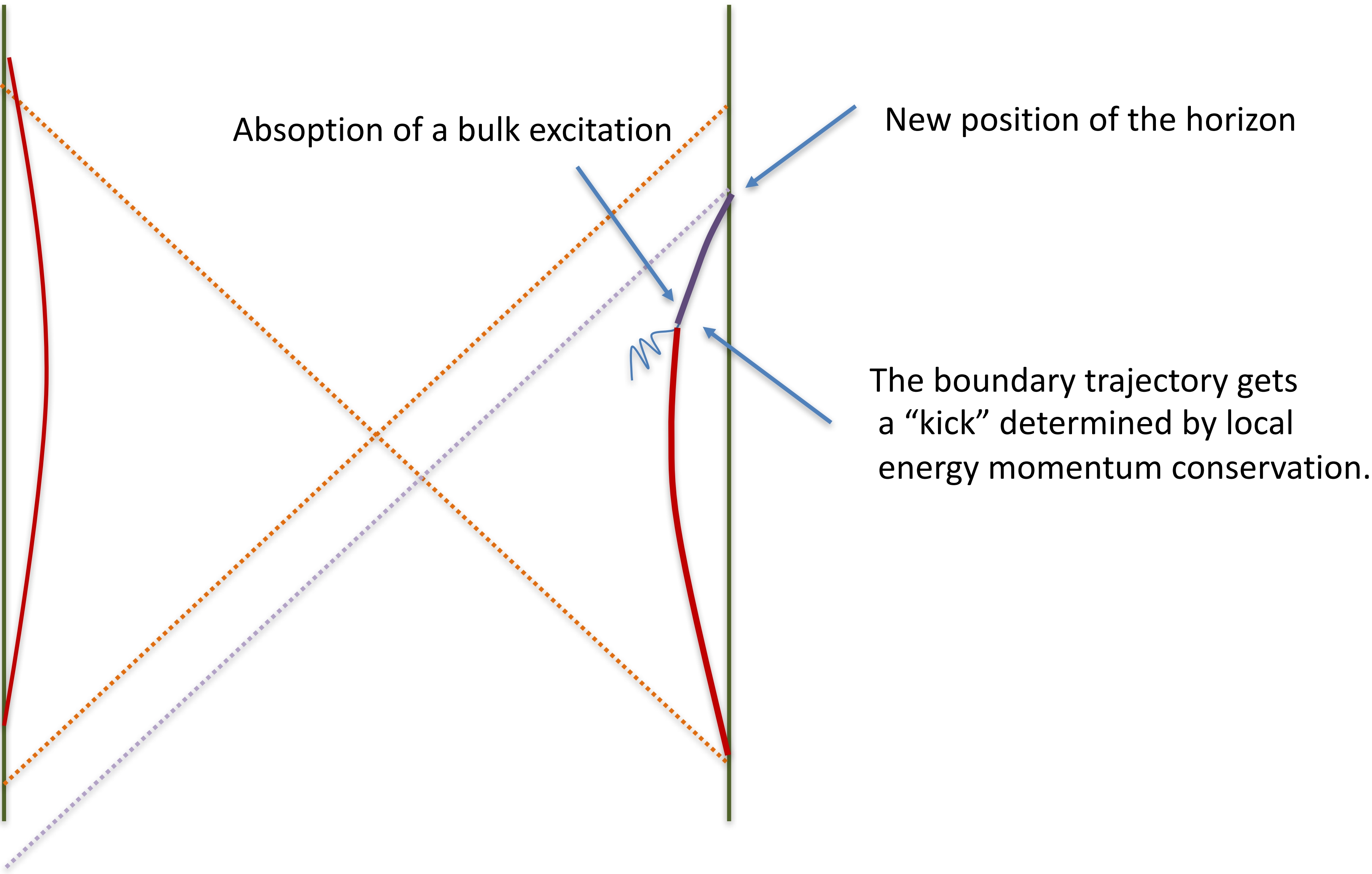
Boundaries also move in a rigid AdS_2 space, following local dynamical laws.

Dynamics



Emission of a bulk excitation
New position of the horizon
The boundary trajectory gets
a "kick" determined by local
energy momentum conservation.

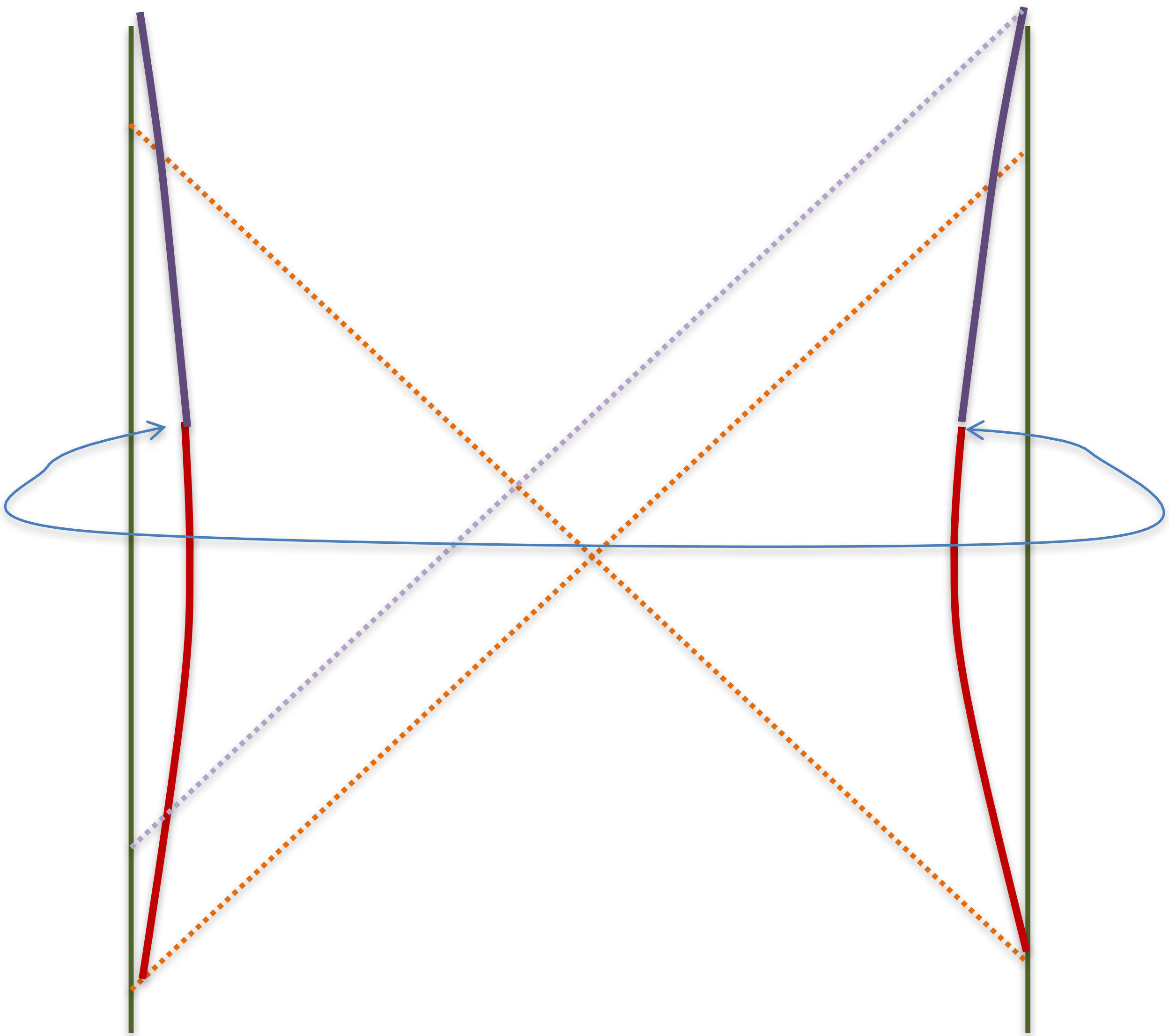
Dynamics



Absoption of a bulk excitation
New position of the horizon
The boundary trajectory gets
a “kick” determined by local
energy momentum conservation.

Interaction between the two boundaries

Gao Jafferis Wall



Insert this in the path integral

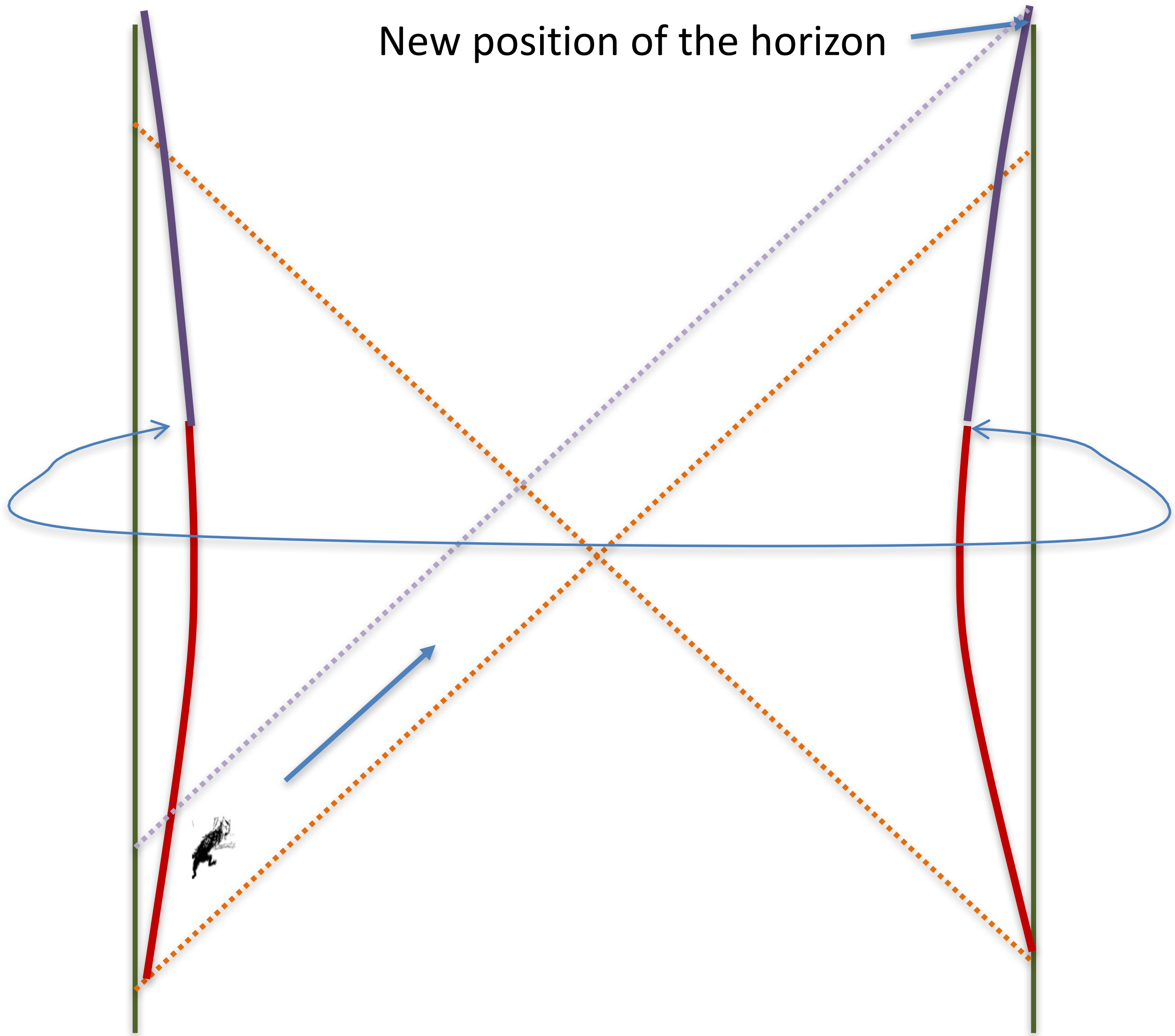
$$e^{ig\phi_L(t_L)\phi_R(t_R)}$$

approximate

$$e^{ig\langle\phi_L(t_L)\phi_R(t_R)\rangle}$$

Force between the two boundaries.
(Can be attractive for the right sign of g).
kicks the trajectories inwards

Interaction makes the wormhole traversable



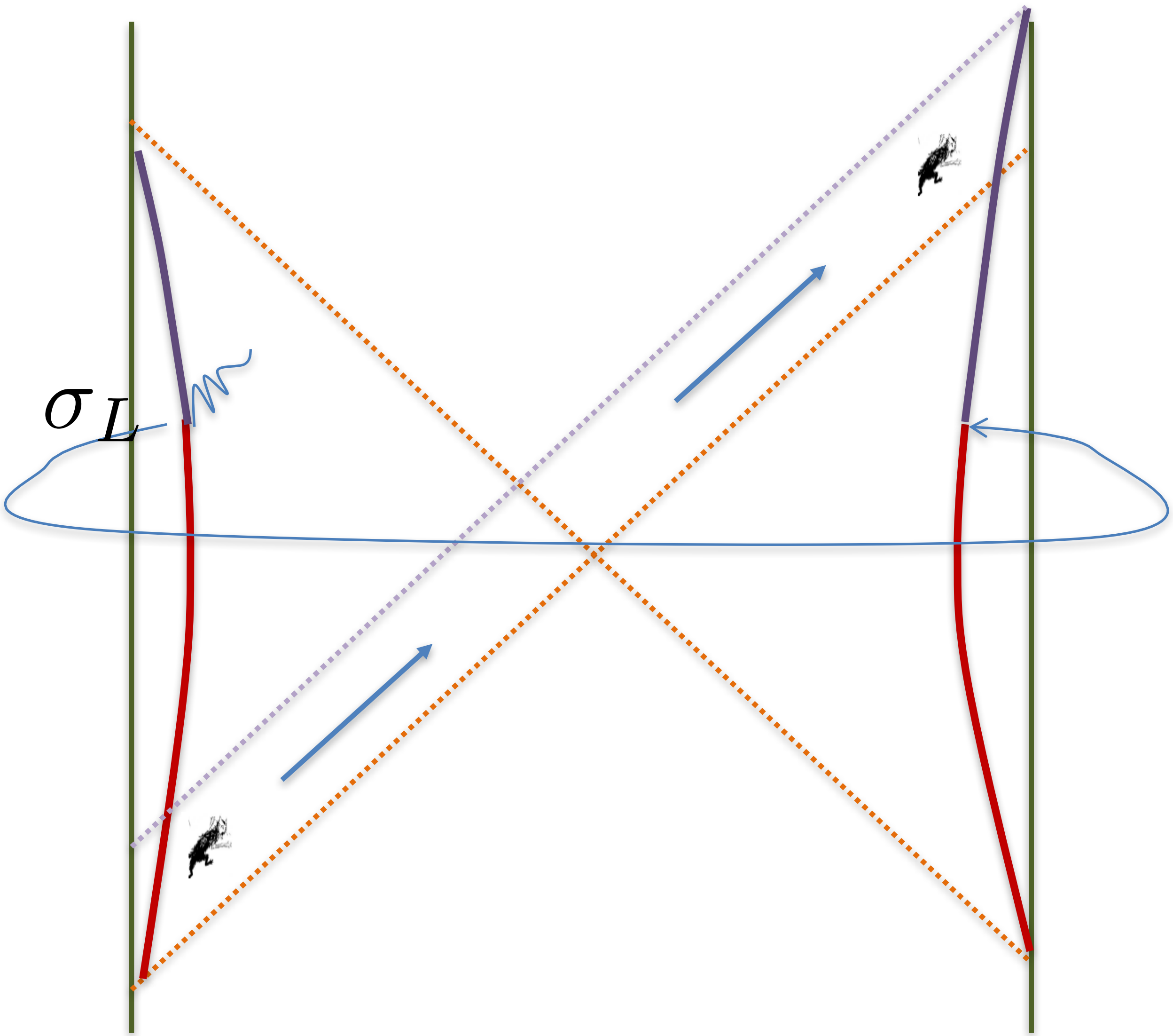
We can now send a signal
from the left to the right.

The wormhole has been
rendered traversable.

No contradiction because
we had a non-local interaction
between the two boundaries.

What does this take ? .

Doing a measurement



Measure

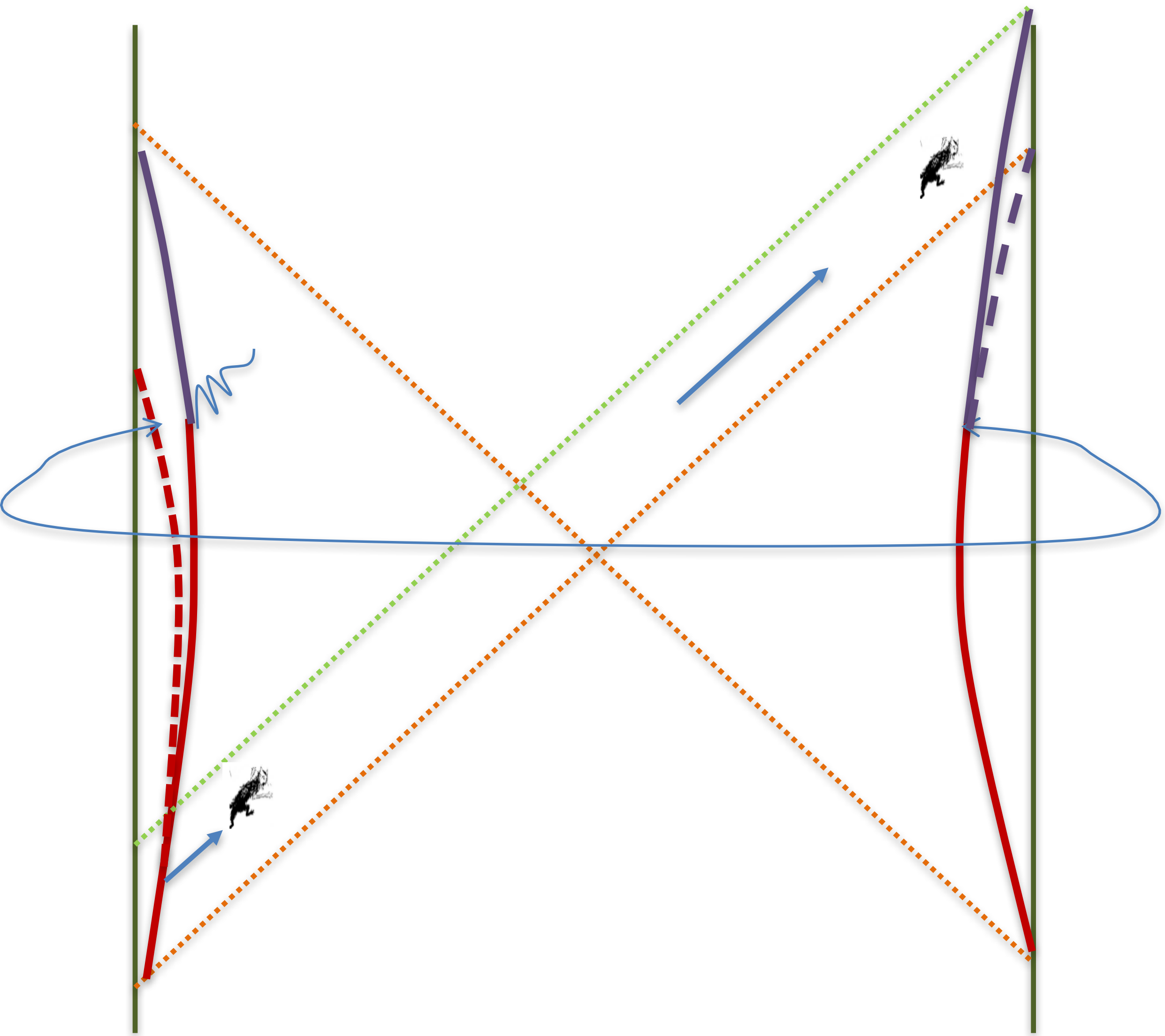
$$\phi_L \longrightarrow \sigma_L$$

Act on the right with

$$e^{ig\sigma_L \phi_R(t_R)}$$

From the point of view of the right we get the same, whether we measure or not.

What if we want to send too much information ?



The insertion of the message also gives a small kick to the trajectory.

Moves the insertion points of the non-local operator away from each other

$\langle \phi_L(t_L)\phi_R(t_R)\rangle$ becomes smaller

Attractive force weakens → no opening of the wormhole.

Precise formula for the 2pt function

$$C = \langle e^{-igV} \chi_R(t) e^{igV} \chi_L(-t) \rangle \ , \qquad V = \phi_L(0)\phi_R(0)$$

- Fourier transform the signals we want to send.
- Correlator of V is evaluated on a background with momentum p → p dependent SL(2) transformation on V .
- Effect is amplified by boosts, or chaos e^t
- Get an extra phase from $\langle V \rangle$

$$C \sim \int dp (p)^{2\Delta - 1} e^{ip} e^{-ig} e^{i \frac{g}{(1+pe^t)^{2\Delta}}}$$

Amount of information we can send is roughly g

$$\langle V \rangle = 1 \ , \quad \langle \phi_L^2(0) \rangle \sim 1$$

Simplified limit

$$C = \langle e^{-igV} \chi_R(t) e^{igV} \chi_L(-t) \rangle \ , \qquad V = \phi_L(0)\phi_R(0)$$

$$C \sim \int dp (p)^{2\Delta-1} e^{ip} e^{-ig} e^{i \frac{g}{(1+G_N p e^t)^{2\Delta}}}$$

$$C \sim \int dp (p)^{2\Delta-1} e^{ip} e^{-i(g 2\Delta G_N e^t) p}$$

Just a simple ``translation'' or one of the operations of SL(2).

The signal does not ``feel'' anything ! Composite objects are simply translated whole !

Shock waves in two dimensions are not felt.

Quantum mechanical model

The SYK model

N Majorana fermions $\{\psi_i, \psi_j\} = \delta_{ij}$

Sachdev Ye Kitaev
Georges, Parcollet

$$H = \sum_{i_1, \cdots, i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

Random couplings, gaussian distribution. $\langle J^2_{i_1 i_2 i_3 i_4} \rangle = J^2/N^3$

To leading order → treat J_{ijkl} as an additional field

J = dimensionful coupling. We will be interested in the strong coupling region

$$1 \ll \beta J, \ \tau J \ll N$$

Define new variable

$$G(\tau,\tau') = \frac{1}{N}\sum_i \langle \psi_i(\tau)\psi_i(\tau')\rangle$$

Integrate out fermions and get an action in terms of a new field G

$$S = N f[G(\tau,\tau')]$$

Is analogous to the full bulk gravity + matter action.

There is a particular G that minimizes the action. It is SL(2) invariant.
(analogous to the vacuum AdS geometry)

Set of low action fluctuations of this solution. Parametrized by a function of a single variable.
Reparametrization mode.

$$G_f = (f'(\tau)f'(\tau'))^{\Delta} G(f(\tau), f(\tau'))$$

Low energy action:

$$S = \frac{N}{J}\int \{f,\tau\} d\tau$$

Same as the action for the UV boundary in the gravity description.

$$H = \sum_{i_1, \cdots, i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

Low energies.

$$S = \frac{N}{J} \int \{f, \tau\} d\tau$$

Same as the action for the UV boundary in the gravity description.

- All that we said before in the wormhole context depended only on the motion of the UV boundary and the propagation in an AdS_2 bulk.
- We got the same action for the boundary. The other modes of G → conformal invariant which lead to correlators as in AdS_2

Same precise formula for the 2pt function

$$C = \langle e^{-igV} \chi_R(t) e^{igV} \chi_L(-t) \rangle \ , \qquad V = g\phi_L(0)\phi_R(0)$$

$$C = \langle e^{-igV} \psi_{j\,R}(t) e^{igV} \psi_{j\,L}(-t) \rangle \ , \qquad V = g \frac{1}{K} \sum_{j=1}^{K} \psi_L(0)\psi_R(0)$$

$$C \sim \int dp (p)^{2\Delta - 1} e^{ip} e^{-ig} e^{i \frac{g}{(1+pe^{t})^{2\Delta}}}$$

Amount of information we can send is roughly g

Conclusions

- Simple picture for the gravitational dynamics of nearly-AdS_2.
- Traversability has a simple origin. Nothing special is felt by the traveler.
- Quantum teleportation = going through the wormhole.
- Same description in quantum mechanics (SYK) and gravity.

Question

- What is this telling us about the interior ?