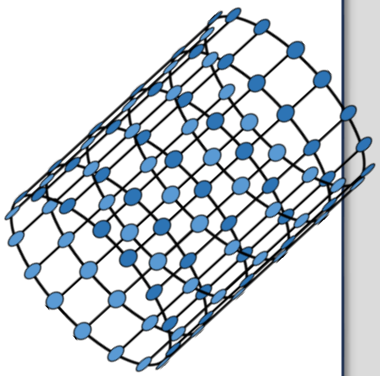
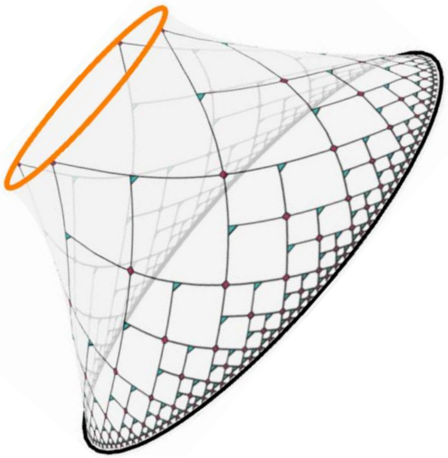
KITP – Santa Barbara October 9th 2017

Frontiers in Quantum Information Physics



Tensor networks as path integral geometry

$$Z = \int D\varphi e^{-S_E[\varphi]}$$



Guifre Vidal

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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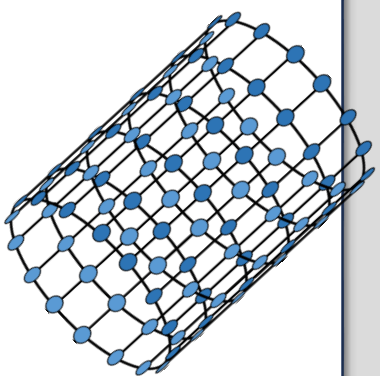
TENSOR NETWORKS INITIATIVE

SIMONS FOUNDATION

NSERC

KITP – Santa Barbara October 9th 2017

Frontiers in Quantum Information Physics



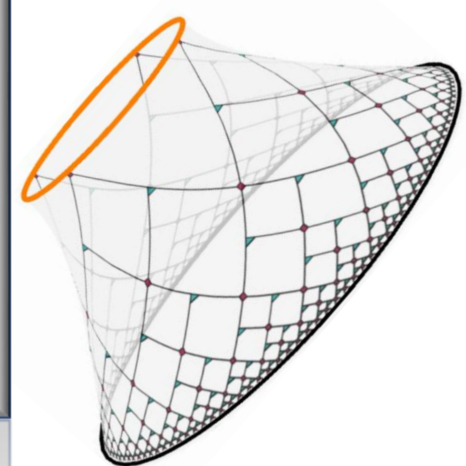
Ten rks

path in metry

Ash Milsted

Perimeter Institute

$$Z = \int D\varphi e^{-S_E[\varphi]}$$



joint work
(*in preparation*)

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TENSOR NETWORKS INITIATIVE

SIMONS FOUNDATION

NSERC

Motivation

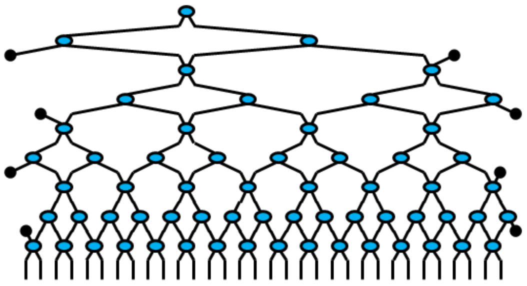
tensor network formalism:

sparse (efficient) representation and manipulation of

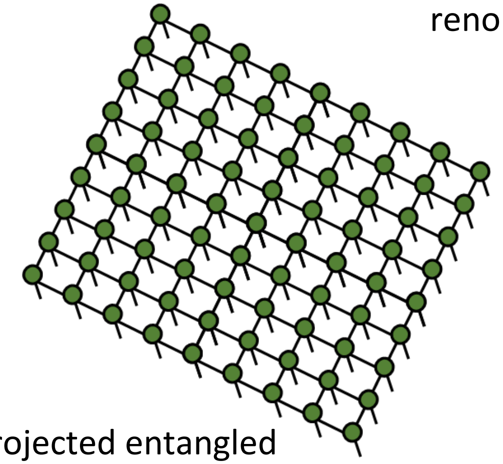
$$|\Psi\rangle \quad =$$

many-body wavefunctions

matrix product state
MPS



multi-scale entanglement renormalization ansatz
MERA



projected entangled pair state
PEPS

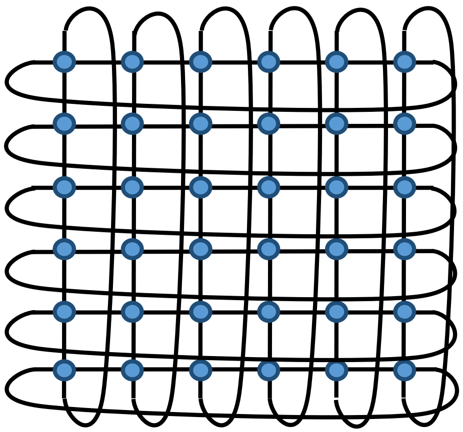
Motivation

tensor network formalism:

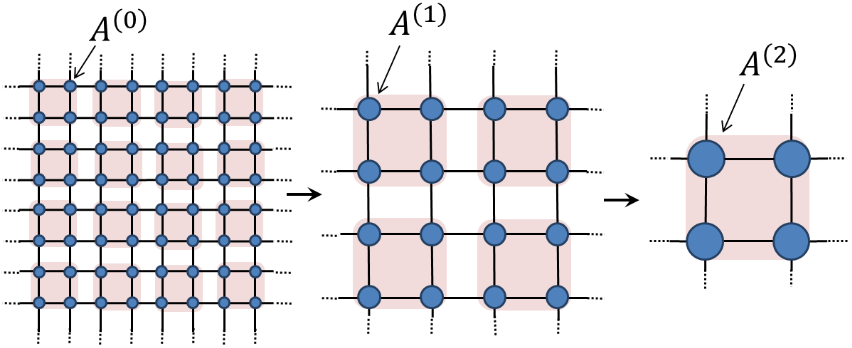
sparse (efficient) representation and manipulation of

$$Z =$$

Euclidean path integral
or
partition function

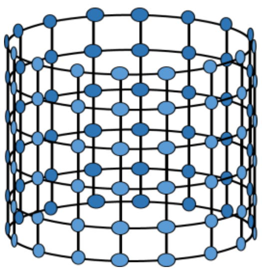


RG flow



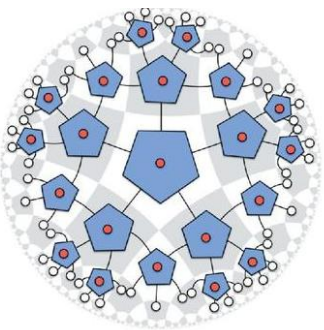
Motivation

tensor network $\sim$ geometry

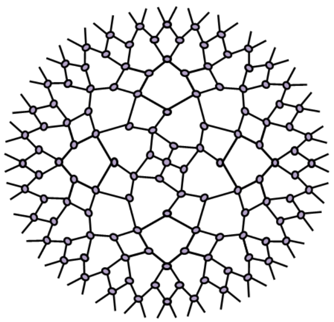


2d partition
function on cylinder

flat space
(flat cylinder)



HaPPY code



MERA

hyperbolic space
(Poincare disk)

Motivation

tensor network $\sim$ geometry

so what?

- geometry contains structural information

$\Psi_{i_1 i_2 \cdots i_N}$

2^N complex coefficients

$|\Psi\rangle$

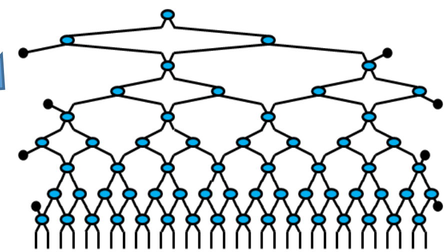
N spins

$C(l) \sim e^{-l/\xi}$

exponential correlations

$S(l) \sim const$

constant entanglement entropy



$C(l) \sim \frac{1}{l^p}$

power-law correlations

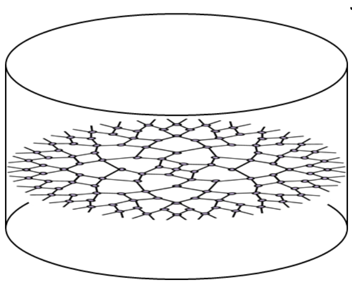
$S(l) \sim \log l$

logarithmic entanglement entropy

- holography AdS_3/CFT_2

MERA = time slice of AdS_3

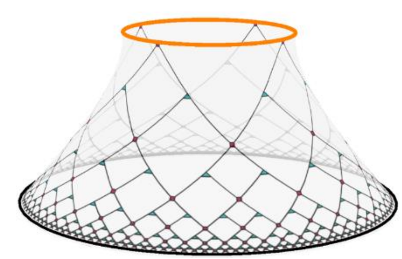
Swingle 2009, 2012



(hyperbolic Disk)

MERA = kinematic space (integral transform)

Czech, Lamprou, McCandlish, Sully, 2015-2016



(de Sitter dS_2)

Outline

tensor networks as …

- geometry

- path integral geometry

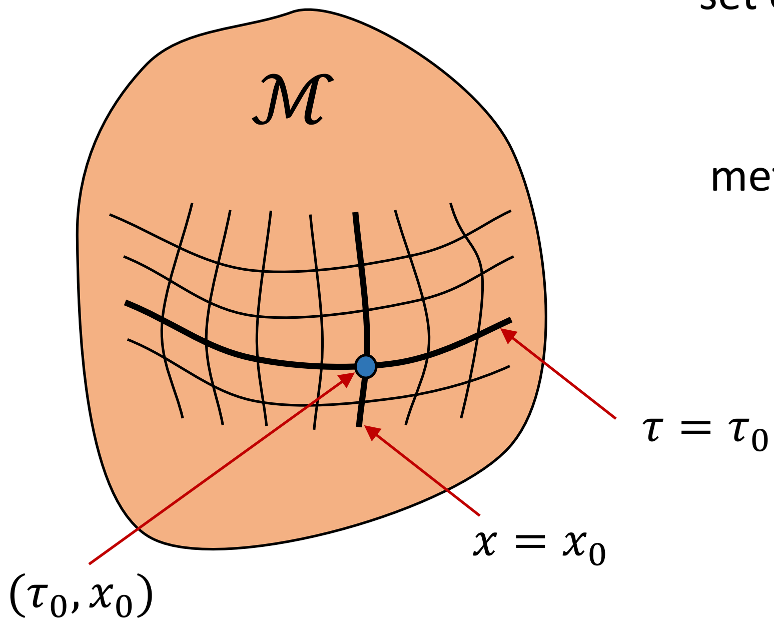
Outline

tensor networks as …

- geometry
- path integral geometry

Manifold

2d manifold $\mathcal{M}$ = Euclidean spacetime



set of coordinates

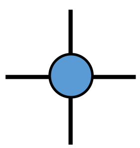
(τ, x)

metric

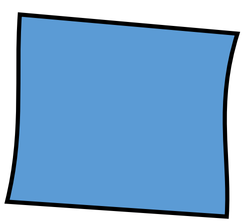
$g_{\mu\nu}(\tau, x)$

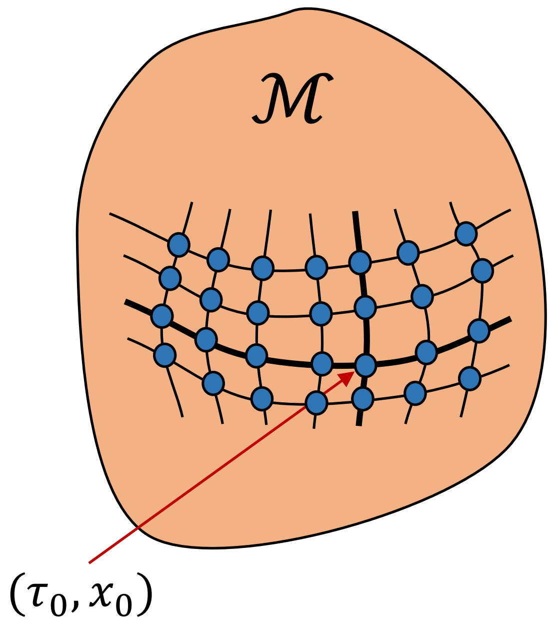
Manifold -> Discretization

2d manifold $\mathcal{M}$ = Euclidean spacetime



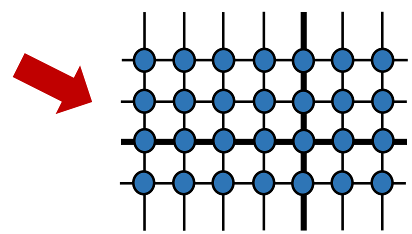
tensor = patch of spacetime





set of coordinates (τ, x)

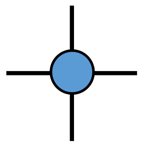
metric $g_{\mu\nu}(\tau, x)$



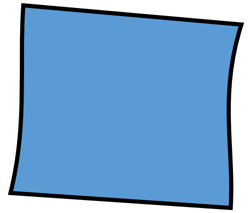
if each tensor is equivalent...

then we lost the metric!

Manifold -> Discretization -> connectivity

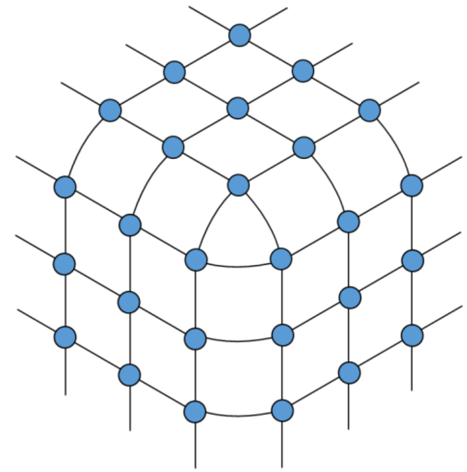


=



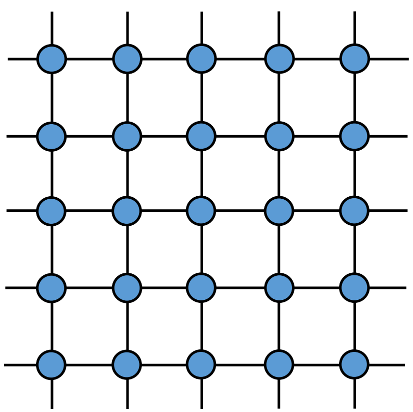
tensor

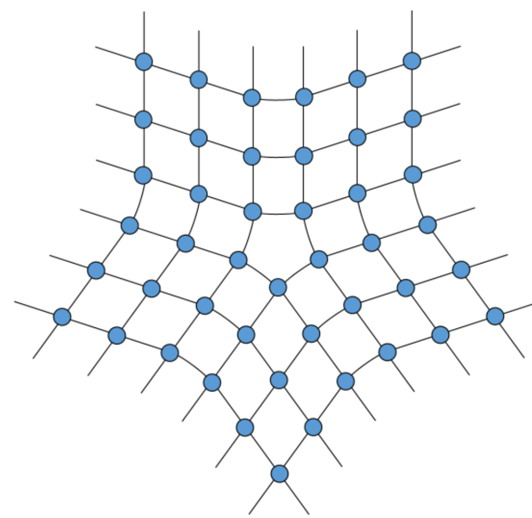
patch of flat spacetime



deficit angle

flat spacetime → curved spacetime



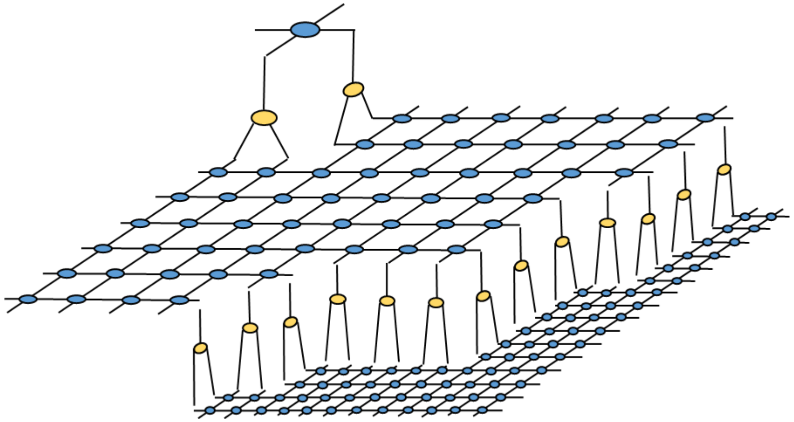


excess angle

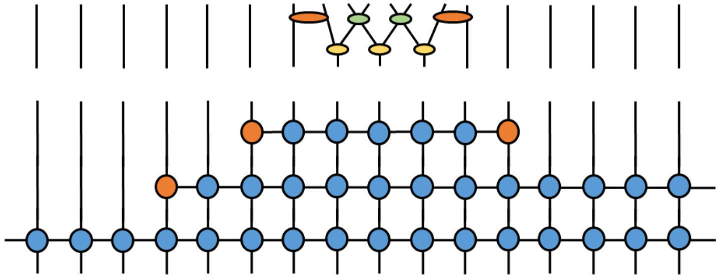
this is NOT what we will do today

In this talk we focus instead in two other types of discretizations

(1) **scale** discretization



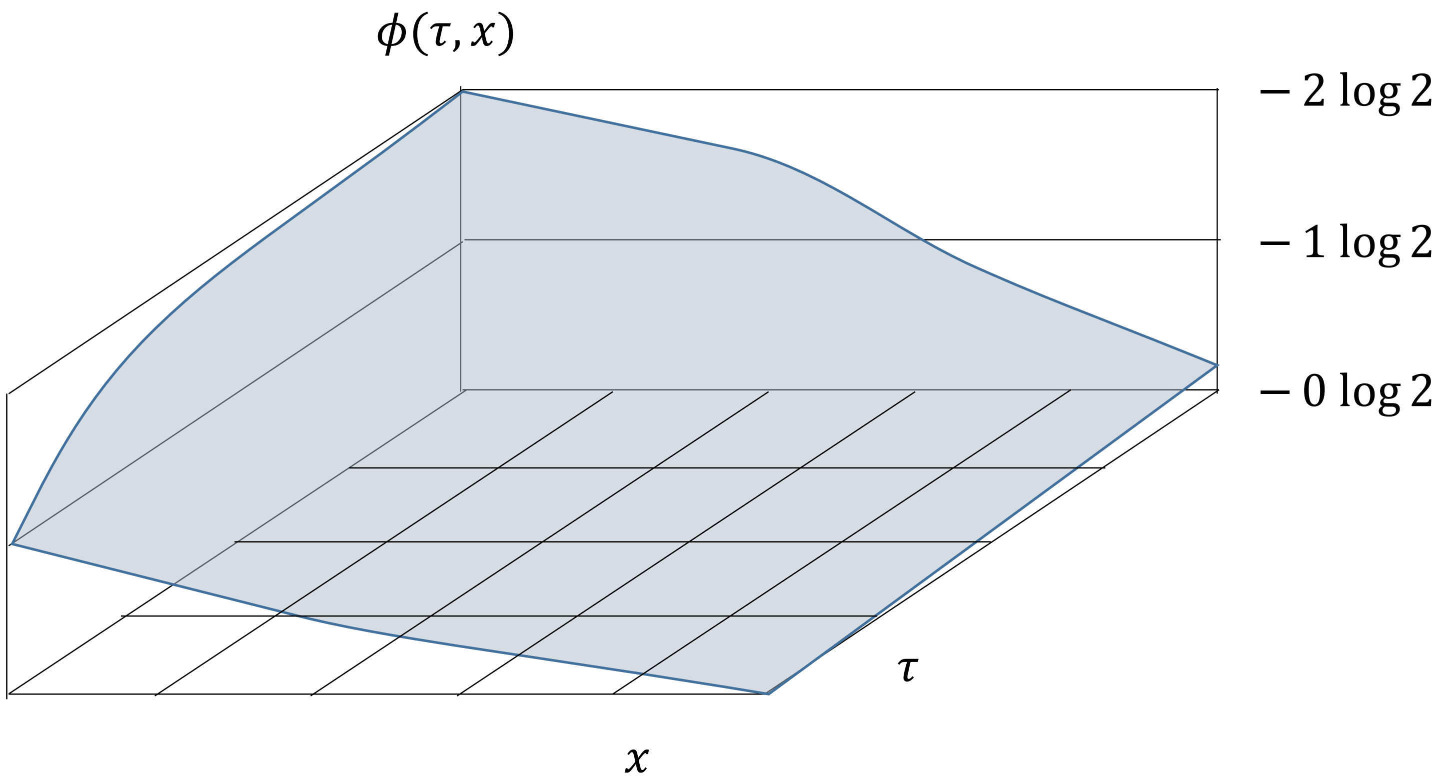
(2) **lapse and shift** discretization



(1) **scale** discretization

2d topological disk is conformally flat $\Rightarrow$ we can find coordinates (τ, x) such that $$g_{\mu\nu}(\tau, x) = e^{2\phi(\tau,x)}\delta_{\mu\nu}$$

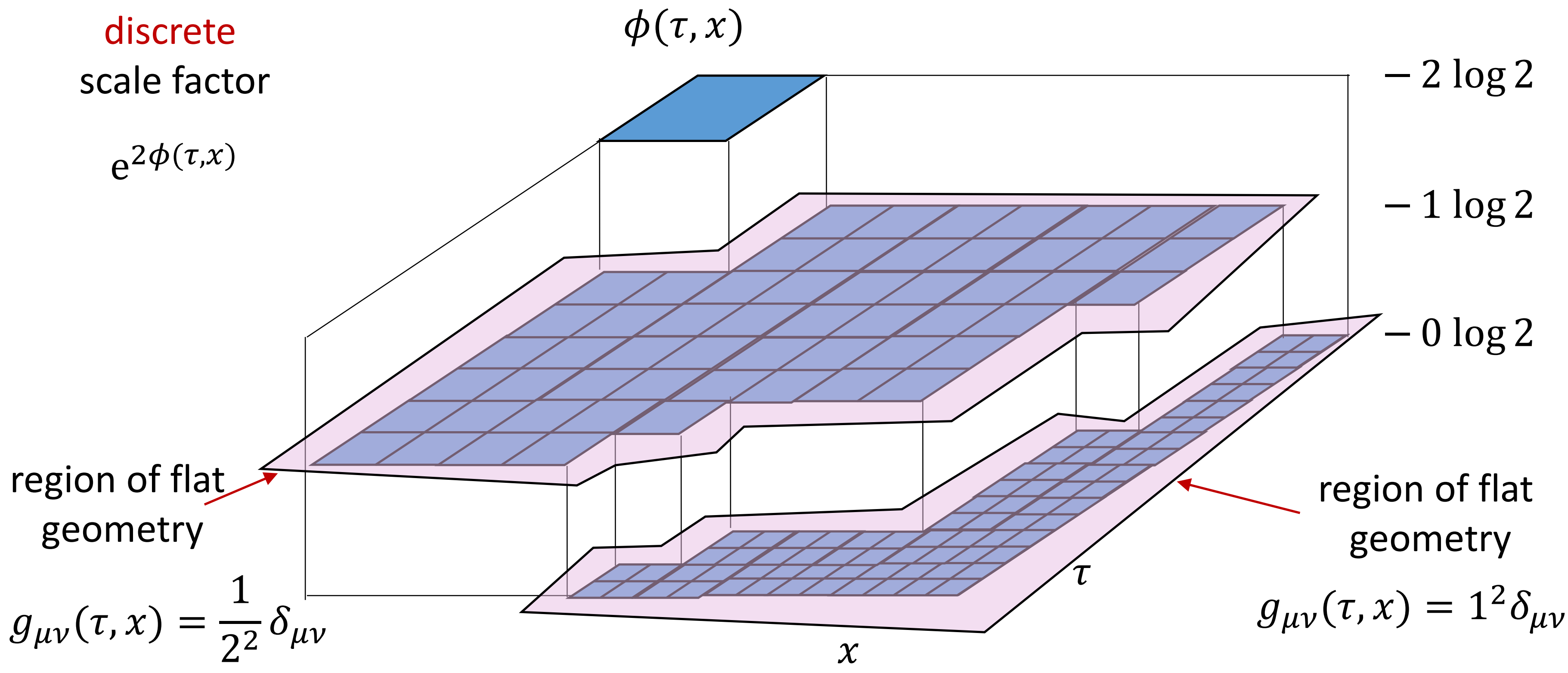
continuous scale factor $e^{2\phi(\tau,x)}$



(1) **scale** discretization

2d topological disk is conformally flat $\Rightarrow$ we can find coordinates (τ, x) such that $g_{\mu\nu}(\tau,x) = e^{2\phi(\tau,x)}\delta_{\mu\nu}$

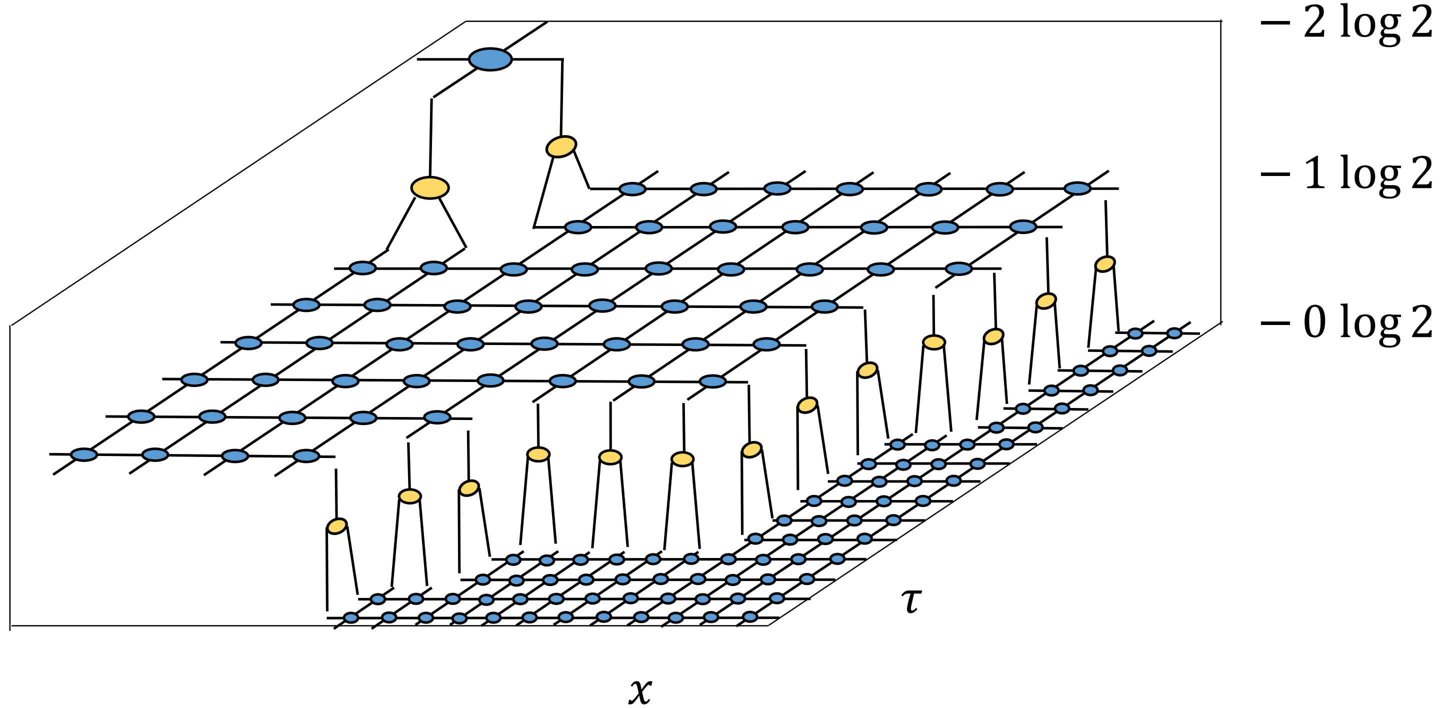
$$g_{\mu\nu}(\tau,x) = 2^{2n(\tau,x)}\delta_{\mu\nu} \qquad n \in \mathbb{Z}$$



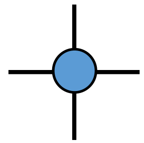
(1) **scale** discretization

2d topological disk is conformally flat $\Rightarrow$ we can find coordinates (τ, x) such that $g_{\mu\nu}(\tau, x) = e^{2\phi(\tau,x)}\delta_{\mu\nu}$

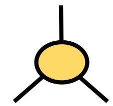
$$g_{\mu\nu}(\tau, x) = 2^{2n(\tau,x)}\delta_{\mu\nu} \qquad n \in \mathbb{Z}$$



two types of tensors:

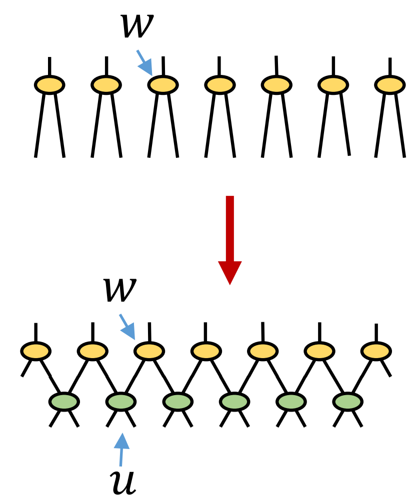


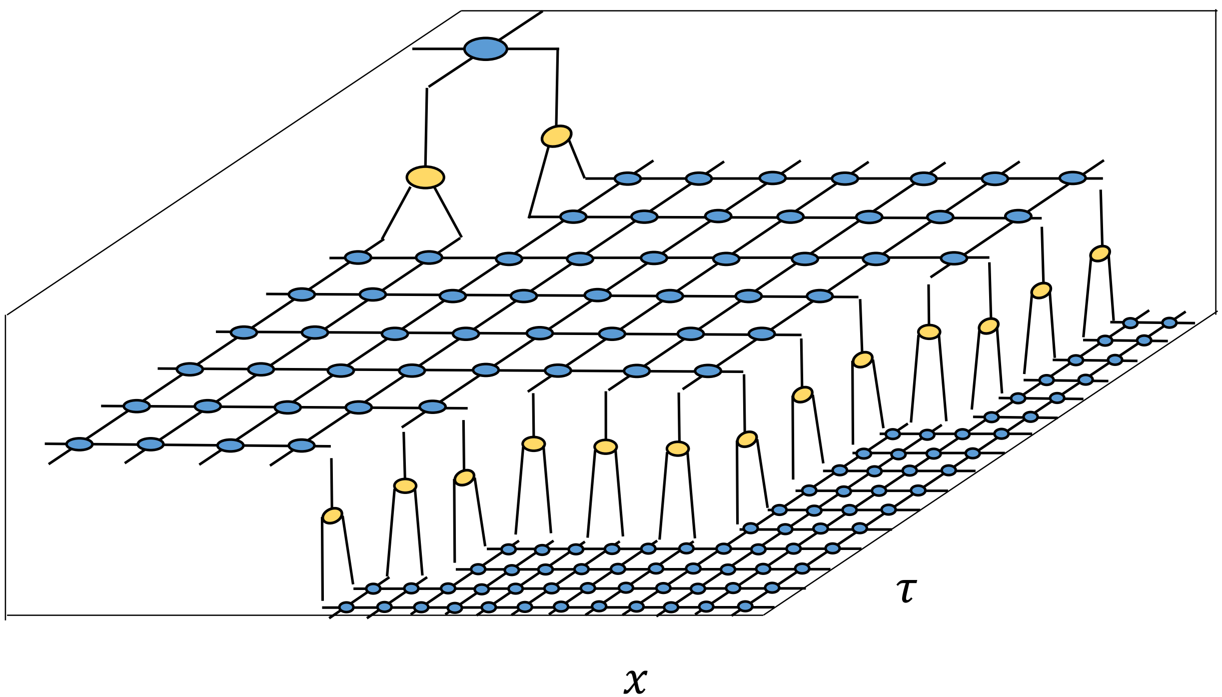
tensor A
= patch of flat
spacetime



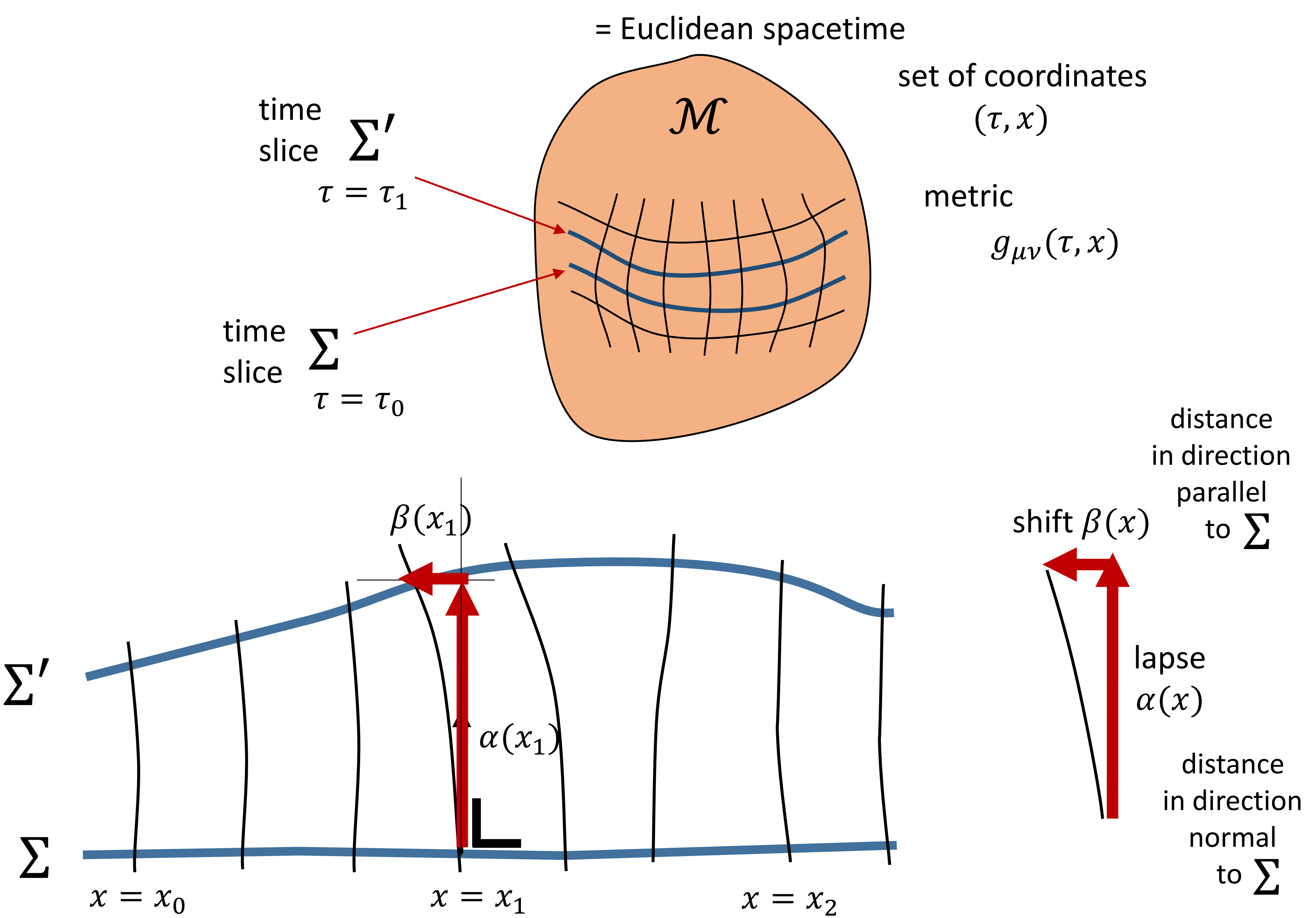
tensor w
= glue connecting
regions
with different
scale factor

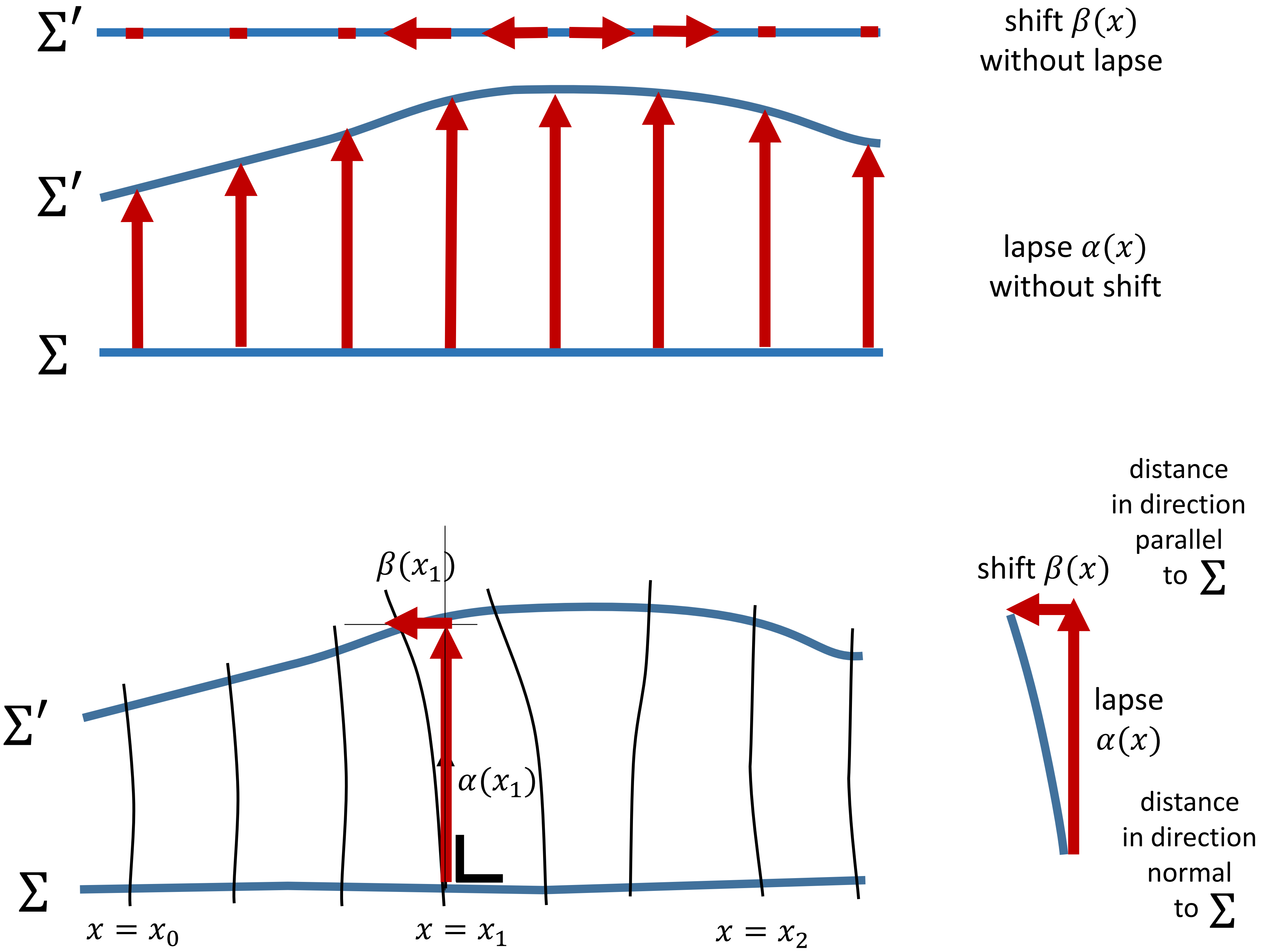
actually, ...



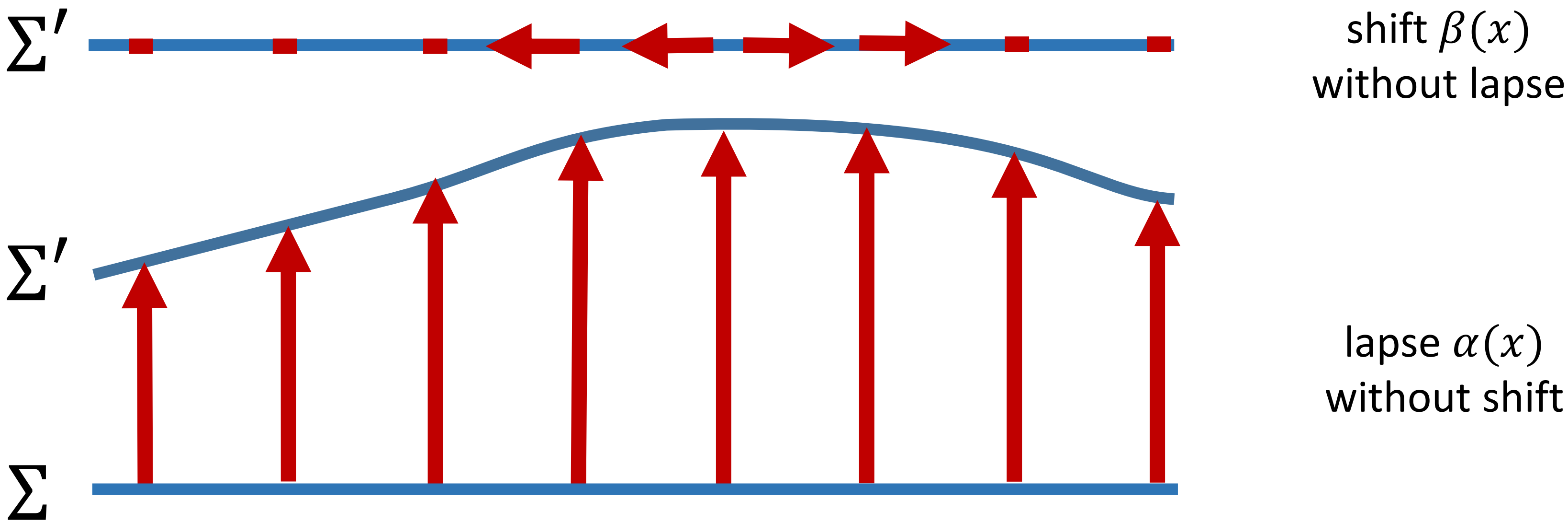


(2) **lapse and shift** discretization

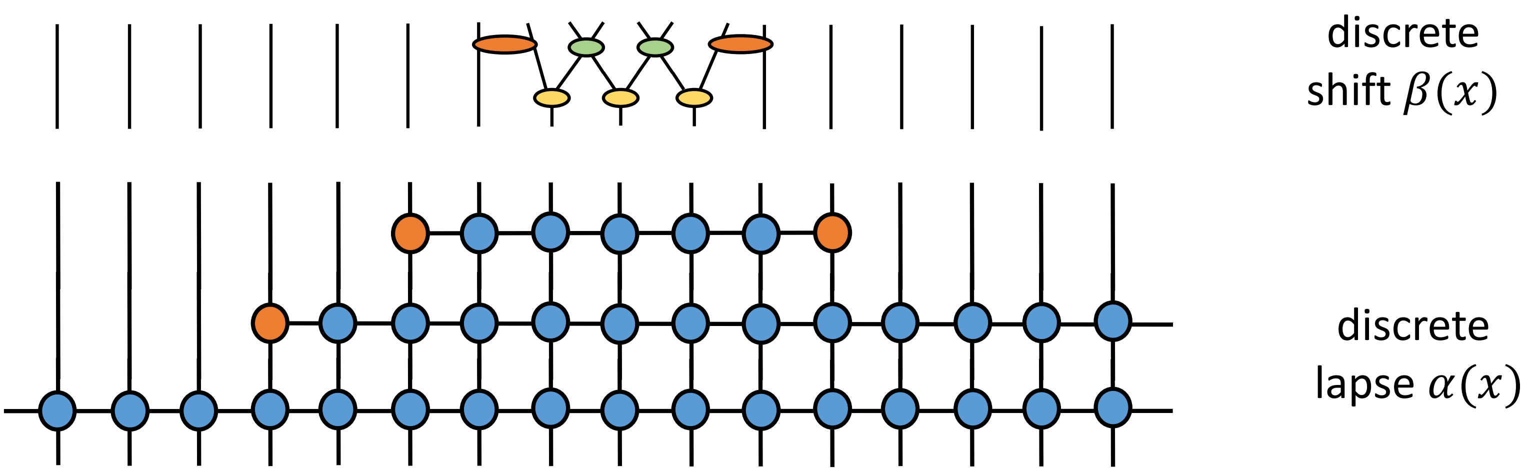




Σ'
shift $\beta(x)$
without lapse
Σ'
lapse $\alpha(x)$
without shift
Σ
distance
in direction
parallel
to Σ
$\beta(x_1)$
shift $\beta(x)$
Σ'
$\alpha(x_1)$
lapse
$\alpha(x)$
distance
in direction
normal
to Σ
Σ
$x = x_0$
$x = x_1$
$x = x_2$



lapse and shift discretization

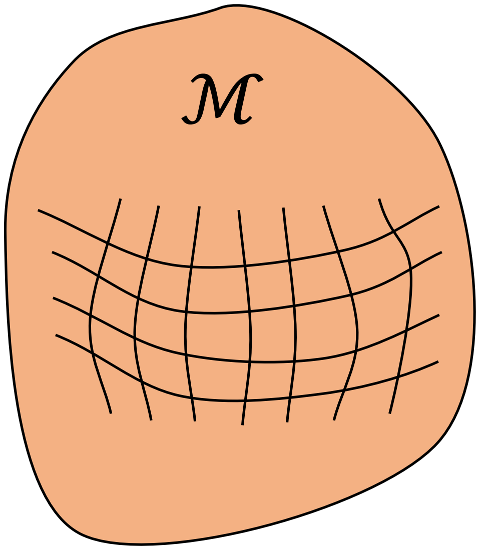


tensor networks as …

- geometry
- path integral geometry

Manifold + Euclidean path integral

2d manifold $\mathcal{M}$ = Euclidean spacetime



set of coordinates

(τ, x)

metric

$g_{\mu\nu}(\tau, x)$

$+$ field

$\varphi(\tau, x)$

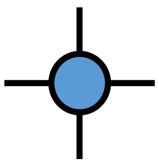
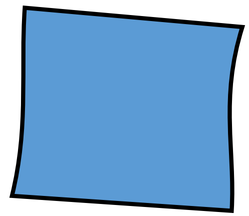
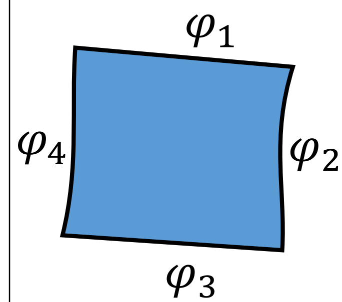
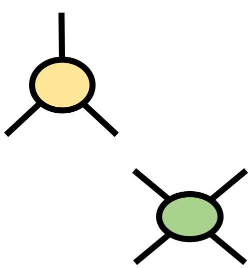
$+$ Euclidean path integral

$$Z = \int D\varphi e^{-S_E[\varphi]}$$

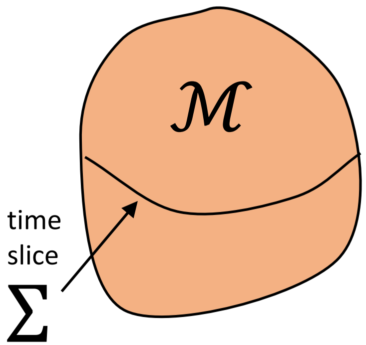
S_E is an Euclidean action

e.g.

$$S_E[\varphi] = \int d\tau dx \sqrt{g}\, g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

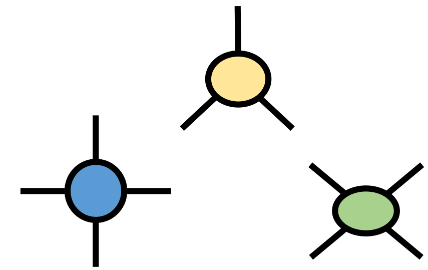
	geometry	**path integral geometry**
 tensor A	 patch of flat Euclidean spacetime	 path integral on patch of flat Euclidean spacetime $Z(\varphi_1, \varphi_2, \varphi_3, \varphi_4) = \int D\varphi e^{-S_E[\varphi]} \Big\vert_{\varphi_1,\varphi_2,\varphi_3,\varphi_4}$
 tensors w, u	rescaling of coordinates at fixed time slice	(MERA tensors) rescaling of coordinates and fields at fixed time slice

From now on, we focus on lapse & shift representation



Claim:

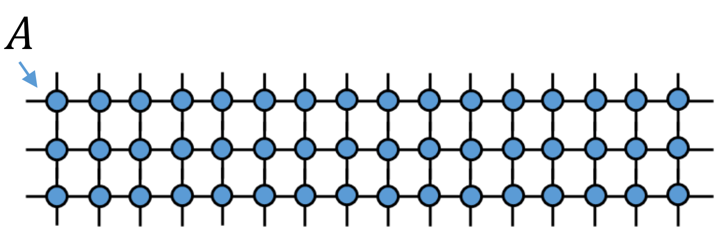
we can use tensors A, w, u
to apply geometric gates
in the Hilbert space V_Σ of a time slice

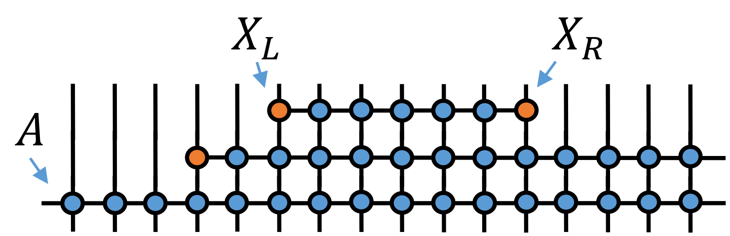


homogeneous
Euclidean time evolution

inhomogeneous
Euclidean time evolution

lapse

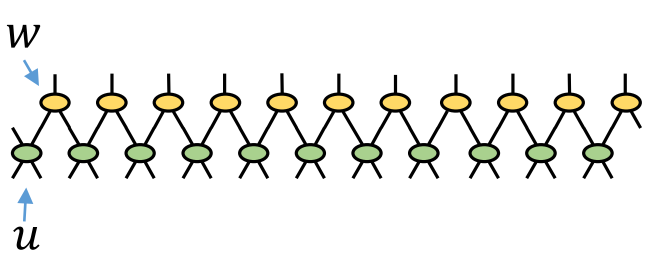


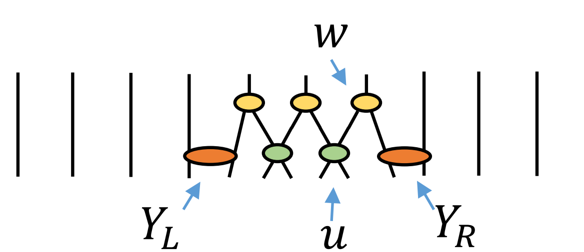


homogeneous
rescaling

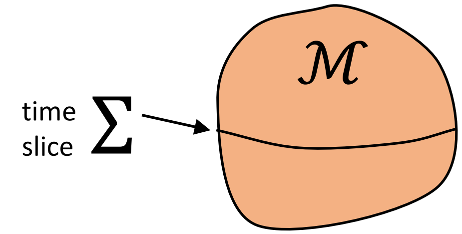
inhomogeneous
rescaling

shift





rest of the talk: provide evidence for this claim

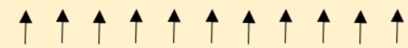


V_Σ

Hilbert
space

plan:

- quantum spin chain (QFT on the lattice)



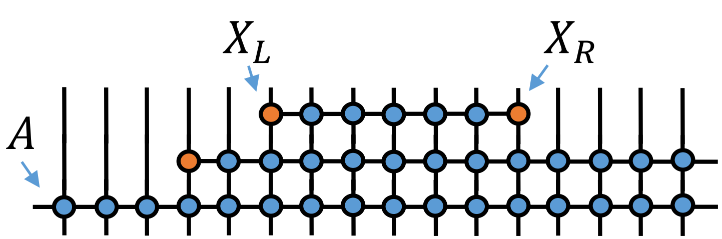
- show that such gates act geometrically on the low energy states V_Σ

problem: how do geometric gates act on low energy states?

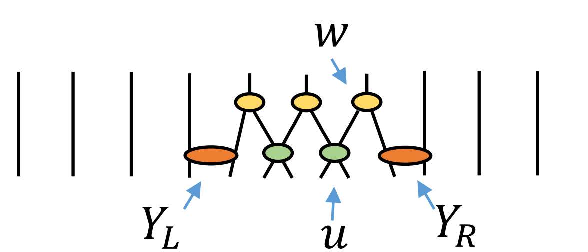
solution: we know the answer for a critical quantum spin chain (= CFT on the lattice)

geometric gates = conformal transformations

inhomogeneous
Euclidean time evolution



inhomogeneous
rescaling



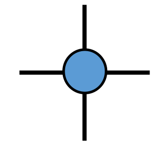
critical
Ising model

from quantum spin
Hamiltonian

$$H = \sum_i \sigma_i^x \sigma_{i+1}^x + \sigma_i^z$$

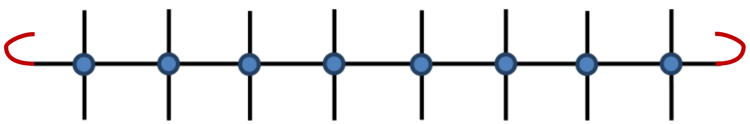
(or from statistical
Boltzmann weights)

algorithm



tensor A

transfer matrix $e^{-\delta\tau\, H}$

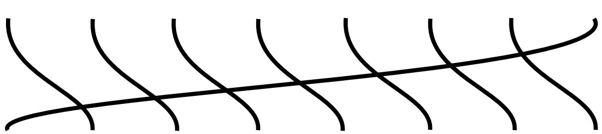


$$H = \frac{2\pi}{L}\left(L_0 + \bar{L}_0 - \frac{c}{12}\right)$$

scaling
dimensions

$$E_\alpha = \frac{2\pi}{L}\left(\boldsymbol{\Delta_\alpha} - \frac{c}{12}\right)$$

translation $e^{-i\delta x\, P}$

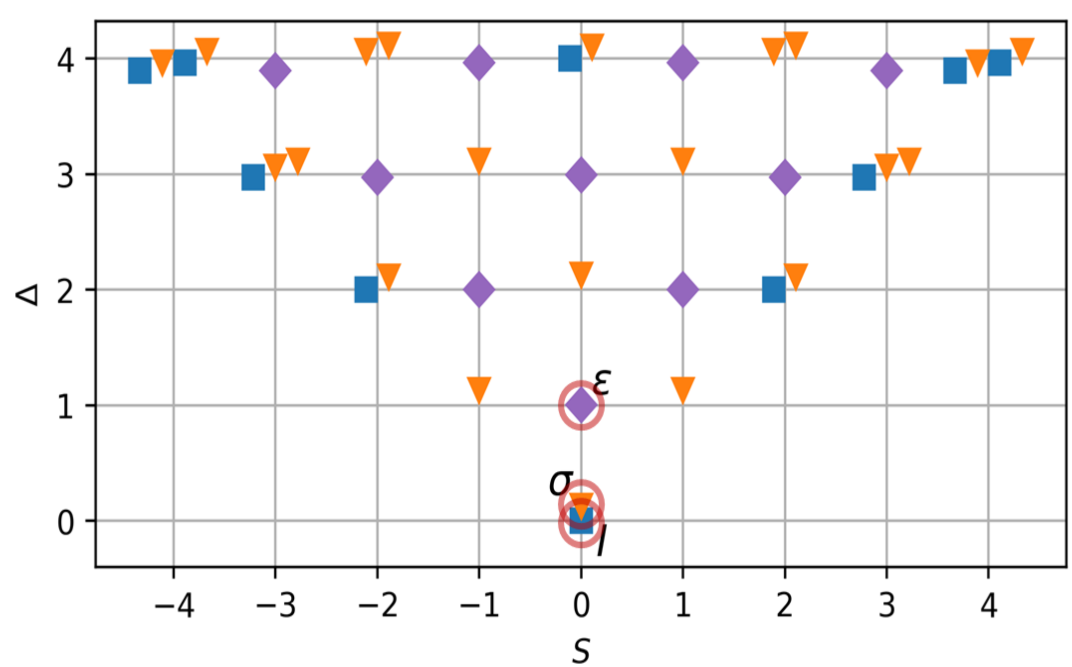


$$P = \frac{2\pi}{L}\left(L_0 - \bar{L}_0\right)$$

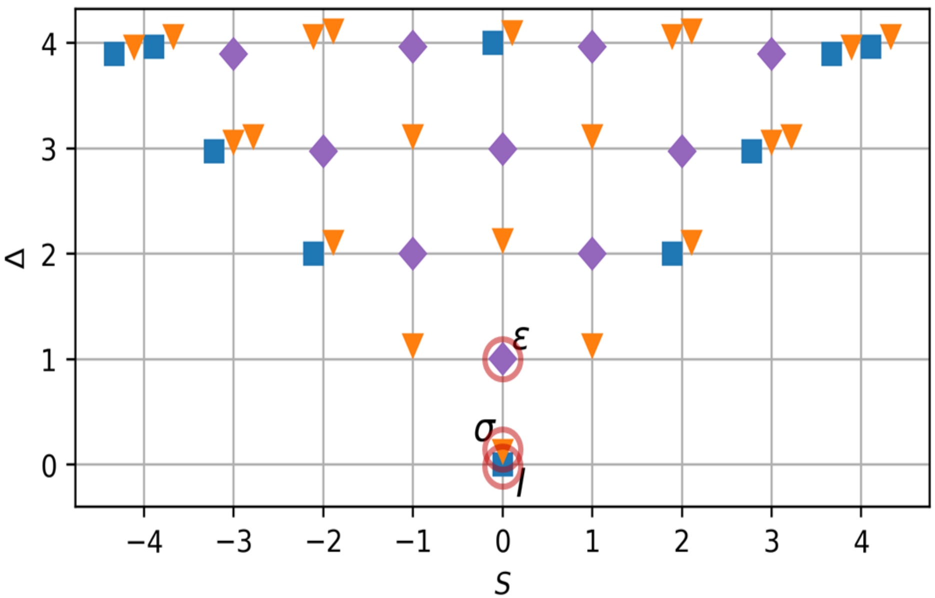
conformal
spins

$$P_\alpha = \frac{2\pi}{L}\boldsymbol{s_\alpha}$$

Example: Ising spin chain on 24 sites



Example: Ising spin chain on 24 sites



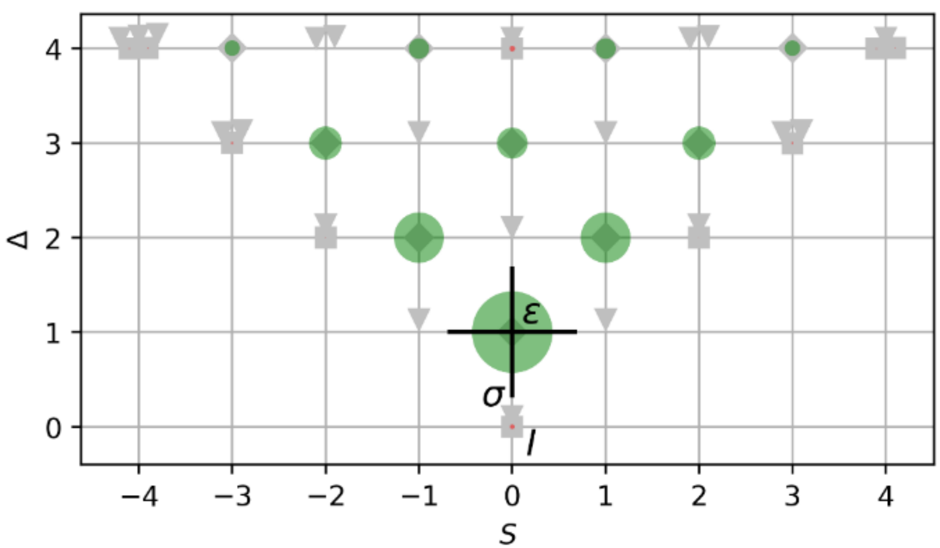
there are 3 conformal towers

towers are not mixed by geometric/conformal transformations

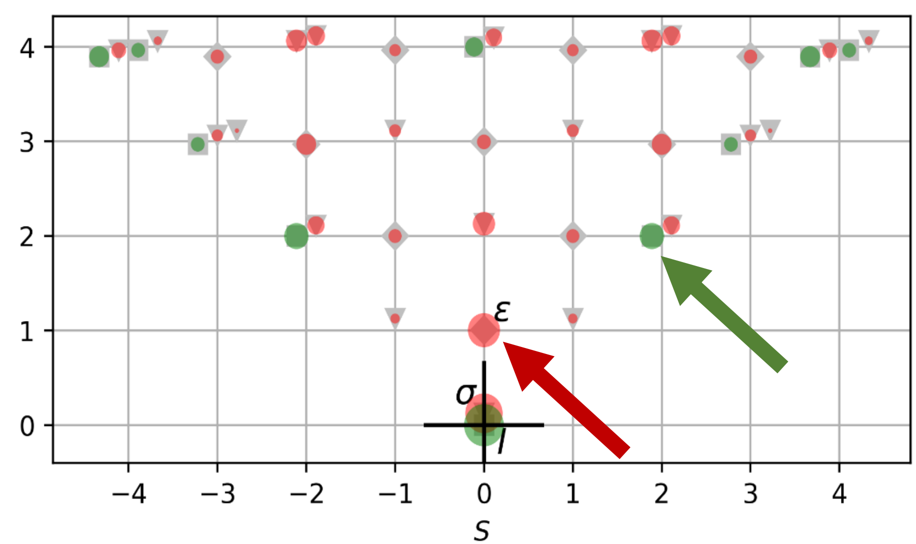
$$|\Psi_\alpha\rangle \longrightarrow G|\Psi_\alpha\rangle = \sum_\beta A_{\alpha\beta}\,|\Psi_\beta\rangle$$

- $|\Psi_\alpha\rangle$: low energy state in one tower
- G: geometric gate
- $|\Psi_\beta\rangle$: low energy states in **same** tower

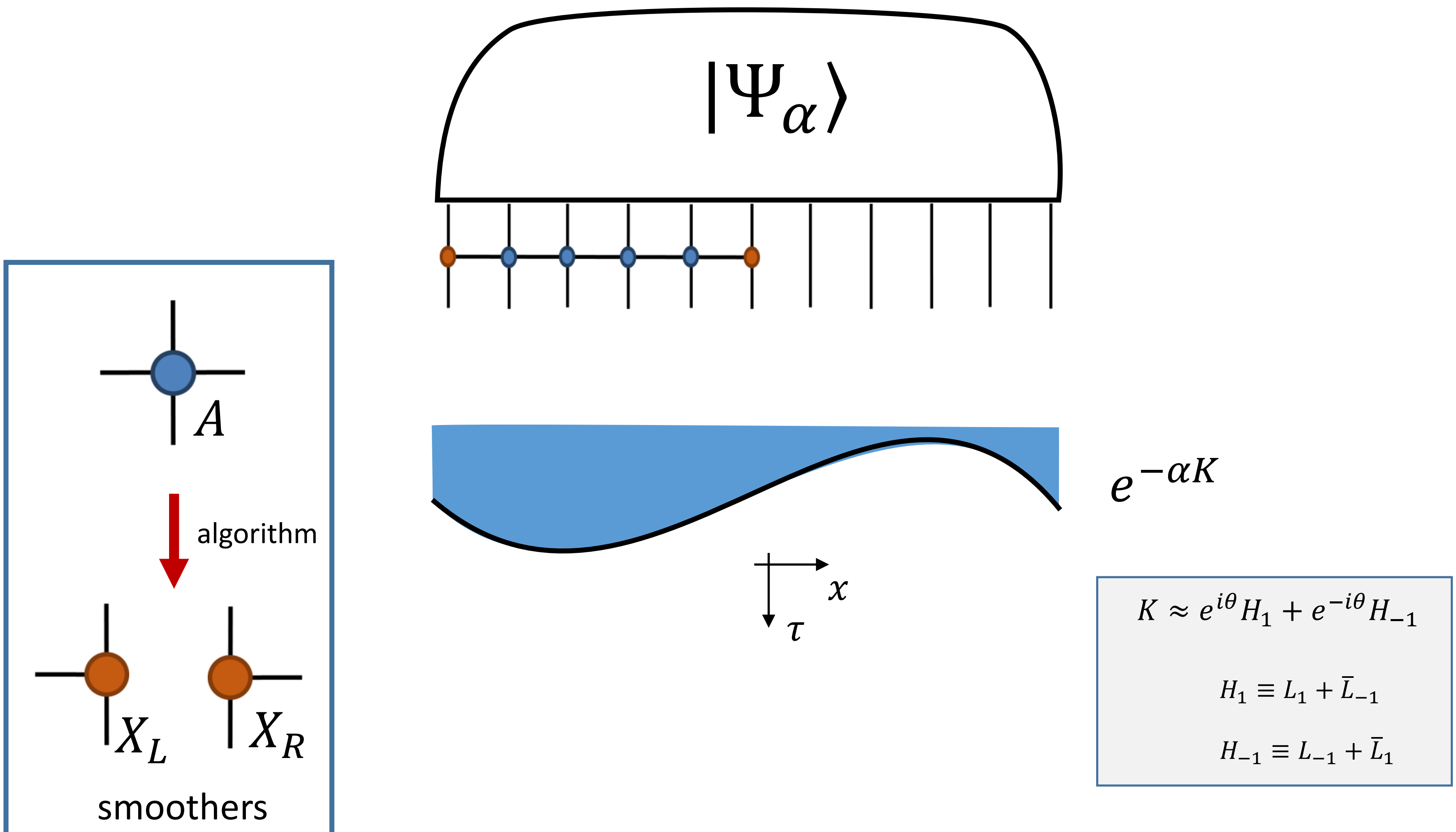
this is a geometric gate

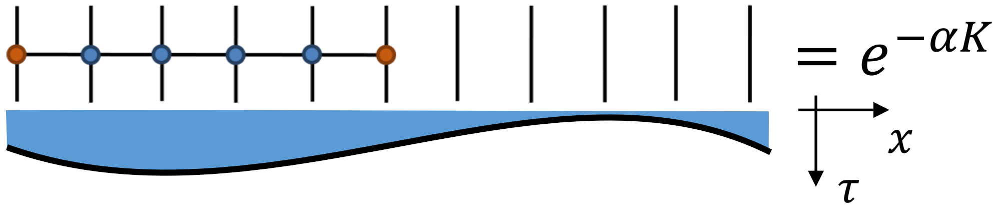


this is not a geometric gate



inhomogeneous
Euclidean time evolution





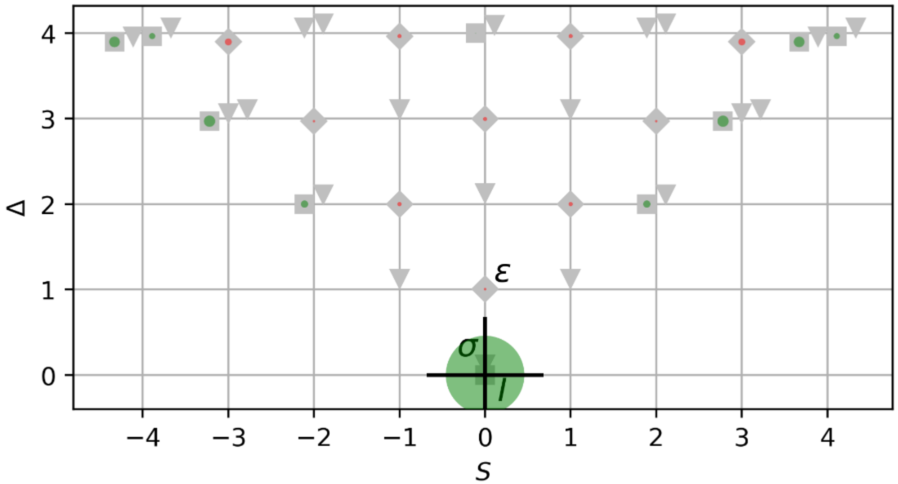
$$K \approx e^{i\theta} H_1 + e^{-i\theta} H_{-1}$$

$$H_1 \equiv L_1 + \bar{L}_{-1}$$

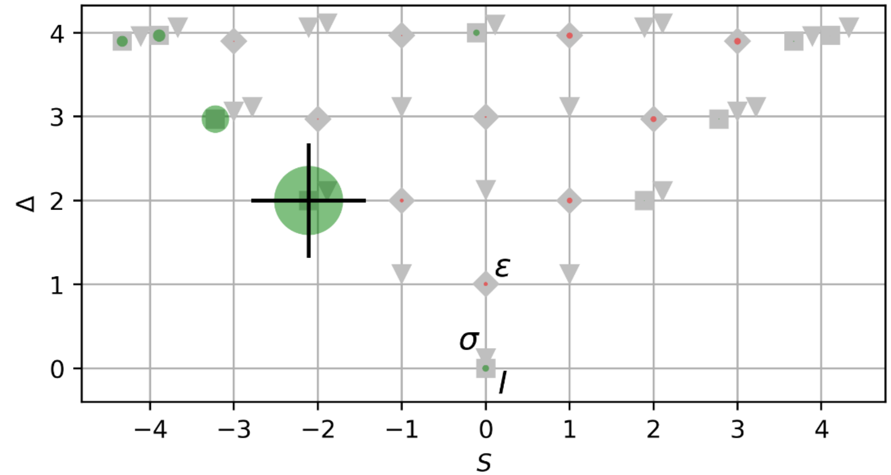
$$H_{-1} \equiv L_{-1} + \bar{L}_1$$

Example: Ising model on 24 sites (12 - 12)

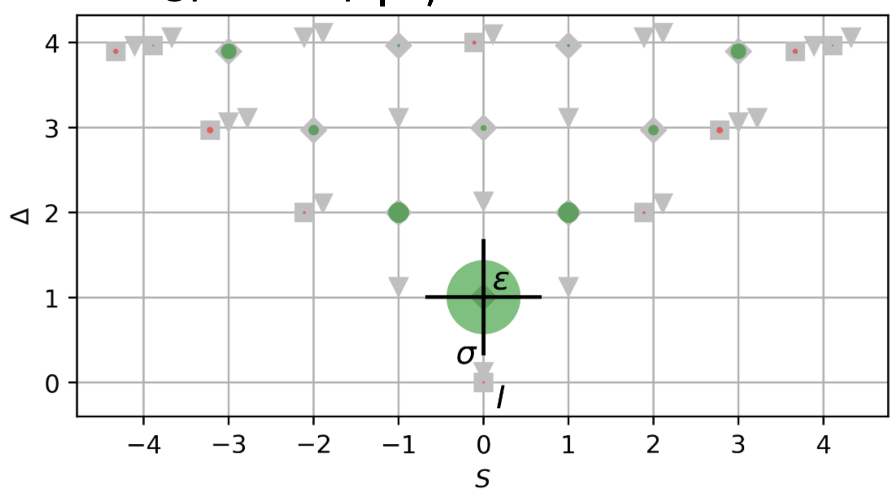
ground state $|\mathbb{I}\rangle$



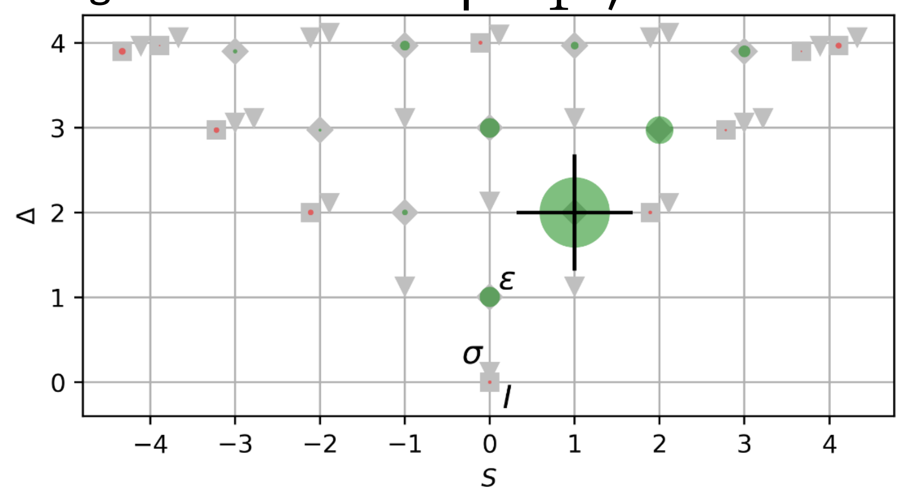
stress tensor $|\bar{T}\rangle = |\bar{L}_{-2}\mathbb{I}\rangle$

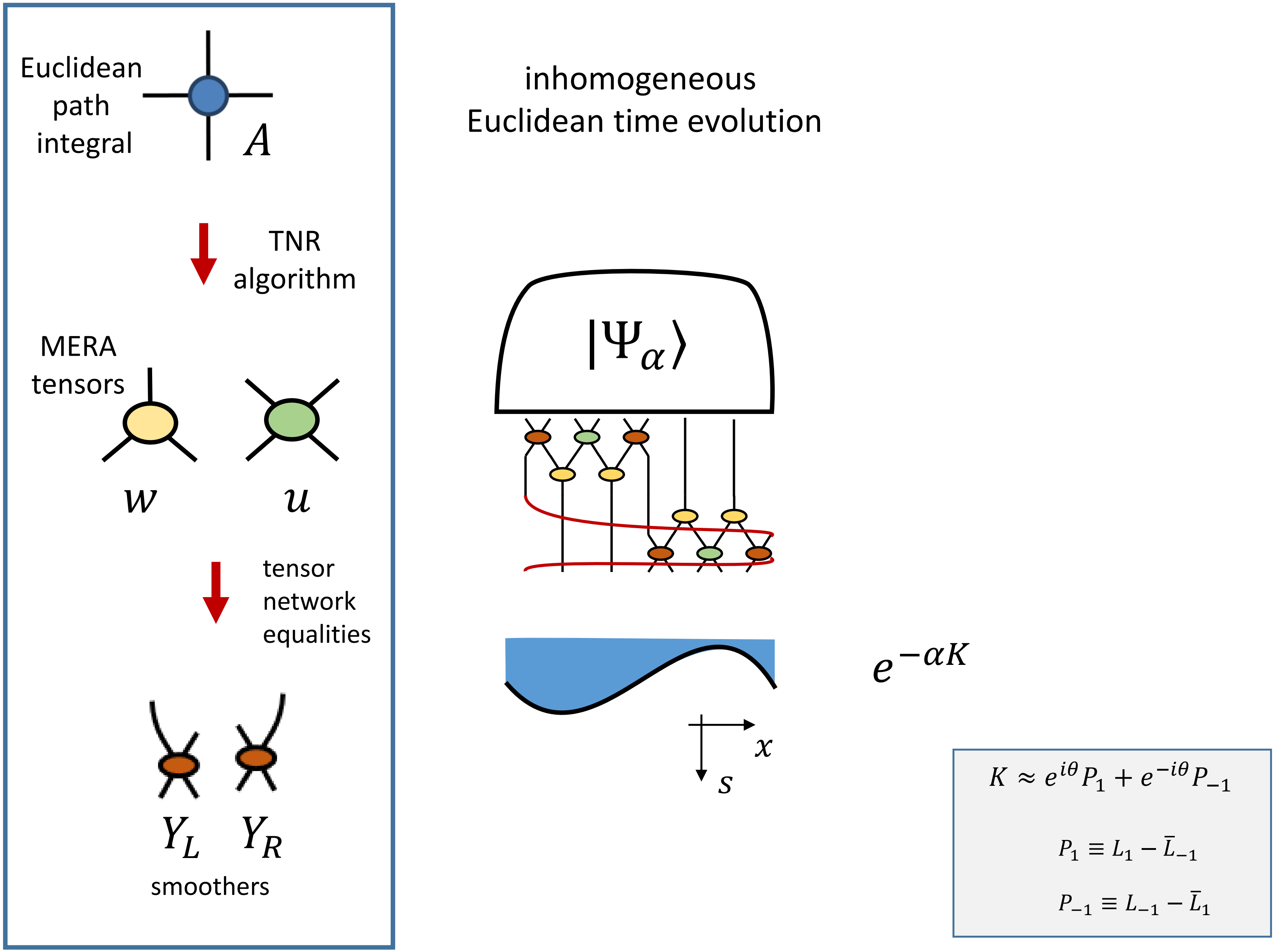


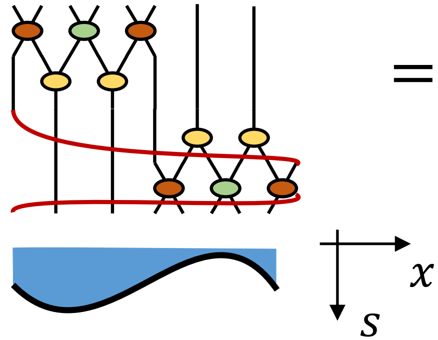
energy density $|\varepsilon\rangle$



global descendant $|L_{-1}\varepsilon\rangle$



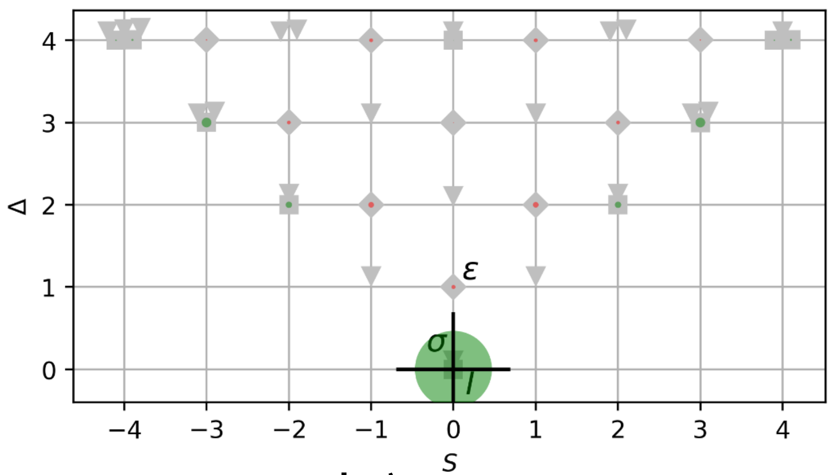

Euclidean path integral
A
TNR algorithm
MERA tensors
w
u
tensor network equalities
Y_L Y_R
smoothers
inhomogeneous Euclidean time evolution
$|\Psi_\alpha\rangle$
$e^{-\alpha K}$
x
s
$K \approx e^{i\theta} P_1 + e^{-i\theta} P_{-1}$
$P_1 \equiv L_1 - \bar{L}_{-1}$
$P_{-1} \equiv L_{-1} - \bar{L}_1$



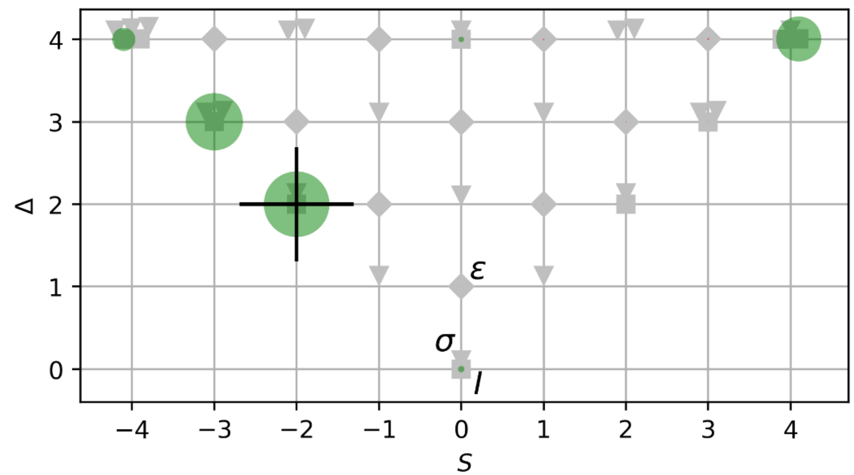
$$= e^{-\alpha K}$$

$$K \approx e^{i\theta} P_1 + e^{-i\theta} P_{-1}$$
$$P_1 \equiv L_1 - \bar{L}_{-1}$$
$$P_{-1} \equiv L_{-1} - \bar{L}_1$$

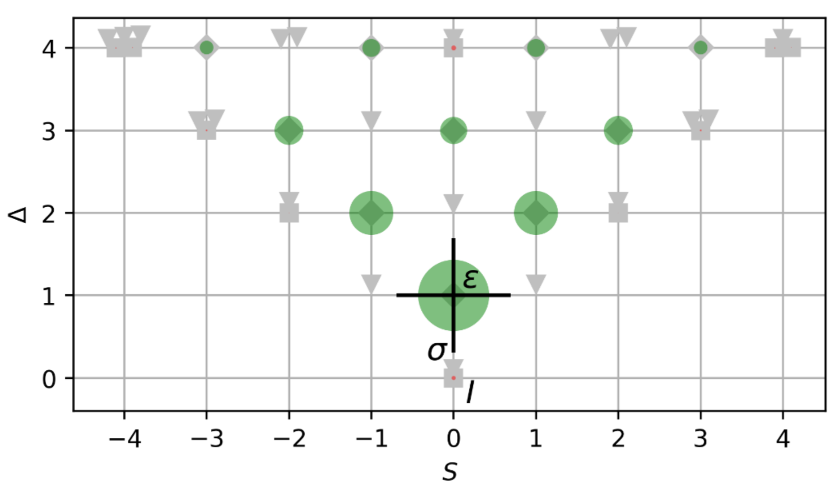
ground state $|\mathbb{I}\rangle$



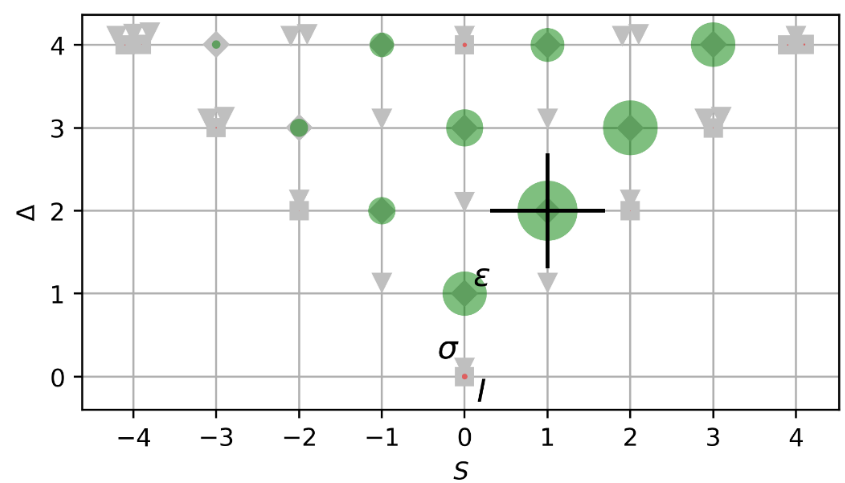
stress tensor $|\bar{T}\rangle = |\bar{L}_{-2}\mathbb{I}\rangle$



energy density $|\varepsilon\rangle$



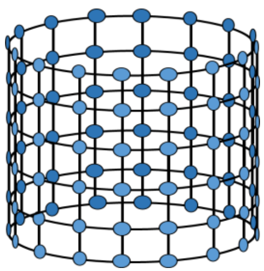
global descendant $|L_{-1}\varepsilon\rangle$



Conclusions

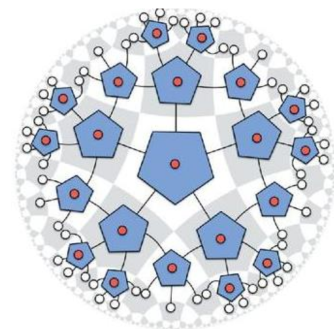
joint work
(*in preparation*)

Ash Milsted
Perimeter Institute

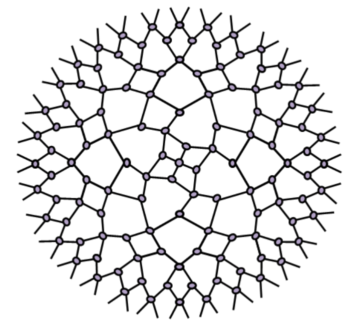


2d partition
function on cylinder

flat space
(flat cylinder)



HaPPY code



MERA

hyperbolic space
(Poincare disk)

tensor network $\sim$ geometry?

Conclusions

joint work
(*in preparation*)

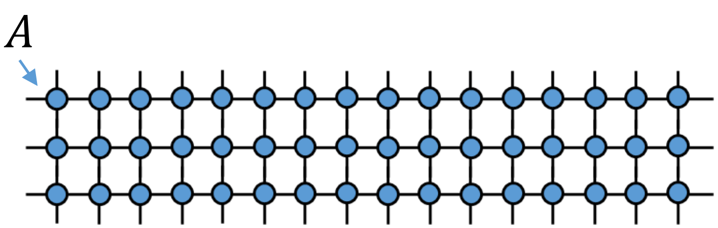
Ash Milsted
Perimeter Institute

(1) tensor networks can be used to represent

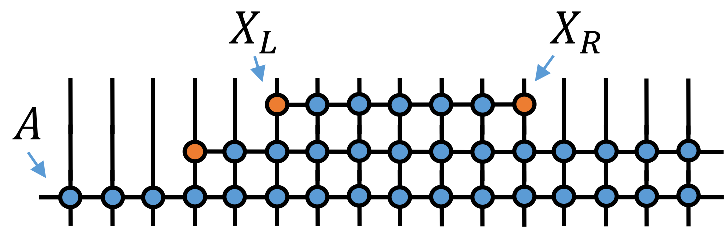
Euclidean path integrals on curved spacetime geometry

(2) tensor networks can implement *geometric transformations*
(in the Hilbert space of a quantum spin chain) corresponding to

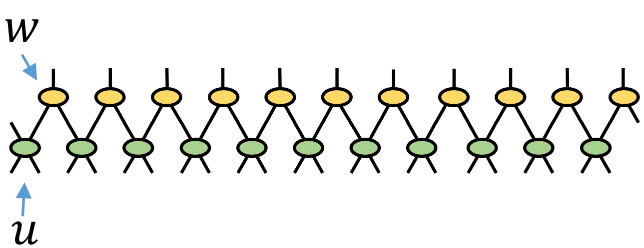
homogeneous
Euclidean time evolution



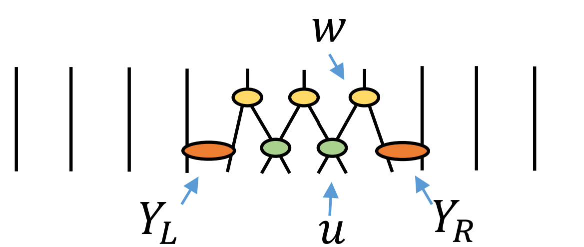
inhomogeneous
Euclidean time evolution



homogeneous
rescaling



inhomogeneous
rescaling



*smoothers X_L, X_R, Y_L, Y_R, are required

Conclusions

joint work
(*in preparation*)

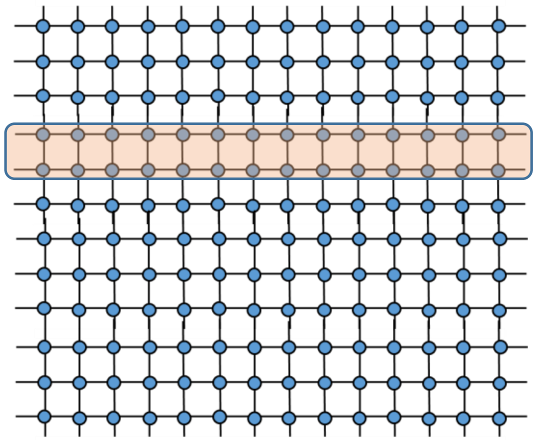
Ash Milsted
Perimeter Institute

(3) from this path integral geometric perspective,

MERA is "rescaling without Euclidean time evolution"

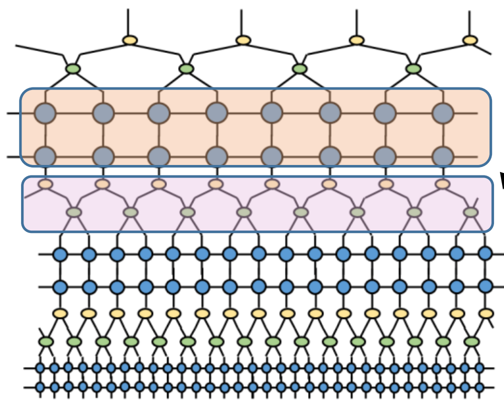
Euclidean time evolution
e^{-H}

change of scale
$e^{-i\,D}$



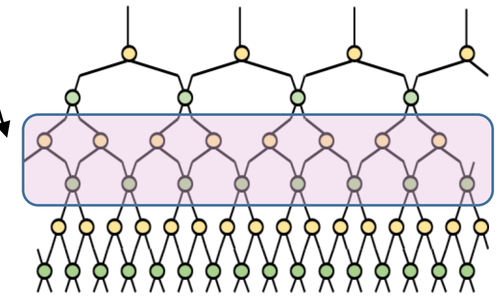
Euclidean path integral in flat spacetime

(only Euclidean time evolution)



Euclidean path integral in hyperbolic space

(both Euclidean time evolution and rescaling)



MERA = limit of shift without lapse

(only rescaling, i.e. no Euclidean time evolution)

THANKS!