



Topology & Entanglement in Driven (Floquet) Many Body Quantum Systems

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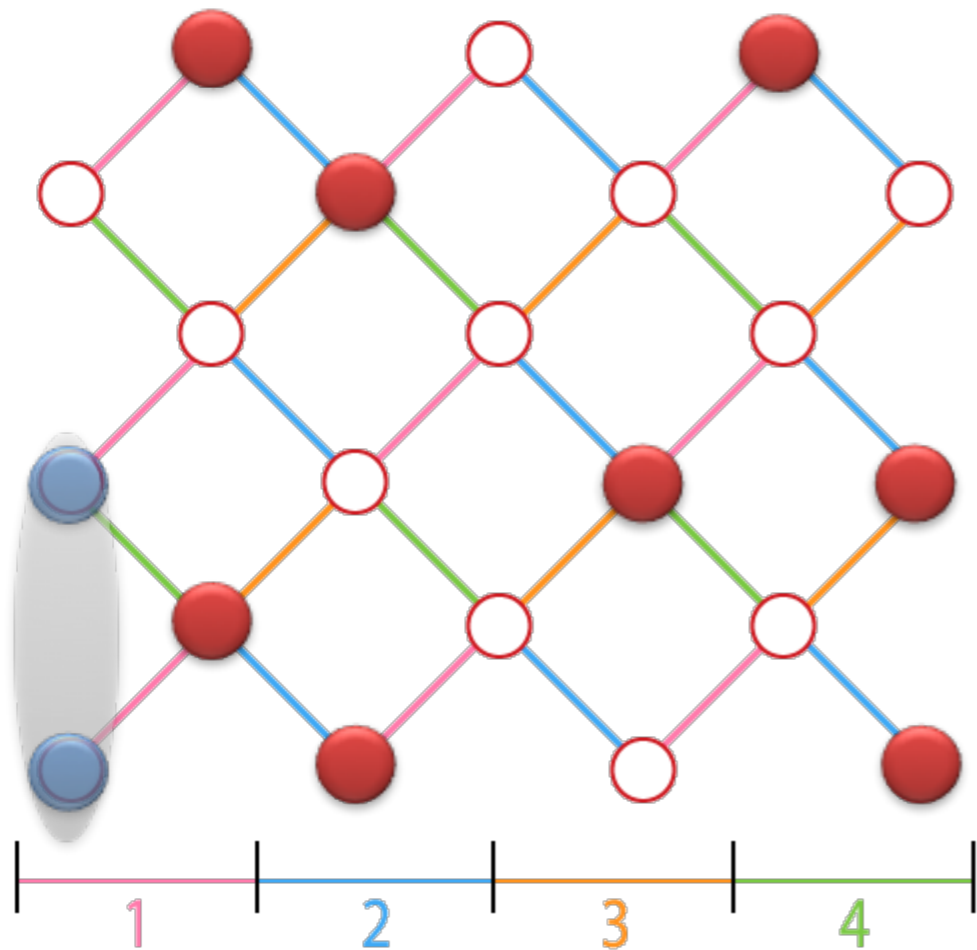


Lukasz Fidkowski
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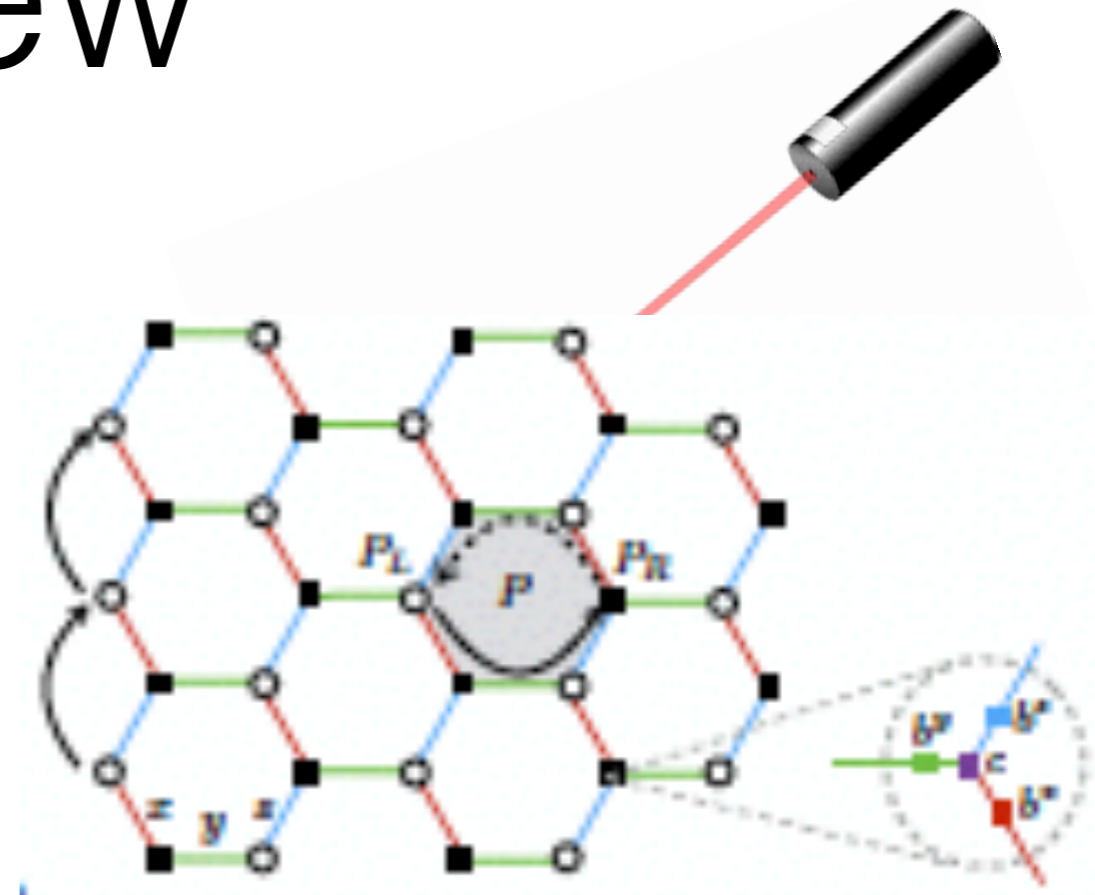
Overview



$$\nu = \log p - \log q$$

Chiral phases in driven systems.

arXiv:1701.01440. Chiral Floquet phases. *Po, Fidkowski, Morimoto, Potter, AV. Phys. Rev. X 6, 041070 (2016)*

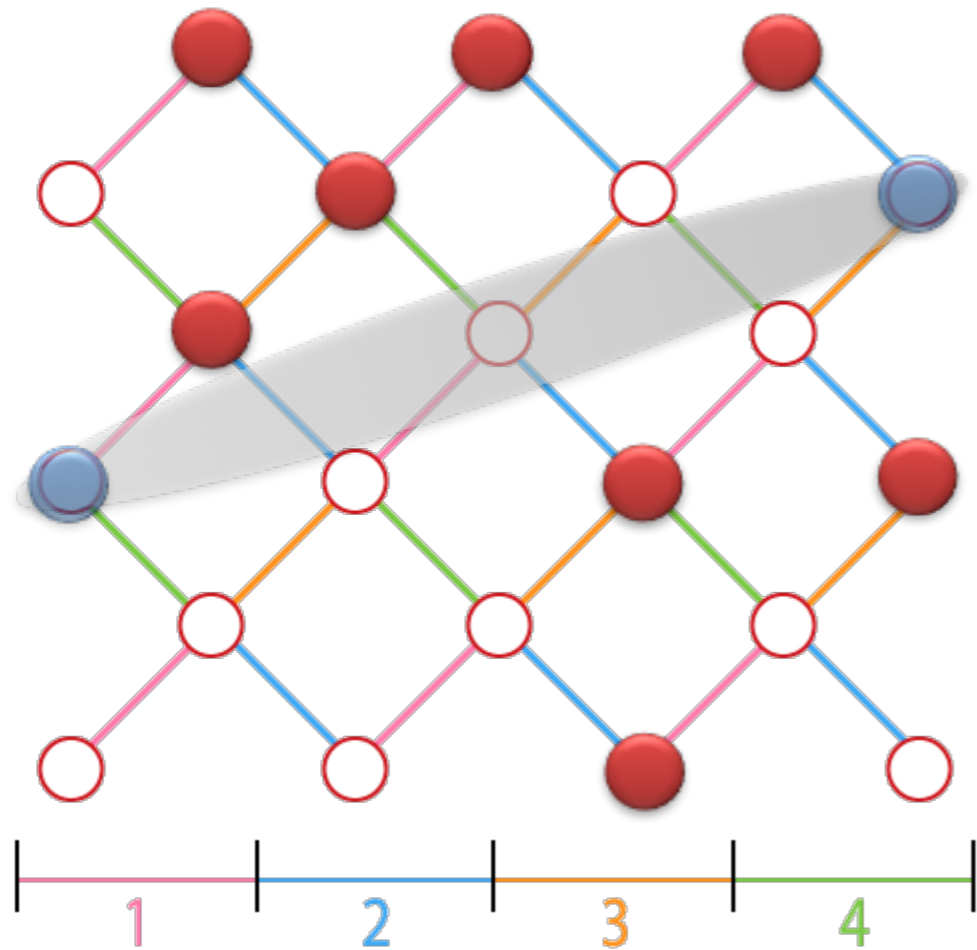


$$\nu = \frac{1}{2} \log 2$$

The Driven Toric code.

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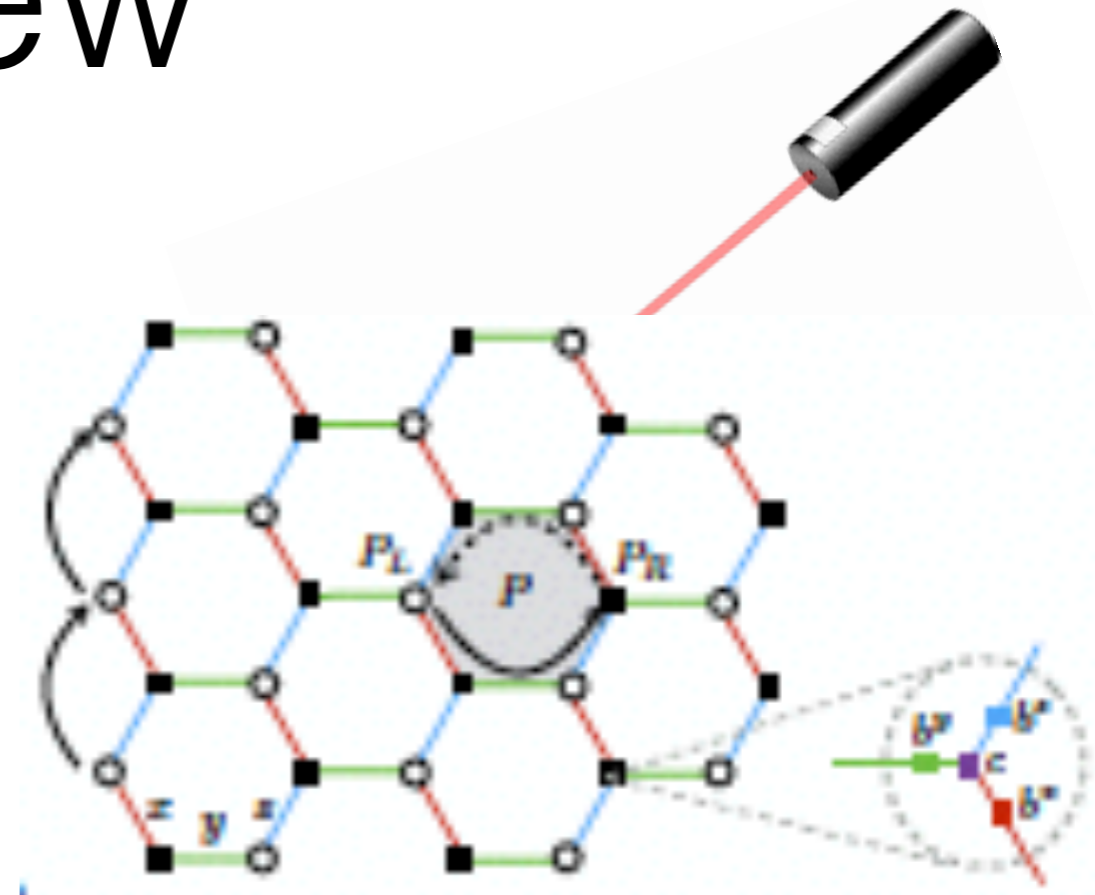
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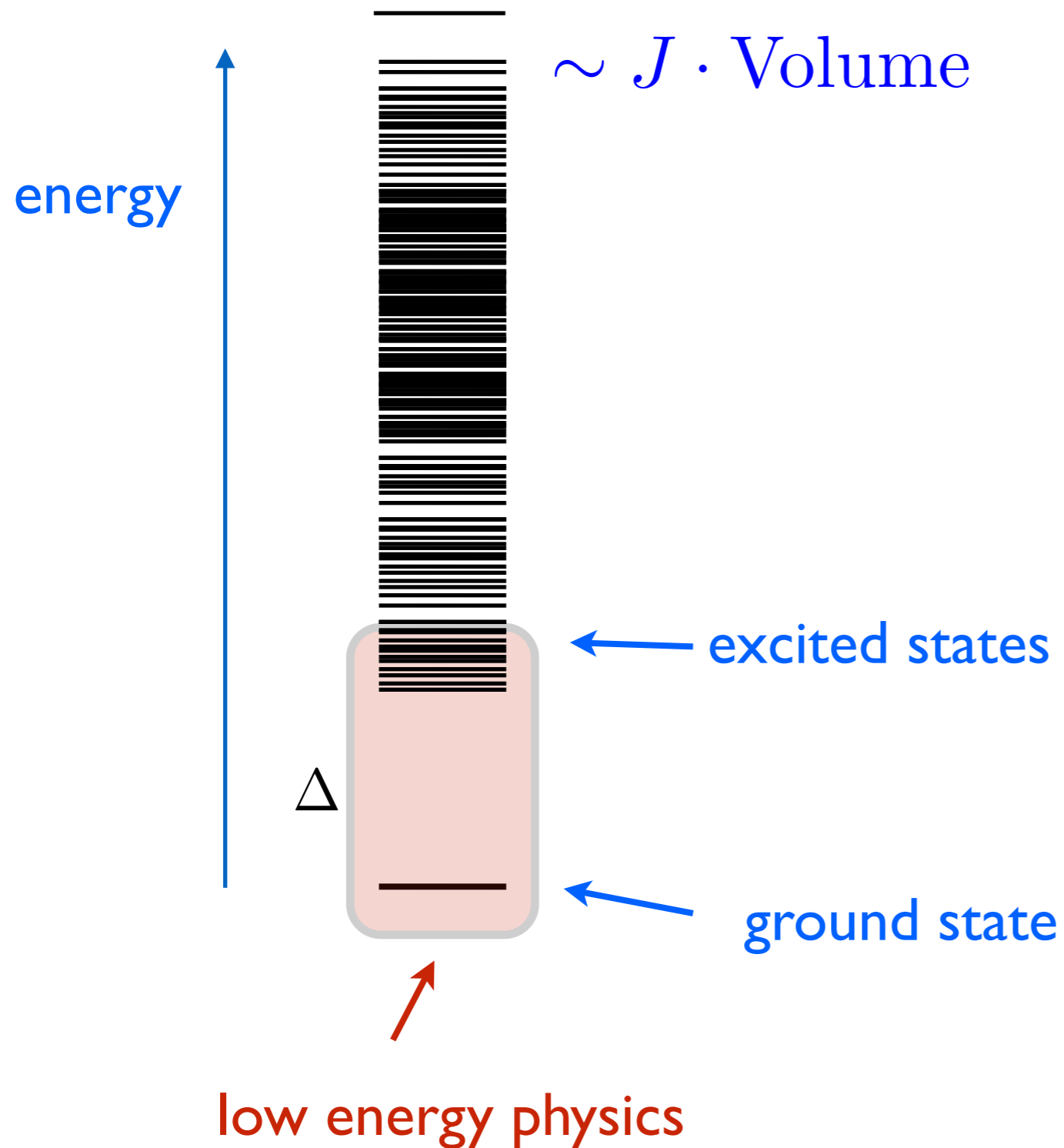


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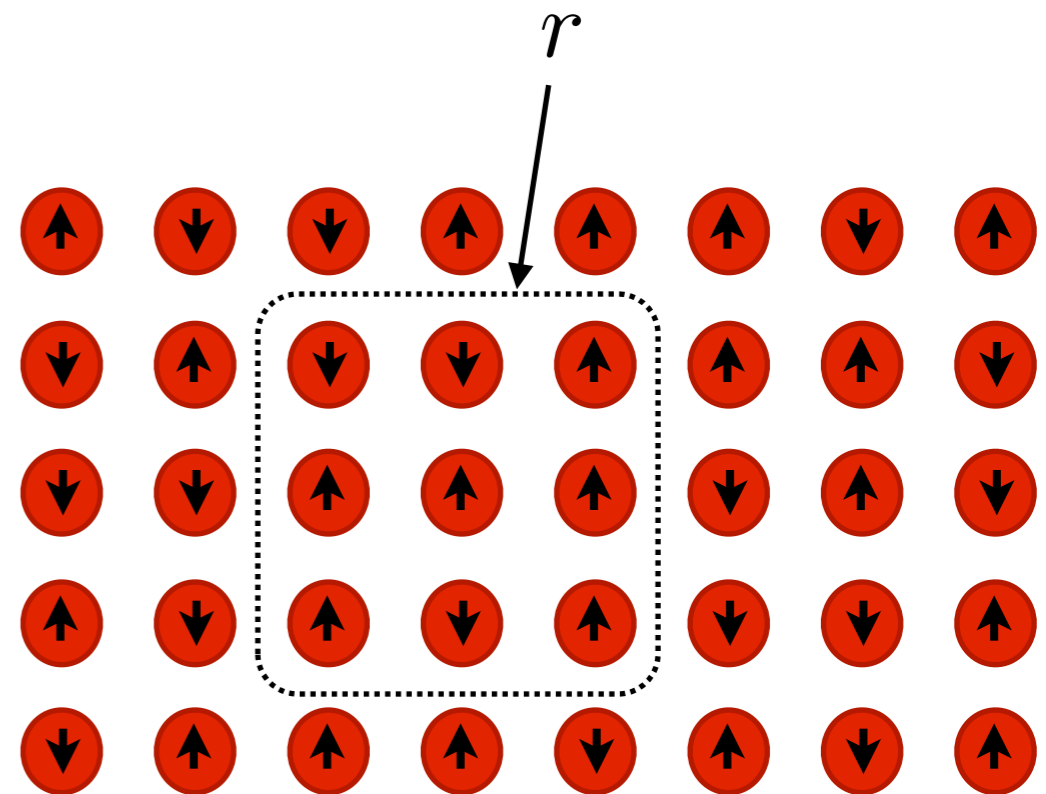
Introduction: A gapped Hamiltonian



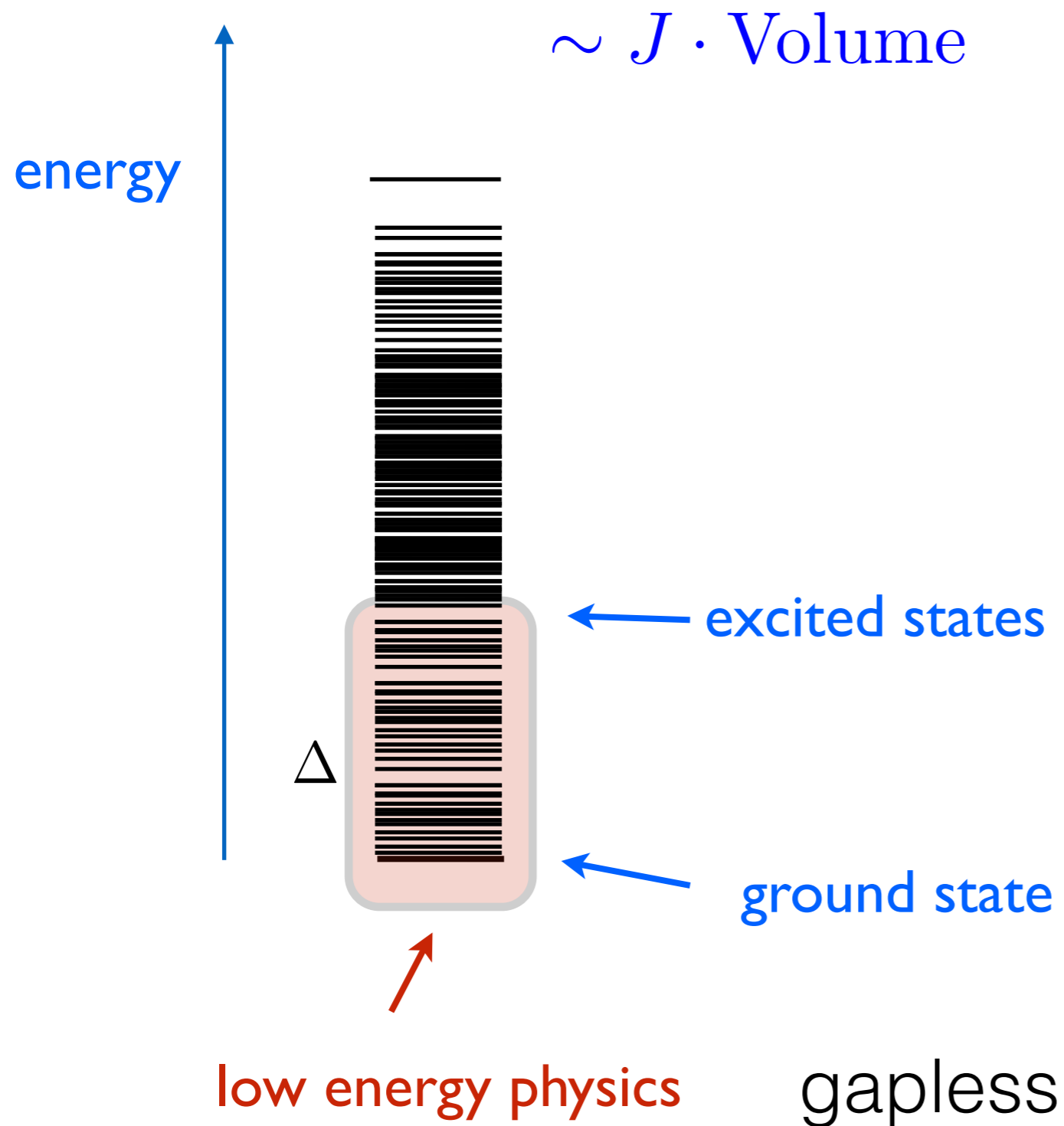
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$$H = \sum_r H_r$$

$$J = \max_r |H_r|$$



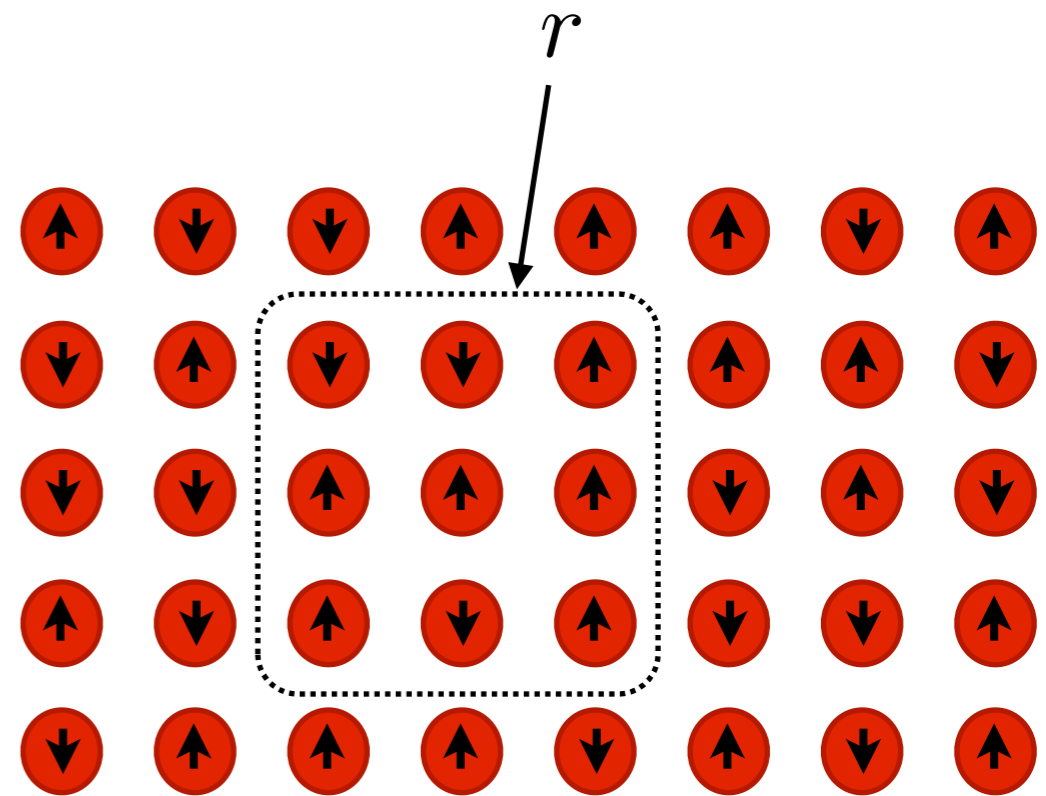
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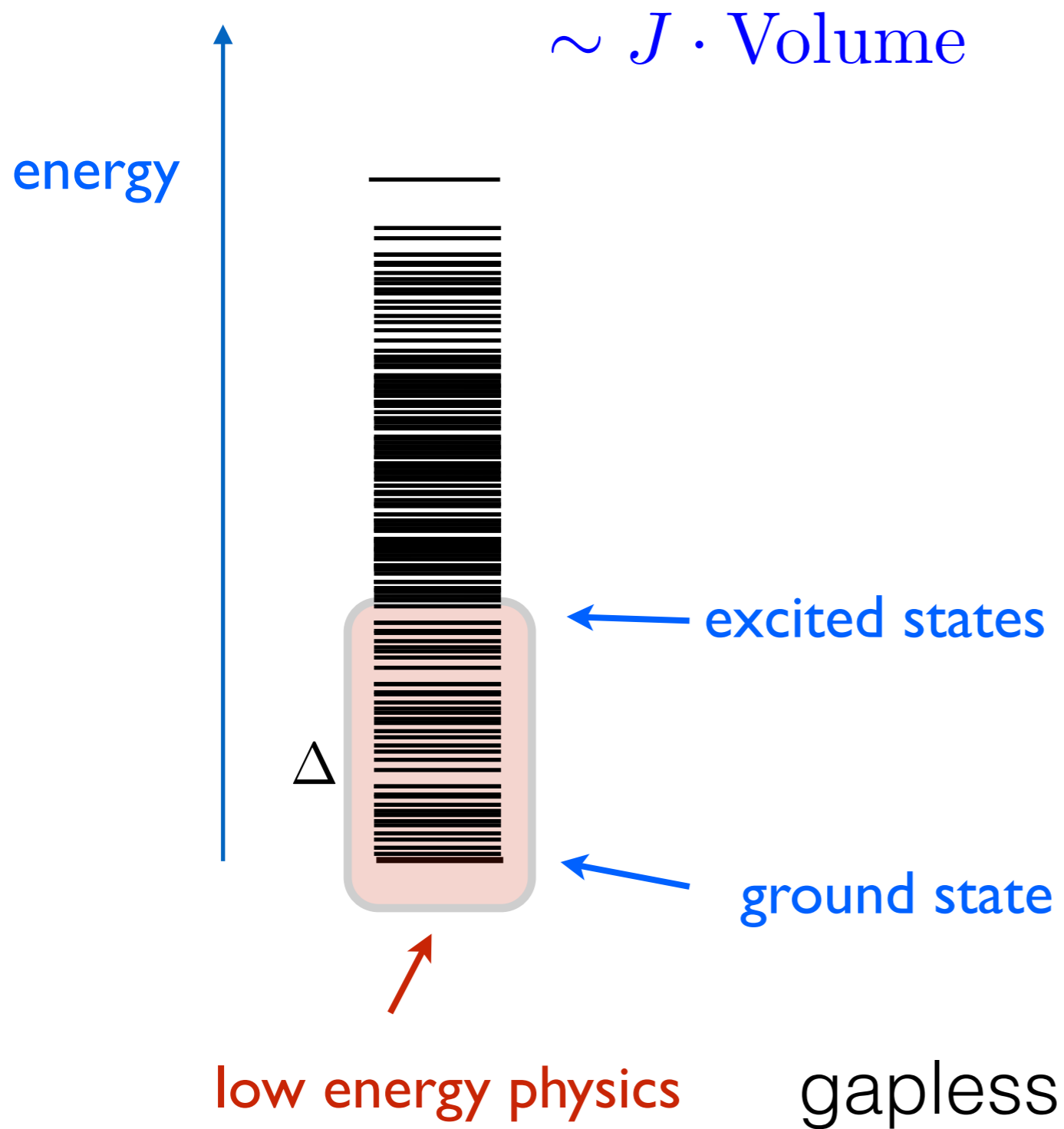
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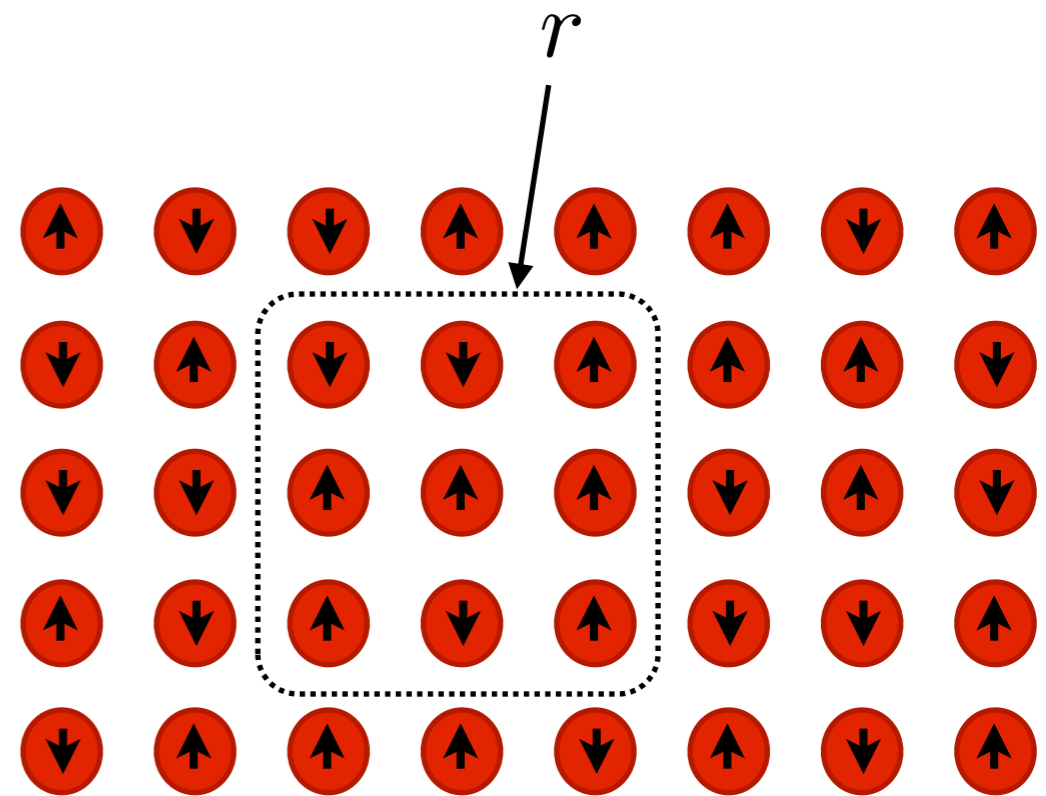
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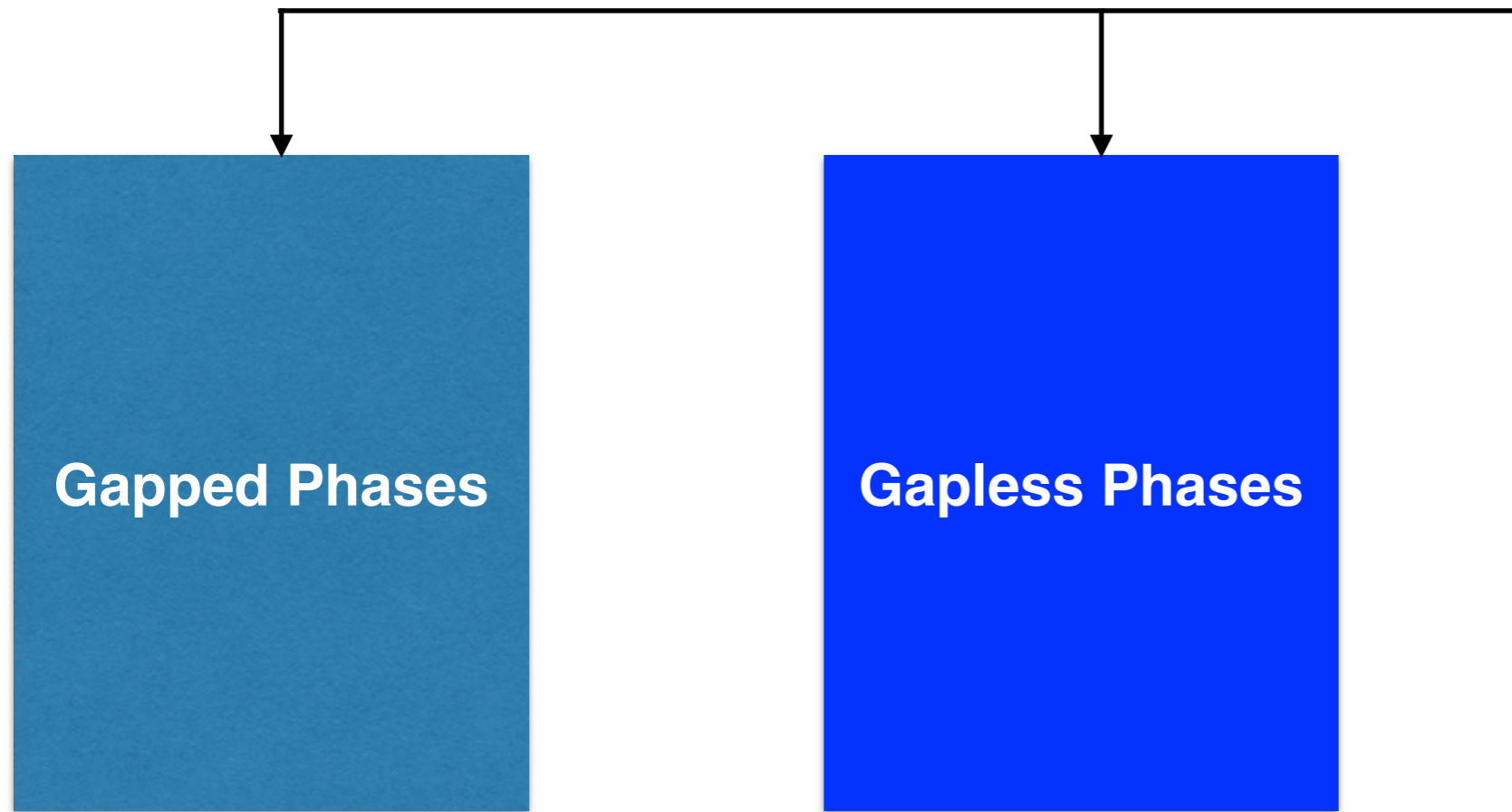
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Need **cooling (T=0)**

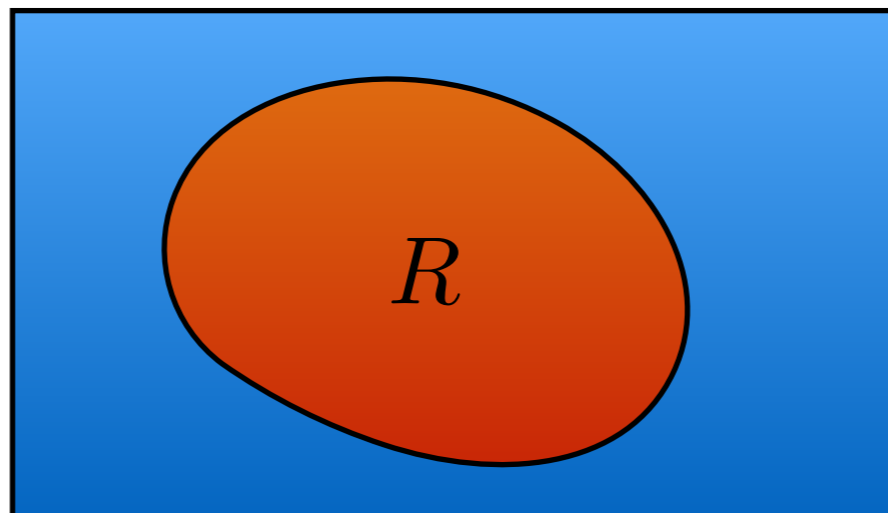
Introduction: Entanglement Signatures



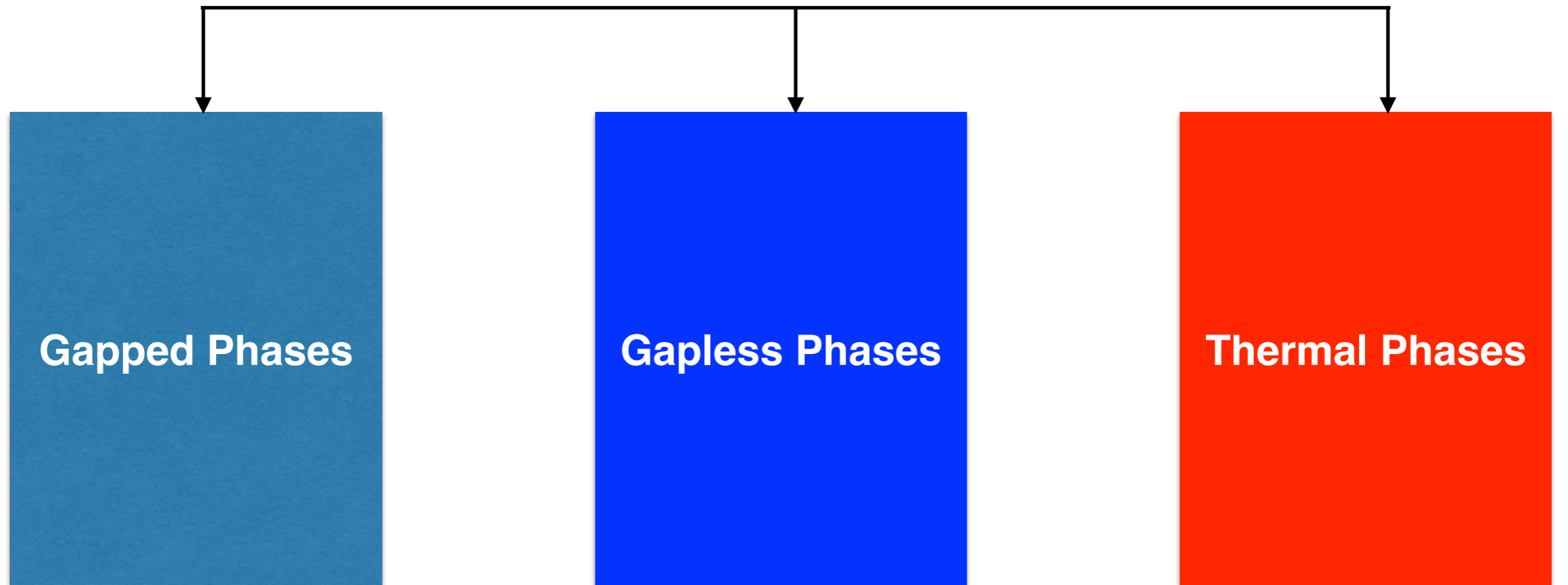
$$S(\rho_R) \sim \alpha |\partial R| - \frac{1}{2} \log D + \dots$$

Area Law * $\log \partial R$

Area Law



Introduction: Entanglement Signatures

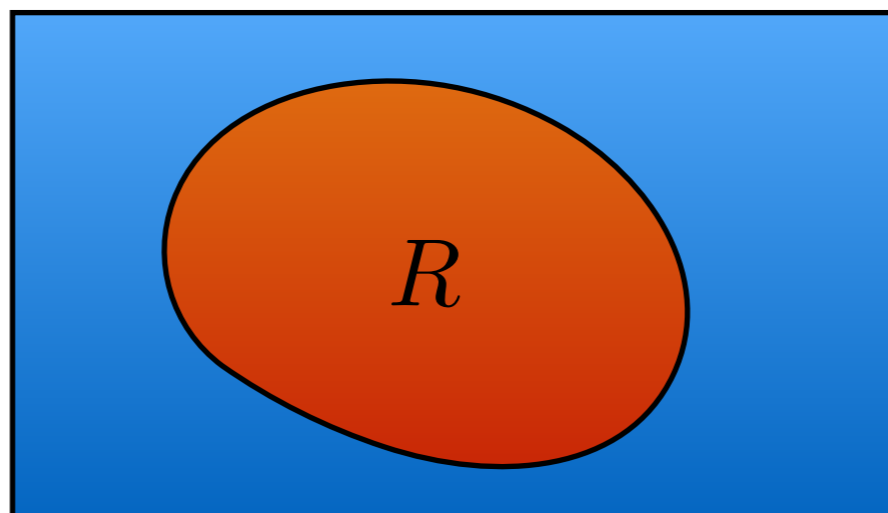


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Area Law * $\log \partial R$

$$S \sim \text{Volume}(R)$$

Area Law



Volume Law

Introduction: Classifying Gapped Quantum Phases

Ground states of gapped local Hamiltonians

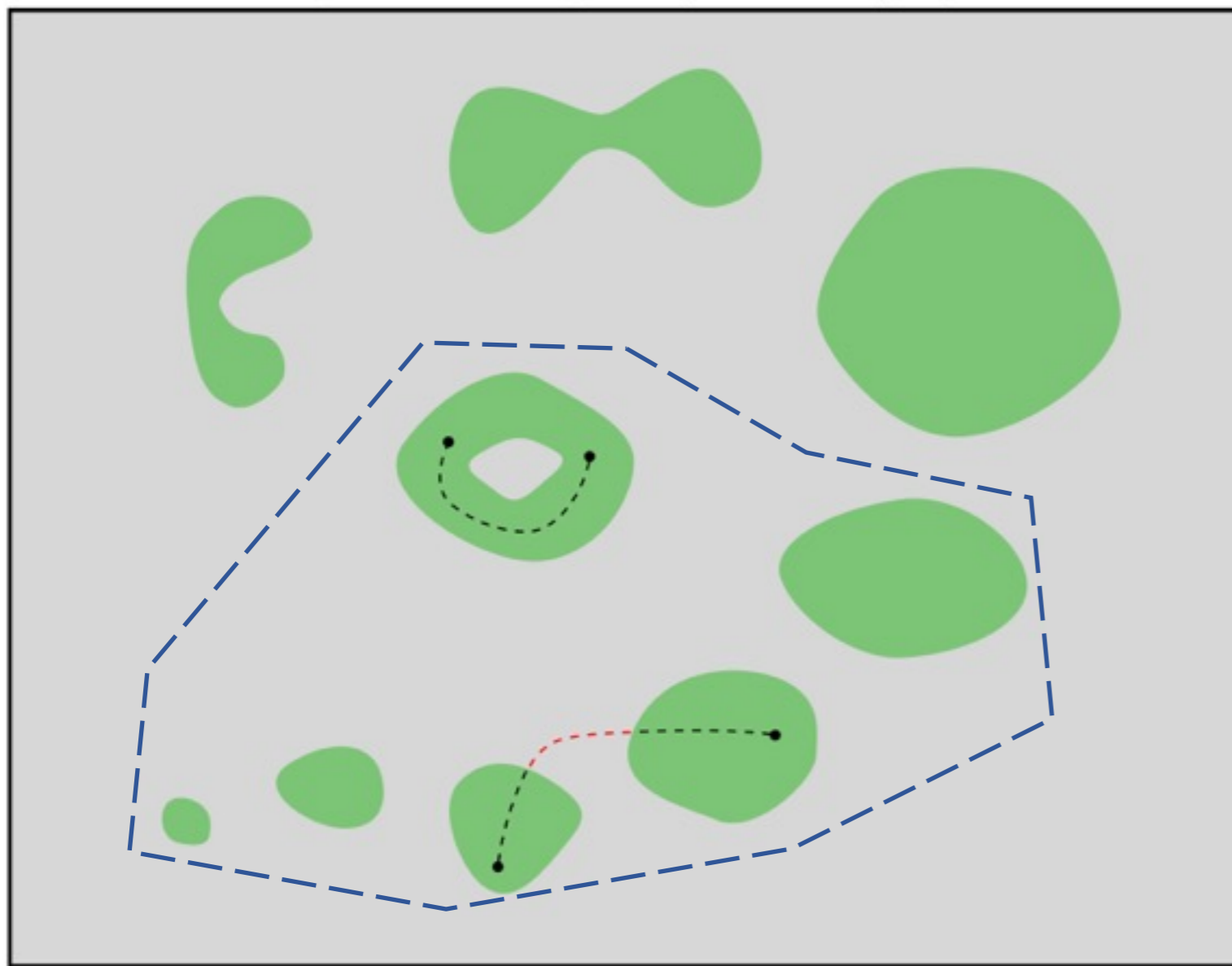
How do we classify
gapped
ground states?

No symmetry except
 $t \rightarrow t+a$



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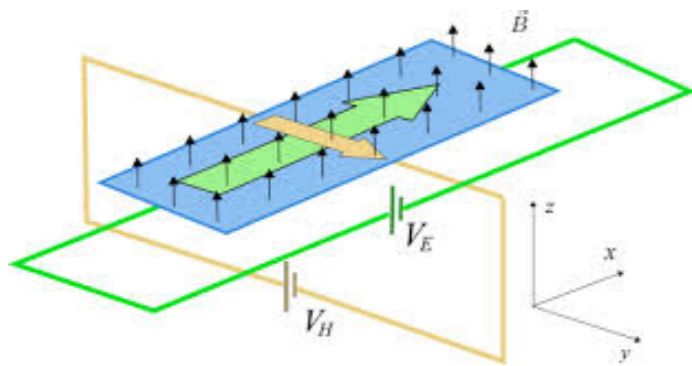


 gapped   gapless 

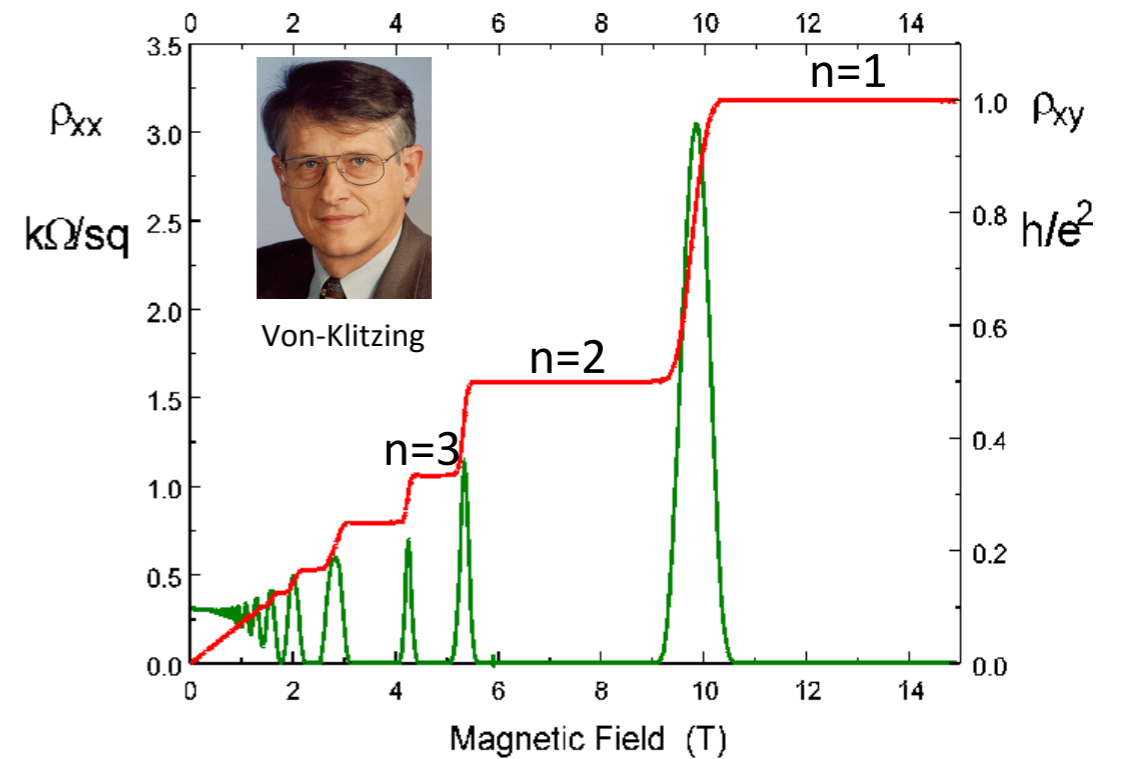
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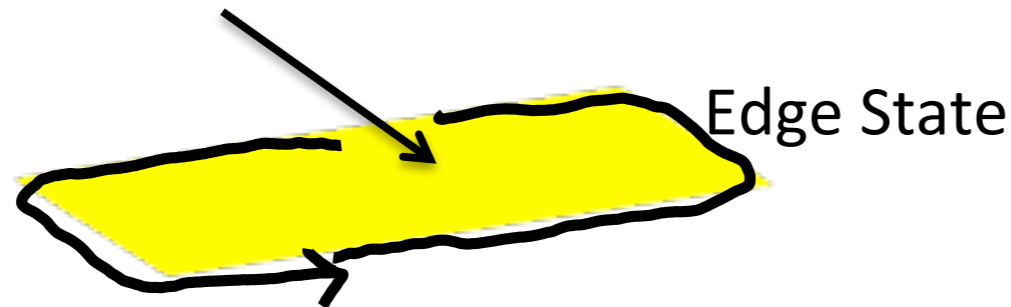
Example 1: Quantum Hall Phases



$$\rho_{xy} = \frac{h}{e^2} \frac{1}{n}$$



Bulk Gapped



Edge State

- Different integers - different phases.
- Need to cool to low temperatures - protected by energy gap.
- Also differentiated by *Thermal-Hall*. Integer in the right units.

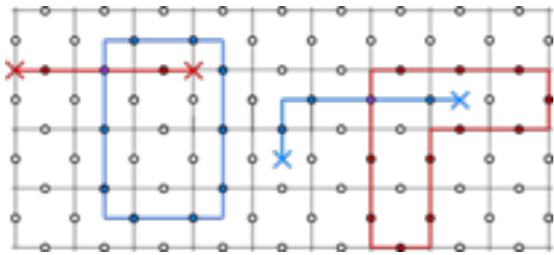
Example 2: Topological Order

- Topological order (Witten, Wen). eg. Fractional Quantum Hall & Toric Code/ Z_2 Gauge theory

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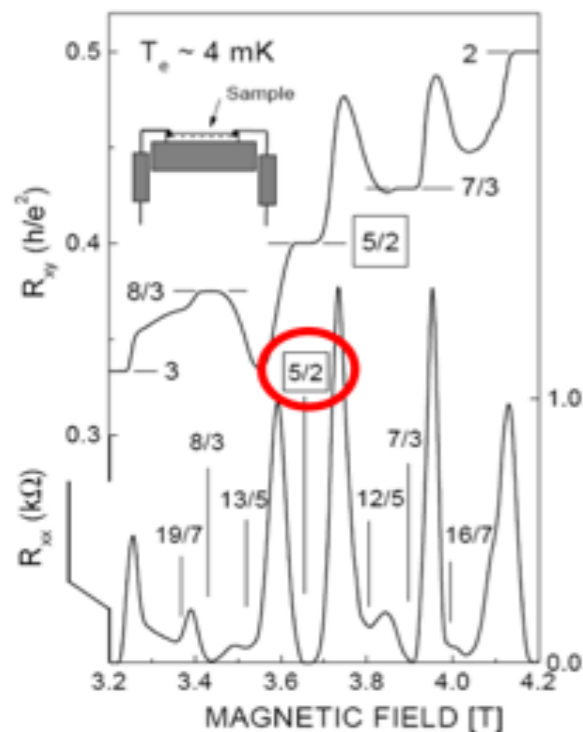


topological ground state degeneracy on a torus

anyonic excitations

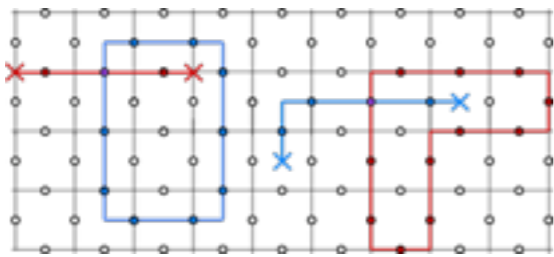
topological entanglement entropy

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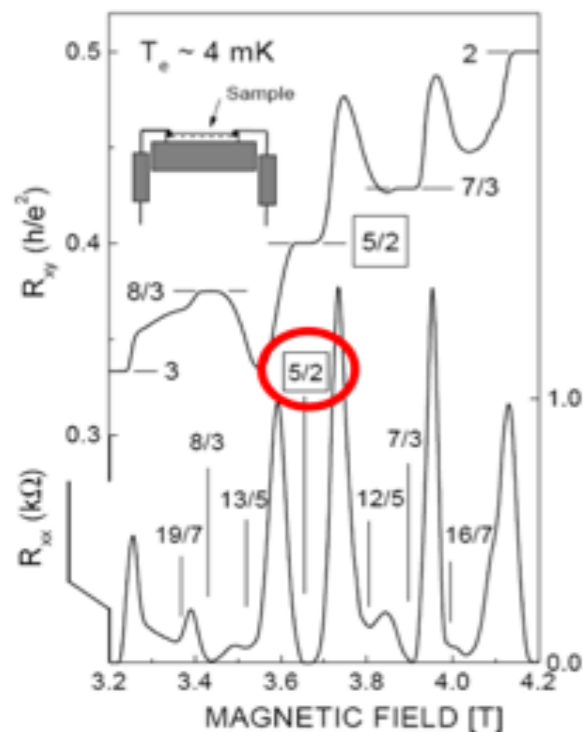


topological ground state degeneracy on a torus

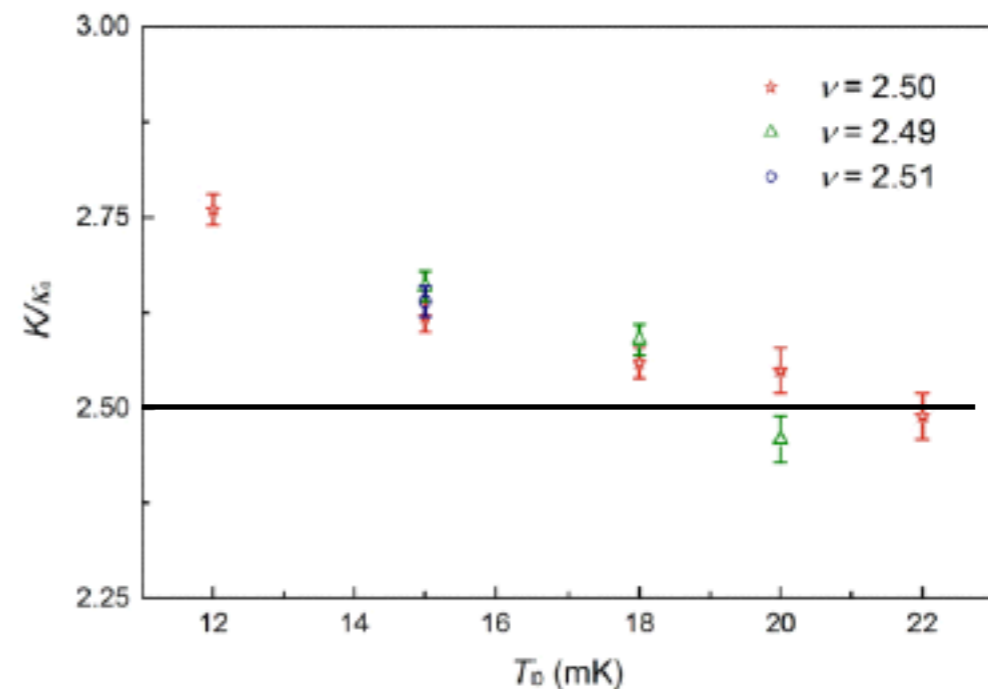
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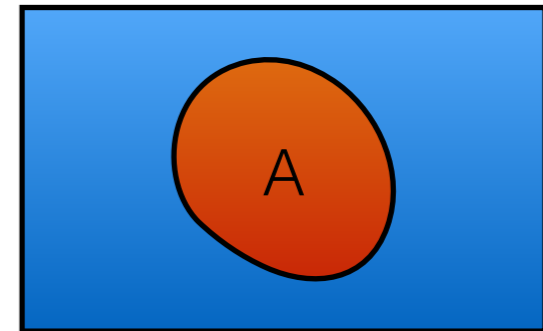
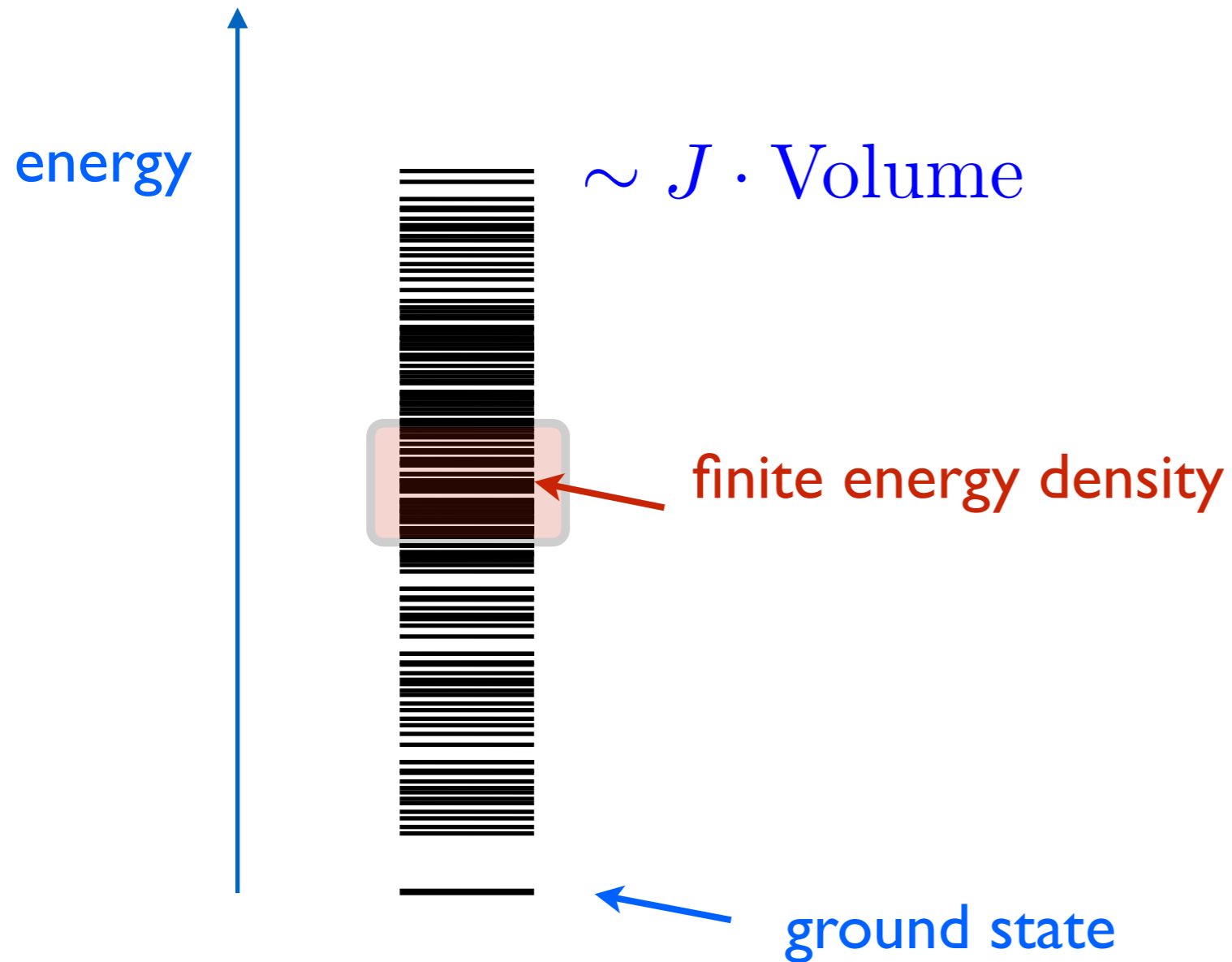
Half quantized
thermal Hall
 $c=5/2$



Banerjee et al.1710.00492

Can we observe topological properties at finite energy density?

Conventional answer: No

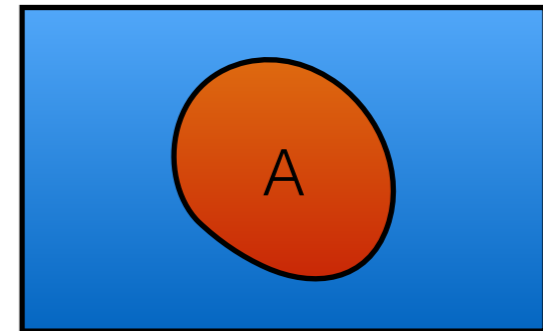
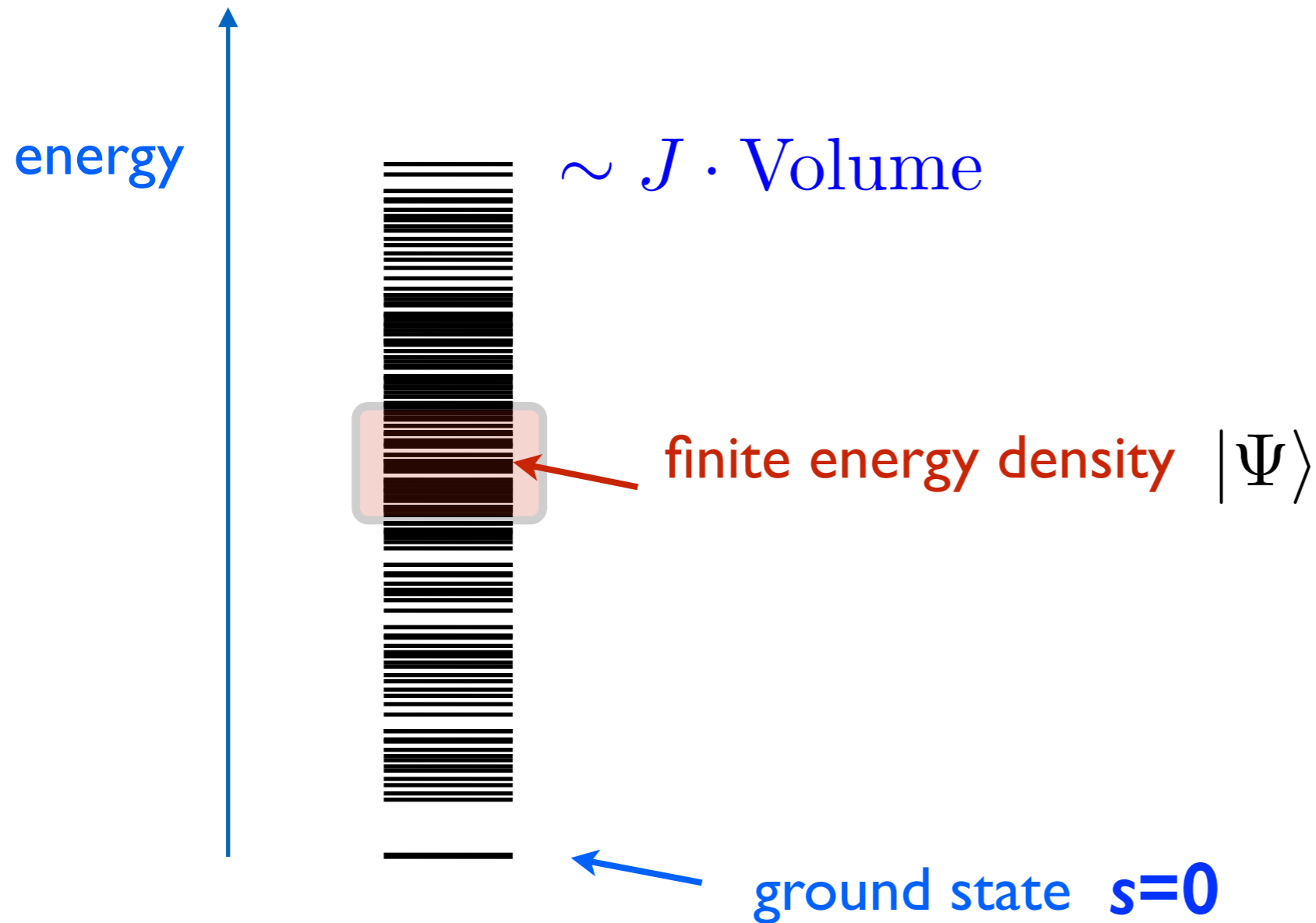


Deutsch 91, Srednicki 94

Eigenstate
Thermalization
Hypothesis
(ETH)

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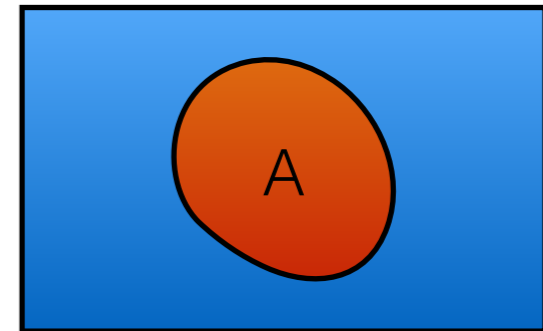
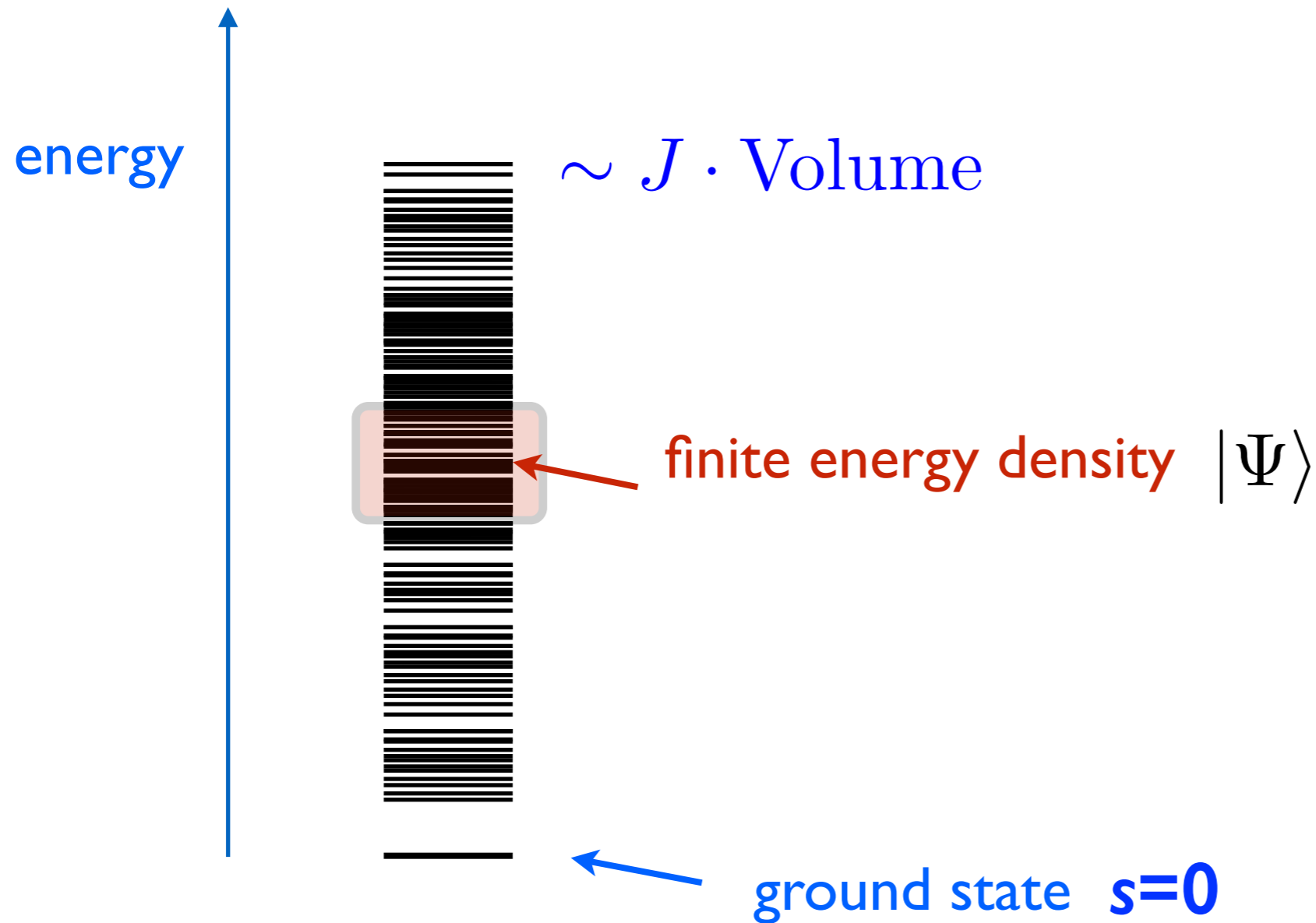


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$$\rho_a = e^{-H/k_B T}$$
$$s = S_A/N_A$$

Deutsch 91, Srednicki 94

Eigenstate
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(ETH)

Avoiding thermalization

- Hamiltonian made of commuting terms:

$$H_0 = h \sum_j \sigma_j^z \quad \leftarrow \text{Pauli z matrix on site } j$$

all eigenstates of H are area law entangled

(could also do e.g. toric code)

- BUT unstable to small perturbations due to translation symmetry:

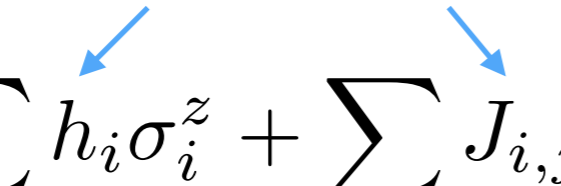
$$H = h \sum_j \sigma_j^z + J \sum_j \sigma_j^x \sigma_{j+1}^x + \dots$$

finite energy density eigenstates are volume law entangled

Avoiding thermalization

- Hamiltonian made of commuting terms with *disordered* coefficients:

disordered couplings

$$H_0 = \sum_i h_i \sigma_i^z + \sum_{i,j} J_{i,j} \sigma_i^z \sigma_j^z + \dots$$


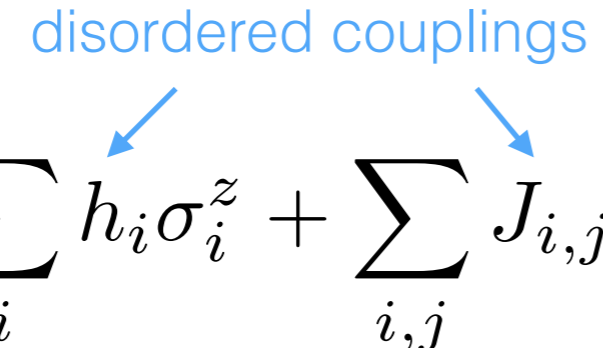
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Avoiding thermalization

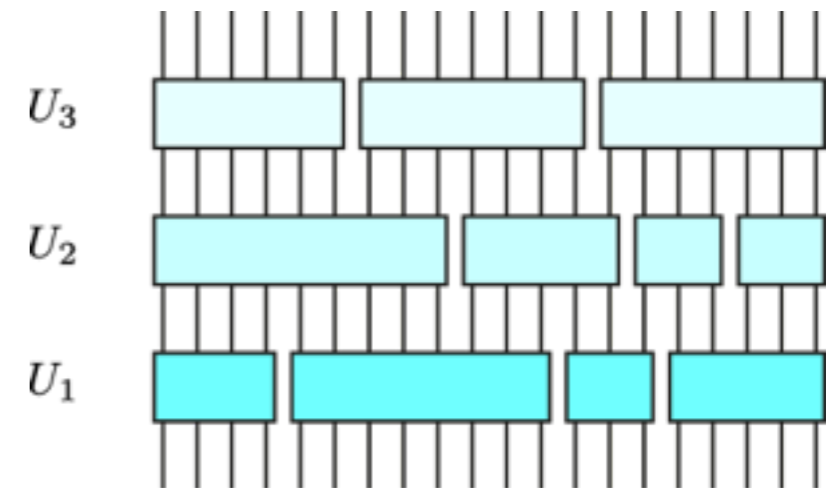
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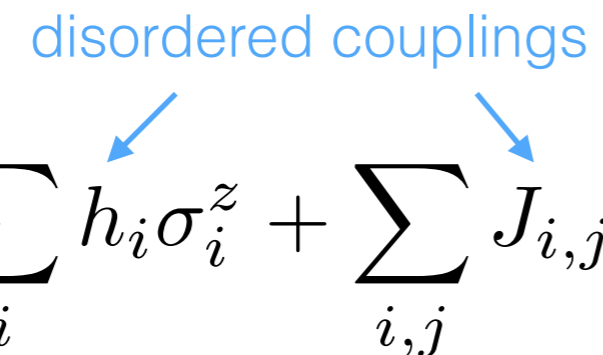


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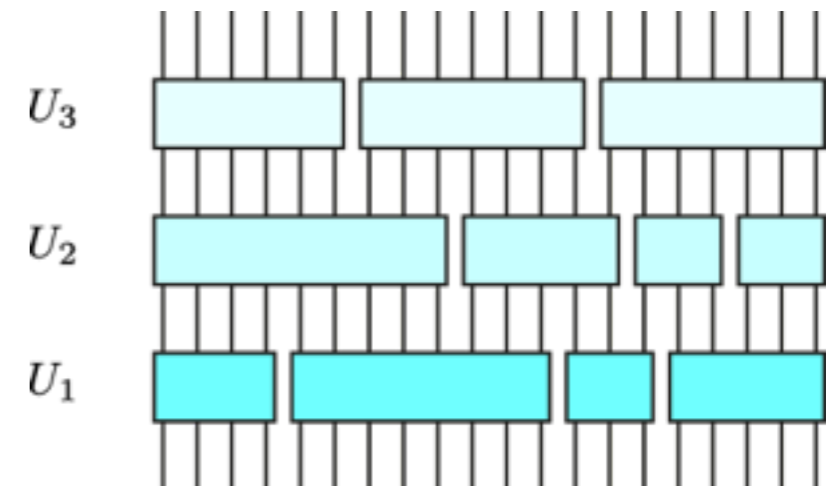
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$$H = H_0 + H_1$$



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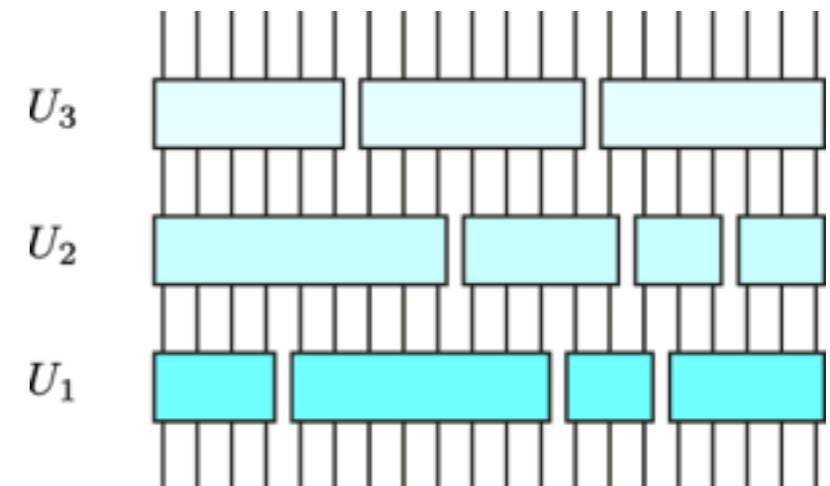
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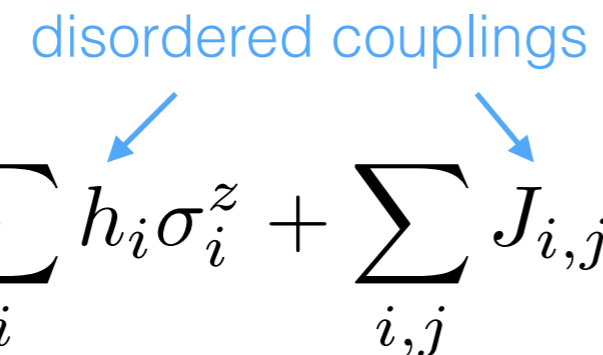


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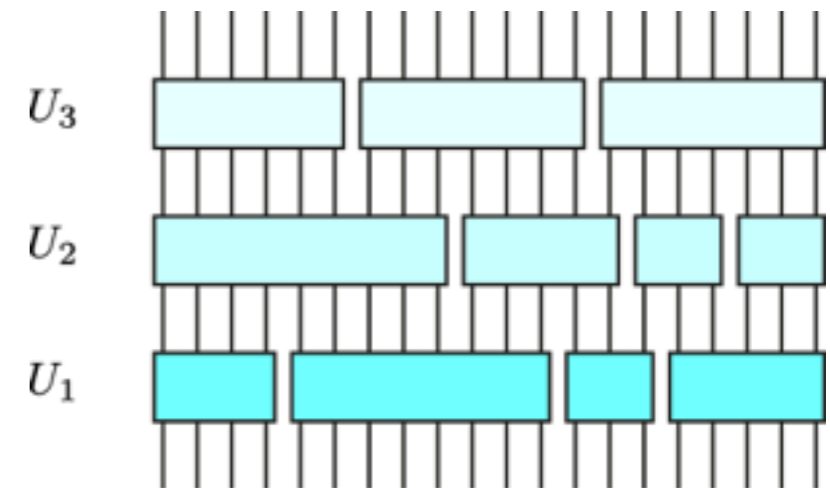
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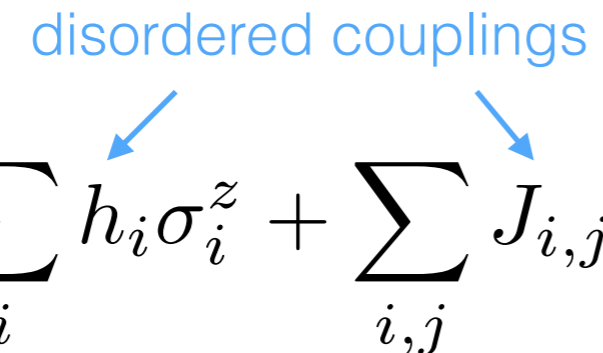
- can make H diagonal in z-basis using finite depth local unitary U

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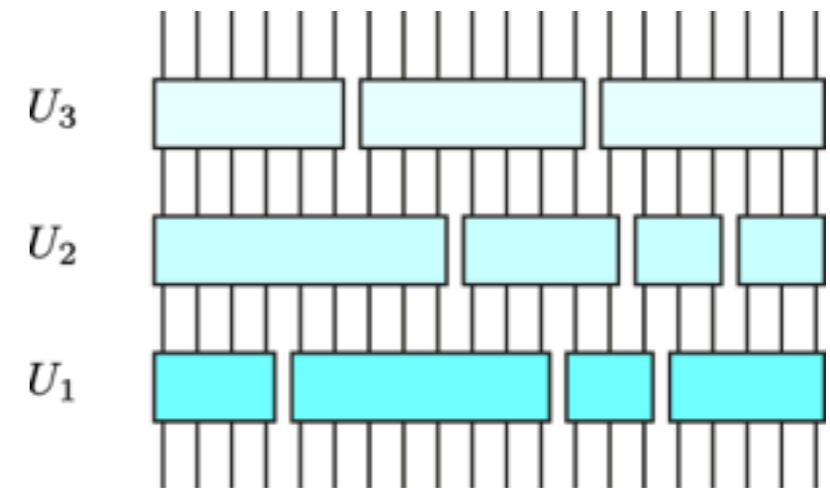
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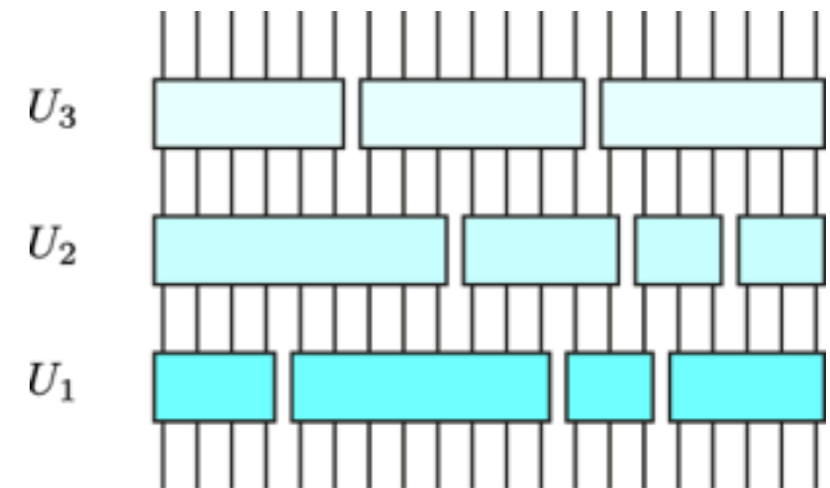
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$$U^\dagger X U$$

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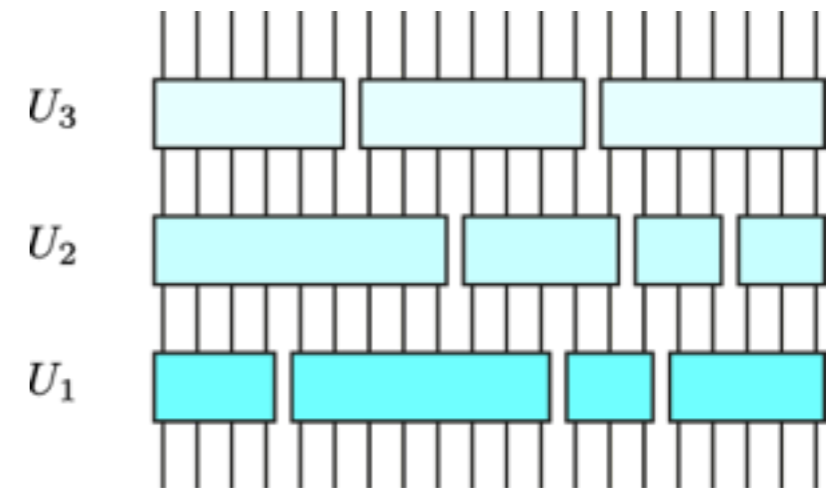
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- can make H diagonal in z-basis using finite depth local unitary U
(Imbrie)

if \mathbf{X} is a local operator, then $U^\dagger \mathbf{X} U$ ‘dressed’ quasi-local operator.

Many-body localization

(Anderson, Mirlin et al., Basko, Aleiner, Altshuler; Oganesyan, Huse, Pal)

- can make H diagonal in z-basis using finite depth unitary U:

$$U^\dagger H U = \sum_i h'_i \tau_i^z + \sum_{ij} J'_{ij} \tau_i^z \tau_j^z + \dots$$

- eigenstates of H are all of the form $U|\psi\rangle$;  product state in z-basis

in particular, area law entangled - excited states 'like' ground states!

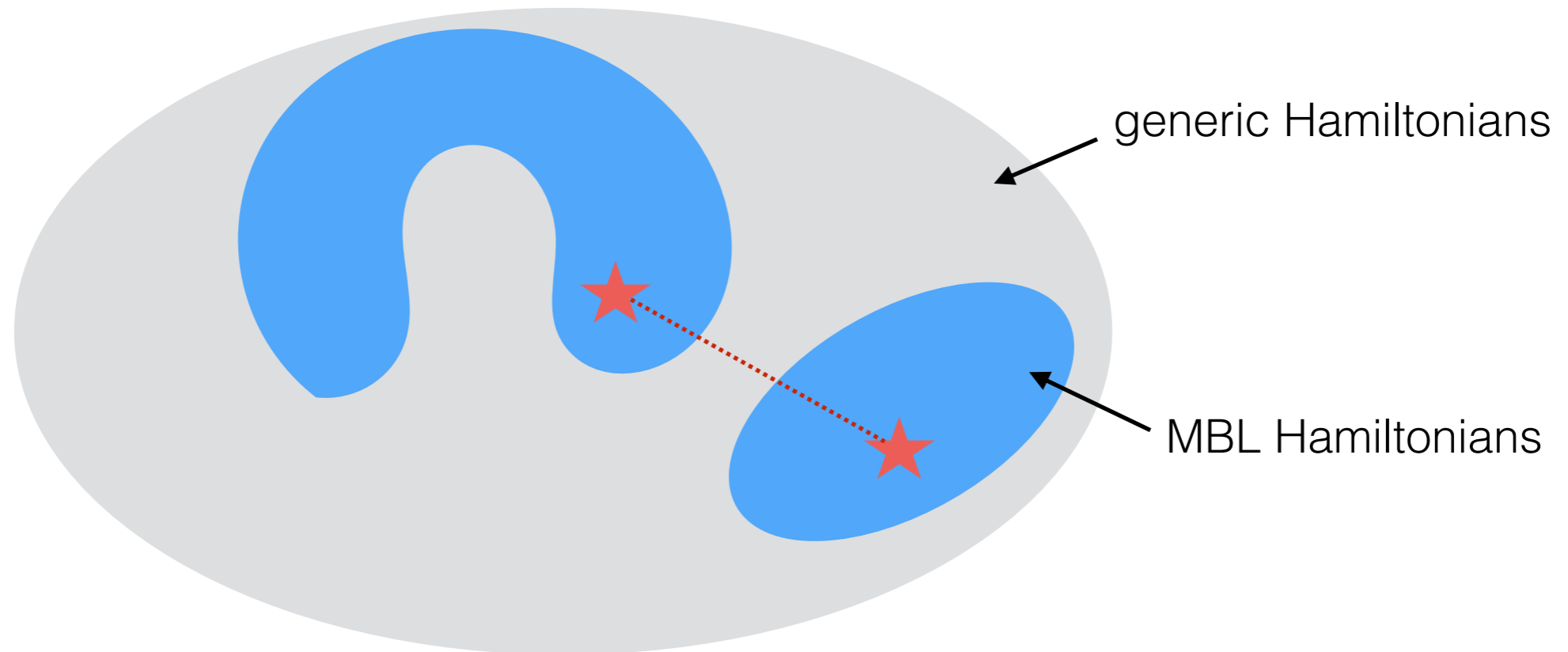
- $U \tau_i^z U^\dagger$ forms complete set of quasi-local conserved quantities

 'l-bits'

(Huse, Nandkishore, Oganesyan
Serbyn, Papić, Abanin,
Vosk&Altman)

Many-body localization and topological phases

- replace 'gapped' with 'many-body localized' (MBL)



- topological order / symmetry protected topological phases at finite energy density

(Bahri, Vosk, Altman, AV; Chandran, Khemani, Laumann, Sondhi, Huse, Nandkishore, Oganesyan, Pal)

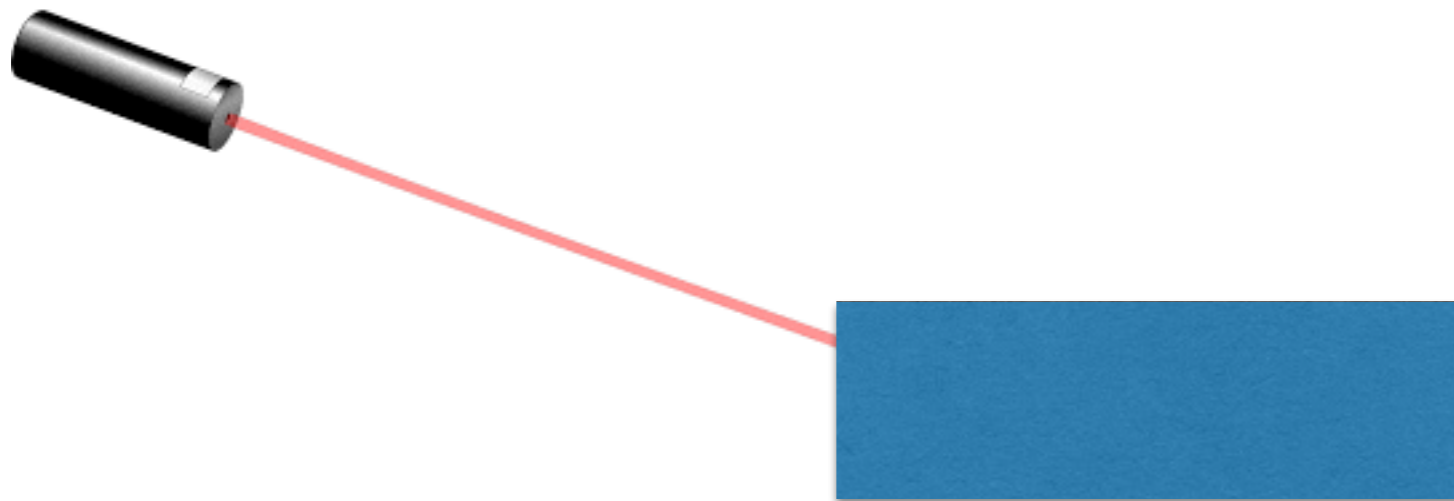
- BUT no chiral (quantum hall) phases allowed in MBL excited states. Commuting projectors incompatible with thermal Hall

(Kitaev, Levin, Potter-AV)

Floquet driving

- periodic time dependent Hamiltonian:

$$H(t + T) = H(t) \quad U_F = T \exp \left(i \int_0^T H(t) dt \right)$$

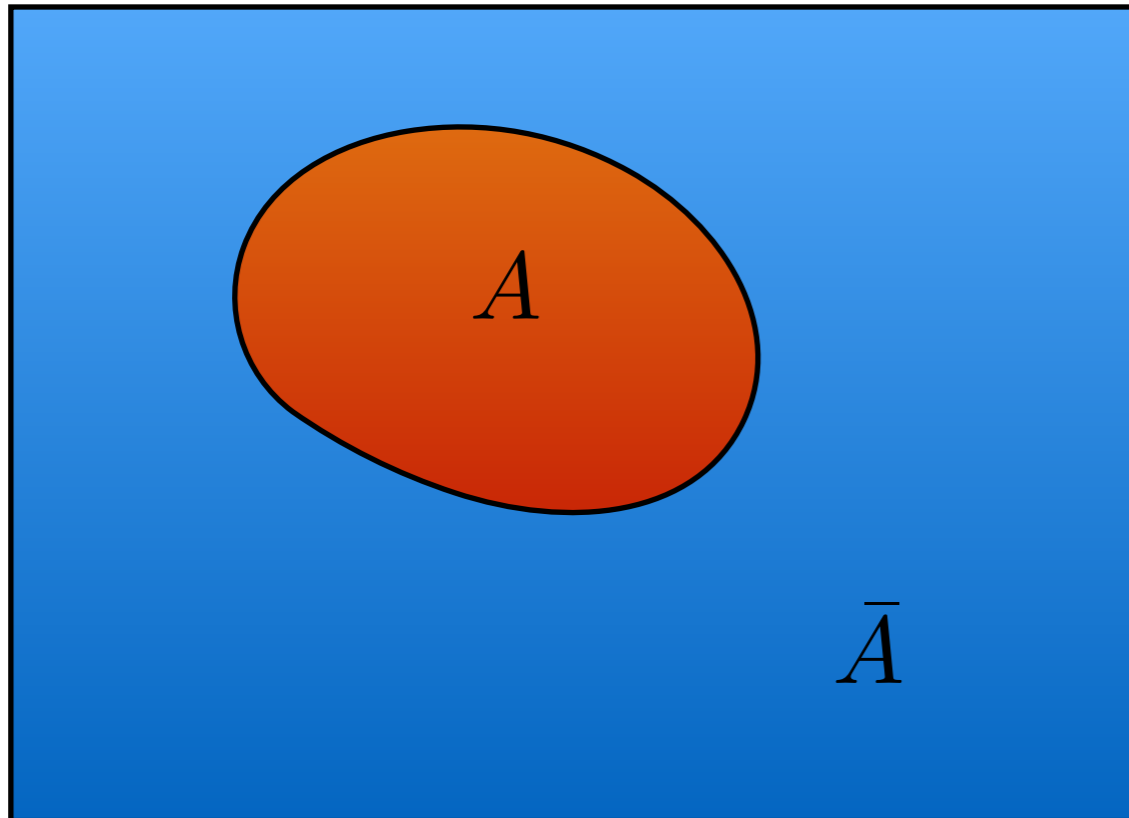


- diagonalize the 'Floquet unitary' U_F
Energy not conserved.

Only 'quasi-energy' mod $\hbar\omega = \frac{h}{T}$

Floquet systems: heating problem

- generically, system will absorb energy until it is at infinite temperature:



$$\rho_A(t) = \text{Tr}_{\bar{A}} |\Psi(t)\rangle\langle\Psi(t)|$$

$$\rho_A(t) \rightarrow \mathbf{1} \quad \left(= \frac{1}{Z} e^{-\beta H} \right)$$

as $\beta \rightarrow 0$

- entropy has been maximized,
- no energy constraint.
- Temperature $\rightarrow \infty$

MBL in Floquet systems:

- MBL can be stable upon turning on a time dependent periodic perturbation:

Ponte, Papic, Huveneers, Abanin;
Lazarides, Das, Moessner

$$U_F = e^{-iT H_{\text{eff}}}$$

with

$$U^\dagger H_{\text{eff}} U = \sum_i h'_i \sigma_i^z + \sum_{i,j} J'_{i,j} \sigma_i^z \sigma_j^z$$

- schematically,

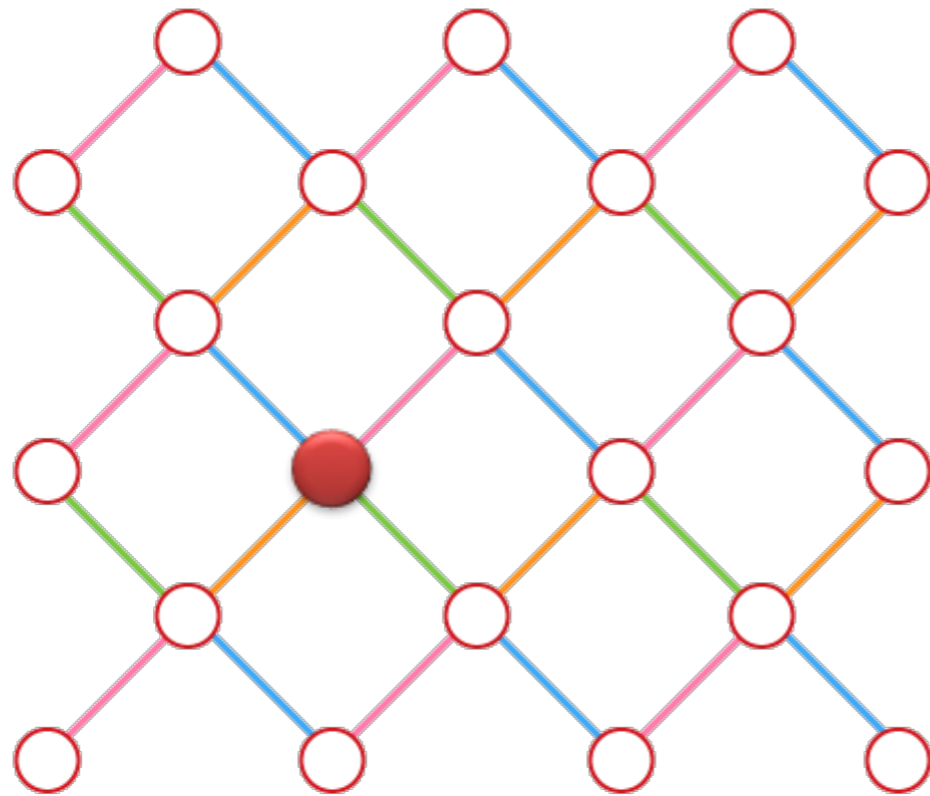
$$U_F = \prod_{\alpha} U_{\alpha} \leftarrow \text{quasi-local commuting unitaries}$$

- **Can we find MBL Floquet phases that have no equilibrium analogue?**
What distinguishes them? No symmetries.

Floquet SPTs in 1D & 3D: Else, Bauer, Nayak. Kayserlingk, Sondhi, Potter, Morimoto, AV. Potter, AV, Fidkowski.
Free fermions (here bosons/Spins): Kitagawa, Demler, Rudner, Lindner, Berg, Levin. Rafael. Harper, Roy.

MODEL of a CHIRAL Floquet phase:

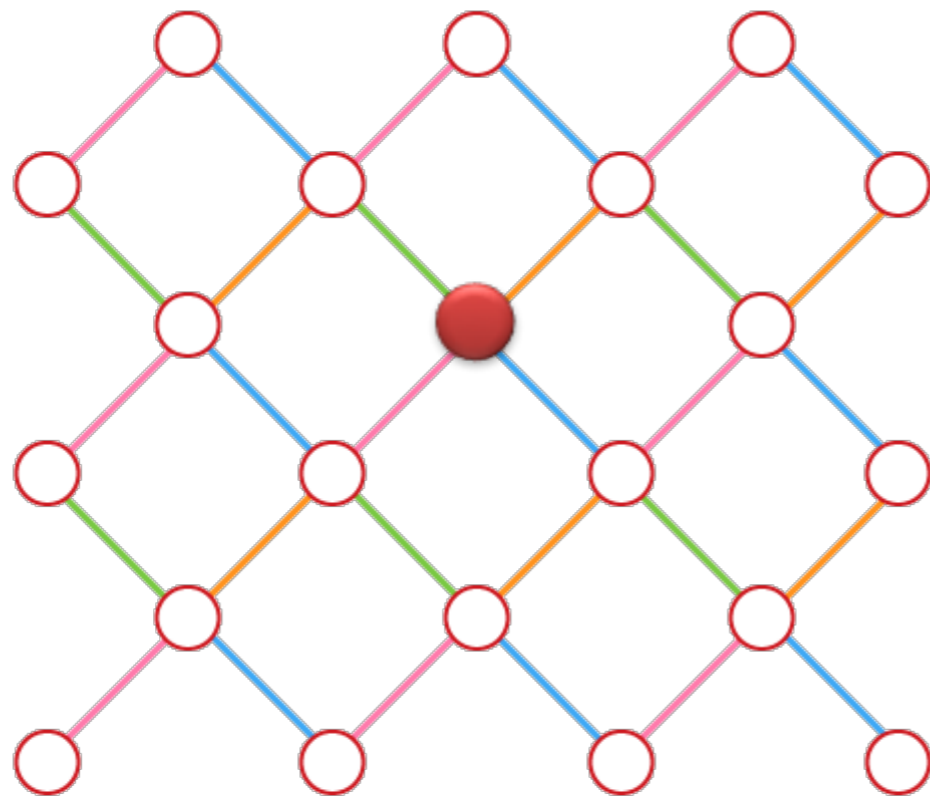
$U = U_1 U_2 U_3 U_4$ SWAP operation along different links



EDGE: Advances one unit
in each period

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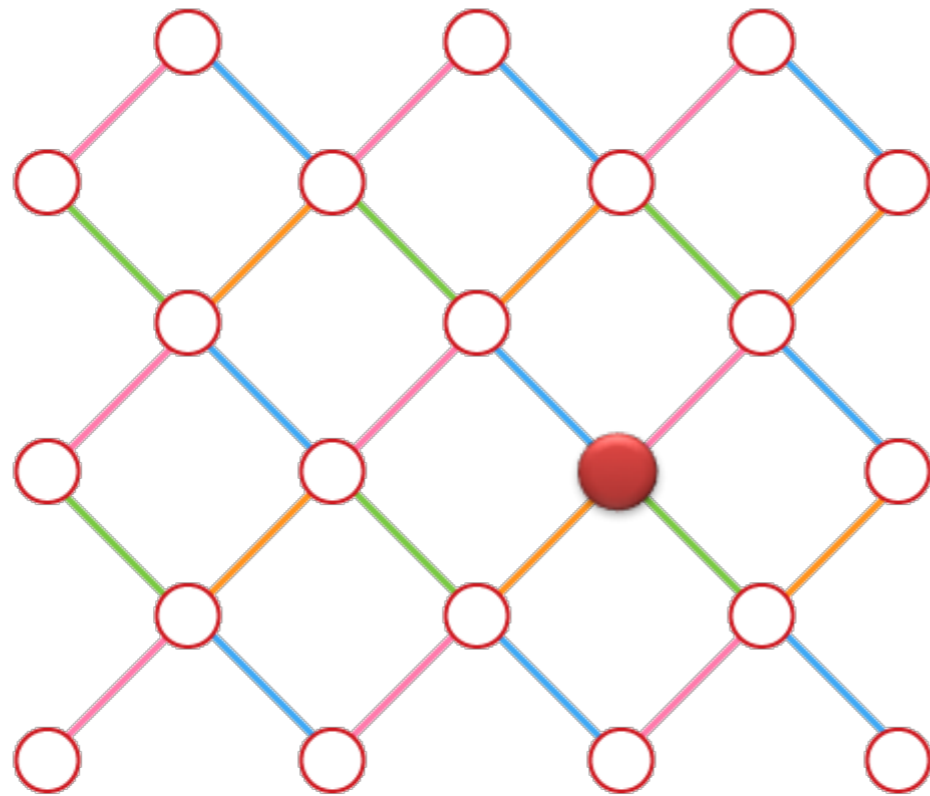
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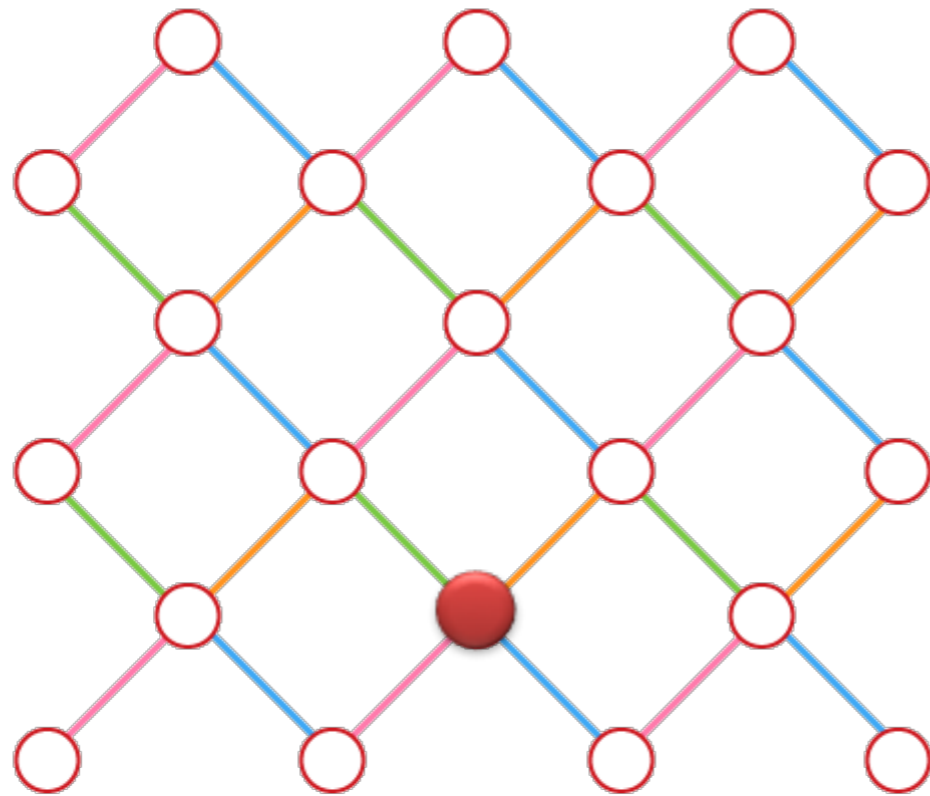
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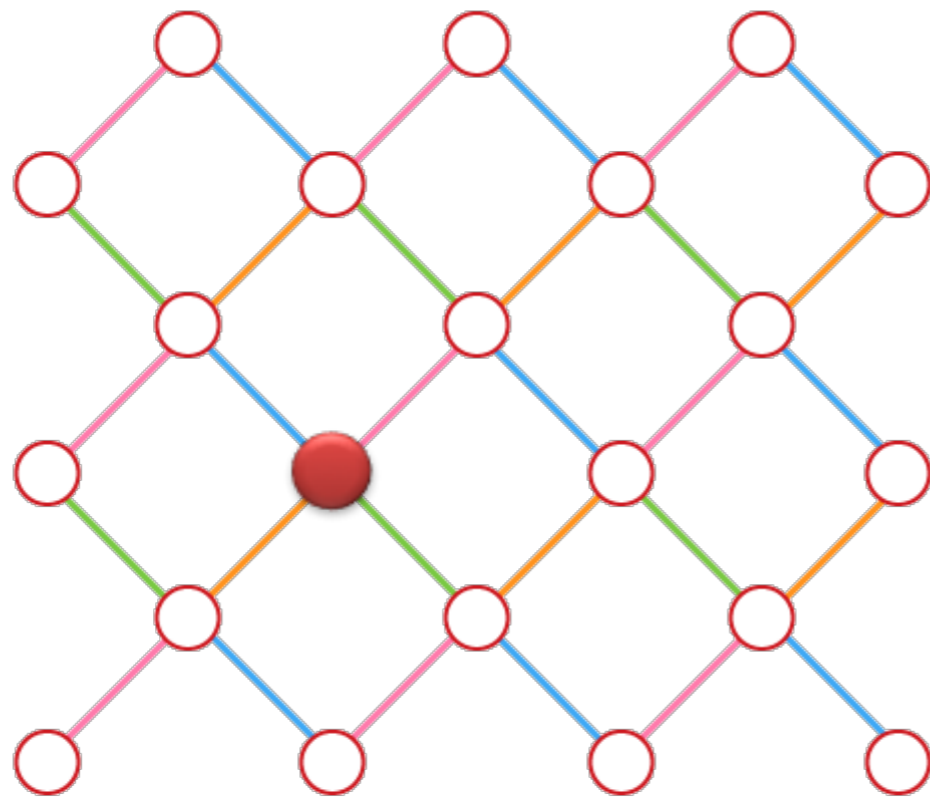
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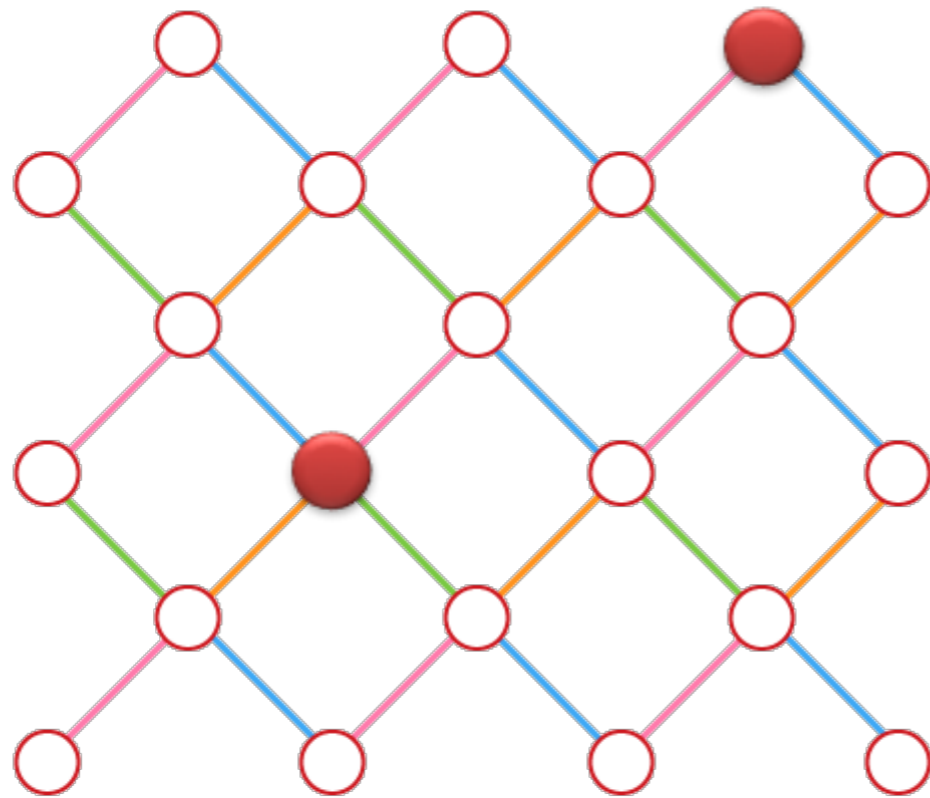
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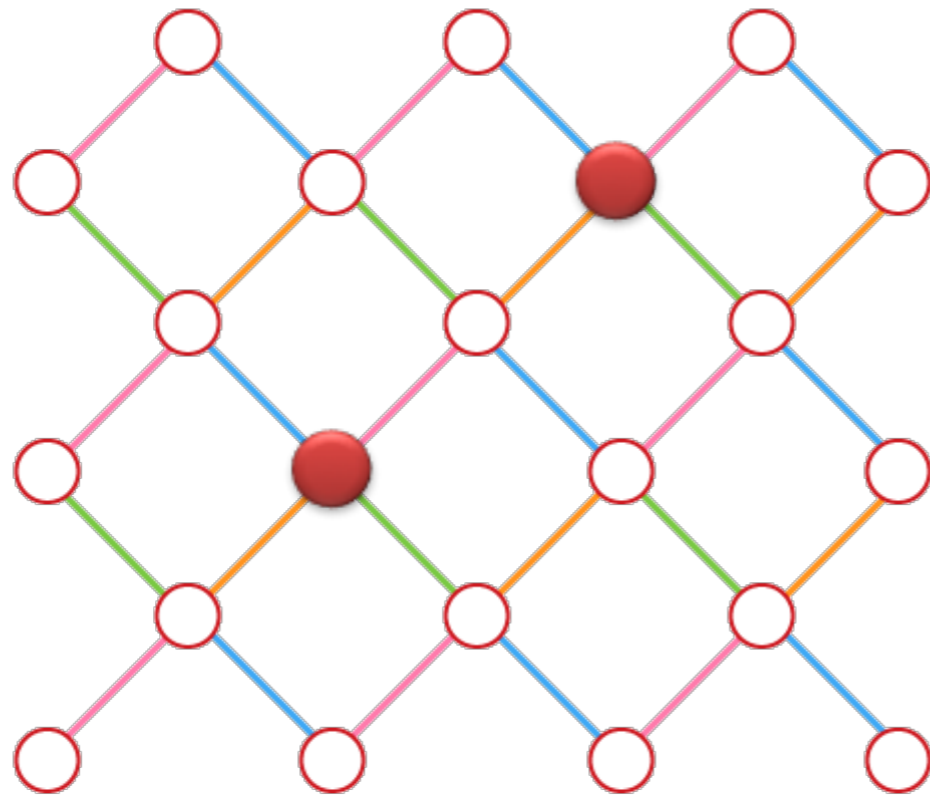
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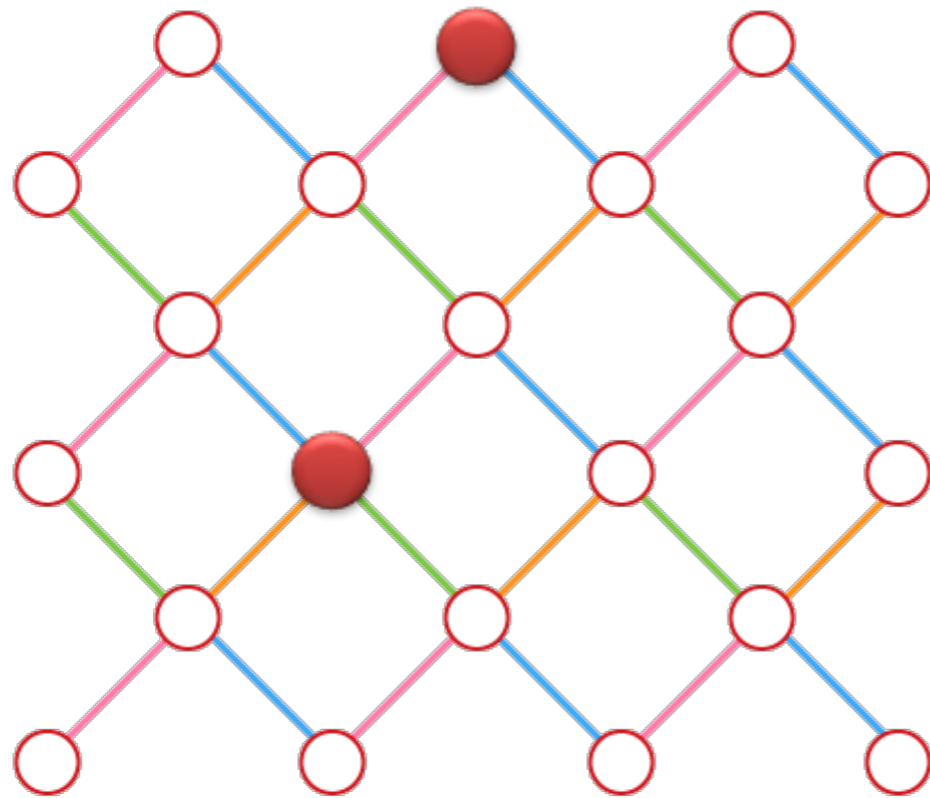
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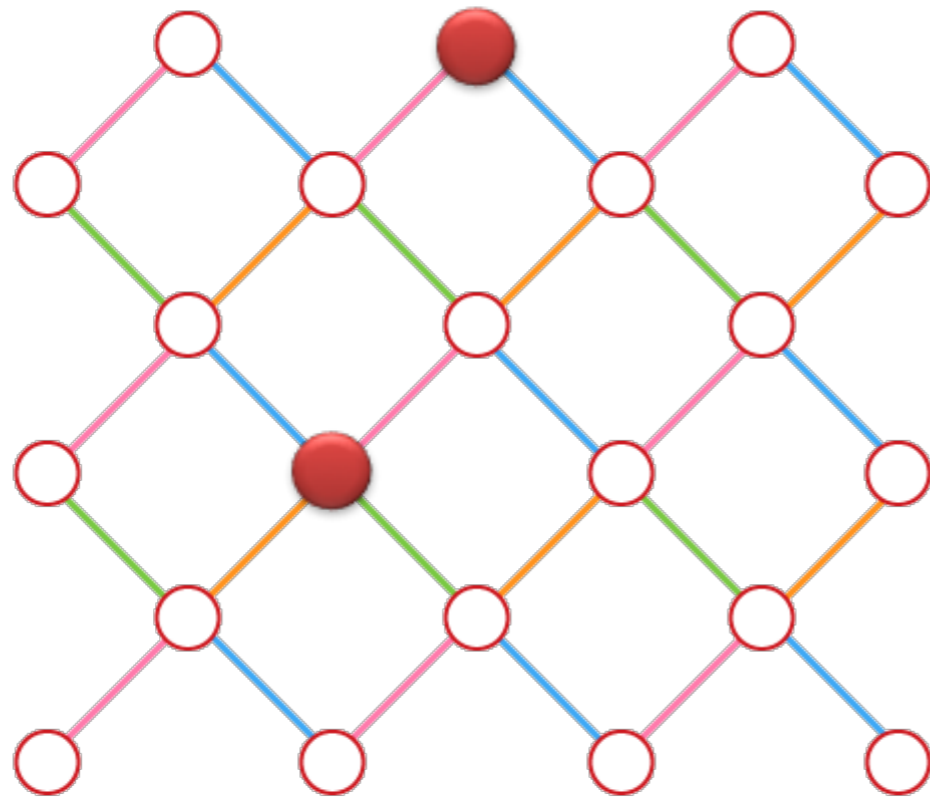
$U = U_1 U_2 U_3 U_4$ SWAP operation along different links



EDGE: Advances one unit
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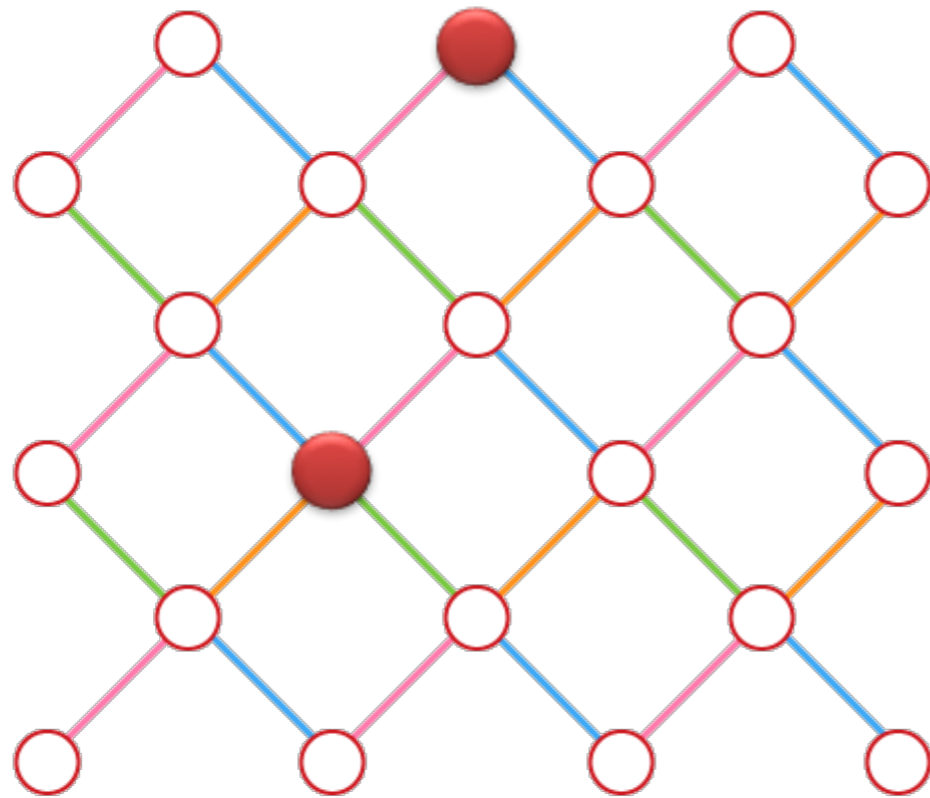
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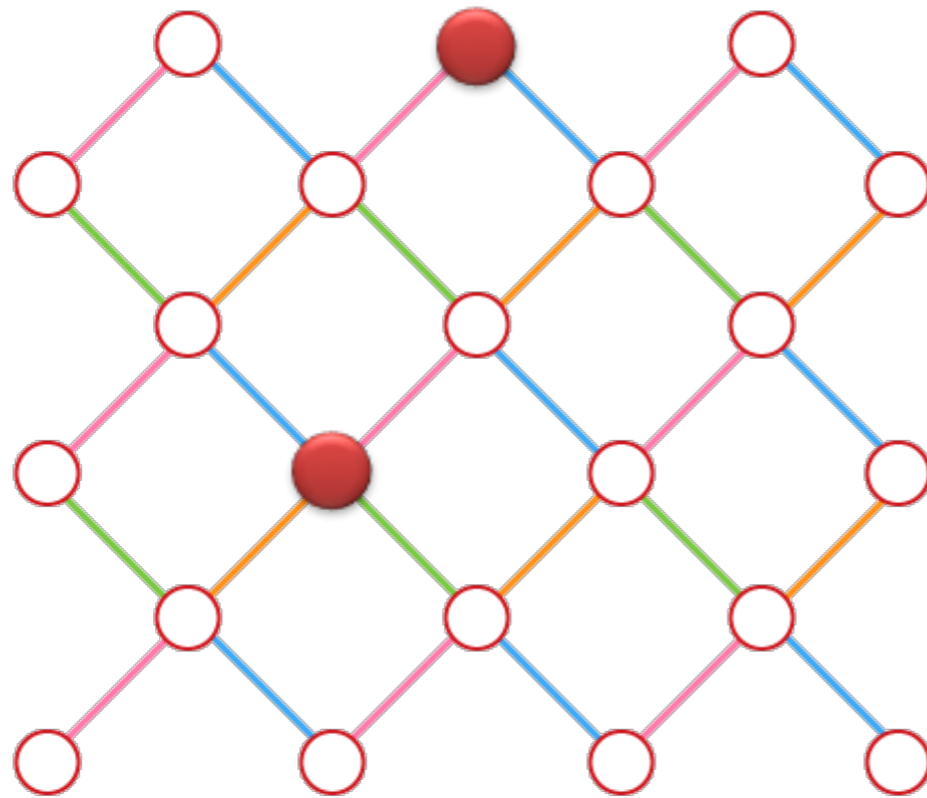


BULK: Reset to original state each period

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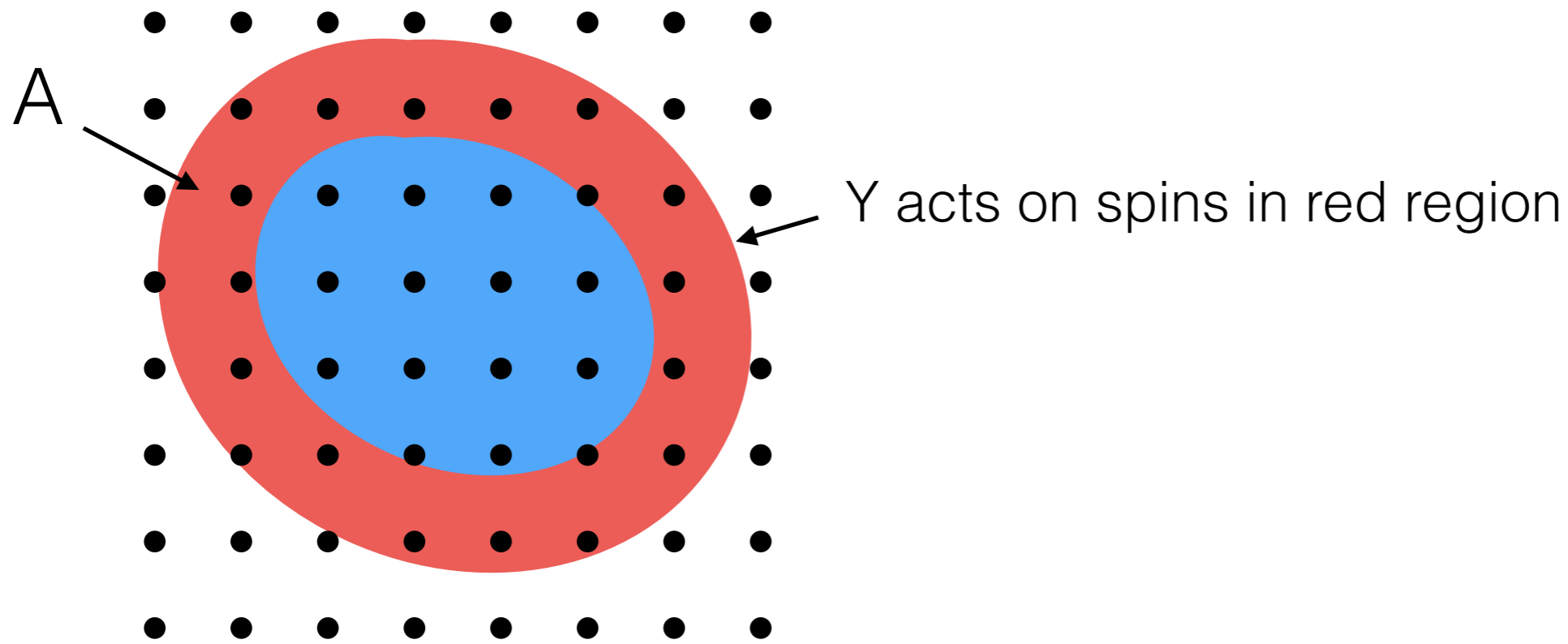
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Independent of
particle distribution!

Note: if SWAPs are not perfect - heating.
MBL gives a stable phase.

Classification of Chiral Unitaries



- Y is locality preserving: for any local operator \mathcal{O} , $Y^\dagger \mathcal{O} Y$ is a (quasi)-local operator supported nearby.
- is Y the Floquet operator of some (quasi)-local 1d Hamiltonian?
- OR is it anomalous - NO local 1D Hamiltonian such that:

$$Y_{1D} = e^{-i \int_0^T H_{1D}(t') dt'}$$

Classification of Locality Preserving 1D Unitaries

Commun. Math. Phys. 310, 419–454 (2012)
Digital Object Identifier (DOI) 10.1007/s00220-012-1423-1

Communications in
**Mathematical
Physics**

Index Theory of One Dimensional Quantum Walks and Cellular Automata

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characterizes the chiral flow of quantum information along the edge, and is a quantized invariant distinguishing different Floquet-MBL phases

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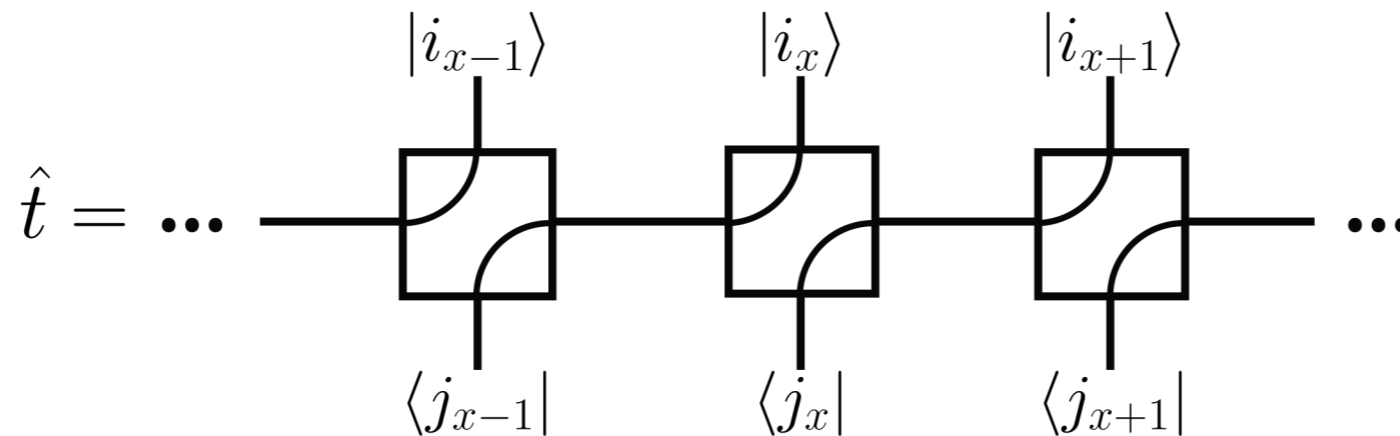
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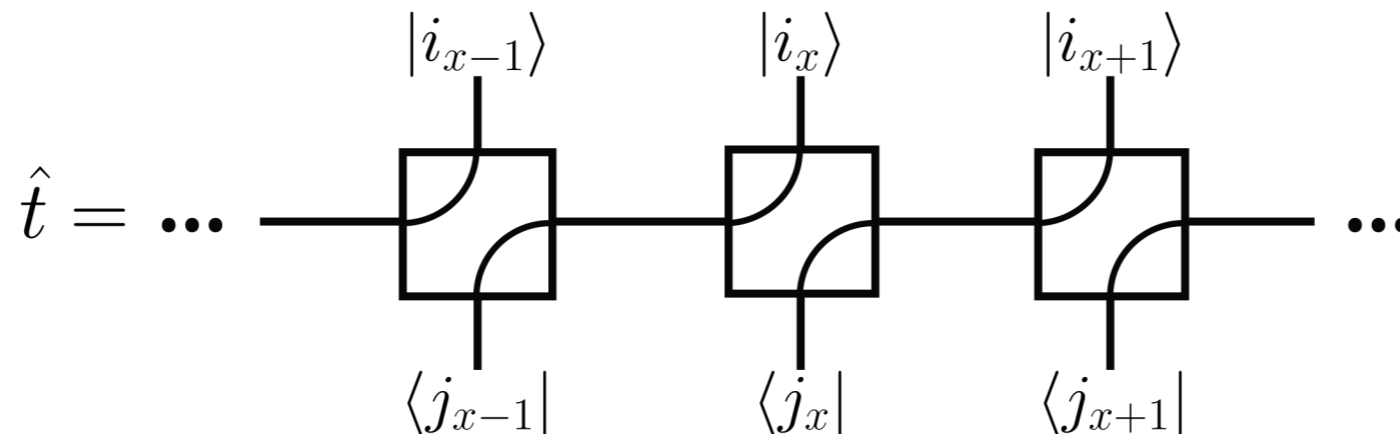
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Dimension of microscopic Hilbert space enters

e.g. $p=2$, $q=3$: a qu**bit** cannot cancel a qu**trit**!

Analogy with Quantum Hall Effect

zero temperature 2d
topological phase

MBL Floquet system

Bulk gap



Bulk MBL

Low energy field
theory for the 1d edge

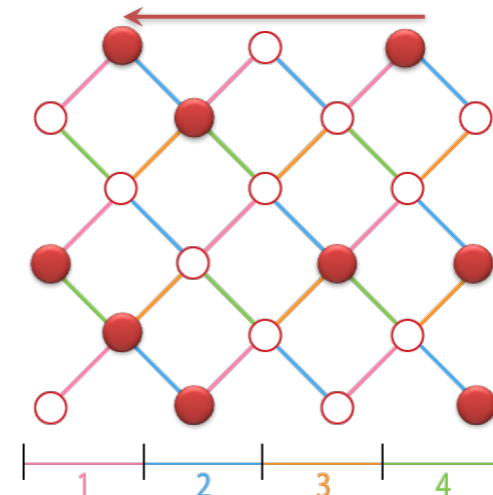
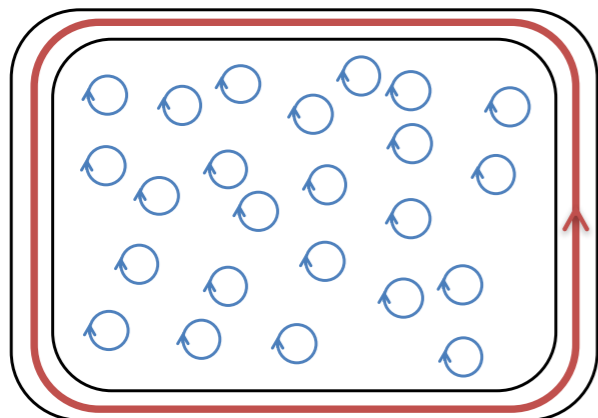


Locality preserving unitary Y on the 1d edge

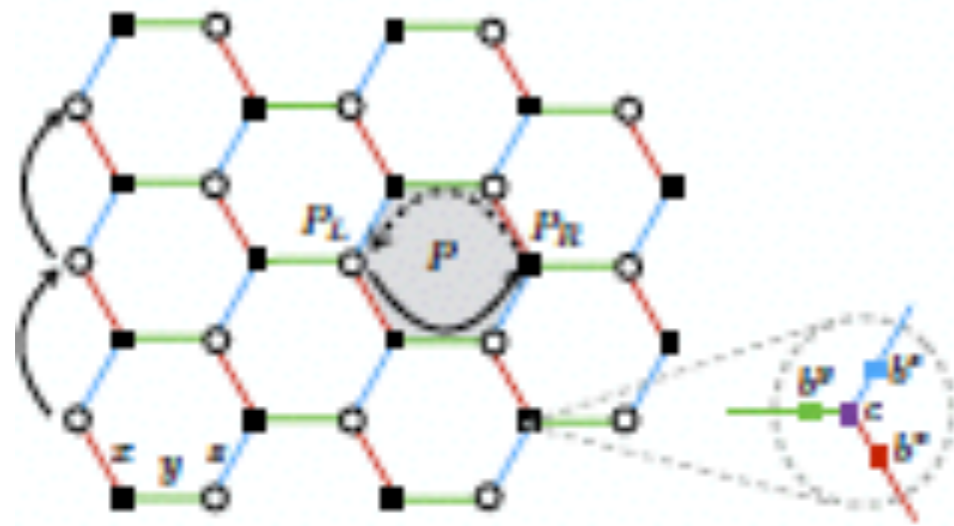
lack of 1d UV
completion for low
energy edge theory
(e.g. chiral anomaly)



Impossibility of writing Y as the Floquet
evolution of a 1d driving Hamiltonian

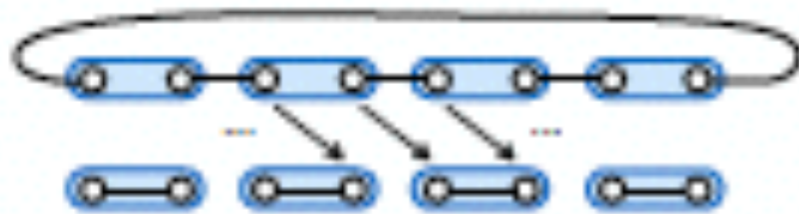


Floquet Enriched Toric Code



$$U(T) = e^{-i\hbar|x|} e^{-i\hbar|y|} e^{-i\hbar|z|}, \quad h^{[j]} = \frac{\pi J}{4} \sum_{\langle rr' \rangle \in j} S_r^j S_{r'}^j$$

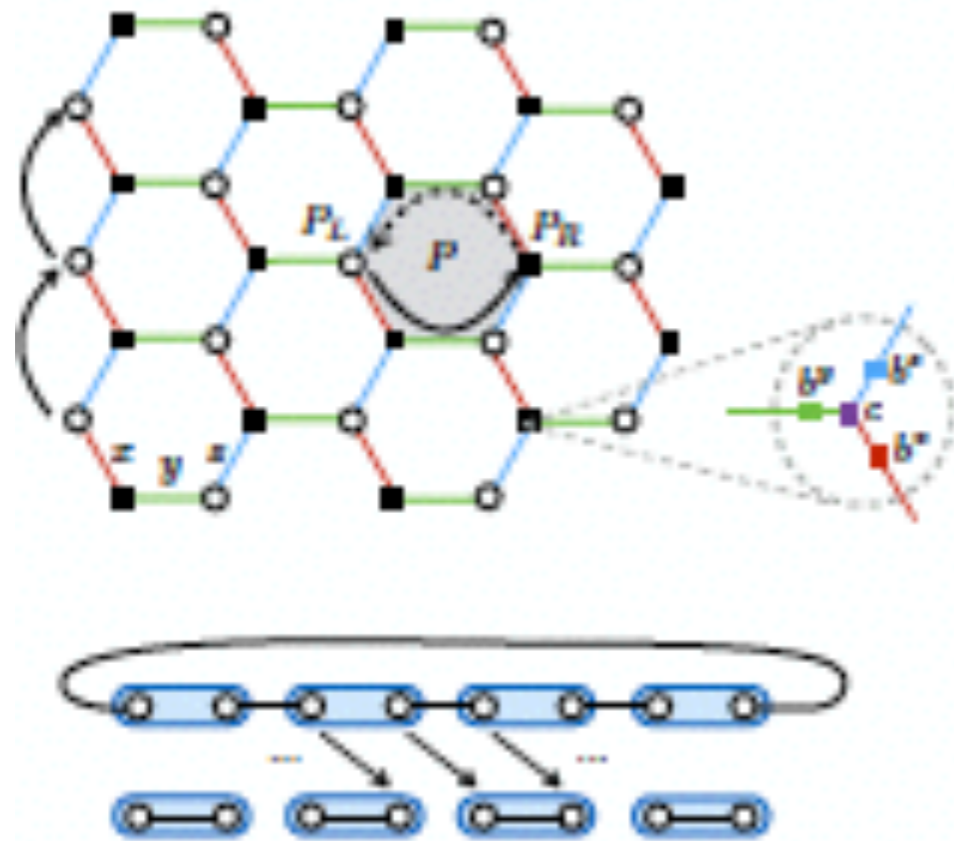
Bulk Eigenstates have Z_2 topological order.



Radical chiral Floquet phases in a periodically driven Kitaev model and beyond

Hoi Chun Po,^{1,2} Lukasz Fidkowski,^{3,4} Ashvin Vishwanath,^{1,2} and Andrew C. Potter⁵

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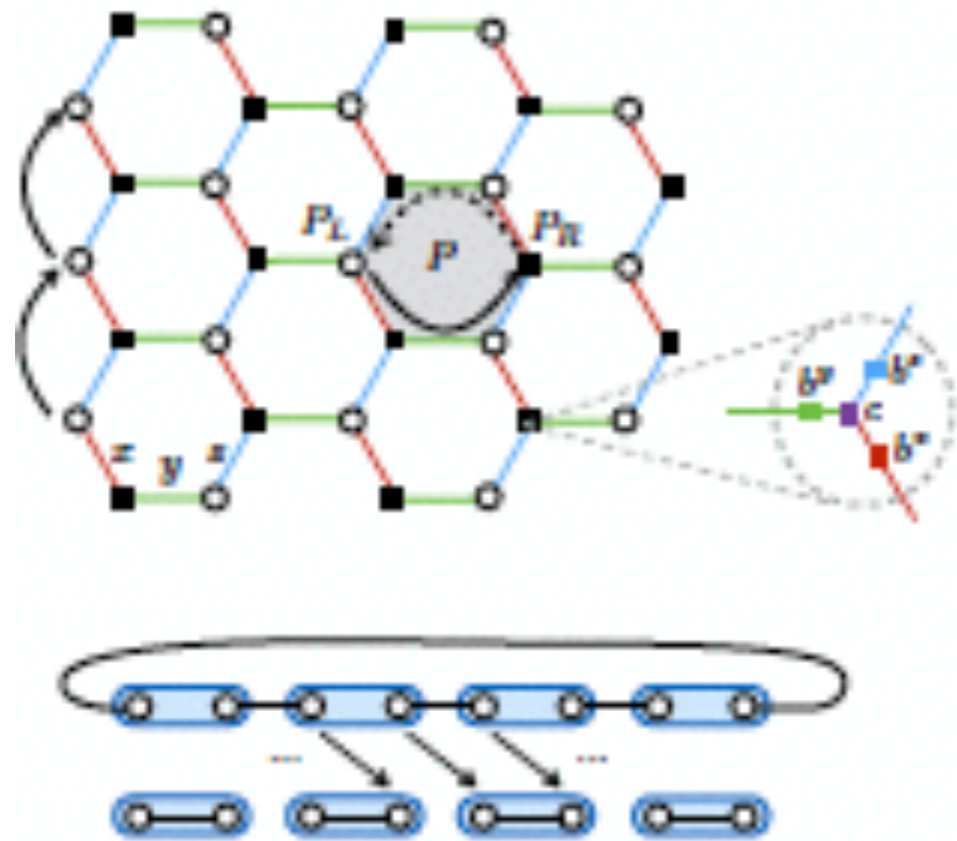
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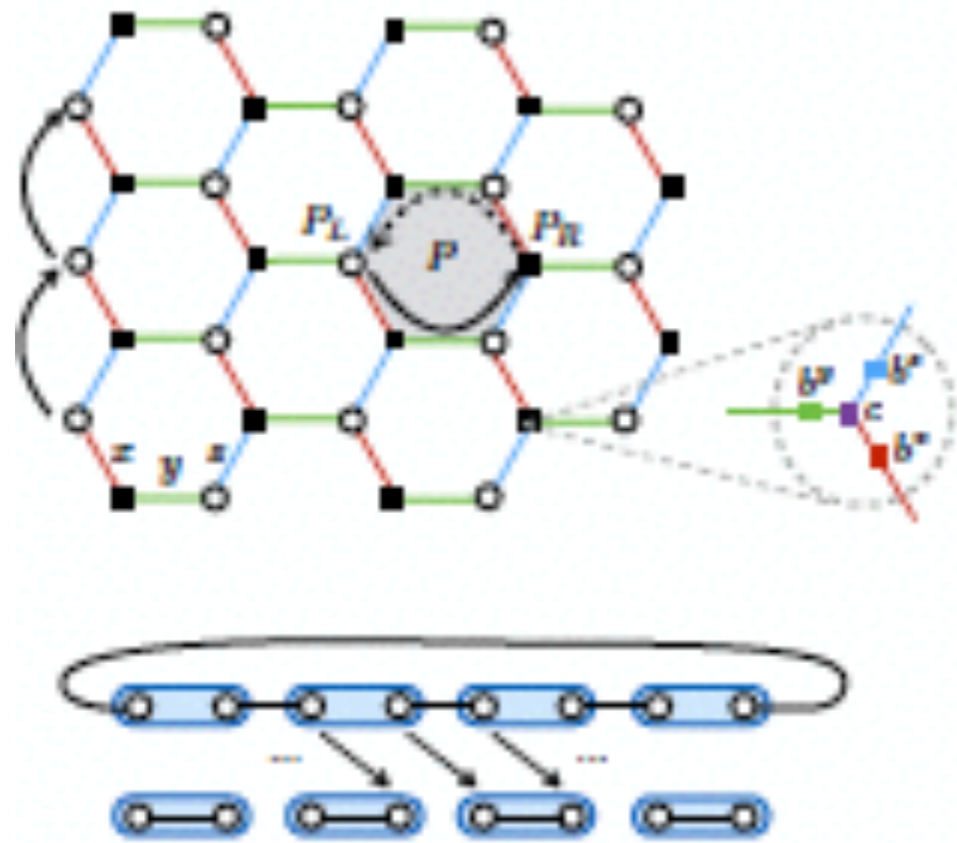
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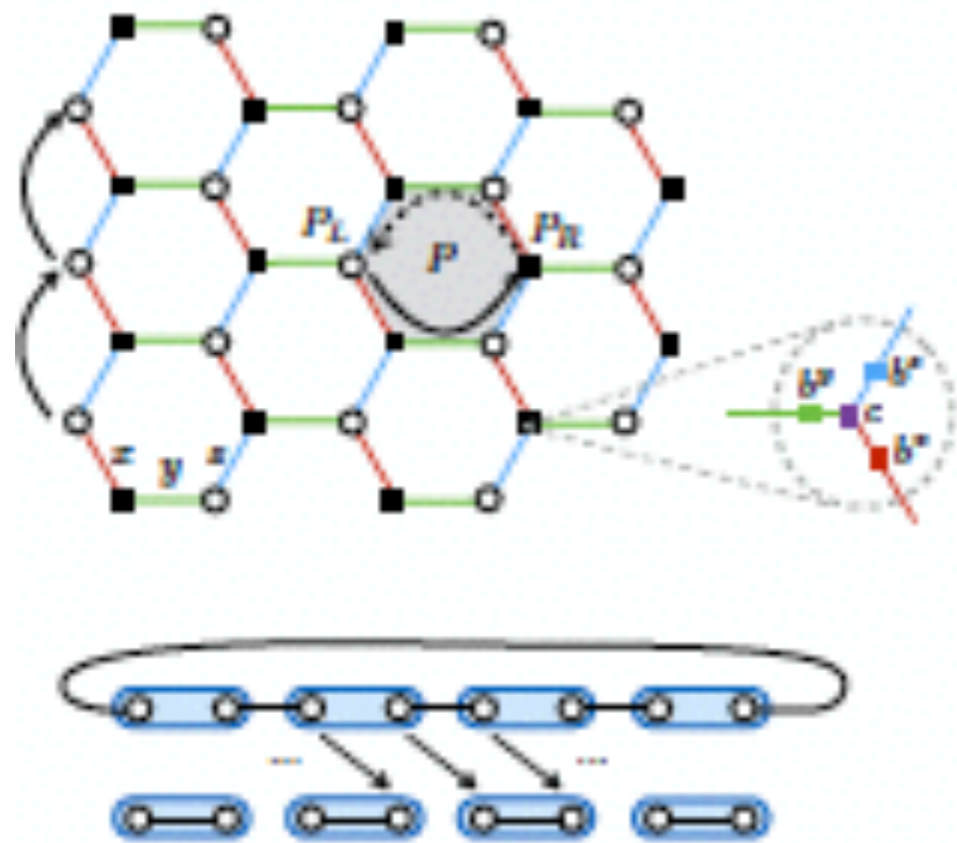
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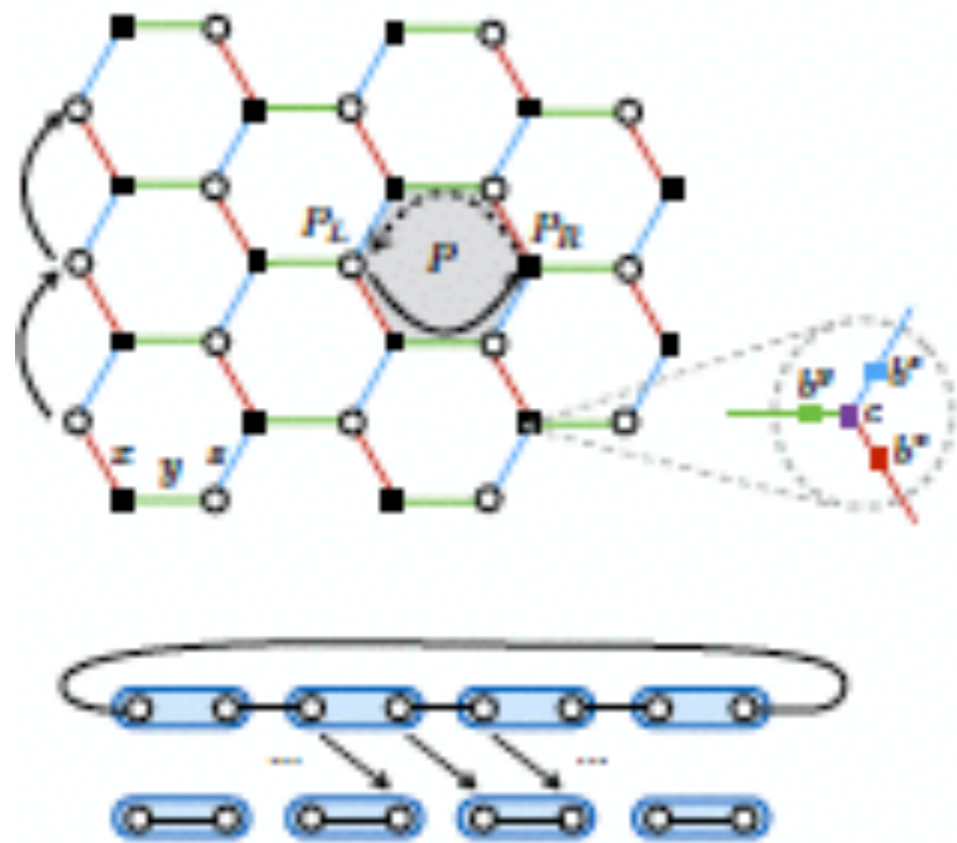
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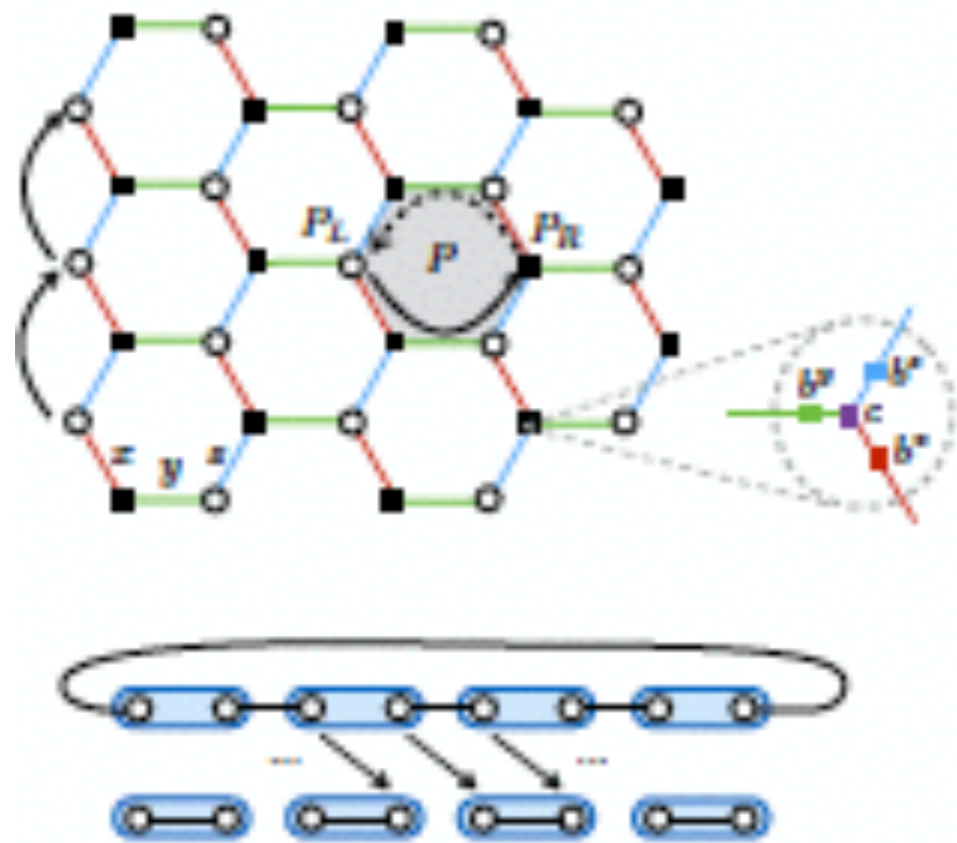
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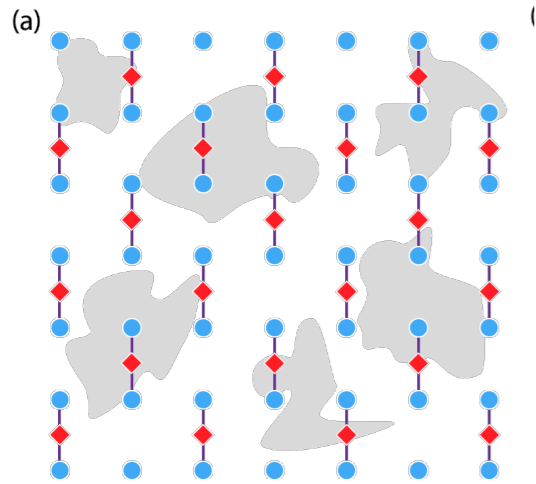
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experiment: Monroe group (trapped ions), Lukin group (NV centers)

Future Directions

Towards Experimental realization - in shaken optical lattices (with quasi periodic disorder). What to measure? Could this be an entanglement `bus`?

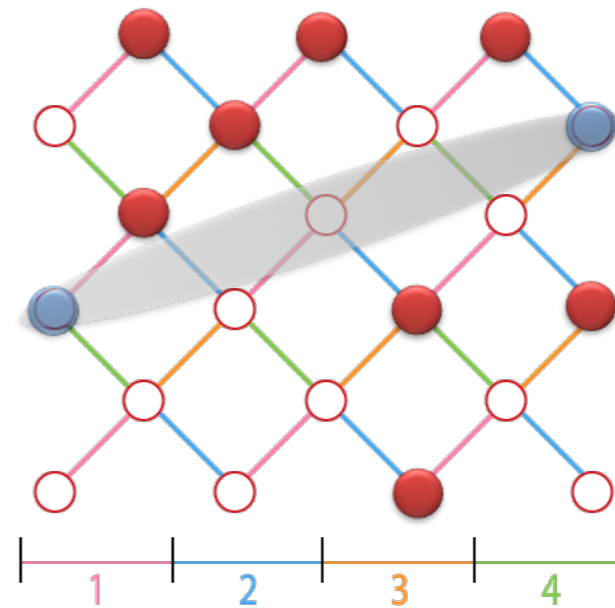
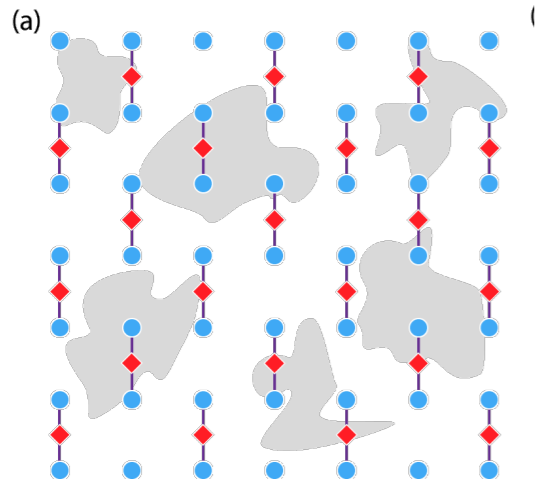


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