

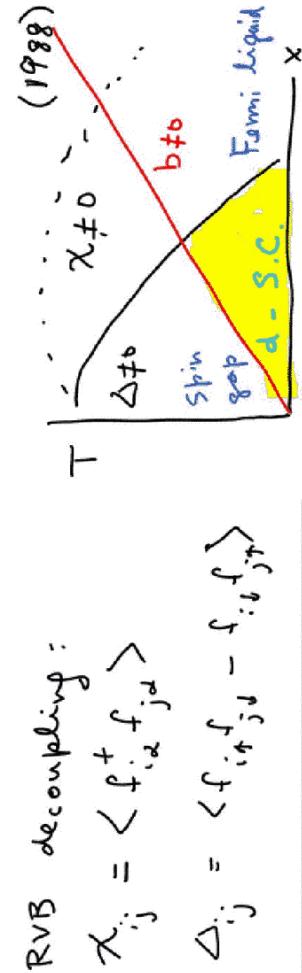
1. Strange metal as spin charge separated state:  
cross-over to pseudo-gap and fermi liquid.
  2. Pseudogap phase is controlled by “nodal” spin liquid.
3. **Has spin liquid been seen experimentally?**  
If so which spin liquid is it?  
Recent work with Sung-Sik Lee:  
Hubbard model as a U(1) gauge theory:  
possible spinon fermi surface in triangular lattice,  
and nodal spin liquid in honeycomb lattice.

High Tc superconductivity is the problem of doping  
a Mott insulator

t-J model: introduce fermions and bosons  $c_\sigma = f_\sigma b^+$  to enforce constraint of no double occupation.

### Early successes:

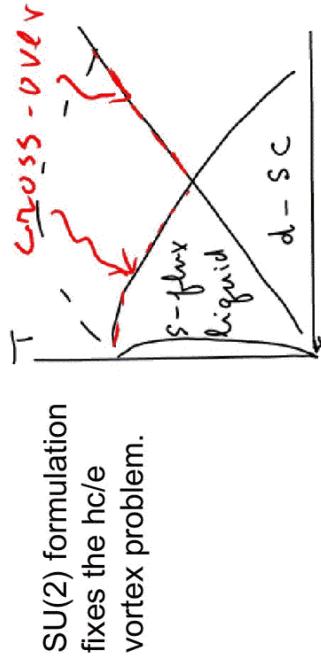
Mean field theory predicted d-SC and spin gap  
Baskaran, Zou and Anderson, solid state comm. (1987)  
Kotliar and Liu, PRB 38,5142 (1988)



$$RvB \text{ decoupling:} \quad \chi_{ij} = \langle f_{i\downarrow}^+ f_{j\downarrow} \rangle$$

$$\Delta_{ij} = \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle$$

Problem: prefer hc/e vortex



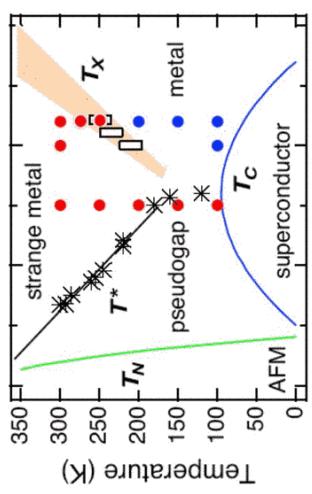
### Strange metal (~1990)

Fermions and bosons coupled to U(1) gauge fields. Spin-charge separation (deconfined gauge theory) at high T, but unstable to recombination (confinement) at low T.

Transverse gauge propagator:  $1/(i\omega/q + \chi q^2)$

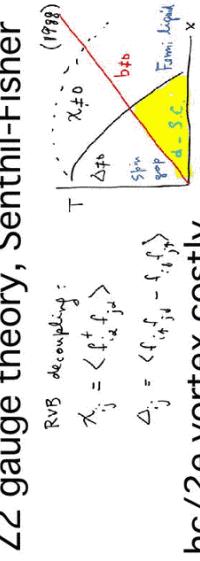
Fermion + gauge :  $C_V = T^{2/3}$ ,  $\sigma_F \sim T^{4/3}$ . Useful for FQHE and spin liquid.  
Boson + gauge : (difficult problem). However, for  $T > T_{BE}$ ,  $\sigma_B \sim T$

D. Kim, D. Lee and P. Lee : quasi-static approximation for gauge fields.  
QMC finds linear T persists to low T.



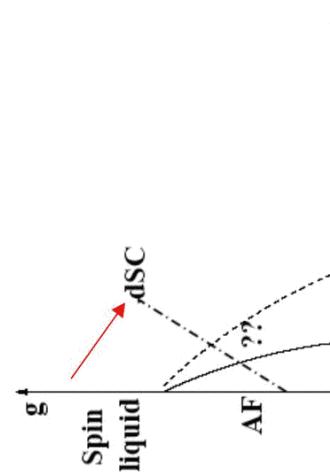
Different suggestions for spin liquid:

1. Dimer , Sachdev RMP  
Holes pair and SC has full gap.
- 2 Z2 gauge theory, Senthil-Fisher



hc/2e vortex costly.

Direct approach:  
Many possible intermediate states such as charge ordering or stripes.  
Connection to SC unclear.



3. “Nodal” spin liquid. (Wen-Lee)  
Possible signature of de-confinement. (Senthil-Lee)

### Gauge field and fermions as emergent properties.

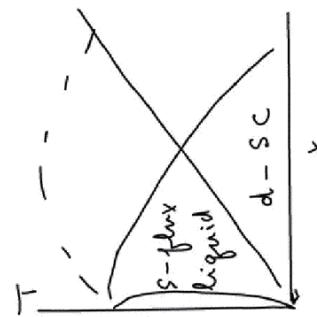
Start with spin models, gauge fields and spinons emerge as low energy excitations:

several exactly soluble models (Kitaev, Wen ,Motrunich-Senthil ...)

Mostly Z(2) gauge theory.

In SU(2) model, staggered flux liquid phase in mean field theory breaks SU(2) down to U(1).

This leads to effective low energy model of Dirac fermions coupled with U(1)gauge field.



However, in QED<sub>3</sub> with N component Dirac fermions De-confined phase is possible for large enough N. (M. Hermel et al, cond-mat )

Note that deconfinement does not mean free fermions.

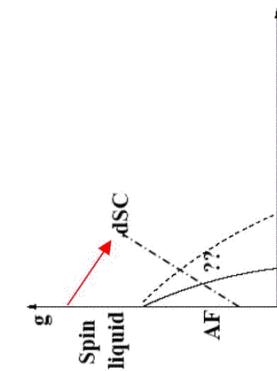
Recent work by Assaad suggests that N=4 Heisenberg model may be an example of de-confined spin liquid.

Half-filling:

Flux phase + gauge fluctuation  
= Dirac fermion + compact gauge fields

Familiar problem in QCD: confinement leads to “chiral symmetry breaking” which translates to Neel order in our case.

**Flux state is our route to AF order.**



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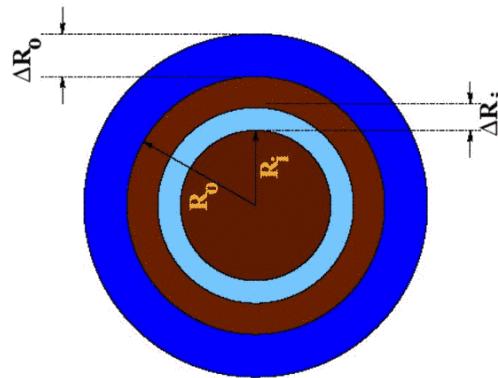
Recent work by Assaad suggests that N=4 Heisenberg model may be an example of de-confined spin liquid.

Signature of de-confinement is conservation of gauge flux, ie absence of instantons.

How to detect it? Senthil and Lee, cond-mat /04

### $hc/2e$ vortex traps – flux quantum of gauge flux

Use this idea to first produce and then detect  
Conserved gauge flux.



$T_{c1}$  of outer ring  $> T_{c2}$  of inner ring.

Cool below  $T_{c1}$  : If ring is thin compared with penetration depth, magnetic field escapes but – gauge flux is trapped in pseudo-gap normal state region. (brown)

Cool below  $T_{c2}$  :

It may be energetically favorable for – physical flux winding to appear in inner ring. Its sign is arbitrary.

Instantons will cause fluctuations between + or – gauge flux. As long as

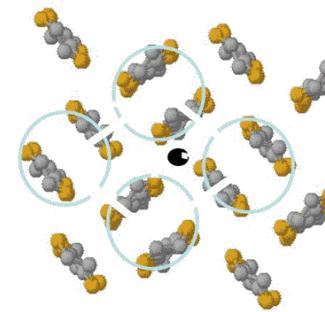
time scale is long compared with time to form vortex, this effect will

Kanoda, PRL, Spin liquid on the insulating side of Mott transition: nonzero  $\chi(T=0)$ ,  $1/T_1$  has linear T component.  
 Imada: numerical evidence for spin liquid  
 Motrunich: projected wave function in t-J plus ring exchange model.

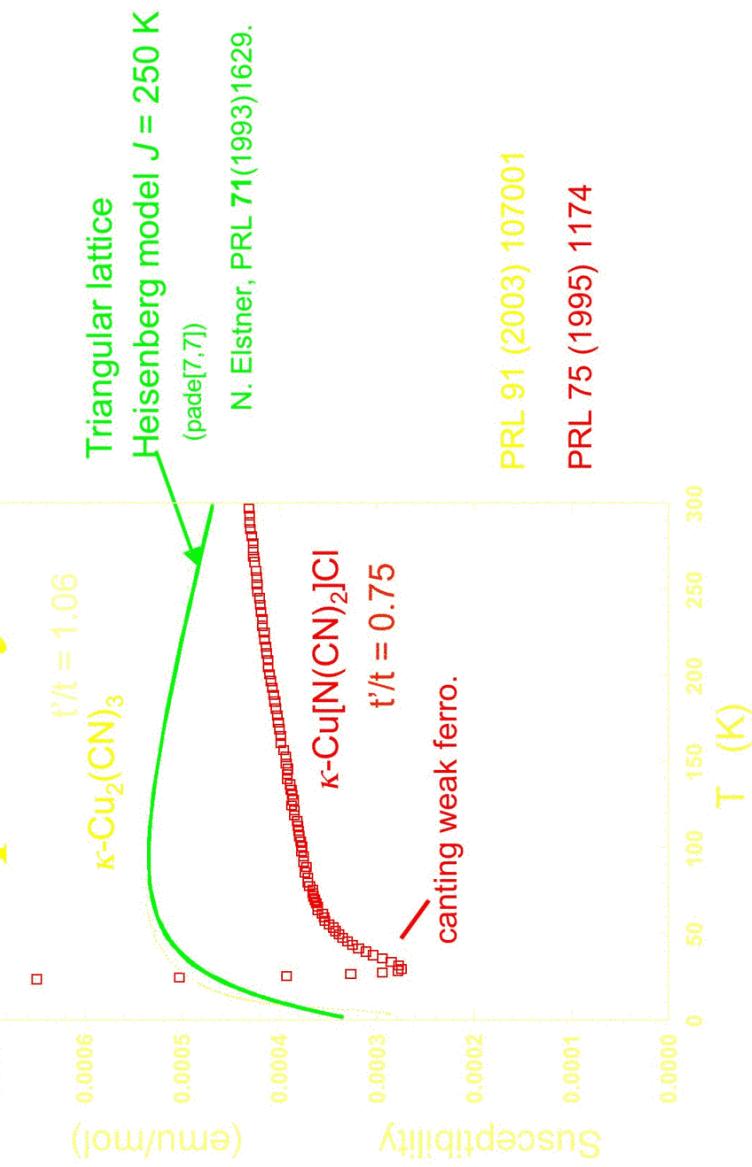
## Spins on triangular lattice in Mott insulator

$K\text{-}(ET)_2X$

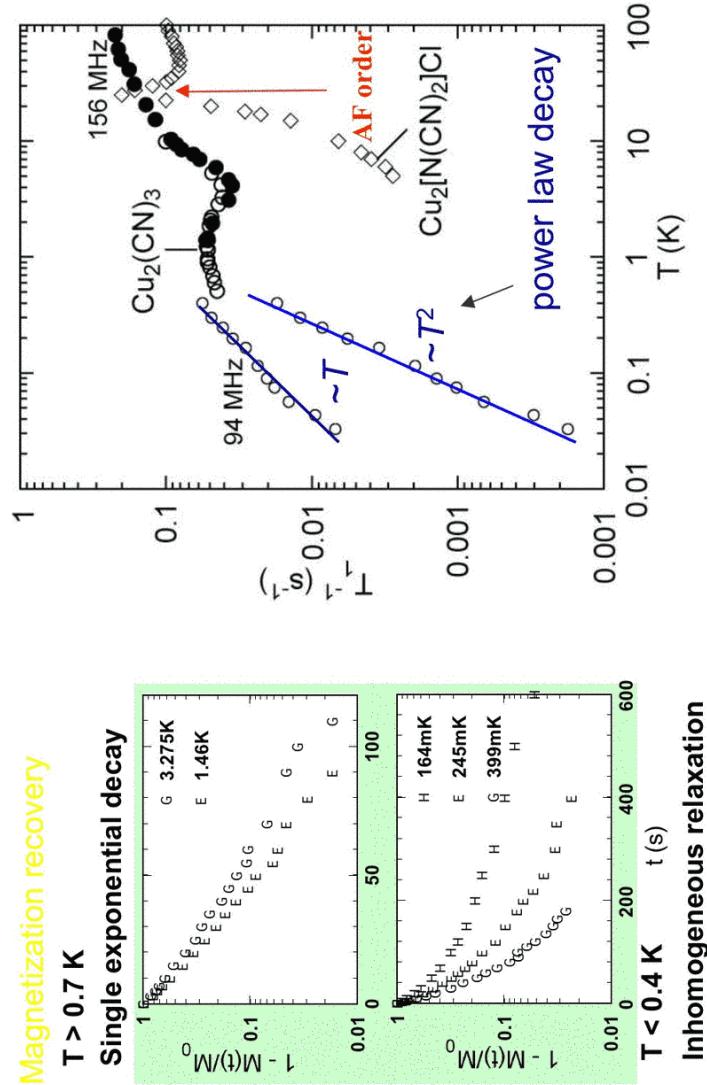
| $X$               | Ground State   | $U/t$ | $t'/t$ |
|-------------------|----------------|-------|--------|
| $Cu_2(CN)_3$      | Mott insulator | 8.2   | 1.06   |
| $Cu[N(CN)_2]Cl$   | Mott insulator | 7.5   | 0.75   |
| $Cu[N(CN)_2]Br$   | SC             | 7.2   | 0.68   |
| $Cu(NCS)_2$       | SC             | 6.8   | 0.84   |
| $Cu(CN)[N(CN)_2]$ | SC             | 6.8   | 0.68   |
| $Ag(CN)_2 H_2O$   | SC             | 6.6   | 0.60   |
| $I_3$             | SC             | 6.5   | 0.58   |



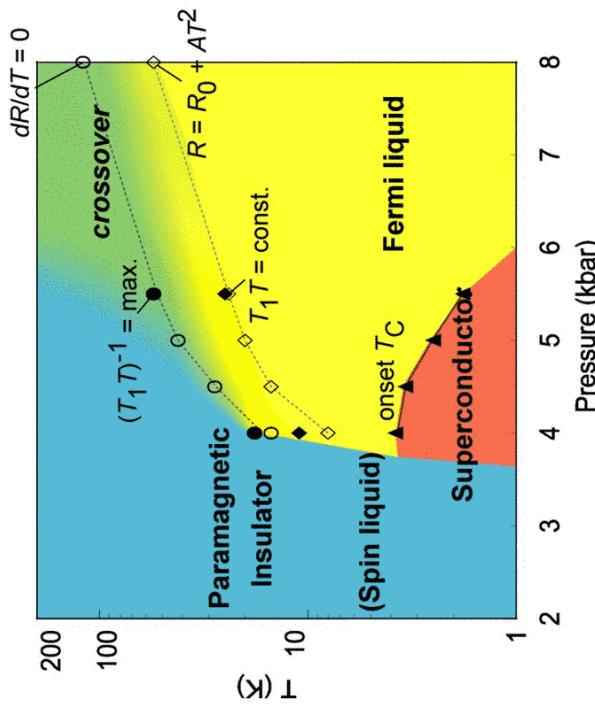
# Magnetic Susceptibility



# $^1\text{H NMR } 1/T_1$



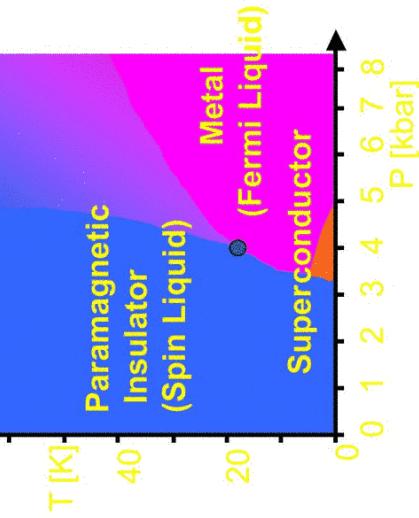
## Phase diagram of $\kappa$ - $(ET)_2Cu_2(CN)_3$



## Concluding Remarks

$\kappa$ -(ET)<sub>2</sub>Cu<sub>2</sub>(CN)<sub>3</sub> : *Mott insulator with triangular lattice*

- Ambient pressure
  - Susceptibility ~ Triangular lattice AF Heisenberg model ( $J \sim 250$  K).
  - No long-range magnetic order down to 32 mK (Spin liquid).
  - Magnetic field induced small inhomogeneous moment at low temperatures.



Hubbard model as a compact U(1) gauge theory.  
(with Sung-Sik Lee).

Florens and Georges, PRB 04, slave rotor formulation.  
Quantum rotor: angular momentum L = charge.

Constraint:

$$\hat{L} = \sum_{\sigma} \left[ f_{\sigma}^{\dagger} f_{\sigma} - \frac{1}{2} \right]$$

$$d_{\sigma} \equiv f_{\sigma} e^{-i\theta}$$

$$H = \sum_{i\sigma} \epsilon_0 f_{i\sigma}^{\dagger} f_{i\sigma} + \frac{U}{2} \sum_i \hat{L}_i^2 - \sum_{ij\sigma} t_{ij} f_{i\sigma}^{\dagger} f_{j\sigma} e^{i(\theta_i - \theta_j)}$$

$$H = \sum_{i\sigma} \epsilon_0 f_{i\sigma}^{\dagger} f_{i\sigma} + \frac{U}{2} \sum_i \hat{L}_i^2 - \sum_{ij\sigma} t_{ij} f_{i\sigma}^{\dagger} f_{j\sigma} e^{i(\theta_i - \theta_j)}$$

We go to path integral formalism and decouple the hopping term.  
Saddle point is the mean field theory of Florens and Georges.  
Low energy Lagrangian is rotor and fermions coupled to  
compact U(1) gauge field.

$$L_F = f_{i\sigma}^{\dagger} (\partial_{\mu} - i\alpha_{\mu} \gamma^5) f_{i\sigma} - \star Q_F^{\circ} (f_{i\sigma}^{\dagger} f_{j\sigma} e^{i(\theta_i - \theta_j)} + \text{c.c.})$$

$$L_{\theta} = \frac{1}{2} U (\partial_x \theta + \alpha_{\mu} \gamma^5)^2 - \star Q_{\theta}^{\circ} (e^{i(\theta_i - \theta_j)} - (a_{ij} e^{i\theta_j} + \text{c.c.}))$$

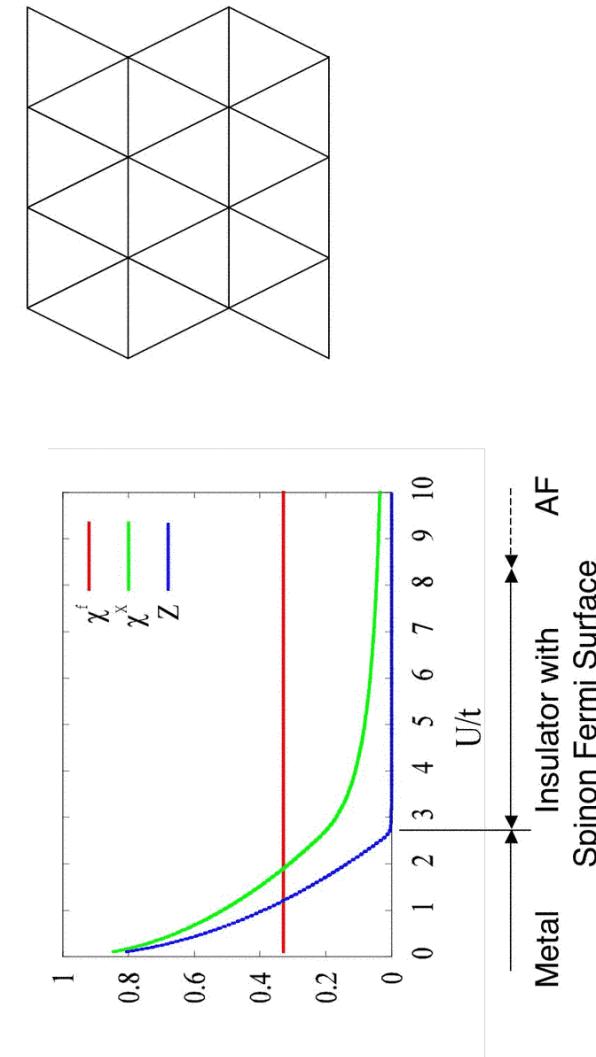
Application: Hubbard model on triangular lattice.

- \_ filled triangular lattice: nearly circular FS, no nesting.
- \_ Mean field theory: spinon FS is stable.
- Gauge theory: rotor is gapped in insulator. Theory reduces to fermion + U(1) gauge field.
- If it is deconfined, we predict:

$$C_V \sim T^{2/3}$$

Thermal conductivity  $\kappa/T \sim T^{4/3}$

## Mean-Field Order Parameters (Triangular Lattice, Half Filling)



Hubbard model in honeycomb lattice

AF vs dimer:  
 Dimer: energy per dimer= -3/4J  
 AF: energy per AF bond= -1/4J

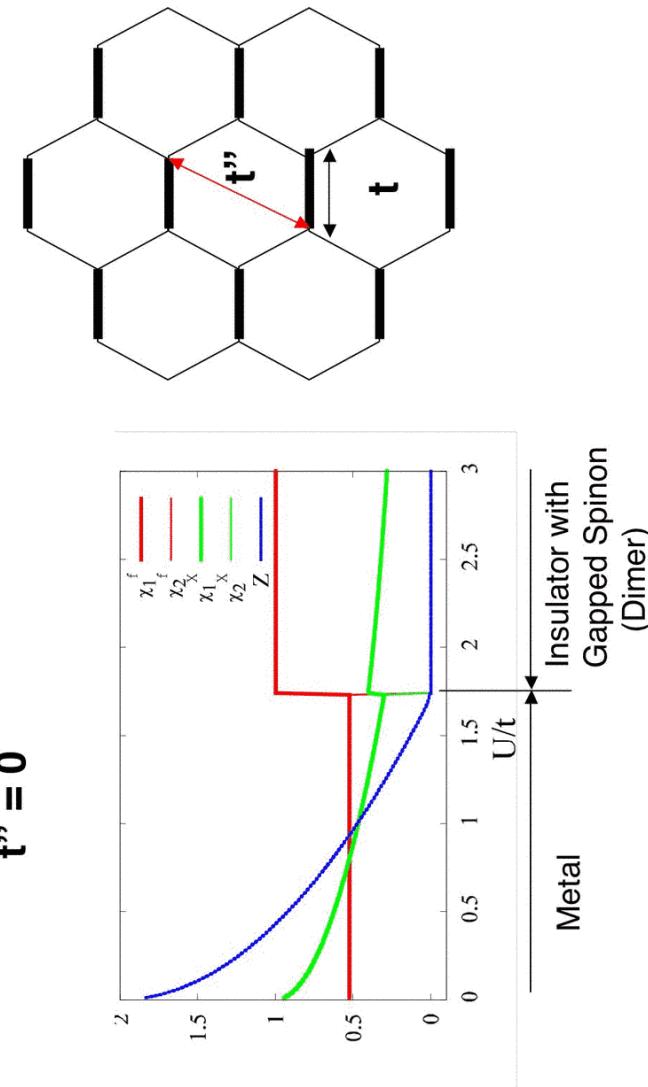
|           | dimer /bond | dimer energy/bond       |
|-----------|-------------|-------------------------|
| 1 dim     | 1/2         | -3/8J                   |
| Square    | 1/4         | -3/16J                  |
| honeycomb | 1/3         | -1/4J <b>same as AF</b> |

Mean field theory: fermions form Dirac nodes. SDW not required.  
 Example of 2 Dirac nodes coupled to U(1) gauge field.

Can this state be stable against dimer?

## Mean-Field Order Parameters (Honeycomb Lattice, Half Filling)

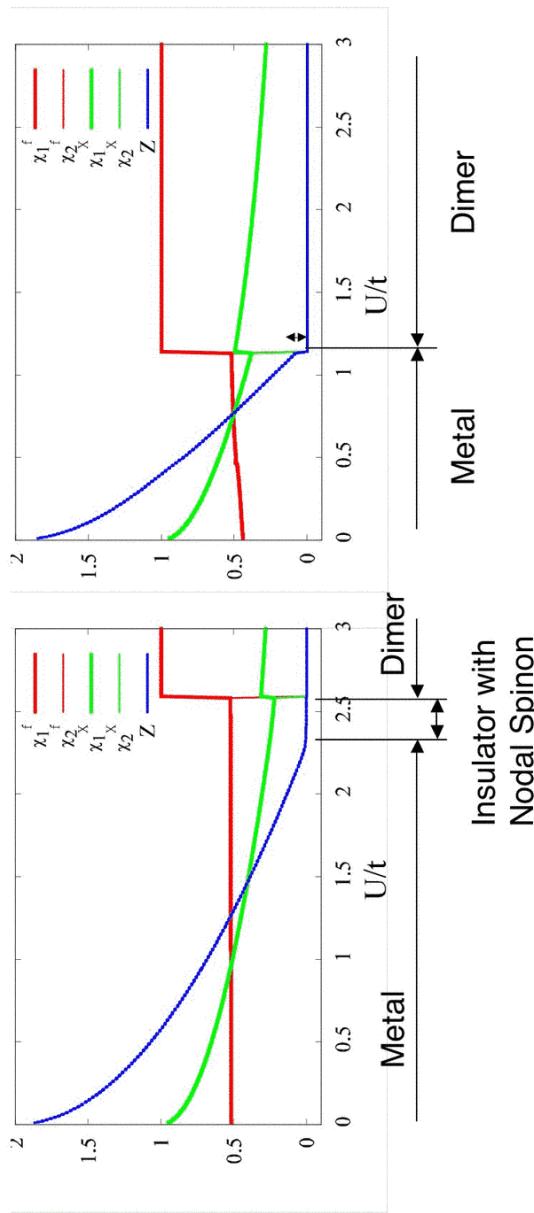
$$t'' = 0$$



# Mean-Field Order Parameters (Honeycomb Lattice, Half Filling)

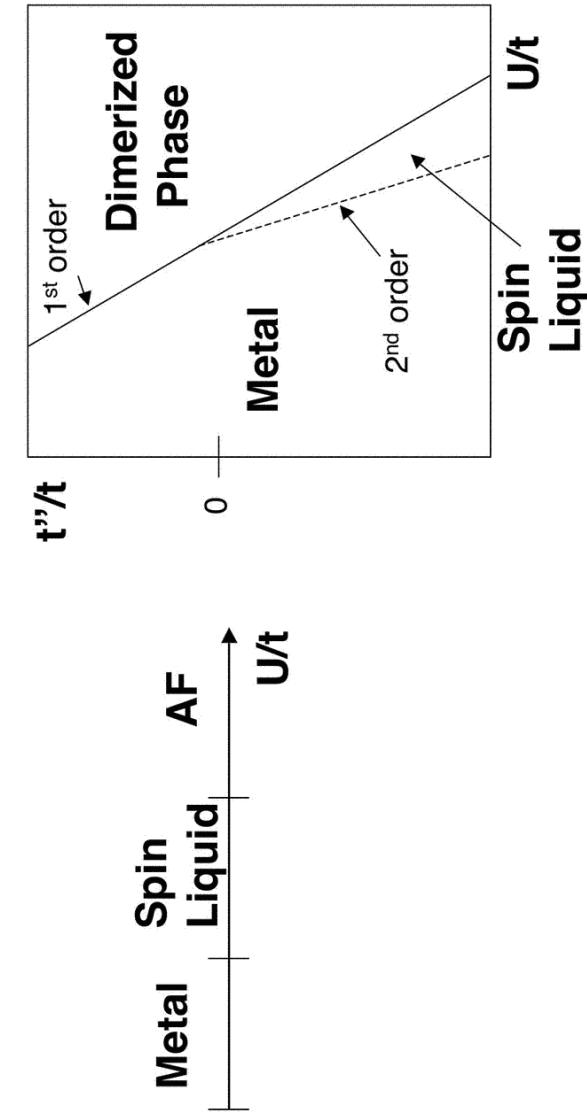
$$t'' = -0.4t$$

$$t'' = 0.4t$$

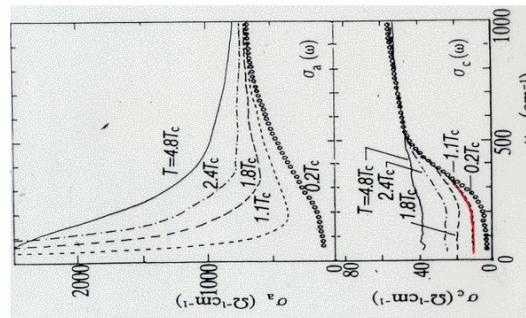


## Schematic Phase Diagram (Half Filling)

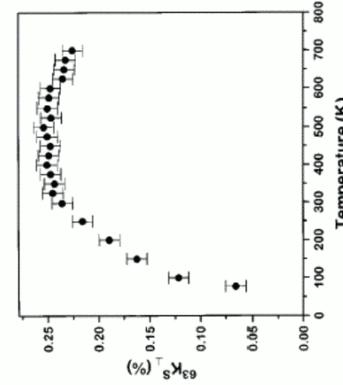
- Triangular lattice
- Honeycomb lattice



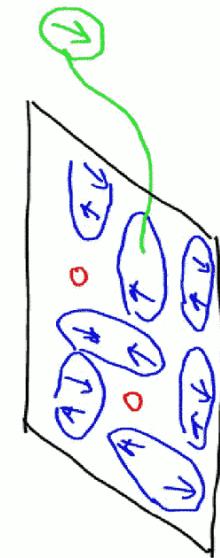
Pseudo-gap is the formation of spin singlet.



ARPES sees gap above  $T_c$ .  
Energy gap  $\gg kT_c$ .

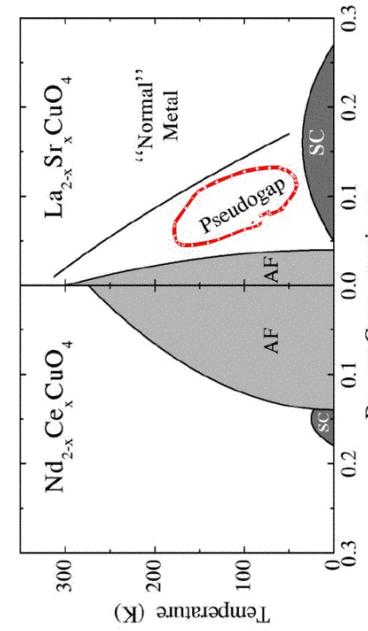


This is the resonating valence bond (RVB) idea of Anderson: Spins form singlets. Superconduct when holes are phase coherent.



Arguments against QCP as dominating the high Temp physics.

1. Unidentified ordered state. No diverging length scale.  
QCP is not a very useful concept unless we know who is it?  
If it is Ising universality class, where is specific heat anomaly?
2. We have nothing against transition in high magnetic field. In fact  
we predicted a Fermi surface rearrangement when vortex cores  
touch. The issue is whether this transition controls temperature  
scale of order  $J$ .
3. Power law is common in gapless fermion systems.  
 $\omega/T$  scaling does not necessarily imply QCP.
4. Violation of basic scaling relation: van der Marel.

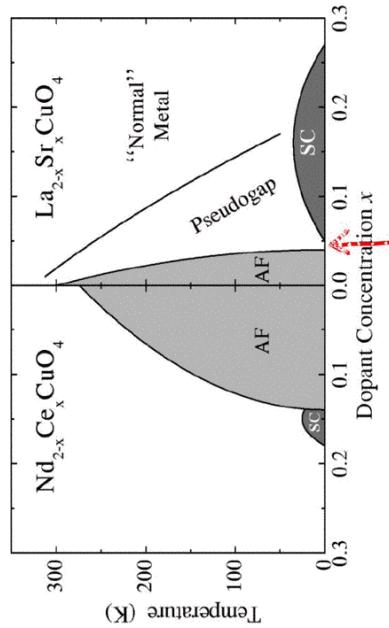


Spin glass QCP dominated by disorder:  
control low energy physics  $\sim 10\text{K}$

What happens in the clean limit?

- first order transition to AF.
- incommensurate states: stripes, Wigner crystals of holes or hole pairs. Low energy scale.
- Metal-SC transition? Nodal metal?

Is the pseudogap controlled by an unstable fixed point?



Spin glass QCP dominated by disorder:  
control low energy physics  $\sim 10\text{K}$

What happens in the clean limit?

- a) first order transition to AF.
- b) incommensurate states: stripes, Wigner crystals of holes or hole pairs. Low energy scale.
- c) Metal-SC transition? Nodal metal?