Fluctuations at the QCP of the Cuprates

I think it is beyond any reasonable doubt that the properties of the Cuprates near the doping for highest Tc are determined by singular fluctuations which are cut-off at low energy by Tc

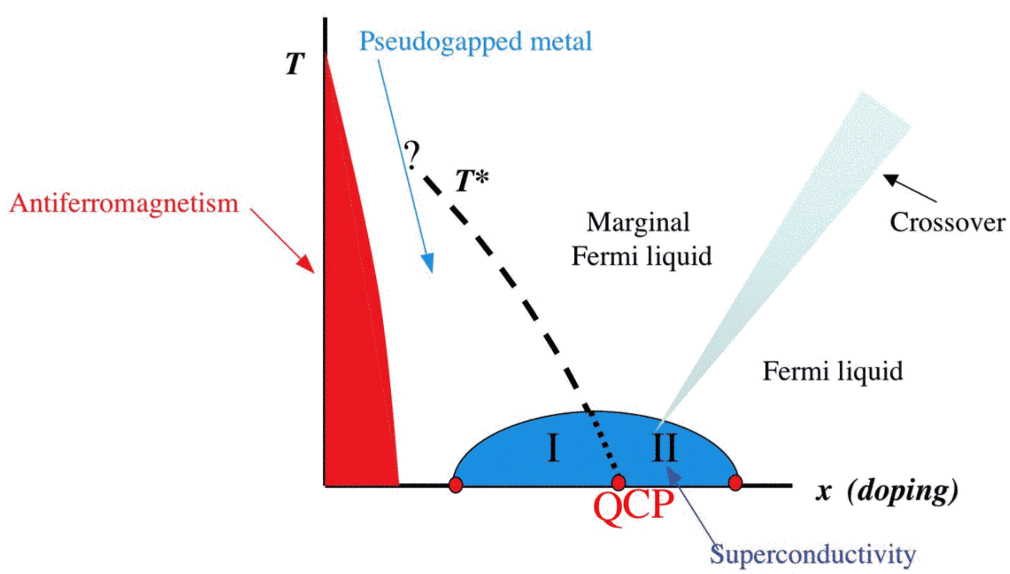
Direct observation of such fluctuations in Raman scattering

What is fluctuating?

How to derive the fluctuation spectra.

Schematic Universal phase diagram of high-T_c superconductors

A Central feature: A putative singularity of the fluctuation spectra in the superconducting dome at T=0.

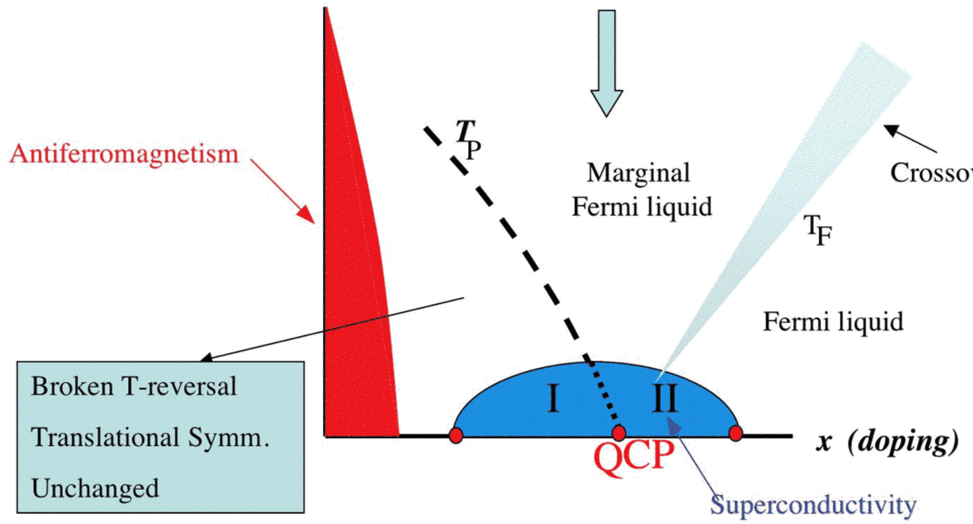


CMV: Los Alamos Proc. ‘93, PR-B ‘97, PRL ‘99

Scale-Invariant Spectrum for both charge/current and magnetic channels

CMV et al., PRL'89: Anomalous properties due to a QCP with critical fluctuations given by:

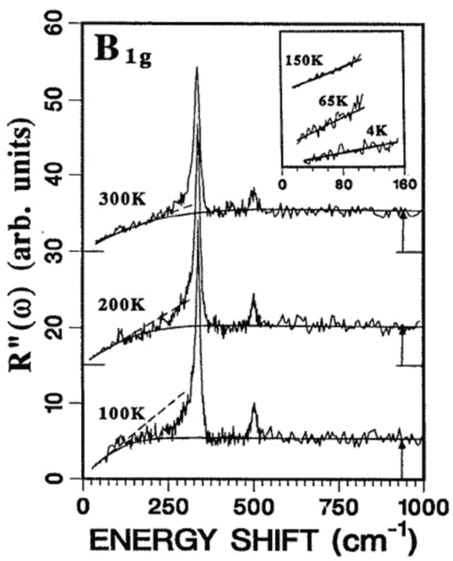
$$Im\chi(\omega,\mathbf{q}) \propto N(0)\omega/T \quad for\ \omega \ll T$$
$$\propto N(0)sgn(\omega) \quad for\ \omega_c \gg \omega \gg T.$$
$$Re\chi(\omega,\mathbf{q}) \propto \ln(\omega_c/x);\quad x \approx max(\omega,T).$$

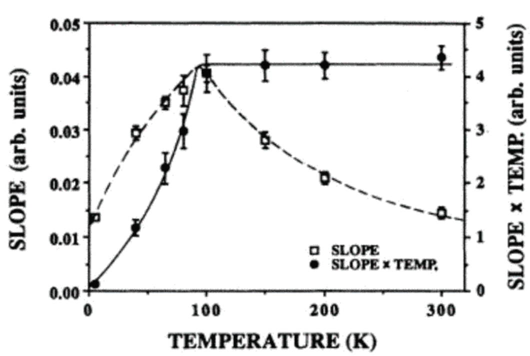


CMV: PR-B '97, PRL '99

Raman Spectra in cuprates near Optimal doping

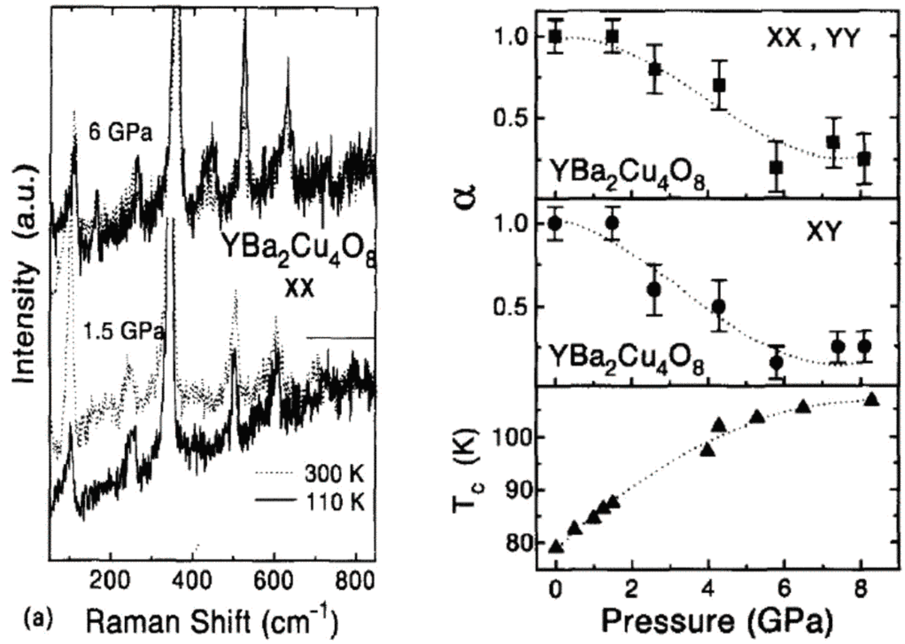
F. Slakey et. al. PRB 43, 3764 (1991)





$$Im\chi(\omega,\mathbf{q}) \propto N(0)\omega/T \quad for\ \omega \ll T$$
$$\propto N(0)sgn(\omega) \quad for\ \omega_c \gg \omega \gg T.$$

- Singular Spectra at Long Wavelengths of the form predicted.
- Look for broken Symmetry with no change in Translational Symmetry
- Fluctuations must be of an unconserved quantity, for example current



Cardona et al., Raman scattering in a sample of 248 under pressure converting it from underdoped towards optimally doped: (SSC, 1996)

Change of the spectrum from one with a "gap' to the predicted scale-invariant form

Fluctuations leading to Marginal Fermi-liquid in Cuprates

- The hypothesized fluctuation spectrum observed directly at long-wavelengths in Raman scattering experiment.
- Explains the temperature and frequency dependence of almost all 'universal' properties of the Cuprates near optimum doping.
- Predict the form of single-particle spectrum observed in ARPES.
- A crossover to a Fermi-liquid exists for x beyond x_c(T).
- And a peculiar phase exists for x less than x_p(T).

What is it that is fluctuating so peculiarly and why?
Look for broken symmetry on the underdoped side at q=0 and an unconserved order parameter. This is mandated by the observed critical fluctuations

Scale-Invariant Spectrum for both charge/current and magnetic channels

CMV et al., PRL'89: Anomalous properties due to a QCP with critical fluctuations given by:

$$Im\chi(\omega,\mathbf{q}) \propto N(0)\omega/T \quad for\ \omega \ll T$$
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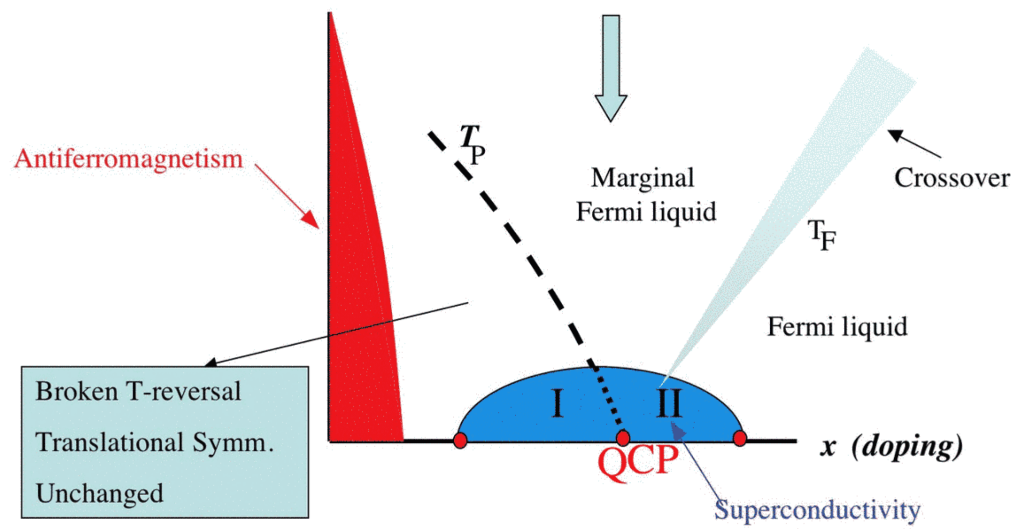


CMV: PR-B '97, PRL '99

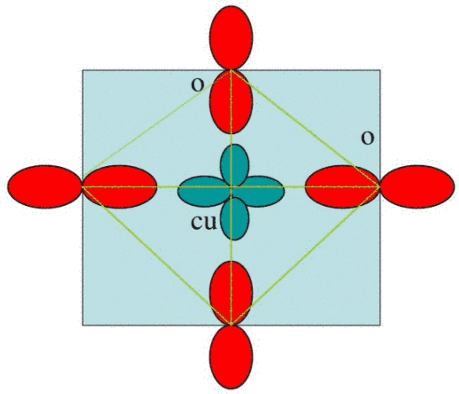
Minimal Model for the Cuprates ?

Must answer the question, why are the Cuprates unique?
Two choices:

1. A two dimensional Hubbard Model captures the essential Physics (Anderson, 1987 and subsequently): the problem is simply that of a doped Mott Insulator.
2. The solid state chemistry of the Cuprates is such that the long-range ionic interactions (besides the local repulsions) change the Physics entirely.

Is there new Physics of quantum correlations beyond Mott, Anderson, Kondo problem, etc.?

$$Minimum\ Hamiltonian\ :\ Space\ of\ three\ orbitals\ (d, p_x, p_y)\ per\ unit\ cell:$$
$$\mathcal{H} = K.E.(t_{pd}, t_{pp}) + Local\ Repulsions\ (U_d, U_p) +\ Ionic\ Interactions\ (V).$$



Ionic Interactions necessary to satisfy screening conditions if the cu-levels are not particle-hole symmetric. Can they change the physics completely?

Deriving the Scale-invariant Spectrum for a local Cu-O problem

Embed a CuO2 cell in a Medium: Exactly solved problem

(Perakis, Ruckenstein, cmv '93; Sire, Giamarchi, Ruckenstein, cmv '94)

Consider Strong-Coupling limit: $U's \to \infty, \quad V \gg t's$

One-electron states in the cell: d-state +bonding combination of p's; non-bonding p's

Lowest energy many-body states: Friedel pair

$$\zeta_\sigma^+|0>=|1\sigma, 0>, \quad \eta_\sigma^+|0>=|0, 1\sigma>.$$

Hybridizing channel, screening channel

Now connect these states to the 'medium' eliminating high energy states through a canonical transformation

One-electron states of the medium: $h_{i,\sigma}^{+}$ and $s_{i,\sigma}^{+}$

Effective Hamiltonian for a CuO2 cell in a 'medium'

$$\mathcal{H} = \delta(\zeta_\sigma^+\zeta_\sigma - \eta_\sigma^+\eta_\sigma) + \hat{t}\zeta_\sigma^+\eta_{\sigma'}s_{1,\sigma'}^+h_{1,\sigma} + Exchange\ (J) + Ionic\ Interactions. \text{ +KE(Medium)}$$

The kinetic energy term connecting local ops. to h's and s's has disappeared!
Why? Only Marginal and Irrelevant terms in the effective hamiltonian.

In the Mixed-Valent Condition : $E(cu^{++}o^{--}) = E(cu^{+}o^{-})$; i.e $\delta = 0$
The local Charge and Current Susceptibilities in Wilson RG calculations are

$$\chi_{local}(\omega,\delta) \propto \ln(\omega) \ \ at\,\delta=0;\ and \propto \delta^2\ at\ \omega=0.$$

Why Currents? Introduce SU(2) space τ_0 *which spans the basis* (ζ^+, η^+)

Term proportional to $\hat{t}$: $\hat{t}(\tau_{o,x}\tau_{1,x} + \tau_{o,y}\tau_{1,y})$

The operator $\tau_{0,y} = i(\zeta^+\eta - \eta^+\zeta)$ represents current in the CuO2 cell

:

Instead of Incoherent screening, current fluctuations which accomplish the same at lower cost in energy and (more elegantly): Why is this special to Cuprates?

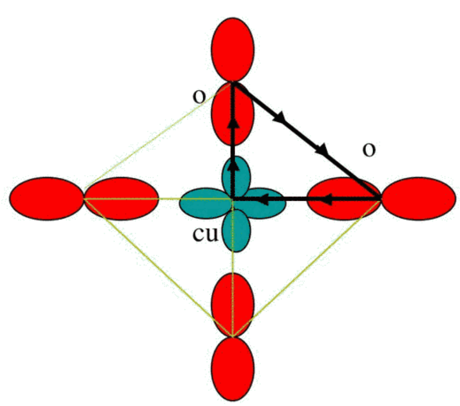
Current Fluctuations

Fluctuating operator: Complex linear combination of the hybridized orbital and the non-bonding orbital:

$$d^+ + p_x^{\ +} + p_y^{\ +} + i\theta(p_x^{\ +} - p_y^{\ +}) \approx d^+ + e^{i\theta}\, p_x^{\ +} + e^{-i\theta}\, p_y^{\ +}$$

Opposite Phase shift between d and p_x orbital and d and p_y orbital.

So, the pattern of current fluctuation in a cell is:



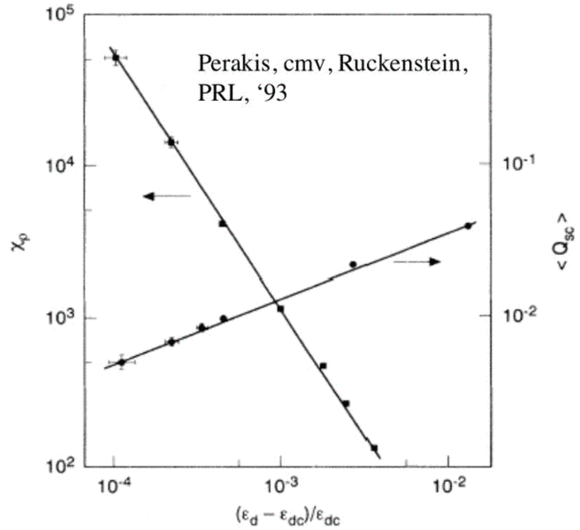


FIG. 1. The charge susceptibility and the charge in the screening channel as a function of $\varepsilon_d - \varepsilon_{dc}$.

Other Results from the Exact Solution of the local Cu-o Model

- Self-energy near the Critical point ~$\omega \ln \omega$
- Local pairing susceptibility in d-wave and ext-s-wave channels ~ $\ln \omega$
- Low energy properties of the Kondo model for small V or away from mixed-valence condition.

Solution for the Lattice

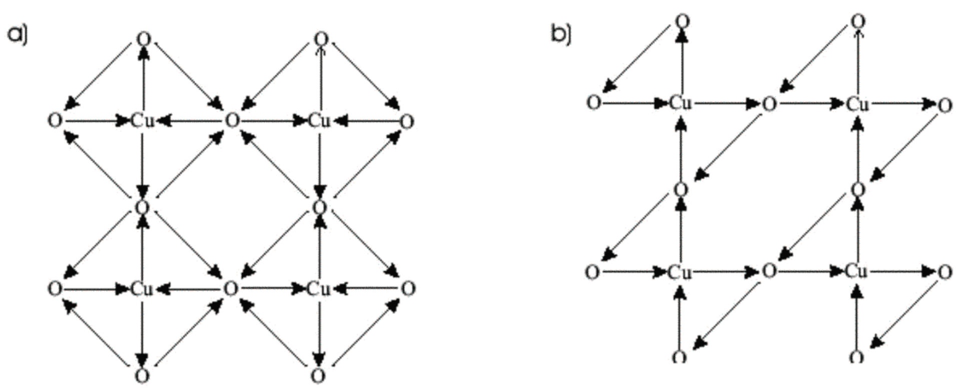
Strong-coupling Hamiltonian for the lattice (cmv '97) in the space of antibonding and nonbonding orbitals per unit cell:

$$\mathcal{H}_{eff} = Constrained\ K.E. + \mathcal{H}_{int} - \mu N$$
$$\mathcal{H}_{int} = \mathcal{J}(1/4 - \sigma_i \cdot \sigma_j)(\mathbf{A} - \tau_i \cdot \mathbf{B} \cdot \tau_j)$$

Mean-field Broken Symmetry Solution With Unaltered Translational Symmetry below critical doping x_c

$$\langle \tau_y \rangle \neq 0$$

representing Time-reversal breaking through spont. currents



Hypothesized current fluctuation spectrum for the lattice (T=0)

Can show it in DMFT; expect to show it is stable.

$$\chi(\omega,q)=\left(M(\omega)+a^2q^2\right)^{-1}$$

$$M(\omega)=\left(\ln(\omega_c/\omega)+i\,\mathrm{sgn}(\omega)\;\right)^{-1}$$

- consistent with experiments (directly observed at q -->0).
- consistent with fluctuations of an unconserved quantity.
- dynamical critical exponent - infinity (some models have been shown to have this (Fisher and Zwerger, Tewari et al.)
- coupling functions of such fluctuations is in irred. rep. of fermi-surface such that d-wave superconductivity promoted.

Validity of theory rests on confirmation of experiments which see the predicted TRV order in the underdoped materials.

Phase diagram of the Cuprates

Over 10^5 Experiments: Systematic and mutually consistent results about the underlying fluctuation spectra and phase diagram with different techniques available.

Focus on properties which are common to all the cuprates and which vary in a systematic fashion along the same lines in the T-x plane of the Cuprates.

Resistivity(T)

Conductivity as a function of frequency.

Raman spectra

NMR relaxation rates

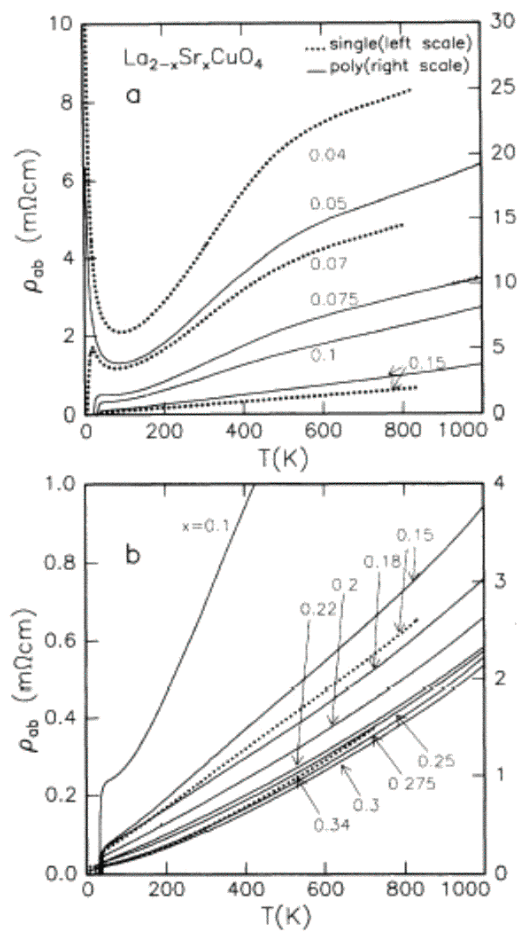
Magnetic susceptibility

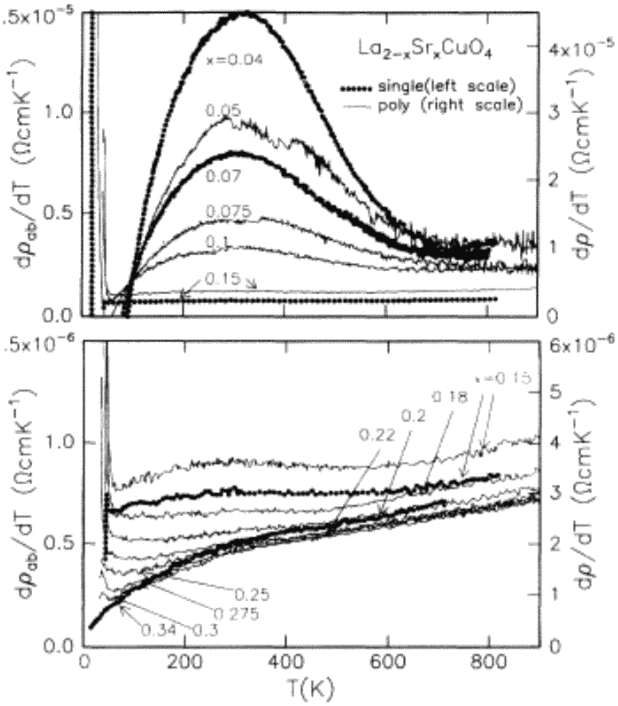
Specific heat

Tunneling and ARPES

(Hall Effect and Nernst Effects)

Takagi et al. (1991)

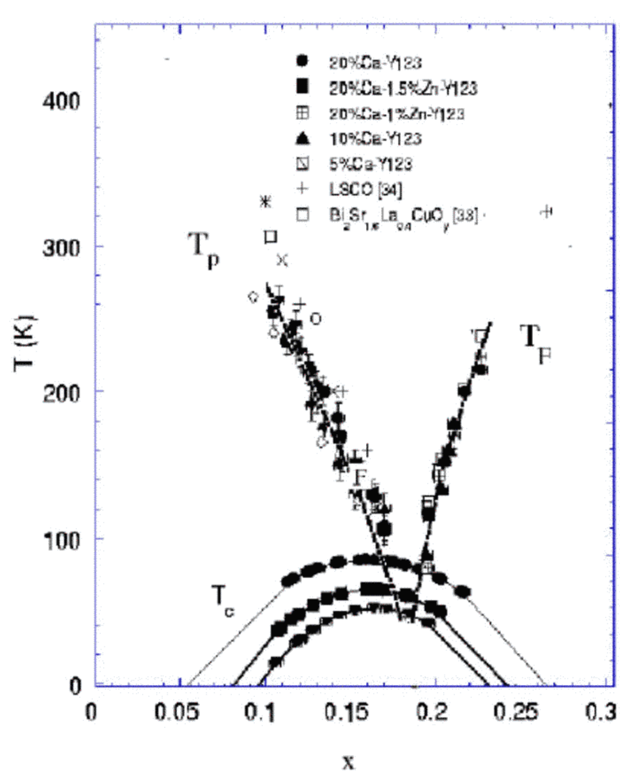




Resistivity ~ T down to lowest temperatures implies breakdown of Fermi-liquid theory and the quasi-particle concept.

Example of Universality of results

From Naqib et al. '2003



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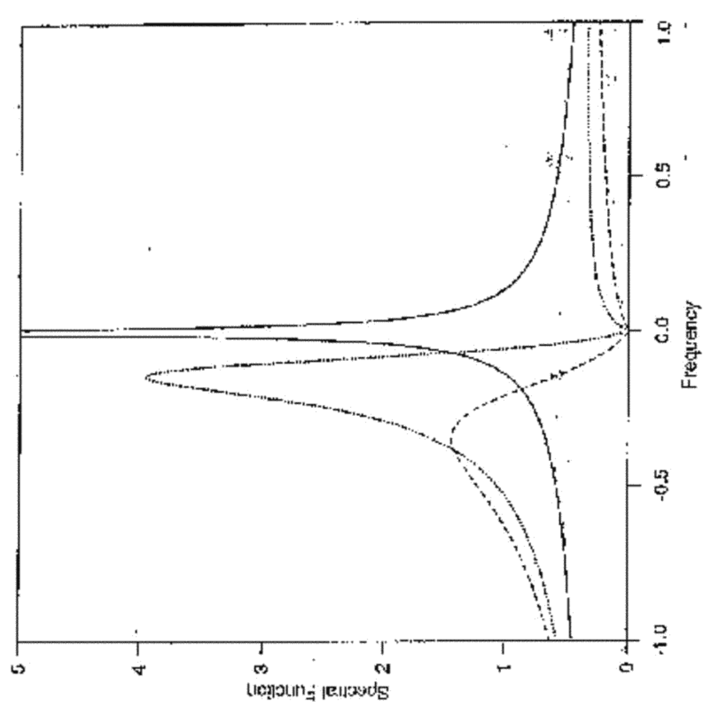
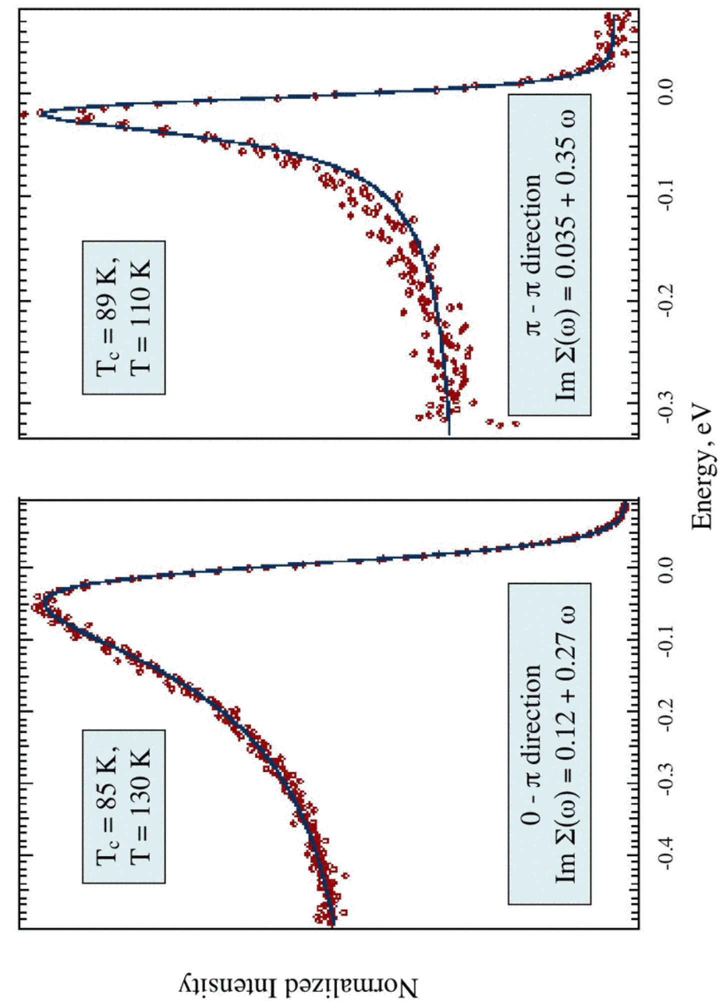
Phenomenology of the Normal State of Cu-O High-Temperature Superconductors

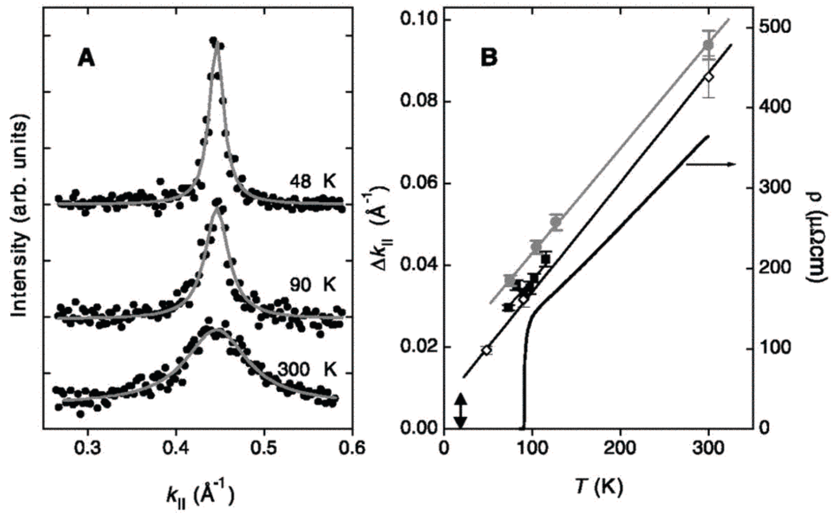
C. M. Varma, P. B. Littlewood, and S. Schmitt-Rink
AT&T Bell Laboratories, Murray Hill, New Jersey 07974
E. Abrahams and A. E. Ruckenstein
Serin Physics Laboratory, Rutgers University, Piscataway, New Jersey 08855
(Received 7 August 1989)

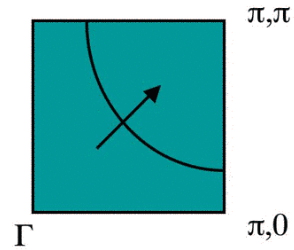


Angle-Resolved Photoemission Bi 2212
Kaminski, et al (ANL)



Momentum Scans (MDC) - II



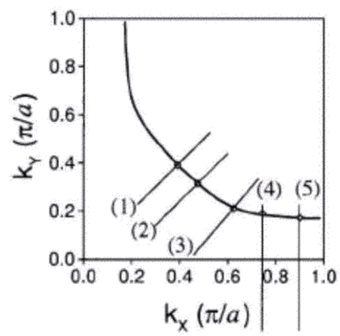


Valla, et al, Science **285,** 2110 (1999) (BNL)

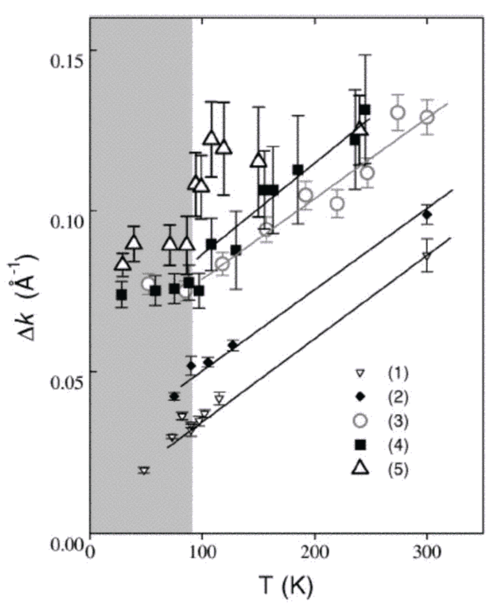
MFL at $\omega = 0$: $\Sigma_1 = 0$, $\Sigma_2 = \lambda\pi T/2$ and the MDC lorentzian is given by

$$A(k,0) = \frac{\lambda T}{2}\frac{1}{\varepsilon_k^2 + (\lambda\pi T/2)^2}$$

Fermi surface anisotropy of MDC widths



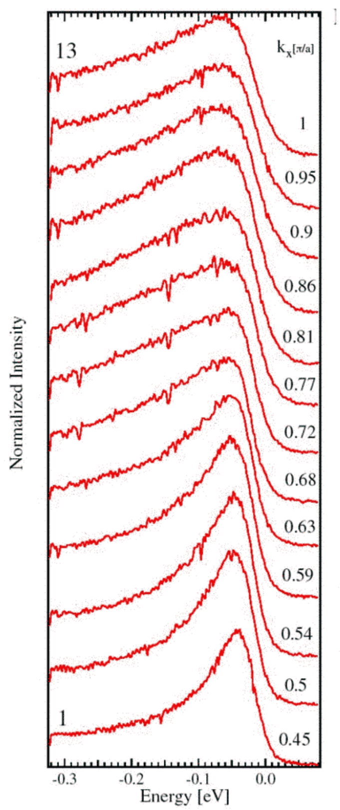
Valla, et al
PRL **85**, 828 (2000)

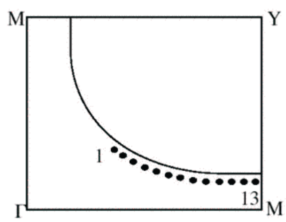


$I\,(\mathrm{k}, \omega{=}0) \propto 1/[(v_0\,\Delta \mathrm{k})^2 + \Sigma_2{}^2]$

$\Sigma_2 \propto \max(\omega, T) + \gamma(\hat{k})$

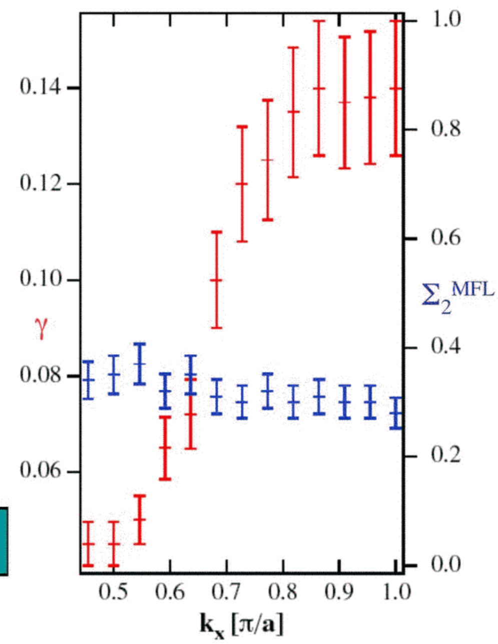
Fermi surface anisotropy of EDC fits to MFL form of Σ



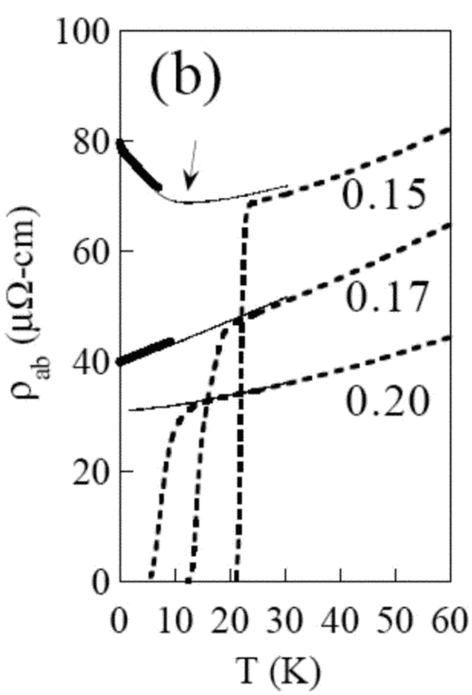


Kaminski, et al (Argonne)

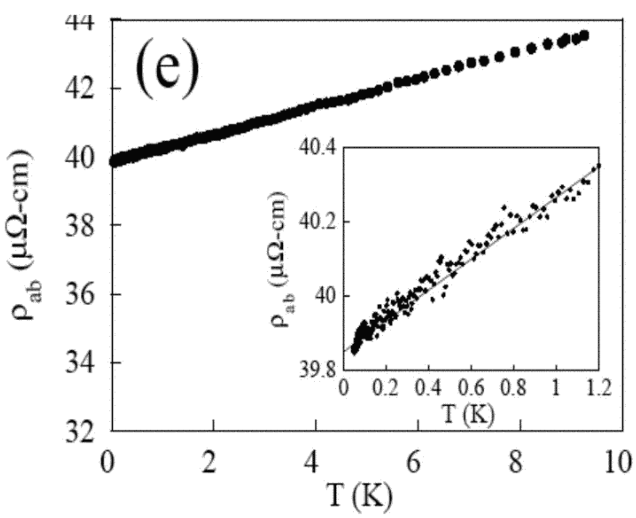
$$\Sigma_2(k,\omega) = \gamma + \Sigma_2^{MFL}$$



ab-plane resistivity for $Pr_{2-x}Ce_xCuO_4$ films with $H>H_{c2}$



Fournier et al., PRL 81,4720 (98)



Loram et al., PRL ‘93

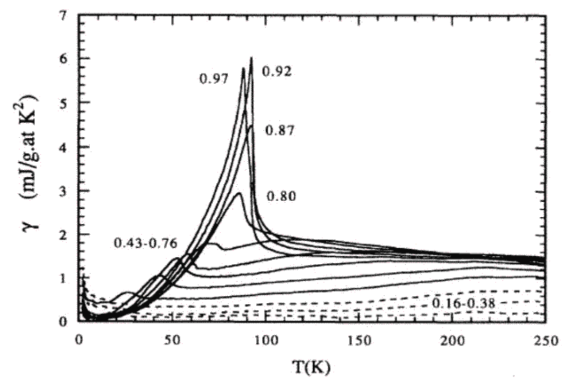
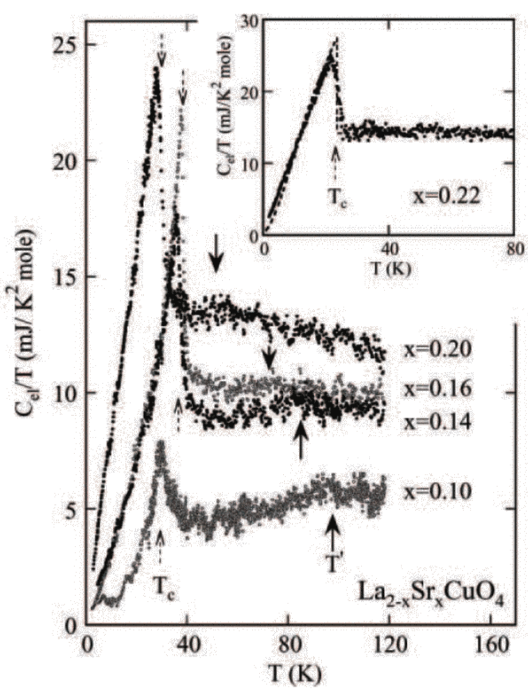


FIG. 4. Electronic specific heat coefficient $\gamma(x,T)$ vs T for $YBa_2Cu_3O_{6+x}$ relative to $YBa_2Cu_3O_6$. Values of x are 0.16, 0.29, 0.38, 0.43, 0.48, 0.57, 0.67, 0.76, 0.80, 0.87, 0.92, and 0.97.

Problem in specific heat measurements: A bump in γ at T higher than where it begins to go down is necessary to conserve entropy.



Oda et al., specific heat of LSCO