http://monqis.physics.monash.edu

MONASH University

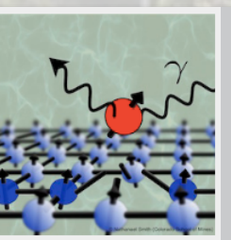
MonQIS

A complete language for non-Markovian quantum phenomena

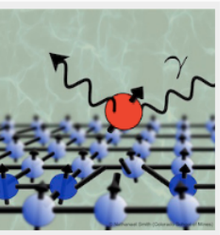
Quantum stochastic processes

chemistry, computers, causality, and complexity

Kavan Modi



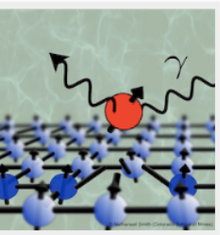
KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium



KITP 2019

Open Quantum System Dynamics:

Quantum Simulators and Simulations Far From Equilibrium

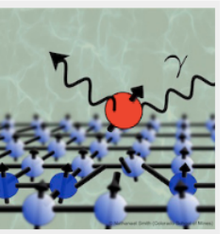


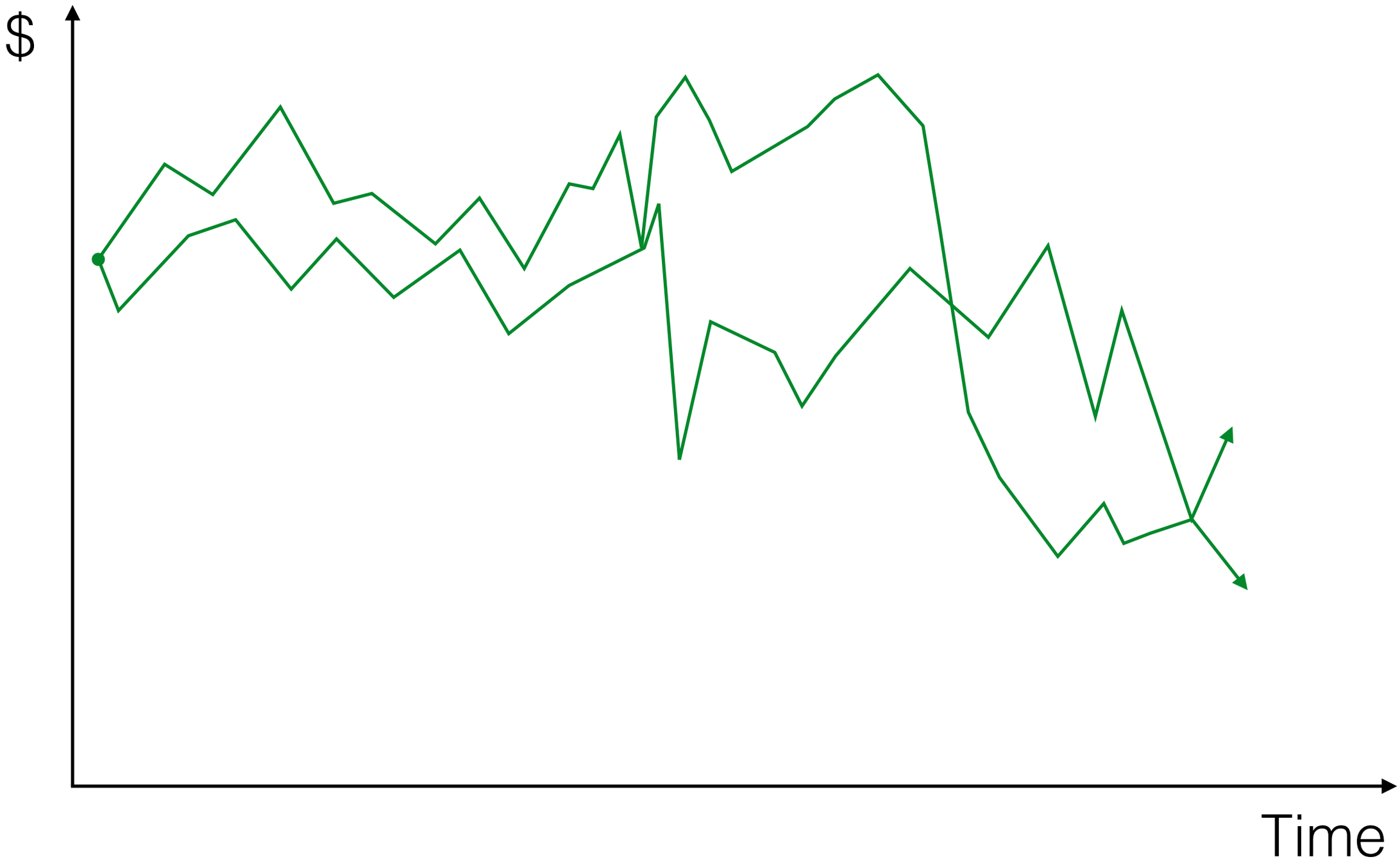
KITP 2019

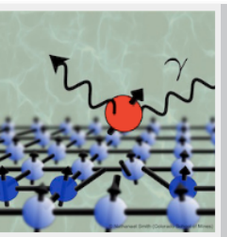
Open Quantum System Dynamics:

Quantum Simulators and Simulations Far From Equilibrium


$
Time

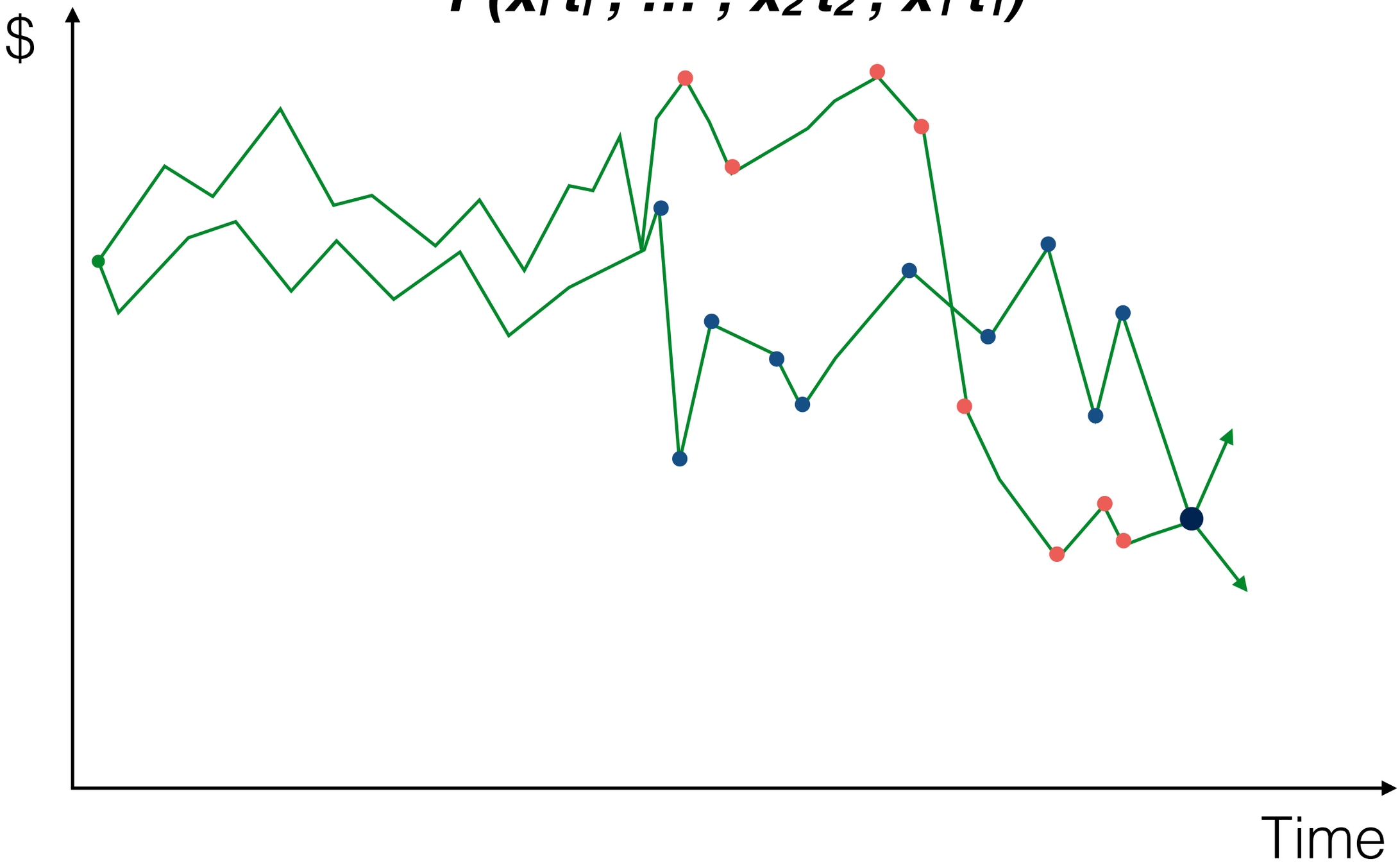


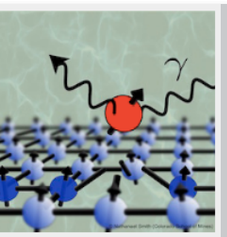




KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

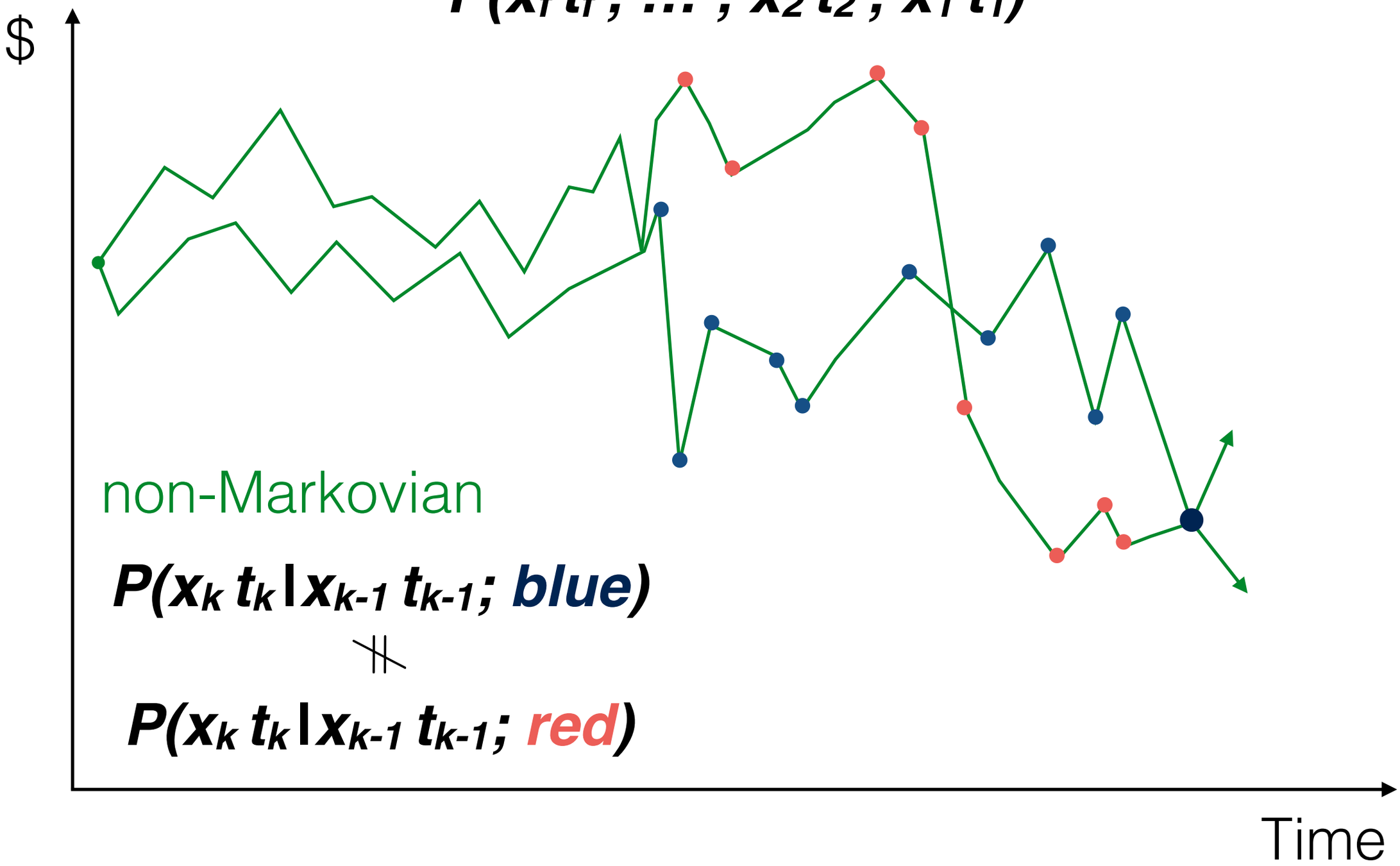
$$P(x_f\, t_f\, ;\, \ldots\, ;\, x_2\, t_2\, ;\, x_1\, t_1)$$

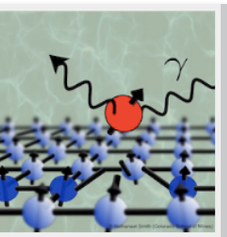




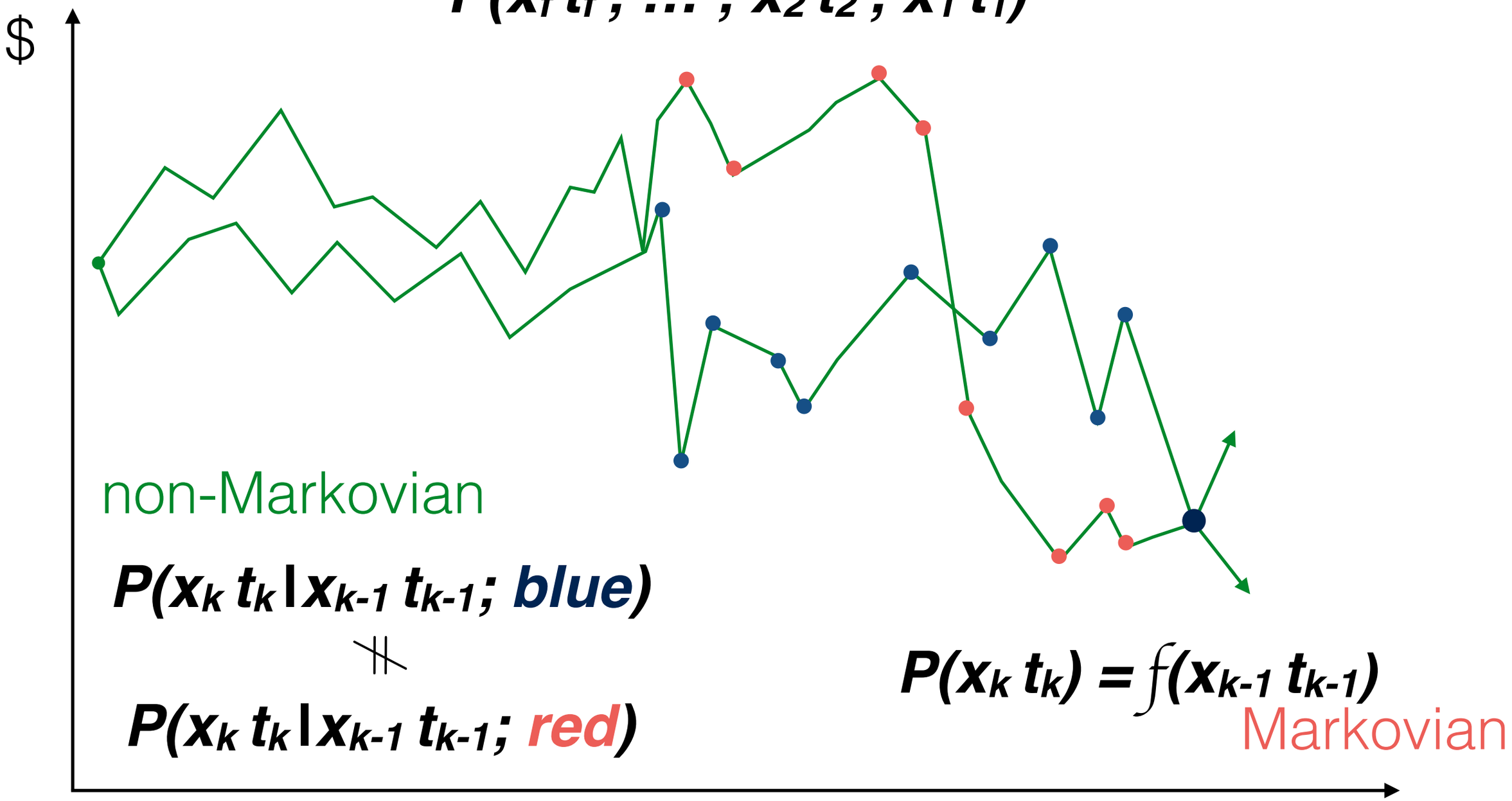
KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

$$P(x_f\, t_f\, ;\, \ldots\, ;\, x_2\, t_2\, ;\, x_1\, t_1)$$

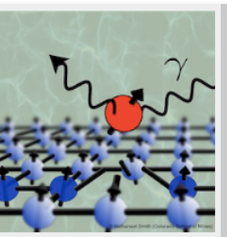


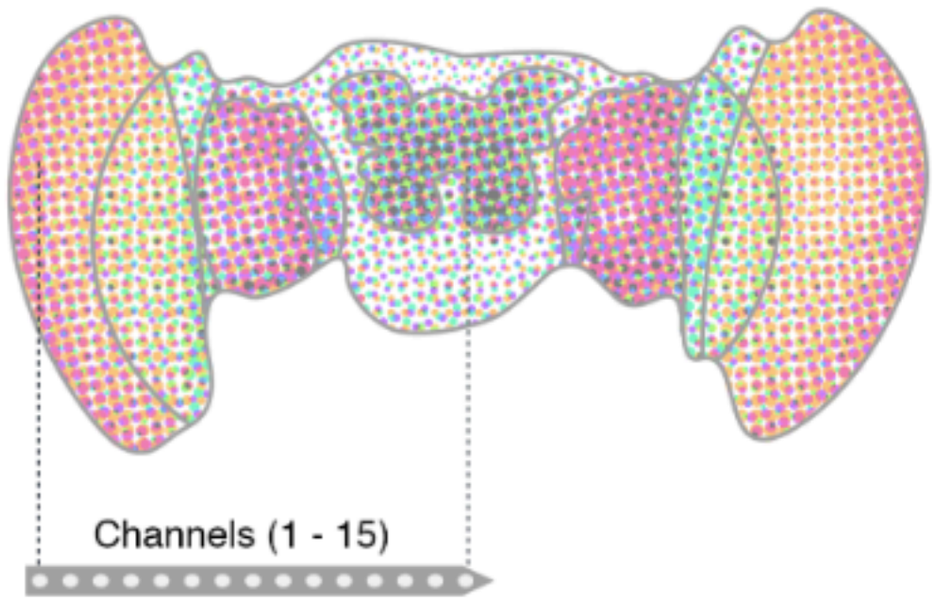


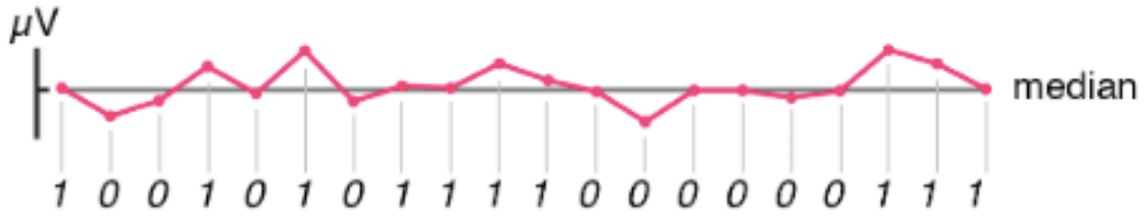
$$P(x_f\, t_f\, ;\, \ldots\, ;\, x_2\, t_2\, ;\, x_1\, t_1)$$

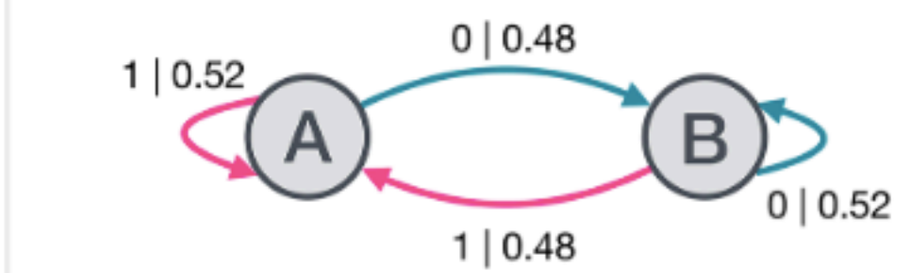


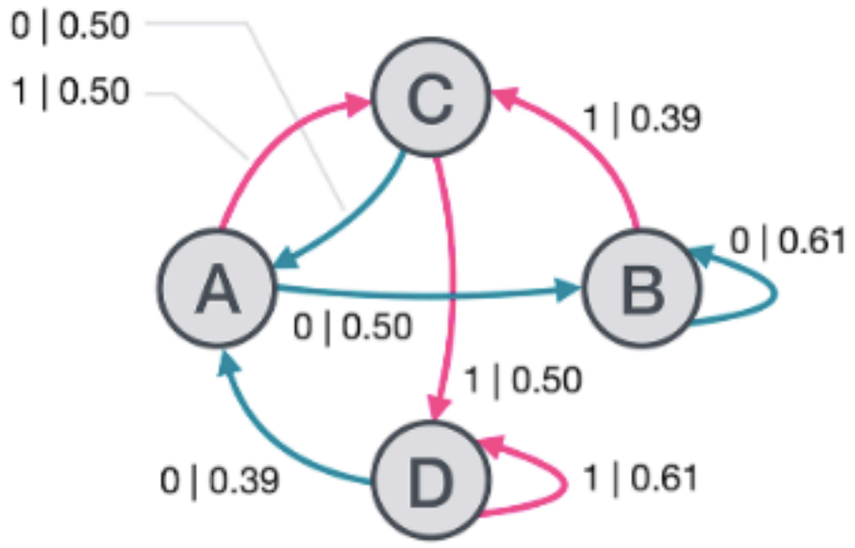
This is more than master equations

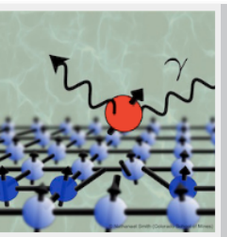




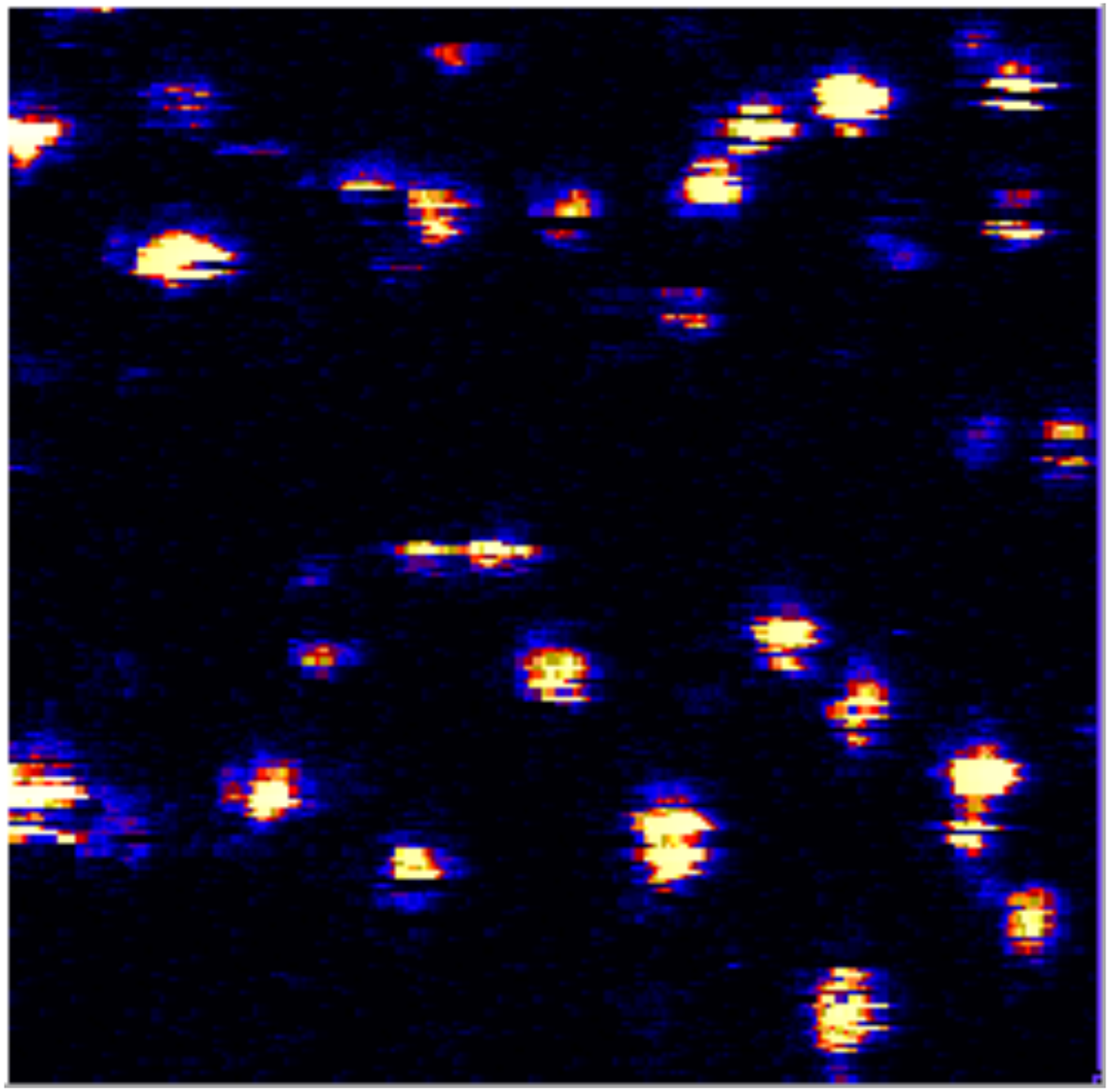


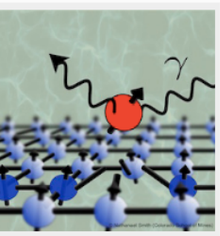






KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

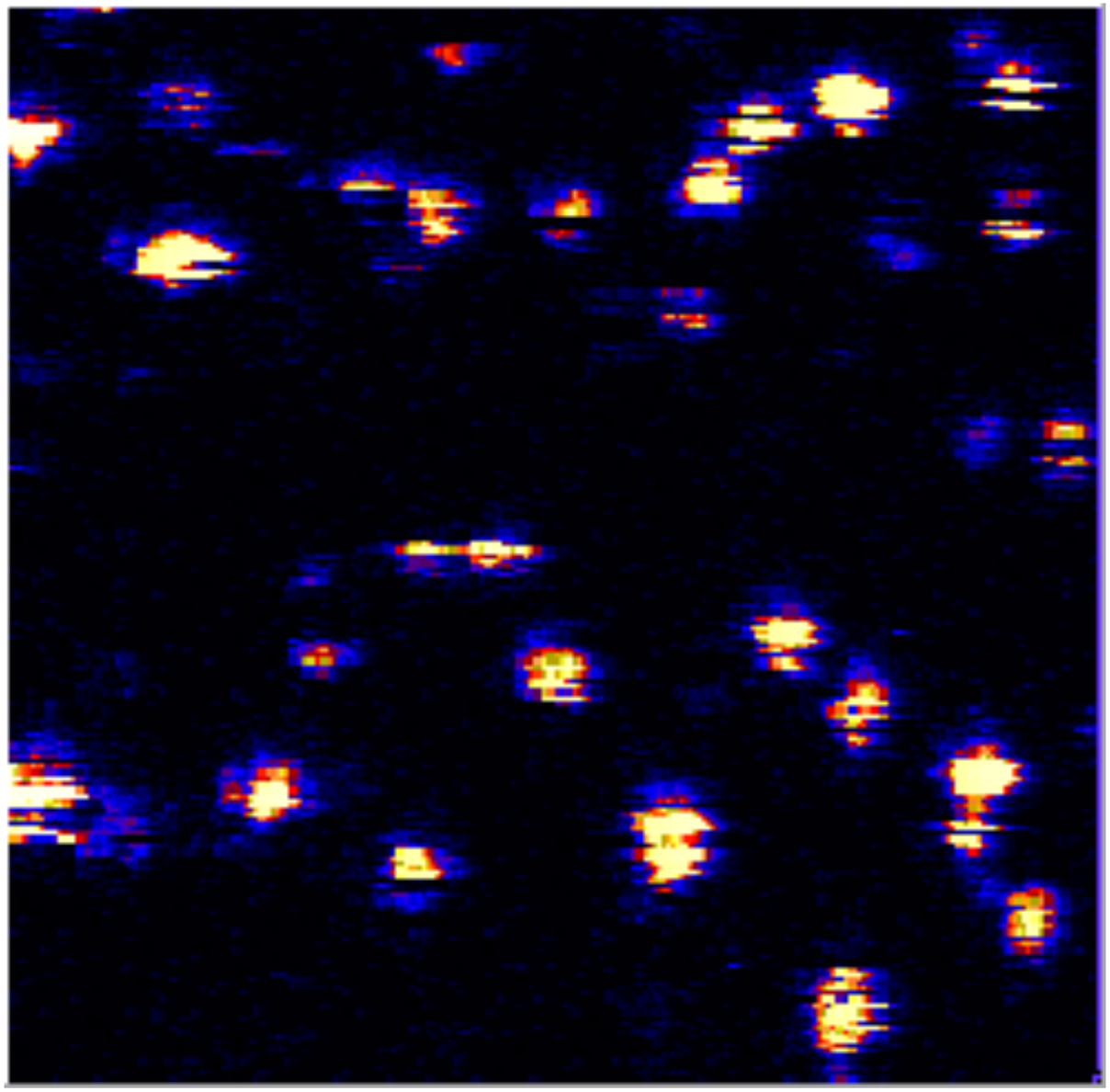


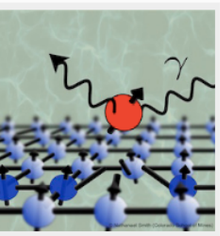


KITP 2019

Open Quantum System Dynamics:

Quantum Simulators and Simulations Far From Equilibrium





KITP 2019

Open Quantum System Dynamics:

Quantum Simulators and Simulations Far From Equilibrium

Stolen from the internet

KITP 2019

Open Quantum System Dynamics:

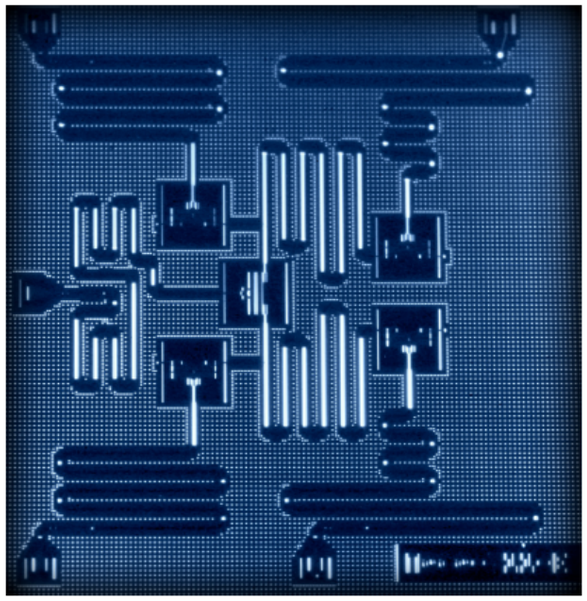
Quantum Simulators and Simulations Far From Equilibrium

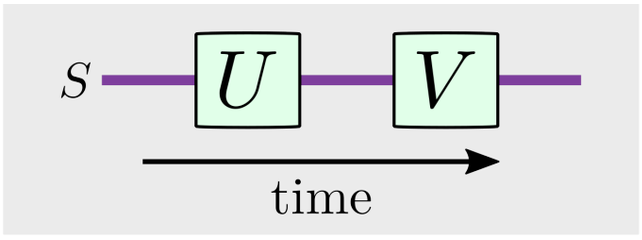
Stolen from the internet

KITP 2019

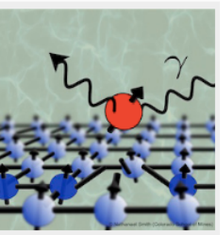
Open Quantum System Dynamics:

Quantum Simulators and Simulations Far From Equilibrium





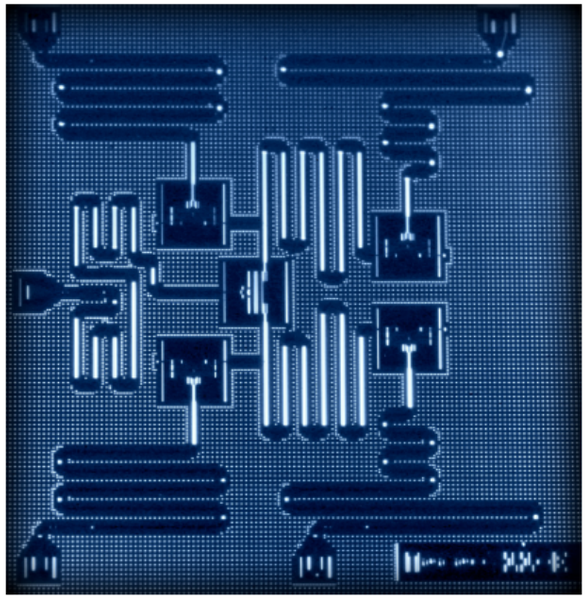
(a)

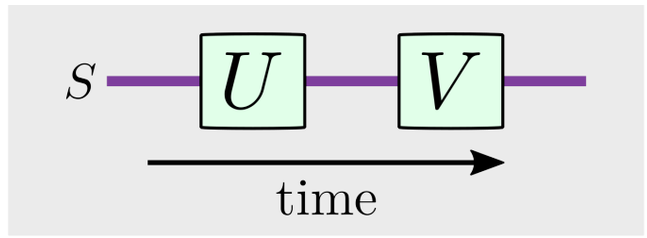


KITP 2019

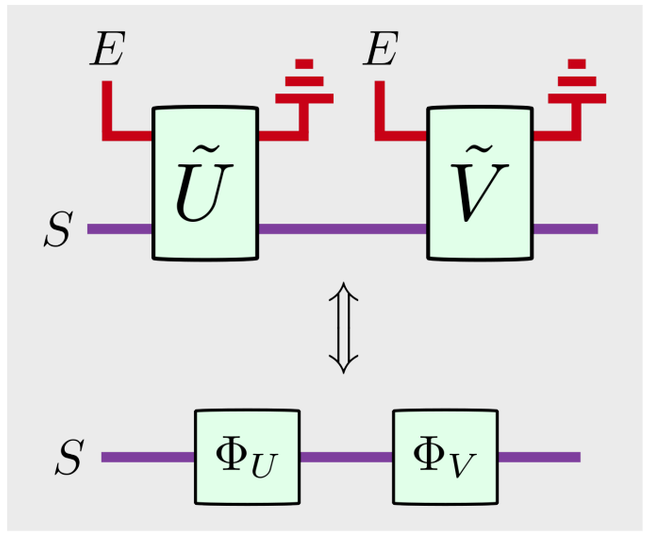
Open Quantum System Dynamics:

Quantum Simulators and Simulations Far From Equilibrium

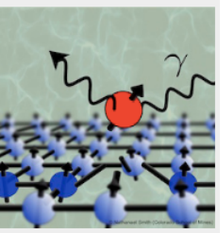




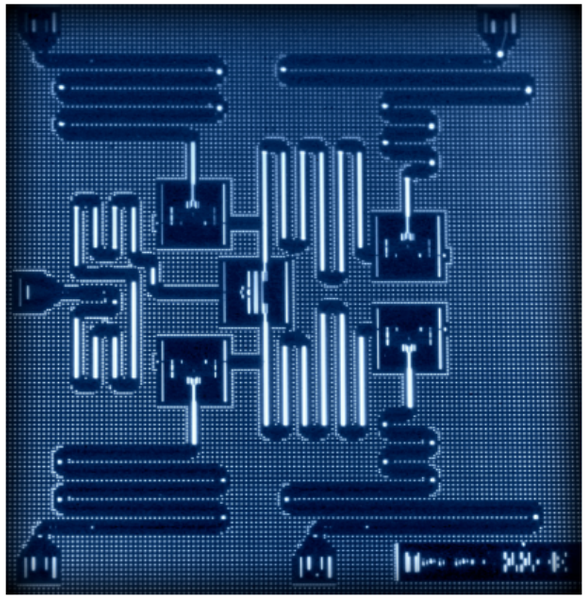
(a)



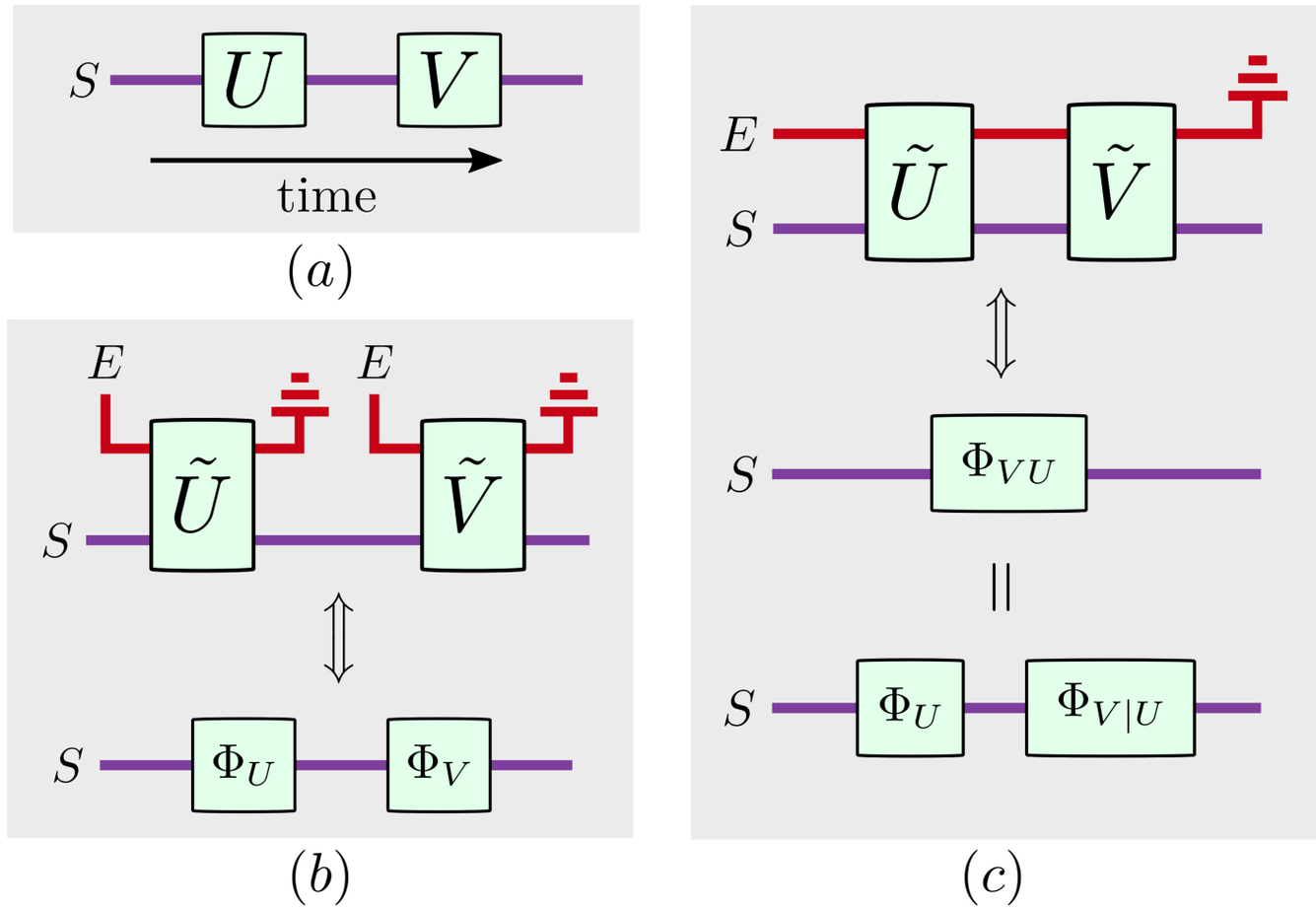
(b)

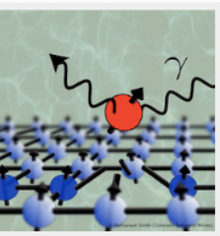


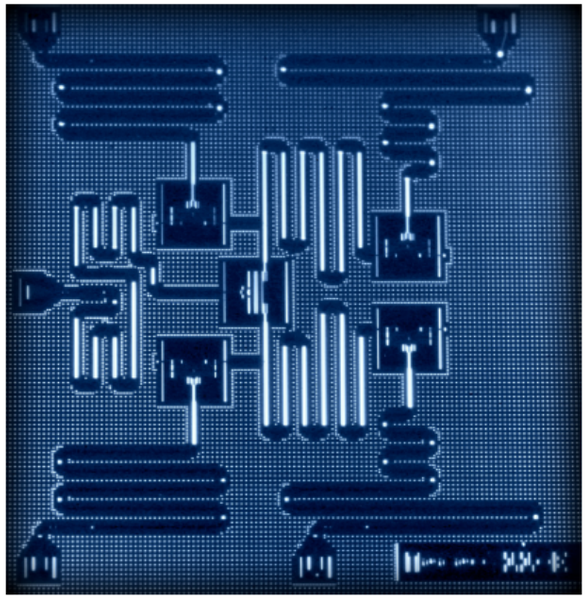
KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium



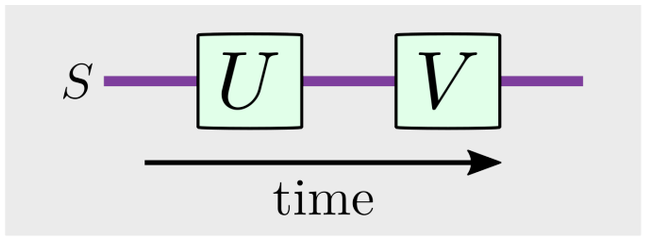
$$\Phi_{V|U} := \Phi_{VU} \circ \Phi_U^{-1}, \quad \text{where} \quad \Phi_U \circ \Phi_U^{-1} = \mathcal{I}$$



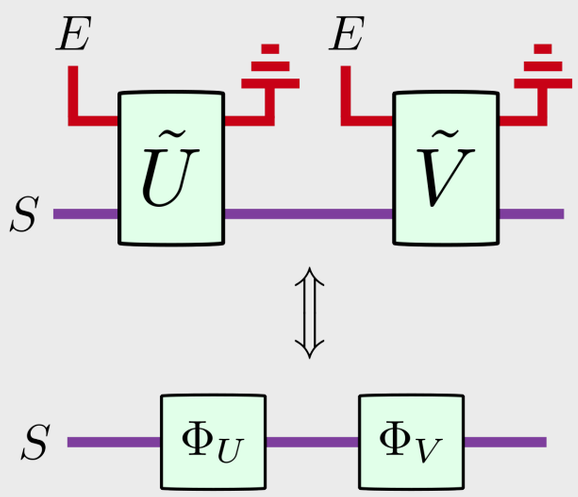




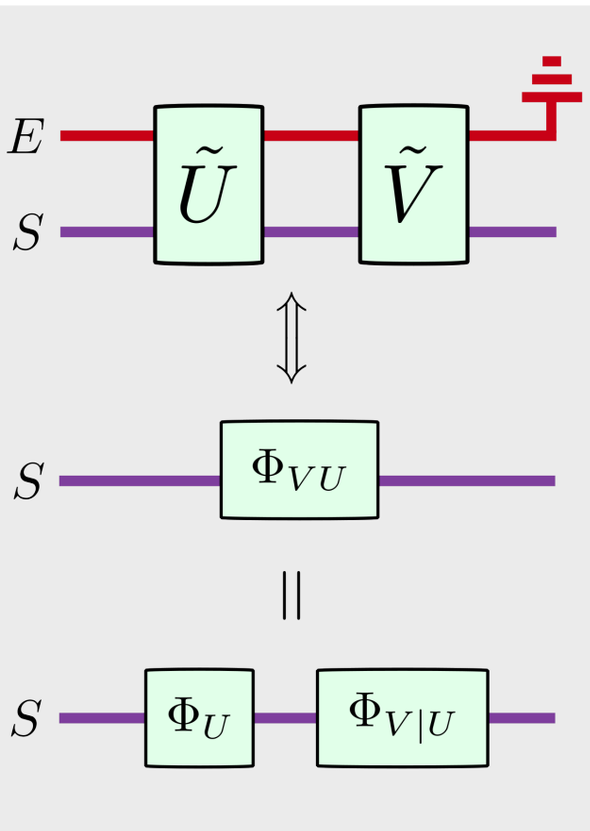
$$\Phi_{V|U} := \Phi_{VU} \circ \Phi_U^{-1}, \quad \text{where} \quad \Phi_U \circ \Phi_U^{-1} = \mathcal{I}$$



(a)

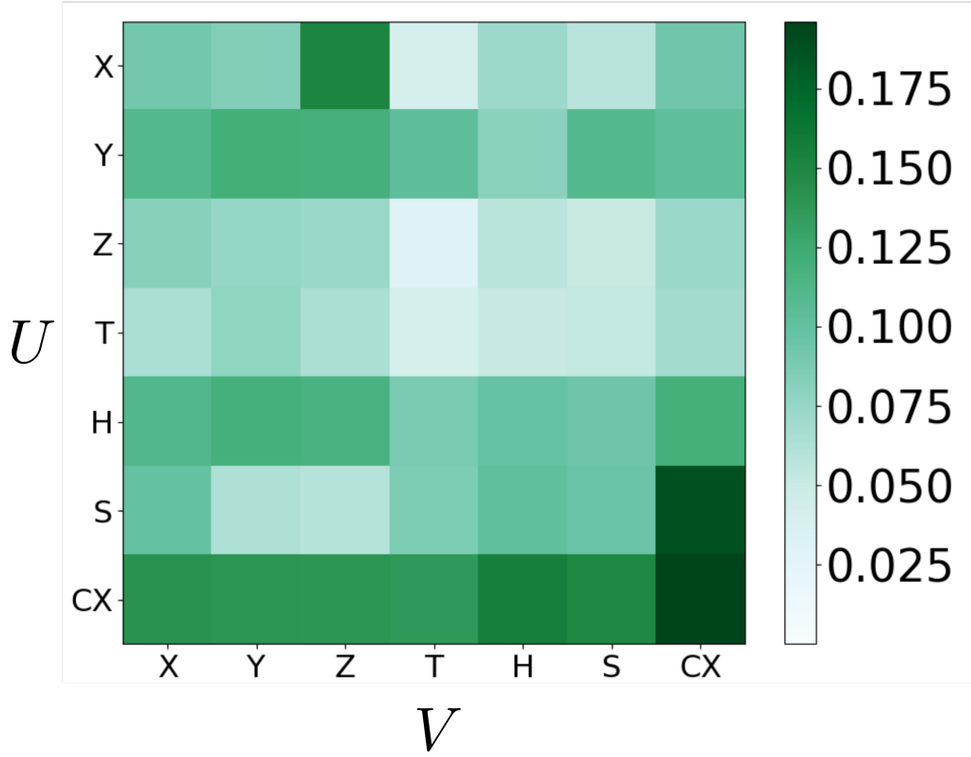


(b)



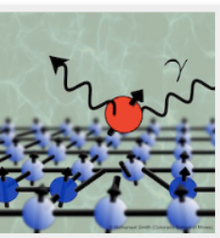
(c)

$$\mathrm{tr}\left|\Phi_{V|U}\right| - 1$$

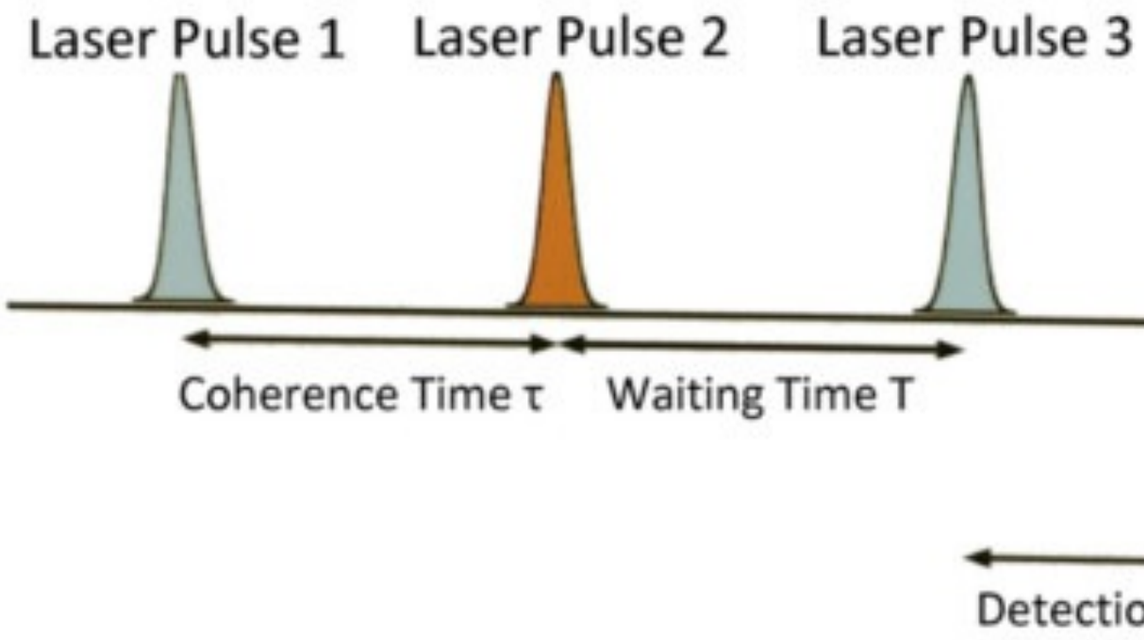


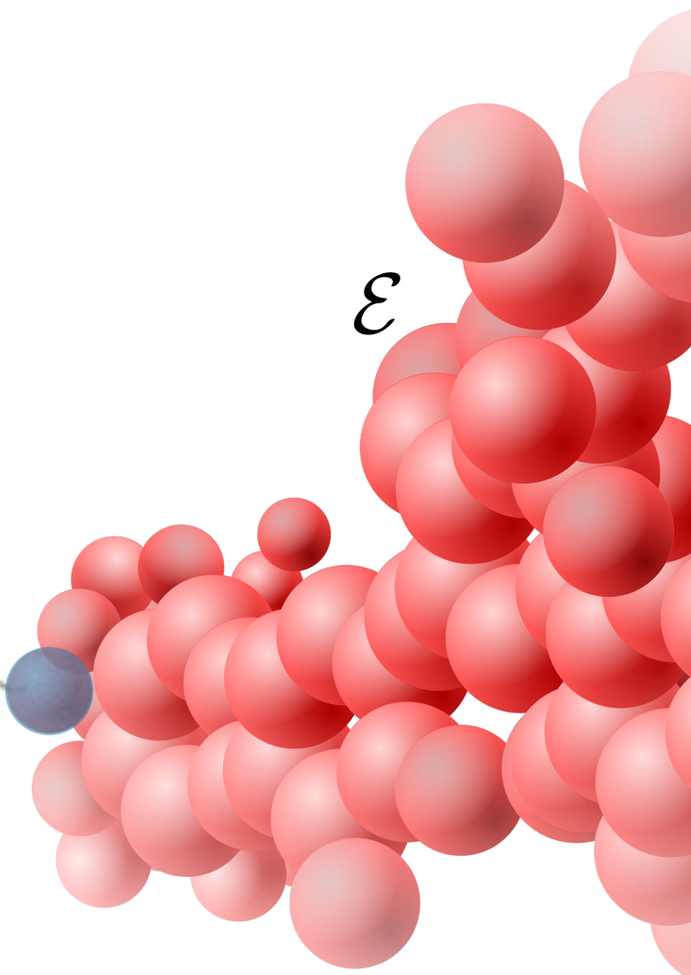
$$U, V \in \mathcal{G} = \{H, S, T, X, Y, Z, C_X\}$$

Morris, Pollock, Modi. *arXiv:1902.07980* (2019)

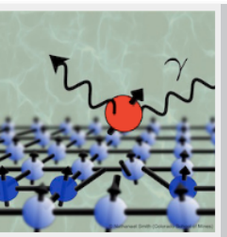


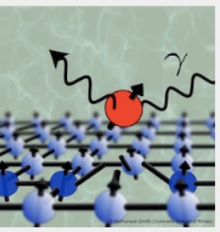
Open quantum systems





http://gaigroup.chem.upenn.edu/images/2DIR_pulse_sequence_small.jpg



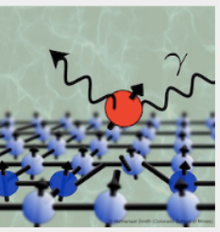


KITP 2019

Open Quantum System Dynamics: Quantum Simulators and Simulations Far From Equilibrium

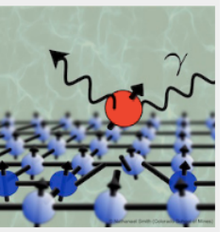
- We usually talk about quantum stochastic processes as master equations or complete positive maps

  What is the relationship between the two?



- We usually talk about quantum stochastic processes as master equations or complete positive maps
  What is the relationship between the two?

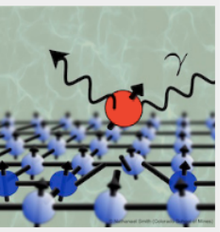
- Can we talk about correlations between multiple time steps in terms of quantum maps?



- We usually talk about quantum stochastic processes as master equations or complete positive maps
  What is the relationship between the two?

- Can we talk about correlations between multiple time steps in terms of quantum maps?

- Are there new approximations and techniques that can make open dynamics more tractable? (classical or quantum)

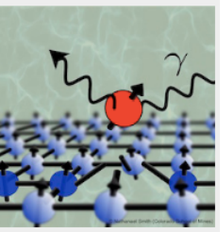


- We usually talk about quantum stochastic processes as master equations or complete positive maps
  What is the relationship between the two?

- Can we talk about correlations between multiple time steps in terms of quantum maps?

- Are there new approximations and techniques that can make open dynamics more tractable? (classical or quantum)

- Can we make open dynamics logically transparent and understand the causal structure?



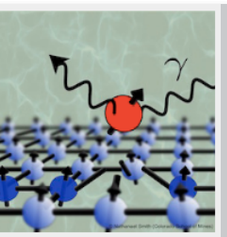
- We usually talk about quantum stochastic processes as master equations or complete positive maps
  What is the relationship between the two?

- Can we talk about correlations between multiple time steps in terms of quantum maps?

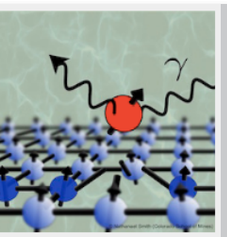
- Are there new approximations and techniques that can make open dynamics more tractable? (classical or quantum)

- Can we make open dynamics logically transparent and understand the causal structure?

- Why are all carbon atoms the same? (fast Markovianisation)



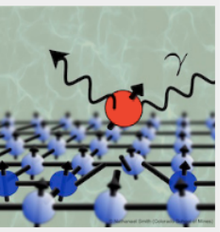
Quantum problem



KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

How do we quantise stuff?

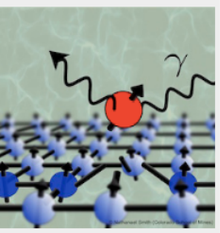
$$P \to \rho$$



How do we quantise stuff?

$$P \rightarrow \rho$$

$$P_{ABC} \rightarrow \rho_{ABC}$$

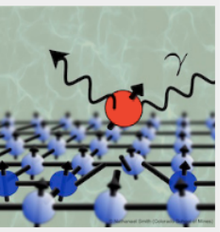


How do we quantise stuff?

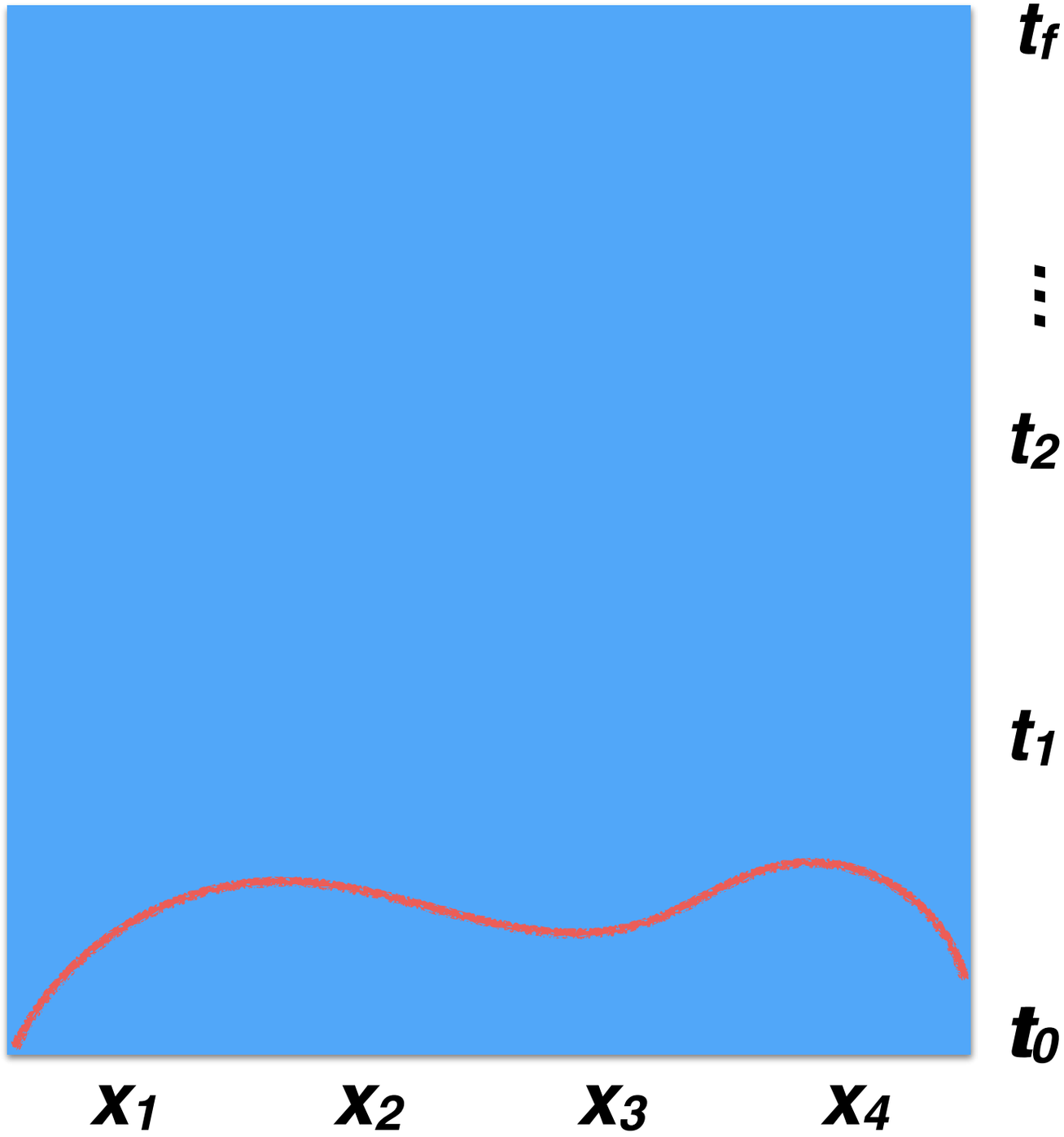
$$P \to \rho$$

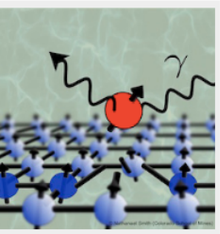
$$P_{ABC} \to \rho_{ABC}$$

$$P_{x_3t_3;x_2t_2;x_1t_1} \xrightarrow{?} \rho_{x_3t_3;x_2t_2;x_1t_1}$$

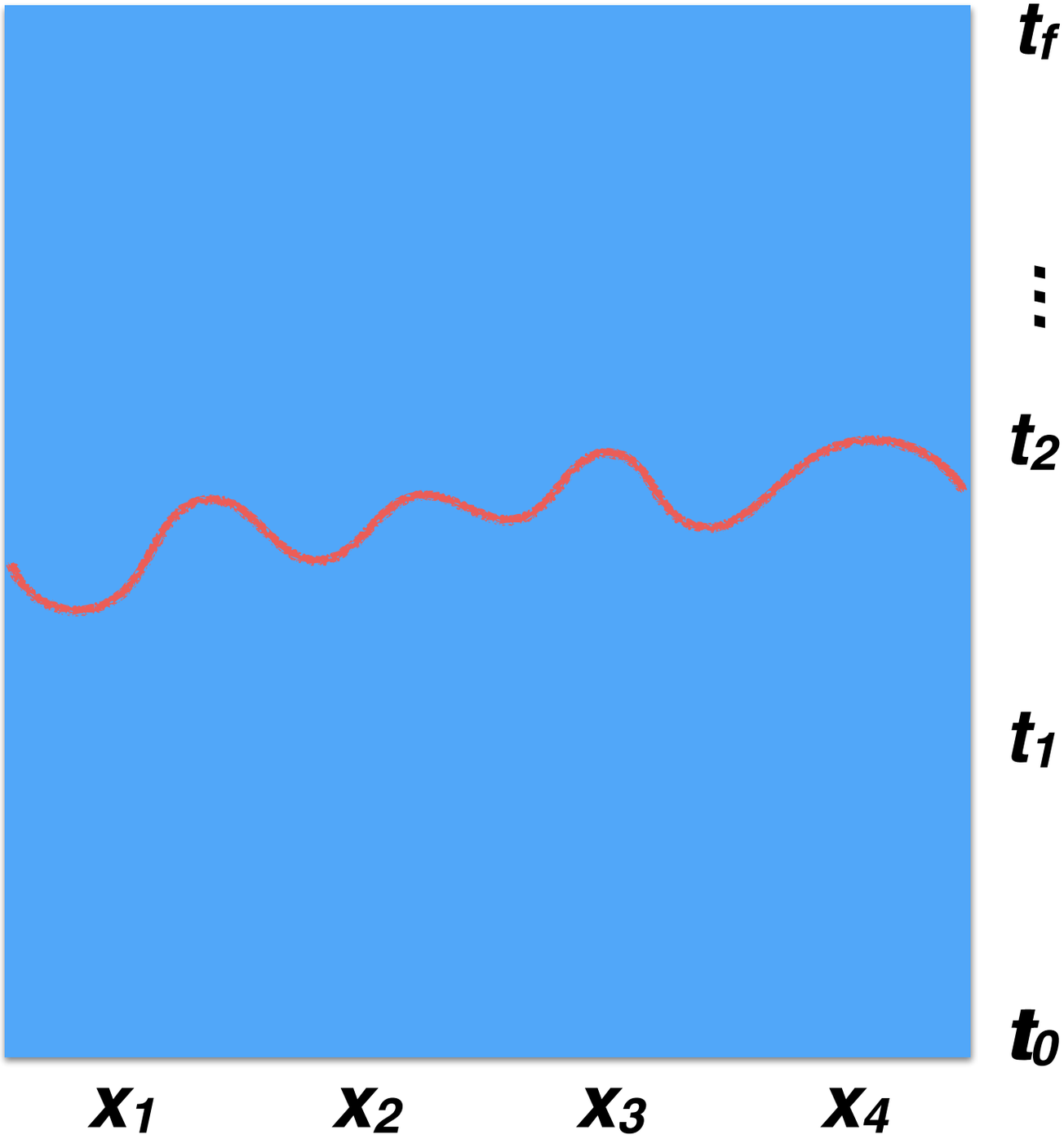


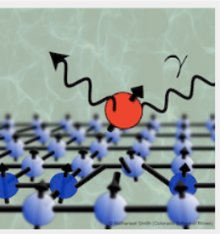
Interventions must be accounted for



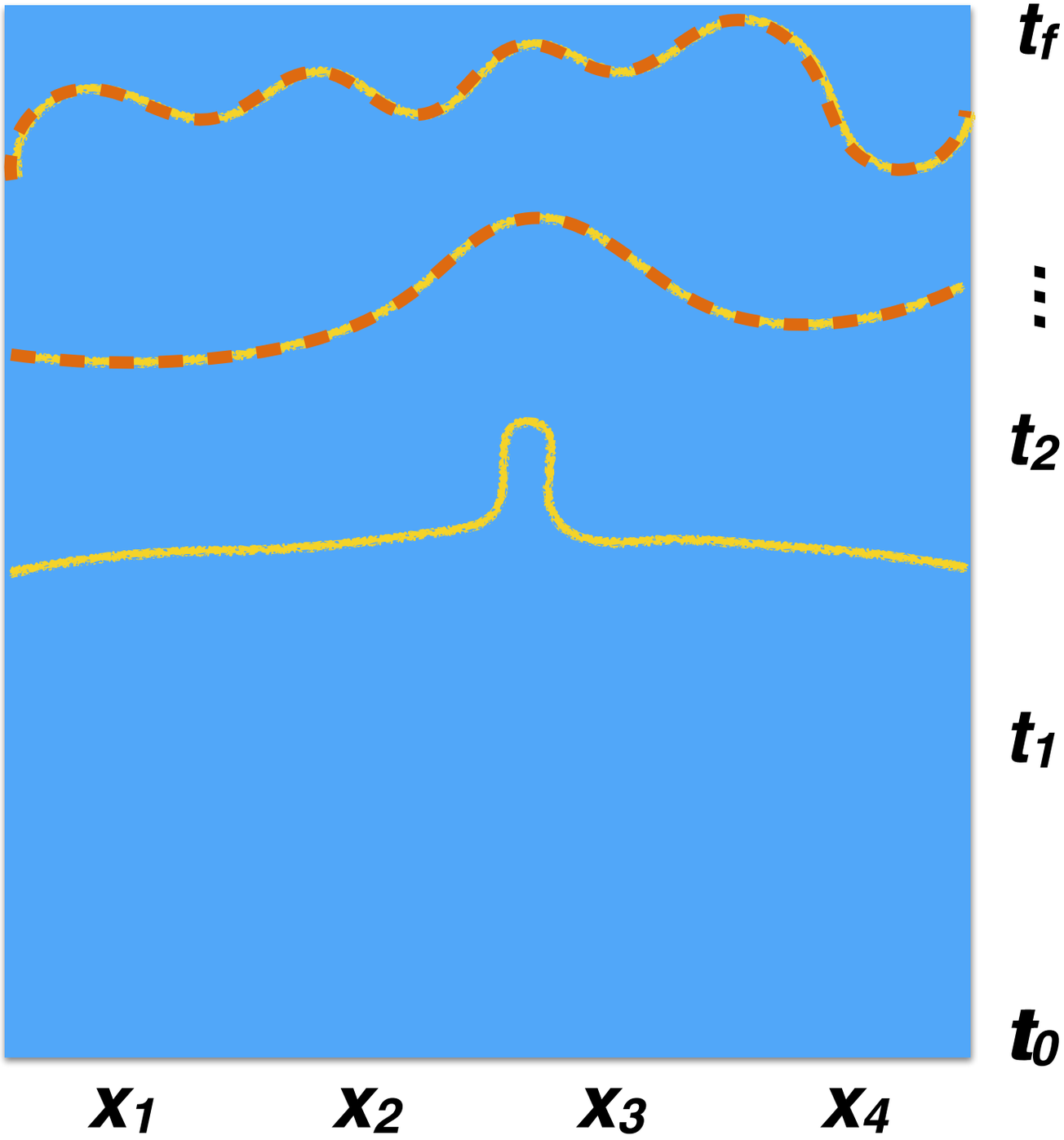


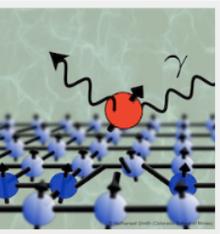
Interventions must be accounted for





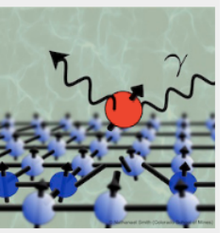
Interventions must be accounted for





Challenge

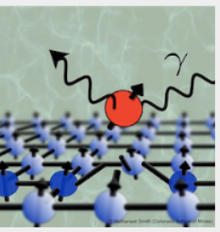
p c r o o n c t e r s o s l



KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

Challenge

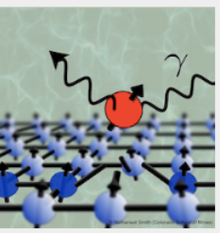
p c r o o n c t e r s o s l



KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

Challenge

p r o c e s s c o n t r o l

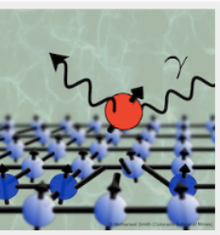


KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

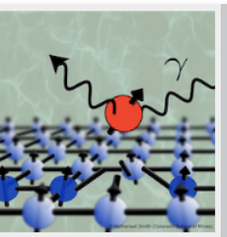
Challenge

p r o c e s s c o n t r o l

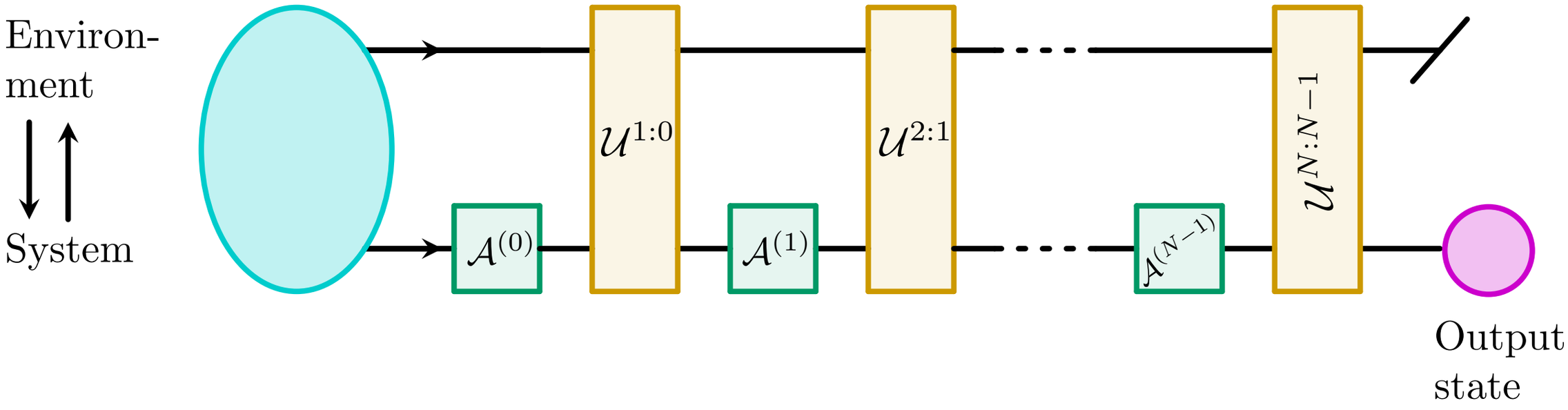
process [control] ⟶ quantum states (probabilities)

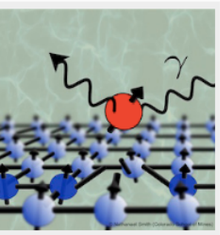


The framework



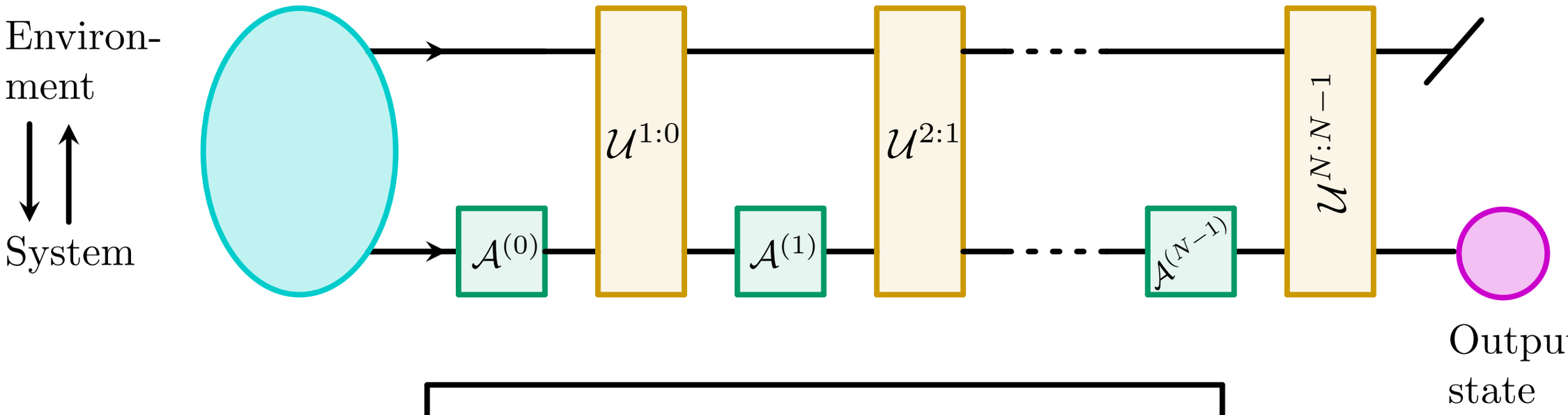
Open quantum mechanics



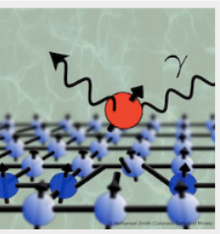


KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

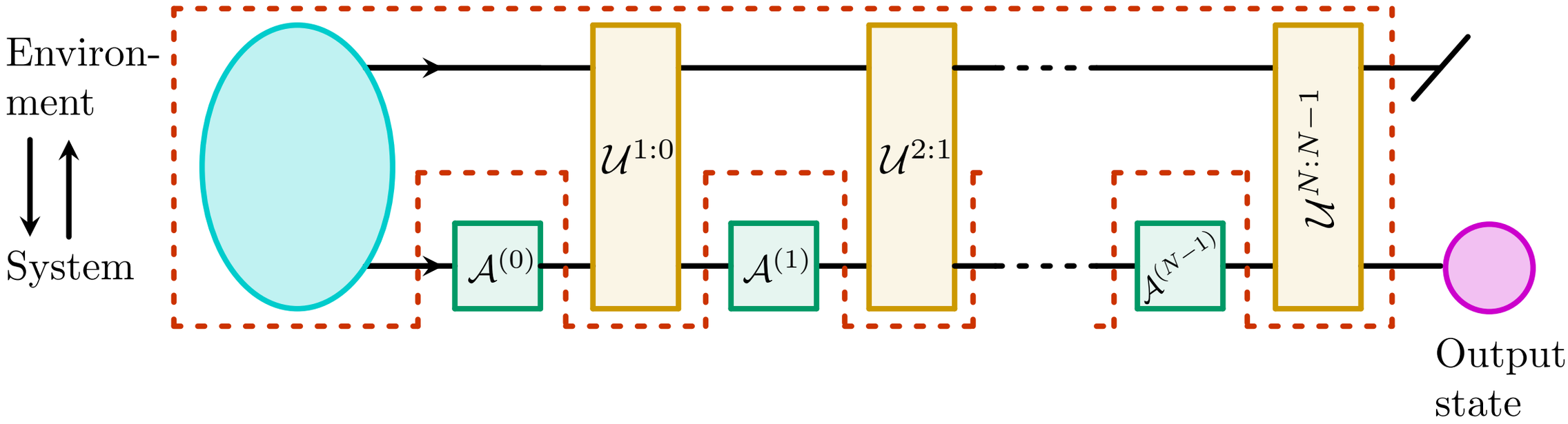
Open quantum mechanics



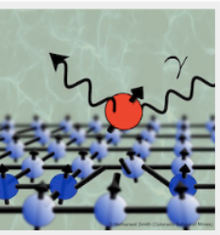
- Measurements
- Laser pulses
- RF pulses
- unitary operations
- .....



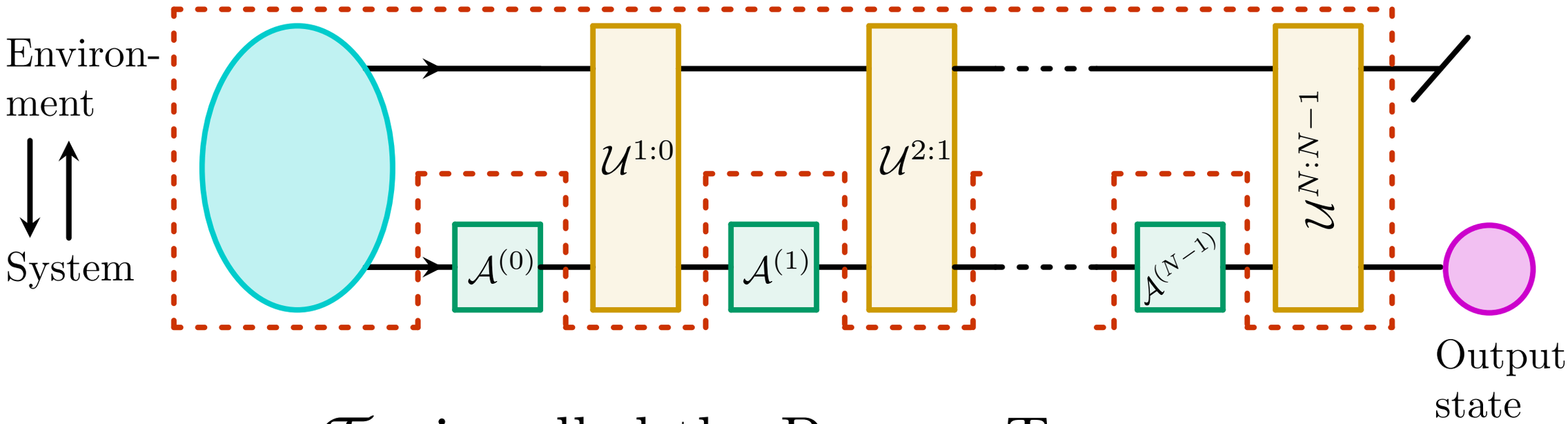
Open quantum mechanics



$$\mathcal{T}^{N:0}\left(\boxed{\mathcal{A}^{(0)}}, \boxed{\mathcal{A}^{(1)}}, \ldots, \boxed{\mathcal{A}^{(N-1)}}\right) = \bigcirc$$



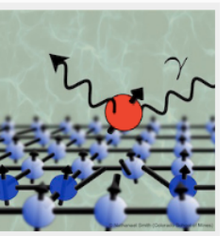
Open quantum mechanics



$\mathcal{T}$ is called the Process Tensor

A mapping from control operations to states

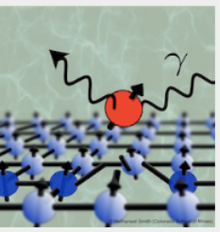
$$\mathcal{T}^{N:0}\left(\mathcal{A}^{(0)}, \mathcal{A}^{(1)}, \ldots, \mathcal{A}^{(N-1)}\right) =$$



How good is this framework?

It’s universal!

Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi. *Phys. Rev. A* 97, 012127 (2018)
(on arXiv since late 2015)

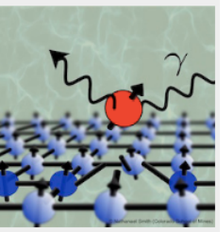


How good is this framework?

It's universal!

Open quantum evolution $\longleftrightarrow \mathcal{T}$

Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi. *Phys. Rev. A* 97, 012127 (2018)
(on arXiv since late 2015)



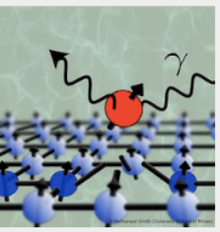
How good is this framework?

It's universal!

Open quantum evolution $\longleftrightarrow$ $\mathcal{T}$

linear and completely positive

Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi. *Phys. Rev. A* 97, 012127 (2018)
(on arXiv since late 2015)

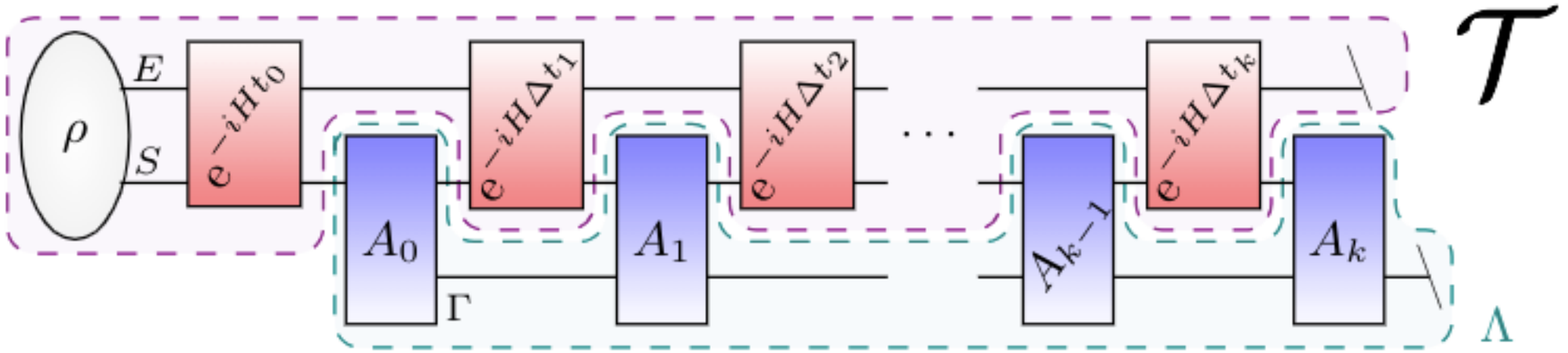


How good is this framework?

It's universal!

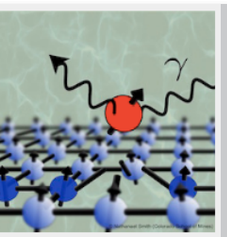
Open quantum evolution $\longleftrightarrow$ $\mathcal{T}$

linear and completely positive



$$p = \mathrm{tr}[\mathcal{T}\Lambda]$$

Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi. *Phys. Rev. A* 97, 012127 (2018)
(on arXiv since late 2015)

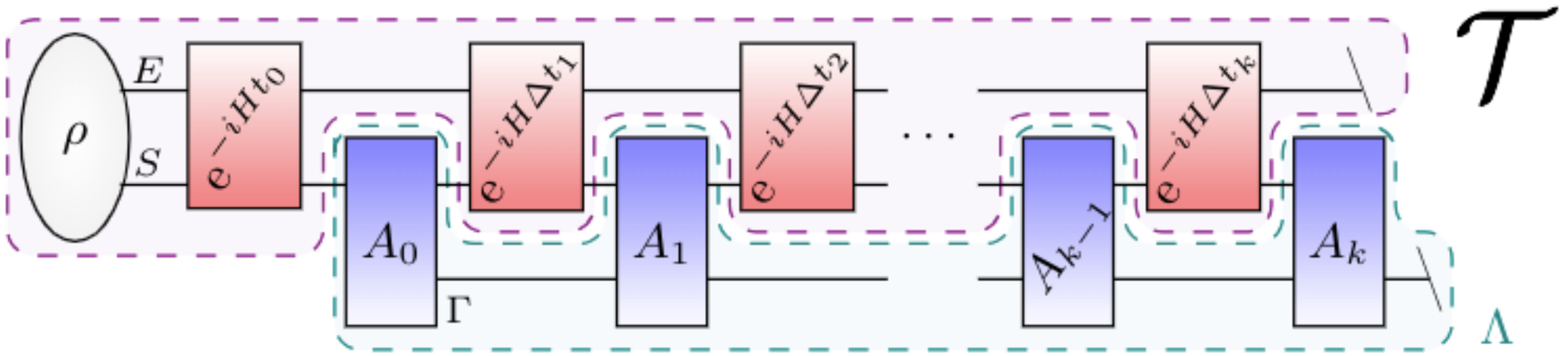


How good is this framework?

It's universal!

Open quantum evolution $\longleftrightarrow$ $\mathcal{T}$

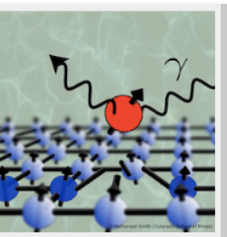
linear and completely positive



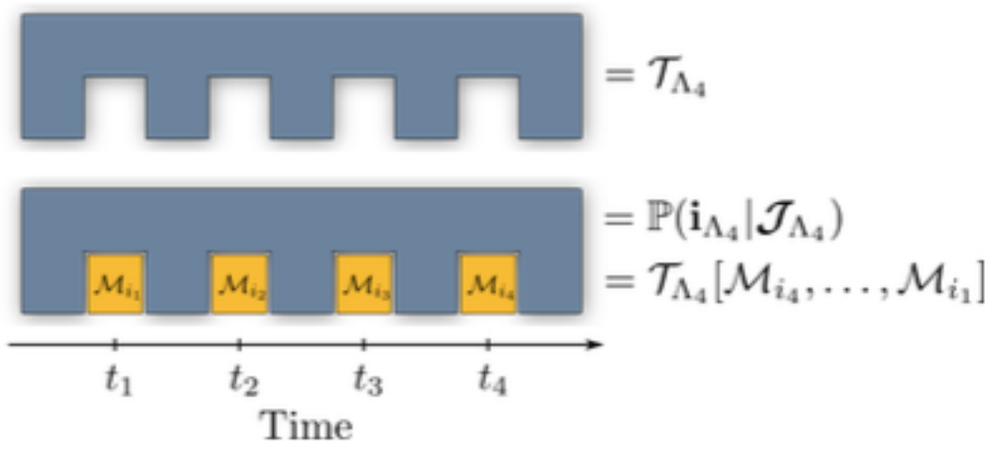
$$p = \mathrm{tr}[\mathcal{T}\Lambda]$$

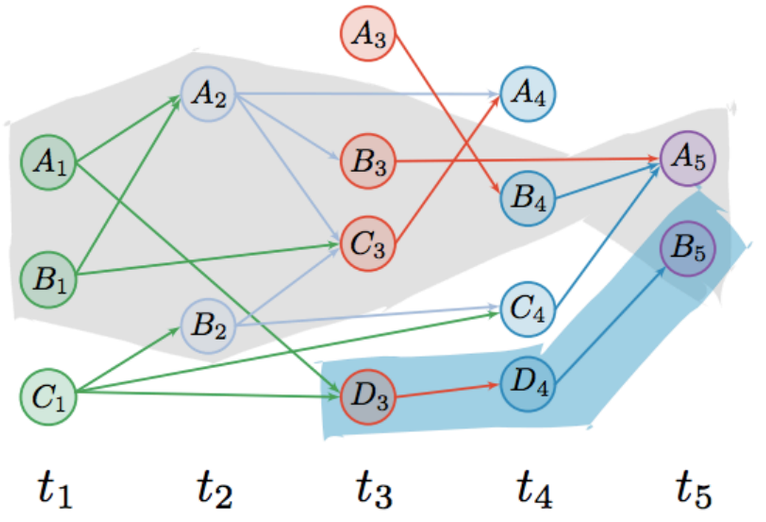
Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi. *Phys. Rev. A* 97, 012127 (2018)
(on arXiv since late 2015)

The language is in terms of maps (quantum combs)

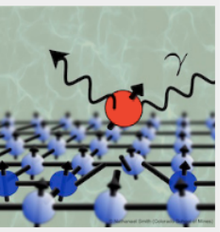


Kolmogorov conditions for quantum causal models

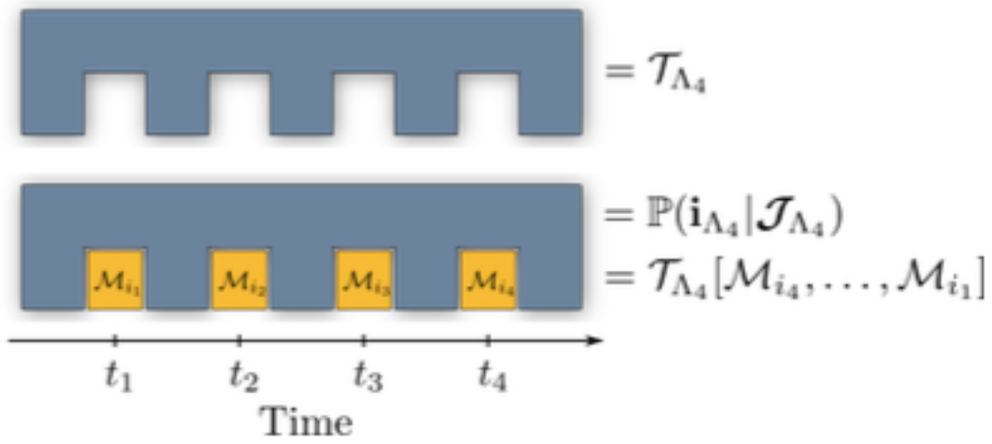


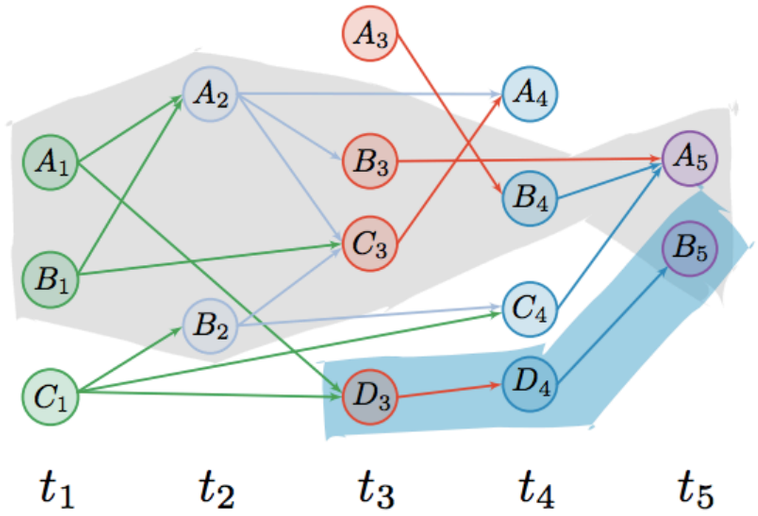


Milz, Sakuldee, Pollock, Modi. *arXiv:1712.02589* (2017)



Kolmogorov conditions for quantum causal models

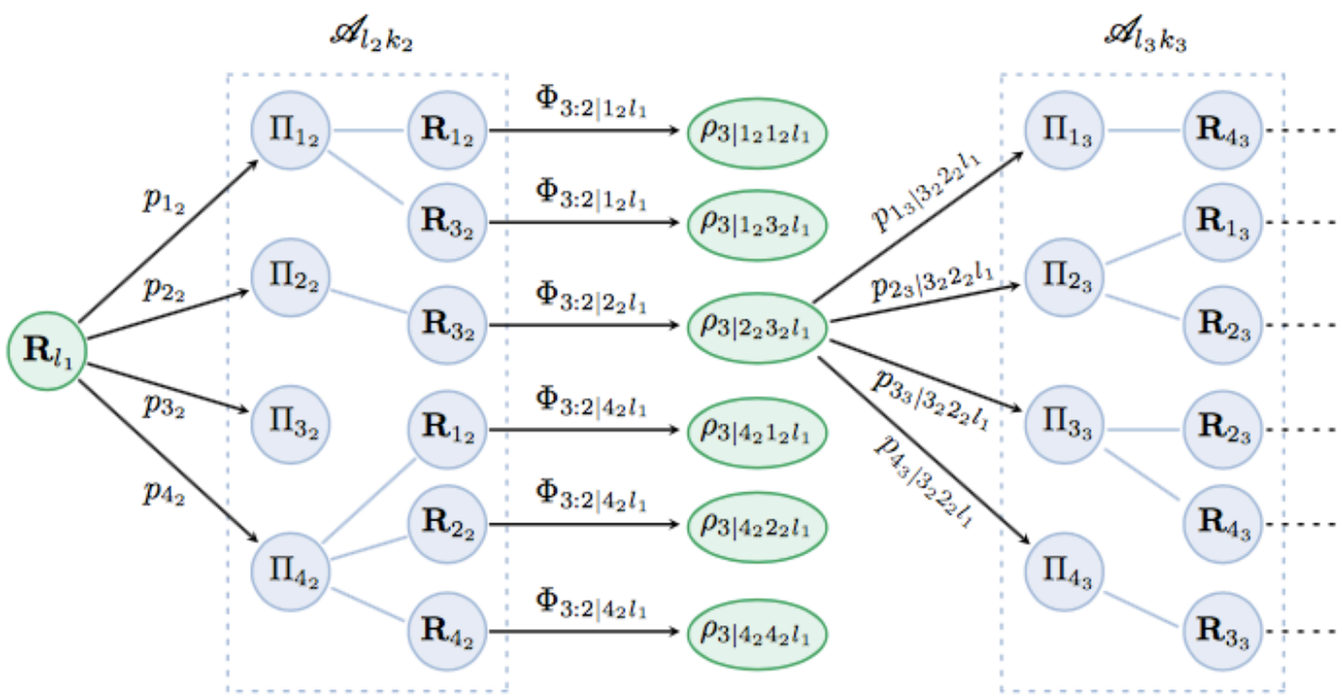


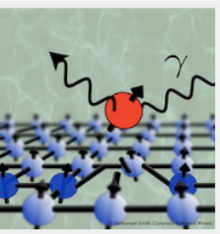


Milz, Sakuldee, Pollock, Modi. *arXiv:1712.02589* (2017)

Construct
coherent quantum trajectories

Sakuldee, Milz, Pollock, Modi. *J. Phys. A* 51, 414014 (2018)





KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

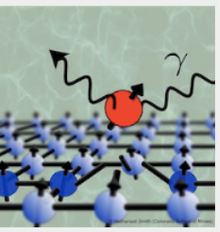
- We usually talk about quantum stochastic processes as master equations or complete positive maps
  What is the relationship between the two?

- Can we talk about correlations between multiple time steps in terms of quantum maps?

- Are there new approximations and techniques that can make open dynamics more tractable? (classical or quantum)

- Can we make open dynamics logically transparent and understand the causal structure?

- Why are all carbon atoms the same? (fast Markovianisation)



Quantum Probability literature

Advances in Mathematics
Volume 20, Issue 3, June 1976, Pages 329–366

Nonrelativistic quantum mechanics as a noncommutative Markof process

Luigi Accardi[1, 2]

Comm. Math. Phys.
Volume 65, Number 3 (1979), 281-294.

Non-Markovian quantum stochastic processes and their entropy

Göran Lindblad

Volume 18, Issue 1, 1982, pp. 97–133 **DOI: 10.2977/prims/1195184017**
Published online: 1982-04-30

Quantum Stochastic Processes

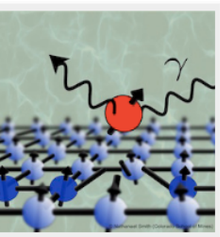
Luigi Accardi[1], Alberto Frigerio and John T. Lewis[2]

Letters in Mathematical Physics
September 1994, Volume 32, Issue 1, pp 75–82 | Cite as

Defining quantum dynamical entropy

Authors | Authors and affiliations

R. Alicki, M. Fannes



KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

Quantum Probability literature

Advances in Mathematics
Volume 20, Issue 3, June 1976, Pages 329–366

Nonrelativistic quantum mechanics as a noncommutative Markof process

Luigi Accardi[1, 2]

Comm. Math. Phys.
Volume 65, Number 3 (1979), 281-294.

Non-Markovian quantum stochastic processes and their entropy

Göran Lindblad

Volume 18, Issue 1, 1982, pp. 97–133 **DOI: 10.2977/prims/1195184017**
Published online: 1982-04-30

Quantum Stochastic Processes

Luigi Accardi[1], Alberto Frigerio and John T. Lewis[2]

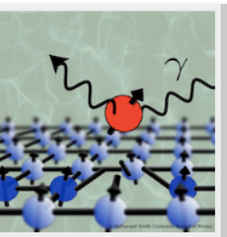
Letters in Mathematical Physics
September 1994, Volume 32, Issue 1, pp 75–82 | Cite as

Defining quantum dynamical entropy

Authors Authors and affiliations

R. Alicki, M. Fannes

But we can make it better!



KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

Quantum Channels with Memory

Dennis Kretschmann, Reinhard F. Werner

Quantum Circuits Architecture

Giulio Chiribella, Giacomo Mauro D'Ariano, Paolo Perinotti

Multiple-time states and multiple-time measurements in quantum mechanics

Y. Aharonov, S. Popescu, J. Tollaksen, L. Vaidman

The Operator Tensor Formulation of Quantum Theory

Lucien Hardy

Non-Markovian quantum processes: complete framework and efficient characterisation

Felix A. Pollock, César Rodríguez-Rosario, Thomas Frauenheim, Mauro Paternostro, Kavan Modi

Causal Boxes: Quantum Information-Processing Systems Closed under Composition

Christopher Portmann, Christian Matt, Ueli Maurer, Renato Renner, Björn Tackmann

Quantum causal modelling

Fabio Costa, Sally Shrapnel

Quantum common causes and quantum causal models

John-Mark A. Allen, Jonathan Barrett, Dominic C. Horsman, Ciaran M. Lee, Robert W. Spekkens

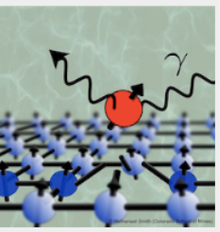
Superdensity Operators for Spacetime Quantum Mechanics

Jordan Cotler, Chao-Ming Jian, Xiao-Liang Qi, Frank Wilczek

Simulation complexity of open quantum dynamics: Connection with tensor networks

I. A. Luchnikov, S. V. Vintskevich, H. Ouerdane, S. N. Filippov

time

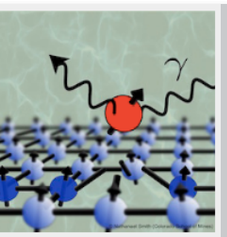


KITP 2019

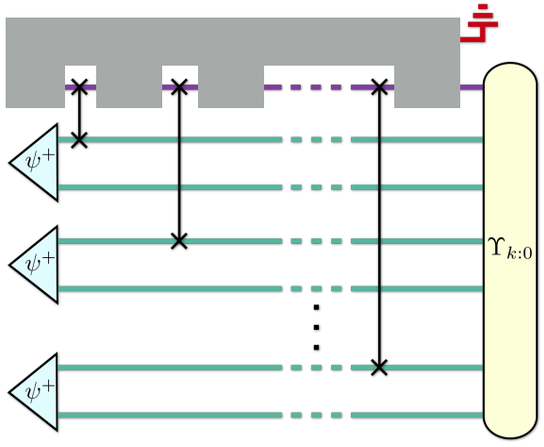
Open Quantum System Dynamics:

Quantum Simulators and Simulations Far From Equilibrium

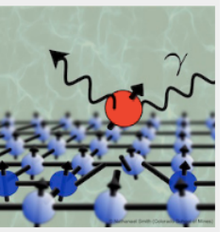
A few problems we can solve with this framework



Processes can be mapped to states with tensor network representation

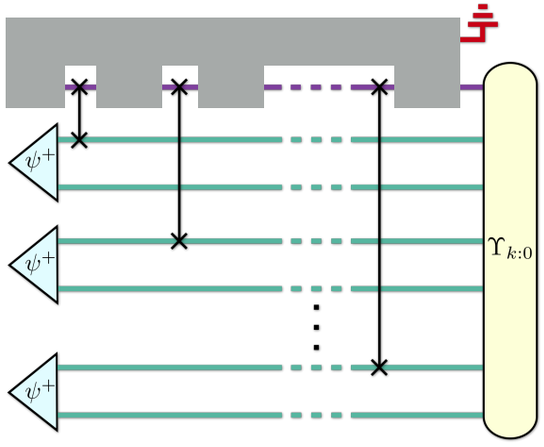


$$\mathcal{T}_{k:0} \quad \Upsilon_{k:0}$$



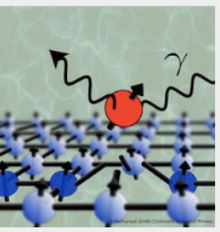
KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

Processes can be mapped to states with tensor network representation

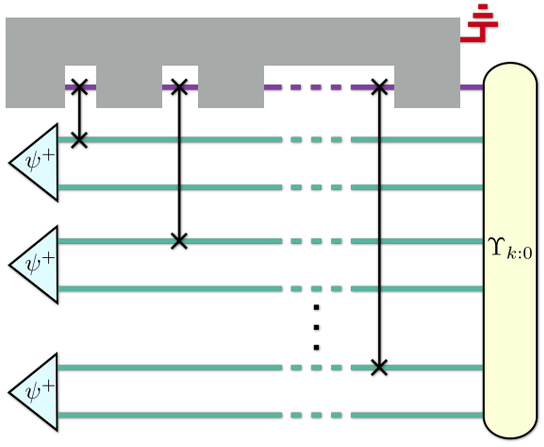


$$\mathcal{T}_{k:0} \longleftrightarrow \Upsilon_{k:0}$$

MPO



Processes can be mapped to states with tensor network representation

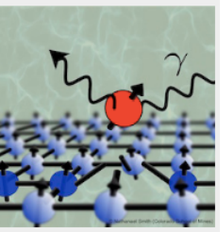


$\mathcal{T}_{k:0}$ ⟷ $\Upsilon_{k:0}$

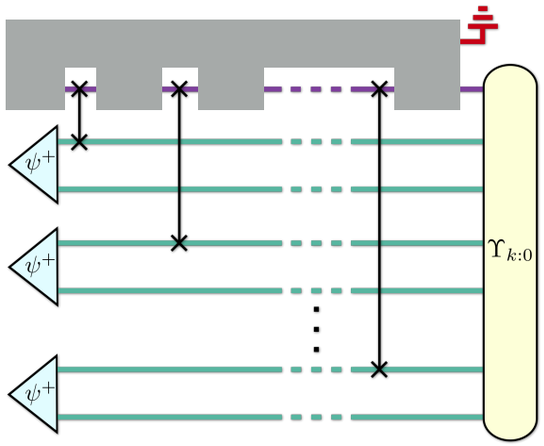
MPO

$P_{x_3t_3;x_2t_2;x_1t_1} \xrightarrow{?} \rho_{x_3t_3;x_2t_2;x_1t_1}$

$P \subset \Upsilon$



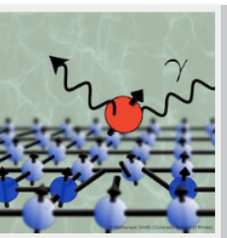
Processes can be mapped to states with tensor network representation



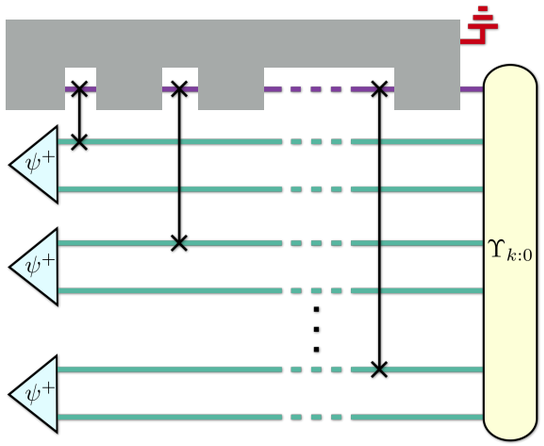
$\mathcal{T}_{k:0}$ ⟷ $\Upsilon_{k:0}$ MPO

$P_{x_3t_3;x_2t_2;x_1t_1} \xrightarrow{?} \rho_{x_3t_3;x_2t_2;x_1t_1}$ $P \subset \Upsilon$

Meaningful measures for non-Markovianity



Processes can be mapped to states with tensor network representation



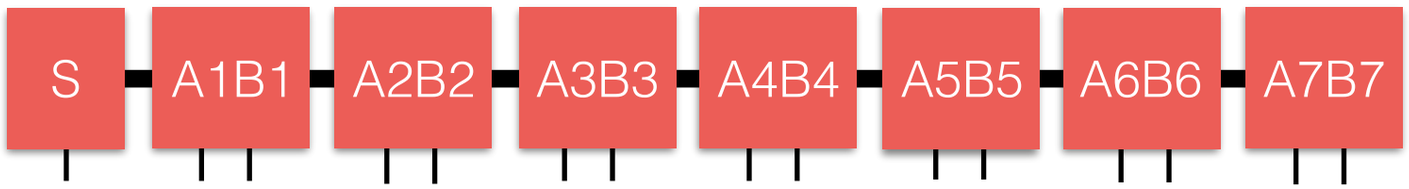
$$\mathcal{T}_{k:0} \longleftrightarrow \Upsilon_{k:0}$$

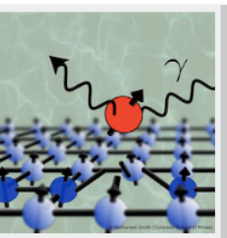
MPO

$$P_{x_3t_3;x_2t_2;x_1t_1} \xrightarrow{?} \rho_{x_3t_3;x_2t_2;x_1t_1} \qquad P \subset \Upsilon$$

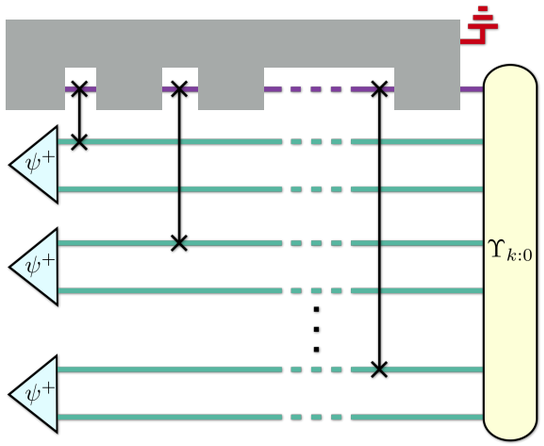
Meaningful measures for non-Markovianity

$\Upsilon_{7:0}$





Processes can be mapped to states with tensor network representation

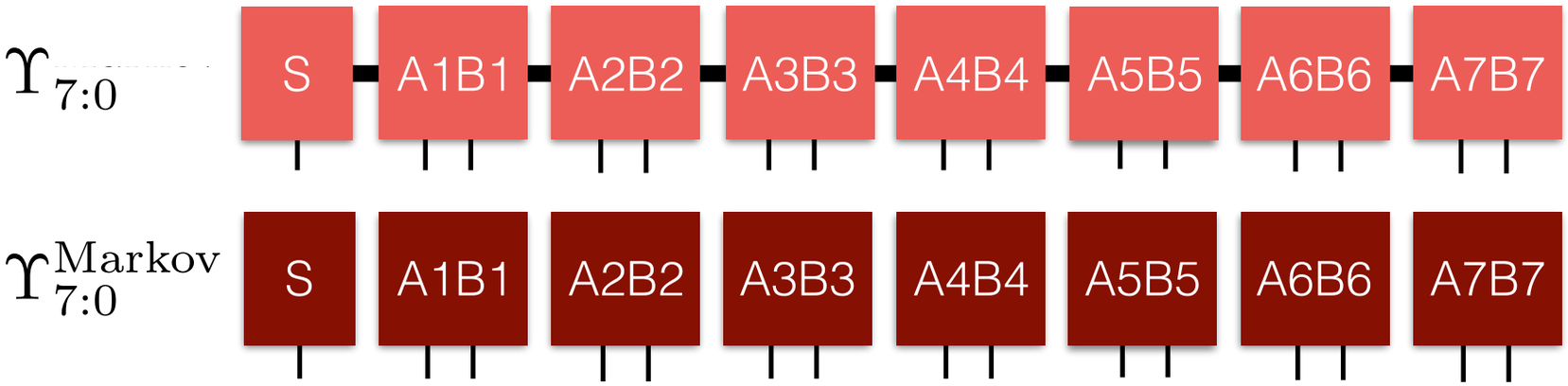


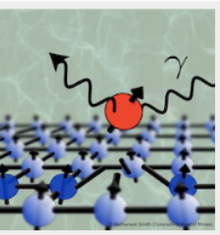
$$\mathcal{T}_{k:0} \longleftrightarrow \Upsilon_{k:0}$$

MPO

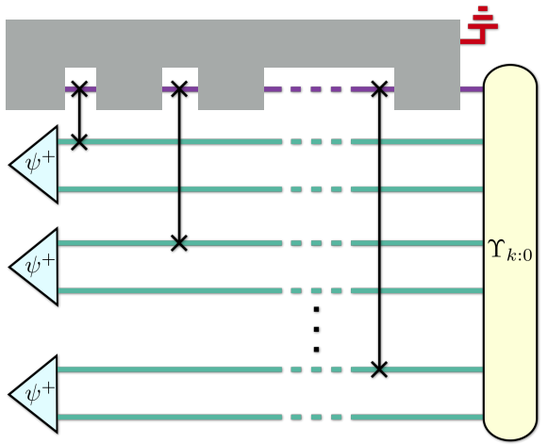
$$P_{x_3t_3;x_2t_2;x_1t_1} \xrightarrow{?} \rho_{x_3t_3;x_2t_2;x_1t_1} \qquad P \subset \Upsilon$$

Meaningful measures for non-Markovianity





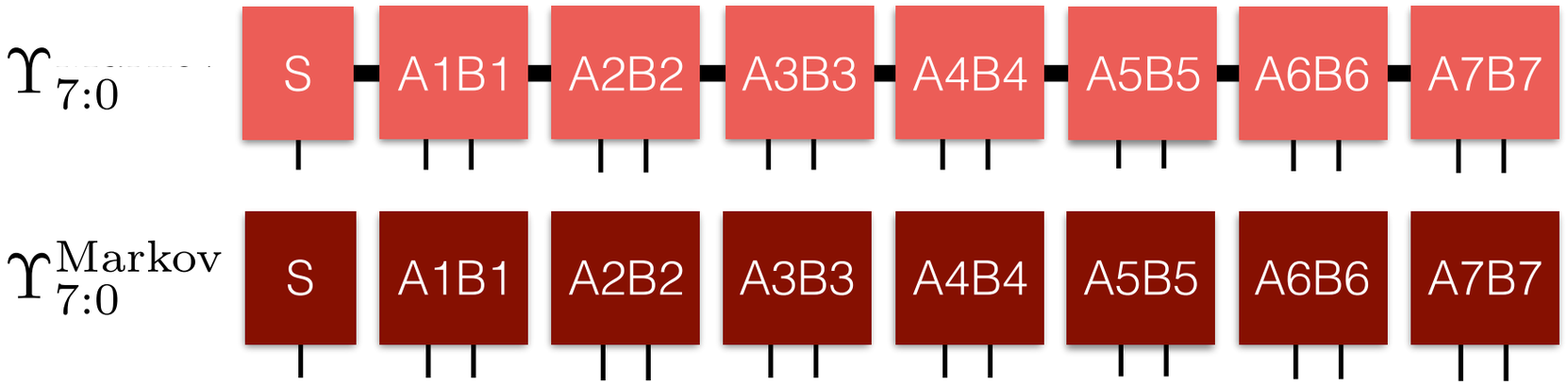
Processes can be mapped to states with tensor network representation



$\mathcal{T}_{k:0}$ ⟷ $\Upsilon_{k:0}$ MPO

$P_{x_3t_3;x_2t_2;x_1t_1} \xrightarrow{?} \rho_{x_3t_3;x_2t_2;x_1t_1}$ $\quad P \subset \Upsilon$

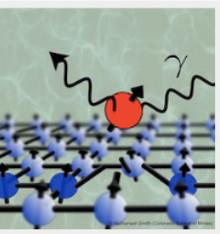
Meaningful measures for non-Markovianity



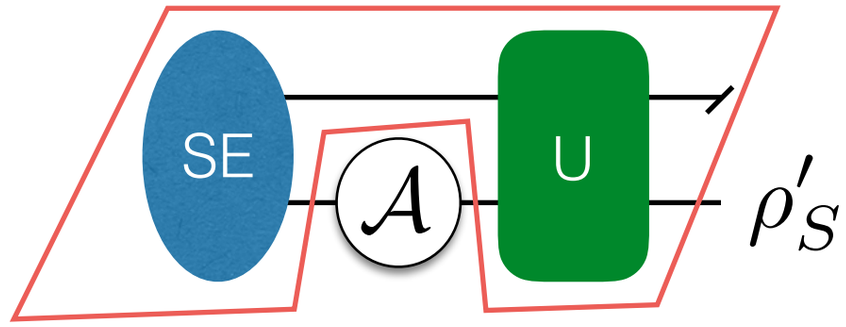
$$\mathcal{N} = R(\Upsilon_{7:0} \| \Upsilon^{\text{Markov}}_{7:0})$$

$$\text{Confusion probability} = e^{-n\mathcal{N}}$$

Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi. *Phys. Rev. A* 97, 012127 (2018) (on arXiv since late 2015)

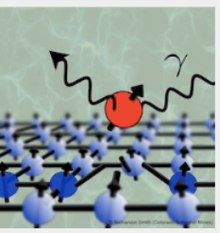


Avoiding non-positivity in presence of *initial correlation* (superchannel)

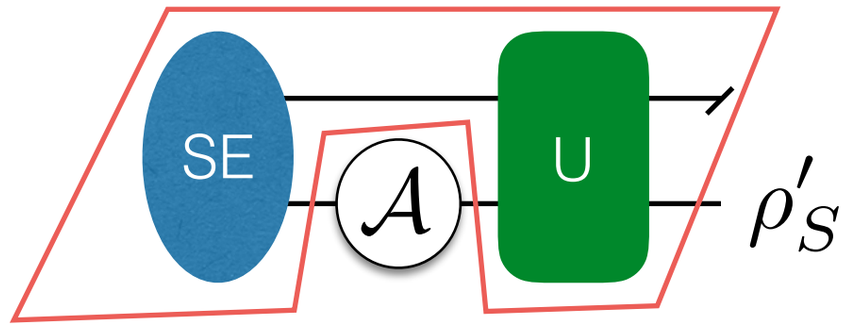


Modi *Sci. Rep.* 2, 581 (2012)
Ringbauer, Wood, Modi, Gilchrist, White, Fedrizzi. *Phys. Rev. Lett.* 114, 090402 (2015)
Vinjanampathy, Modi. *Phys. Rev. A* 92, 052310 (2015)

Paz Silva, Hall, Wiseman. *arXiv:1810.12540* (2018)



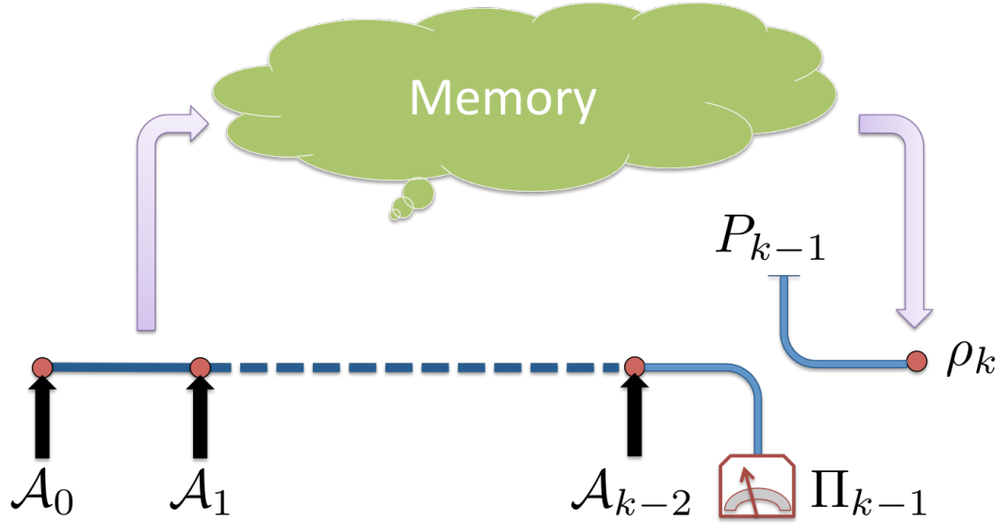
Avoiding non-positivity in presence of *initial correlation* (superchannel)



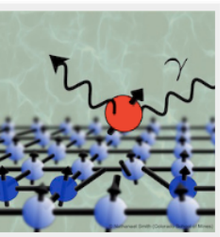
Modi *Sci. Rep.* 2, 581 (2012)
Ringbauer, Wood, Modi, Gilchrist, White, Fedrizzi. *Phys. Rev. Lett.* 114, 090402 (2015)
Vinjanampathy, Modi. *Phys. Rev. A* 92, 052310 (2015)

Paz Silva, Hall, Wiseman. *arXiv:1810.12540* (2018)

Unambiguous notion of *quantum Markov processes*

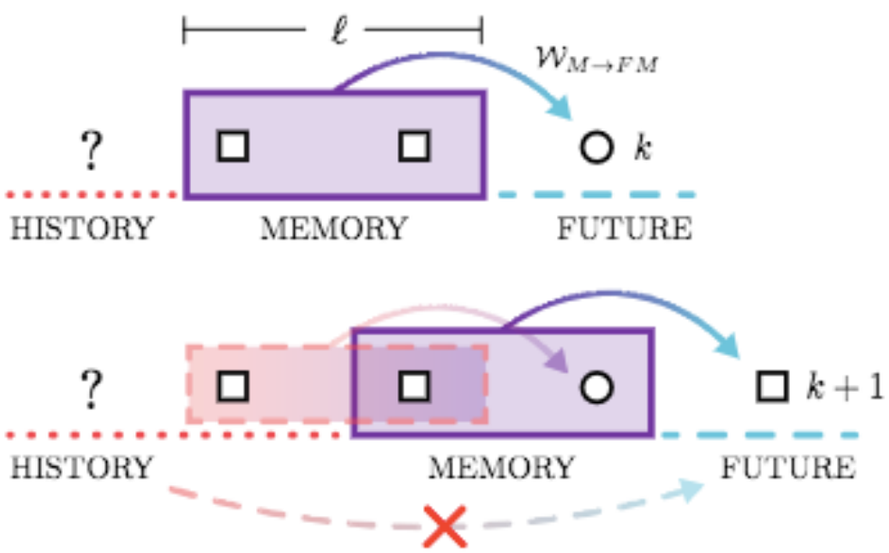


Pollock, Rodríguez-Rosario, Frauenheim, Paternostro, Modi. *Phys. Rev. Lett.* 120, 040405 (2018)

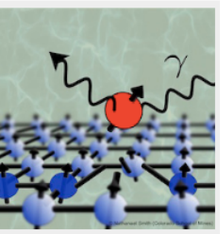


How long is memory

Quantum Markov order

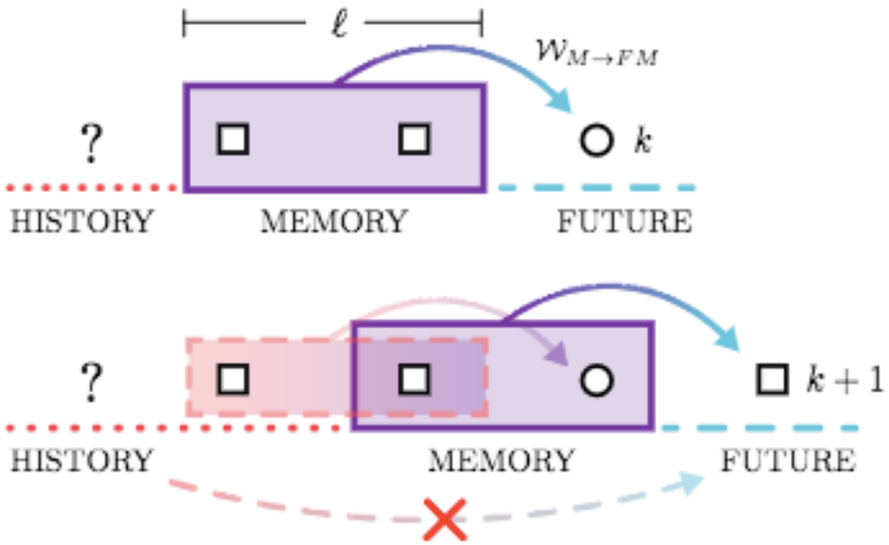


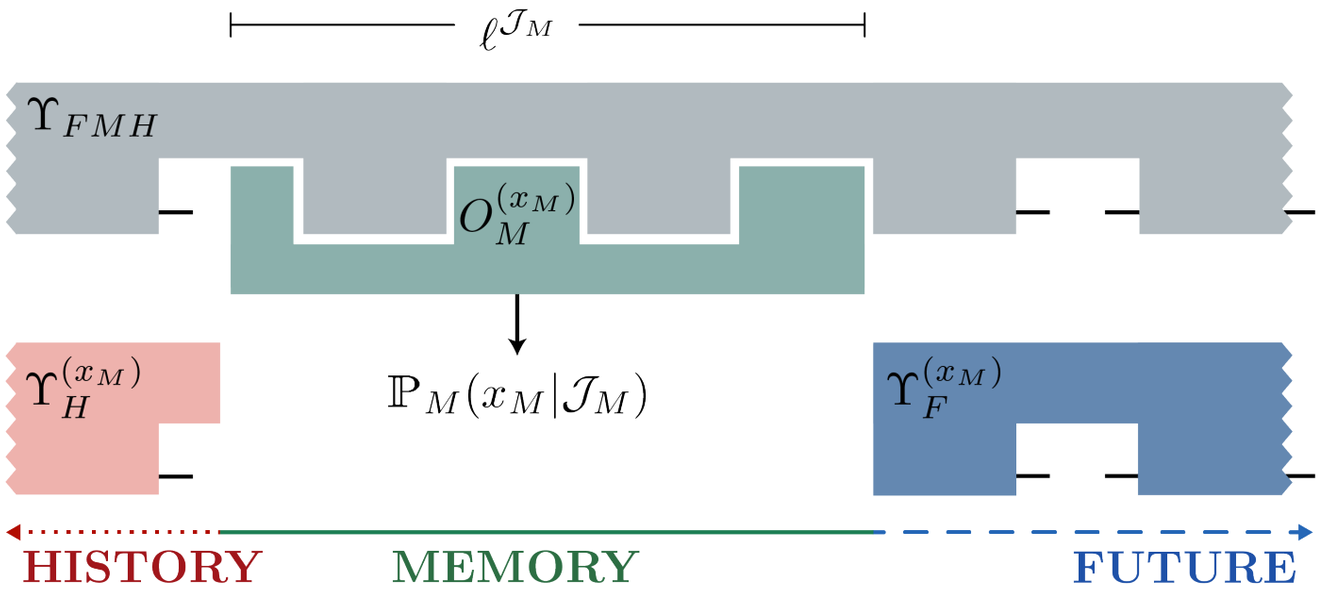
Taranto, Pollock, Milz, Tomamichel, Modi. *arXiv:1805.11341* (To appear in PRL 2019)
Taranto, Milz, Pollock, Modi. *arXiv:1810.10809* (To appear in PRA2019)



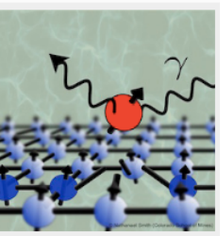
How long is memory

Quantum Markov order





Taranto, Pollock, Milz, Tomamichel, Modi. *arXiv:1805.11341* (To appear in PRL 2019)
Taranto, Milz, Pollock, Modi. *arXiv:1810.10809* (To appear in PRA2019)



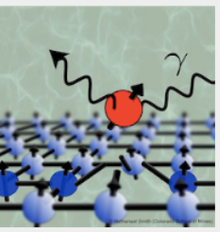
- We usually talk about quantum stochastic processes as master equations or complete positive maps
  What is the relationship between the two?

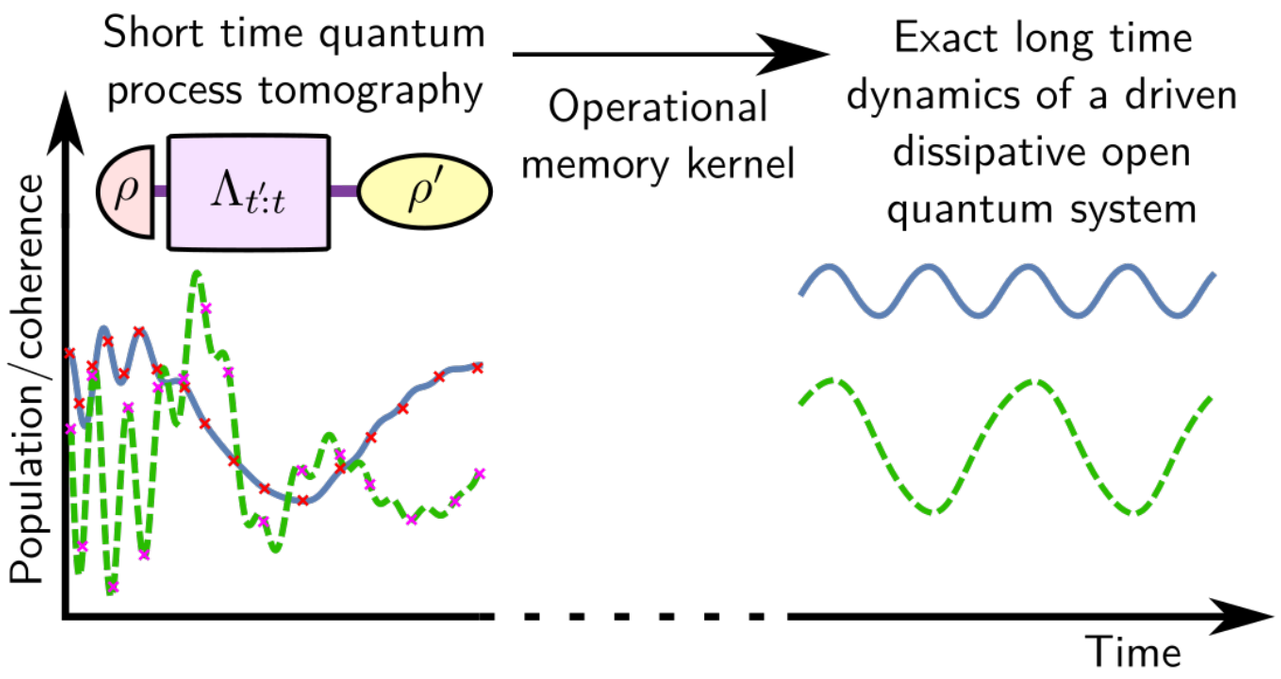
- Can we talk about correlations between multiple time steps in terms of quantum maps?

- Are there new approximations and techniques that can make open dynamics more tractable? (classical or quantum)

- Can we make open dynamics logically transparent and understand the causal structure?

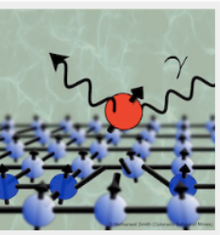
- Why are all carbon atoms the same? (fast Markovianisation)



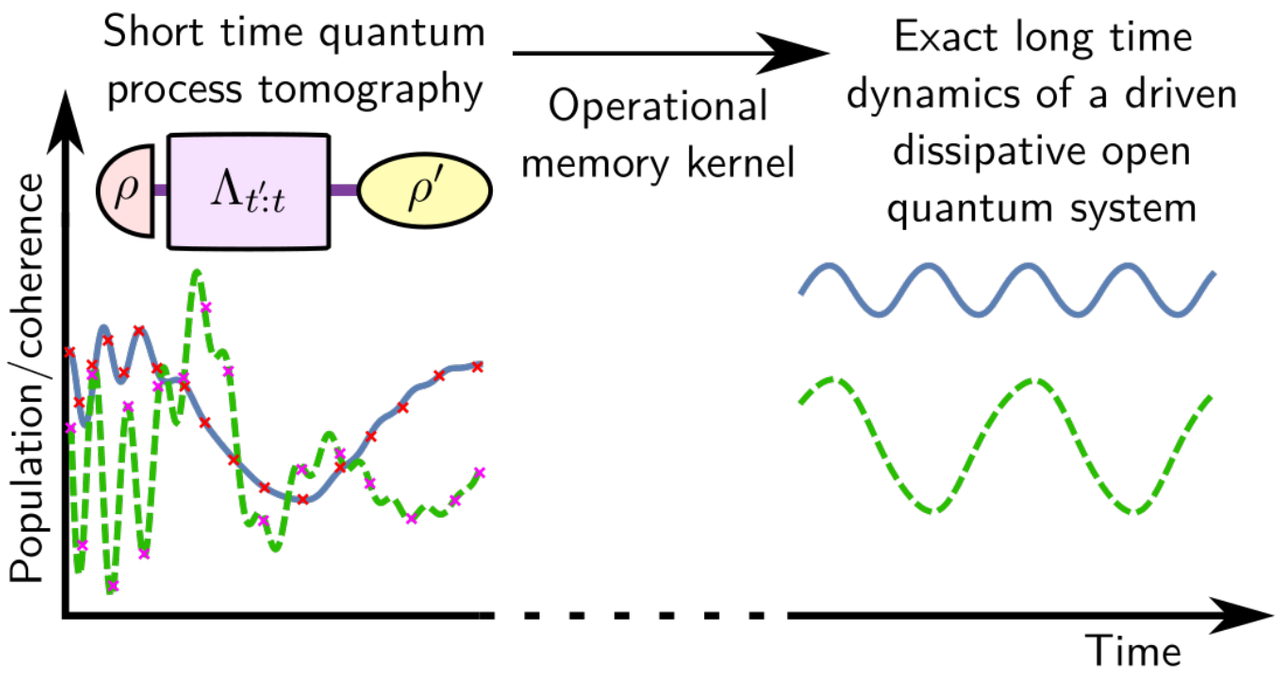


Reconstruct *Nakajima-Zwanzing master equations* (Chemistry)

Pollock, Modi. *Quantum 2, 76* (2018)



KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium



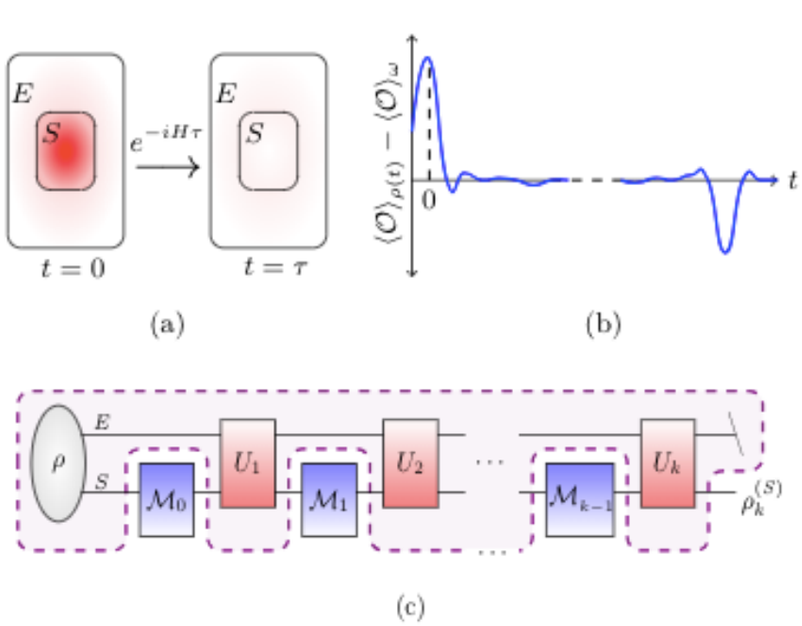
Reconstruct *Nakajima-Zwanzing master equations* (Chemistry)

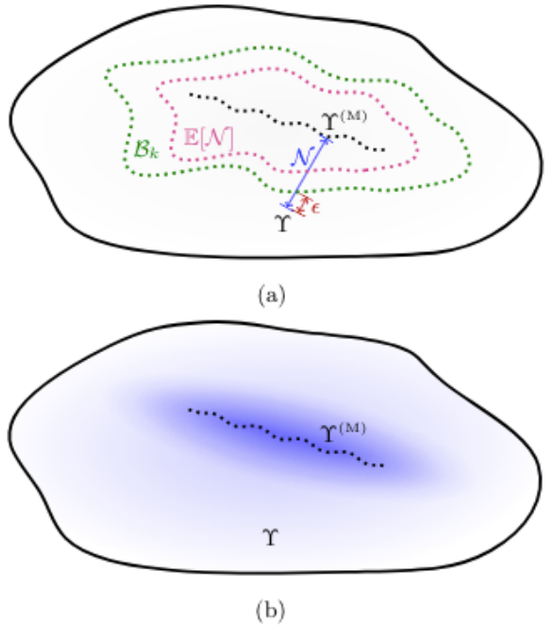
Pollock, Modi. *Quantum 2, 76* (2018)

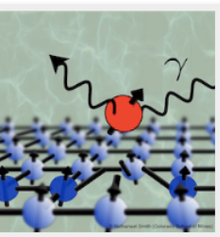
Statistical abundance of Markov processes

Figueroa-Romero, Modi, Pollock. *arXiv:1802.10344* (2018)

$$\mathbb{P}[\mathcal{N} \geqslant \mathcal{B}_k(d_E, d_S) + \epsilon] \leqslant \exp\left\{\frac{-k}{4}\frac{d_E}{d_S^{2k-1}}\right\}$$

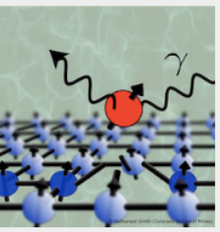






KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

- We usually talk about quantum stochastic processes as master equations or complete positive maps
  What is the relationship between the two?
- Can we talk about correlations between multiple time steps in terms of quantum maps?
- Are there new approximations and techniques that can make open dynamics more tractable? (classical or quantum)
- Can we make open dynamics logically transparent and understand the causal structure?
- Why are all carbon atoms the same? (fast Markovianisation)



Funding provided by

Australian Government
Australian Research Council

CIÊNCIA SEM FRONTEIRAS

Ministry of Education
SINGAPORE

DAAD
Deutscher Akademischer Austauschdienst
German Academic Exchange Service
Australia

THE ROYAL SOCIETY

MONASH University
THE UNIVERSITY OF WARWICK

John Templeton Foundation

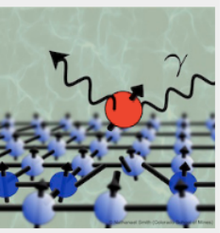
FQXi
FOUNDATIONAL QUESTIONS INSTITUTE

MONASH University

SAMSUNG

Australian Government
Department of Industry, Innovation and Science

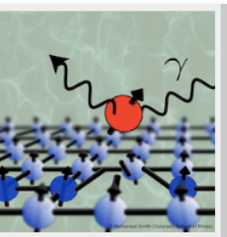
APPLIED
Technology, Engineering & Science



KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

Conclusions

We have a universal descriptor for arbitrary quantum stochastic process and whole lot of applications…



KITP 2019
Open Quantum System Dynamics:
Quantum Simulators and Simulations Far From Equilibrium

http://monqis.physics.monash.edu

(Review paper) S. Milz, F. Pollock, K. Modi Open Sys. Info. Dyn. 24, 1740016 (2017)
L. Li, M. Hall, H. Wiseman Physics Reports, 759, 1 (2018)