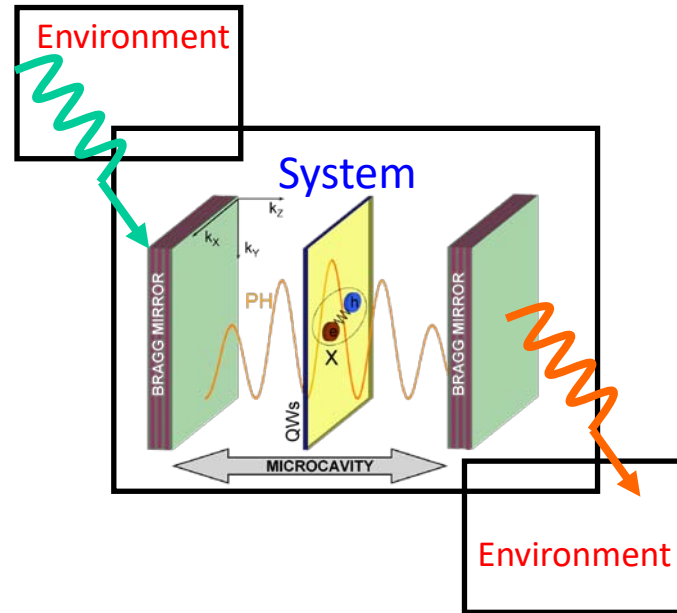


Driven-dissipative quantum fluids of light

Marzena Szymańska

KITP, April 2019





2D Light-matter condensates with drive and decay

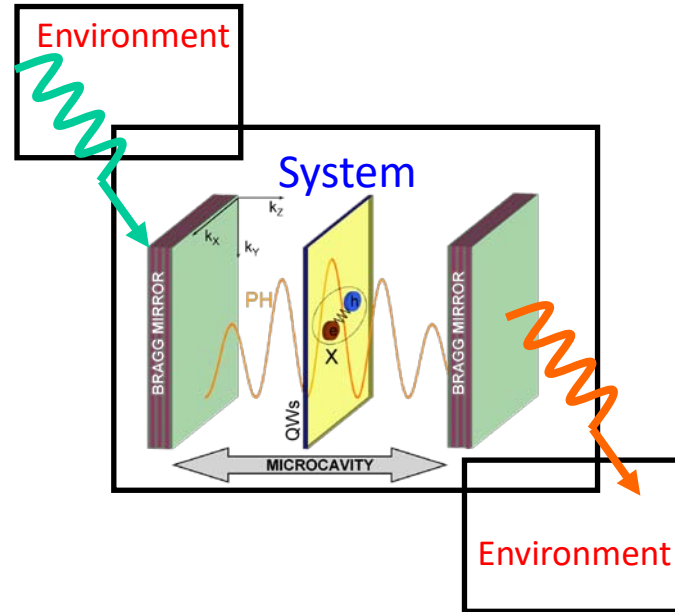
Polaritons

Photon BEC

Circuit QED systems

Atoms in cavities

....



2D Light-matter condensates with drive and decay

Can thermal equilibrium be achieved?

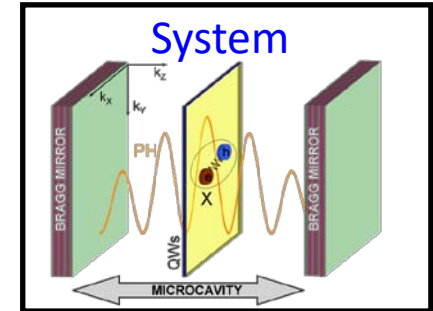
Can non-equilibrium but **non-trivial** phases be engineered?

Closed System

- ✧ Dimensionality: 2D
- ✧ Modes: linear
- ✧ Occupation: thermal

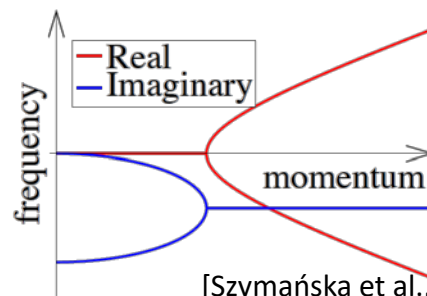
$$|g^{(1)}(x, -x)| = A|2x|^{-\alpha}$$

$$\alpha_{s,t} = k_B T / n_s < 1/4$$

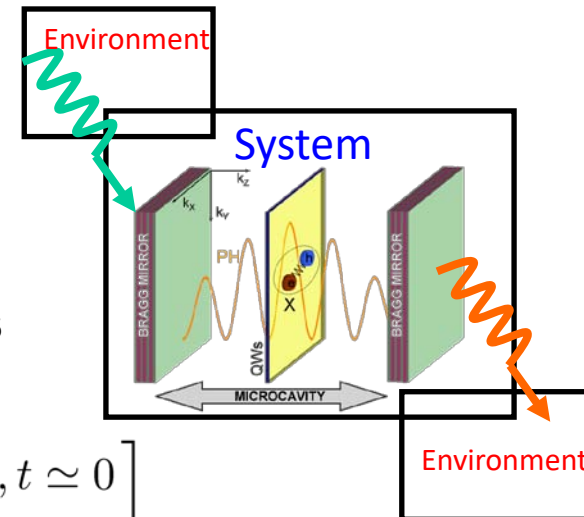


Driven-dissipative System

- ✧ Dimensionality: 2D
- ✧ Modes: diffusive
- ✧ Occupation: non-thermal



[Szymańska et al., *PRL* 2006
Wouters & Carusotto *PRA* 2007]



From Keldysh field theory

$$\langle \psi^\dagger(\mathbf{r}, t) \psi(0, 0) \rangle \simeq |\psi_0|^2 \exp \left[-\alpha \begin{cases} \ln(r/r_0) & r \rightarrow \infty, t \simeq 0 \\ \frac{1}{2} \ln(c^2 t / \gamma_{\text{tot}} r_0^2) & r \simeq 0, t \rightarrow \infty \end{cases} \right]$$

$$\alpha_t = 1/2 \alpha_s$$

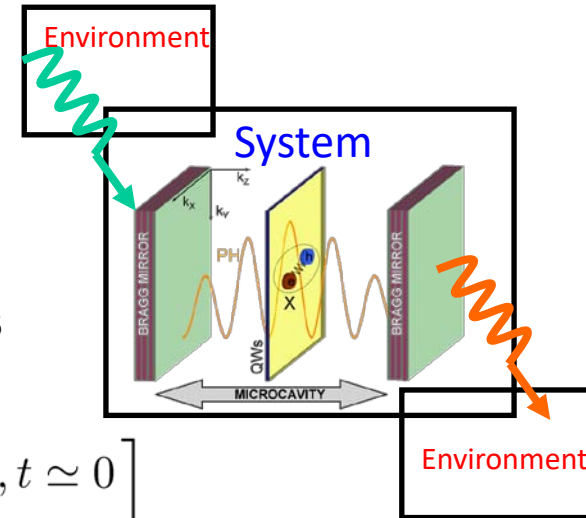
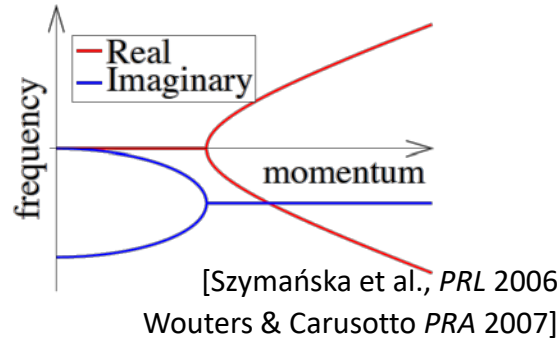
α (pump, decay, density)

[Szymańska et al., *PRL* 2006; *PRB* 2007]

Spatial and Temporal Coherence

Driven-dissipative System

- Dimensionality: 2D
- Modes: diffusive
- Occupation: non-thermal



From Keldysh field theory

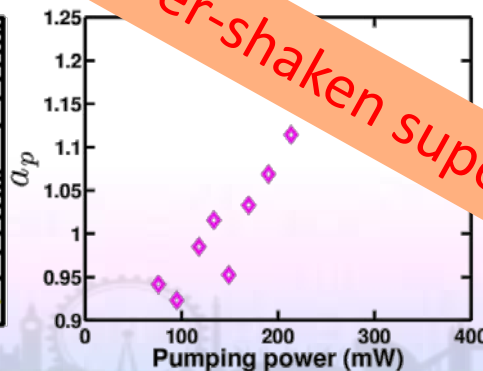
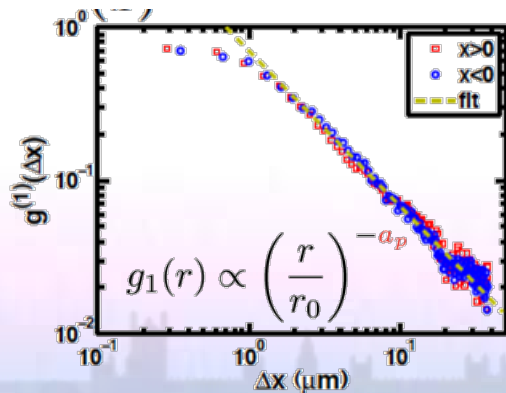
$$\langle \psi^\dagger(\mathbf{r}, t) \psi(0, 0) \rangle \simeq |\psi_0|^2 \exp \left[-\alpha \begin{cases} \ln(r/r_0) & r \rightarrow \infty, t \simeq 0 \\ \frac{1}{2} \ln(c^2 t / \gamma_{\text{tot}} r_0^2) & r \simeq 0, t \rightarrow \infty \end{cases} \right]$$

α (pump, decay, density)

[Szymańska et al., *PRL* 2006; *PRB* 2007]

$$\alpha_t = 1/2\alpha_s$$

From stochastic dynamics to KMT and early experiments

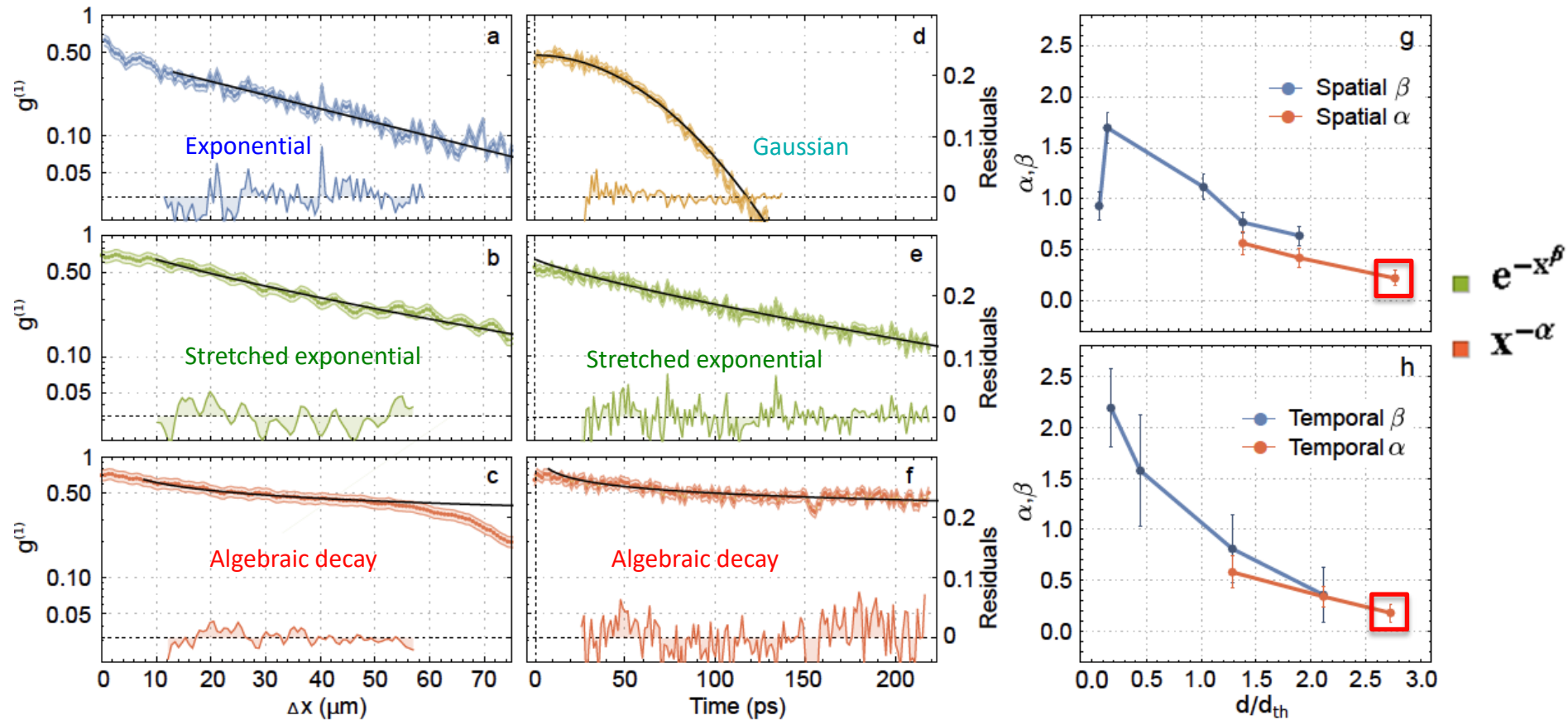


Faster decay possible
than equilibrium
upper limit

[*PNAS* (2012), *PRX* (2015)]

Over-shaken superfluid

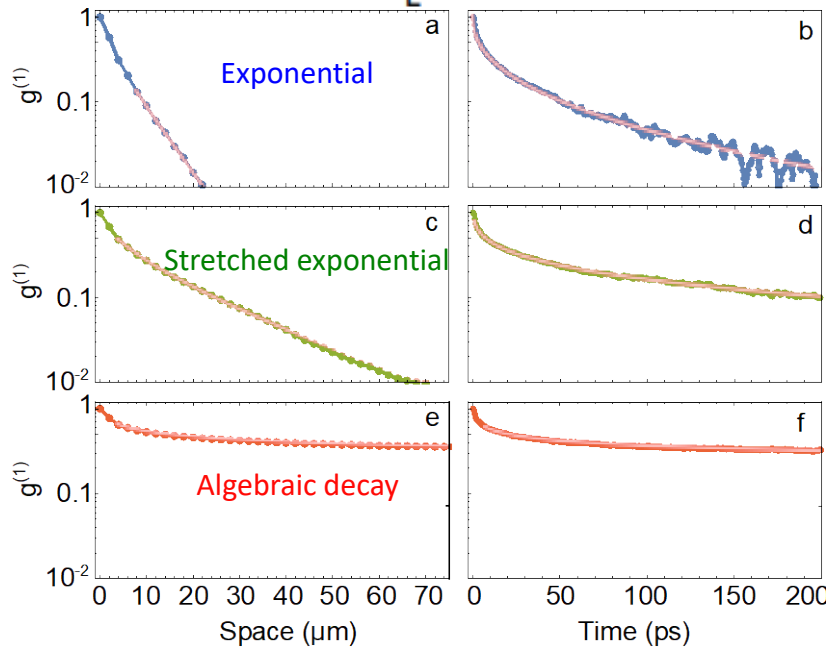
Spatial and Temporal Coherence - Experiment



$$\alpha_s = \alpha_t < 1/4$$

Crossover from exponential to algebraic decay of coherence with equilibrium exponents

$$id\psi(\mathbf{r}, t) = \left[-\frac{\nabla^2}{2m} + g|\psi(\mathbf{r}, t)|^2 + i(\gamma - \kappa - \Gamma|\psi(\mathbf{r}, t)|^2) \right] \psi(\mathbf{r}, t)dt + dW$$



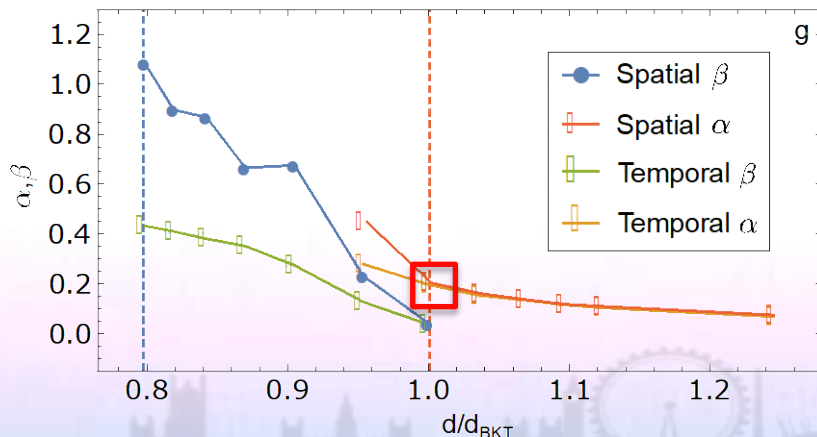
$$\frac{\gamma}{1 + \frac{|\psi(\mathbf{r}, t)|^2}{n_s}} \quad \text{more non-linear drive}$$

$$\langle dW^*(\mathbf{r}', t)dW(\mathbf{r}, t) \rangle = \frac{\gamma + \kappa + \Gamma|\psi(\mathbf{r}, t)|^2}{dV} \delta_{\mathbf{r}, \mathbf{r}'} dt$$

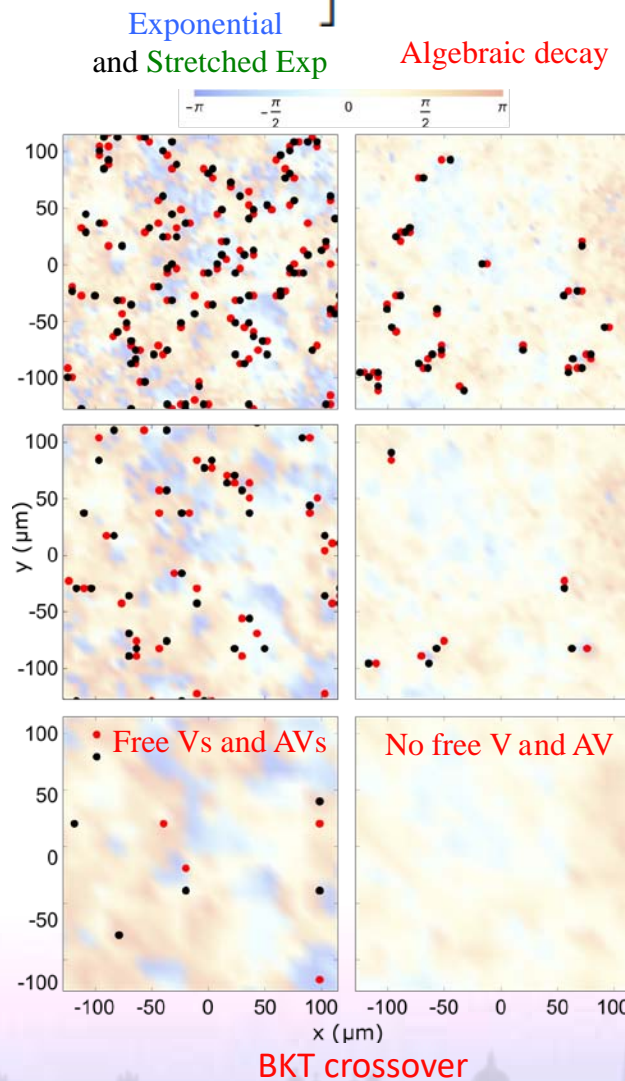
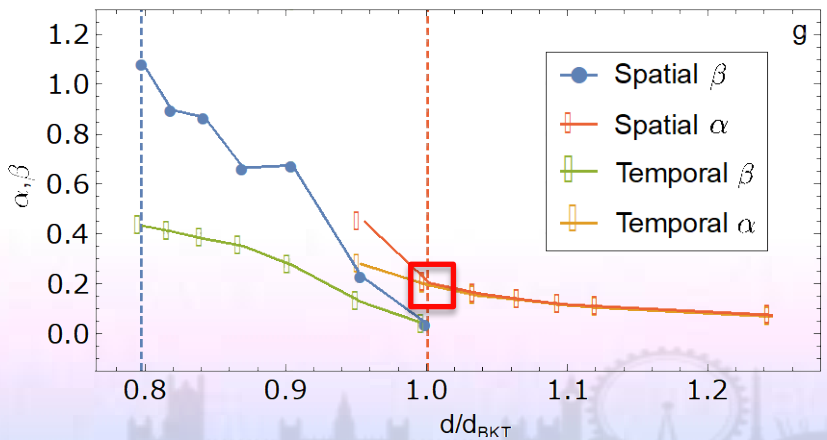
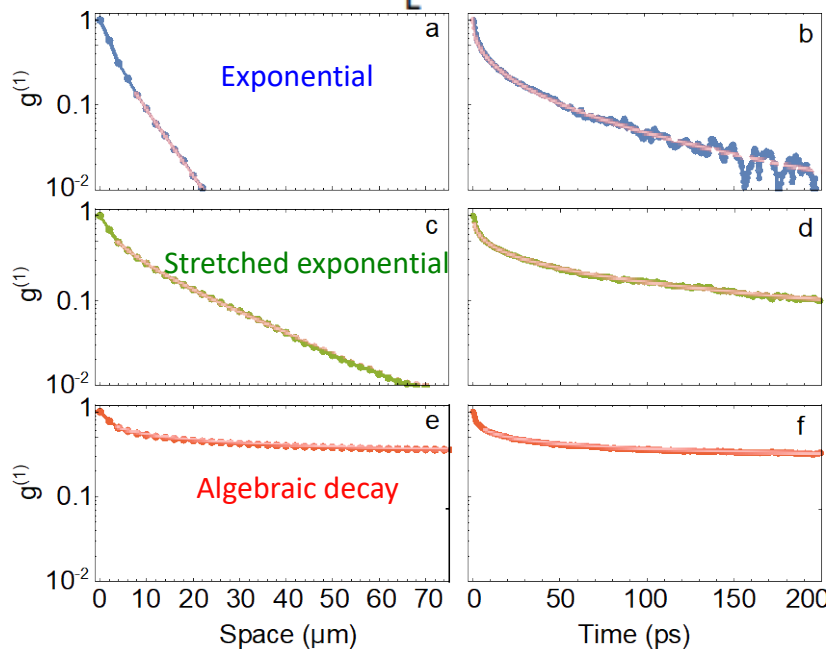
$$\frac{\gamma}{1 + \frac{|\psi(\mathbf{r}, t)|^2}{n_s}} + \kappa$$

For experimental parameters

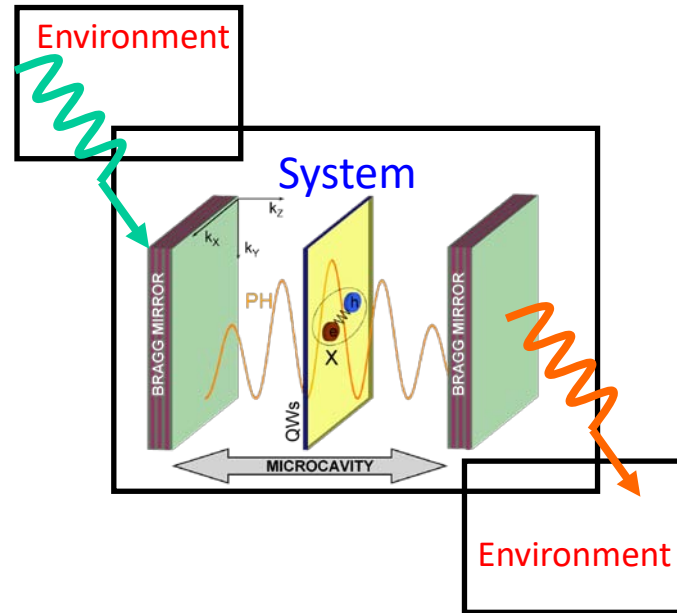
$$\alpha_s = \alpha_t < 1/4$$



$$i d\psi(\mathbf{r}, t) = \left[-\frac{\nabla^2}{2m} + g|\psi(\mathbf{r}, t)|^2 + i(\gamma - \kappa - \Gamma|\psi(\mathbf{r}, t)|^2) \right] \psi(\mathbf{r}, t) dt + dW$$



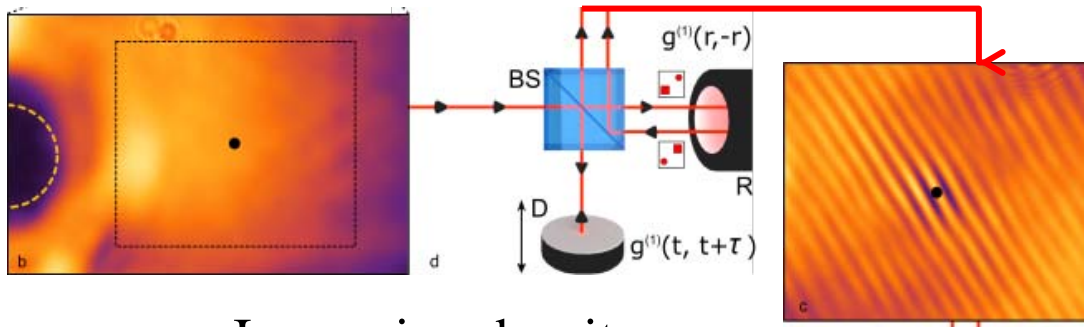
BKT crossover



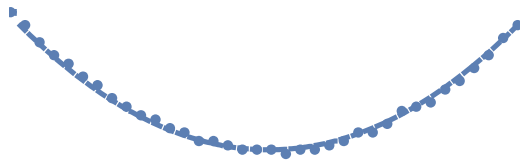
2D Light-matter condensates with drive and decay

Can thermal equilibrium **YES** be achieved?

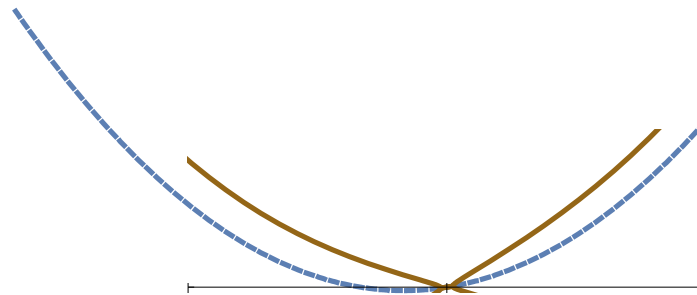
Can non-equilibrium but **non-trivial** phases be engineered?



Increasing density \rightarrow



$$\omega_{bog} = -i\Gamma/2 \pm \sqrt{\frac{k^2}{2M} \left(\frac{k^2}{2M} + 2g - \frac{\Gamma^2}{4} \right)}$$



$$\omega_{offset} = v_c * k_c$$

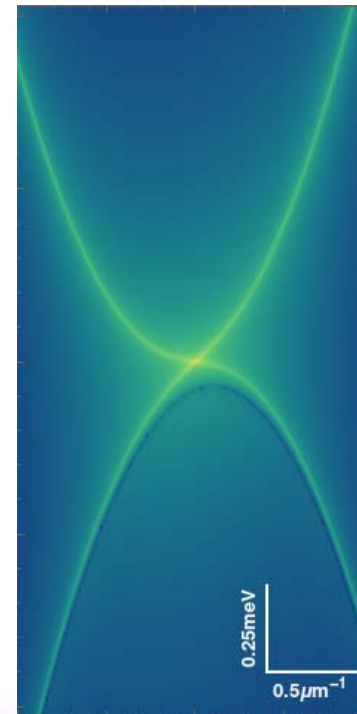
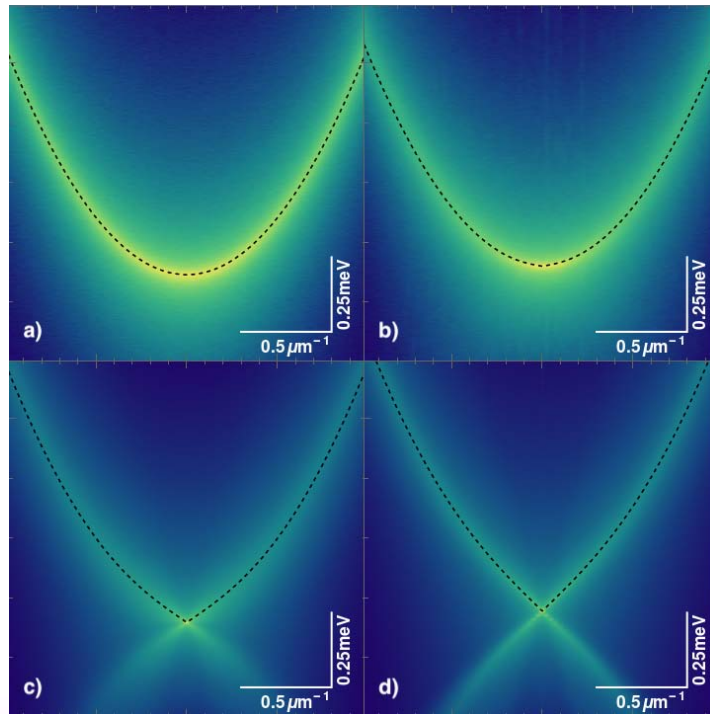
$$\omega_{doppler} = (k - k_c) * v_c$$

Theoretical Spectrum from

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = \left[-\frac{\hbar^2 \nabla^2}{2m} + i\frac{\hbar}{2} \left(\frac{\gamma(\mathbf{r})}{1 + \frac{|\psi(\mathbf{r}, t)|^2}{n_s}} - \kappa \right) + g|\psi(\mathbf{r}, t)|^2 + V(\mathbf{r}) \right] \psi(\mathbf{r}, t)$$

$k_c=0$

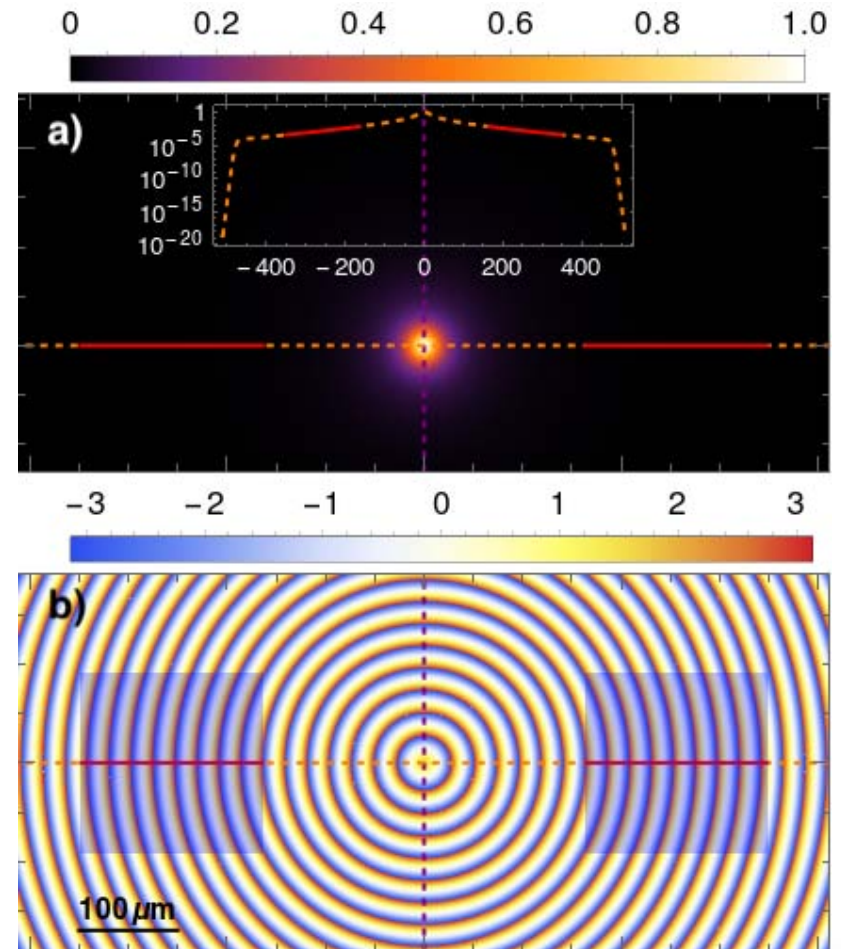
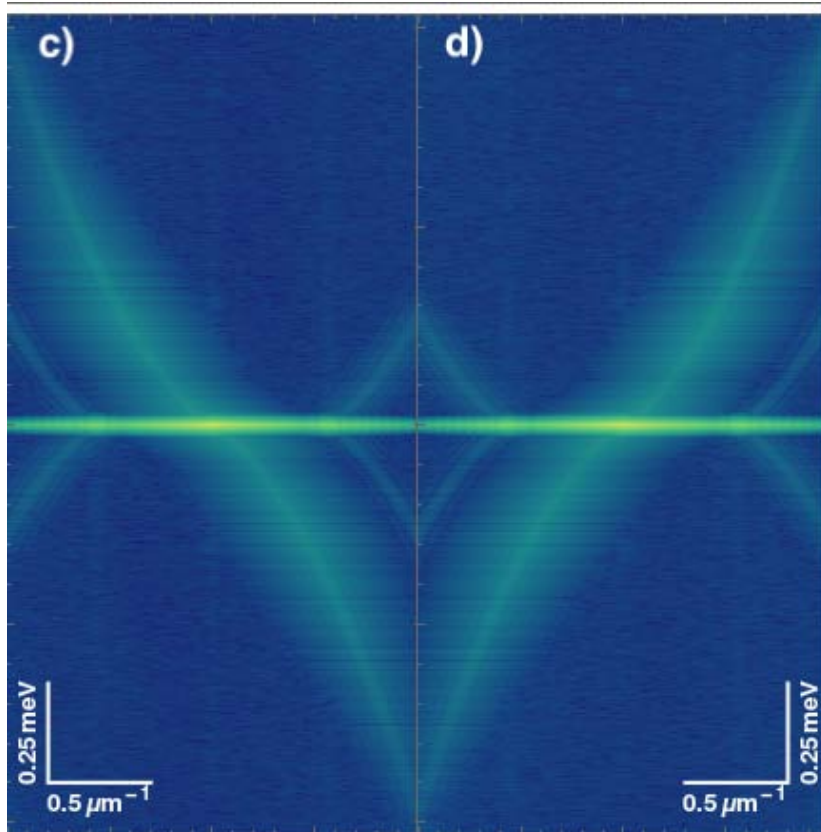
k_c finite

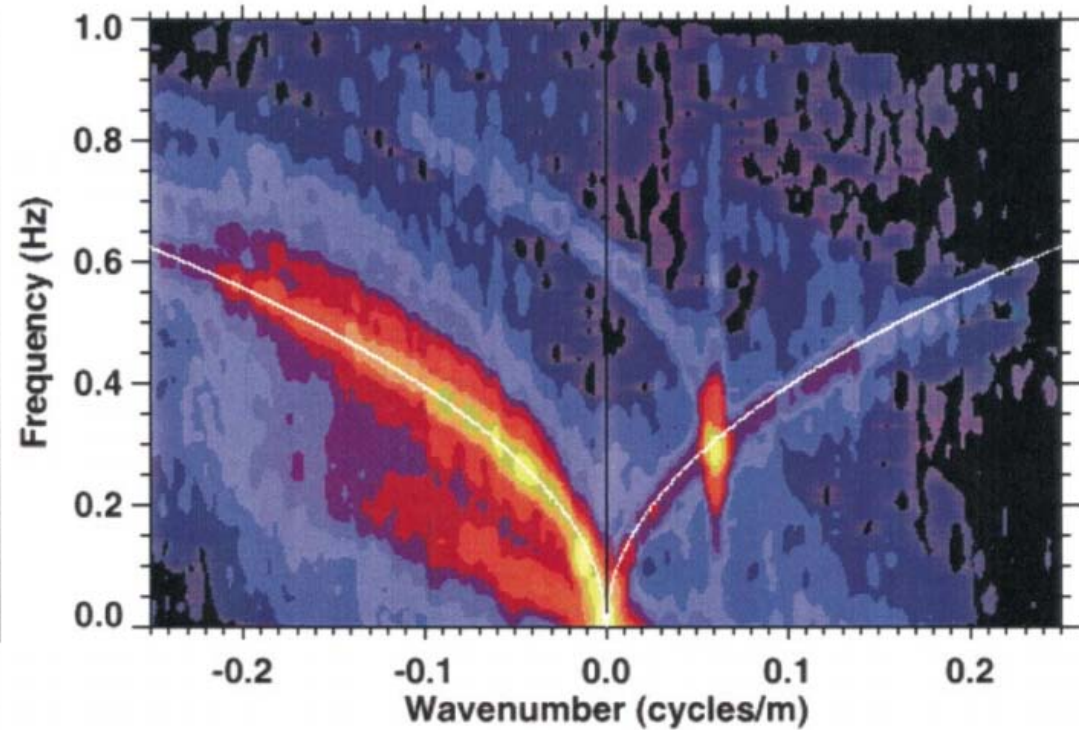
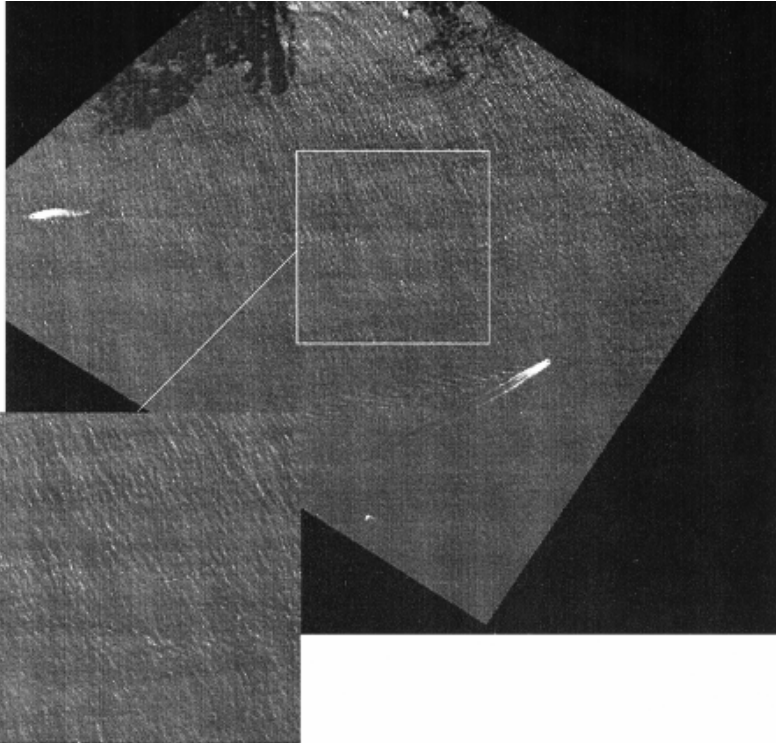


From Truncated Wigner and Keldysh Field theory

[Ballarini et al. 2019]

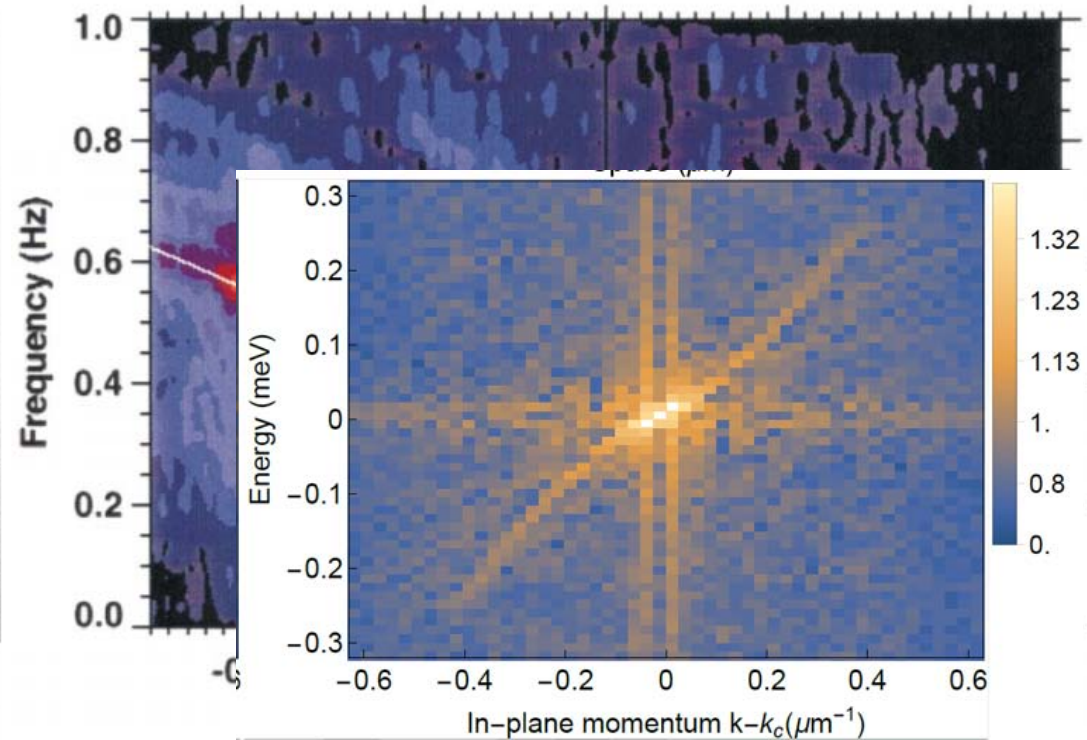
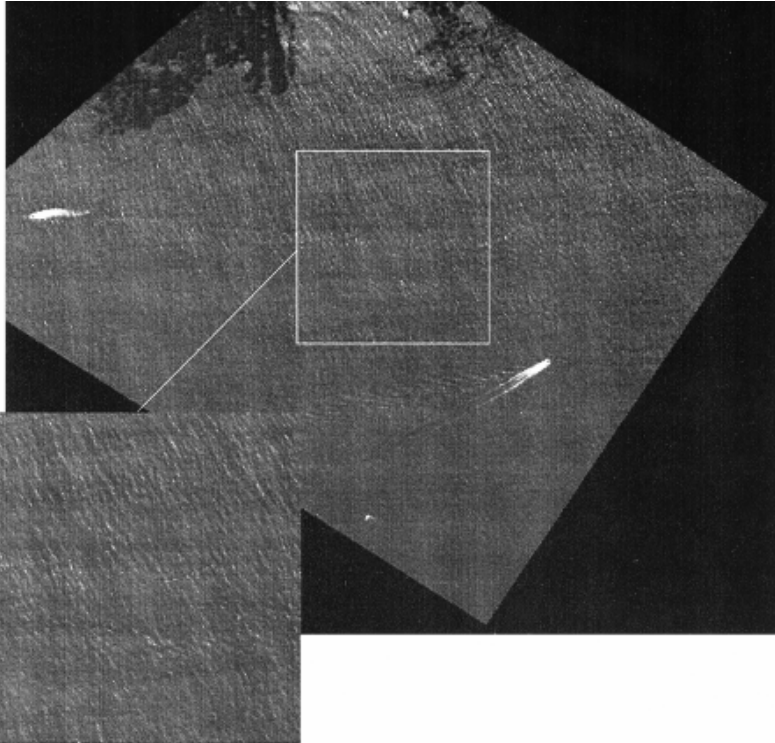
Theoretical Spectrum





Airborne optical measurements of surface gravity wave dispersion

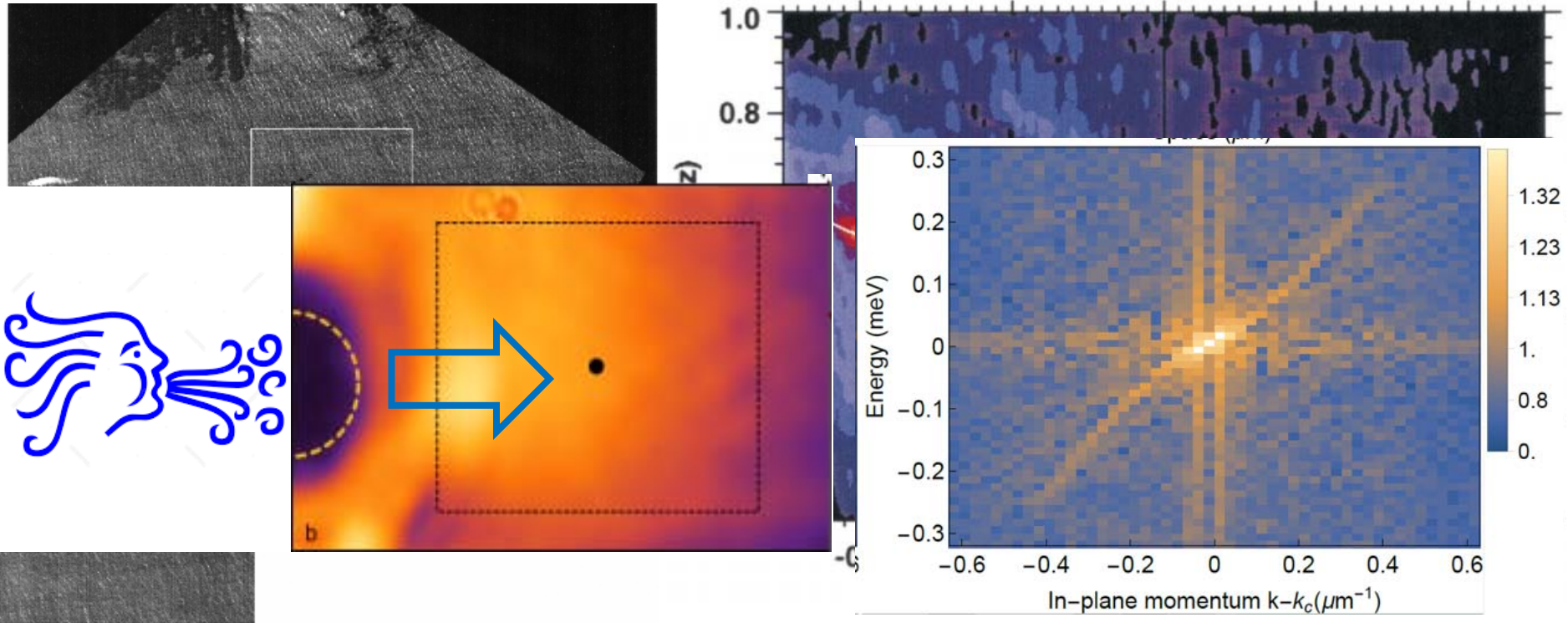
[J. P. Dugan, et al., Journal of Geophysical Research (2001)]



Airborne optical measurements of surface gravity wave dispersion

[J. P. Dugan, et al., Journal of Geophysical Research (2001)]

“Wind” Effect of the Asymmetric Reservoir



Airborne optical measurements of surface gravity wave dispersion

[J. P. Dugan, et al., Journal of Geophysical Research (2001)]

Treating phase fluctuations exactly

[Altman et al, *PRX* 2015]

$$\partial_t \theta = D_x \partial_x^2 \theta + D_y \partial_y^2 \theta + \frac{\lambda_x}{2} (\partial_x \theta)^2 + \frac{\lambda_y}{2} (\partial_y \theta)^2 + \bar{\zeta}(\mathbf{x}, t)$$

At large distances

$$\langle \phi^*(r) \phi(0) \rangle \sim e^{-r^{2\chi}}, \quad \chi \approx 0.37$$

KPZ non-linearity

Stretched exponential (faster than algebraic) decay of coherence but **superfluidity survives**

Incoherently pumped microcavity:

L_1 unrealistically large away from BKT

KPZ order

killed by vortices \Rightarrow exponential decay of correlations

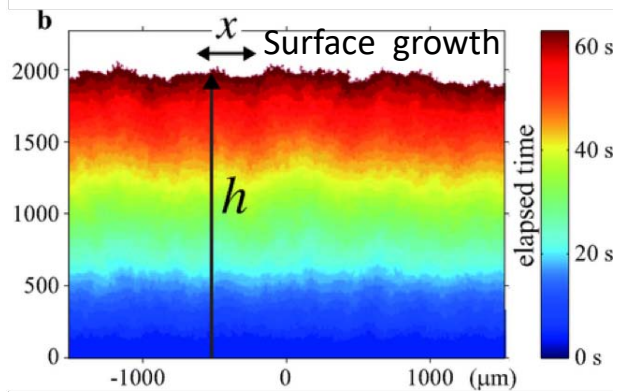
or shows up on too large lengthscales \Rightarrow power-law decay of correlations within system size

KPZ – why interesting?

Solving KPZ equation Martin Hairer, Warwick



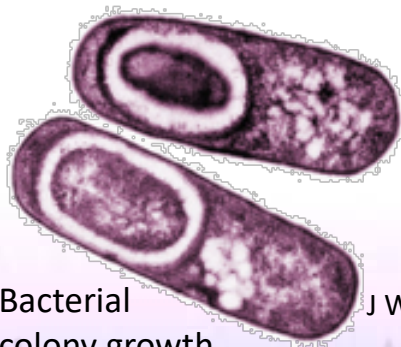
Universality class for a wide range of non-equilibrium phenomena in 1D



Takeuchi et al, 2011

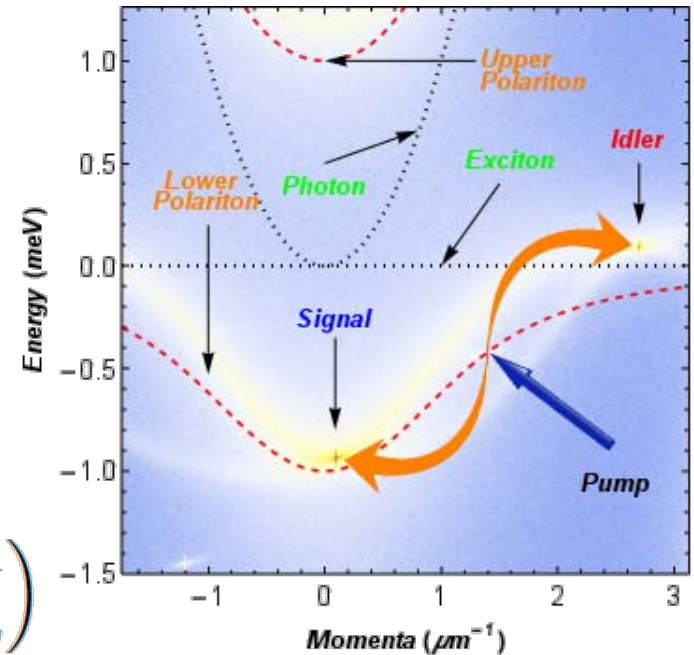
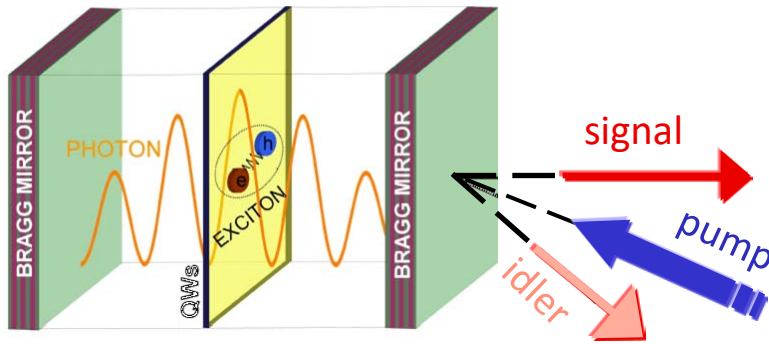


J Maunuksela et al, PRL, 1997



J Wakita et al, 1997

To date no experimental realisation of KPZ in 2D



$$\hat{H}_S = \int d\mathbf{r} \begin{pmatrix} \hat{\psi}_X^\dagger & \hat{\psi}_C^\dagger \end{pmatrix} \begin{pmatrix} \frac{-\nabla^2}{2m_X} + \frac{g_X}{2} |\hat{\psi}_X|^2 & \frac{\Omega_R}{2} \\ \frac{\Omega_R}{2} & \frac{-\nabla^2}{2m_C} \end{pmatrix} \begin{pmatrix} \hat{\psi}_X \\ \hat{\psi}_C \end{pmatrix}$$

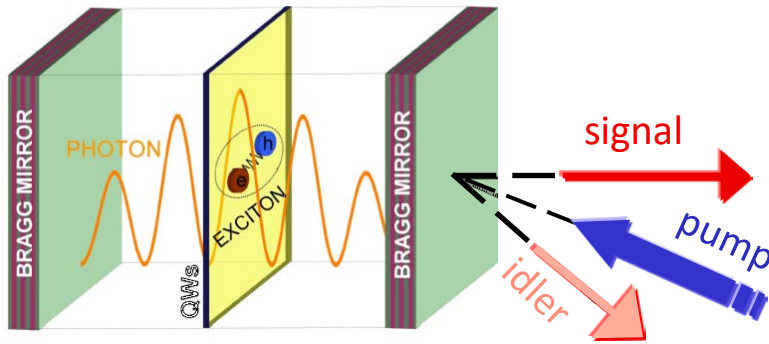
$$\hat{H}_{SB} = \int d\mathbf{r} \left[F(\mathbf{r}, t) \hat{\psi}_C^\dagger(\mathbf{r}, t) + \text{H.c.} \right] + \sum_{\mathbf{k}} \sum_{l=X,C} \left\{ \zeta_{\mathbf{k}}^l \left[\hat{\psi}_{l,\mathbf{k}}^\dagger(t) \hat{B}_{l,\mathbf{k}} + \text{H.c.} \right] + \omega_{l,\mathbf{k}} \hat{B}_{l,\mathbf{k}}^\dagger \hat{B}_{l,\mathbf{k}} \right\}$$

✧ Non-thermal occupation

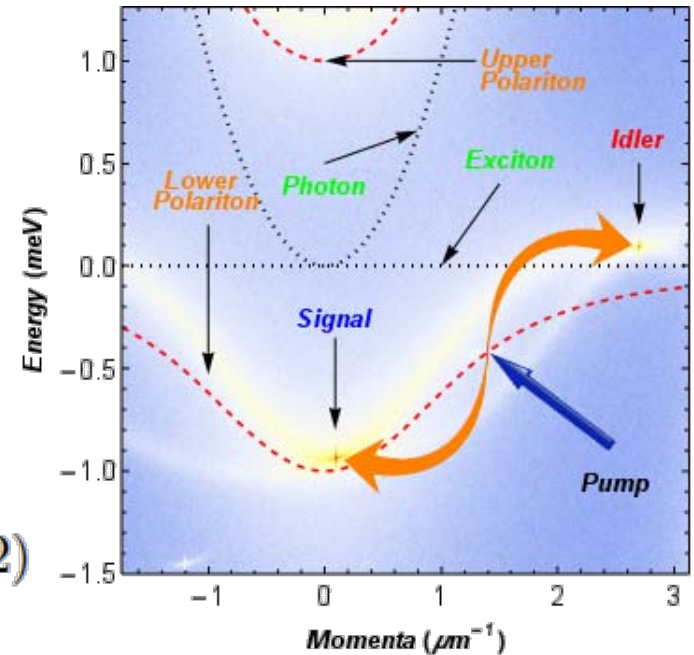
✧ Signal phase is completely free and idler phase locked to signal via pump

$$2\varphi_p = \varphi_s + \varphi_i$$

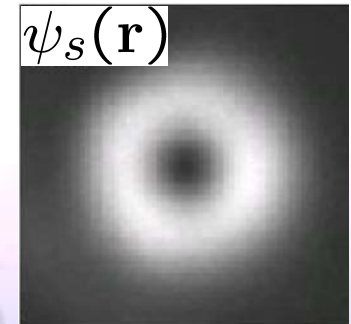
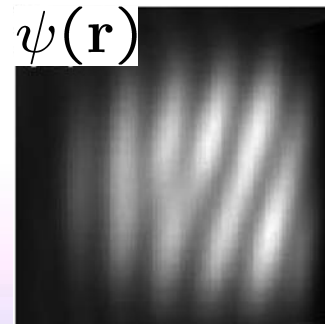
✧ Spontaneous U(1) symmetry breaking gapless and diffusive Goldstone mode



$$\begin{aligned}
 |\psi_{LP}(\mathbf{r}, t)|^2 = & \sum_j \rho_j^2 \\
 & + 2 \left(\sqrt{\rho_s \rho_p} \cos(\phi_s - \phi_p + (\mathbf{k}_{si} \cdot \mathbf{r} - \omega_{si}t)/2) \right. \\
 & + \sqrt{\rho_p \rho_i} \cos(\phi_i - \phi_p - (\mathbf{k}_{si} \cdot \mathbf{r} - \omega_{si}t)/2) \\
 & \left. + \sqrt{\rho_s \rho_i} \cos(\phi_s - \phi_i + \mathbf{k}_{si} \cdot \mathbf{r} - \omega_{si}t) \right)
 \end{aligned}$$



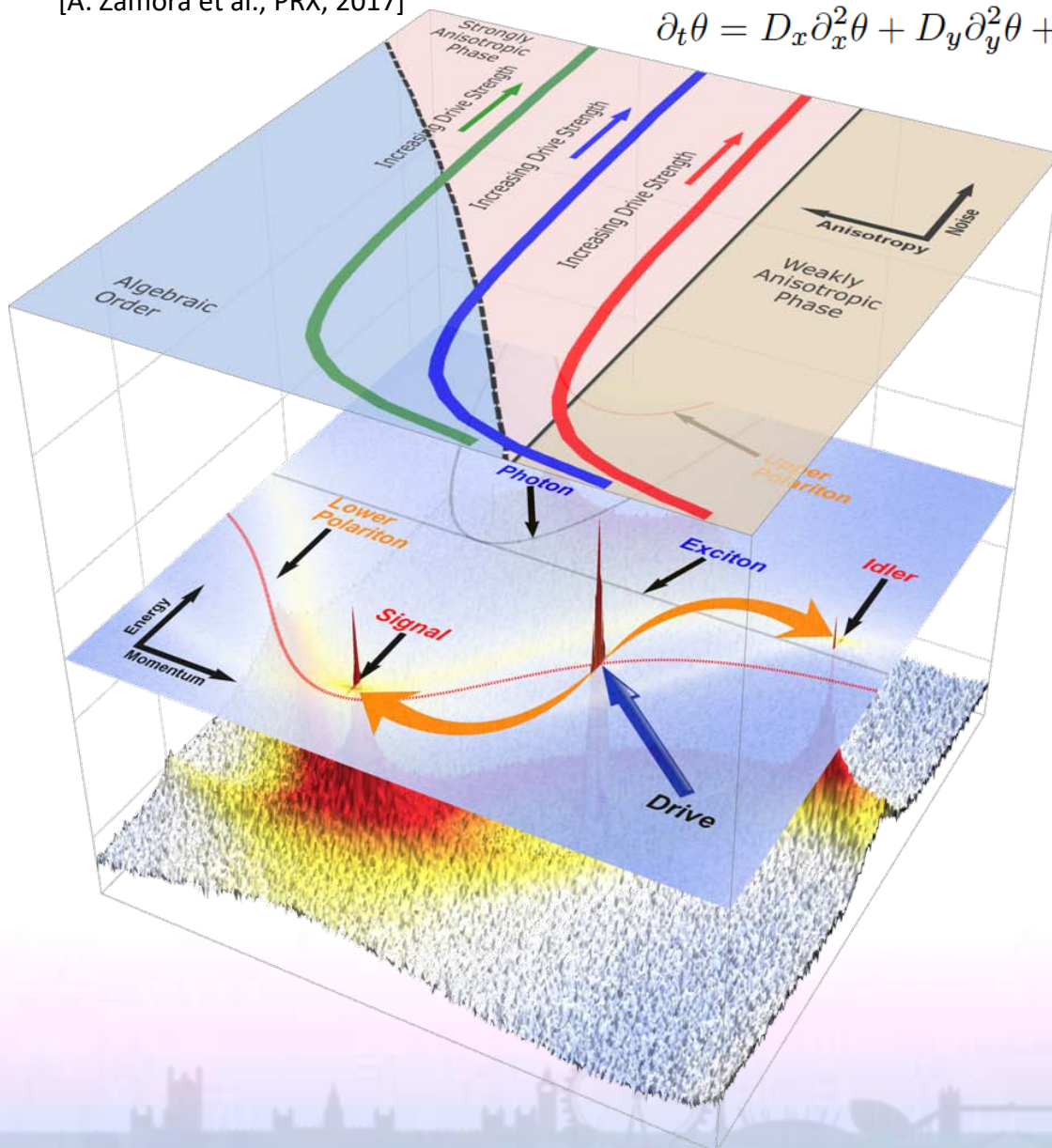
- ✧ Time crystal
- ✧ Vortices: dislocations in density wave and time crystal
- ✧ After filtering in momentum: usual vortices



Tuning Across Universalities with OPO

[A. Zamora et al., PRX, 2017]

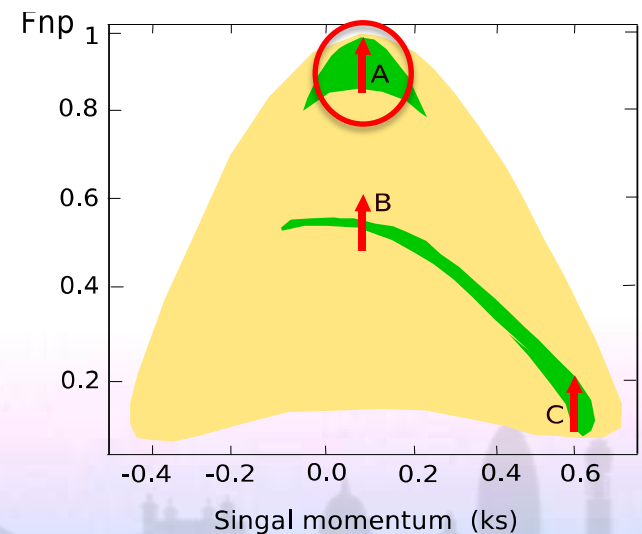
$$\partial_t \theta = D_x \partial_x^2 \theta + D_y \partial_y^2 \theta + \frac{\lambda_x}{2} (\partial_x \theta)^2 + \frac{\lambda_y}{2} (\partial_y \theta)^2 + \bar{\zeta}(\mathbf{x}, t)$$



Negative detuning

By increasing drive we move from **non-equilibrium** to **equilibrium** fixed point

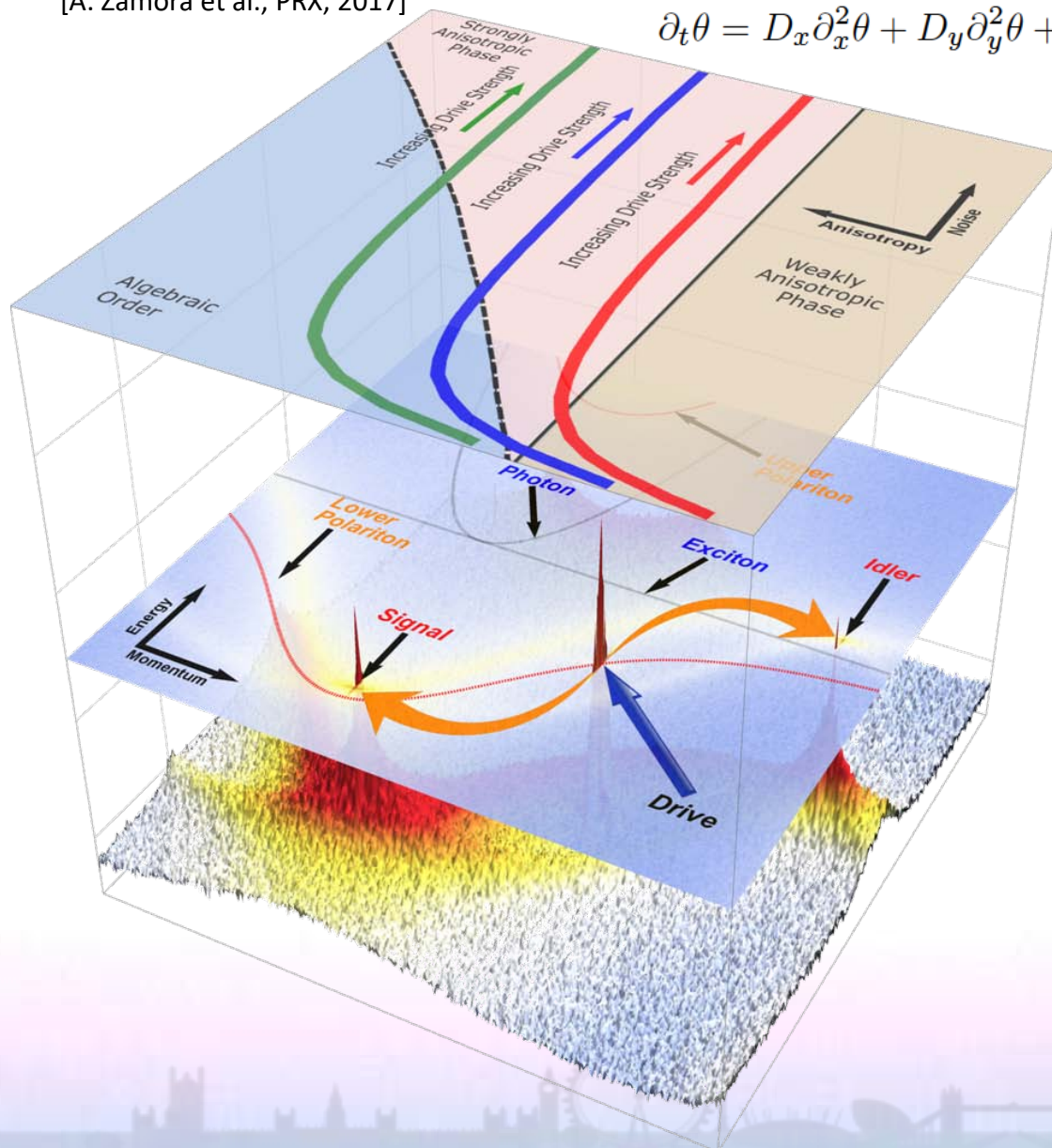
Two different universality classes as the drive is increased



Tuning Across Universalities with OPO

[A. Zamora et al., PRX, 2017]

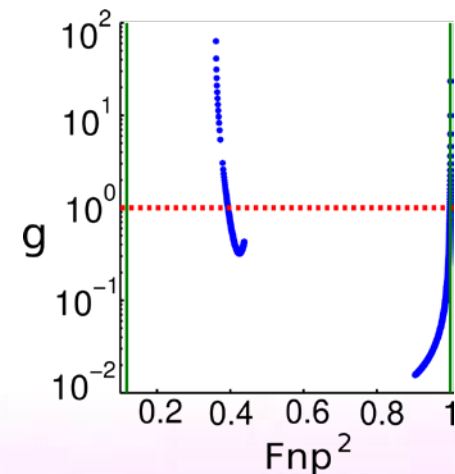
$$\partial_t \theta = D_x \partial_x^2 \theta + D_y \partial_y^2 \theta + \frac{\lambda_x}{2} (\partial_x \theta)^2 + \frac{\lambda_y}{2} (\partial_y \theta)^2 + \bar{\zeta}(\mathbf{x}, t)$$



Negative detuning

By increasing drive we move from **non-equilibrium** to **equilibrium** fixed point

Two different universality classes as the drive is increased

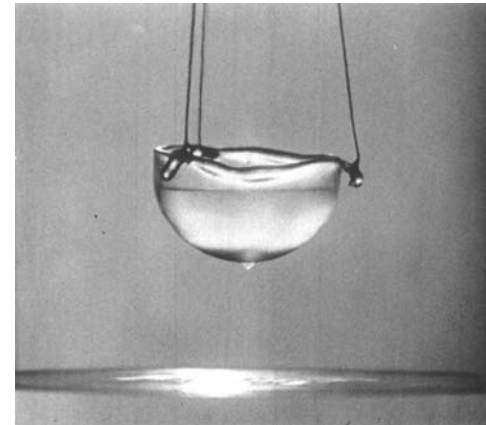


KPZ at all length-scales?

What is a superfluid?

Defined by flow properties:

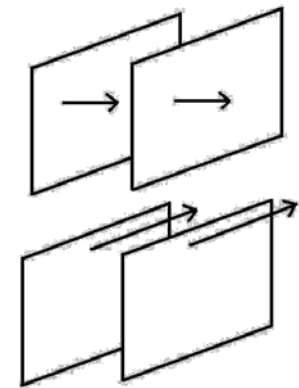
- No viscosity
- **No transverse response**
- Quantised vortices
- Metastable persistent flow



Current-current response function: $\delta j_i(\mathbf{q}) = \chi_{ij}(\mathbf{q})\delta f_j(\mathbf{q})$

Long wavelength limit:

- Transverse direction first: **longitudinal response**
- Longitudinal direction first: **transverse response**

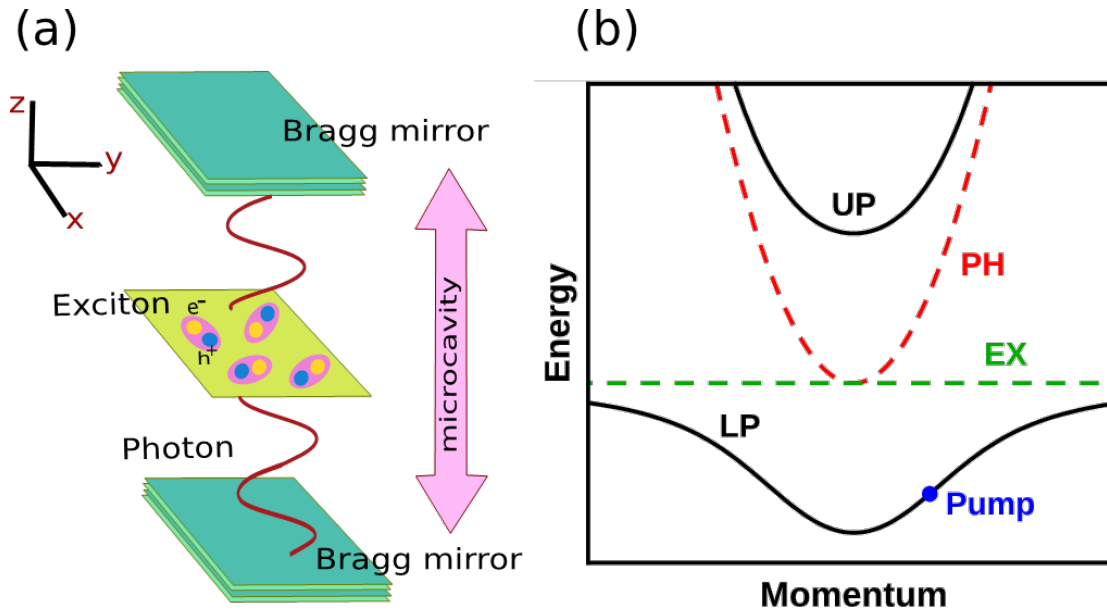


Superfluid component responds to longitudinal but not transverse perturbations

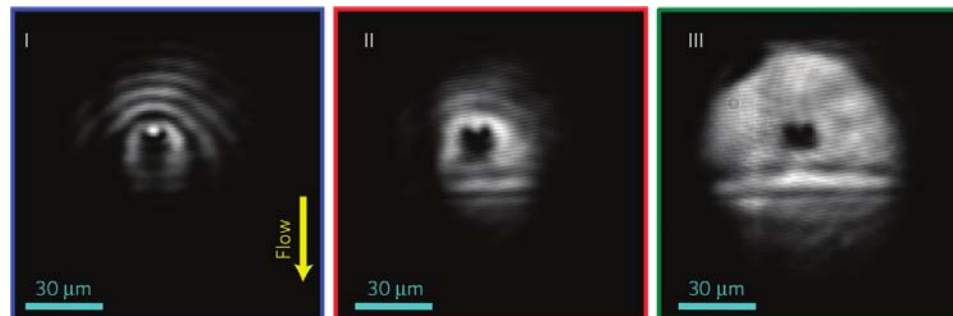
Superfluid density:

$$\rho_S = m \lim_{\mathbf{q} \rightarrow 0} (\chi_L(\mathbf{q}) - \chi_T(\mathbf{q}))$$

Coherently Driven Polaritons




Nearly dissipationless flow observed



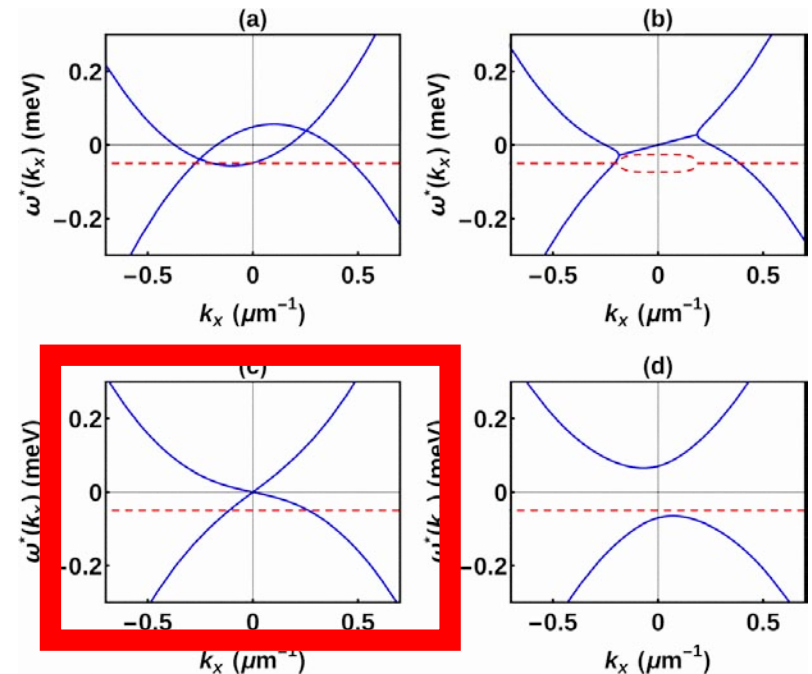
Amo et al. Nature Phys. 5 (2009)

Increasing Drive



$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \hat{a}_{\mathbf{k}}^{\dagger} \hat{a}_{\mathbf{k}} + \frac{F_p}{\sqrt{2}} (\hat{a}_0^{\dagger} + \hat{a}_0) + \frac{V}{2} \sum_{\mathbf{k}, \mathbf{k}', \mathbf{q}} \hat{a}_{\mathbf{k}-\mathbf{q}}^{\dagger} \hat{a}_{\mathbf{k}'+\mathbf{q}}^{\dagger} \hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{k}'} + \sum_p \omega_p^A \hat{A}_p^{\dagger} \hat{A}_p + \sum_{\mathbf{k}, p} \zeta_{\mathbf{k}, p} (\hat{a}_{\mathbf{k}}^{\dagger} \hat{A}_p + \hat{A}_p^{\dagger} \hat{a}_{\mathbf{k}})$$

$$\omega_{\mathbf{k}}^{*, \pm} = \frac{\alpha_{\mathbf{k}}^+ - \alpha_{\mathbf{k}}^-}{2} - i\kappa \pm \frac{1}{2} \sqrt{(\alpha_{\mathbf{k}}^+ + \alpha_{\mathbf{k}}^-)^2 - 4V^2 |\psi_0|^4}$$



- Phase fixed by the pump so
spectrum always gapped

- Blue detuning $\Delta = V |\psi_0|^2 \rightarrow$ Landau criterion fulfilled in real part (c)

Superfluid density from Keldysh: $\rho_S = m \lim_{q \rightarrow 0} (\chi_L(\mathbf{q}) - \chi_T(\mathbf{q})) = 0$

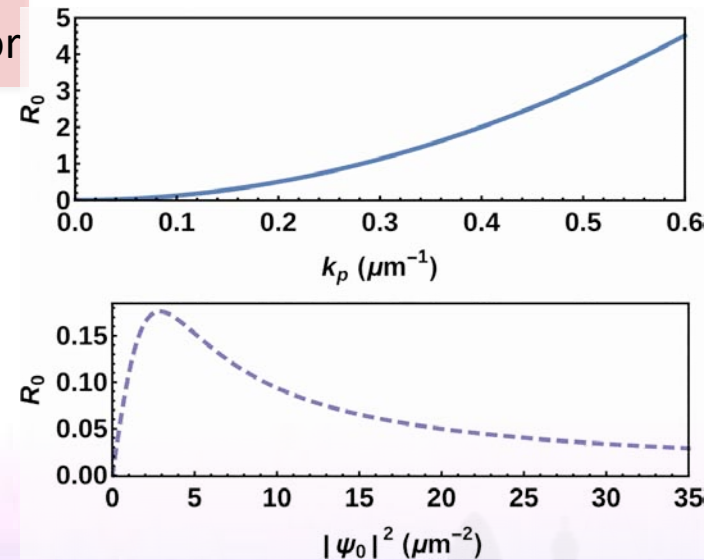
Coherently pumped polaritons are not superfluid

but ...

New quantum state

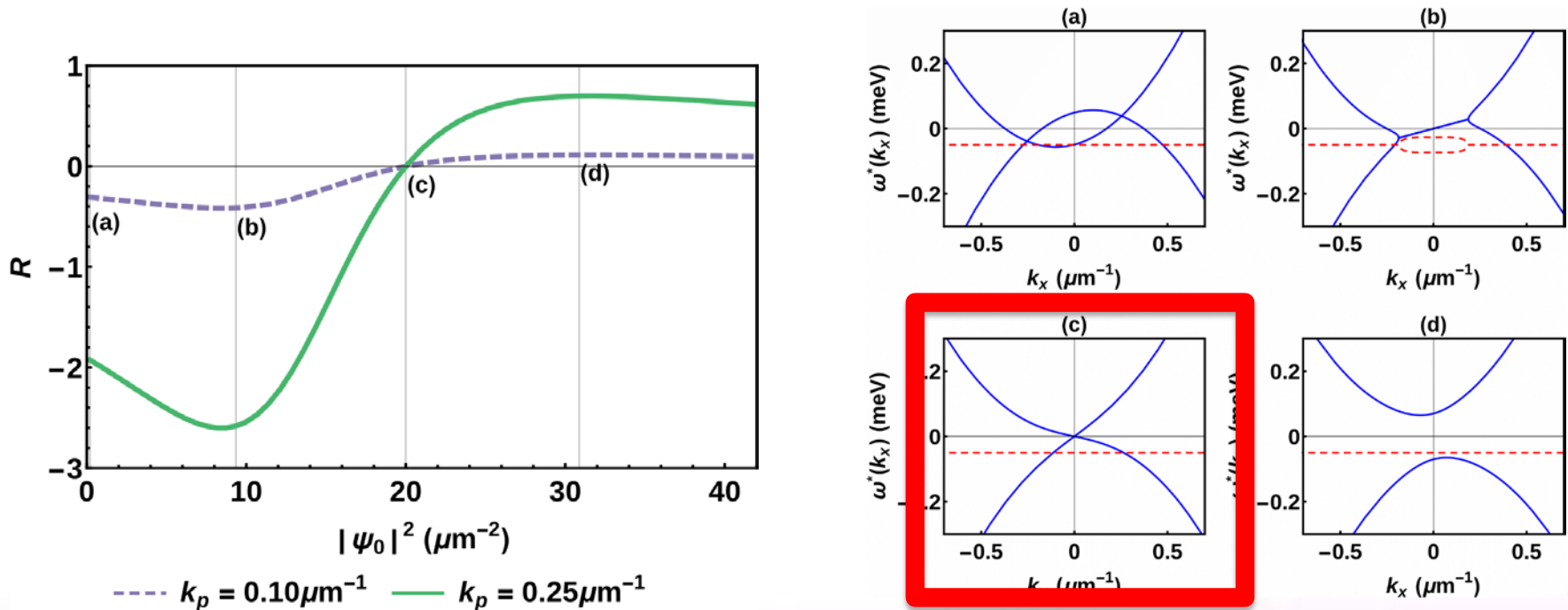
- Most of the system does not respond to neither longitudinal nor transverse forces
- **no superfluid but no normal fluid either**
- Rigid state fixed by the external pump k_p
- Some **normal** response dependent on pump vector

Rigid state fits well with inability to form vortices and solitons



Detuning and the response function

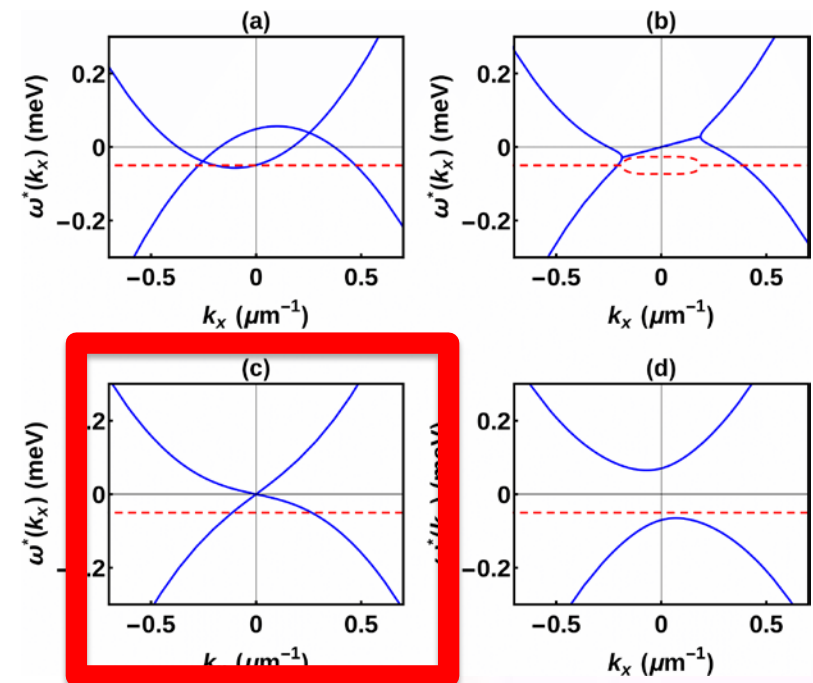
- Vary pump strength for a given detuning
- Normal response goes to zero when the real spectrum fulfils the Landau criterion



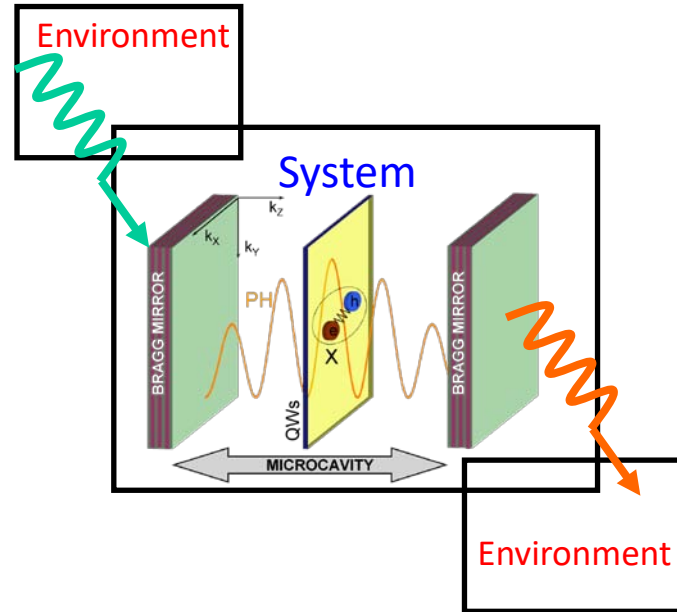
Experiment records the non-fluid rather than superfluid state

Detuning and the response function

- Vary pump strength for a given detuning
- **Normal response goes to zero** when the real spectrum fulfils the **Landau criterion**



Experiment records the non-fluid rather than superfluid state



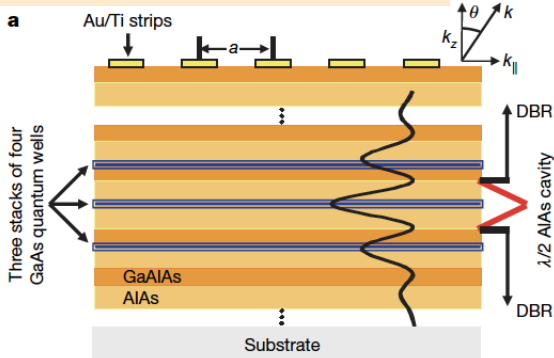
2D Light-matter condensates with drive and decay

Can thermal equilibrium be achieved?

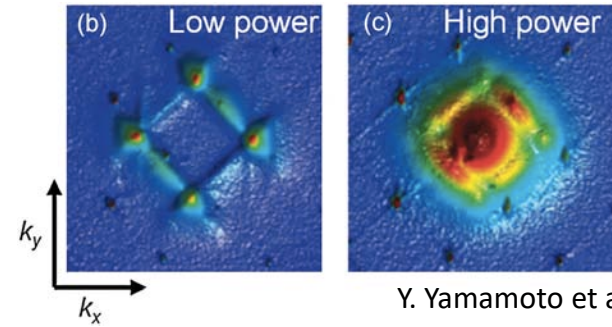
YES

Can non-equilibrium but non-trivial phases be engineered?

Gold deposition



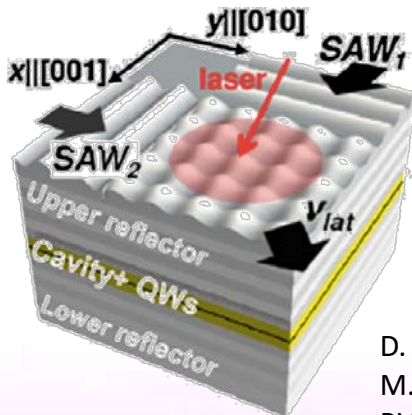
Energy of photons spatially modulated



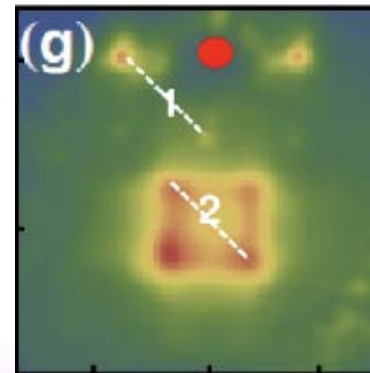
Y. Yamamoto et al.

Polaritons in a weak periodic potential

Surface acoustic waves

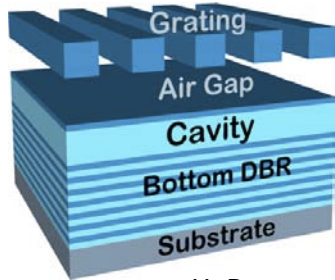


D. N. Krizhanovskii,
M. S. Skolnick,
P.V Santos et al.

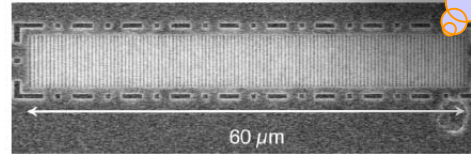


Polaritons in a weak periodic moving potential

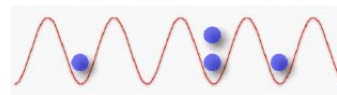
Sub-wavelength gratings



H. Deng et al.

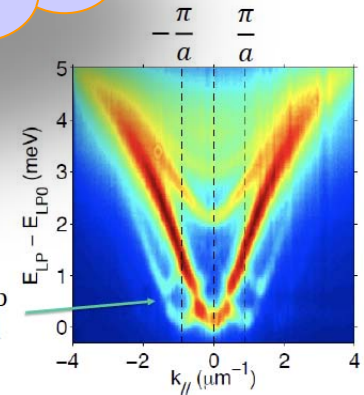


Polaritons modulated by periodic potentials
(period $a = 7 \mu\text{m}$)

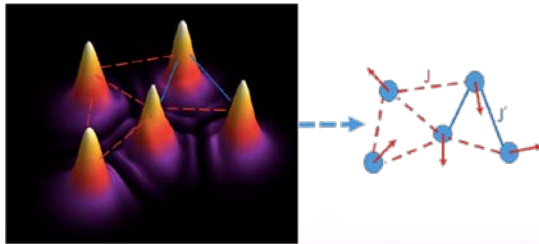


bandgap opened

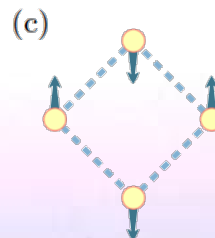
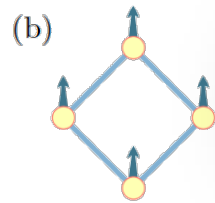
Towards Bose-Hubbard model



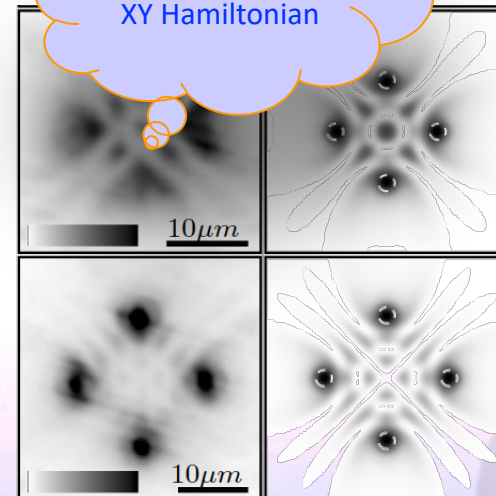
Pump modulation



P. G. Lagoudakis
N. Berloff

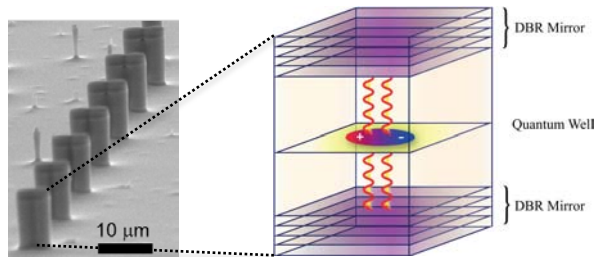


Simulating XY Hamiltonian

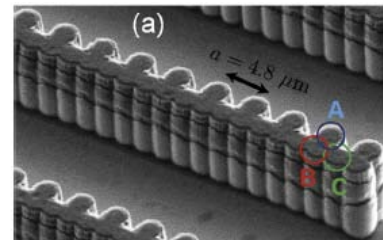


Ferromagnetic order Antiferromagnetic

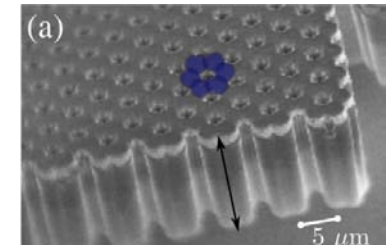
Micropillars



1D lattices

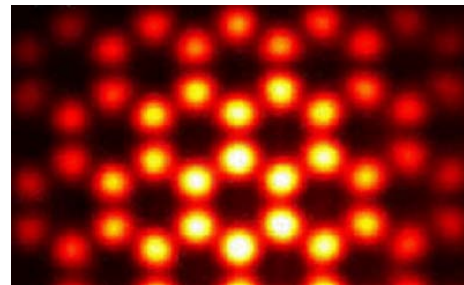


2D lattices



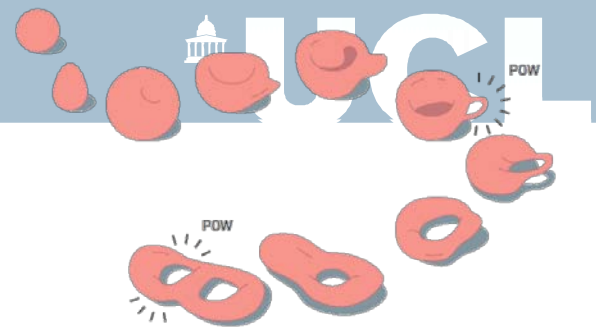
A. Amo, J. Bloch et al.

Honeycomb lattices (like in Graphene)

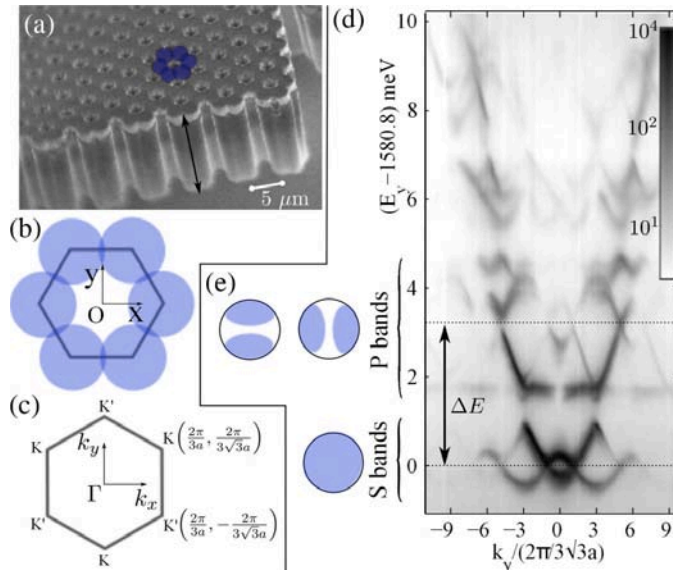


A. Amo, J. Bloch et al.

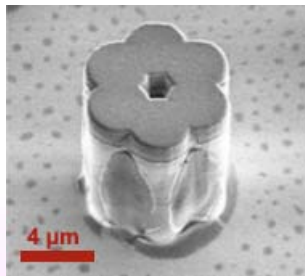
Topological Photons?



Building “photonic graphene” – Dirac cones

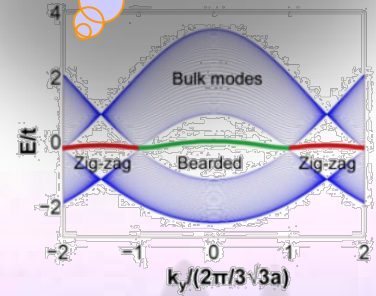
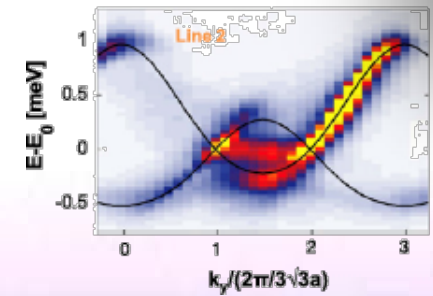
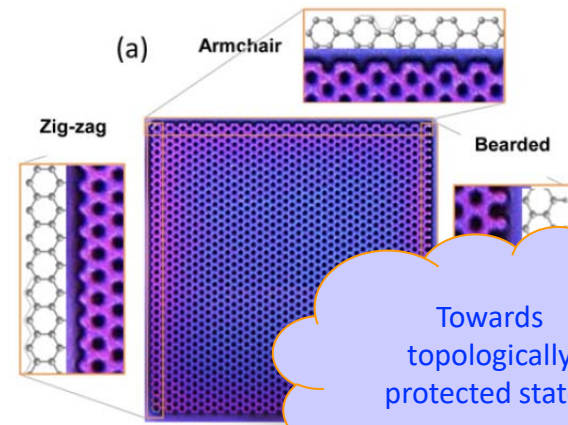


Spin-orbit coupling for photons



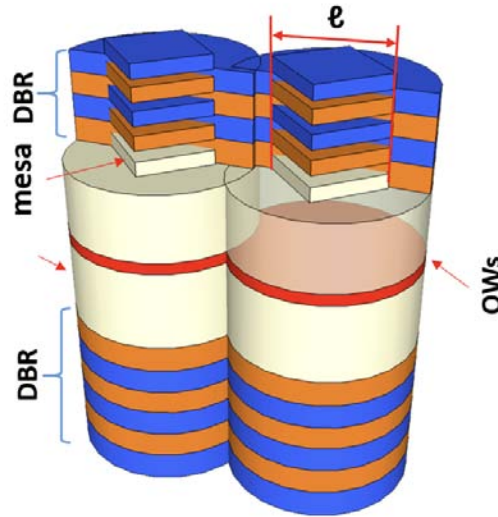
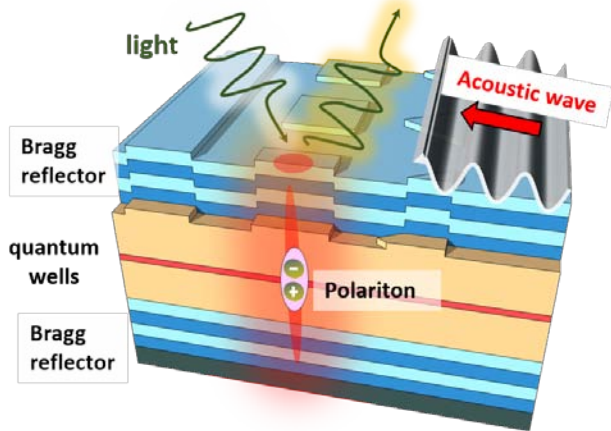
polarization-dependent confinement and tunneling of photons between adjacent micropillars

Edge states

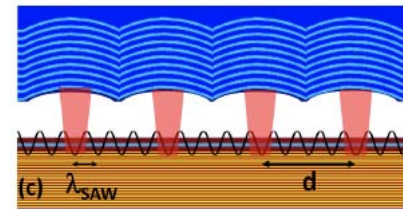


Optimising confinement

Polariton microcavity

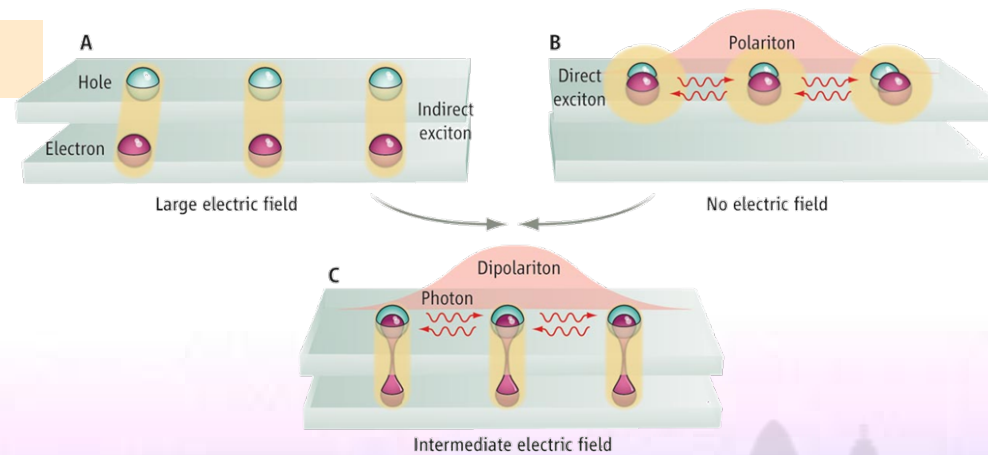


Quanteria Interpol consortium
<https://interpol-quanteria.org>



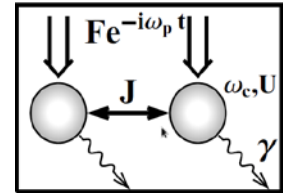
Increasing exciton-exciton interactions

- Dipolar polaritons but in lattice potentials
- Exciton Feshbach resonance



$$\hat{H} = \sum_{i=1,2} (-\Delta \hat{a}_i^\dagger \hat{a}_i + U \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i + F(\hat{a}_i^\dagger + \hat{a}_i)) - J(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_1 \hat{a}_2^\dagger)$$

$$\partial_t \hat{\rho} = -i[\hat{H}, \hat{\rho}] + D(\rho)$$



Local:

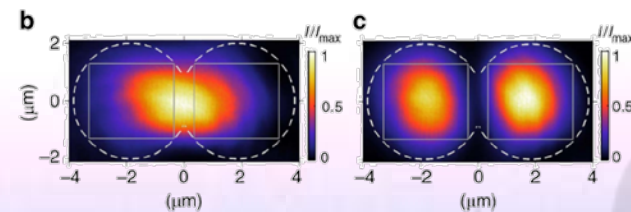
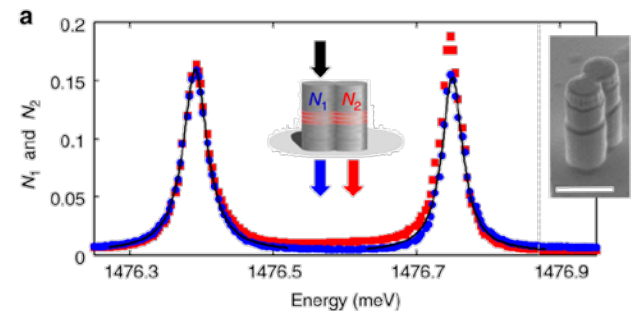
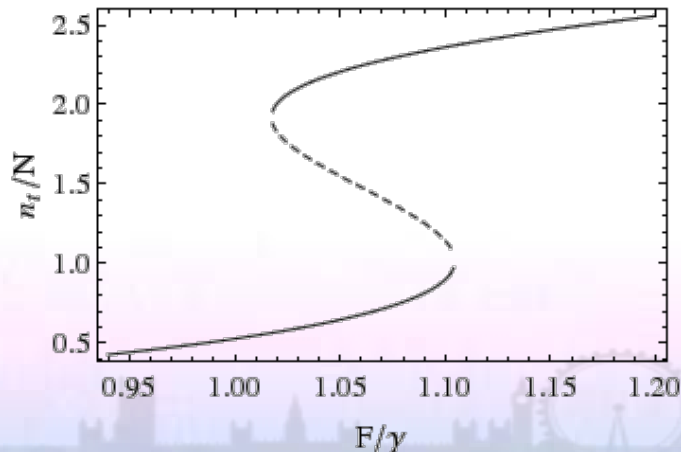
$$\frac{\gamma}{2} \sum_{i=1,2} 2\hat{a}_i \hat{\rho} \hat{a}_i^\dagger - \{\hat{a}_i^\dagger \hat{a}_i, \hat{\rho}\}$$

Non-local:

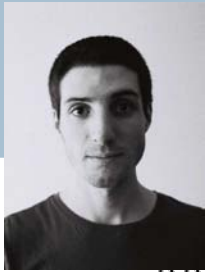
$$\frac{\gamma}{2} \left(2(\hat{a}_1 + \hat{a}_2) \rho (\hat{a}_1^\dagger + \hat{a}_2^\dagger) - \{(\hat{a}_1^\dagger + \hat{a}_2^\dagger)(\hat{a}_1 + \hat{a}_2), \hat{\rho}\} \right)$$

Semi-classical regime:

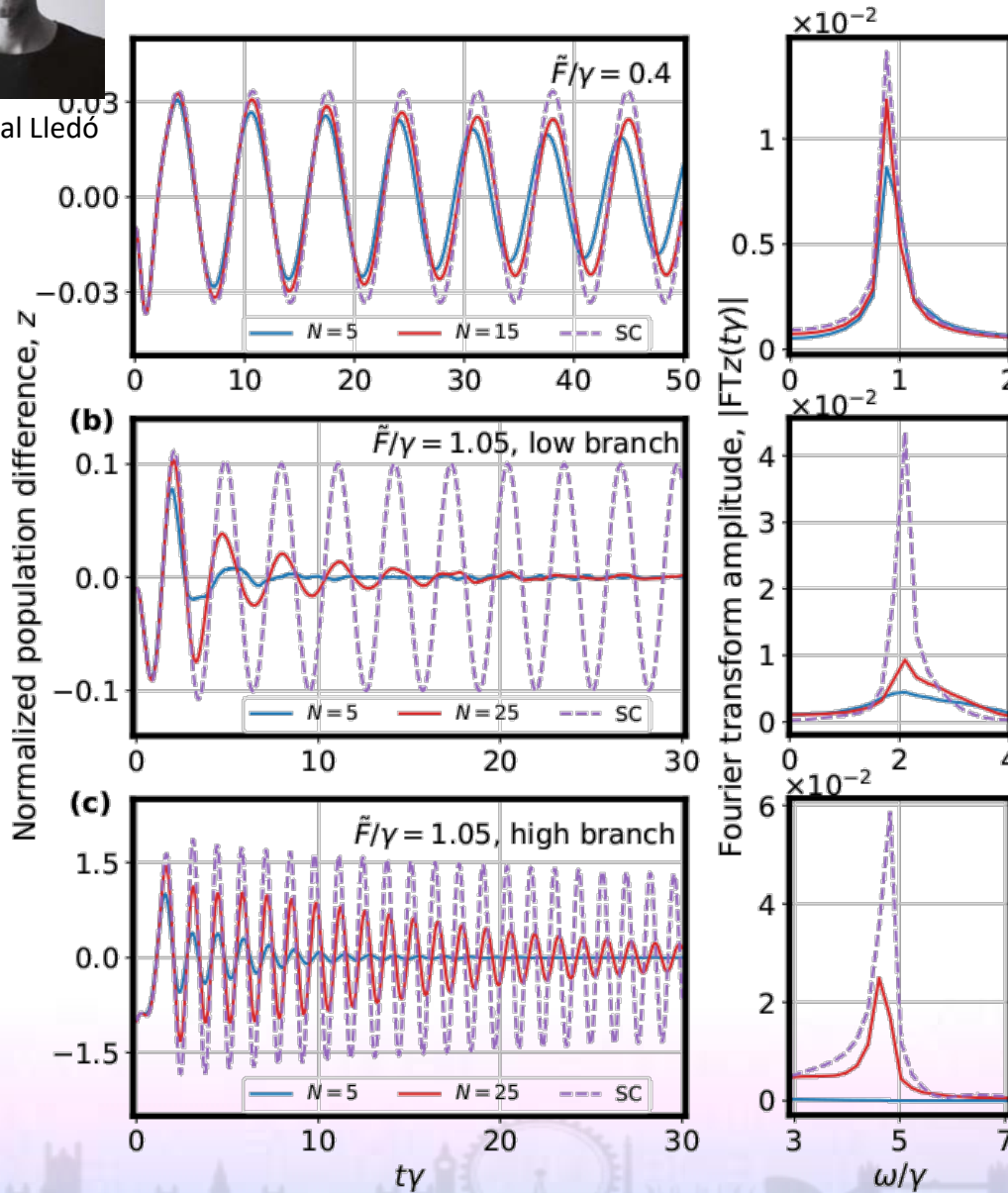
Bistability



Bistable Time-Crystal



Cristóbal Lledó



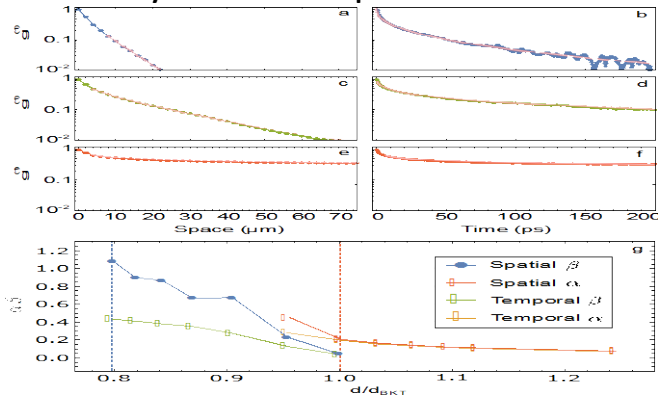
✧ Non-local dissipation leads to oscillatory solutions – bistable in bistability window

✧ Frequency of oscillations robust even for finite system and in the presence of perturbations

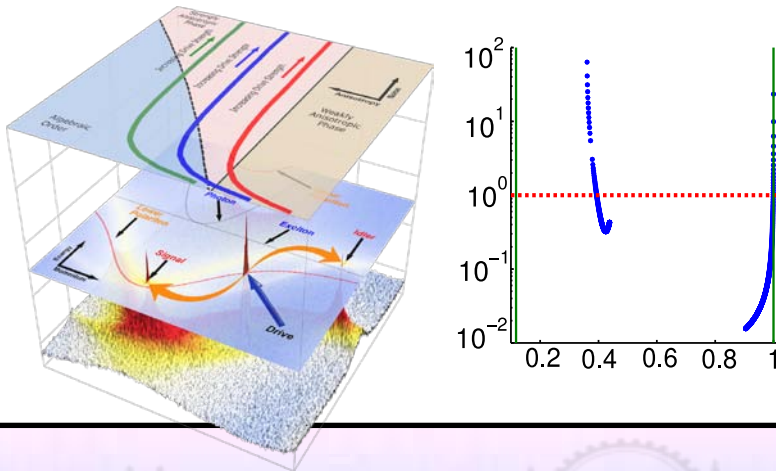
[Lledo et al. arXiv:1901.04438 (2019)]

Conclusions

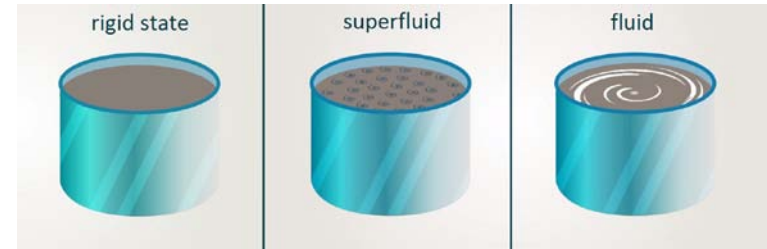
- Best microcavities indistinguishable from closed systems in equilibrium



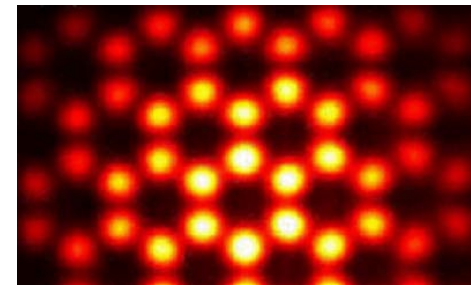
- Anisotropy and dissipation – as in OPO – different phases, KPZ order at all lengthscales?



- Coherent drive – new quantum state with novel flow properties



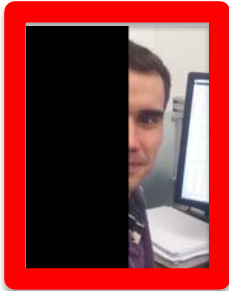
- Lattice systems: correlated and topological states



Acknowledgements



Group:



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D. Ballarini



S. Diehl



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